HEADSTART

Mathematics

TEACHER'S GUIDE

GRADE

9

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Contents

Introduction

How this Teacher's Guide should be used

Mathematics Grade 9 Teacher's Guide is a Mathematics course that provides rich resources to ensure complete curriculum coverage and the successful development of mathematical concepts and skills.

The Teacher's Guide supports you by:
- defining subject, the National Curriculum and Assessment Policy Statement (CAPS) and teaching terminology
- indicating pacing, content and resources in content *overviews*, and structuring the course into lessons with *clear teaching guidelines* according to the CAPS
- providing *background information* (prior knowledge and skills covered in previous grades and/or interesting subject/topic background
- providing suitable *remediation and extension* activities for each lesson
- providing *suggested answers* for every activity
- providing Formal Assessment Tasks as well as *marking guidelines and rubrics* as required by the CAPS
- providing the chapter and unit at the bottom of the page to enable easy referencing between components
- offering additional resources that support core content.

Five easy-to-navigate sections

Sections A: How this Teacher's Guide should be used and how the course works
- Introduction and the CAPS
- An overview of the CAPS
- The CAPS for Mathematics

Section B: Planning and assessment in the Senior Phase
- Planning tools and teaching plans
- Types of assessment, including the Formal Programme of Assessment for Mathematics
- Recording and reporting assessment
- Teaching and learning Mathematics in the Senior Phase
- Inclusivity

Section C: Lesson plans for the teaching of Mathematics

Section D: Resources for the teaching of Mathematics

How this course works

The __________ series meets the requirements of the CAPS for the Senior Phase. In Grade 9, Mathematics consists of two core components: a Teacher's Guide and a Learner's Book.

The Learner's Book

The 2-colour Learner's Book provides content knowledge, core concepts and skills development. It includes activities for learners to develop, practise and consolidate their knowledge and skills. Teachers receive guidance on how to teach important concepts. Written texts are supported by illustrations that help to explain content. All examples, activities and illustrations are representative of all cultural groups.

Activities steadily become more challenging so that learners progressively develop their understanding of concepts.

The Teacher's Guide

The Teacher's Guide provides you, the teacher, with all the planning, teaching and assessment tools you need to successfully teach this subject.

Curriculum and Assessment Policy Statement

An overview of the Mathematics Curriculum and Assessment Policy Statement

This series is based on the *National Curriculum Statement Grades R–12* (NCS, January 2012) which is the policy document for learning and teaching in South Africa. The NCS consists of three documents, namely:

- Curriculum and Assessment Policy Statements (CAPS) for all approved subjects for Grades R–12
- *National Policy pertaining to the Programme and Promotion Requirements of the National Curriculum Statement Grades R–12*
- *National Protocol for Assessment Grades R–12* (January 2012).

Each CAPS document has four sections:

- Section 1 – Introduction to the Curriculum and Assessment Policy Statements for the specific subject
- Section 2 – The specific subject's aims, time allocations and requirements to offer it as a subject
- Section 3 – Overview of topics, teaching plan and content clarification for the specific subject
- Section 4 – Assessment guidelines in the specific subject.

Sections 2, 3 and 4 of the CAPS document, together with the National Policy pertaining to the Programme and Promotion Requirements of the NCS, represent the norms and standards of the *National Curriculum Statement Grades R–12*. Together these documents are the basis for determining minimum outcomes, processes and procedures for the assessment of learner achievement in public and independent schools.

Instructional time allocation

The instructional time in the Senior Phase is as follows:

Subject	Teaching hours per week
Home Language	5
First Additional Language	4
Mathematics	4,5
Natural Sciences	3
Social Sciences	3
Technology	2
Economic Management Sciences	2
Life Orientation	2
Creative Arts	2
Total	27,5

The CAPS for Mathematics

Each CAPS document provides:
- an overview of topics and content areas for its subject (see below)
- the weighting prescribed for each content area (see below)
- a teaching plan for the subject (see Section C – Planning and assessment).

The following content areas comprise the Senior Phase Mathematics curriculum:
- Numbers, Operations and Relationships
- Patterns, Functions and Algebra
- Space and Shape (Geometry)
- Measurement
- Data Handling.

Each content area has a prescribed weighting to ensure complete curriculum coverage.

Content Area	Grade 7	Grade 8	Grade 9
Numbers, Operations and Relationships	30%	25%	15%
Patterns, Functions and Algebra	25%	30%	35%
Space and Shape (Geometry)	25%	25%	30%
Measurement	10%	10%	10%

Content Area	Grade 7	Grade 8	Grade 9
Data Handling (Statistics)	10%	10%	10%
Total	100%	100%	100%

Topic overview

	Grade 7	Grade 9	Grade 9
Term 1	Mental calculations Order and compare whole numbers (9 digits) Properties of whole numbers Calculations with whole numbers: Addition and subtraction (6 digits) Multiplication and division (4-digit by 2-digit) Multiples and factors (of 2- and 3-digit whole numbers) Prime factors LCM and HCF (3-digit whole numbers) Solve problems (ratio and rate; percentages, decimal fractions; financial context) Exponents Measure angles Construct geometric figures Classify 2D shapes Similar and congruent 2D shapes Solve problems	Mental calculations Order and compare whole numbers (prime numbers to 100) Properties of whole numbers Calculations with whole numbers Multiples and factors Solve problems (ratio and rate; percentages, decimal fractions; financial context) Count, order and compare integers Calculations with integers Properties of integers Solve problems Represent numbers in exponential form Calculations in exponential form Laws of exponents Numeric and geometric patterns Input and output values or rules for patterns and relationships Equivalent forms Algebraic language Expand and simplify algebraic expressions Set up equations and solve by inspection	Mental calculations Properties of whole numbers Calculations with whole numbers Multiples and factors Solve problems (ratio and rate; direct and indirect proportion; percentages, decimal fractions; financial context) Calculations with integers Properties of integers Solve problems Common fractions Decimal fractions Exponents Calculations in exponential form Solve problems Numeric and geometric patterns Input and output values or rules for patterns and relationships Equivalent forms Algebraic language Expand and simplify algebraic expressions Equations (use factorisation; of the form where a product of factors = 0)

Term 2	Mental calculations Common fractions Percentages Decimal fractions Equivalent forms Solve problems Input and output values for patterns and relationships Equivalent forms (verbal, flow diagrams, tables, formulae, number sentences) Area and perimeter of 2D shapes Convert SI units Surface area and volume of 3D objects	Mental calculations Algebraic language Expand and simplify algebraic expressions Set up equations and solve by using additive and multiplicative inverses Construct and investigate geometric figures Classify 2D shapes Similar and congruent triangles Angle relationships Solve problems	Mental calculations Investigate properties of geometric figures by construction Classify 2D shapes Similar and congruent triangles Solve problems Angle relationships Use the Theorem of Pythagoras Area and perimeter of 2D shapes (polygons and circles)
Term 3	Mental calculations Numeric and geometric patterns Input and output values for patterns and relationships Equivalent forms Algebraic language Number sentences Interpret and draw graphs Transformations Classify 3D objects Build 3D models	Mental calculations Common fractions Percentages Decimal fractions The Theorem of Pythagoras Area and perimeter of 2D shapes Surface area and volume of 3D objects Solve problems Data handling	Mental calculations Input and output values or rules for patterns and relationships Algebraic language Expand and simplify algebraic expressions Factorise algebraic expressions Equations Draw and interpret graphs Draw linear graphs from given equations Surface area and volume of 3D objects (include cylinders)
Term 4	Mental calculations Integers Numeric and geometric patterns Input and output values for patterns and relationships Algebraic language Number sentences Data handling Probability	Mental calculations Input and output values or rules for patterns and relationships Equivalent forms Solve algebraic equations Interpret and draw graphs Transformations Enlargements and reductions Classify 3D objects Build 3D models Probability	Mental calculations Transformations Enlargements and reductions Classify 3D objects Build 3D models Data handling Probability

Planning and assessment

Types of planning tools

The following planning tools are provided:

- A teaching plan that follows the CAPS time allocations
- Sample lesson plans

Teaching plan for Mathematics Grade 9

This teaching plan shows:

- the pacing of the topics for the course by term
- where to find the relevant content and activities in the Learner's Book
- when Formal Assessment takes place, cross-referenced to suitable activities in the Learner's Book.

TERM 1					
Chapter	**Content/topics (as per the CAPS)**	**Learner's Book**	**Time allocation**	**LB page ref**	**TG page ref**
1	**Whole numbers**	Unit 1 Properties of whole numbers Unit 2 Calculations with whole numbers Unit 3 Multiples and factors Unit 4 Ratio, rate and proportion Unit 5 Financial Mathematics	4,5 hours	7	25
Chapter 1 Revision				74	64
POA Investigation 1: Pulse rates				45	49
POA Assignment 1: Ratio and rate				48	51
2	**Integers**	Unit 1 Properties of integers Unit 2 Calculations with integers	4,5 hours	76	66
Chapter 2 Revision				96	79
3	**Common fractions**	Unit 1 Revision: Fractions Unit 2 Equivalent forms of fractions Unit 3 Calculations with fractions	4,5 hours	97	80
Chapter 3 Revision				129	95
4	**Decimal fractions**	Unit 1 Revision Unit 2 Calculations with decimal fractions Unit 3 Equivalent forms for decimal fractions	4,5 hours	130	96
Chapter 4 Revision				144	105
5	**Exponents**	Unit 1 Revision: Comparing and representing numbers in exponential form Unit 2 Scientific notation Unit 3 Calculations with numbers in exponential form Unit 4 Exponents in problem-solving	5 hours	145	106
Chapter 5 Revision				167	120
POA Investigation 2: Powers of 2: Paper folding				164	119

14	**Functions and relationships**	Unit 1 Input and output values Unit 2 Equivalent forms of a relationship	5 hours	354	256
Chapter 14 Revision				363	266
15	**Algebraic expressions**	Unit 1 Algebraic expressions and factors Unit 2 Common factors Unit 3 The difference of two squares Unit 4 Trinomials Unit 5 Simplifying algebraic fractions	9 hours	364	268
Chapter 15 Revision				384	284
16	**Algebraic equations**	Unit 1 Exponential equations Unit 2 Quadratic equations Unit 3 Substitution and ordered pairs	9 hours	385	285
Chapter 16 Revision				397	295
17	**Graphs**	Unit 1 Interpreting graphs Unit 2 Drawing graphs Unit 3 Graphs showing proportion	12 hours	398	296
Chapter 17 Revision				421	328
18	**Surface area and volume of 3D objects**	Unit 1 Surface area of 3D objects Unit 2 Volume and capacity of 3D objects Unit 3 The effect on surface area or volume if dimensions are doubled	5 hours	423	319
Chapter 18 Revision				442	328
POA Project 1: Volume and surface area				441	**328**
Term 3 test					329
TERM 4					
19	**Transformation geometry**	Unit 1 Transformations with points on a coordinate plane Unit 2 Transformations with line segments and geometric figures on a coordinate plane Unit 3 Enlargements and reductions	9 hours	443	331
Chapter 19 Revision				446	341
20	**Geometry of 3D objects**	Unit 1 Properties and definitions of the Platonic solids Unit 2 Properties of spheres and cylinders Unit 3 Models of cube, prisms, pyramids and cylinders	9 hours	467	342
Chapter 20 Revision				482	350
21	**Data handling and probability**	Unit 1 Collecting and recording data Unit 2 Organising, ordering and summarising data Unit 3 Representing data Unit 4 Analysing, interpreting and reporting data Unit 5 Probability	15 hours	483	351
Chapter 21 Revision				528	369
Exam exemplar (November) Memorandum					372
Additional examination exemplar and memorandum (November)					378

Sample lesson plan for Grade 9

Some may find daily lesson plans useful, although these are not a formal policy requirement. An example of how to complete a lesson plan is below.

Date:	**Grade:** 7	**Term:** 1
Unit: 2	**Unit title:** Number sentences	**Contact time:** 3 hours
Content/concept: (Teacher to complete)	**Activities:** 1–5	**Resources required:** Learner's Book; classwork book; place-value cards
Activity 1:		
Links with previous activity: n/a		
Links with next activity: (Teacher to complete)		
Teaching plan (Teacher to complete; tips provided below) Mental mathematics Check prior knowledge: class discussion Teaching procedures (methods you will use): Differentiation (how you will deal with extension/remedial tasks): Progress checks:		
Assessment: Teacher, peer		
Confidential information/Teacher reflection: (Teacher to complete)		

Assessment in the Senior Phase

Assessment is about collecting evidence of the learners' learning. It is an integral part of teaching and learning, and should be planned when planning the lesson content. Assessment helps to identify the needs of the learners. It also provides evidence of progress, enables teachers to reflect on what they are doing and provides for feedback and reporting to all stakeholders. Good assessment practice in Mathematics includes:

- assessing whether skills and aims are applied to content knowledge
- determining whether learners can apply this knowledge to procedures and problems
- providing feedback.

The four steps of assessment

1. Generating and collecting evidence of achievement
2. Evaluating the evidence
3. Recording the findings
4. Using the findings to guide future learning and teaching.

Types of assessment

Type of assessment	Description
Baseline assessment	Establishes whether learners meet basic skills and knowledge level required Helps teacher plan for the year and for each learner Is administered at the beginning of the year and before a particular topic Results are used as a guide for teaching and not for promotion purposes.
Diagnostic assessment	Informs the teacher about certain specific problem areas that may hinder performance May help determine whether a learner's problems are content or psycho-social based Appropriate interventions should follow on from diagnostic assessment Results should inform interventions and not be used for promotion purposes.
Formative assessment	Used to aid the learning process and not for promotion purposes Usually informal, to provide the teacher and learner with a more frequent account of where the learner is at Teachers can use this form of assessment to modify and adapt their own teaching.
Summative evaluation	Carried out after completion of a topic or cluster of topics Is an assessment of learning that has taken place Recorded and used for promotion This is usually formal assessment, making up the Formal Programme of Assessment.

Informal or daily assessment

Informal assessment is a daily monitoring of learners' progress. This is done through observations, discussions, practical demonstrations, learner-teacher conferences, informal classroom interactions, etc. The Learner's Book is packed with activities that can be used for informal assessment, once learners have mastered the basic concepts.

The CAPS tells us that informal assessment should be used to provide feedback to the learners and to inform planning for teaching, but need not be recorded or taken into account for promotion. It should not be seen as separate from learning activities taking place in the classroom. Learners or teachers can mark these assessment tasks.

Self-assessment and peer assessment actively involve learners in assessment. This is important as it allows learners to learn from and reflect on their own performance.

Learners often experience difficulty completing extended writing.

Learners should read and write regularly, starting with sentences and paragraphs and building up to extended pieces of work. Much of this may be structured by working through activities.

Formal assessment

All assessment tasks that make up a Formal Programme of Assessment for the year are regarded as formal assessment. Formal assessment tasks are marked and formally recorded by the teacher for progression and certification purposes. All formal assessment tasks are subject to moderation for the purpose of quality assurance and to ensure that appropriate standards are maintained.

Formal assessment provides teachers with a systematic way of evaluating how well learners are progressing in a grade and in a particular subject. Examples of formal assessments include tests, examinations, practical tasks, projects, oral presentations, demonstrations, performances, etc. Formal assessment tasks form part of a year-long Formal Programme of Assessment in each grade and subject.

Formal assessment requirements of Mathematics

The forms of assessment used should be appropriate for the learners' ages and developmental levels.

Learners must complete formal assessments each term. Formal assessments include formally assessed tasks, along with projects and examinations.

The Formal Programme of Assessment as prescribed by the CAPS is shown below. This Programme of Assessment is generic across the three grades in the Senior Phase, and lists the types of formal assessments required each term.

Minimum requirements for formal assessment Senior Phase: Mathematics

	Forms of assessment	Minimum requirements per term				Number of tasks per year	Weighting
		Term 1	Term 2	Term 3	Term 4		
SBA (School-based Assessment)	Tests	1	1	1		3	40%
	Examination		1			1	
	Assignment	1		1	1	3	
	Investigation		1		1	2	
	Project			1		1	
	Total	2	3	3	2	10*	
Final examination		End of year				1	60%

*To be completed before the final examination at the end of the year

Types of formal assessment for Mathematics

Tests and examinations

These are individual assessment tasks. Tests and examinations for formal assessment should cover a substantial amount of content. Tests and examinations must be completed under strictly controlled conditions.

Each test and examination must cater for a range of cognitive levels in the correct allocation (see the table below).

Cognitive level	**Description of skill to be demonstrated**	**%**
Knowledge	Estimation and appropriate rounding of numbers Straight recall Identification and direct use of correct formula Use of mathematical facts Appropriate use of mathematical vocabulary	≈25
Routine procedures	Performing of well-known procedures Simple applications and calculations which might involve many steps Derivation from given information may be involved Identification and use (after changing the subject) of correct formulae generally similar to those encountered in class	≈45
Complex procedures	Problems involving complex calculations and/or higher-order reasoning Investigate elementary axioms to generalise them into proofs for straight line geometry, congruence and similarity No obvious route to the solution Problems not necessarily based on real-world contexts Making significant connections between different representations Require conceptual understanding	≈20
Problem-solving	Unseen non-routine problems (which are not necessarily difficult) Higher-order understanding and processes are often involved Might require the ability to break the problem down into its constituent parts	≈10

Projects

Learners complete *one* project in Mathematics in each grade. Projects can be used to test a range of skills and competencies. It is prescribed that learners complete a project in Term 3 of each grade. Projects must provide learners with the ability to demonstrate their understanding of a mathematical concept and apply it to a real-life situation. Be wary of prescribing projects that are beyond the cognitive level of the learners, or that will simply involve duplicated facts and data from reference material.

Assignments

An assignment is also an individual task, similar to tests and examinations. However, the assignment should be an extended piece of work with a focus on more demanding work than that covered in class. *Three* assignments per year are required by the CAPS. The assignment can include past questions, but should also include more challenging aspects encouraging the learner to use additional material to help them. The assignment can be completed at home.

Investigations

An investigation should be used to discover rules or concepts. It is recommended that learners should conduct investigations in class as much as possible, and that the final written task definitely be done in class. Rubrics are used to assess investigations. *Two* investigations per year are required by the CAPS.

The skills involved in investigations include:

- organising and recording ideas and discoveries in tables and diagrams
- explaining ideas in appropriate forms
- showing clear understanding of concepts and procedures through calculations
- generalising and drawing conclusions.

Tests and examinations

Assessment tasks should be designed to cover the content and concepts of the subject and include a variety of activities selected to assess the identified aims and skills.

Before handing out an assessment task to learners, teachers should ensure that they are able to answer all the questions themselves. When teachers set an assessment task, they should draw up a memorandum of answers and/or a rubric for the assessment. Refer to the seven-point rating code or scale of achievement in this Teacher's Guide when constructing a rubric.

Feedback should acknowledge strengths and identify areas of weakness for learners' developmental needs. Action plans on how learners will be supported should accompany this feedback. It is important that the feedback provided to learners encourages them to do better, and builds their self-confidence.

Formal Programme of Assessment (POA) plan for Mathematics

We have provided a full Formal Programme of Assessment plan. These assessments cater for a range of cognitive levels and abilities as required by the CAPS, as described in this section.

Programme of Assessment for ________ Mathematics

Forms of assessment	**Term 1**	**Term 2**	**Term 3**	**Term 4**
*Revisions	At the end of every chapter. Can be used as assignments.			
Tests	TG p. 183		TG p. 329	
Exam exemplar (June) Exam exemplar: (**additional**) June Exam exemplar Paper 1 (November) Exam exemplar Paper 2 (November) Exam exemplar: memorandum November Exam exemplar: (**additional**) November		LB p349 TG p248		LB p532 TG p372 TG p378
*Assignment 1 *Assignment 2 *Assignment 3 *Assignment 4	LB p48; TG p51 LB p165; TG p119 LB p166; TG p120 LB p187; TG p133			
Investigation 1 Investigation 2 Investigation 3 Investigation 4	LB p45; TG p49 LB p164; TG p119		LB p345; TG p240	
Project 1			LB p441; TG p328	
Note: *Revision exercises at the end of every chapter can be used for consolidation and/or as a form of assessment (assignment). **Assignments 1–4 can be spread across four terms				

Inclusive assessment

Teachers need to develop adaptive and alternative methods to assess learners with barriers to learning, so that learners are given opportunities to demonstrate competence in ways that suit their needs. Here are some examples of how to assess these learners, while still maintaining the validity of the assessment.

- Some learners may need concrete apparatus for a longer time than their peers.

- Assessment tasks, especially written tasks, may have to be broken up into smaller sections for learners who cannot concentrate or work for a long time, or short breaks may be given during the tasks. Learners can also be given extra time to complete tasks.
- Some learners may need to do their assessment tasks in a separate venue to limit distractions.
- A variety of assessment instruments should be used, as a learner may find that a particular assessment instrument does not allow them to show what they can do.
- Learners who cannot read can have tasks read to them and they can orally dictate answers. Assessment can also include a practical component in which learners can demonstrate their competence without having to use language.
- A sign-language interpreter can be used.
- Assessment tasks can be available in Braille or enlarged with bolded text.
- Assessment can include the use of dictaphones or computers with voice synthesisers.
- The forms of assessment used should be appropriate for age and developmental levels. The design of these tasks should cover the content of the subject and include a variety of tasks designed to achieve the objectives of the subject.

Recording and reporting assessment

Recording

Recording documents the level of a learner's performance in a specific assessment task. It indicates learner progress towards the achievement of the knowledge as prescribed in the curriculum. Records of learner performance should be used to verify the progress made by teachers and learners in the teaching and learning process.

Reporting

Learners' performance can be reported in a number of ways. These include report cards, parents' meetings, school visitation days, parent-teacher conferences, phone calls, letters, class or school newsletters, etc. Teachers in all grades report in percentages against the subject. The various achievement levels and their corresponding percentage bands are as shown in the table below.

Rating code	Description of competence	Marks %
7	Outstanding achievement	80–100
6	Meritorious achievement	70–79
5	Substantial achievement	60–69
4	Adequate achievement	50–59
3	Moderate achievement	40–49
2	Elementary achievement	30–39
1	Not achieved	0–29

Teaching and learning Mathematics

What is Mathematics?

Mathematics is a language. It uses symbols and notations to describe relationships. Mathematics is a human activity that involves observing, representing and investigating patterns and relationships in both the physical and social dimensions. Mathematics helps develop key mental processes such as logical and critical thinking, accuracy and problem-solving. All of these processes contribute to a learner's decision-making ability.

The specific aims of Mathematics

The aims of Mathematics are to develop:
- a critical awareness of mathematical relationships
- confidence and competence in Mathematics without fear of the subject
- curiosity and a love of Mathematics
- appreciation for the beauty and elegance of Mathematics
- recognition of the subject as a creative art
- the deep conceptual understanding required to understand Mathematics
- the acquisition of specific skills in order to apply Mathematics
- study related subject matter and study further in Mathematics.

Specific skills required in Mathematics

In order to develop the essential skills the learner should:
- develop correct mathematical language use
- develop number vocabulary, number concept and application skills
- learn to listen, communicate, think, reason logically and apply the knowledge gained
- learn to investigate, analyse, represent and interpret information
- learn to pose and solve problems
- be aware that Mathematics plays a key role in real-life situations.

Content area focus in the Senior Phase

Content area	General focus	Specific focus
Numbers, Operations and Relationships	Meaning of different kinds of numbers Relationships between different kinds of numbers Relative sizes of different numbers	Represent numbers in a variety of ways and move flexibly between representations
	Representations of numbers in various ways Operations with numbers Estimation and checking solutions	Recognise and use properties of operations with different number systems Solve a variety of problems, use an increased range of numbers and are able to perform multiple operations correctly and fluently

Patterns, Functions and Algebra	Achieve efficient manipulation skills which will carry over into other domains of the subject Describe patterns and relationships through the use of symbolic expressions, graphs and tables Identify and analyse regularities and change in patterns Make predictions and solve problems	Investigate numerical and geometric patterns to establish the relationships between the variables Express rules governing patterns in algebraic language or symbols Develop algebraic manipulative skills to recognise the equivalence between different representations of the same relationship Analyse situations in a variety of contexts Use of different and equivalent representations – algebraic language, formulae, expressions, equations and graphs
Space and Shape (Geometry)	Properties of shapes and objects Relationships between these properties Orientations, positions and transformations of two-dimensional shapes and three-dimensional objects	Draw and construct a wide range of geometric figures and solids using appropriate instruments Appreciate the use of constructions to investigate the properties of geometric figures and solids Develop clear and precise descriptions and classification categories of geometric figures and solids Solve geometric problems drawing on known properties of geometric figures and solids
Measurement	Select and use appropriate units, instruments and formulae Make sensible estimates Be aware of sensibleness and reasonableness of measurement and results	Use formulae for measuring area, perimeter, surface area and volume of geometric figures and solids Select and convert between units of measurement Use the Theorem of Pythagoras to solve problems involving right-angled triangles
Data Handling	Ask questions and find answers in order to describe events and the social, technological and economic environment Collect, organise, represent, analyse, interpret and report data Enabled to make informed predictions by the study of probability Describe randomness and uncertainty	Pose questions for investigation Collecting, summarise and represent and critically analyse data Interpret, report and make predictions about situations Probability – include single and compound events and their relative frequency in simple experiments

Teaching Mathematics in the Senior Phase

Grade 9 is usually the final year in primary school, though it is the beginning of the Senior Phase. The Grade 9 a learners are the “senior” learners in the school and as such they feel quite important.

The transition to Grade 9 is an important one, and a BIG step. For many learners , for many different reasons, going to ”high school” is a combination of being exciting, yet challenging and even stressful.

Learners in the Senior Phase do Mathematics every day. They must be challenged to think abstractly and critically, and not to merely copy formulae and do substitutions.

The writing of formal tests and exams becomes even more important. The Mathematics teacher must spend time developing exam techniques which include unpacking terminology used in exams, such as *determine, identify, deduce, predict, present, summarise, expand, suggest, illustrate*, and so on.

The Learner’s Book provides many built-in opportunities for learners to engage with these. Section E of the Teacher’s Guide also provides a list of important terminology.

Presenting answers, time management, exam stress management, etc. are all important areas in which learners must receive constant coaching. Mathematics teachers should work very closely with Life Orientation teachers in order to support learners with these issues.

The volume of work increases in Grade 9, and the expectations are higher. Learners are also expected to mark their own work (from the board) and this is new to many Grade 9 learners.

Grade 9 is the most challenging grade in this phase. While being on the cusp of the final and another extremely important phase of their high school career, learners are also dealing with other issues important to them, such as sexuality. Girls develop faster than boys. Some schools separate the boys and girls in grade 9 in order to address this issue.

Grade 9 is a crucial year in the teaching of Mathematics: learners are required to make a choice between Mathematics and Mathematical Literacy in Grade 10. This choice will be based on their experience and level of success achieved in Grade 9. For learners who have some idea of their future career, the choice between these two might be somewhat more straightforward.

It is of upmost importance that teachers lay a good foundation for basic algebra and geometry in Grades 8 and 9 in order to facilitate suitable and appropriate subject choice.

Inclusive teaching

What is inclusive teaching?

In the Senior Phase, it is crucial that learners find themselves in an environment where they can develop an interest in learning and the belief that they can learn. Inclusive education is defined as a learning environment that promotes the full personal, academic and professional development of all learners irrespective of race, class, gender, disability, religion, culture, sexual preference, learning styles and language.

Inclusion is about acknowledging and respecting that:

- all children have the right to learn
- all children are able to learn
- all learners need support
- all learners are unique and have different, but equally valued, learning needs
- all learners need the opportunity to build on their own unique strengths
- the learner is the centre of the teaching and learning process
- there are differences in learners, for example, age, gender, language, culture, learning styles, disabilities, HIV status and so on.

Inclusion is also about:

- enabling educational structures, systems and learning methodologies to meet the needs of all learners
- more than just formal schooling – it embraces learning that occurs in the home, community and so on
- changing attitudes, behaviour, methodologies and environments to meet the needs of all learners
- ensuring maximum participation of all learners in the culture and curriculum of all educational institutions
- identifying and minimising barriers to learning that can occur at any level of the system.

Some of the learners in your class may already suffer from exclusion or think negatively about education. There is no reason for their exclusion from class activities. It is the responsibility of the teacher to ensure the inclusion of these learners. This means adapting activities to suit their needs and capabilities. It is equally important that the class is not divided because of this. Rather, learners with these challenges should be accepted and helped where possible by their peers. Learners should at all times be discouraged from teasing, bullying or ignoring learners with special needs. When these attitudes are directed towards a learner, they create a barrier to learning in that learner.

Practical guidelines for inclusive teaching

- Have a true understanding of each learner's background, strengths, unique abilities, needs and barriers. Then use this information to inform your planning and give a clearer focus.
- Remember that the teacher is a facilitator of learning.
- Keep the content and material as relevant as possible.
- Break down learning into small, manageable and logical steps. Keep instructions clear and short (plan beforehand).
- Grade activities according to the different levels and abilities of learners. Try to ensure that learners remain challenged enough without undue stress.
- Develop a balance between individual teaching, peer tutoring, cooperative learning and whole-class teaching.
- Use learners to help one another in the form of group types, peer-assisted learning, buddy systems and so on. Ensure that learners feel included and supported in the classroom by both the teacher and their peers.
- Set up pairs and groups of learners where members can have different tasks according to strengths and abilities. Promote self-management skills and responsibility through group roles and the types of tasks you set.
- Motivate learners and affirm their efforts and individual progress. Build confidence. Encourage questioning, reasoning, experimentation with ideas and risking opinions.
- Determine the learner's Zone of Proximal Development (ZPD) and use it for effective teaching and learning. Vygotsky described the ZPD as the distance between what the learner already knows and understands and what they can understand with adult support. Learning is thus a social interaction as the teacher mediates and supports the learner as they understand a new concept.
- Spend time on consolidating new learning. Use different ways to do this until all learners understand the concept. Make time to go back to tasks so that learners can learn from their own and others' experiences and methods.
- Use and develop effective language skills (expressive and receptive, verbal and non-verbal).
- Experiment with a variety of teaching methods and strategies to keep learners interested and to cater for and develop different learning styles. Use games, cooperative group work, brainstorming, problem-solving, debates, presentations and so on.

Learners with barriers to learning

A barrier to learning is anything that prevents a learner from participating fully and learning effectively. This includes learners who were formerly disadvantaged and excluded from education because of the historical, political, cultural and health challenges facing South Africans. Some other examples of barriers to learning may be learners who are visually or hearing impaired, or learners who are intellectually challenged. Barriers to learning cover a wide range of possibilities and learners may often experience more than one barrier. Some barriers, therefore, require more than one adaptation in the classroom and varying types and levels of support.

These learners may require and should be granted more time for:

- completing tasks
- acquiring thinking skills (own strategies)
- assessment activities.

Teachers need to adapt the number of activities to be completed without interfering with the learners gaining the required language skills.

CHAPTER

1 Whole numbers

Chapter overview

Learner's Book pages 7 to 74
Recommended pacing: 4,5 hours

This chapter focuses on the following:	
Unit 1: Properties of whole numbers	30 minutes
Revision: The properties of (zero) 0 and 1	
The real number system	
Unit 2: Calculations with whole numbers	30 minutes
Estimating solutions	
Revision	
Calculators	
Unit 3: Multiples and factors	30 minutes
Multiples and Lowest Common Multiple (LCM)	
Factors and Highest Common Factor (HCF)	
Finding the LCM and the HCF by prime factorising	
Unit 4: Ratio, rate and proportion	50 minutes
Ratio, equivalent ratios and percentages	
Problem-solving	
Direct and indirect proportion	
Rate, including speed	
POA Investigation Pulse rates	
POA Assignment 1: Ratio and rate	
Unit 5: Financial Mathematics	1 hour 10 minutes
Percentages	
VAT (Value-Added Tax)	
Profit, loss, and discounts	
Loans, simple and compound interest	
Hire purchase	
Commission and rentals	
Bank and exchange rates	
Income, budgets and accounts	
Chapter 1 Revision	35 minutes

UNIT

1 Properties of whole numbers

Unit overview

Learner's Book page 8
Recommended pacing: 30 minutes

This unit focuses on the following:

- Manipulating odd and even numbers
- Deciding whether the answer of a certain calculation is zero or undefined
- Justifying answers
- Working with consecutive numbers
- Distinguishing between prime and composite numbers
- Describing the real number system by recognising, defining and distinguishing properties of natural numbers, whole numbers, integers, rational and irrational numbers.

Resources: Learner's Book; calculator; exercise book

Background information

Learners were introduced to the following properties of numbers in Grades 7 and 8:

- Prime factors of numbers to at least 100
- Listing prime factors of numbers to at least 3-digit whole numbers
- Finding the LCM and HCF of at least 3-digit whole numbers by inspection and factorisation
- 0 in terms of its additive property (identity element for addition)
- 1 in terms of its multiplicative property (identity element for multiplication
- Recognised the division property of 0, where any number divided by 0 is undefined).

Teaching guidelines

In Grade 9, learners consolidate their number knowledge and calculation techniques for whole numbers. These techniques should have been developed over a number of years already and are revised in this unit.

Make sure that the key properties of zero and 1 are well-understood. This is very important in the understanding of especially algebraic fractions later in the year.

This unit also revises prime and composite numbers; and then discusses how rational numbers are defined as numbers with decimal fractions that either end of recur.

Because only 4,5 hours are allocated for the entire chapter, the revision will have to be fairly quick. If learners are struggling with any of the concepts, allow them some time to do a few examples and assign some for homework.

Because of the wide scope of the whole chapter, it can also be used as a reference chapter when a particular topic is being studied, for example, when fractions are studied in algebra, it may be useful to revise prime numbers and division by zero again.

Revision: The properties of zero (0) and 1; The real number system; Natural numbers

The properties of 0; The properties of 1; Whole numbers

Activity 1 Work with odd and even numbers

Learner's Book page 10

Guidelines for implementing this activity

- Work through the examples in the Learner's Book with the class.
- Discuss the different types of number systems that the learners know and revise the properties of zero.
- Discuss odd and even numbers and their properties.

Remedial and extension

Remedial: If learners struggle, let them copy the table in the Learner's Book. Working in pairs they should give at least five more examples of each type of number. They can check each other's work which will reinforce their learning of the names of the different types of numbers.
Extension: Challenge learners to draw a large number line (–2 to 2) and fill in the different types of number systems on the number line in different colours. Once they have filled in the types of number systems already covered, ask them to think about what gaps there would be on the number line and what could cause the gaps.

Suggested answers

1 $\times 2$
2 -1
3 **a** $2(10) = 20$ **b** $2(71) = 142$ **c** $2n$ **d** $250 \div 2 = 125$th
4 **a** $2(10) - 1 = 19$ **b** $2(71) - 1 = 141$
c $2(n) - 1$ **d** $(325 + 1) \div 2 = 163$rd

Activity 2 Work with prime and composite numbers

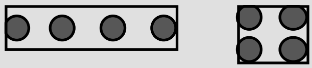
Learner's Book page 11

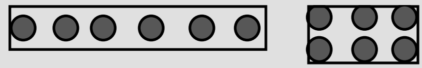
Guidelines for implementing this activity

- Ask the learners to call out the prime numbers that are less than 100 and list these on the board.
- Use the prime numbers that have been listed in the Learner's Book to help the learners answer the questions in the activity.
- Make sure that learners understand that prime numbers only have two factors: 1 and the number itself. This will help learners explain why 2 is a prime number even though it is even, and why 1 is not a prime or a composite number.

Remedial and extension

Remedial: A concrete way of showing smaller prime and composite numbers is to ask learners to arrange dots or objects in rectangles. 1 only has one arrangement; 2, 3 and 5 also only have one arrangement each. 4 and 6 can have two types of arrangements:

4: 

6: 

- The numbers with only one type of rectangle only have two factors and are prime numbers.
- The numbers with more than one type of rectangle have more than two factors and are composite.
- The numbers that form squares are the square numbers.

Extension: Learners investigate the statement that apart from 2 and 3, all prime numbers are either one more or one less than a multiple of 6, (but the converse is not true).

Suggested answers

1 No, every other even number has a factor of 2 as well.
2 No, 2 is a prime number but all the other prime numbers do end in these numbers.
3 Yes, 2 + 5 = 7; 11 + 2 = 13; 11; 13 or 17; 19 or 29; 31 or 41; 43. These numbers would give the twin primes and will always involve adding 2 to another prime number.
4 16; 18; 20; 21; 22; 24; 25; 26;27; 28
5 Only 2 and 3

Integers

Activity 3 Work with recurring decimals

Learner's Book page 12

Guidelines for implementing this activity

- Work through the examples in the Learner's Book with the class.
- This exercise is a good opportunity for learners to use their calculators.
- Identify all the different types of calculators in class. There may be a range of different calculators from scientific to non-scientific calculators. It is useful to have learners with the same type of calculators work together as they can assist each other with the workings of each type.
- Explain the convention of using the dot above the recurring number. When there is more than one recurring number, the dots would be placed on the outermost repeating digits, e.g. 0,3964964 … = $0,\dot{3}96\dot{4}$. Some books use a horizontal line (called a vinculum) above the repeating digits.

Remedial and extension

Remedial: Give learners more examples to try on their calculators. It is important that they practise inserting fractions on their particular type of calculator. It is also important to practise the decimal dot placement. Use single repeating digits at first.

Extension: Challenge learners to investigate which types of numbers in the denominators lead to recurring numbers, for example, do all prime numbers in the denominator give recurring decimals? Do all multiples of 3 in the denominator lead to recurring decimals?

Suggested answers

1 a $0{,}\dot{1}$; $0{,}\dot{2}$; $0{,}\dot{3}$ etc... b $0{,}\dot{0}\dot{1}$; $0{,}\dot{0}\dot{2}$; $0{,}\dot{0}\dot{3}$ etc... c $0{,}\dot{0}0\dot{1}$; $0{,}\dot{0}0\dot{2}$; $0{,}\dot{0}1\dot{5}$; $0{,}\dot{4}9\dot{7}$

2 a $\frac{2}{9}$ b $\frac{2}{99}$ c $\frac{13}{99}$ d $\frac{7}{999}$

3 If there is one recurring number, then there is one 9 in the denominator. If there are 2 recurring numbers, then there are two 9s in the denominator. If there are 3 recurring numbers, then there are three 9s in the denominator

4 a $\frac{121}{999}$ b $1\frac{6}{99}$

Activity 4 Work with rational and irrational numbers

Learner's Book page 13

Guidelines for implementing this activity

- Work through the notes in the Learner's Book with the class.
- Make sure that learners understand that rational numbers are fractions that either give decimal fractions that have a fixed number of digits behind the comma, OR the fractions give recurring decimal numbers.
- Irrational numbers are therefore the fractions that give decimal numbers that do not end or recur.

Remedial and extension

Remedial: Make sure that learners ask the following questions when deciding whether a number is rational or irrational:

- Does the decimal part of the number end?
- Does the decimal part of the number recur?
- If it does either of these, it is rational. If it does neither it is irrational.

Extension: Question **8** will challenge learners to think about the numbers that do not satisfy the criteria discussed so far. This will also link to the number line in the suggestions for Extension in Activity 1 in the Learner's Book. Learners could research when the different number systems came into existence and then see why they were developed.

Suggested answers

1 Rational
2 Irrational
3 Rational
4 Rational
5 Irrational
6 3,1415926… ≈ 3,141593 (correct to 6 decimal places)
7 Pi is the ratio of the circumference of a circle to its diameter. It seems that who discovered pi is not well documented, but it has been known for about 4 000 years. The 16th letter of the Greek alphabet has been used to name the ratio since the 1800s. More recent work is better documented though.
*8 Learners can list any two square roots of negative numbers, or any other even powered square roots (4th root, 6th root, etc.) of negative numbers. These would be part of the non-real or imaginary numbers and together with the real numbers form the complex numbers.

UNIT

2 Calculations with whole numbers

Unit overview

Learner's Book page 14
Recommended pacing: 30 minutes

This unit focuses on the following:
- Calculations with whole numbers
- Estimating solutions
- Revising calculations using all four operations on whole numbers using Mental Maths
- Revising calculations using all four operations on whole numbers using columns.
- Revising estimation and rounding off of whole numbers.
- Using calculators in operations.

Resources: Learner's Book; calculator; exercise book

Background information

In Grade 7 and 8, learners revised calculations using all four operations on whole numbers, estimating and using calculators where appropriate. In Grade 6, learners worked with:
- addition and subtraction of whole numbers to at least 6-digit numbers
- multiplication of at least 4-digit by 2-digit whole numbers
- division of at least 4-digit by 2-digit numbers
- performing calculations using all four operations on whole numbers, estimating and using calculators where appropriate.

Teaching guidelines

Although learners have been using these calculations since the primary school, it is a good idea to revise the basic operations again. Emphasise that learners should always start a calculation by estimating the answer even if they are using a calculator to do the calculation. This will assist them in making fewer errors.
Mental Maths is important with estimating. Remind learners that calculators are an excellent tool in Mathematics but it should not become a crutch.

Estimating solutions

Rounding off to the nearest ten, hundred and thousand; Rounding off: getting more accurate estimates; Using doubling and halving to estimate; Rounding off measurements

Activity 1 Estimate solutions

Learner's Book page 16

Guidelines for implementing this activity

- Explain that estimating is not a random guess. Learners should be encouraged to estimate answers before doing any calculation because it gives an immediate way of checking whether the answer is of the right order.
- Mental Maths is a great tool for estimating.
- Work through the examples in the Learner's Book on the board.

Remedial and extension

Remedial: Give learners daily practice at estimating. Include a five-minute session at the beginning of a lesson. Concentrate on one aspect at a time, for example, ask learners to round off to the nearest ten or hundred. You can use the same numbers for both exercises. Have Mental Maths sessions of doubling and halving, and then use this technique to estimate calculations.
Extension: Ask learners to work in pairs and challenge each other to estimate as close as possible to the actual answers. They should also explain their techniques to each other as they develop them.

Suggested answers

1 **a** **i)** 3 490 **ii)** 3 500 **iii)** 3 000
b **i)** 14 850 **ii)** 14 900 **iii)** 15 000
c **i)** 92 640 **ii)** 92 600 **iii)** 93 000

2 **a** **i)** Min. = 66,5 **ii)** Max. = 67,499...
b **i)** Min. = 39,5 **ii)** Max. = 40,499...
c **i)** Min. = 0,3865 **ii)** Max. = 0,3874999...

3 **a** 1 160 **b** 8 900 **c** 3 300 **d** 70 600 **e** 1 930 800

4 **a** 1 156 **b** 8 893 **c** 3 303 **d** 70 556 **e** 1 930 832

5 **a** $540 \times 50 = 270 \times 100 = 27\,000$
b $600 \times 116 = 300 \times 232 = 69\,600$
c $229 \times 48 \approx 230 \times 50 \approx 115 \times 100 \approx 11\,500$ (Actual answer = 10 992)
d $198 \times 22 \approx 200 \times 20 \approx 4\,000$ (Actual answer = 4 356)

Solving problems using rounding and compensation

Activity 2 Estimate solutions

Learner's Book page 17

Guidelines for implementing this activity

- Rounding and compensation use the rounding skills practised in the previous activity. The compensation 'repays' any of the adjustments made in the rounding off part.
- Work through the example in the Learner's Book on the board.
- More examples may be needed before the learners do the activity.

Remedial and extension

Remedial: Give learners extra practice. It helps to ask them to think of getting change. If something costs R367 and you pay with R370 how much change would you get back? This has to be subtracted from the final answer. If learners work in pairs giving and receiving change, it makes it easier for them to grasp the concept.
Extension: Challenge learners to set up a Mental Maths test of 20 questions (with answers). They should swop this with a partner and race each other to see who finishes first. They also need to check one another's work.

Suggested answers

1 24 × R10 ≈ R240 (Actual answer = R239,76)
2 11 432 + 9 876 ≈ 11 400 + 9 900 ≈ 21 300 (Actual answer = 21 308)
3 R275 × 12 ≈ 300 × 12 ≈ 3 600 (Actual answer = 3 300)
4 R1 256 + R2 990 + R890 + 1 100 ≈ 1 300 + 3 000 + 900 + 1 100 ≈ 6 300 (R6 236)

Revision: Addition and subtraction of whole numbers

Checking solutions using the inverse rule; Revision: Addition and subtraction methods

Activity 3 Add and subtract in columns

Learner's Book page 18

Guidelines for implementing this activity

- Work through the notes in the Learner's Book and do the examples on the board.
- Show the learners how to line up the units, tens and hundreds columns correctly.

Remedial and extension

Remedial: Make sure learners understand how to line up the digits in the correct place value columns. Give them more practice to improve their skills.
Extension: Ask learners to add four, five and six 6-digit numbers together. Ask them to see how quickly they can complete their work accurately.

Suggested answers

1 14 940

2 378

3 620

4
$$\begin{array}{r} 7\ 4\ 3\ 8\ 2 \\ 1\ 7\ 8\ 3\ 6\ 5 \\ \hline 2\ 5\ 2\ 7\ 4\ 7 \\ \hline \end{array}$$

5
$$\begin{array}{rr} & 1\ 3\ 4\ 7\ 9\ 8 \\ - & 7\ 3\ 9\ 8\ 0 \\ \hline & 6\ 0\ 8\ 1\ 8 \\ \hline \end{array}$$

6 10 999 714

Revision: Multiplication and division of whole numbers

Multiplying, dividing and checking solutions

Activity 4 Multiply and divide in columns

Learner's Book page 19

Guidelines for implementing this activity

- Work through the notes in the Learner's Book and do the examples on the board.
- Although learners have been using these methods since Grade 6, it is important to consolidate it.
- Show the learners how to line up the units, tens and hundreds columns correctly.

Remedial and extension

Remedial: Use some of the primary school concepts when completing the four operations in columns and help learners to develop a technique on how to break numbers up and to work in columns. Allow learners to use easier examples when rounding off numbers.
Extension: Add variables into the questions when using the four operations and then test whether learners remember to add the like and unlike terms. Provide more examples of rounding off with decimals. Ask learners to make up their own questions for others in the class to do, using their calculators.

Suggested answers

1 **a** 14 651 **b** 40 192

c

$$\begin{array}{r} 1\,5\,5\,6 \\ \times \quad 3\,2\,4 \\ \hline 6\,2\,2\,4 \\ 3\,1\,1\,2\,0 \\ 4\,6\,6\,8\,0\,0 \\ \hline 5\,0\,4\,1\,4\,4 \\ \hline \end{array}$$

d 7 954 668

2 735

3 729

Mixed operations

Revision: The order of operations

Activity 5 Work with mixed operations

Learner's Book page 20

Guidelines for implementing this activity

- Work through the examples in the Learner's Book with the learners and do some additional examples on the board.
- Spend some time revising BODMAS and provide additional examples if needed.
- Revise what a term is and let learners identify the terms and then simplify each term separately.

Remedial and extension

Remedial: Sometimes learners struggle with rules like BODMAS. There are alternate ways like identifying terms that can be used. Ask learners to use a pencil or a highlighter and draw a vertical line on the plus and minus signs that divide terms. Once they have done this, they should then do the calculation between the term lines first and add and subtract last. It is important that learners consolidate this before starting Algebra later in the term.
Extension: Question **6** is more challenging than the other questions. You can also ask learners to see how many different answers they can get by putting brackets into different places in a number sentence like the following: $13 \times 12 + 53 - 14 \div 7 \times (15 - 2) + \sqrt[3]{64\ 000}$

Suggested answers

1 139 2 73 3 2 364

4 $2 \times 25 + 2 - 12 = 50 + 2 - 12 = 40$

5 $18 \div 6 - \frac{(4^2 + 10^2)}{2^2} = 3 - \frac{(16 + 100)}{4} = 3 - \frac{(116)}{4} = 3 - 29 = -26$

*6 -476

Calculators

Using the Casio fx-82ZA PLUS calculator

Activity 6 Work with a scientific calculator

Learner's Book page 23

Guidelines for implementing this activity

- Group learners with different calculators together so that they can help each other.
- The instructions given here are for the Casio calculator, but most scientific calculators work similarly.
- It is useful to use an emulator with a data projector so learners can actually see which keys you are pressing. If you do not have these, a large poster of a calculator (obtainable from suppliers) is also very useful.
- Work through the notes in the Learner's Book and do the examples on the board.
- More examples may be needed to illustrate the full use of the calculator.
- If some learners have different calculators to this one, they may need assistance.

Remedial and extension

Remedial: It sometimes helps to give learners square paper to do these calculations. They should fill in the numbers on the squared paper from the units. This prevents learners from lining up the wrong place value in the wrong column, but some learners may struggle to list the numbers 'in reverse'.
Extension: Ask learners to make up five 6-digit division calculations and five multiplication calculations. They should always estimate, calculate and then check their answers. Ask them to also check their answers on their calculators.

Suggested answers

Learners check their answers for Activity 5.

1 139	**2** 73	**3** 2 364
4 40	**5** –26	**6** –476

UNIT

3 Multiples and factors

Unit overview

Learner's Book page 24
Recommended pacing: 30 minutes

This unit focuses on the following:

- Distinguishing between multiples and factors.
- Finding the highest common factor between numbers
- Using prime factorisation of numbers to find LCM and HCF.

Resources: Learner's Book; calculator; exercise book

Background information

In Grade 8, learners revised:

- prime factors of numbers to at least 3-digit whole numbers
- finding the LCM and HCF to at least 3-digit whole numbers by inspection and factorisation.

Teaching guidelines

Revise that multiples are about repeated addition (or multiplication) and factors are about division. Make sure that learners can distinguish between these terms. Often the stumbling block at this level of Mathematics is the large amount of new vocabulary that learners are faced with, especially if they are not first language English speakers. Challenge the learners who finish the activities quickly to design some posters with easy-to-remember and punchy definitions for some of the vocabulary and put these up in the Mathematics classroom.

Multiples

Activity 1 Work with multiples

Learner's Book page 24

Guidelines for implementing this activity

- Work through the examples in the Learner's Book with the class.
- Remind learners that while they work through the activities, if they cannot answer a question, they are to substitute a number instead of the variable and then try to answer the question.
- Do a few examples of the times tables.

Remedial and extension

Remedial: Practise some 'skip counting' starting at different points. Remind learners that the number itself is also a multiple. If they struggle with the variables in the activity ask learners to substitute numbers in and test whether the statements are true.
Extension: Question **4** is more challenging and learners can try the same with more than three consecutive numbers.

Suggested answers

1 **a** 30; 36; 42; 50(6) = 300
  b 45; 54; 63; 50(9) = 450
  c 65; 78; 91; 50(13) = 650
2 **a** True, for example 32 is a multiple of 8 and 32 is also a multiple of 4
  b False, 12 is a multiple of 4 but not a multiple of 8
  c True, because $3 \times 5 = 15$, and any multiples of these will be divisible by 15.
  d False, as $4 + 9 = 13$ which is not a multiple of 5.
3 42
4 **a** 6; 9; 12
  b They are all multiples of 3
  c $x + x + 1 + x + 2 = 3x + 3$
  d $3(x + 1)$ i.e. it will always be a multiple of 3

Lowest Common Multiple (LCM)

Activity 2 **Find the LCM**

Learner's Book page 25

Guidelines for implementing this activity

- Work through the notes in the Learner's Book and then work through the examples on the board with the class.
- Discuss what the words 'lowest' and 'common' mean in this context.

Remedial and extension

Remedial: Let learners use their calculators to add the same number on to generate the multiples. On a non-scientific calculator, the key strokes are, for example:
$\boxed{6}$ $\boxed{+}$ $\boxed{6}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$
On a scientific calculator, the key strokes are, for example:
$\boxed{6}$ $\boxed{+}$ $\boxed{6}$ $\boxed{=}$ $\boxed{+}$ $\boxed{6}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$
Extension: Give learners larger numbers to find the LCM of. They can either use factorisation or their calculators to generate multiples, but it would be more challenging to be able to do the larger numbers using factorisation.

Suggested answers

1 48
2 42
3 84
4 72

Factors

Activity 3 Find factors

Learner's Book page 25

Guidelines for implementing this activity

- Work through the notes in the Learner's Book and then work through the examples on the board with the class.
- This should be revision for Grade 9s, but it is necessary to consolidate before proceeding with algebra in later chapters.

Remedial and extension

Remedial: Revise the 'dolphin method' used in Grade 8 of this course. Start with 1 and ask how many times it divides into the number – both 1 and the number are factors. Then check if the number is odd or even; if even, divide by 2; then 3 until the factors are the same as ones that have already been listed.
Extension: Give learners examples of 3- and 4-digit numbers and find their factors of.

Suggested answers

1 **a** 1; 2; 3; 4; 6; 12 **b** 1; 2; 3; 4; 6; 8; 12; 16; 24; 32; 48; 64; 96; 192
c 1; 2; 3; 4; 6; 8; 12; 24 **d** 1; 3; 7; 9; 21; 63
2 3; 5
3 **a** {1; 3; 5; 9; 15; 45}
b {1; 2; 3; 6; 11; 22; 33; 66}
c {1; 2; 3; 4; 5; 6; 8; 10; 12; 15; 20; 24; 30; 40; 60; 120}
d {1; 2; 3; 4; 5; 6; 9; 10; 12; 15; 18; 20; 30; 36; 45; 60; 90; 180}

Highest Common Factor (HCF)

Activity 4 Find the HCF

Learner's Book page 26

Guidelines for implementing this activity

Work through the notes in the Learner's Book and then work through the examples on the board with the class.

Remedial and extension

Remedial: Make sure learners list all the factors of the numbers before finding the HCF. Give them more practice with smaller numbers and encourage them to go step-by-step using the 'dolphin method'. They can also check that they have an even number of factors for a non-square number and an odd number of factors for a square number.
Extension: Give learners three and four numbers to find the HCF of. These numbers can be a mixture of 2- and 3-digit numbers.

Suggested answers

1 $F_{25} = \{1; \mathbf{5}; 25\}$; $F_{40} = \{1; 2; 4; \mathbf{5}; 8; 10; 20; 40\}$; The HCF of 25 and 40 is 5.
2 $F_{18} = \{1; \mathbf{2}; \mathbf{3}; \mathbf{6}; 9; 18\}$; $F_{30} = \{1; \mathbf{2}; \mathbf{3}; 5; \mathbf{6}; 10; 15; 30\}$
There are three common factors and the HCF of 18 and 30 is 6.
3 $F_{24} = \{1; \mathbf{2}; 3; \mathbf{4}; 6; \mathbf{8}; 12; 24\}$; $F_{56} = \{1; \mathbf{2}; \mathbf{4}; 7; \mathbf{8}; 14; 28; 56\}$
There are three common factors and the HCF of 24 and 56 is 8.
4 $F_{15} = \{1; 3; \mathbf{5}; 15\}$; $F_{40} = \{1; 2; 4; \mathbf{5}; 8; 10; 20; 40\}$
5 $F_{50} = \{1; 2; \mathbf{5}; 10; 50\}$; $F_{72}\{1; 2; 3; 4; \mathbf{6}; 8; 9; 12; 18; 24; 36; 72\}$

Finding the LCM and HCF by prime factorising

Activity 5 Prime factorise

Learner's Book page 27

Guidelines for implementing this activity

- Revise what prime numbers are and explain the concept of factorisation.
- Show the learners either a tree diagram or the ladder method in the Learner's Book.

Remedial and extension

Remedial: Give learners more practice using the ladder method. They can also make a list of the prime numbers at the top of their page and check each one as the move down the ladder.
Extension: Once learners have completed the activity, give them some larger numbers as well as some examples with variables.

Suggested answers

1 $2 \times 3^2 \times 5$ 2 3×5^3 3 $2^2 \times 7^2$ 4 $2^2 \times 3 \times 7 \times 13$

Using prime factorising to find the HCF; Using prime factorising to find the LCM

Activity 6 Find the HCF and LCM prime factorising

Learner's Book page 28

Guidelines for implementing this activity

- Work through the notes in the Learner's Book and do the examples on the board.
- It is useful for learners to write the prime factors in expanded form, but also encourage them to use the exponential form.
- Explain what 'highest' and 'lowest' mean in this context.

Remedial and extension

Remedial: Practise the times tables using flash cards. This will help learners to cope better with multiples. Use divisibility rules to help the learners to find factors. Show learners how to use their calculators with division calculations.
Extension: Give learners more complex numbers to prime factorise. Ask the class to find a method on their calculators to help them to prime factorise a number. (Give them a hint about the FACT function.)

Suggested answers

1 $9 = 3^2$; $24 = 2^3 \times 3$. Thus the HCF is 3 and LCM is 72.
2 $15 = 5 \times 3$; $40 = 2^3 \times 5$. Thus the HCF is 5 and LCM is 120.
3 $18 = 3^2 \times 2$; $24 = 2^3 \times 3$. Thus the HCF is 6 and the LCM is 72.

UNIT 4

Ratio, rate and proportion

Unit overview

Learner's Book page 29
Recommended pacing: 50 minutes

This unit focuses on the following:

- Calculating the ratio of one quantity to another quantity and expressing it in fraction form
- Calculating quantities resulting from dividing a given quantity into a given ratio
- Interpreting and solving relevant word problems
- Forming ratios equivalent to given ratios (including ratios with three ratio numbers)
- Expressing ratios in percentage form
- Expressing ratios given in mixed ratio numbers or decimals as ratios in whole numbers in the simplest form
- Calculating quantities resulting from dividing a given quantity into a given ratio expressed in percentage form
- Calculating percentage increases and decreases of prices and other quantities
- Interpreting and solving relevant word problems.
- Distinguishing between direct and indirect proportion
- Calculating the scale factor for a direct proportion
- Solving problems about direct and indirect proportion.
- Recognising rates expressed in various units
- Calculations involving rates
- Calculations involving exchange rates.

Resources: Learner's Book; calculator; exercise book

Background information

In Grade 8 learners solved problems involving:

- comparing two or more quantities of the same kind (ratio)
- comparing two quantities of different kinds (rate)
- sharing in a given ratio where the whole is given
- increasing or decreasing of a number in a given ratio.

Teaching guidelines

Remind learners about the differences between ratio and rate.

- A ratio compares two or more quantities of the same kind. The two quantities would have the same unit and these would cancel out, so a ratio has no units.
- A rate is a comparison of two different types of quantities. There are many rates that we use in everyday life such as speed, electricity consumption, water usage, prices per kilogram of food, exchange rates and so on. Rates will always have units of the form km per hour; kW per hour; R39 per kilogram, and so on. 'Per' means *divide*.

Ratios

What is ratio?

Activity 1 **Calculate ratios**

Learner's Book page 31

Guidelines for implementing this activity

- Write 'Ratio is a way of comparing quantities of the same kind.' on the board. Discuss the statement while providing examples.
- Discuss the worked examples in the Learner's Book and emphasise the note at the end of the second example.
- Let the learners study the two Think-Do examples and then discuss these as a class.
- Let the learners do questions **1** to **3** in their exercise books.

Remedial and extension

Remedial: Give learners examples with smaller numbers where they can actually use concrete objects to arrange in groups, for example if there are 24 dots on the board and they need to be coloured in red and green in the ratio 5 : 1. Learners can make groups of 5 + 1 = 6 and see how many of them there are in 24. Once they have mastered smaller concrete examples start giving examples with larger numbers.
Extension: Challenge learners to find at least two examples of ratios in every day life and to use them to find larger or smaller quantities. Most learners are aware of mixing juice concentrates and water in certain ratios. The nutritional information on foodstuffs are also ratios and can be used to find out what the quantities would be in different sized servings.

Suggested answers

1 **a** Number of male teachers = 18 – 13 = 5
Ratio female teachers : male teachers = 13 : 5

b Learners : teachers = 450 : 18 = 25 : 1 (after dividing by 18)
c Average number of learners per teacher = 25

2 a Side length of floor : side length of tile = 2,4 m : 200 mm
= 2 400 mm : 200 mm
= 12 : 1

b Area floor : Area tile = $(2,4 \text{ m})^2 : (0,2 \text{ m})^2$
= $5,76 \text{ m}^2 : 0,04 \text{ m}^2$
= 576 : 4
= 144 : 1

c Number of tiles required to cover the floor = 144

3 a Pocket money of Kulani : Pocket money of Bolani = R70 : R42 = 10 : 6 = 5 : 3
b Kulani's savings : Kulani's pocket money = R21 : R70
= $3 : 10 = \frac{3}{10}$
c Bulani's savings : Bulani's pocket money = R14 : R42
= $2 : 6 = \frac{1}{3}$
d Kulani's total monthly savings = 4 × R21 = R84
Bolani's total monthly savings = 4 × R14 = R56
e Kulani's monthly pocket money : Bolani's monthly pocket money
= (4 × R70) : (4 × R42)
= 70 : 42 (after dividing by 4)
= 5 : 3 (after dividing by 14)

More application of ratio

Activity 2 Calculate ratios of quantities

Learner's Book page 33

Guidelines for implementing this activity

- Make a rough sketch on the board depicting the first example.
- Discuss the Think-Do part with the learners.
- Let the learners study examples 2 and 3 and discuss them in pairs.
- Learners do questions 1 to 3 in their exercise books, checking their partners.

Remedial and extension

Remedial: Ask the learners to give examples of quantities in order to review this concept. Make sure that they understand the concept 'quantities of the same kind'.
Extension: Discuss partnerships in small businesses and the criteria for dividing the profits between the partners.

Suggested answers

1 How many parts? 5 + 4 = 9. How much is one part? $\frac{1}{9}$ of R252,90 = R28,10
Mulalo gets 5 parts = 5 × R28,10 = R140,50; Tsakami gets 4 parts = 4 × R28,10 = R112,40

2 a Baskets A : Baskets B : Baskets C = 48 : 54 : 42
= (8 × 6) : (9 × 6) : (7 × 6)
= 8 : 9 : 7 after dividing by 6

b Number of parts = 8 + 9 + 7 = 24
1 part of R425 = of R425 = R425 ÷ 24 = R17,71
Day wage for A = 8 × R17,71 = R141,68
Day wage for B = 9 × R17,71 = R159,39
Day wage for C = 7 × R17,71 = R123,97

3 **a** Dosage for ants. Insecticide : water = 20 ml : 10 000 ml = 1 : 500
b Dosage for spiders. Insecticide : water = 100 ml : 10 000 ml = 1 : 100
c In the mixture for spiders the water ratio is 2,5 ℓ : 10 ℓ = 1 : 4 = $\frac{1}{4}$
Thus insecticide to be mixed with 2,5 ℓ water = $\frac{1}{4}$ of 100 ml = 25 ml
d In the mixture for army worms the water ratio is 5 ℓ : 10 ℓ = 1 : 2 = $\frac{1}{2}$
Thus insecticide to be mixed with 5 ℓ water = $\frac{1}{2}$ of 2,5 ml = 1,25 ml
***e** Dosage for army worms : Dosage for ants = 2,5 ml : 20 ml = 25 : 200
= 12,5 : 100 = 12,5%

Equivalent ratios and percentages

Equivalent ratios; Percentages

Activity 3 Write equivalent ratios

Learner's Book page 36

Guidelines for implementing this activity

- Write the sentence 'Any ratio can be transformed to an equivalent ratio by multiplying (or dividing) the quantities by the same factor.' on the board. Discuss the statement as you give learners various examples on the board.
- Discuss examples 1 and 2 and emphasise the various ways of writing a ratio. Do similar examples of your own on the board.
- Work through the Think-Do example 3 and then ask learners the question, "What percentage of the 100 cm stick is the 75 cm stick?"
- Learners study the Think-Do worked example 4.
- Learners do questions **1** and **2** in their exercise books.
- Work through the Think-Do worked example 5 with learners.
- Learners do question **3** in their exercise books.

Remedial and extension

Remedial: Ratios can also be expressed as fractions. Learners are used to changing equivalent fractions by multiplying and dividing. Show learners how the ratio is an equivalent way of writing fractions and to transform ratios, you have to multiply (or divide) all the parts by the same amount. Give learners extra examples to practise.
Extension: Give learners examples of ratios that have variables to simplify.

Suggested answers

1 **a** Divide by HCF 11. Then 55 : 121 = 5 : 11
b 1,5 hours : 50 minutes = 90 minutes: 50 minutes (divide by HCF 10) = 9 : 5

c 6 cm : 126 mm : 12 cm
= 60 mm : 126 mm : 120 mm (divide by HCF 6 and drop the units) = 10 : 21 : 20

d 560 g : 1,05 kg : 1,47 kg = 560 g : 1 050 g : 1 470 g
= 56 : 105 : 147 (after dividing by 10)
= 8 : 15 : 21 (after dividing by 7)

e 108% : 84% : 48% = 108 : 84 : 48 (divide by HCF 12) = 9 : 7 : 4 (after dividing by 12)

2 a 375 ml : 1 ℓ = 375 ml : 1 000 ml (divide by HCF 125)
$= 3 : 8 = \frac{3}{8} \times 100\% = 37{,}5\%$

b R12,50 : R500 = R25 : R1 000 (multiply by 2 to get whole numbers)
= 5 : 200 (after dividing by 5; units dropped)
= 1 : 40 (after dividing by 5)
(Instead of twice dividing by 5 you can divide once by 25.)
1 : 40 = 2,5%

c 96 mm : 6 cm = 96 mm : 60 mm (divide by 12)
= 8 : 5 100% = 8 × 20% = 160%

d 99c : R1,65 = 99c : 165c
= 9 : 15 (after dividing by 11)
= 3 : 5 (after dividing by 3)
= 60%

e 2,5 kg : 750 g = 2 500 g : 750 g (divide by HCF 250)
$= 10 : 3 = \frac{10}{3} \times \frac{100}{1}\% = \frac{1\ 000}{3}\% \times 333\frac{1}{3}\%$

3 a 7, 5 : 12,5 = 75 : 125 (after × 10) = 3 : 5 (after ÷25)

b 0,24 : 1,08 : 0,84 = 24 : 108 : 84 (after ×100) = 2 : 9 : 7 (after ÷ 12)

c $5\frac{2}{3} : 8\frac{1}{3} = \frac{17}{3} : \frac{25}{3}$ (× 3 to get whole numbers)= 17 : 25

d $1\frac{7}{8} : 2\frac{3}{8} : 3\frac{1}{8}$ (× 8 to get whole numbers) $\frac{15}{8} : \frac{19}{8} : \frac{25}{8}$ = 15 : 19 : 25

Problem-solving

Activity 4 Solve word problems

Learner's Book page 36

Guidelines for implementing this activity

- Show an advertisement showing the "Was… Now…" prices and use it as an example to calculate the percentage by which the "Was" price was reduced.
- Read through the worked example in the Learner's Book and then learners do questions **1** and **2**.

Remedial and extension

Remedial: Remind learners that percentage means 100 parts. They should follow the same patterns they developed for ratios just using 100 as the total.
Extension: Ask learners to design two questions based on percentage ratios. They can use data from other subjects like Natural Sciences or Geography to make up interesting contexts.

Suggested answers

1 Lot A = 40% of 150 kg of flour = $\frac{40}{100} \times \frac{150}{1}$ kg = 60 kg

Lot B = 35% of 150 kg of flour = $\frac{35}{100} \times \frac{150}{1}$ kg = 52,5 kg

Lot C = 25% of 150 kg of flour = $\frac{1}{4} \times \frac{150}{1}$ kg = 37,5 kg

2 Kulani's share = 40% of R12 450 = $\frac{40}{100}$ × R12 450 = 4 × R1 245 = R4 980

Lungisani's share = $33\frac{1}{3}$% of R12 450 = $\frac{1}{3}$ × R12 450 = R4 150

Msizi's share = $26\frac{2}{3}$% of R12 450 = $\frac{80}{300}$ × R12 450 = $\frac{8}{3}$ × R1 245 × 8 = R415 = R3 320

Direct and indirect proportion

Direct proportion

Activity 5 Work with direct proportion

Learner's Book page 38

Guidelines for implementing this activity

- Work through examples 1 and 2 with the class making sure that the learners understand the meaning of *scale factor*.
- On the board, draw a table like the one in example 1, but adapt it to the A4-A5-A6 example. Fill in the lengths and breadths and, using a calculator, find the ratio $L : b$ for each of the three sheet sizes. It should be about 1 : 4, which is the scale factor of this direct proportion.
- Write the opening sentence of Activity 1 on the board and link it to examples 1 and 2 and the A4-A5-A6-example.
- Let the learners do questions **1** to **4** in their exercise books.

Remedial and extension

Remedial: Give learners extra practice with scale factors. This is a topic that we often understand intuitively and often use practically when doing crafts.
Extension: Ask learners to explain how the direct proportion works in the distance-speed-time formula. Are all the variables in direct proportion? Which are not?

Suggested answers

1 Learners complete the table.

Rectangle	A	B	C	D
	l = 4 cm b = 2 cm	l = 6 cm b = 3 cm	l = 8 cm b = 4 cm	l = 30 cm b = 15 cm
Perimeter $P = 2(l + b)$	12 cm	18 cm	24 cm	90 cm
Ratio $P : l$	12 : 4 = 3 : 1	18 : 6 = 3 : 1	24 : 8 = 3 : 1	90 : 30 = 3 : 1
Ratio $P : b$	12 : 2 = 6 : 1	18 : 3 = 6 : 1	24 : 4 = 6 : 1	90 : 15 = 6 : 1

- **a** Perimeters are in direct proportion to their lengths; scale factor = 3
- **b** Perimeters are in direct proportion to their breadths; scale factor = 6

2
- **a** 7,5 ℓ: 1,25 ℓ = 750 : 125 = 6 : 1
- **b** 500 ml
- **c** Water to be added to 50 ml concentrate = 50 ml × 6 = 300 ml.

3
- **a** Learners copy and complete the table. Let them use the π button on their calculators.

	S	M	L
Radius: r	14 mm	21 mm	35 mm
Circumference: $C = 2\pi r$	87,964 mm	131,947 mm	219,911 mm
Area: πr^2	615,752 mm^2	1 385,442 mm^2	3 848,451 mm^2
Ratio of $C : r$	6,28	6,28	6,28
Ratio of $A : r$	43,98	65,97	109,96

- **b** Circumferences of disks are in direct proportion to their radii; scale factor is 6,3.
- **c** Areas of disks are in direct proportion to their radii; no scale factor.

4
- **a** Interest : Period = 80 : 1 for each of the five periods.
- **b** Amount of interest is directly proportional to the period of investment. Reason: The ratio Interest : Period is constant for any period.

Indirect proportion

Activity 6 Work with indirect proportion

Learner's Book page 41

Guidelines for implementing this activity

- Read through example 1 with the learners and then show them on the board that the equation $l \div b = 12$ can also be written in the form $l = \frac{12}{b}$ or the form $b = \frac{12}{l}$.
- Work through example 2 with the learners. Point out that there is a difference in example 2 of Activity 1, where the petrol price per litre was fixed. In this example, the total purchase price for petrol is fixed at R200.
- Let the learners do questions **1** and **2** in their exercise books.
- Discuss question 3 (packing of apples) with the class. Let learners first do question **3a** and then discuss the answer with them. Let learners do question **3b** and discuss the answer. Learners do question **3c**.

Remedial and extension

Remedial: Take an A4 sheet of paper (297 mm long and 210 mm wide), fold it in half so that each half is A5 size (210 mm long and 148,5 mm wide). Cut the sheet along the fold to make two A5 sheets. Fold one A5 sheet in half and cut it to make two A6 sheets (148,5 mm long and 105 mm wide). Paste a new A4 sheet, one A5 sheet and one A6 sheet on a poster or the board. Ask the question: "Are the three rectangular sheets identical or just similar?"

Remedial and extension continued

Answer: they are not identical but they are similar. The three rectangles are in direct proportion. Provide the learners with sheets of cm^2-grids and instruct them to work in pairs to draw various rectangles on the grids, so that the area of every rectangle is 12 cm^2. The results should be 4 cm long by 3 cm wide, 6 cm by 2 cm , 12 cm by 1 cm , 8 cm by 1,5 cm, etc. Write the equation $l \div b = 12$ on the board and emphasise that this equation is true for every correct rectangle they drew. Tell learners that these rectangles are indirectly proportional. Two quantities are indirectly proportional when one quantity decreases as the other increases.
Extension: Refer learners back to the distance-speed-time formula and to say which values are indirectly proportional.

Suggested answers

1 a

b	20	16	10	8	5	4
h	2	2,5	4	5	8	10

b $b \times h = 2 \times 20 = 40$ which can also be written as $b = \frac{40}{h}$

c b is indirectly proportional to h.
Reason: $b \times h = 40$, a constant number. As b decreases, h increases.

2 a

p	1 m	0,5 m	1,5 m	2 m	2,4 m	3 m
n	12	2	8	6	5	4

b $p \times n = 12$

c The number of pieces, n, are indirectly proportional to p, the length of every piece.

3 a The number of boxes per shift (B): 100 Small, 40 Medium and 16 Large.
b is indirectly proportional to n, the number of apples per box.
Reason: $b \times n = 2\,400$, a constant product.

b Small box $n : t = 24 : 16 = 3 : 2 = 1,5$; Medium box $n : t = 60 : 40 = 3 : 2 = 1,5$
Large box $n : t = 150 : 100 = 3 : 2 = 1,5$
n is directly proportional to t.
Reason: $n : t$ is the constant rate of 1,5 apples/sec.

c Number of apples per box (a) : time (t) = 3 : 2. a is directly proportional to t.

d Small box $w : t$ = 40c : 16 sec = 2,5c per sec; Medium box $w : t$ = 100c : 40 sec = 2,5c per sec.
Large box $w : t$ = 250c : 100 sec = 2,5c per sec.; w is directly proportional to t.;
Reason: $w : t$ is the constant rate 2,5c per sec.

Rate and speed

Rate

Activity 7 Calculate rates

Learner's Book page 43

Guidelines for implementing this activity

- Work through the worked example on the board.
- Let the learners work in pairs to measure each other's pulse rates, as in question **3**.
- Work through the worked examples in the activity with the learners.
- Let the learners do question **3** in their exercise books.

Remedial and extension

Remedial: Give learners more examples of simple rates from everyday life. Allow learners to work in pairs so that they can explain the different rates to each other and decide which quantity is the numerator and which is the denominator.
Extension: Question **3** is more challenging for learners.

Suggested answers

1 **a** Danielle was paid R92 ÷ 8 h = R11,50 per hour when she started to work.
b Danielle's new wage = R11,50 + 10% of R11,50 per hour = R11,50 + R1,15 = R12,65/h
c Danielle's pay for an 8-hour day = 8 × R12,65 = R101,20
Danielle's pay for a 5 1×2 -day week = 5,5 × R101,20 = R556,60

2 **a** Rate per 1 kg of flour = R33,95 ÷ 2,5 kg = R13,58 per kg which is cheaper than R18,99/kg.
b Rate per litre of cooking oil:
750 ml bottle: R10,45 ÷ 0,75 ℓ = R13,93 per litre
2 ℓ bottle: R19,19 ÷ 2 ℓ = R9,60 per litre, which is the cheapest rate.
c Rate per dozen eggs: $\frac{1}{2}$ doz. pack: R7,19 ÷ $\frac{1}{2}$ = R14,38 per doz.
$2\frac{1}{2}$ doz. pack: R33,95 ÷ 2,5 = R13,58 per doz., the cheapest rate.

3 Mr Dlamini buys some British pounds from Mrs C at an exchange rate of R12,785 per £1.
So Mrs C receives 400 × R12,785 = R5 114.
Mr Dlamini sells some of his pounds to Mr D at an exchange rate of R13,4767 per £1.
So Mr D receives 5 000 ÷ 13,4767 = £371,01.
Mr Dlamini buys euros from Mrs E at an exchange rate of R11,278 per €1.
So Mrs E receives 4 500 × R11,278 = R50 751.
Mr Dlamini sells some euros to Mr F at an exchange rate of R11,9060 per €1.
So Mr F receives 3 700 ÷ 11,906 = €310,77.
Mr Dlamini buys pula from Ms G at an exchange rate of R0,932 per pula.
So Ms G receives 1 000 × R0,932 = R932.
Mr Dlamini sells pula to Mr H at an exchange rate of R0,808 per pula.
So Mr H receives 1 000 ÷ 0,808 = R1 237,62.

Pulse rates

Learner's Book page 45

According to the Senior Phase Mathematics CAPS, investigations should promote critical and creative thinking. They can be used to discover rules or concepts and may involve inductive reasoning, identifying or testing patterns or relationships, drawing conclusions, and establishing general trends.

To avoid having to assess work which is copied without understanding, it is recommended that whilst initial investigation could be done at home, the final write-up should be done in class, under supervision, without access to any notes. Investigations are assessed with rubrics, which can be specific to the task, or generic, listing the number of marks awarded for each skill.

The following rubric may be used, or one that you find appropriate to use for this investigation.

	0–1	**2–3**	**4–5**	**6–7**
Organising and recording ideas and discoveries using, for example, diagrams and tables	No evidence of strategy or procedure to record the data	Partial completion of the data	Recorded all data	Recorded all data clearly and accurately
Communicating ideas with appropriate explanations	No description of the method required for the task	Partial completion of the descriptions of the method	Descriptions of the method are effective	Descriptions of the method are effective and appropriate
Calculations showing clear understanding of mathematical concepts and procedures	No evidence of understanding the concepts required for the task	Minimal evidence of understanding of mathematical concepts or big ideas	Evidence of understanding of the big ideas	Provided evidence of in depth understanding of concepts and procedures
Generalising and drawing conclusions	No conclusion stated or no data recorded	Conclusions partly supported by the data	Appropriately used data to support conclusions	Interpretation of data supports conclusions and raised new questions

Distance, time and speed

Activity 8 Work with speed

Learner's Book page 48

Guidelines for implementing this activity

- Write the opening sentence (the definition of speed) of this activity on the board.
- Emphasise the fact that speed is a rate as shown by the word *per* in kilometres *per* hour (km/h), metres *per* second (m/s) etc.
- Discuss the relationship between distance (d), time (t) and speed (s), as expressed by the formula $d = s \times t$ and its variants $S = \frac{d}{t}$ and $t = \frac{d}{s}$.
- Draw the triangular 'reminder' on the board and demonstrate how to use it to write each of the three formulae above.
- Do example 1 on the board to show details of the calculation.
- Learners study example 2 and then do the same calculation without looking at the solution in the Learner's Book.
- Let the learners try to do examples 3 and 4 without looking at the solutions in the Learner's Book. Afterwards, they may consult the Learner's Book to assess their own attempts.
- Learners do the questions **1** to **5** on their own.

Remedial and extension

Remedial: Discuss the four examples of rates in the Learner's Book. Emphasise the function of the term 'per' used in the units of most rates. Ask the class the question: "If you ride with a bicycle from home to school, do you travel at one speed all the way or does your speed vary?" Discuss the likely circumstances that will influence the speed of the bicycle.

Extension: Ask the learners to cite more examples of rates. Ask the question: "Say the distance is 5 km and the time you take to travel this distance is half an hour. What was your average speed in km/h?" Emphasise the fact that most things that move, do so at variable speeds and that calculated speeds are usually *average* speeds.

Suggested answers

1. 54 minutes = (54 × 60) h = 0,9 h
   Distance that train travelled: $d = s \times t = (75 \times 0{,}9)$ km = 67,5 km
2. Driving time: $t = \frac{d}{s} = \frac{1\,500 \text{ km}}{90 \text{ km/h}} = 16\frac{2}{3}$ h = 16 h 40 min
3. Average speed of snail: $S = \frac{d}{t} \times \frac{125 \text{ cm}}{4{,}5 \text{ min}}$ = 27,8 cm/min
4. **a** Jabu's speed in m/s = (40 km × 1 000) × (1 h × 60 × 60) m/s
   = (40 000 × 3 600) m/s
   $= \frac{100}{9}$ m/s = 11,11 m/s

   b Jabu's distance = $s \times t = \frac{100}{9}$ m/s × 45 sec = (100 × 5) m × 500 m

5 a $5 \text{ km} = 5 \times 1\,000 \text{ m} = 5\,000 \text{ m}$
$1 \text{ min and } 48 \text{ sec} = (60 + 48) \text{ sec} = 108 \text{ sec} = (108 \times 60) \text{ min} = (1{,}8 \times 60) \text{ h} = 0{,}03 \text{ h}$
Average speed of racing car $= \frac{d}{t} \times (5\,000 \text{ m}) \times (108 \text{ sec}) = 46{,}3 \text{ m/s}$
In km/h: Average speed $\times (5 \text{ km}) \times (0{,}03 \text{ h}) = 166{,}7 \text{ km/h}$

b Time at 200 km/h $= \frac{d}{s} = \frac{5 \text{ km}}{200 \text{ km/h}} = 0{,}025 \text{ h} = 0{,}025 \times 60 \times 60 \text{ sec} = 90 \text{ sec}$

PoA | Assignment 1 Ratio and rate

Learner's Book page 48

Suggested answers

1 a 18 : 35 b 20 : 1 c 4 : 1 d 3 : 2 : 1 (6)
2 $2 + 7 + 1 = 10$; $45 \div 10 = 4{,}5$
blue $= 2 \times 4{,}5 = 9$ litres; yellow $= 7 \times 4{,}5 = 31{,}5$ litres; white $= 1 \times 4{,}5 = 4{,}5$ litres (5)
3 a 1: 1: 1
b x units (6)
4 a $425 \text{ g} \div 100 \text{ g} = 4{,}25$ servings b $18{,}8 \times 4{,}25 = 79{,}9$ g
c $5{,}4 \times 4{,}25 = 22{,}95$ g d $370 \times 4{,}25 = 1\,572 \text{ mg} \approx 1{,}57$ g
e $513 \times 4{,}25 = 2\,307{,}75$ kJ (5)
5 a $S = \frac{D}{T} = \frac{24}{10{,}5} = 137{,}14$ km/h b $D = S \times T = 140 \times \frac{24}{60} = 56$ km (6)
6 a $85 \times 120 = 10\,200$ litres $= 10{,}2$ kl in two hours
b $2\,000 \div 85 = 23{,}52$ minutes $= 23$ min $31{,}76$ sec (6)
7 a $1\,000 \times 8{,}1075 = \text{R}8\,107{,}50$
b $10\,000 \div 8{,}4571 = \$1\,182{,}44$
c $2\,000 \times 0{,}9320 = 1\,864$ (6)
[40 marks]

UNIT

5 Financial Mathematics

Unit overview

Learner's Book page 50
Recommended pacing: 1 hour and 10 minutes

This unit focuses on the following:

- Working with percentage increases and decreases
- Solving word problems that involve percentages
- Working out the amount of VAT owed from a transaction
- Solving financial problems involving profit, loss, discount and VAT
- Solving financial problems involving interest
- Solving financial problems involving simple and compound interest
- Solving financial problems involving loans
- Solving financial problems involving higher purchase
- Solving financial problems involving commission and rentals
- Solving financial problems involving banking and exchange rates
- Solving financial problems involving budgets, earnings and accounts

Resources: Learner's Book; calculator; exercise book

Background information

In Grade 8 learners learnt how to solve problems that involve whole numbers, percentages and decimal fractions in financial contexts such as:

- profit, loss, discount and VAT
- budgets
- accounts
- loans
- simple interest
- hire purchase and
- exchange rates.

Teaching guidelines

Compound interest and the use of formulae for simple interest and compound interest calculation are new to Grade 9. The other topics in Financial Mathematics are a revision of work done in Grade 8. The clarification notes in CAPS recommends that learners do sufficient repeated calculations for simple and compound interest before using the formulae for these calculations.

Percentages

Activity 1 Revise common fractions and percentages

Learner's Book page 50

Guidelines for implementing this activity

- Revise the four operations with fractions before starting with percentages. Remind learners that percentage means per 100 and that the denominator in a percentage would be 100. Revise equivalent fractions and how to convert fractions to percentages and percentages to fractions.
- Revise decimal fractions and how to convert decimals to percentages.

Remedial and extension

Remedial: Some learners struggle with fractions and should be given another opportunity to go through the basic operations. It is also useful to show learners how to do fractions on the calculator. Some scientific calculators have the vertical fraction display which makes it easier for learners to relate what they have on the page to what is being entered on the calculator. Five or ten minutes of Mental Maths practice on these conversions every day will also give learners more practice.
Extension: Let learners set each other Mental Maths tests on converting between fraction, percentage and decimals

Suggested answers

1	b	**2**	c	**3**	d	**4**	b	**5**	d
6	a	**7**	a	**8**	b	**9**	a	**10**	b

Percentages of whole numbers

Activity 2 Find a percentage of a whole number

Learner's Book page 52

Guidelines for implementing this activity

- Work through the examples in the Learner's Book with the class.
- Encourage the learners to discuss whether the answers they get are realistic or not (this is to encourage critical thinking).

Remedial and extension

Remedial: Encourage learners to learn the most common percentages used like: 50% is a half; 25% is a quarter; 10% is one–tenth; 1% is one hundredth; 20% is one–fifth. The more they use these, the more confident they will become with them. Give extra practise in finding these percentages.
Extension: Challenge learners to think about percentages that are greater than 100%. What does 200% mean? What does inflation of 900% mean?

Suggested answers

1 a $\frac{50}{100} \times 300 = R150{,}00$ b $\frac{25}{100} \times 80 = R20{,}00$

c $\frac{1}{100} \times 30 = R0{,}30$

2 a $\frac{5}{100} \times 180 = R9{,}00$ b $\frac{20}{100} \times 25 = R5{,}00$

c $\frac{25}{100} \times 3 = R0{,}75$ d $\frac{75}{100} \times 42 = R31{,}50$

3 a $\frac{15}{100} \times 340 = R51{,}00$ b $\frac{60}{100} \times 75 = R45{,}00$

c $\frac{80}{100} \times 50 = R40{,}00$ d $\frac{2{,}5}{100} \times 880 = R22{,}00$

4 $\frac{58}{100} \times 234 = R135{,}72$

5 a $\frac{12}{100} \times 17 = R2{,}04$ b $\frac{8}{100} \times 38 = R3{,}04$

c $\frac{32}{100} \times 340 = R108{,}80$ d $\frac{2}{100} \times 76{,}40 = R1{,}53$

Percentage increase and decrease

Activity 3 Calculate percentage increase and decrease

Learner's Book page 53

Guidelines for implementing this activity

- Work through the examples in the Learner's Book with the class. Stress the words *increase* and *decrease* to the learners.
- Learners complete the activity on their own.

Remedial and extension

Remedial: Remind learners that *increase* is adding on. If the amount that they have already is 100% and they then add an amount on, for example 6%, the new total is 100% + 6% = 106%. They can use this as an alternate method of finding the new total by dividing by 100 and multiplying by the original amount. Give learners more examples to try out. Similarly, *decreasing* is subtracting and they can subtract the percentage decrease from 100% and calculate the new total.

Suggested answers

1 a $\frac{10}{100} \times 350 = R35 \therefore R350 + R35 = R385$

b $\frac{5}{100} \times 74 = R3{,}70 \therefore R74{,}00 - R3{,}70 = R70{,}30$

c $\frac{5}{100} \times 524 = R26{,}20 \therefore R524{,}00 + R26{,}20 = R550{,}20$

d $\frac{17{,}5}{100} \times 960 = R168 \therefore R960 - R168 = R792$

2 a $\frac{17}{100} \times 340 = R57{,}80 \therefore R340{,}00 + R57{,}80 = R397{,}80$

b $\frac{42}{100} \times 905 = R380{,}10 \therefore R905{,}00 - R380{,}10 = R524{,}90$

c $\frac{4{,}7}{100} \times 1680 = R78{,}96 \therefore R1\ 680{,}00 + R78{,}96 = R1\ 758{,}96$

d $\frac{14{,}5}{100} \times 2\ 990 = R433{,}55 \therefore R2\ 990{,}00 - R433{,}55 = R2\ 556{,}45$

3 $\frac{10}{100} \times 185 = R18{,}50 \therefore \text{Price} = R185{,}00 - R18{,}50 = R166{,}50$

4 $\frac{28,3}{100} \times 295\ 000 = \text{R}83\ 485$

$\therefore$ New value = R295 000 + R83 485 = R378 485

5

Karabo :	R15 400	R770	R16 170
Wilma :	R11 350	R567,50	R11 917,50
Palisa :	R10 625	R531,25	R11 156,25
Cindy :	R9 475	R473,75	R9 948,75
Lance :	R8 720	R436	R9 156

Value–Added Tax (VAT)

Activity 4 Calculate VAT

Learner's Book page 54

Guidelines for implementing this activity

- Work through the examples in the Learner's Book with the class.
- Explain to learners that each time we buy something; we have to pay a type of tax called VAT. When we see a price that excludes VAT, we have to add 14% to the price to get the full price we will have to pay. Each time we see a price that includes VAT, we know that 14% has already been added to the original price.

Remedial and extension

Remedial: VAT is an application of percentage increase. A fixed percentage of 14% is being added on or the original amount is being calculated. Make sure learners have grasped the percentage increase well before starting with VAT. The activity only asks learners to add the VAT on to prices.
Extension: Ask learners to investigate whether it matters if the tax is added on before or after a percentage discount.

Suggested answers

1 Cellphone: $\frac{14}{100} \times 1\ 499 = \text{R}209,86$ Selling Price = R1 499,00 + R209,86 = R1 708,86

2 Vitamins: $\frac{14}{100} \times 89,79 = \text{R}12,57$ Selling Price = R89,79 + R12,57 = R102,36

3 Calculator price = $126,95 \times \frac{14}{100} = \text{R}17,77$ Selling Price = R126,95 + R17,77 = R144,72

4 Clock: $\frac{14}{100} \times 14,45 = \text{R}18,82$ Selling Price = R134,45 + R18,82 = R153,27

Profit, loss and discounts

Activity 5 Calculate profit and loss

Learner's Book page 56

Guidelines for implementing this activity

- Work through the example in the Learner's Book with the class. Explain the examples leading up to this activity to the class.

- Explain to learners that businesses spend money in order to keep their business running. When the money received at the end of the month is more than the money spent, we say that the business made a *profit*. When the total money received at the end of the month is less than the money spent, we call this a *loss*.

Remedial and extension

Remedial: It is often easier to photocopy some paper money and get learners to actually see that if they buy an item for R100, they must sell it for more than R100 to make a profit. Let learners work in pairs as a 'shopkeeper' and 'customer' until the concept is well understood. Now show learners again that we would like to convert that to percentage by making an equivalent fraction of profit/cost price.
Extension: Ask learners to calculate a series of calculations like with cost price, mark up, VAT, discount of 10% to get final prices for five items.

Suggested answers

DVD: R95 $\times \frac{20}{100}$ = R19. Selling Price = R95 + R19 = R114

DVD player: R399,95 $\times \frac{12}{100}$ = R47,99. Selling Price = R399,95 – R47,99 = R351,96

Interest

Earning interest; Paying interest

Activity 6 **Calculate interest rates**

Learner's Book page 57

Guidelines for implementing this activity

- Work through the example in the Learner's Book with the class.
- Remind learners how to use their calculators for this activity.
- While learners complete this activity as a class, encourage discussions relating to the interest rates found. Encourage feedback from learners, asking questions such as, "Is this a very high interest rate or not?"

Remedial and extension

Remedial: Interest is also an application of percentages. Give learners an opportunity to redo any questions that they get wrong and give some additional exercises for them to practise.
Extension: Give learners an exercise with larger amounts of money and challenge them to do the calculations without the use of a calculator.

Suggested answers

1 18%

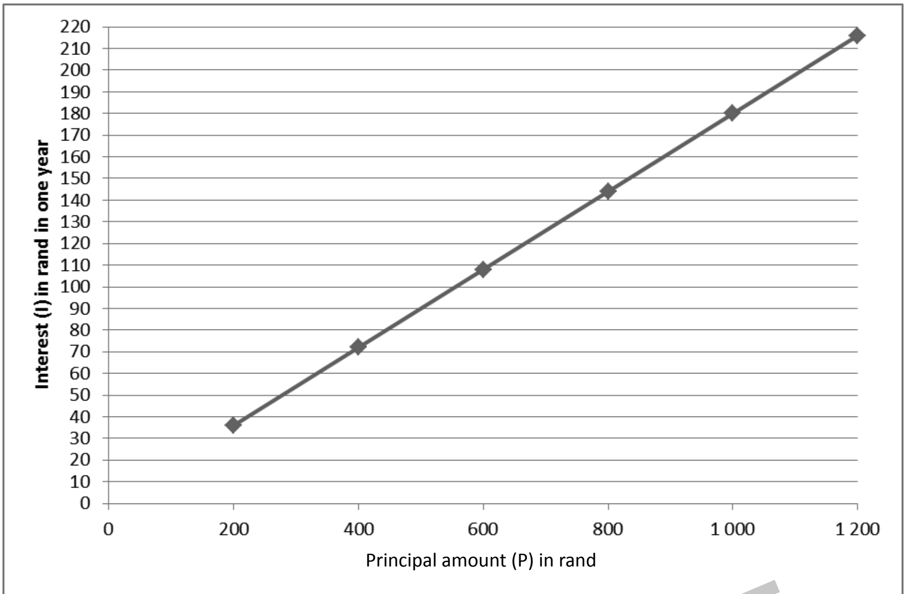
2 $\frac{36}{300} \times 100 = 12\%$

3 $\frac{55}{500} \times 100 = 11\%$

4 $\frac{9,50}{50} \times 100 = 19\%$

5 $\frac{973,50}{9\ 260} \times 100 = 10,51\%$

6



7 **a** R90 *__b__ $y = \frac{18}{100} \times x$ (Answer given in tip on page 57).

c $y = \frac{18}{100} \times 400 = \text{R}72$

Activity 7 Calculate interest rates

Learner's Book page 58

Guidelines for implementing this activity

- Hand out graph paper/block paper to each learner and complete the example in the Learner's Book with the class.
- Advise the learners to develop similar formulae to the one in the Learner's Book.

Remedial and extension

Remedial: Remind learners of the method of using a 'Clue board' to work out the numbers in the tables, for example, if R8 is paid per R100 rand, then R0,80 will be paid for R10 (one tenth). R50 is half of R100 and so on. This is also a ratio and learners can be shown to treat it like a ratio if they find that easier.
Extension: Ask learners to draw the graphs of the two tables.

Suggested answers

1

Principal (*P*)	10	50	200	300	500	2 000
Interest (*I*)	R0,80	R4	R16	R24	R40	R160

2

Principal (P)	200	300	600	750	1 000	2 000
Interest (I)	R19	R28,50	R57	R71,25	R95	R190

Simple and compound interest

Simple interest

Activity 8 Calculate simple interest

Learner's Book page 59

Guidelines for implementing this activity

- Explain to learners that simple interest is an application of percentage increase or decrease. The same methods can be used but this can be time consuming when there are several years to calculate. It is quicker to use the formula for simple interest given on page 58 of the Learner's Book.
- Work through the example in the Learner's Book with the class.
- Learners do the activity by themselves.

Remedial and extension

Remedial: Give learners more practise with the substitution into the formula.
Extension: Ask learners to investigate percentage decrease or depreciation as used in tax calculations.

Suggested answers

1 S.I. = 9 000 $\times \frac{6,5}{100} \times 3$ = R1 755

2 S.I. = 13 500 $\times \frac{7,4}{100} \times 4$ = R3 996

Will have R13 500 + R3 996 = R17 496

Compound interest

Activity 9 Calculate compound interest

Learner's Book page 61

Guidelines for implementing this activity

- Work through the examples in the Learner's Book with the class.
- Check that at the end of each year of the question, that the learners have the correct amount; otherwise the answers will be very different at the end of the question.

Suggested answers

1 1st year: R12 500 $\times \frac{6,5}{100}$ = R812,50. Total = R13 312,50

2nd year: R13 312,50 $\times \frac{6,5}{100}$ = R865,31. Total = R14 177,81

3rd year: R14 177,81 $\times \frac{6,5}{100}$ = R921,56. Total = R15 099,37

2 R4 300 × $\frac{11}{100}$ = R473   New Value = R4 773

3 R92 000 × $\frac{9}{100}$ = R8 280   New Value = R69 328,53

Loans

Loan tables

Activity 10 Work with loans

Learner's Book page 64

Guidelines for implementing this activity

- Work through the examples in the Learner's Book with the class.
- Advise learners to use the table in the Learner's Book to complete this activity.

Remedial and extension

Remedial: Show the learners how the tables work again and then ask them to call out a number of practise examples to show on the table on the board or projector. Let them redo any of the calculations that are not correct.
Extension: Questions **3** and **4** are more challenging. Learners can also work in pairs and make up amounts that their colleagues have to look up in the tables.

Suggested answers

1 Monthly repayment = 32,5204 × 9 = R292,68

2 a $2\frac{1}{2}$ years i.e. 30 months × 38,0443
To get from 1 000 to 85 000, × by 85 = 38,0443 × 85 = R3 233,74 per month
b 25,0647 × To get from 1 000 to 16 000, × by 16
25,0647 × 16 + R401,04 per month

3 a 36 months = 32,7387 × 12,5 = R409,23 per month
b R14 732,28

4 a 27,0763 × 15,75 = R426,45 per month
b Total: R20 469,69

Hire purchase

Activity 11 Work with hire purchase

Learner's Book page 65

Guidelines for implementing this activity

- Bring in a pamphlet from a retail store that offers hire purchase agreements and ask learners to comment on the prices and viability of the offers.
- Work through the worked examples in the Learner's Book to assist learners to answer the questions.

Remedial and extension

Remedial: Group learners together so that they can assist each other with the exercise. It is very good for learners who can complete the exercise quickly to explain to another learner the steps required to do the activity. This then consolidates their own knowledge and extends their understanding.

Suggested answers

1 a Total = 229 + 18 × 135,60 = R2 669,80
  b Save R2 669,80 – R2 298 = R371,80
2 a Total = $\frac{10}{100}$ × 1 598 + 48 × 37,15 = R1 943
  b Save R1 943 – R1 598 = R345
3 Total = $\frac{15}{100}$ × 879 + 26 × 39,98 = R1 171,33
4 a R200 + 24 × R74,69 = R1 992,56
  b R1 992,56 – R1 499 = R493,56
  c R493,56 ÷ 2 = R246,78.
5 a R300 + 36 × R199,53 = R7 483,08
  b R7 483,08 – R5 890 = R1 593,08
  c R1 593,08 ÷ 3 = R531,03
6 a R500 + 48 × R187,58 = R9 503,84
  b R9 503,84 – R5 990 = R3 513,84
  c R3 513,84 ÷ 4 = R878,46
  d R878,46 ÷ 52 = R16,89

Commission and rentals

Commission

Activity 12 Work with commission

Learner's Book page 67

Guidelines for implementing this activity

- Work through the example in the Learner's Book on the board with the class.
- Remind learners that commission is an application of a percentage of a total amount.

Remedial and extension

Remedial: Learners practise by redoing questions they get wrong and doing additional exercises.
Extension: Question **6** is more challenging.

Suggested answers

1 R16 000 × $\frac{5}{100}$ = R800
2 R92 000 × 2 × $\frac{2}{100}$ = R3 680
3 a R660 000 × 7,5 ÷ 100 = R49 500
  b R320 000 × 7,5 ÷ 100 = R24 000
  c R160 000 × 7,5 ÷ 100 = R12 000

4 **a** R9 000 × 18 ÷ 100 = R1 620 **b** R5 400 × 18 ÷ 100 = R972
c R2 300 × 18 ÷ 100 = R414
5 **a** R15 000 × 2,5 ÷ 100 = R375 **b** R27 000 × 2,5 ÷ 100 = R675
6 **a** R0 because it is less than R10 000.
b R24 000 – R10 000 = R14 000
R14 000 × 17,5 ÷ 100 = R2 450

Rentals

Activity 13 Work with rentals

Learner's Book page 68

Guidelines for implementing this activity

- Work through the worked example in the Learner's Book with the class.
- Learners complete the activity in pairs and then write their answers on their own.

Remedial and extension

Remedial: Rentals are quite straight forward income, but when a commission has to be paid to a letting agent or rental company there percentage calculations involved. Give learners a few extra examples if they do not manage the activity.
Extension: Provide an additional activity that contains questions with more months, with the rent increasing after a certain amount of months as well as a commission for the rental agency.

Suggested answers

1 2 × 1 750 = R3 500; $\frac{6}{100}$ × 3 500 = R210 commission **2** R3 500 + R210 = R3 290

Bank and exchange rates

Bank statements; Credit cards

Activity 14 Work with a bank statement

Learner's Book page 69

Guidelines for implementing this activity

- Work through the bank statement from the Learner's Book with the class.
- Learners answer all questions and discuss the information shown on the bank statement.

Remedial and extension

Remedial: There is a large language component in this section and it may be difficult for some learners to understand all the new terms. Use simple language and analogies to explain the vocabulary.
Extension: Ask learners to compile a glossary of the terms used in this section. They should then use simple language to explain what each term means. These can be used to assist learners who require remedial teaching.

Suggested answers

1 **a** 7 April to 7 May 2014 **b** 19% p.a. **c** R10 000
d R3 923,94 **e** 1 June 2014 **f** R559,90
2 **a** R6 844,06 **b** R8 200,00
3 Yes
4 R559,90 ÷ R6 076,06 × 100 = 9,21%
5 Advantages: • buy now, pay later • don't carry cash • 55 interest-free days
Disadvantages: • careful of overspending • must budget to pay back money

Exchange rates

Activity 15 Work with exchange rates

Learner's Book page 71

Guidelines for implementing this activity

- Work through the worked example in the Learner's Book with the class.
- Learners use the exchange rates from the Learner's Book to answer questions.

Remedial and extension

Remedial: The learners need to be able to work with percentages so do a revision on percentage. Make up a very basic bank statement and ask some questions on this. Ask learners what their expenses are each month and ask them to make an example of a budget based on this.
Extension: Give learners a pamphlet from a shop and ask them to work out the purchase price of the product advertised on the hire purchase agreement. Make up questions and assign marks to them as an example to learners of what to expect. Obtaining forms from a bank for learners to fill in as an example may pose a fun challenge. Bring a newspaper to school and show learners where to find exchange rates for that particular day.

Suggested answers

1 **a** R2 000 × 13,34 = R26 680 **b** R6 500 × 6,52 = R42 380
2 **a** $\frac{5\ 000}{1{,}04}$ = 4 807,69 (HK dollars) **b** $\frac{11\ 200}{11{,}27}$ = 993,79 (euros)

Income, budgets and accounts

Income

Activity 16 Work with income

Learner's Book page 72

Guidelines for implementing this activity

Work through the notes and example in the Learner's Book with the class.

Remedial and extension

Remedial: Make sure that learners who struggle follow the steps as given to them in the example. Learners can work in pairs where needed, and in this way support each other.
Extension: Learners should find question **3** more challenging.

Suggested answers

1 R940 – 280,78 – 37,62 – 49,75 – 9,40 = R572,45 per week
2 R42,80 × 36 = R1 540,80 per week
R1 540,80 – 536,11 – 51,25 – 43,50 – 15,41 – 7,60 – 50 = R836,93 per week
3 R63,70 × 38 = R2 420,60
R2 420,60 – 871,42 – 44,90 – 217,85 – 24,21 – 5 – 70 = R1 187,19 per week

Budgets

Activity 17 Work with budgets

Learner's Book page 73

Guidelines for implementing this activity

- Work through the budget in the Learner's Book with the class and discuss what income and expenses are.
- Ask learners for examples of what makes up income and expenses and write these as two lists on the board.

Remedial and extension

Remedial: Explain the concepts using basic examples that learners can identify with. Break it down for them using simple basic income examples like earning pocket money and how they can use a budget to plan how to save for an event or a larger purchase.
Extension: Ask learners to develop a budget for a class outing. They must include all costs and work out how much each learner would have to pay.

Suggested answers

Income	R25 550
+	R3 050
	R28 600
Expenses	
Home loan	R2 826
Food and groceries	R3 500
Transport	R600
Insurance	R930
Electricity	R1 380
School fees	R1 246
Entertainment	R850
Emergencies	R750
	R12 082
Remaining	**R16 518**

Chapter 1 Revision

Learner's Book page 74

Encourage learners to review the content covered before attempting the revision activities. The revision activities should be used to assess learners' progress thus far, and to assess where remediation may be required.

Suggested answers

1 a Rational
b Rational
c Rational
d Irrational
2 a 9; 18; 27; 36; 45
b 7; 14; 21; 28; 35; 42; 49; 56; 63; 70
3 a 1; 2; 3; 6
b 2; 3
c 6; 12; 18; 24
d None, they are all even and are also multiples of 6 and so will be divisible by 6.
4 a $2 \times 2 \times 2 \times 2 \times 5$
b $2 \times 2 \times 2 \times 2 \times 3 \times 5$
5 a $18 : 35$ $(\frac{18}{35})$
b $20 : 1$
6 $450 \times \frac{4{,}75}{100} \times \frac{5}{12} = \text{R}8{,}91$
7 a Total cost = R295 + 24 × 84 = R2 311
b R2 311 – 1 895 = R416
c Learners' own answers

CHAPTER

2 Integers

Chapter overview

Learner's Book pages 76 to 95
Recommended pacing: 4,5 hours

This chapter focuses on the following:

Unit 1:	**Properties of integers**	1,5 hours
	The commutative property	
	The associative property	
	The distributive property	
	The additive inverse	
	The multiplicative inverse	
Unit 2:	**Calculations with integers**	3 hours
	Revision: Ordering and comparing integers	
	Adding and subtracting integers	
	Multipling and dividing integers	
	Problem-solving	
	Calculations with squares and square roots	
	Calculations with cube numbers and cube roots	
Chapter 2 Revision		35 minutes

UNIT

1 Properties of integers

Unit overview

Learner's Book page 76
Recommended pacing: 1,5 hours

This unit focuses on the following:

- Revising the commutative property of addition and multiplication of integers
- Revising the associative property of addition and multiplication of integers
- Revising the distributive property of addition and multiplication of integers
- Revising the additive inverse of a number
- Revising the multiplicative inverse of a number.

Resources: Learner's Book; exercise book; calculator

Background information

In Grades 7 and 8, learners were expected to recognise and use the commutative, associative and distributive properties of addition and multiplication for integers as well as the additive and multiplicative inverses for integers.

Teaching guidelines

These properties of integers have been studied in Grade 7 and Grade 8 so should not be new to learners.
This unit should not take too much time to complete as it is revision and should just serve as a warm-up for Unit 2 Calculations with Integers.
It is a good idea to remind learners at different stages during the year which of these properties are being used at different stages so that it becomes part of their regular mathematical vocabulary.
Make sure that you give simple definitions to learners so that language does not impair their understanding.

Revision: Integers; The commutative property; The associative property; The distributive property

Activity 1 Work with the properties of integers

Learner's Book page 77

Guidelines for implementing this activity

- Work through the distributive, associative and commutative properties with the class.
- Learners might need additional number examples to illustrate these properties.

- Learners need to be able to perform these properties more than they need to know the terminology.

Remedial and extension

Remedial: Give learners more number examples for each of the properties. Explain the words in simpler language: commutative means the order does not matter; associative means we can regroup terms to make it easier to calculate; and distributive means either multiplying into brackets, or factorising which means each term is divided by a factor.

Extension: Give learners some exercises that involve variables and ask them to identify the property demonstrated in each step, for example:

$17 + 5y - 4x + y - 2x - 10$	original statement
$17 - 10 + 5y + y - 4x - 2x$	[CP]
$(17 - 10) + (5y + y) - (4x + 2x)$	[AP]
$(7) + y(5 + 1) - 2x(2 + 1)$	[DP]
$7 + y^6 - 2(3)x$	[CP]
?	[CP]

Suggested answers

1 a $-3 - 6 = -9$ $2 - 11 = -9$ Thus they are the same.
  b $-4 \times -21 = +84$ $-28 \times -3 = +84$ Thus they are the same.
  c 47 47 Thus they are the same.
  d -31 and -31 Thus they are the same.
  e $+66$ $+66$ Thus they are the same.

2 a $5 - 22 = -17$ and $22 - 5 = 17$.
  b $-17 - (+12) = -29$ and $+12 - (-17) = 29$.
  c $-8 - (-19) = 11$and $-19 - (-8) = -11$
  a, b and c show that the commutative law does not hold for subtraction.

3 a $-3(-11 + 5) = (-3 \times -11) + (-3 \times 5) = (33) + (-15) = 18$
  b $4(-6 - 12) = (4 \times -6) + (4 \times -12) = (-24) + (-48) = -72$
  c* $-2 - (3 + 5) = -2 + (-1 \times 3) + (-1 \times 5) = -2 + (-3) + (-5) = -10$

The additive inverse

Activity 2 Work with the additive inverse

Learner's Book page 78

Guidelines for implementing this activity

- Work through the notes in the Learner's Book on the additive inverse.
- The identity for addition is zero.
- An additive inverse is a number that will change another number to zero when it

is added. It has the same numeric value, but the signs are opposite.
- Additive inverses are used in solving equations and this topic can be revised again then.
- Since this is revision learners can do the activity as quickly as possible after working through the example.

Remedial and extension

Remedial: Remind learners that they just have to change the sign because when they add the new number to the old one, the result will be zero. They can confirm this on their calculators. Make sure learners correct any answers that they did not get right the first time.
Extension: Question 9 is a little more challenging.

Suggested answers

1 -11	2 700	3 0
4 $-4x$	5 11	6 $-7\ 435$
7 $99\ 999$	8 $12y$	*9 $7x^2$

The multiplicative inverse

Activity 3 Work with the multiplicative inverse

Learner's Book page 79

Guidelines for implementing this activity

- Work through the notes in the Learner's Book on the multiplicative inverse.
- The identity for multiplication is 1.
- For multiplication, "the identity" is 1, because multiplying by 1 does not change anything.
- The multiplicative inverse is the same number, but on the opposite side of the fraction line, for example, the inverse of -6 when you are multiplying is $-\frac{1}{6}$, because $(-6)\left(-\frac{1}{6}\right) = 1$. The sign stays the same.
- Ask learners to do the activity as quickly as possible.

Remedial and extension

Remedial: Give additional questions on multiplicative inverse if necessary.
Extension: Give learners some examples with algebraic fractions and exponents.

Suggested answers

1 $\frac{1}{11}$	2 $-\frac{1}{700}$	3 n/a
4 $\frac{1}{4x}$	5 $-\frac{1}{7y}$	6 $\frac{1}{1\ 252}$
7 $\frac{1}{99\ 999}$	8 $\frac{2}{5}$	*9 $-\frac{1}{7x}$

UNIT
2 Calculations with integers

Unit overview

Learner's Book page 80
Recommended pacing: 3 hours

This unit focuses on the following:
- Revising the calculations involving all four operations with integers
- Revising the calculations involving all four operations with numbers that involve the squares of integers
- Revising the calculations involving all four operations with numbers that involve the square roots of integers
- Extending the calculations to include square roots of variables
- Revising the calculations involving all four operations with numbers that involve the cubes of integers
- Revising the calculations involving all four operations with numbers that involve the cube roots of integers
- Extending the calculations to include cube roots of variables.

Resources: Learner's Book; exercise book; calculator; packs of cards

Background information

Learners have already learnt how to:
- add and subtract with integers
- multiply and divide with integers
- perform calculations involving all four operations with integers
- perform calculations involving all four operations with numbers that involve squares, cubes, square roots and cube roots of integers.

Teaching guidelines

All the work covered in this unit is based on revision of Grade 8 work.
It is very important that learners consolidate the work with integers before working with algebra.
It may be necessary to do another quick revision of this section as a warm-up activity before starting with algebra later in the term.

Revision: Order and compare integers

Activity 1 Compare and order integers

Learner's Book page 80

Guidelines for implementing this activity

- Work through the notes in the Learner's Book on the board with the class.
- Draw a number line on the board and show learners how to use this to add and subtract integers.

- Use the concept of language negatives to help learners to understand the concept of negative numbers multiplied by negative numbers, for example, "you must not not do your homework". Ask learners what this sentence means.

Remedial and extension

Remedial: Use the number line with more basic questions to add and subtract numbers. Remind learners that adding a number is moving it to the right on a number line and subtracting a number is moving it to the left on a number line. Draw a number line on the floor using chalk and ask the learners to physically move up and down the number line.
Extension: Change the questions completed in the examples to include variables and then ask learners to add and subtract the terms.

Suggested answers

1 a –13; –12; –6; 0; 15; 17 b –7; –6; –5; –3; –2; 0
c –142; –121; –101; –99; –98 d – 6; –5,8; –5,7; –5,4; –5,1; –5
2 a 31 b –11 c 4
d 2,5 e 7,5 f –0,5
3 a True b True c True
d False: it is warmer because it is closer to zero
e False: we added 2 each time
f False: we subtracted 4 each time
g True h True
i False it is warmer, but you cannot say it is twice as warm as it is not absolute temperature.
j False: zero is neither positive nor negative
4 a 0 °C b –2 234 589 c Tuesday
d –R1 120 e –R605
5 a Add 120 to each term. b Subtract 7 from each term.
c Subtract 10 from each term.

Adding and subtracting integers

Adding and subtracting positive numbers

Activity 2 **Add and subtract positive numbers**

Learner's Book page 82

Guidelines for implementing this activity

- Learners should be comfortable adding positive numbers and subtracting when the first number is larger than the second number.
- Use the number line to show these familiar operations first and then use the number line to show how a negative number results from a subtraction where the second number is the larger of the two.
- Ask learners to do the activity as quickly as possible (speed test).

Remedial and extension

Remedial: Encourage learners to use the number line to allow them to count on and count back until they have formed a rule in their mind and don't need to use the number line any longer. It is also useful to draw the number line vertically so it reminds learners of a thermometer.
Extension: Give learners larger numbers as well as terms with variables to add together.

Suggested answers

1 16	**2** 28	**3** 91
4 128	**5** 7	**6** –7
7 –1	**8** –6	**9** –12

Adding and subtracting negative numbers

Activity 3 Add and subtract negative numbers

Learner's Book page 83

Guidelines for implementing this activity

- This topic is also revision for learners.
- Make sure that learners can use the number line to add and subtract numbers if they have not worked out a rule for themselves yet.
- Ask learners to do the activity as quickly as they can.

Remedial and extension

Remedial: It is useful for learners to do repeated subtractions or additions on the calculator and to write down the patterns. The key strokes for a non-scientific calculator are: $\boxed{1}\boxed{7}$ $\boxed{-}$ $\boxed{5}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$
On a scientific calculator: $\boxed{1}\boxed{7}$ $\boxed{-}$ $\boxed{5}$ $\boxed{=}$ $\boxed{-}$ $\boxed{5}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ or
$\boxed{1}\boxed{7}$ $\boxed{-}$ $\boxed{(}$ $\boxed{-}$ $\boxed{5}$ $\boxed{)}$ $\boxed{=}$ $\boxed{-}$ $\boxed{(}$ $\boxed{-}$ $\boxed{5}$ $\boxed{)}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$
Extension: Give learners exercises with larger integers and some that involve variables.

Suggested answers

1 10	**2** –22	**3** –83
4 90	**5** 21	**6** 23
7 29	**8** 12	**9** –26

Activity 4 Calculate temperatures

Learner's Book page 84

Guidelines for implementing this activity

- Prepare the class for this activity by using temperatures taken from the newspaper weather report and asking learners to increase/decrease the temperatures shown.
- Work through the notes in the Learner's Book with the class.
- Learners do this activity on their own.

Remedial and extension

Remedial: Temperatures are an everyday application of negative and positive numbers and most learners understand this intuitively. Encourage learners to draw a vertical number line to represent a thermometer when they are doing these calculations and other integer addition and subtraction calculations. Let learners redo any calculations that they do not get correct.

Extension: In Physical Science, the absolute zero of temperature is at –273 °C on the Kelvin scale. Ask learners to find a way of working out what freezing point (0 °C); room temperature (25 °C) and boiling point (100 °C) would be on the Kelvin scale.

Suggested answers

From	To	Temperature change
Harare	Cape Town	–8
Harare	Rio de Janeiro	+3
Sydney	Cape Town	+8
London	Beijing	–5
London	Moscow	–7
Beijing	Moscow	–2
London	Cape Town	+20
Windhoek	London	–24

Activity 5 Add and subtract integers

Learner's Book page 85

Guidelines for implementing this activity

- Draw a long number line on the board and ask some questions similar to those in this activity.
- Show learners how to count forwards from the smaller number and backwards from the bigger number to find the number exactly between these two numbers.
- Using the same number line, show learners that they can count forward for a positive number and backwards for a negative number on the number line.
- Write the functions: +(–); –(–); –(+) on the board and ask learners to call out what the rules for each mean when each is pointed at, for example: "add and subtract equals subtract"; "subtract and subtract equals add". This will assist learners in getting used to the rules before attempting them on their own.

Remedial and extension

Remedial: Let learners draw and count on a number line. Provide as much practice working with integers, especially negative integers, as possible.
Extension: Ask learners to work in pairs and design a test (with memo) with larger integers.

Suggested answers

1	a	–7	b	–11	c	–18	d	–31
	e	8	f	–9	g	–6	h	$-19y$
2	a	8	b	–13	c	12	d	3
	e	4	f	–23	g	–9	h	–9

Multiplying and dividing integers

Using number lines to multiply and divide negative numbers

Activity 6 Multiply and divide integers

Learner's Book page 87

Guidelines for implementing this activity

- Work through the worked examples in the Learner's Book with the class on the board.
- Advise learners to use a number line to assist them with this activity.

Remedial and extension

Remedial: Revise the times tables with learners to assist them with the multiplication and division work without using their calculators.
Extension: Add variables to the worked examples and to the activity and assess how learners manage with multiplying or dividing integers and variables.

Suggested answers

1	a −2	b 2	c 3	
	d 3	e −4	f 4	
2	a −6	b −30	c 32	d −63
	e 72	f 35	g −8	h 5
	i −6	j 5	k 10	l −12

3 a A positive times a negative is a negative
b Correct
c A negative times a negative is a positive
d A negative times a negative is a positive
e Correct
f 8 times 5 is 40
g A positive times a negative is a negative
h A negative divided by a negative is a positive
i Correct
j A negative times a negative is a positive

4	a −147	b −133	c 414
	d −40	e −13	*f 6

Problem-solving

Activity 7 Work with integers

Learner's Book page 87

Guidelines for implementing this activity

Learners complete this activity on their own.

Remedial and extension

Remedial: Use real-life examples to help learners understand this concept. For example, we have R120 and want to buy a pair of jeans for R150, how much money do we need? Card games involving addition and subtraction are also helpful.
Extension: Use a pack of cards and assign a variable to each suit. Learners add only the variables, that is the same suits, together.

Suggested answers

1 a 120 + 5 times 120 = R720
b 3 times −23 = −69

2

Top row	$2{,}821109907 \times 10^{12}$						
6th row	746496	3779136					
5th row		−576	−1 296	−2 916			
4th row	24	−24	54	−54			
3rd row	6	4	−6	−9	6		
2nd row	−3	−2	−2	3	−3	−2	
Bottom row	−3	1	−2	1	3	−1	2

Mixed operations

The order of operations

Activity 8 Use the correct order of operations

Learner's Book page 88

Guidelines for implementing this activity

- Revise the rules of integers with the class.
- Provide some easy examples and work through these as a class.
- Remind learners that we work inside brackets first, and then from left to right when working with mixed calculations.

Remedial and extension

Remedial: Learners should redo the questions they got wrong until they can get the correct answers on their own. They should be provided with ample opportunities to practise.
Extension: Challenge learners to do questions **6** and **7** and fill in brackets in different places to see how many different answers they can get.

Suggested answers

1 13	**2** –4	**3** –5	**4** –4
5 12	***6** $14x$	***7** $-3 - 6x$	

Problem-solving

Activity 9 Play a game

Learner's Book page 88

Guidelines for implementing this activity

- Learners complete this activity in groups and groups who get the most number of questions correct may be rewarded. A few packs of cards are needed for this activity.

Remedial and extension

This is a fun activity where learning takes place. Learners often learn very quickly in a game situation and their classmates will correct them if they make a mistake.

Suggested answers

Learners play card games as explained in the Learner's Book.

Calculations with square numbers and square roots

Revision: Square numbers

Activity 10 Investigate square numbers

Learner's Book page 89

Guidelines for implementing this activity

- Work through this activity with the class and discuss the concepts observed.

Remedial and extension

Remedial: Have learners revise their times tables and in particular the square numbers to at least 122. They can write these as a pattern on the top of their page when working with square numbers.
Extension: Challenge learners to find and learn all the square numbers to 252.

Suggested answers

1 16; 9; 4; 1
2 They are all perfect square numbers
3 The number of rows and columns are the same for each picture
4 Learners draw picture with 5 × 5 stars
5 25; 36; 49; 64; 81
6 x^2

Square roots; Square roots with variables; The rules for square numbers and square roots

Activity 11 Do calculations with squares and square roots

Learner's Book page 92

Guidelines for implementing this activity

- Work through the notes in the Learner's Book.
- Show learners where to find the square and the square root keys on the calculator and how these functions work.
- Learners complete this activity on their own.

Remedial and extension

Remedial: Ask learners to write down the first 15 square numbers and learn these. This will help them to identify them more easily.
Extension: Provide similar activities to those in the Learner's Book.

Suggested answers

1 **a** No **b** No **c** No
d Yes **e** No **f** No
2 **a** 9 + 16 = 25 **b** 25 + 144 = 169 **c** 36 **d** 64
3 The answers are all perfect squares themselves.
4 **a** False **b** False

5	a	100	b	9	c	–64	d	–144
	e	–100	f	16	g	–40		
6	a	9	b	8	c	1	d	No solution
7	a	3	b	No solution	c	6	d	6
	e	The answers are the same.						
8	a	4	b	3	c	5	d	7
	e	No, they are not the same.						
9	a	x^5	b	x^2y^3	c	xy^4		

Calculations with cube numbers and cube roots

Revision: Cube numbers

Activity 12 Find volumes of cubes

Learner's Book page 93

Guidelines for implementing this activity

- Work through this activity with the class as a whole group and generate group discussions regarding the findings.

Remedial and extension

Remedial: It is useful to get learners to see a box of square cubes and how many fit into it, or two build cubes with smaller cubes. Once they have built a few cubes ask learners how they could go about calculating the number of cubes without having to build them each time.
Extension: Ask learners to construct a strong 1 dm^3 box and to check how much water it will hold. Remind learners to line the box with a plastic bag before pouring water into it.

Suggested answers

1	a	1	b	8	c	27	d	64
	e							

2 They are perfect cube numbers

3 125; 216; 343

*4 $V = l \times b \times h$

*5 $V = x^3$

Cube roots; Cube roots with variables; The rules for cube numbers and cube roots

Activity 13 Find cube numbers and cube roots

Learner's Book page 95

Guidelines for implementing this activity

- Work through the notes in the Learner's Book.
- Show learners where to find the cube and cube root keys on the calculator and how these functions work.
- Learners complete this activity on their own.

Remedial and extension

Remedial: Ask learners to write down the first ten cube numbers and learn these. This will assist them in identifying these cube numbers more easily.
Extension: Provide similar activities to those in the Learner's Book.

Suggested answers

1 a 5 b 6 times 6 times 6 c 1
d −1 e 8 times 8 = 64
2 a 9 b 125 + 21 = 146
3 343
4 1 000
5 a 13,82 b 4 784,09 c −0,001
d 7 e 1,97 f −6
6 a y^4z^5 b ab^6 c $3xy$ d $-p^2q^3$

Chapter 2 Revision

Learner's Book page 96

Encourage learners to review the content covered before attempting the revision activities. The revision activities should be used to assess learners' progress thus far, and to assess where remediation may be required.

No calculators may be used in this exercise.

Suggested answers

1 a −27 b 0 c 0 d −11

2

Before	Situation changes	After
−5 °C	Temperature rises by 8 °C	3 °C
+R370	Withdrawal of R700	−R330
−10 cm	Water level drops by 27cm	−37 cm
−17 °C	The temperature rises by 15 °C	−2 °C
−R60	Deposit R130	+R70
+R60	Withdraw R230	−R170
7 mm	Water level drops by 36mm	−29 mm

3 a −2 b 40 c −13 d 9
4 a R400 b Yes, R10 c 125 − 10 = R115
d Kristen's expression is correct and Richards' is also correct. Thus they are both correct.
5 a −7 b −7

CHAPTER

3 Fractions

Chapter overview

Learner's Book pages 97 to 129
Recommended pacing: 4,5 hours

This chapter focuses on the following:

Unit 1: Revision: Fractions 1,5 hours

- Types of fractions
- Equivalent fractions
- Mixed numbers and improper fractions
- Simplifying square numbers, square roots, cube numbers and cube roots
- Simplifying squares, cubes, square roots and cube roots of simple algebraic fractions

Unit 2: Equivalent forms of fractions 1,5 hours

- Converting common fractions to decimal numbers and percentages
- Converting decimal fractions to common fractions and percentages
- Converting percentages to common fractions and decimal fractions
- Rounding off decimal numbers and percentages

Unit 3: Calculations with fractions 1,5 hours

- Adding and subtracting fractions
- Adding and subtracting algebraic fractions
- Multiplying and dividing fractions
- Multiplying and dividing algebraic fractions
- Problem solving

Chapter 3 Revision 35 minutes

UNIT

1 Revision: Fractions

Unit overview

Learner's Book page 97
Recommended pacing: 1,5 hours

This unit focuses on the following:
- Types of fractions
- Equivalent fractions
- Finding equivalent fractions:
  - simplifying fractions
  - comparing fractions
- Mixed numbers and improper fractions:
  - converting between mixed numbers and improper fractions
- Simplifying square numbers. square roots, cube numbers and cube roots
- Simplifying squares, cubes, square roots and cube roots of simple algebraic fractions
  - simplifying squares and cubes of algebraic fractions
  - simplifying square roots and cube roots of algebraic fractions

Resources: Learner's Book; exercise book; calculator

Background information

In Grade 8 learners should have revised the following:
- addition and subtraction of common fractions, including mixed numbers;
- finding fractions of whole numbers
- multiplication of common fractions, including mixed numbers
- division of whole numbers and common fractions by common fractions
- calculations with the squares, cubes, square roots and cube roots of common fractions

Teaching guidelines

Most of the work done in this unit is revision of the Grade 8 work. Many learners struggle with the concept of fractions, so it is a good idea to redo the basics again. The calculations can be done without a calculator. A good understanding of fractions will be necessary when looking at algebraic fractions later in the chapter.
When adding and subtracting fractions, learners will probably use the equivalent fraction method used in the earlier grades. They will also find that it is easier to use the Lowest Common Denominator (LCD) rather than larger numbers. With some practice, learners soon pick up how to speed up the process.

Although learners may have other methods for division, they should be reminded that division is the inverse of multiplication. In other words division is the same as multiplying with the reciprocal.

Types of fractions; Equivalent fractions

Finding equivalent fractions

Activity 1 Find equivalent fractions

Learner's Book page 99

Guidelines for implementing this activity

- Revise the vocabulary associated with fractions. Make sure that all the learners understand each of the terms given.
- Remind learners about equivalent fractions – the fractions might look different but still have the same value.
- Using 1 as the identity for multiplication and division, the fraction can be changed to another fraction by multiplying the numerator and the denominator by the same number. Similarly both can be divided by the same number, effectively dividing by 1.
- Ask learners to do the exercise as quickly as possible.

Remedial and extension

Remedial: Ask learners to copy the list of the vocabulary into their exercise books. They should write at least five more examples next to each one.
Extension: Give learners that finish quickly a few more challenging questions to do.

Suggested answers

1 $\frac{4}{6}$	2 $\frac{16}{20}$	3 $\frac{35}{40}$	4 $\frac{35}{45}$
5 $\frac{9}{21}$	6 $\frac{30}{100}$	7 $\frac{75}{100}$	8 $\frac{80}{100}$

Simplifying fractions

Activity 2 Simplify fractions

Learner's Book page 100

Guidelines for implementing this activity

- Work through the examples with learners.
- Ask learners to do the activity as quickly as they can as this is revision.

Remedial and extension

Remedial: Revise factorising again from Chapter 1. Give learners a few examples to just factorise before giving them more fractions to work with.
Extension: Give learners much larger numbers in the fractions and ask them to use prime factorisation to find the HCF of the numerators and denominators. Give learners a few examples with variables to simplify. Questions **2g–h** are also more challenging.

Suggested answers

1 **a** $\frac{12}{15}$ **b** $\frac{4}{5}$

2 **a** $\frac{3}{4}$ **b** $-\frac{5}{8}$ **c** $\frac{1}{3}$ **d** $\frac{1}{8}$ **e** $-\frac{67}{100}$

f $\frac{71}{180}$ ***g** 1 ***h** $\frac{9}{10}$ ***i** -1 ***j** 1

Comparing fractions

Activity 3 Compare fractions

Learner's Book page 101

Guidelines for implementing this activity

- Revise Lowest Common Denominator from Chapter 1 briefly before starting.
- Once the LCD has been found, learners then need to convert the fractions to equivalent fractions with the same denominator.
- Work through the worked examples with learners.
- Challenge learners to do the activity as quickly as possible showing all their calculations.

Remedial and extension

Remedial: Make sure that learners can find the LCDs. Provide extra practice exercises finding these. Learners should redo any of the questions that they get wrong in the activity. Encourage them to verbalise their mistakes in order to be aware of them and reduce the risk of repeating these mistakes.
Extension: Give learners a few examples of more challenging comparisons like question **1c** and questions **2c** and **2d**.

Suggested answers

1 **a** $\frac{3}{5} < \frac{2}{3}$ **b** $\frac{7}{5} < \frac{10}{7}$ ***c** $\frac{9}{2} > \frac{11}{4}$

d $\frac{2}{3} < \frac{3}{4}$ **e** $\frac{19}{3} > \frac{19}{4}$

2 Arrange the following in ascending order.

a $\frac{13}{28} < \frac{1}{2} < \frac{4}{7}$ **b** $\frac{2}{3} < \frac{3}{4} < \frac{5}{6}$

***c** $\frac{19}{8} < \frac{8}{3} < \frac{13}{4}$ ***d** $\frac{7}{2} < \frac{18}{5} < \frac{19}{5}$

Mixed numbers and improper fractions

Converting between mixed numbers and improper fractions

Activity 4 **Convert between mixed numbers and improper fractions**

Learner's Book page 102

Guidelines for implementing this activity

- Work through the examples in the Learner's Book with learners.
- This is revision, but should be consolidated.
- The new scientific calculators allow converting fractions to mixed fractions and back. Ask learners to check their answers using their calculators.

Remedial and extension

Remedial: Remind learners about the different parts of a fraction and what they mean. Once they understand this, provide them with more practice in converting.
Extension: Ask learners to do larger numbers and also to check their answers on their calculators.

Suggested answers

1 **a** Learners shade in 9 blocks. **b** 9 **c** $\frac{3}{12}$ or $\frac{1}{4}$

2 **a** $\frac{20}{9}$ **b** $-\frac{41}{7}$ **c** $\frac{8}{3}$
d $\frac{507}{100}$ **e** $\frac{413}{50}$ **f** $-\frac{68}{5}$

3 **a** $1\frac{4}{7}$ **b** $4\frac{3}{5}$ **c** $8\frac{3}{8}$
d $2\frac{7}{65}$ **e** $\frac{251}{1\,000}$ **f** $1\frac{333}{1\,000}$

Simplifying square numbers, square roots, cube numbers and cube roots

Simplifying square numbers

Activity 5 **Simplify square numbers**

Learner's Book page 103

Guidelines for implementing this activity

- Show learners the two ways that squaring fractions can be done.
- Work through the worked examples in the Learner's Book.
- Learners do the activity by themselves.

Remedial and extension

Remedial: Remind learners about the signs when multiplying. Make sure that learners redo any questions that they get wrong in the activity. Encourage them to verbalise their mistakes in order to be aware of them and reduce the risk of repeating these mistakes.
Extension: Ask learners to explain why all the squares are positive.

Suggested answers

1 $\frac{144}{16} = 9$

2 $\frac{16}{25}$

3 $\frac{961}{16} = 60\frac{1}{16}$

4 $\frac{25}{81}$

5 $\frac{169}{16} = 10\frac{9}{16}$

6 $\left(2\frac{2}{3}\right)^2 = \frac{8}{3} = \frac{64}{9}$

7 $\left(\frac{17}{5}\right)^2 = \frac{289}{25}$

8 $\left(\frac{25}{6}\right)^2 = \frac{625}{36}$

9 $(\frac{16}{3})^2 = \frac{256}{9}$

10 $\frac{81}{100}$

*11 $12 \times (\frac{-2}{5})^2 = 12 \times \frac{4}{25} = \frac{48}{25}$

*12 $-11 \times (\frac{5}{7})^2 = -11 \times \frac{35}{49} = \frac{-55}{7} = -7\frac{6}{7}$

Simplifying square roots

Activity 6 Simplify square roots

Learner's Book page 104

Guidelines for implementing this activity

- Explain the two routes that learners can choose when finding square roots as discussed in the Learner's Book.
- Show the same example using the different methods and let learners choose which one is most appropriate for them.
- Learners should do the activity without the use of a calculator.

Remedial and extension

Remedial: Make sure that learners can find the square roots. It is useful for learners to learn the first 12 perfect squares and their roots. It will make an activity like this a lot quicker.
Extension: Challenge learners to do the exercise in less than one minute! Ask them to design another 10-question test and working with a partner, see who can complete the other's test faster.

Suggested answers

1 $\frac{2}{3}$

2 $\frac{4}{2} = 2$

3 $\frac{4}{3}$

4 $\frac{5}{2}$

5 $\frac{12}{10} = \frac{6}{5}$

6 $\frac{10}{25} = \frac{2}{5}$

*7 $-\frac{1}{2}$ or $\frac{1}{2}$

*8 $-\frac{8}{7}$

*9 n/a (Square root of a negative number)

*10 $\sqrt{5}\,\frac{5}{9}$

Simplifying cube numbers; Simplifying cube roots

Activities 7–9 Simplify cube numbers; Simplify cube roots; Calculate squares and square roots, cubes and cube roots

Learner's Book pages 104–105

Guidelines for implementing these activities

- Show the same worked examples using the different methods and let learners choose which one is most appropriate for them in each case.
- Remind learners about the signs remaining the same when cubing numbers and taking square roots.

- Let learners test their skills in Activity 9. They should work on their own.

Remedial and extension

Remedial: It is useful for learners to learn the first ten cubes and their roots.
Extension: Activity 7 questions **5** to **9** and Activity 8 questions **6** to **10** are more challenging. Learners should do all the activities without the use of a calculator.

Suggested answers

Activity 7

1 $\frac{1}{27}$	**2** $\frac{512}{19\ 683}$	**3** $\frac{8}{27}$
4 $\frac{-8}{125}$	**5** $\frac{1\ 331}{8}$	**6** $-\frac{729}{64}$
7 $-\frac{125}{8}$	**8** $-\frac{27}{64}$	**9** $-\frac{512}{27}$

Activity 8

1 $\frac{2}{3}$	**2** $\frac{-2}{4} = \frac{-1}{2}$	**3** $\frac{10}{\sqrt[3]{-9}}$	**4** $\frac{\sqrt[3]{9}}{2}$
5 $\frac{3}{2}$	**6** -1	***7** $-\frac{4}{5}$	***8** $\frac{2}{-3}$
***9** $\frac{3}{5}$	***10** $\frac{1}{-100}$		

Activity 9

1 $\frac{225}{16} = 14\frac{1}{16}$	**2** $1\frac{15}{49}$	**3** $\frac{1}{64}$	**4** $\frac{27}{125}$
5 $\frac{3}{16}$	**6** $-\frac{1}{3}$	**7** $\frac{4}{8} = \frac{1}{2}$	**8** $\frac{5}{9}$
9 $\frac{3}{2}$	**10** $\frac{-2}{4} = -\frac{1}{2}$	**11** $\frac{-1}{10}$	**12** -1

Simplifying squares, cubes, square roots and cube roots of simple algebraic fractions

Simplifying squares and cubes of algebraic fractions
Simplifying square roots and cube roots of algebraic fractions

Activities 10–11 **Simplify squares and cubes of algebraic fractions; Simplify square roots and cube roots of algebraic fractions**

Learner's Book pages 107–108

Guidelines for implementing these activities

- Revise the vocabulary that is used in algebraic expressions.
- Work through the revision examples on the board.
- Make sure that learners understand each step in the worked examples.
- Allow learners to work in pairs for the first three questions in the activities.
- The rest of the activities should be done individually.

Remedial and extension

Remedial: Learners use the expanded notation if they find this challenging. Once they have expanded all terms, they can collect like terms together and multiply out the numbers. Give learners several more examples to consolidate their knowledge.
Extension: Give learners a few more challenging questions such as those in questions **7** to **12**.

Suggested answers

Activity 10

1	$\frac{x^2}{y^2}$	**2**	$\frac{x^2}{y^2}$	**3**	$\frac{16x^4}{25y^6}$	**4**	$\frac{8x^3}{27y^3}$
5	$\frac{a^3}{b^3}$	**6**	$125\frac{p^3}{q^3}$	**7**	$\frac{a^2}{b^2}$	**8**	$-\frac{x^3}{y^3}$
9	$\frac{a^6}{b^6}$	**10**	$\frac{25x^2}{49y^2}$	**11**	$\frac{9x^2}{25y^2}$	**12**	$-8\frac{a^3}{b^3}$

Activity 11

1	$\frac{4p}{5q^2}$	**2**	$\frac{5x^3}{6y^4}$	**3**	$\frac{4a^3}{b^2}$
4	$\frac{8p^4}{b^6}$	**5**	$\frac{4a^2}{b^3}$	**6**	$\frac{3a^2}{4b^{10}}$
7	$\frac{12a^{25}}{3b^2}$	**8**	$\frac{4a^4}{2b^3} = 2\frac{a^4}{b^3}$	**9**	$\frac{5x^5}{4y^4}$

UNIT 2

Equivalent forms

Unit overview

Learner's Book page 109
Recommended pacing: 1,5 hours

This unit focuses on the following:

- Converting common fractions to decimal fractions and percentages
- Converting decimal fractions to common fractions and percentages
- Converting percentage to common fractions and decimal fractions
- Rounding off decimal numbers and percentages.

Resources: Learner's Book; calculator; exercise book

Background information

In Grade 8 learners should have revised the following:

- Common fractions (fractions where one denominator is a multiple of the other)
- Common fraction and decimal fraction forms of the same number

- Common fraction, decimal fraction and percentage forms of the same number.

Teaching guidelines

Many learners struggle with the concepts of fractions, but are quite confident with decimals because of their almost intuitive understanding of money. Percentages are also often practised when learners get their marks for a test or other assessment.

Most of the work done in this unit is revision of the Grade 8 work. Make the links between the three types of fractions very clear at the outset. Let learners practise with a table where they convert from one form to the other two forms. The calculations can be done without a calculator, but it is a good idea to give some that are done on the calculator. This is a skill that will be used in everyday life.

Converting common fractions to decimal fractions and percentages; Converting decimal fractions to common fractions and percentages; Converting percentages to common fractions and decimal fractions

Converting common fractions to decimal fractions; Converting common fractions to percentages; Converting decimal fractions to common fractions; Converting decimal fractions to percentages; Converting percentages to common fractions; Converting percentages to decimal fractions

Activities 1–2 **Write equivalent forms; Problem-solving**

Learner's Book page 112

Guidelines for implementing these activities

- Work through each section and then give learners a chance to practise by doing the appropriate question of the activity.
- Alternating with activity questions will break the revision up a little, because learners might find it difficult to work through all the theory in one sitting without losing focus.
- Learners must do Activity 2 on their own.

Remedial and extension

Remedial: Provide additional practice exercises. Learners can take each of the examples in Activity 1 and convert to both the other forms and not just one. This will provide double the amount of practice.
Extension: Give learners more challenging examples of the types of questions in Activity 2.

Suggested answers

Activity 1

1	**a** 0,08	**b** 0,625	**c** 0,45	**d** 0,28
2	**a** 40%	**b** 85%	**c** 24%	**d** 44%

3	a	$\frac{16}{25}$	b	$\frac{9}{20}$	c	$\frac{27}{25} = 1\frac{2}{25}$	d	$\frac{25}{8} = 3\frac{1}{8}$
4	a	72%	b	3,5%	c	125%	d	137,2%
5	a	$\frac{2}{5}$	b	$\frac{9}{10}$	c	$\frac{13}{20}$	d	$\frac{181}{50} = 3\frac{31}{50}$
6	a	0,07	b	1,89	c	0,008	d	2,5

Activity 2

1 a 60 people for all three because $\frac{1}{5}$ is equivalent to 0,2 and 20%.
  b 30 m for all three because $\frac{3}{4}$ is equivalent to 0,75 and 75%.
  c 210 g for all three because $\frac{21}{30}$ is equivalent to 0,7 and 70%.

2 a Both 57,6 apples because $\frac{3}{5}$ is equivalent to 0,6.
  b Both R28 because $\frac{2}{5}$ is equivalent to 40%.
  c Both are 850 m or 0,85 km because 0,85 and 85% are equivalent.

Rounding off decimal numbers and percentages

Rounding off to the nearest percentage; Rounding off correct to one decimal place; Rounding off correct to two decimal places

Activity 3 Round off numbers

Learner's Book page 114

Guidelines for implementing this activity

- Rounding off is a really important skill for many other subjects.
- It is very useful for estimation which should be done before calculations, as well as approximation done at the end of certain calculations (for example, when an answer is 1, 9 people we round off to 2 people in order to make the answer more meaningful).
- It often seems simple to us as teachers, but we forget that it is still confusing to learners.
- Let learners do the activity on their own.

Remedial and extension

Remedial: Try other ways if learners do not remember the formal method. If they agree that halfway between 30 and 40 is 35 then anything less than 35 must be rounded down to 30 and equal to or more than 35 to 40. Similarly if 150 is the number halfway between 100 and 200, then anything smaller must be rounded down to 100 and anything equal or bigger must be rounded up.
Extension: Ask learners to round off a number with seven random digits after the comma to one decimal place, then two, three, and so on. Ask learners to find the difference between the rounded numbers. Is there a pattern? There is also an extension challenge at the end of Activity 2 for learners who complete the exercises quickly.

Suggested answers

Activity 3

1	a	44,0%	b	76,7%	c	63,6%
2	4					
3	3,85					
4	2-hundredths					
5	a	48,4	b	48,351		
6	a	13 600	b	4 060		

*Extension: Challenge yourself

1 8,301

2 Learners shade in $\frac{6}{8}$

3 12 metres

4 $\frac{13}{52} \times 100 = 25\%$

UNIT 3

Calculations with fractions

Unit overview

Learner's Book page 116
Recommended pacing: 1,5 hours

This unit focuses on the following:
- Adding and subtracting fractions with the same denominator
- Adding and subtracting fractions with different denominators
- Adding and subtracting algebraic fractions with the same denominators
- Adding and subtracting algebraic fractions with different denominators
- Adding and subtracting algebraic fractions with numerators that have two terms
- Multiplying and dividing fractions.
- Multiplying and dividing algebraic fractions

Resources: Learner's Book; calculator; exercise book

Background information

In Grade 8, learners revised the following:
- Addition and subtraction of common fractions, including mixed numbers
- Finding fractions of whole numbers
- Multiplication of common fractions, including mixed numbers.

In Grade 8, learners were introduced to the following:
- Division of whole numbers and common fractions by common fractions
- Calculations with the squares, cubes, square roots and cube roots of common fractions.

Teaching guidelines

This section is revision of previous work done with fractions. It is really necessary to ensure that learners have consolidated their understanding of fractions before moving on to algebraic fractions.
The Think-Do examples assist in crystallising exactly what is happening at each step. If learners do these steps once they start the algebra, they will find it a lot easier to transfer their numeric knowledge of fractions to algebra.
However, once learners start the algebraic fractions and they are not sure that what they are seeing is true, they can substitute numbers into the expression to validate it, for example; if learners substitute $a = 4$ and $b = 3$ into the equation, they will find that the LHS is equal to the right hand side. $\frac{1}{a} + \frac{1}{b} = \frac{(a + b)}{ab}$.

Adding and subtracting fractions; Adding and subtracting fractions with different denominators

Adding and subtracting fractions with the same denominator; Adding fractions with different denominators; Subtracting fractions with different denominators

Activities 1–2 **Add and subtract fractions; Add and subtract fractions**

Learner's Book pages 117–118

Guidelines for implementing these activities

- Use the instructions in the Think-Do examples to take learners through the basic steps again.
- The first activity and the examples before it deal with denominators that are the same, so the numerators just have to be added or subtracted.
- In the second activity, learners find the LCD, equivalent fractions and then add or subtract.
- Let learners work through the activities by themselves because this is revision and they should check how well they are doing before proceeding with algebraic equations.

Remedial and extension

Remedial: Make sure that learners understand every step in the Think-Do examples. Provide additional exercises if necessary.
Extension: Learners fractions with bigger numbers and challenge them not to use their calculators to find the LCDs.

Suggested answers

Activity 1

1	a	$\frac{5}{8}$	b	$\frac{8}{11}$	c	$\frac{3}{17}$	d	$\frac{7}{23}$
2	a	$\frac{26}{12} = \frac{13}{6} = 2\frac{1}{6}$	b	$\frac{19}{9} = 2\frac{1}{9}$	c	$\frac{1}{3}$	d	$\frac{25}{7} = 3\frac{4}{7}$

Activity 2

1	a $\frac{37}{20} = 1\frac{17}{20}$	b $\frac{18}{35}$	c $\frac{2}{45}$
	d $\frac{1}{3}$	e $\frac{1}{6}$	f $\frac{1}{20}$
2	a $\frac{21}{10} = 2\frac{1}{10}$	b $\frac{31}{12} = 2\frac{7}{12}$	c $\frac{31}{15} = 2\frac{1}{15}$
	d $\frac{19}{12} = 1\frac{7}{12}$	e $\frac{133}{30} = 4\frac{13}{30}$	f $\frac{182}{55} = 3\frac{17}{55}$

3 He had $\frac{3}{10}$ left of his pocket money.

Adding and subtracting algebraic fractions

Simplifying algebraic fractions; Adding and subtracting algebraic fractions

Activities 3–4 **Simplify algebraic fractions; Add and subtract algebraic fractions**

Learner's Book pages 119–121

Guidelines for implementing these activities

- Make sure that learners remember all the vocabulary that we use for algebraic expressions.
- Work through the examples with the class.
- Learners do the activities on their own.

Remedial and extension

Remedial: Work through the steps of the examples again and make sure learners understand each step. Ask them to write the steps next to the first three simplifications that they do so that they get to know them. Let learners redo any exercises that they get wrong, and verbalise their mistakes in order to become more aware of these and reduce the risk of repeating them.
Extension: Ask learners to do Activity 4 as quickly as possible and then to move on to Activity 5. Give learners a few more complicated simplifications such as questions **9** to **14** of Activity 5.

Suggested answers

Activity 3

1 x	2 $-q$	3 1	4 x
5 5	6 $2p$	7 $-2s$	8 $\frac{1}{4}$
9 $-\frac{1}{5}$	10 $\frac{1}{9}$		

Activity 4

1 $\frac{4a}{5}$	2 $\frac{5x}{9}$	3 $\frac{x}{8}$	4 $\frac{4m}{12} = \frac{m}{3}$

5 $\frac{29x}{14}$ 6 $\frac{11a}{12}$ 7 0 8 $\frac{29x}{10}$

9 $\frac{8z}{11}$ 10 $\frac{28ab}{15}$ 11 0 12 $\frac{xy}{12}$

13 $-\frac{10p}{18} = -\frac{5p}{9}$ 14 $\frac{(11a + 5b)}{10}$

Adding and subtracting algebraic fractions with numerators that have two terms

*Activities 5–6 Work with more complex fractions; Add and subtract algebraic fractions with numerators with that have two terms

Learner's Book pages 122–123

Guidelines for implementing these activities

- Learners encounter more complex fractions with variables in the denominator as well as in the numerator.
- Show learners that the principles are still the same.
- Learners do Activities 5 and 6.

Remedial and extension

Remedial: Give learners simpler fractions at first and make sure that they have understood the worked examples. Once they are more confident, let them work through the activities. Make sure that they redo any simplifications that they do not get right, and verbalise their mistakes in order to become more aware of these and reduce the risk of repeating them.
Extension: Both Activity 5 and Activity 6 are more challenging.

Suggested answers

Activity 5

1 $\frac{2 + 3x}{6x}$ 2 $\frac{6 + y}{8y}$ 3 $\frac{7 + 12x}{21}$ 4 $\frac{55 - 14a}{22a}$

5 $\frac{16 - 49b}{28b}$ 6 $\frac{27 - 10a}{30a}$

Activity 6

1 $\frac{2(2x - 1) - (x - 1)}{6} = \frac{3x - 1}{6}$ 2 $\frac{2(a + 1) + (3a - 2)}{14} = \frac{5a}{14}$

3 $\frac{2(4 + y) + 9(3 + 2y)}{18} = \frac{35 + 20y}{18}$ 4 $\frac{8b + 4c - 15b - 9c}{12} = \frac{-7b - 5c}{12} = -\frac{7b + 5c}{12}$

5 $\frac{2y - 4x + 4x + 5y}{4} = \frac{7y}{4}$ 6 $\frac{3(6x + 2) + 4(3x - 2)}{12} = \frac{30x - 2}{12}$

Multiplying and dividing fractions

Multiplying fractions; Dividing fractions

Activities 7–8 Multiply fractions; Divide fractions

Learner's Book pages 124–125

Guidelines for implementing these activities

- Once multiplication of fractions has been revised, remind learners that division by the fraction $\frac{a}{b}$ is the same as multiplying by the reciprocal, $\frac{b}{a}$.
- Work through the worked example with the class.
- Learners do the activities by themselves.

Remedial and extension

Remedial: Give learners more practice with multiplication and division. Make sure that they understand every step in the Think-Do examples.
Extension: Activity 7 questions **8** to **10** are more challenging because of the size of the numbers involved. Ask learners to do the exercises as quickly as possible and without the use of a calculator. They should also design another five questions with solutions and swap them with a partner.

Suggested answers

Activity 7

1 $\frac{1}{2}$	2 $\frac{1}{2}$	3 $\frac{3}{40}$	4 $\frac{119}{15} = 7\frac{14}{15}$
5 $\frac{110}{21} = 5\frac{5}{21}$	6 $\frac{336}{25} = 13\frac{11}{25}$	7 $\frac{793}{125} = 6\frac{43}{125}$	8 $\frac{893}{12} = 74\frac{5}{12}$
9 $\frac{119}{12} = 9\frac{11}{12}$	10 $\frac{9\,251}{432} = 21\frac{179}{432}$		

Activity 8

1 $\frac{8}{15}$	2 $\frac{49}{8} = 6\frac{1}{8}$	3 $\frac{77}{40} = 1\frac{37}{40}$	4 $\frac{25}{64}$
5 $\frac{27}{28}$	6 2	7 $\frac{27}{8} = 3\frac{3}{8}$	8 $\frac{ad}{bc}$

Multiplying and dividing algebraic fractions; Problem-solving

Activities 9–10 Multiply and divide algebraic fractions; Solve problems with fractions

Learner's Book page 127

Guidelines for implementing these activities

- Use question **8** of Activity 8 as the opener for discussing multiplication.

- Substitute numbers into the expression so that learners can see that it confirms what they have been doing with numbers.
- Work through the Think-Do worked examples carefully.
- Learners do Activities 9 and 10.

Remedial and extension

Remedial: Quite often learners do not realise that the variables that we work with in Algebra actually represent numbers. Do several similar numerical fraction multiplications and ask learners to substitute one, then two, three variables for the numbers and to build up an algebraic rule for what they have been doing numerically. Once they have developed the rule, they can use it for their other simplifications.
Extension: Ask the learners to develop algebraic rules for more complex multiplications and division.

Suggested answers

Activity 9

1 $\frac{p}{r}$ 2 1 3 $\frac{x}{y^2}$

4 $\frac{b}{2a}$ 5 $\frac{2s}{3}$ 6 $\frac{(x+3)}{(x+1)}$

7 $\frac{(x+4)}{x(x-2)}$ 8 $x(x-5)$ 9 $(x+2)$

Activity 10

1 a $\frac{1}{4} + \frac{1}{5} = \frac{9}{20} \rightarrow 1 - \frac{9}{20} = \frac{11}{20}$. Eleven-twentieths of the class play rugby.

b 11 learners c Nine learners

2 $\frac{3}{8} \times 592 = 222$ kg

3

Colour	Common Fraction	Decimal fraction	Percentage
Red	$\frac{4}{25}$	0,16	16%
Blue	$\frac{1}{5}$	0,2	20%
Black	$\frac{1}{10}$	0,1	10%
Green	$\frac{7}{50}$	0,14	14%
Orange	$\frac{9}{50}$	0,18	18%
Purple	$\frac{11}{50}$	0,22	22%

Chapter 3 Revision

Learner's Book page 129

Encourage learners to review the content covered before attempting the revision activities. The revision activities should be used to assess learners' progress thus far, and to assess where remediation may be required.

Suggested answers

1 $\frac{1}{3}$ **2** $5\frac{1}{4}$ **3** $\frac{31}{5}$ **4** $\frac{4}{7} < \frac{5}{8} < \frac{9}{14}$ **5** $\frac{85}{68}$

6 **a** 24% **b** 116%

7 $54 - 36 = 18$

$\frac{18}{54} \times 100 = 33{,}3\%$

8 **a** 449,46 **b** 10 000

9 **a** $\frac{19x}{10}$ **b** $\frac{-7a}{20}$ **c** $\frac{15}{4x}$ **d** $\frac{11}{2c}$

10 **a** $\frac{1}{5}$ **b** $\frac{2}{x}$ **c** $\frac{2a}{d}$

CHAPTER

4 Decimal fractions

Chapter overview

Learner's Book pages 130 to 144
Recommended pacing: 4,5 hours

This chapter focuses on the following:

Unit 1:	**Revision**	40 minutes
	Revision: Decimal fractions	
Unit 2:	**Calculations with decimal fractions**	2 hours 20 minutes
	Addition and subtraction	
	Multiplication and division	
Unit 3:	**Equivalent forms for decimal fractions**	1,5 hours
	Decimal numbers and common fractions	
	Further operations	
Chapter 4 Revision		35 minutes

UNIT

1 Revision

Unit overview

Learner's Book page 131
Recommended pacing: 40 minutes

This unit focuses on the following:

- Place value, comparing and ordering decimal fractions
- Rounding off and estimation
- Measurements and estimation
- Use estimation and a calculator.

Resources: Learner's Book; calculator; exercise book

Background information

In Grade 8, learners revised the following:

- Ordering, comparing and place value of decimal fractions to at least 3 decimal places

- Rounding off decimal fractions to at least two decimal places
- Learners used calculation techniques:
  - Using knowledge of place value to estimate the number of decimal places in the result before performing calculations
  - Using rounding off and a calculator to check results where appropriate

Teaching guidelines

Emphasise that decimals are fractions with a different notation from common fractions. The denominators for decimals are powers of 10. The convenient thing about decimals is that money uses a decimal system and so we already have a large amount of knowledge about doing the operations based on the two decimal places of money.
The other form of fractions is the percentage form where the denominator is 100. It is very useful in many applications especially financial mathematics. Decimal fractions are much easier to use to compare the size of different fractions.
Calculators make their use very simple, but it is very important to be able to estimate before using the calculator so that answers obtained from calculator are more meaningful.

Revision: Decimal fractions

Place value, comparing and ordering decimal fractions

Activity 1 **Revise decimal fractions**

Learner's Book page 132

Guidelines for implementing this activity

- Work through the notes in the Learner's Book with the class.
- Take a piece of string measuring say, 2 600 mm and ask learners how they would divide it into three equal pieces. (Divide the total measurement by 3). Explain that this is an example of why it is sometimes necessary to have approximate answers.
- Revise the thousands, hundreds, tens, units, tenths, hundredths and thousandths columns with the class before they complete this activity.

Remedial and extension

Remedial: Use money as a context for learners who find decimal fractions challenging. Most people can cope with money and are able to do all the operations with money. Extend the concept to thousandths using the place-value tables.
Extension: Because this is revision, learners should find it quite straightforward. Challenge them to do the exercise as quickly as possible without the use of a calculator. They can check their answers once they have completed the whole activity.

Suggested answers

1

		Tth	T	H	T	O	,	t	h	th
e.g.	4 474,921			4	4	7	,	9	2	1
a	325,226			3	2	5	,	2	2	6
b	21,589				2	1	,	5	8	9
c	1 452,023		1	4	5	2	,	0	2	3
d	98 326,325	9	8	3	2	6	,	3	2	5
e	12 256,235	1	2	2	5	6	,	2	3	5

2 **a** 21, 589 **b** 98 326,325 **c** 325,226 **d** 12 256,235
e 21,589 **f** 447,921 **g** 21,589

3 **a** < **b** > **c** < d > e < f >

4 **a** 9,322; 9,223; 8,700; 8,007; 5,002; 2,339
b 2,980; 2,890; 2,098; 2,090; 2,089; 2,080; 2,009

5 **a** 21,589; 325,226; 447,921
b 1 452,823; 12 256,235; 98 326,325

6 **a** fifty **b** five **c** five thousand
d five million **e** five-tenths **f** five ten-thousandths

Rounding off and estimation

Activity 2 Round off decimal fractions

Learner's Book page 134

Guidelines for implementing this activity

- Remind the learners how to round off numbers to a certain number of decimal places.
- Work through the notes in the Learner's Book with the class.
- Learners do this activity on their own.

Remedial and extension

Remedial: Work through the examples in the Learner's Book again. If learners cannot remember the method for rounding off, ask them what half the distance is between the two hundredths for example, and if the number is less than the halfway mark it must be rounded down. If it is equal to the halfway mark or larger, it should be rounded up.

Suggested answers

1 **a** 6,5 **b** 48,5 **c** 9,1 **d** 0,3
2 **a** 3,85 **b** 19,63 **c** 0,51 **d** 71,46
3 **a** 4,684 **b** 0,183 **c** 72,561 **d** 0,054
4 **a** 38,3 **b** 38,27 **c** 38,268
5 **a** 17 **b** 82 **c** 237 **d** 106

Activity 3 Estimate

Learner's Book pages 134–135

Guidelines for implementing this activity

- Work through the notes in the Learner's Book. Discuss examples with the class about what the smallest value is that we can have so as to round down and similarly, what the largest value is that we can have in order to round up.
- Learners do this activity on their own.

Remedial and extension

Remedial: Give learners a few more examples so that they can work out the pattern.
Extension: Challenge learners to investigate percentage error in measurements.

Suggested answers

1 a 75   b 84
2 a 350   b 449

Use estimation and a calculator

Activity 4 Estimate

Learner's Book page 135

Guidelines for implementing this activity

- Work through the notes in the Learner's Book. Discuss examples of what would or would not be a good estimate for a number. Write up a few examples on the board and ask learners to give the best estimate for each.
- Learners do this activity on their own.

Remedial and extension

Remedial: Revise rounding off and estimation work from previous grades.
Extension: Add variables to some of the decimal questions and remind learners to only add/subtract like terms but to multiply any terms. Give learners some more challenging questions on estimation.

Suggested answers

1 5 times 6 = 30; Actual = 30,4776
2 18 + 22 = 40; Actual = 39,8
3 $\frac{(6 \times 3)}{9} = 2$; Actual = 2,029
4 35 – 10 = 25; Actual = 25, 02
5 $\frac{(33 \times 5)}{3} = 55$; Actual 50,96
6 $\frac{7 \times 3}{4} = 5{,}25$; Actual = 4,65

UNIT

2 Calculations with decimal fractions

Unit overview

Learner's Book page 136
Recommended pacing: 2 hours 20 minutes

This unit focuses on the following:
- Completing multiple operations with decimals
- Using a calculator where appropriate
- Solving problems in context involving decimal places.

Resources: Learner's Book; calculator; exercise book

Background information

In Grade 8 learners revised the following:
- Addition, subtraction, multiplication and of decimal fractions to at least 3 decimal places
- Division of decimal fractions by whole numbers

Learners were introduced to the following:
- Extending multiplication to 'multiplication by decimal fractions' not limited to one decimal place
- Extending division to 'division of decimal fractions by decimal fractions'
- Calculating the squares, cubes, square roots and cube roots of decimal fractions.

Teaching guidelines

The decimal form is very much easier than fractions. It follows the same place-value system as the whole numbers do. You can therefore add them as you do with whole numbers. Encourage learners to use the place-value system until they are very confident about all the operations with decimal fractions and decimal numbers.

Addition and subtraction

Mental addition and subtraction

Activity 1 **Add and subtract decimal fractions mentally**

Learner's Book page 136

Guidelines for implementing this activity

- Work through the notes in the Learner's Book with the class.
- Show the learners a number of different ways to break down the numbers, for example 4,8 can be 4,2 + 0,6. 4,2 will easily add to 5,8 to give 10. This makes it easier to then add 0,6.
- Learners do this activity on their own.

Remedial and extension

Remedial: Encourage learners to think of the decimal values as money. If there is only one place after the comma, they should add a zero and imagine they are adding or subtracting money.
Extension: Give learners a few examples with algebraic variables and larger decimals to challenge them.

Suggested answers

1 **a** 0,6 **b** 0,69 **c** 2,7 **d** 1,45

2

+	**2,9**	**4,8**	**3,9**	**2,5**	**5,9**	5,7
3,1	6	7,9	7	5,6	9	8,8
6,5	9,4	**11,3**	10,4	9	12,4	**12,2**
8,2	11,1	13	12,1	10,7	14,1	13,9
4,9	7,8	9,7	8,8	7,4	10,8	10,6
7,1	10	11,9	11	9,6	13	12,8
9,6	12,5	14,4	13,5	12,1	15,5	15,3

3 **a**

25
12,2 | 12,3
5,3 | 6,9 | 5,4
3,2 | 2,1 | 4,8 | 0,6

b

18,7
8,7 | 10
5,2 | 3,5 | 6,5
4,6 | 0,6 | 2,9 | 2,6

Addition and subtraction in columns

Activity 2 Add and subtract decimal numbers

Learner's Book page 138

Guidelines for implementing this activity

- Remind learners how to add and subtract decimals in columns and to line up the digits in the correct columns.
- Work through the notes in the Learner's Book.
- Learners do this activity on their own.

Remedial and extension

Remedial: Give leaners squared paper to use or ask them to draw parallel lines on their pages to make columns. They should line the commas of the numbers up first and then the other digits position in the correct place-value columns.
Extension: Give learners a few examples of much larger decimal numbers to add in columns. Make sure that they estimate their answers before starting to calculate.

Suggested answers

1	a	42,1	b	27,9	c	141,99	d	18,68
	e	22,08	f	47,34	g	53,47		
2	84,73 kg							

Multiplication and division

Mental multiplication; Mental division

Activity 3 Multiply and divide decimal numbers

Learner's Book page 139

Guidelines for implementing this activity

- Work through the notes in the Learner's Book.
- Let learners explain how they do these exercises mentally. This will give other learners a bigger range of choices when they do the calculations themselves.
- Learners do this activity on their own.

Remedial and extension

Remedial: Ask learners to imagine that they are multiplying or dividing money. They can add on zeros to the number if there is only one digit after the decimal comma. Give learners more examples and allow them to write a certain amount down, but to try and do as much as possible mentally.
Extension: Give learners larger decimal numbers to multiply. Ask them to write their estimates down first and then to calculate (mentally) the difference between their estimates and their actual answers.

Suggested answers

1	460	2	17,6	3	716,1	4	96
5	15,3	6	14	7	30	8	6

Multiplication and division methods

Activity 4 Multiply and divide decimal numbers

Learner's Book page 140

Guidelines for implementing this activity

- Revise how to multiply and divide decimals in columns and to ensure that they line up the digits in the correct columns.
- Remind learners that if we multiply a numerator by a number, we must do the same to the denominator.
- Work through the notes in the Learner's Book.
- Learners do this activity on their own.

Remedial and extension

Remedial: Revise long multiplication and division. Remind learners to line up digits in columns when adding and subtracting.
Extension: Add variables to some of the decimal numbers and then remind learners only to add/ subtract like terms but to multiply any terms together.

Suggested answers

1 a 649,7 b 38,4 c 27,68
d 59 e 13,178 f 8,391
2 $32 \times 73{,}2 = 2\ 342{,}4$ seconds = 39,04 minutes = 0 hours 39 minutes and 2,4 seconds.

UNIT
3 Equivalent forms for decimal fractions

Unit overview

Learner's Book page 141
Recommended pacing: 1,5 hours

This unit focuses on the following:
- Decimal numbers and common fractions
- Further operations.

Resources: Learner's Book; calculator; exercise book

Background information

In Grade 8, learners revised equivalent forms between:
- common fraction and decimal fraction forms of the same number
- common fraction, decimal fraction and percentage forms of the same number

Teaching guidelines

Remind learners that decimal fractions are fractions with denominators that are powers of 10, that is, 10; 100; 1 000; and so on. It is easy enough to convert them first to a fraction with one of these denominators and then to simplify them if there are common factors. Converting a decimal to a fraction requires that learners multiply the decimal by 100.

Decimal numbers and common fractions

Activity 1 Work with equivalent forms

Learner's Book page 142

Guidelines for implementing this activity

- Before starting to convert a decimal fraction to a common fraction, learners should always check whether the decimal fraction is a rational number.
- Does the decimal end or recur? If it does it is rational and can be converted to a common fraction of the same value.
- Work through the examples in the Learner's Book.
- Learners should do the activity on their own.

Remedial and extension

Remedial: Make sure that learners understand the steps in the examples given in the Learner's Book. Give learners a few more examples and ask them to write the steps out in words as well as do the calculation. If they struggle to do this, write part of the instruction and ask them to complete it.
Extension: Give learners a few examples with recurring numbers. They should revise the work done in the earlier units.

Suggested answers

1	a	$\frac{4\,831}{1\,000} = 4\frac{831}{1\,000}$	b	$\frac{1\,137}{50} = 22\frac{37}{50}$	c	$11\frac{9}{100}$	d	$268\frac{3}{1\,000}$
2	a	1,116	b	462,648	c	39,13	d	0,166
3	a	0,666…	b	0,1666…	c	0,555…	d	0,375

Further operations

Activity 2 Work with decimal numbers

Learner's Book page 143

Guidelines for implementing this activity

- Work through the examples in the Learner's Book.
- Learners may need a few extra examples so it is useful to have some ready.
- Let learners do the activity on their own and without a calculator.
- Once they have completed the activity, they should check their answers on the calculator. This will give them practice at using their calculator for this type of calculation too.

Remedial and extension

Remedial: Make sure that learners redo any of the questions that they may get wrong. Provide opportunities for extra practice if necessary.
Extension: Question **1h** is more challenging. Give learners a few more questions like this.

Suggested answers

1 a $\sqrt{\frac{1}{100}} = 0{,}1$
b $0{,}2 \times 0{,}2 \times 0{,}2 = 0{,}008$
c $3{,}2 \times 3{,}2 = 10{,}24$
d 15
e $\sqrt{\frac{144}{25}} = \frac{12}{5} = 2{,}4$
f $-\frac{833}{500} = -1\frac{333}{500} = -1{,}666$
g $1{,}5 + 2{,}6 = 4{,}1$
h $\frac{17^2}{10^2} + \frac{14^2}{10^2} + \sqrt[3]{\frac{216}{1\,000}} = \frac{289}{100} + \frac{196}{100} + \frac{6}{10} = \frac{289 + 196 + 60}{100} = \frac{545}{100} = \frac{109}{20} = 5\frac{9}{20} = 5{,}45$

2 a $2{,}3^2(-5{,}75^3 + \sqrt{1{,}44}) = -999{,}33$
b $2{,}3^2(-5{,}75^3 - \sqrt{1{,}44}) = -1\,012{,}03$
c $2{,}3^2 \times -5{,}75^3 + 2{,}32 \times \sqrt{1{,}44} = -999{,}33$
d $(2{,}3^2 + -5{,}75^3)(\sqrt{1{,}44} - \sqrt[3]{1{,}25}) = -420{,}87$
e $(2{,}3^2 - -5{,}75^3)(\sqrt{1{,}44} + \sqrt[3]{1{,}25}) = 444{,}97$

Chapter 4 Revision

Learner's Book page 144

Encourage learners to review the content covered before attempting the revision activities. The revision activities should be used to assess learners' progress thus far, and to assess where remediation may be required.

Suggested answers

1 a two units b two hundredths c two thousandths
2 a 8 463 b 8 500
3 a 38,27 b 38,3
4 162,5 cm to 163,4 cm
5 $\approx [10 + (9 - 0{,}5)]^2 \approx [10 + 9]^2 \approx [19]^2 \approx 361$ [Actual answer = 344,47]
6 (Learners may use other mental maths methods).
a $16{,}2 - 1{,}2 - 0{,}7 = 15{,}0 - 0{,}7 = 14{,}3$
b $7{,}2 \times 10 + 7{,}2 = 72{,}0 + 7{,}2 = 79{,}2$
c 51,1
7 $298{,}17 - 76{,}37 =$ R221,80
8 $2{,}32 + 1{,}8 + 1{,}75 = 5{,}87$ m
9 $7{,}5 \times 12 = 90$ kg
10 $4\,250 \div 4 =$ R1 062,50

CHAPTER

5 Exponents

Chapter overview

Learner's Book pages 145 to 167
Recommended pacing: 5 hours

This chapter focuses on:

Unit 1: Revision: Comparing and representing numbers in exponential form	1 hour
Expanding numbers in exponential form	
Writing numbers in exponential form	
Simplifying numbers in exponential form	
Unit 2: Scientific notation	1,5 hours
Writing large numbers in scientific notation	
Writing small decimal numbers in exponential notation	
Converting scientific notation to normal notation	
Using a calculator	
Unit 3: Calculations with numbers in exponential form	1,5 hours
Multiplying powers with the same base (Law 1)	
Dividing powers with the same base (Law 2)	
Raising a power to a power (Law 3)	
Finding a power of a product in brackets (Law 4)	
Unit 4: Exponents in problem-solving	1 hour
Solving simple exponential equations	
POA Investigation 2: Powers of 2: Paper Folding	
POA Assignment 2: Powers of 2: Calculate a target	
POA Assignment 3: Consecutive numbers	
Chapter 5 Revision	40 minutes

UNIT 1

Revision: Comparing and representing numbers in exponential form

Unit overview

Learner's Book page 146
Recommended pacing: 1 hour

This unit focuses on the following:
- Expanding numbers in exponential form
- Writing numbers in exponential form
- Simplifying numbers in exponential form

Resources: Learner's Book; calculator; exercise book

Background information

In Grade 8 learners did the following:
- Revised compared and represented whole numbers in exponential form
- Compared and represented integers in exponential form
- Compared and represented numbers in scientific notation, limited to positive exponents

Teaching guidelines

This unit is revision of the work done on exponents in the previous grades. It is best to make sure that the concepts are well consolidated numerically before moving onto algebra. Remind learners that multiplication is a short form of repeated addition, and that the exponential form is a short form of writing repeated multiplications of the same base. Writing in exponential and expanded form is a useful way of seeing how the exponent laws work in Unit 3 of this chapter. It might be useful to revise the two forms again before starting with exponent laws.

Expanding numbers in exponential form

Activity 1 **Expand numbers in exponential form**

Learner's Book page 146

Guidelines for implementing this activity

- Ensure that learners do not use any shortcuts. They must expand all the questions as this will then help them understand that when multiplying variables you count how many variables there are and then this number becomes the exponent.
- When adding, only change the coefficient. The variable does not change.
- Explain that a number can sometimes be treated as a base.
- Work through the examples in the Learner's Book and then let learners complete the activity in their exercise books.

Remedial and extension

Remedial: Provide learners with several examples to practise writing in the expanded form. Make sure that they understand that if an exponent is not present above a variable, there is an implied 1 and that that variable needs to be listed in the expanded form.
Extension: Provide learners with more challenging examples such as Activity 1 question **6**. Also give them some examples with 0 and negative numbers as exponents.

Suggested answers

1 $7 \times 7 \times 7$
2 $d \times d \times d \times d \times d \times d$
3 $b \times b \times b \times b \times c \times c$
4 $5 \times x \times x \times x$
5 $6 \times x \times x \times x \times x \times y \times y$
6 $5 \times 5 \times 5 \times 5 \times 5 \times 5$

Writing numbers in exponential form

Activity 2 Write numbers in exponential form

Learner's Book page 148

Guidelines for implementing this activity

- Work through the examples in the Learner's Book.
- Make sure that learners understand how to write expressions in exponential form even though this is revision of work done in Grade 8.
- Emphasize that the number of times that a number or variable it multiplied by itself is represented in the exponent.
- Learners do the activity on their own as quickly as possible.

Remedial and extension

Remedial: Remind learners that if a factor only appears once it has an exponent of 1, but we do not write the 1 although it is implied. Variables next to each other without spaces imply multiplication. Provide learners with additional examples until they are confident with the concept and can do the exercise by themselves.
Extension: Provide learners with more examples such as questions **4d** to **4f**.

Suggested answers

1 **a** x^4y^2
b x to the power of 4 multiplied by y squared (to the power of 2).
c x and y
d 4 and 2

2 **a** -6^4 **b** b^3c^2 **c** a^9

3 **a** $3r^2p^2$ **b** $15a^2b^2$ **c** $24abxy$
d $24axy^2$ **e** $-3x^2y$ **f** $-8px^3$

4

	Question	Coefficient	Base	Exponent
a	$5b^5$	5	b	5
b	$-8r^{12}$	−8	r	12
c	$2(xyz)^2$	2	xyz	2
d*	$4y$	4	y	1
e*	$-y^3$	−1	y	3
f*	$9p^4r^2$	9	p and r	4 and 2

Simplifying numbers in exponential form

Activity 3 Simplify numbers in exponential form

Learner's Book page 150

Guidelines for implementing this activity

- Work through the examples in the Learner's Book.
- When simplifying fractions ensure that learners expand all the exponents and cancel out (simplify) as shown in the example. Learners must not move the denominator to the top. The denominator and numerator remain in their positions.
- Learners must also understand that if everything cancels in the numerator there is a 1 in the numerator.
- Work through the summary with the class as a form of revision, or let learners work through the summary in pairs for 5 minutes and allow pairs to volunteer sharing orally their own understanding of each point in the summary (and give examples where appropriate).

Remedial and extension

Remedial: Make sure that learners expand the factors completely and then simplify. In the process, they will be reminded of the exponent rules of multiplying the same base (add the exponents).
Extension: Provide learners with a few examples with more mixed coefficients and variables.

Suggested answers

1 a $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 = 5^{10}$
b $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^{12}$
c $y \times y \times y \times y \times y \times y \times y \times y \times y \times y \times y \times y \times y = y^{13}$
d $a \times a \times a \times a \times a \times a \times a \times a \times a = a^9$
e $x \times x \times x \times x \times x \times x = x^6$
f $b \times b \times b \times b = b^4$
g $4 \times 4 \times 3 \times 3 \times 4 \times 4 \times 4 \times 3 \times 3 \times 3 \times 3 = 4^5 \times 3^6$
h $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 2^{10} \times 3^3$
i $p \times p \times q \times q \times q \times p \times p \times p \times p \times p \times q \times q = p^7q^5$
j s^7t^4

2 a True b False 7^5 c False x^3 d False x^8

3 a $\frac{3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3}{3 \times 3 \times 3 \times 3 \times 3} = 3^2$ b $\frac{7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7}{7 \times 7 \times 7 \times 7} = 7^4$

c $\frac{x \times x \times x \times x \times x \times x \times x \times x \times x}{x \times x} = x^7$ d $\frac{y \times y \times y \times y}{y} = y^3$

e $\frac{y \times y \times y}{y \times y \times y} = 1$ f $\frac{p \times p \times q \times q \times q \times q \times q}{p \times q \times q \times q} = pq^2$

g $\frac{s \times s \times s \times s \times s \times s \times s \times s \times t \times t \times t \times t}{s \times s \times s \times t \times t \times t \times t} = s^5$ h $\frac{x \times x \times x}{x \times x \times x \times x \times x \times x} = \frac{1}{x^3}$

UNIT 2

Scientific notation

Unit overview

Learner's Book page 151
Recommended pacing: 1 hour 30 minutes

This unit focuses on the following:

- Writing large numbers in scientific notation
- Writing small decimal numbers in scientific notation
- Converting scientific notation to normal notation
- Using a calculator.

Resources: Learner's Book; calculator; exercise book

Background information

- In Grade 8 learners compared and represented numbers in scientific notation, limited to positive exponents.
- In Grade 9, learners extend scientific notation to include negative exponents.

Teaching guidelines

Scientific notation is a useful tool for writing very large and very small numbers. It is also used in other subjects such as Physical Science and the Life Sciences. Scientific notation can describe numbers from the infinitesimal to the almost infinite. Learners are always very fascinated with actual numbers such as the circumference of the earth or the number of people in the world. Use the Internet to find a few numbers that will interest your particular class.
Use the Think-Do examples to ensure that learners learn each step of the process before tackling the activities.

Writing large numbers in scientific notation; Writing small decimal numbers in scientific notation

Activities 1–2 Identify and write large numbers in scientific notation; Identify and write small decimal numbers in scientific notation

Learner's Book pages 153–154

Guidelines for implementing these activities

- Work through the introduction and the steps in the Learner's Book.
- Emphasise that the decimal number in the scientific notation should always be between 1 and 10.
- Ensure that learners understand each step, and repeat the steps if necessary. Once they understand the steps the Think-Do examples will be a little easier to work through. Learners can even be encouraged to work through the Think-Do examples independently or in pairs.
- It is important to ensure that learners understand that the digit 3 moves seven decimal places to the right, *not the comma*.
- Before working through the examples in the Learner's Book ask the following: Is this number written in scientific notation (for example, 300 000 000 m/s)? How can we write it in scientific notation? Let learners close their textbooks and attempt doing the first example, and one or two additional examples in their exercise books and volunteer their answers. Allow them to verbalise what they did, and ensure that they are using the correct vocabulary (for example the digit shifts, not the comma). Let them demonstrate on the board the decimal place change. Go through example 2 with them as a means of consolidating the concept.
- Work through Example 3 and allow learners to do additional examples such as Example 3 in their exercise books before doing the activity.
- Work through the steps and examples for Activity 2 in the same way. Provide similar, additional examples where rounding off is required.

Remedial and extension

Remedial: Assist learners with the place value change when writing numbers in scientific notation. An easy way to convert very small numbers is to count the number of zeros including the zeros to the left of the comma. This becomes the negative exponent.
Extension: Challenge learners to find four interesting very large numbers and four interesting infinitesimal numbers. They should write these in scientific notation.

Suggested answers

Activity 1

1 **a** Yes **b** No **c** No

2 The missing numbers are:

10^0	10^1	10^2	10^3	10^4	10^5	10^6
1	10	100	1 000	10 000	100 000	1 000 000

3 a $5,90 \times 107$ (Make sure that the zero is also written to 2 dp).
b $2,90 \times 10^{6}$ c $1,28 \times 10^{4}$ d $2,00 \times 10^{5}$
e $3,14 \times 10^{6}$ f $8,20 \times 10^{3}$

Activity 2

1 a No b Yes c No d No
2 a $8,43 \times 10^{-5}$ b $2,18 \times 10^{-6}$ c $3,60 \times 10^{-2}$
d $1,00 \times 10^{-13}$ e $3,00 \times 10^{-4}$ f $8,52 \times 10^{-9}$

Converting scientific notation to normal notation; Using a calculator

Activities 3–4 Write numbers in normal notation; Work with scientific and normal notation on a calculator

Learner's Book page 155

Guidelines for implementing this activity

- Work through the examples in the Learner's Book with the class.
- Let learners do a few additional examples in their exercise books as practice and volunteer their answers. Let them verbalise and/or illustrate their methods and address any misunderstandings that you notice.
- Also, let learners practise a few calculator exercises in their books before attempting the activity. Walk around the class to monitor whether they are doing these correctly, and to assist with the steps if necessary.
- Learners must complete this activity on their own in their exercise books.
- Work through the summary with the class as a form of revision, or let learners work through the summary in pairs for 5 minutes and allow pairs to volunteer sharing orally their own understanding of each point in the summary (and give examples where appropriate).

Remedial and extension

Remedial: Remind learners about the place value shifts that the digit in the number makes when a number is written in normal notation. Also remind learners how to multiply and divide numbers by powers of 10. Refer back to the chapter on decimal fractions if necessary and/or let learners practise with basic exercises that involve multiplying/dividing numbers by powers of 10. These basics can be given as homework.
Extension: Ask learners to research where scientific notation is used in other subjects and ask them to find interesting facts about very large and very small numbers, for example, the size of a neuron, the circumference of the world in km, and so on. Ask learners to swap the very large and small numbers that they had collected earlier in the unit and to write these in normal notation.

Suggested answers

Activity 3

1	a	30 000	b	2 960 000	c	942	d	38 700
2	a	0,02	b	0,0038	c	0,00279	d	0,0002872

Activity 4

1	a	$3{,}72 \times 10^{5}$	b	$7{,}80 \times 10^{-5}$	c	$6{,}1 \times 10^{-3}$
	d	$4{,}39 \times 10^{-3}$	e	$8{,}75 \times 10^{4}$	f	$8{,}2 \times 10^{-6}$
2	a	5 200	b	0,0000709	c	3 256 000
	d	0,000000307	e	0,0271	f	0,000000004876

3 a $9\ 223{,}68 \times 100 + 0{,}00489 = 922\ 368{,}1649$

b $86\ 700 - 0{,}424 = 86\ 699{,}58$

c $0{,}031 \times 84\ 000 = 2\ 604$

d $0{,}00698 \div 0{,}000000004 = 174\ 500$

4 a $1{,}29 \times 10^{5}$ b $2{,}6 \times 10^{-7}$

UNIT 3

Calculations with numbers in exponential form

Unit overview

Learner's Book page 156
Recommended pacing: 1 hour 30 minutes

This unit focuses on the following:

- Exponent Law 1: Multiplying of powers with the same base
- Exponent Law 2: Dividing of powers with the same base
- Exponent Law 3: Raising a power to a power
- Exponent Law 4: Finding a power of a product in brackets

Resources: Learner's Book; calculator; exercise book

Background information

This unit aims to further establish the general laws of exponents, limited to natural number exponents.

- $a^m \times a^n = a^{m+n}$
- $a^m \div a^n = a^{m-n}$, if $m > n$
- $(am)^n = a^{m \times n}$
- $(a \times t)^n = a^n \times t^n$
- $a^0 = 1$

Teaching guidelines

The aim of this unit is to help learners revise the laws of exponents and to progress them beyond the very tedious expansion and contraction methods when multiplying and dividing exponents. However, these methods may still be used as a form of remediation to help learners discover the shortcuts.

Learners will hopefully have picked up the patterns as they were completing Unit 1, and articulate and use the laws themselves.

Although this is revision, learners require a lot of practice because they sometimes get confused between when to add and subtract exponents and when to multiply them. A systematic and careful approach is essential, with many additional practice exercises to help consolidate understanding and application of the Exponent Laws.

Exponent Law 1: Multiplying of powers with the same base

Activity 1 Work with Exponent Law 1

Learner's Book page 157

Guidelines for implementing this activity

- Work through the examples in the Learner's Book.
- Let learners do quick, similar examples in their exercise books after working through each example with them in the Learner's Book. Monitor their understanding in this way.
- Remind learners to use Exponent Law 1 to complete this activity. Emphasise that when there is no visible exponent it implies an exponent of 1.
- Encourage learners to work as quickly as possible through the activity as this is revision.

Remedial and extension

Remedial: If learners cannot use the exponent laws at first, let them continue expanding the factors. Point out that it is very time consuming and it is more effective to use the exponent laws. Provide extra practice if necessary.
Extension: Provide learners with a few more challenging examples similar to those in questions **10** to **12**. Also include some different variables and fractions in the examples.

Suggested answers

1 m^9	**2** s^8	**3** a^{11}	**4** n^{10}	**5** k^2	**6** b^{27}
7 3^5	**8** 2^5	**9** $30k^7$	**10** $81m^{12}$	**11** $-64a^7$	**12** $24a^8$
13 $-24m^{15}$	**14** $40p^{13}$				

Exponent Law 2: Dividing powers with the same base

Fractions where the larger exponent lies in the numerator; Fractions where the exponents in the numerator and denominator are equal; Fractions where the larger exponent lies in the denominator

Activities 2–4 **Work with Exponent Law 2; Work with powers with exponents that are equal to 0; Work with powers with negative exponents**

Learner's Book pages 158–161

Guidelines for implementing these activities

- Revise Exponent Law 2 by working through each introduction in the Learner's Book.
- Remind learners of the following:
  - Any base to the power zero equals 1. Remember, the base may not be equal to 0.
  - Always simplify the coefficients first and then apply the division law for powers with the same base.
  - We cannot use the exponent law to simplify examples such as $\frac{m^6}{n^2}$ because the bases are not the same, therefore we cannot subtract the exponents.
  - A negative exponent such as $3^{-3} = \frac{1}{3^3}$
- Let learners do quick, similar examples in their exercise books after working through each example with them in the Learner's Book. Monitor their understanding and progress in this way.
- Remind learners to use Exponent Law 2 to complete all the activities. Learners complete each activity individually.

Remedial and extension

Remedial: If learners cannot use the exponent laws at first, let them continue expanding the factors. Point out that it is very time-consuming and it is more effective to use the exponent laws. Provide extra practice exercises if necessary. Doing short exercises after each example as described above (before learners attempt the activity) is also a good form of remediation and consolidation.
Extension: Provide learners with a few more challenging exercises. Include some different variables and fractions in the examples.

Suggested answers

Activity 2

1 $x^{6-3} = x^3$	**2** $x^{5-1} = x^4$	**3** $c^{7-7} = c^0 = 1$
4 $y^{21-19} = y^2$	**5** $r^{12-9} = r^3$	**6** $a^{4-2} = a^2$
7 5^2	**8** 2	**9** 1
10 a^2	**11** x^6	**12** $5x^5$
13 $-3y^2$	**14** $-7x$	

Activity 3

1	1	**2**	1	**3**	6	**4**	$5 + 2 = 7$
5	$1 + 1 = 2$	**6**	$9 - 3 = 6$	**7**	$1 - 6 = -5$	**8**	1
9	9	**10**	$1 + 1 = 2$	**11**	$3 - 1 = 2$	**12**	1

Activity 4

1 **a** $\frac{1}{x^5}$ **b** $\frac{4}{y^{-5}}$ **c** $\frac{y^3}{x^2}$ **d** $\frac{1}{7^3}$

2 **a** 3^2 **b** $7^{8-4} = 7^4$ **c** y^3 **d** 1

e $\frac{1}{x^3}$ **f** $\frac{-1}{a^3}$ **g** pq^2 **h** s^5

i $\frac{2}{-b^2}$ **j** $-50x^8$

3 $0{,}0000673 = \frac{6{,}73}{10\,000} = \frac{6{,}73}{10^5} = 6{,}73 \times 10^{-5}$.

a It is. **b** The number is very small.

Exponent Law 3: Raising a power to a power

Activity 5 Work with powers of powers

Learner's Book page 162

Guidelines for implementing this activity

- Work through the examples with learners.
- Let learners do quick, similar examples in their exercise books after working through each example with them in the Learner's Book. Monitor their understanding and progress in this way.
- Let learners use the expanded notation method to help them see why Exponent Law 3 is actually correct.
- Provide a few additional examples and let learners first do them with expanded notation and then using the law. Encourage learners to use the law when doing the activity.

Remedial and extension

Remedial: If learners are mixing up the rules or laws, get them to go back to basics and expand and then cancel. They will eventually discover the shortcuts.
Extension: Provide similar examples but make the exponents much larger and add more negative exponents. The unit on practical exponents is geared towards extension.

Suggested answers

1 **a** $3^{2 \times 3} = 3^6$ **b** $2^{3 \times 2} = 2^6$ **c** $2^{2 \times 5} = 2^7$ **d** $4^{4 \times 3} = 4^{12}$

2 **a** $p^{7 \times 5} = p^{35}$ **b** $a^{-2 \times 6} = a^{-12} = \frac{1}{a^{12}}$ **c** $a^{6 \times 2} = a^{12}$ **d** $t^{5 \times -6} = t^{-30} = \frac{1}{t^{30}}$

e $k^{10 \times 4} = k^{40}$ **f** $c^{-2 \times -8} = c^{16}$ **g** $c^{5 \times 3} \times c^{2 \times 2} = c^{15+4} = c^{19}$

h $\frac{-a^{24}}{a^{20}} = -a^4$ **i** $p^{10}q \times \frac{q^4}{2p^8} = \frac{p^{10-8}q^{1+4}}{2} = \frac{p^2q^5}{2}$ **j** $-21a^4b$

Exponent Law 4: Finding a power of a product in brackets

Activity 6 Work with powers of products in brackets

Learner's Book page 162

Guidelines for implementing this activity

- Exponent Law 4 is an application of Exponent Law 3 powers of powers.
- Work through the examples with the learners, and let learners do quick, similar examples in their exercise books after working through each example with them in the Learner's Book. Monitor their understanding and progress in this way.
- It is important to emphasise that a base that appears not to have a power is raised to the power of 1 and not zero.
- Learners do the activity by themselves.
- Work through the summary with the class as a form of revision, or let learners work through the summary in pairs for 5 minutes and allow pairs to volunteer sharing orally their own understanding of each point in the summary (and give examples where appropriate).

Remedial and extension

Remedial: Let learners use the expanded form until they are more confident of the result. They can also use a highlighter or pencil to write in all the implied 1s (exponents) as well as the multiplication of exponents that needs to take place (as shown in the examples). This will help them keep track of the process and the calculations that are required.
Extension: Provide learners with additional examples containing more variables, some negative exponents and fractions.

Suggested answers

1 $2^6 \times 5^4 = 40\ 000$
2 $a^9b^9c^3$
3 $p^4q^4r^8$
4 $3^{10}m^{10}$
5 x^8y^4
6 $2x^6y^6$
7 $\frac{y^{25}}{y^{20}} = y^5$
8 $a^{20} \times a^{12} = a^{32}$
9 $9a^8$
10 $m^{12}n^{18}$
11 $16x^8y^{12}$

UNIT

4 Exponents in problem-solving

Unit overview

Learner's Book page 163
Recommended pacing: 1 hour

This unit focuses on the following:
- Solving simple exponential equations.

Resources: Learner's Book; calculator; exercise book

Background information

In Grade 9, learners solve problems in contexts involving numbers in exponential forms.

Teaching guidelines

Learners are shown how to solve simple exponential equations. This is then applied to some practical investigations.

Solving simple exponential equations

Activity 1 Solve simple exponential equations

Learner's Book page 163

Guidelines for implementing this activity

- Work carefully through the examples in the Learner's Book and provide quick additional exercises for learners to do in their exercise books after each example. Let them volunteer their answers and monitor and check their understanding in this way. How quickly and accurately they can do these exercises should determine how fast you work through the examples. Ensure that learners who find this concept more challenging are not left behind. Let them volunteer their answers as well and be aware if they are hesitant to do this in front of the class, in which case they have to be earmarked for individual or small-group monitoring and support using a similar approach.
- Learners can also pair up with a partner when doing these quick exercises.
- Emphasise that we use equivalent forms of powers when solving simple exponential equations.

Remedial and extension

Remedial: Do a few more examples slowly so that learners can see the principle. Working through additional exercises after each example or in small target groups as explained above is also a good form of remediation.
Extension: Learners could find this quite challenging especially the questions with negative exponents. The more challenging exercises serve as extension.

Suggested answers

1 $2^x = 2^2 \rightarrow x = 2$
2 $2^x = 2^{-3} \rightarrow x = -3$
3 $3^x = 3^3 \rightarrow x = 3$
4 $3^x = 3^{-2} \rightarrow x = -2$
5 $4^x = 4^2 \rightarrow x = 2$
6 $4^x = 4^{-1} \rightarrow x = -1$
7 $5^x = 5^3 \rightarrow x = 3$
8 $5^x = 5^{-2} \rightarrow x = -2$

PoA | Investigation 2 Powers of 2: Paper folding

Learner's Book page 164

Guidelines for implementing this activity

- Give each group an A4 page and show the class how to fold the page according to the diagrams and instructions in the Learner's Book.
- You could give a prize to the group that manages to fold the paper the most times.
- Go through this activity with the class once all the groups have finished and discuss their findings.

Suggested answers

No. of folds	0	1	2	3	4	5	6
No. of pieces	1	2	4	8	16	32	64

1 1 fold and 2 pieces
2 2 folds and 4 pieces
3 3 folds and 8 pieces
4 4 folds and 16 pieces; 5 folds and 32 pieces
5 The folds are too thick and the pieces are too small.
6 1 024 pieces
7

No. of parts	1	2	4	8	16	3^2	64
Base 2	2^0	2^1	2^2	2^3	2^4	2^5	2^6

8 $2n$

PoA | Assignment 2 Powers of 2: Calculate a target

Learner's Book page 165

Guidelines for implementing this activity

- Get a copy of a chess board, or make one beforehand. Discuss how many blocks make up the board.
- Start off the class by placing 1c on the first block and then 2c on the second block. Continue until the class picks up the pattern and then let learners carry on in their groups to complete the activity.

Suggested answers

1

Chess square	1	2	3	4	5	6	7	8
Rand	1	2	4	8	16	32	64	128

2 $y = 2^{x-1}$ where y is the amount of money Daniel has to put on any square and x is the number of the chess square.
3 $y = 2^{12-1} = 2^{11} = 2\,048$

4 $y = 2^{64-1} = 2^{63} = 9{,}22 \times 1018$ c = 9223372087000000000 c. This is a very large amount of money!

5 Learners will have their own responses, e.g. it is always a good idea to save money and possibly David should set a target of the amount of money that he needs and then calculate how many days it will take to save that amount.

PoA | Assignment 3 Consecutive numbers

Learner's Book page 166

Guidelines for implementing this activity

- This assignment is best completed in a group. Remind the learners about consecutive numbers by working through the example, and then start the assignment with the class.
- The groups must then complete the assignment and then hand in one copy of their answers to be marked.

Tip

Pair up learners in such a way that they are able to support each other.

Suggested answers

1 $1 = 0 + 1$; $3 = 2 + 1$; $5 = 2 + 3$; $6 = 3 + 2 + 1$; $7 = 4 + 3$; $9 = 4 + 5 = 2 + 3 + 4$
$10 = 4 + 3 + 2 + 1$

2 2; 4; 8; …

3 16; 32; 64; …

Chapter 5 Revision

Learner's Book page 167

Encourage learners to review the content covered before attempting the revision activities. The revision activities should be used to assess learners' progress thus far, and to assess where remediation may be required.

Suggested answers

1 a y^{27} b k^6 c p^{14}
d $\frac{t^{15}}{t^7} = t^8$ e $125m^{12}$ f $6a^6b^9$

2 a 1 b $7(1) = 7$ c 1 d $-3(1) + 3 = 0$

3 a $4(1) = 4$. False b $a^{2-5} = a^{-3} = \frac{1}{a^3}$. False
c $n^2 \times n = n^3$. False d $\frac{6m^4}{3m^4} = 2m$. False

4 D

5 $6{,}43 \times 10^6$

6 0,00032

7 a $3{,}36 \times 10^{14}$ b $7{,}5 \times 10^{-19}$

8 300 000 km/s = $1{,}08 \times 10^9$ km/h

CHAPTER

6 Numeric and geometric patterns

Chapter overview

Learner's Book pages 168 to 188
Recommended pacing: 4,5 hours

This chapter focuses on:

Unit 1:	**Investigating and extending numeric and geometric patterns**	2 hours
	Patterns with a constant difference	
	Patterns with a constant ratio	
	Patterns without a constant difference or ratio	
Unit 2:	**Analysing patterns and predicting their terms**	2,5 hours
	Finding the general rule for patterns with a constant difference	
	Finding the general rule for simple patterns with a constant ratio (enrichment)	
	Using the general rule to predict positions of terms in a pattern (enrichment)	
POA Assignment 4: Matchstick patterns		
Chapter 6 Revision		35 minutes

UNIT

1 Investigating and extending numeric and geometric patterns

Unit overview

Learner's Book page 169
Recommended pacing: 2 hours

This unit focuses on the following:

- Continuing a number pattern
- Finding the missing terms in a sequence of numbers
- Writing up a general statement
- Using algebra and variables to help them draw up a general statement
- Completing an input and output table
- Completing a flow diagram
- Justifying the pattern that they have identified.

Resources: Learner's Book; calculator; exercise book

Background information

In Grade 8 learners did the following:

- Investigating and extending numeric and geometric patterns looking for relationships between numbers, including patterns:
  - represented in physical or diagram form
  - not limited to sequences involving a constant difference or ratio
  - of learner's own creation
  - represented in tables
  - represented algebraically.

In Grade 9 learners continue to investigate patterns using all the different representations used in Grades 7 and 8. Learners will be taken through steps using patterns and tables to find the general rule. The general rule, T_n, is then used to find the position of a value in a particular sequence or what the value will be at a given position.

Teaching guidelines

Numeric and geometric patterns were covered in Grades 7 and 8. In Grade 9, the emphasis is on finding the general rule so that terms of the form T_n can be calculated if the position in the sequence is known. Some consolidation of the work done in Grade 8 is advised. This involves working with continuing or completing patterns, describing the pattern in words using tables, and finding formulae. Patterns can involve integers, fractions and can be in exponential form.

In Grade 9, learners again investigate numeric and geometric patterns through continuing/completing patterns, describing how they did this in their own words, and can also draw up tables and graphs to help find a formula. Learners are expected to find common differences or ratios if they exist.

Patterns with a constant difference

Numeric patterns with a constant difference

Activity 1 **Extend numeric patterns with a constant difference**

Learner's Book page 170

Guidelines for implementing this activity

- Discuss what numeric sequences/patterns are and that they can be formed by adding/subtracting numbers from a previous number.
- They can also be formed by multiplying/dividing numbers from the previous number.
- Do as many patterns on the board as possible.
- Remember to give at least four numbers of a pattern because it *could* be ambiguous if you do not. For example, in question **4a** learners are asked to find the constant difference which is easily calculated to be 1. However, without specifically asking for the constant difference, the third term of 1; 2; 3; … could arise like this: $1 + 2 = 3$. This means that the fourth term could be $2 + 3 = 5$, and so on.
- Remember to retain a fun aspect to patterns.

Remedial and extension

Remedial: Give learners another set of basic examples where they just have to extend the pattern. Once they have extended the patterns, ask them to describe in their own words how they extended the pattern. Once they are more confident, they should tackle the activity again on their own.
Extension: Give learners more complex patterns with algebraic variables, two or more patterns combined and some without constant differences or ratios. Also, ask learners to design at least three of their own patterns (constant difference, constant ratio and one without either).

Suggested answers

1 –1; 5; 11; 17; 23

2 1; 4; 7; 10; 13 and 3; 5; 7; 9; 11

3	**a**	i	25; 33	ii	add 8 each time		
	b	i	11; 27	ii	add 4 each time		
	c	i	–11; 1	ii	add 3 each time		
4	**a**	i	Add 1 each time	ii	1	iii	4; 5; 6
	b	i	Add –2 each time	ii	–2	iii	–9; –11; –13
	c	i	Add 5 each time	ii	5	iii	25; 30 35
	d	i	Add –5 each time	ii	–5	iii	–24; –29; –34
	e	i	Add –6 each time	ii	–6	iii	–18; –24; –30
	f	i	Add 8 each time	ii	8	iii	50; 58; 66
	g	i	Add –9 each time	ii	–9	iii	–54; –63; –72
	h	i	Add $\frac{1}{2}$ each time	ii	$\frac{1}{2}$	iii	$3\frac{1}{2}$; 4; $4\frac{1}{2}$
	i	i	Add $-\frac{1}{4}$ each time	ii	$-\frac{1}{4}$	iii	$2\frac{1}{4}$; 2; $1\frac{3}{4}$
	j	i	Add 2,5 each time	ii	2,5	iii	13,5; 16; 18,5
	k	i	Add $1\frac{1}{2}$ each time	ii	$1\frac{1}{2}$	iii	$1\frac{1}{2}$; 3; $4\frac{1}{2}$
	l	i	Add $\frac{1}{3}$ each time	ii	$\frac{1}{3}$	iii	2; $2\frac{1}{3}$; $2\frac{2}{3}$

Geometric patterns with a constant difference

Activity 2 Extend geometric patterns with a constant difference

Learner's Book page 172

Guidelines for implementing this activity

- Work through the example with the class.
- Geometric patterns are visual and can also be physical. Provide pencils, sticks or burnt matches for learners to build some of the patterns themselves.
- Toothpicks work well, although they are sharp and you need to caution learners about working with them.
- Let them build the patterns given in the worked examples and then give leaners an opportunity to answer the questions themselves before explaining it on the board.
- Allow them some time to develop patterns by themselves with the sticks.

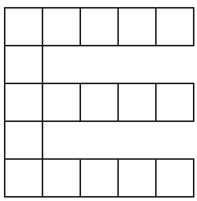
- If necessary give another example to make sure that everyone understands the steps.
- Let learners complete the activity on their own.

Remedial and extension

Remedial: If possible, give learners sticks or used matches to actually build the patterns and then write the pattern out numerically or in the form of a table. In the case of questions that require drawing, make sure learners do draw the patterns. Seeing the patterns develop make the questions less abstract.
Extension: Ask learners to use sticks or to draw geometric patterns of their own. They swap these with a partner so that they can work out each other's patterns.

Suggested answers

1 **a**



b

Term (n)	1	2	3	4	5
Number of squares (T_n)	8	11	14	17	20

c Add three squares each time. **d** $T_{15} = 3(15) + 5 = 50$

2 **a**

b

Term (n)	1	2	3	4	5
Number of crosses (T_n)	10	14	18	22	26

c Add two more crosses to each of the horizontal rows of crosses, i.e. add four crosses.
d $T_{21} = 4(21) + 6 = 90$

3 **a** 9; 11
b Add two more squares each time.
c $T_{50} = 2(50) + 1 = 101$

Patterns with a constant ratio

Numeric patterns with a constant ratio; Geometric patterns with a constant ratio

Activities 3–4 **Extend numeric patterns with a constant ratio; Extend geometric patterns with a constant ratio**

Learner's Book pages 173–175

Guidelines for implementing these activities

- Do a brief revision of fractions and ratios. Use some questions from Chapters 1 and 3 as revision.
- Once learners are comfortable with working with ratios, explain how ratios are found in some patterns.
- Use the examples in the Learner's Book to show how to find the ratios in this type of pattern.
- Make sure to show the difference between these patterns and the patterns with a constant difference.
- Give learners a chance to try one example of each type by themselves.
- Before attempting Activity 4, remind learners how to calculate perimeters and areas of rectangles.
- They should do the activities as quickly as possible.

Remedial and extension

Remedial: Let learners use their calculators if they find completing the patterns mentally challenging. For a pattern like: 2; 6; 18; 54; … Ratio is 6 ÷ 2 = 3.

Keystrokes for a non-scientific calculator: 2 $\boxed{\times}$ 3 $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$
Keystrokes for a scientific calculator: 2 $\boxed{\times}$ 3 $\boxed{\times}$ 3 $\boxed{=}$ $\boxed{=}$ $\boxed{=}$
Extension: Challenge learners to continue the patterns by another four terms instead of three as in the activity. They should work as quickly as possible without the use of a calculator.

Suggested answers

Activity 3

1	a	Double the last term	b	×2	c	8; 16; 32
2	a	Multiply by −2	b	×(−2)	c	16; −32; 64
3	a	Multiply the last term by three.	b	×3	c	81; 243; 729
4	a	Multiply by negative three.	b	× (−3)	c	81; −243; 729
5	a	Multiply by four	b	× 4	c	256; 1 024; 4 096
6	a	Multiply by negative five.	b	× (−5)	c	625; −3 125; 15 625
7	a	Multiply by $\frac{1}{2}$	b	$\times \frac{1}{2}$	c	$\frac{1}{16}; \frac{1}{32}; \frac{1}{64}$
8	a	Multiply by negative $\frac{1}{2}$.	b	$\times \left(-\frac{1}{2}\right)$	c	$\frac{1}{16}; -\frac{1}{32}; \frac{1}{64}$
9	a	Multiply by $\frac{1}{3}$	b	$\times \frac{1}{3}$	c	$\frac{1}{81}; \frac{1}{243}; \frac{1}{729}$

10 a Multiply by negative $\frac{1}{3}$. b $\times\left(-\frac{1}{3}\right)$ c $-\frac{1}{81}; \frac{1}{243}; -\frac{1}{729}$

11 a Multiply by $\frac{1}{4}$. b $\times \frac{1}{4}$ c $\frac{1}{256}; \frac{1}{1\,024}; \frac{1}{4\,096}$

12 a Multiply by negative $\frac{1}{5}$. b $\times\left(-\frac{1}{5}\right)$ c $\frac{1}{625}; -\frac{1}{3\,125}; \frac{1}{15\,625}$

Activity 4

1 a 4; 12; 36; …
b 4; 12; 36; 108; 324; 972
c Each line segment splits into three to form the next level; Each term therefore has three times more line segments than the previous term (Number of line segments × 3)

2 a

Perimeter Term 1	Perimeter Term 2	Perimeter Term 3
$2(l + b) = 2(2 + 1)$ $= 6$ cm	$2(l + b) = 2(4 + 2)$ $= 12$ cm	$2(l + b) = 2(8 + 4)$ $= 24$ cm

b Pattern: The perimeter doubles from one term to the next.
c 2×3; $2^2 \times 3$; $2^3 \times 3$; $2^4 \times 3$
d Perimeter of rectangle no. 10: $= 2(l + b) = 2(2^{10} + 2^9) = 2(1\,024 + 512) = 3\,072$ cm (The ratio of the longest sides of the rectangle from one term to the next is 2^1; 2^2; 2^3; 2^4; and so on and the shortest sides are 2^0; 2^1; 2^2; 2^3; and so on)

2 a

Area Term 1	Area Term 2	Area Term 3
$A = l \times b = 2$ cm $\times$ 1 cm $= 2$ cm^2	$A = l \times b = 4$ cm $\times$ 2 cm $= 8$ cm^2	$A = l \times b = 8$ cm $\times$ 4 cm $= 32$ cm^2

b Pattern: The area is 4 times more from one term to the next.
c 2^1; 2^3; 2^5; …
d $A = L \times b = 2^{10} \times 2^9 = 1\,024 \times 512 = 524\,288$ cm^2 (The ratio of the longest sides of the rectangle from one term to the next is 2^1; 2^2; 2^3; 2^4; and so on, and the shortest sides are 2^0; 2^1; 2^2; 2^3; and so on) OR $2^{19} = 524\,288$

Patterns without a constant difference or ratio

Patterns with square numbers; Patterns with cube numbers; Patterns in nature

Activity 5 Extend patterns without a constant difference or ratio

Learner's Book page 178

Guidelines for implementing this activity

- The patterns in this section have neither a constant difference nor a constant ratio.
- It is a good opportunity to revise square and cube numbers and test learners' knowledge of the first ten square and cube numbers.
- Use the examples in the Learner's Book to show how to work with this type of pattern.
- Give learners a chance to try one example of each type by themselves, for example, let them do Activity 5 question **1h** and then volunteer sharing their answers.

- Do additional activities in which you provide a starting number and the rule, for example, 2; … (rule: $3n + 2$) or 1; … (rule: $n^2 + 2$); and so on. Let learners write the first five terms, describe the pattern and present the pattern in a table. (The latter can be done for homework.)

Suggested answers

1 **a** **i** Square numbers

ii

n	1	2	3	4	5
$\mathbf{T}_n$	1	4	9	16	25

iii 25; 36; 49 **iv** $T_n = n^2$

b **i** $\frac{1}{\text{square number}}$

ii

n	1	2	3	4	5
$\mathbf{T}_n$	$\frac{1}{1} = 1$	$\frac{1}{4}$	$\frac{1}{9}$	$\frac{1}{16}$	$\frac{1}{25}$

iii $\frac{1}{25}; \frac{1}{36}; \frac{1}{49}$ **iv** $T_n = \frac{1}{n^2}$

c **i** One more than a square number.

ii

n	1	2	3	4	5
$\mathbf{T}_n$	2	5	10	17	26

iii 26; 37; 50 **iv** $T_n = n^2 + 1$

d **i** cube numbers

ii

n	1	2	3	4	5
$\mathbf{T}_n$	1	8	27	64	125

iii 216; 343; 512 **iv** $T_n = n^3$

e **i** $\frac{1}{\text{cube number}}$

ii

n	1	2	3	4	5
$\mathbf{T}_n$	$\frac{1}{1} = 1$	$\frac{1}{8}$	$\frac{1}{27}$	$\frac{1}{64}$	$\frac{1}{125}$

iii $\frac{1}{125}; \frac{1}{216}; \frac{1}{343}$ **iv** $T_n = n^3$

f **i** One more than a cube number

ii

n	1	2	3	4	5
$\mathbf{T}_n$	2	9	28	65	126

iii 217; 344; 513 **iv** $T_n = n^3 + 1$

g i Double the previous term and minus one OR two to the power of the term number plus one.

ii

n	1	2	3	4	5
T_n	3	5	9	17	33

iii 65; 129; 257 iv $T_n = 2^n + 1$

h i Term number multiplied by the consecutive term number OR term number squared plus term number.

ii

n	1	2	3	4	5
T_n	2	6	12	20	30

iii 42; 56; 72 iv $T_n = n^2 + n = n(n + 1)$

2 a Add the previous two terms together to get the next term.

b 0; 1; 1; 2; 3; 5; 8; 13; 21; 34; 55; 89; 144; 233; 377

3 Learners create their own pattern without a constant difference or ratio. They can for example using different numbers to start a Fibonacci sequence, for example, 24; 7; 31; 38; 69; 107; 176; …

UNIT 2

Analysing patterns and predicting their terms

Unit overview

Learner's Book page 179

Recommended pacing: 2 hours 30 minutes

This unit focuses on the following:

- Finding the general rule for patterns with a constant difference
- Finding the general rule for simple patterns with a constant ratio (enrichment)
- Using the general rule to predict positions of terms in a pattern.

Resources: Learner's Book; calculator; exercise book

Background information

In Grade 8, learners described and justified the general rules for observed relationships between numbers in own words or in algebraic language.
In Grade 9, learners are shown step-by-step methods to develop a general rule.

Teaching guidelines

Learners are shown how to analyse the different terms of pattern by looking at the constant difference and how it creates a progressive pattern. This can then be used to develop a general rule. Once learners have developed the pattern and formula, they will be able to calculate any other term. Tables are used for the development of more difficult formulae.

This is a very important skill and shows learners how algebra can be developed from numbers into a rule. This rule then becomes portable in the sense that they can then use the rule to find out other facts about the pattern they are investigating. Quite often algebra is introduced in a separate chapter and there are not enough links to numbers and rules.
The work done in Grade 9 is a springboard for work that will be done in the FET phase. Remind learners how important it is to ensure that they are able to use algebra to formulate general rules of patterns found in numerical form.

Finding the general rule for patterns with a constant difference

Activity 1 **Find the general rule for patterns with a constant difference**

Learner's Book page 182

Guidelines for implementing this activity

- Work through all the examples in the Learner's Book with the learners.
- The work revises using tables and writing down the rules in words.
  - Remind learners how to find the constant difference. This number is multiplied by the term number, for example, $T_n = 4(n)$.
  - Ask them to find out how far the above answers (the product of the term number and the constant difference) are from each actual number in the pattern. This value is what we add in the rule: $T_n = 4(n) + 1$. This is the general rule.
- The examples show a few different types of patterns with a constant difference.
- If learners are confident, they can do some of the examples by themselves first and then check their answers.
- Learners do the activity by themselves.

Remedial and extension

Remedial: Make sure that learners understand all the steps in the examples. They should redo any of the questions in the activity that they do not get correct. Let them verbalise their mistakes so that they become aware of where their mistakes are (and possible patterns of repeated mistakes), focus on how to correct these and reduce the risk of repeating the same mistakes.
Extension: Give learners a few more challenging examples, such as questions **1d** and **1e**.

Suggested answers

1 a i

n	1	2	3	4	5
$\mathbf{T}_n$	2	5	8	11	14

ii $T_n = 3n - 1$
iii $T_1 = 3(1) - 1 = 2$; $T_2 = 3(2) - 1 = 5$; $T_3 = 3(3) - 1 = 8$; $T_4 = 3(4) - 1 = 11$
iv $T_{150} = 3(150) - 1 = 449$

b i

n	1	2	3	4	5
$\mathbf{T}_n$	150	165	180	195	210

ii $T_n = 15(n) + 135$
iii $T_1 = 15(1) + 135 = 150$; $T_2 = 15(2) + 135 = 165$; $T_3 = 15(3) + 135 = 180$
$T_4 = 15(4) + 135 = 195$
iv $T_{150} = 15(150) + 135 = 2\,385$

c i

n	1	2	3	4
$\mathbf{T}_n$	1 020	1 050	1 080	1 110

ii $T_n = 30n + 990$
iii $T_1 = 30(1) + 990 = 1\,020$; $T_2 = 30(2) + 990 = 1\,050$; $T_3 = 30(3) + 990 = 1\,080$
$T_4 = 30(4) + 990 = 1\,110$
iv $T_{150} = 30(150) + 990 = 5\,490$

d i

n	1	2	3	4
$\mathbf{T}_n$	–9	–6	–3	0

ii $T_n = 3n - 12$
iii $T_1 = 3(1) - 12 = -9$; $T_2 = 3(2) - 12 = -6$; $T_3 = 3(3) - 12 = -3$; $T_4 = 3(4) - 12 = 0$
iv $T_{150} = 3(150) - 12 = 438$

e i

n	1	2	3	4
$\mathbf{T}_n$	–4	–8	–12	–16

ii $T_n = -4n$
iii $T_1 = -4(1) = -4$; $T_2 = -4(2) = -8$; $T_3 = -4(3) = -12$; $T_4 = -4(4) = -16$
iv $T_{150} = -4(150) = -600$

2 a i $T_n = 5n$
ii $T_1 = 5(1) = 5$; $T_2 = 5(2) = 10$; $T_3 = 5(3) = 15$; $T_4 = 5(4) = 20$
iii $T_{200} = 5(200) = 1\,000$
iv Input $\rightarrow \times 5 \rightarrow$ output

b i $T_n = 4n + 1$
ii $T_1 = 4(1) + 1 = 5$; $T_2 = 4(2) + 1 = 9$; $T_3 = 4(3) + 1 = 13$; $T_4 = 4(4) + 1 = 17$
iii $T_{200} = 4(200) + 1 = 801$
iv Input $\rightarrow \times 4 \rightarrow +1 \rightarrow$ output

c i $T_n = 6n + 1$
ii $T_1 = 6(1) + 1 = 7$; $T_2 = 6(2) + 1 = 13$; $T_3 = 6(3) + 1 = 19$; $T_4 = 6(4) + 1 = 7$
iii $T_{200} = 6(200) + 1 = 1\,201$
iv Input $\rightarrow \times 6 \rightarrow +1 \rightarrow$ output

d i $T_n = 4n - 3$

ii $T_1 = 4(1) - 3 = 1$; $T_2 = 4(2) - 3 = 5$; $T_3 = 4(3) - 3 = 9$; $T_4 = 4(4) - 3 = 13$
iii $T_{200} = 4(200) - 3 = 797$
iv Input → ×4 → −3 → output

3 a 6; 12; 18; 24 b $T_n = 6n$
c The *n*th term will have 6 times *n* number of matches

4 a 6; 11; 16; ... b $T_n = 5n + 1$
c The *n*th term will have five times the term number plus 1.

Finding the general rule for simple patterns with a constant ratio (enrichment)

Activity 2 Find the general rule for patterns with a constant ratio

Learner's Book page 184

Guidelines for implementing this activity

- This is an enrichment activity so spend time with the learners that have completed the constant difference exercise.
- Make sure that learners who need additional practice with general rules for patterns with constant differences have enough practice exercises to do while you work with the learners who will be attempting this exercise.
- Work through the examples in the Learner's Book. Give learners an opportunity to figure out, on their own, how to write the general term of each pattern.
- Once learners have given one or two hypotheses, ask them to test these by using some patterns as examples.
- Once they have established that the pattern has the general form of T_n = (constant ratio)^n, they should have no problem working out the general rules of the questions.
- Choose two or three of the questions for learners to do quickly before they attempt the rest of the activity. Discuss their findings.
- Learners can work in pairs.

Remedial and extension

This is an enrichment activity. Make the exercise a little more challenging by either subtracting or adding a constant amount from a pattern with a constant ratio, for example, instead of 4; 16; 64; … give them 5; 17; 65; ….
Ask learners to give a few of their own patterns.

Suggested answers

1 a $T_1 = 4^1 = 4$; $T_2 = 4^2 = 4 \times 4 = 16$; $T_3 = 4^3 = 4 \times 4 \times 4 = 64$
b $T_n = 4^n$ c $T_{10} = 4^{10} = 1\ 048\ 576$

2 a $T_1 = 5^1 = 5$; $T_2 = 5^2 = 5 \times 5 = 25$; $T_3 = 5^3 = 5 \times 5 \times 5 = 125$
b $T_n = 5^n$ c $T_{10} = 5^{10} = 9\ 765\ 625$

3 a $T_1 = 6^1 = 6$; $T_2 = 6^2 = 6 \times 6 = 36$; $T_3 = 6^3 = 6 \times 6 \times 6 = 216$
b $T_n = 6^n$ c $T_{10} = 6^{10} = 60\ 466\ 176$

4 a $T_1 = 7^1 = 7$; $T_2 = 7^2 = 7 \times 7 = 49$; $T_3 = 7^3 = 7 \times 7 \times 7 = 343$

b $T_n = 7^n$ c $T_{10} = 7^{10} = 282\ 475\ 249$

5 a $T_1 = 8^1 = 8$; $T_2 = 8^2 = 8 \times 8 = 64$; $T_3 = 8^3 = 8 \times 8 \times 8 = 512$

b $T_n = 8^n$ c $T_{10} = 8^{10} = 1\ 073\ 741\ 824$

6 a $T_1 = 9^1 = 9$; $T_2 = 9^2 = 9 \times 9 = 81$; $T_3 = 9^3 = 9 \times 9 \times 9 = 729$

b $T_n = 9^n$ c $T_{10} = 9^{10} = 3\ 486\ 784\ 401$

7 a $T_1 = 10^1 = 10$; $T_2 = 10^2 = 10 \times 10 = 100$; $T_3 = 10^3 = 10 \times 10 \times 10 = 1\ 000$

b $T_n = 10^n$ c $T_{10} = 10^{10} = 1 \times 10^{10}$

Using the general rule to predict positions of terms in a pattern (enrichment)

Activity 3 Predict terms and positions in patterns

Learner's Book page 186

Guidelines for implementing this activity

- Once we have a general term, it can be used to find the value of a particular term or the position of a particular value in a sequence.
- Explain to learners that what we are trying to do is to prevent having to add a particular value 99 times to get the 100th term. The general term gives us a shortcut method that can save a lot of time and effort.
- It is important to emphasise that this is the value of algebra – writing formulae and equations that provide quick ways of giving us the information that we need.
- The mathematical skills used for finding the positions or values of terms are:
- substituting numbers into an algebraic rule and
- solving for the unknown value.
- It may be necessary to do a quick revision of substitution and solving equations. Have a short exercise ready that you can hand out to learners to refresh these skills again. You can use an exercise in Chapter 9 as the basis for this.
- Learners do the activity by themselves.

Remedial and extension

Remedial: Learners have had a lot of practice with patterns but might still find it difficult to work out the general rule. It is sometimes useful to pair learners in mixed pairs so that the one can learn from the other. It may be a different method but just as effective.
Extension: Let learners show other learners how they develop the general rules.

Suggested answers

1 a 5; 7; 9; 11 b $T_n = 2n - 5$

c $T^{56} = 2(56) - 5 = 107$ d $59 = 2(n) - 5 \rightarrow n = 32$

2 a $T_n = 5n + 2$, therefore $T_{10} = 5(10) + 2 = 52$; $T_{15} = 5(15) + 2 = 77$

$T_{20} = 5(20) + 2 = 102$

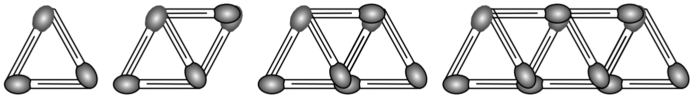
b $317 = 5(n) + 2 \rightarrow n = 63$

3 a $T_n = 4n - 2$

b $T_{50} = 4(50) - 2 = 198$; $T_{100} = 4(100) - 2 = 398$; $T_{200} = 4(200) - 2 = 798$

c $4\ 226 = 4(n) - 2 \rightarrow n = 1\ 057$

4 a



b 15 c 51 ($T_n = 2n + 1$ therefore $T_n = 2(25) + 1 = 51$

d $T_n = 2n + 1$ therefore: $303 = 2n + 1$

$$303 - 1 = 2n + 1 - 1$$

$$\frac{302}{2} = \frac{2n}{2}$$

$$\frac{302}{2} = n$$

$$151 = n$$

PoA | Assignment 4 Matchstick patterns

Learner's Book page 187

Suggested answers

Introduction: The purpose of the assignment is to:

1 construct certain shapes
2 notice the patterns
3 determine formulae.

1

Term number (n)	Number of small squares (s)	Perimeter of L-shape (P)	Number of matches (m)
1	3	8	10
2	5	12	16
3	7	16	22
4	9	20	28
5	11	24	34

2 a $s = 2n + 1$ b $P = 4n + 4$ c $m = 6n + 4$

3 a The number of small squares increases by two each time, therefore the coefficient of n is 2. The number of squares in the first term minus 2 is 1, therefore the constant is 1.

b The number of matches in the perimeter increases by 4 in each term, therefore the coefficient of n is 4. The perimeter of the first term minus 4 is 4, therefore the constant is 4.

c The number of matches for each term increases by 6 in each term, therefore the coefficient of n is 6. The number of matches in the first term minus 6 is 4, therefore the constant is 4.

Chapter 6 Revision

Learner's Book page 188

Encourage learners to review the content covered before attempting the revision activities. The revision activities should be used to assess learners' progress thus far, and to assess where remediation may be required.

Encourage learners to review the content covered before attempting this activity.

Suggested answers

1 a 31; 24; 17 b 23; 29; 31 c $\frac{\sqrt{10}}{81}; \frac{\sqrt{12}}{121}; \frac{\sqrt{14}}{169}$ d 21; 28; 36

2 3; −2; −7; −12; −17

3 a 1; 10 b Add 3 each time

4 Input → ×4 − 1 → Output

5 a 10
  b 15
  c 1; 3; 6; 10; 15; 21; 28; 36; 45; 55

6 a 4; 7; 10; 13; 16
  b Number of dots = $3n + 1$
  c 1 dot in the middle and 3 branches of n dots

CHAPTER

7 Functions and relationships

Chapter overview

Learner's Book pages 189 to 204
Recommended pacing: 4 hours

This chapter focuses on the following:

Unit 1: Input and output values		2 hours
	Finding input values	
Unit 2: Equivalent forms		2 hours
	Describe relationships using equivalent forms	
Chapter 7 Revision		25 minutes

UNIT

1 Input and output values

Unit overview

Learner's Book page 190
Recommended pacing: 2 hours

This unit focuses on the following:

- Finding output values
- Drawing a flow diagram and writing an equation
- Finding input values
- Writing a formula

Resources: Learner's Book; calculator; exercise book; block paper

Background information

In Grade 8 learners did the following:

- Determined input values, output values or rules for patterns and relationships using flow diagrams, tables, formulae and equations
- Determined equivalence of different descriptions of the same relationship or rule presented verbally, in flow diagrams, in tables, by formulae, by equations and by graphs on the Cartesian plane.

In the Intermediate Phase, the focus was on patterns with numbers and/or pictures. Learners examined patterns to find the relationships between the terms. They used words to describe the rules of the patterns.

In the Senior Phase learners have been exposed to more opportunities to write rules in algebraic form when investigating patterns. In the previous chapter, learners looked at numeric and geometric patterns and described the patterns in words, tables, algebraic sentences, flow diagrams and graphs. This gave learners the opportunity to learn the conventions of the algebraic or symbolic form when they simplified the expressions.

Teaching guidelines

In Chapter 7, Functions and Relationships, the development of the concepts of using patterns to identify relationships is continued. The chapter starts with input and output values. This is revision for learners as they have been dealing with flow diagrams since the Foundation Phase.

In Functions and Relationships, the process is taken a step further, where the steps in the flow diagram are used to formalise the process into an equation (or general rule as it was called when dealing with patterns). It is important to point out to learners that the equation is similar to the general rule and that conventionally we use different variables like x and y where in the formulae for patterns, the y-value was T_n and the x-value was n. x and y represent the input and output values of the flow diagrams in relationships. It is important that these concepts are well understood before looking at the inverse processes. The inverse processes are used to find input values when output values are given. Again, learners have done these processes informally since the Foundation and Intermediate Phase. Some revision and consolidation of vocabulary is necessary.

The second unit of this chapter looks at the equivalent forms of relationships. Learners have been using all the different equivalent forms in patterns and this provides an excellent consolidation of moving between the different aspects of relationships in words to flow diagrams, tables, formulae and equations, as well as graphs.

Use this section to constantly remind learners that Mathematics is a tool that we can use to make sense of things that are in our everyday lives. Once we can describe a relationship like speed and fuel consumption in words, it forms the basis of making it into a formal algebraic equation that can be used to make predictions for further use.

Finding output values

Writing a formula; Drawing a flow diagram and writing an equation; Using tables to find output numbers

Activity 1 **Find output values**

Learner's Book page 192

Guidelines for implementing this activity

- Introduce *input* and *output* values by working through the introduction in the Learner's Book. Check learners' understanding of the meaning of *relationship*, *input value*, *rule* and *output value*. Learners who volunteer explanations may want to draw a flow diagram on the board since this may be what they are most familiar with. Allow a learner to do this, or even encourage it by asking: How can we show/present a relationship? What do you remember about showing/presenting a relationship? Use the learner's flow diagram as the starting point.

- Work through the first example in the Learner's Book by linking it to the flow diagram on the board and deriving the formula from this. The number of people is the *input*; the rule is (×2) and the *output* is the number of scoops (for example, 4).
- Now write the following flow diagram and equation on the board by writing variables in place of the actual values. Remind learners that a variable is a letter of the alphabet that represents a number.
  - Flow diagram: $x \rightarrow \boxed{\times 2} \rightarrow y$
  - Equation: $y = 2x$. Remind learners of the following algebraic convention: $x \times 2$ is the same as $2x$.
- Work through the second example. Explain to learners that we can substitute x with any value. This will give us the value of y.
- Let learners do additional substitutions in their exercise books, for example: If $x = 9$; 70; 110; and so on, what would the value of y be?
- Remind learners that they have used substituted input values (x) and applied a rule (×2) to find the corresponding output (y) values.
- Work through Example 3 (or let learners work through the example independently or in pairs and report back what they have learnt from it).
- Test to see whether learners can do Example 4 on their own or in pairs, and report back what they have learnt.

Remedial and extension

Remedial: Learners have been working with flow diagrams since the Foundation Phase. There may be a problem with understanding the language of functions and relationships, but once the concepts are consolidated the language should be easier to grasp. Ensure that the language and terminology are intentionally repeated throughout. The more learners hear it, the better. The additional examples and worked examples that learners are advised to do independently and in pairs in the guidelines above should help to consolidate the concepts. Provide learners with additional examples if necessary and make sure that they do all the questions in the activity.
Extension: Give learners a few examples with multi-step flow diagrams. Ask them to show how they would construct the flow diagram if they were given the output values and had to find the input values.

Suggested answers

1 **a** $1 \rightarrow \boxed{\times 3} \rightarrow 3$; $2 \rightarrow \boxed{\times 3} \rightarrow 6$; $4 \rightarrow \boxed{\times 3} \rightarrow 12$; $7 \rightarrow \boxed{\times 3} \rightarrow 21$; $9 \rightarrow \times 3 \rightarrow 27$

b $1 \rightarrow \boxed{+2} \rightarrow 3$; $2 \rightarrow \boxed{+2} \rightarrow 4$; $4 \rightarrow \boxed{+2} \rightarrow 6$; $7 \rightarrow \boxed{+2} \rightarrow 9$; $9 \rightarrow \boxed{+2} \rightarrow 11$

c $1 \rightarrow \boxed{\times 3} \rightarrow +1 \rightarrow 4$; $2 \rightarrow \boxed{\times 3} \rightarrow +1 \rightarrow 7$; $4 \rightarrow \boxed{\times 3} \rightarrow +1 \rightarrow 13$;
$7 \rightarrow \boxed{\times 3} \rightarrow +1 \rightarrow 22$; $9 \rightarrow \boxed{\times 3} \rightarrow +1 \rightarrow 28$;

d $1 \rightarrow \boxed{\times 4} \rightarrow -1 \rightarrow 3$; $2 \rightarrow \boxed{\times 4} \rightarrow -1 \rightarrow 7$; $4 \rightarrow \boxed{\times 4} \rightarrow -1 \rightarrow 15$;
$7 \rightarrow \boxed{\times 4} \rightarrow -1 \rightarrow 27$; $9 \rightarrow \boxed{\times 4} \rightarrow -1 \rightarrow 35$

2 **a** Output values = –2; 1; 4; 7; 10; 13 **b** Output values = 2; 5; 10; 17; 26; 37

c Output values = 4; 9; 16; 25; 36; 49

d 4; 28; 52; 76; 100; 124

3 a Output values =80; 150; 200

b Output values = R40; R72; R112

4 a

Input	1	2	3	4	5	6	7	8	9	10
Output	5	9	13	**17**	**21**	**25**	**29**	**33**	**37**	**41**

b

Input	1	2	3	4	5	6	7	8	9	10
Output	2	4	**6**	**8**	**10**	12	**14**	**16**	**18**	**20**

c

Input	1	2	3	4	5	6	7	8	9	10
Output	2	4	8	**16**	**32**	**64**	**128**	**256**	**512**	**1024**

Finding input values

Writing a formula; Drawing a flow diagram and writing an equation; Using tables to find input and output values

Activity 2 Find input values

Learner's Book page 194

Guidelines for implementing this activity

- Revise the terminology of a formula, flow diagram, table and equation. Also make sure that you remind learners about inverse operations:
  - Addition is the inverse of subtraction, and vice versa.
  - Multiplication is the inverse of division, and vice versa.
  - Squaring is the inverse of square rooting a number, and vice versa.
- Work through Example 1 in the Learner's Book. Point out that the forward relationship uses multiplication as the operation and the inverse relationship uses division.
- Remind learners that when we reverse something we have to do the inverse operation: instead of walking forward, we would walk back; if we lift something up in the forward movement, we would be putting it down in the reverse (or inverse) action. It can be fun to show learners what a movie clip looks like if it is played in reverse, if you are able to.
- Help them solve for the input value of x by using inverse operations.
- Work through worked examples 2 and 3 in the Learner's Book. Spend more time on example 3. Many learners will be able to solve this intuitively, but challenge them to think of the inverse operations that they employ in that process.
- Example 4 is an extension of example 3, where the values are placed in a table.
- Learners complete this activity on their own.

- Mark this activity to see exactly how learners are dealing with these questions.

Remedial and extension

Remedial: Work through the examples again. Ask learners to think of a series of actions that they could do in reverse, for example walking to their desks, sitting down, taking out their Maths textbooks and writing materials. What would this sequence look like in reverse? Point out the connection between the reverse sequence and doing everything inversely: sit down becomes get up; get book out of bag becomes put book in bag. In Mathematics, subtraction becomes addition when we go in reverse because it is the inverse operation. Remind them again about the inverse operations: addition is the inverse of subtraction; and multiplication is the inverse of division. Let learners write the inverse operations in pencil or preferably a highlighter and then draw the flow diagram.
Extension: Give learners a few multi-stepped examples to reverse.

Suggested answers

1 a $3 \rightarrow \boxed{\div 3} \rightarrow 1$; $15 \rightarrow \boxed{\div 3} \rightarrow 5$; $24 \rightarrow \boxed{\div 3} \rightarrow 8$; $33 \rightarrow \boxed{\div 3} \rightarrow 11$

b $3 \rightarrow \boxed{-2} \rightarrow 1$; $5 \rightarrow \boxed{-2} \rightarrow 3$; $16 \rightarrow \boxed{-2} \rightarrow 14$; $29 \rightarrow \boxed{-2} \rightarrow 27$

c $7 \rightarrow \boxed{-1} \rightarrow \boxed{\div 3} \rightarrow 2$; $16 \rightarrow \boxed{-1} \rightarrow \boxed{\div 3} \rightarrow 5$; $25 \rightarrow \boxed{-1} \rightarrow \boxed{\div 3} \rightarrow 8$;
$31 \rightarrow \boxed{+1} \rightarrow \boxed{\div 3} \rightarrow 10$

d $7 \rightarrow \boxed{+1} \rightarrow \boxed{\div 4} \rightarrow 2$; $15 \rightarrow \boxed{+1} \rightarrow \boxed{\div 4} \rightarrow 4$; $19 \rightarrow \boxed{+1} \rightarrow \boxed{\div 4} \rightarrow 5$;
$26 \rightarrow \boxed{+1} \rightarrow \boxed{\div 4} \rightarrow \frac{27}{4} = 6\frac{3}{4}$

2 a $12 \rightarrow \boxed{\div 3} \rightarrow 4$; $21 \rightarrow \boxed{\div 3} \rightarrow 7$; $36 \rightarrow \boxed{\div 3} \rightarrow 12$; $66 \rightarrow \boxed{\div 3} \rightarrow 22$

b $12 \rightarrow \boxed{-1} \rightarrow 11$; $21 \rightarrow \boxed{-1} \rightarrow 20$; $36 \rightarrow \boxed{-1} \rightarrow 35$; $66 \rightarrow \boxed{-1} \rightarrow 65$

c $12 \rightarrow \boxed{+20} \rightarrow 32$; $21 \rightarrow \boxed{+20} \rightarrow 41$; $36 \rightarrow \boxed{+20} \rightarrow 56$; $66 \rightarrow \boxed{+20} \rightarrow 86$

3 a $48 \rightarrow \boxed{\div 8} \rightarrow 6$; $64 \rightarrow \boxed{\div 8} \rightarrow 8$; $100 \rightarrow \boxed{\div 8} \rightarrow 12\frac{1}{2} \rightarrow 13$ tables

b $20 \rightarrow \boxed{\div 5} \rightarrow 4$; $55 \rightarrow \boxed{\div 5} \rightarrow 11$; $70 \rightarrow \boxed{\div 5} \rightarrow 14$ chocolate bars

4 a

Input	1	2	3	4	5	**6**	**7**	8	**9**	**10**
Output	3	7	11	**15**	**19**	23	27	**31**	37	39

b

Input	1	2	3	**4**	5	7	**8**	20	**25**	50
Output	3	5	**7**	**9**	**11**	15	**17**	**41**	**51**	**101**

c

Input	1	2	3	4	**5**	6	7	**8**	9	**10**
Output	2	4	8	**16**	32	**64**	128	256	**512**	1 024

UNIT

2 Equivalent forms

Unit overview

Learner's Book page 196
Recommended pacing: 2 hours

This unit focuses on the following:

- Matching equivalent forms
- Matching equations, flow diagrams, tables and verbal descriptions of relationships
- Graphs, tables and equations.

Resources: Learner's Book; calculator; exercise book; drawing materials for graphs including long rulers, pencils, erasers and possibly squared paper

Background information

In Grade 8 learners determined, interpreted and justified equivalence of different descriptions of the same relationship or rule presented:

- verbally
- in flow diagrams
- in tables
- by formulae
- by equations.

In Grade 9, learners consolidate work with input and output values done in Grade 8. They continue to find input or output values in flow diagrams, tables, formulae and equations. Learners also show relationships or rules in the form of graphs on a Cartesian plane.

Teaching guidelines

Use every opportunity to show learners the equivalence of describing patterns and relationships particularly when it comes to tables, mathematical expressions and graphs. As learners gain more experience with the different forms, they will be able to choose the most appropriate way to represent a particular situation.

Also, as they see the relationships develop between tables, the formulae and graphs they will understand graphs and their features better. Being able to see how the different aspects of Mathematics are interlinked is vital for good understanding and being able to apply and solve mathematics in other situations, especially in everyday life.

Matching equivalent forms

Matching equations, flow diagrams, tables and verbal descriptions of relationships

Activity 1 Find equivalent forms

Learner's Book page 197

Guidelines for implementing this activity

- Revise the words and terms *verbal descriptions* (a relationship in words); *flow diagrams*; *input and output values*; *formulae* and *equations*. Explain the word *equivalent* again: it may look different but it represents the same relationship.
- Write the verbal descriptions 1 and 2 in the Learner's Book on the board and ask learners to write these in their exercise books. Underline the words "multiplied" and "is subtracted". Ask learners to write in the mathematical symbols for these words above the words in a different colour.
- Write "Input value" on the board. Ask learners to fill in the mathematical operators and numbers that follow this word in the verbal sentence.
  - Input value $\rightarrow \times 5 - 3 \rightarrow$ output value.
  - Similarly, input value $\rightarrow \times 3 - 1 \rightarrow$ output value
- Once learners have converted the word sentence into a flow diagram, they may find it easier to identify other equivalent forms.
- Ask learners to match the flow diagrams, tables and equations with each of these before looking at the answers given.
- Follow up on the board to make sure that everyone has time to see the differences between the correct ones and the distractors.
- You could ask learners to work in pairs for questions **1** to **3** and to do questions **4** and **5** individually depending on your unique classroom situation.

Remedial and extension

Remedial: Work through the examples in the Learner's Book with learners again. If writing on the board, use colour chalk to show learners how the different forms are linked.
Extension: Ask learners to work through the activity as quickly as possible and without the use of a calculator. Give them two or three additional questions with more complex equations involved. Question **5** is also quite challenging and you can ask learners to make up similar ones and test their friends.

Suggested answers

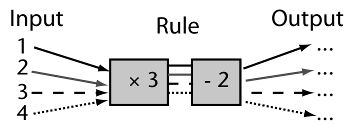
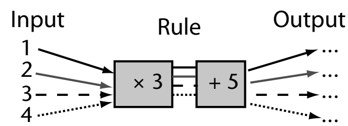
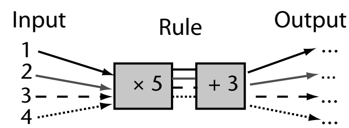
1

	COLUMN A		COLUMN B
a	Double the input number subtract 3	4	$x \rightarrow \times 2 \rightarrow -3 \rightarrow$
b	Multiply the input number by 2 and add 3	1	$x \rightarrow \times 2 \rightarrow +3 \rightarrow$
c	Think of a number, add 3 and double the answer	2	$x \rightarrow +3 \rightarrow \times 2 \rightarrow$
d	Subtract 3 from the input number and multiply by 2	3	$x \rightarrow -3 \rightarrow \times 2 \rightarrow$

2 **a** $y = 2x - 3$ **b** $y = 2x + 3$

c $y = (x + 3) \times 2 = 2x + 6$ **d** $y = (x - 3) \times 2 = 2x - 6$

3

<table>
<tr><th colspan="2">Flow diagram</th><th>Equation</th><th>Table</th></tr>
<tr><td>a</td><td>
</td><td>$y = 3x - 2$</td><td><table><tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>y</td><td>1</td><td>4</td><td>7</td><td>10</td></tr></table></td></tr>
<tr><td>b</td><td>
</td><td>$y = 3x + 5$</td><td><table><tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>y</td><td>8</td><td>11</td><td>14</td><td>17</td></tr></table></td></tr>
<tr><td>c</td><td>
</td><td>$y = 5x + 3$</td><td><table><tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>y</td><td>8</td><td>13</td><td>18</td><td>23</td></tr></table></td></tr>
</table>

4

Equation	Flow diagram	Correct input value (x)
$2x + 7 = 25$	$x \rightarrow \times 2 \rightarrow +7 \rightarrow 25$	$x = 9$
$7x + 2 = 23$	$x \rightarrow \times 7 \rightarrow +2 \rightarrow 23$	$x = 3$
$2x - 7 = 23$	$x \rightarrow \times 2 \rightarrow -7 \rightarrow 23$	$x = 15$
$7x - 2 = 26$	$x \rightarrow \times 7 \rightarrow -2 \rightarrow 26$	$x = 4$
$\frac{x}{2} + 7 = 23$	$x \rightarrow \div 2 \rightarrow +7 \rightarrow 23$	$x = 32$

5 Let x be the number then:

$$[(2x + 17) \div 3 + 4] \times 4 = 60 \text{ or } 4\left(\frac{(2x + 17)}{3} + 4\right) = 60$$

$$\begin{aligned} (2x + 17) \div 3 + 4 &= 15 \\ (2x + 17) \div 3 &= 11 \\ (2x + 17) &= 33 \\ 2x &= 16 \\ x &= 8 \end{aligned}$$

Graphs, tables and equations

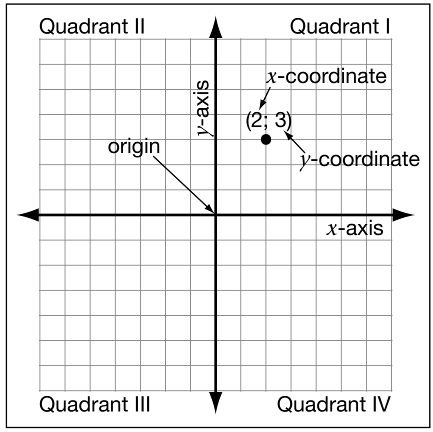
Activity 2 Find the relationship

Learner's Book page 201

Guidelines for implementing this activity

- Remind learners what a Cartesian plane is. Revise the vocabulary that is associated with Cartesian planes like x-axis, y-axis, origin, quadrants.

- Revise coordinate pairs with learners. The *x*-value is a move horizontally and the *y*-value is a vertical move. The coordinate pair shows the *x*-value first and the *y*-value second.
- Ask learners to read the coordinate pairs before looking at the answers. This will give them good practice. You can also ask five learners to call out a coordinate pair each and five other volunteers to plot them on a Cartesian plane drawn on the board.
- Once the learners are comfortable with coordinate pairs again, ask them to plot the ones from the graph in a table as in Step 2 of Example 1 in the Learner's Book.



- Explain to learners that graphs can be used to plot input and output values.
- The values on a graph can be written in table.
- The table and/or the graph can be used to write a formula or an equation.
- In Example 2, learners draw an equation to generate a table. The values on the table are then plotted onto a graph. Make sure that learners maintain very high standards of accuracy and neatness when drawing graphs. They must have sharpened pencils and a long ruler.
- Examples 3 and 4 revise using equivalent forms of relationships.
- Learners complete the activity on their own.

Remedial and extension

Remedial: Some learners find it easier to start with values in a graph and to plot them. Give learners a few examples like this. The important thing is that they realise that we can make a graph from a table or make a table from a graph. Both these equivalent forms help to find the formula or equation for the relationship.
Extension: Ask learners to draw a graph of the type of relationship in question **5** of Activity 1. Give learners a graph with a negative gradient and ask them to draw up the table and find the formula.

Suggested answers

1 **a** $y = 2x + 2$

b

x	1	2	3	4	5	6	10	15	50	100
y	4	6	8	10	12	**14**	**22**	**32**	**102**	**202**

2 **a**

x	1	2	3	4	5	6	10	15	50	100
y	**1**	**5**	**9**	**13**	**17**	**21**	**37**	**57**	**197**	**397**

b $y = 4x - 3$

3 **a**

x	–5	–4	–3	–2	–1	0	1	2	3	4	5
y	–9	–7	–5	–3	–1	1	3	5	7	9	11

b

x	1	2	3	4	5	6	7	8	9	10
y	3	5	7	9	11	13	15	17	19	21

c

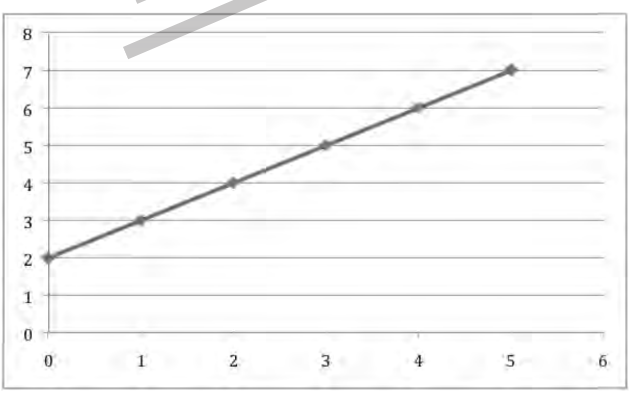
x	-2	$-1\frac{3}{4}$	$-1\frac{1}{2}$	$-1\frac{1}{4}$	-1	$-\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	0	$\frac{1}{4}$	$\frac{1}{2}$
y	3	$-2\frac{1}{2}$	-2	$-1\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$1\frac{1}{2}$	2

x	$\frac{3}{4}$	1	$1\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{3}{4}$	2
y	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4	$4\frac{1}{2}$	5

4 $y = -4x - 7$

x	**–5**	**–3**	**–1**	**0**	**2**	**10**
y	13	5	− 3	− 7	− 15	− 47

5 **a**



b $y = x + 2$

Chapter 7 Revision

Learner's Book page 203

Encourage learners to review the content covered before attempting the revision activities. The revision activities should be used to assess learners' progress thus far, and to assess where remediation may be required.

Suggested answers

1 **a** $-10 \rightarrow \boxed{\times 2} \rightarrow -20$; $-6 \rightarrow \boxed{\times 2} \rightarrow -12$; $-1 \rightarrow \boxed{\times 2} \rightarrow -2$;
$0 \rightarrow \boxed{\times 2} \rightarrow 0$; $3 \rightarrow \boxed{\times 2} \rightarrow 6$; $5 \rightarrow \boxed{\times 2} \rightarrow 10$; $11 \rightarrow \boxed{\times 2} \rightarrow 22$; $20 \rightarrow \boxed{\times 2} \rightarrow 40$

b $-10 \rightarrow \boxed{\div 2} \rightarrow -5$; $-6 \rightarrow \boxed{\div 2} \rightarrow -3$; $-1 \rightarrow \boxed{\div 2} \rightarrow -\frac{1}{2}$;
$0 \rightarrow \boxed{\div 2} \rightarrow 0$; $3 \rightarrow \boxed{\div 2} \rightarrow 1\frac{1}{2}$; $5 \rightarrow \boxed{\div 2} \rightarrow 2\frac{1}{2}$; $11 \rightarrow \boxed{\div 2} \rightarrow 5\frac{1}{2}$; $20 \rightarrow \boxed{\div 2} \rightarrow 10$

c $-10 \rightarrow \boxed{\times 2} \rightarrow \boxed{-3} \rightarrow -23$; $-6 \rightarrow \boxed{\times 2} \rightarrow \boxed{-3} \rightarrow -15$;
$-1 \rightarrow \boxed{\times 2} \rightarrow \boxed{-3} \rightarrow -5$; $0 \rightarrow \boxed{\times 2} \rightarrow \boxed{-3} \rightarrow -3$; $3 \rightarrow \boxed{\times 2} \rightarrow \boxed{-3} \rightarrow 3$;
$5 \rightarrow \boxed{\times 2} \rightarrow \boxed{-3} \rightarrow 7$; $11 \rightarrow \boxed{\times 2} \rightarrow \boxed{-3} \rightarrow 19$; $20 \rightarrow \boxed{\times 2} \rightarrow \boxed{-3} \rightarrow 37$

d $-10 \rightarrow \boxed{+2} \rightarrow \boxed{\times 2} \rightarrow -16$; $-6 \rightarrow \boxed{+2} \rightarrow \boxed{\times 2} \rightarrow -8$;
$-1 \rightarrow \boxed{+2} \rightarrow \boxed{\times 2} \rightarrow 2$; $0 \rightarrow \boxed{+2} \rightarrow \boxed{\times 2} \rightarrow 4$; $3 \rightarrow \boxed{+2} \rightarrow \boxed{\times 2} \rightarrow 10$;
$5 \rightarrow \boxed{+2} \rightarrow \boxed{\times 2} \rightarrow 14$; $11 \rightarrow \boxed{+2} \rightarrow \boxed{\times 2} \rightarrow 26$; $20 \rightarrow \boxed{+2} \rightarrow \boxed{\times 2} \rightarrow 44$

2

	x	**−2**	**0**	**2**	**4**	**6**
a	$y = 4x - 3$	−11	−3	5	13	21
b	$y = x^2 - 1$	3	−1	3	15	35
c	$y = (x - 1)^2$	9	1	1	9	25
d	$y = (x - 1)(x + 1)$	3	−1	3	15	35

3 **a** input $\rightarrow \boxed{\times 4} \rightarrow \boxed{-1} \rightarrow$ output

b Multiply the input value by four and then subtract one.

c

x	− 3	− 2	− 1	0	1	2	3	4	5
y	−13	−9	−5	−1	3	7	11	15	19

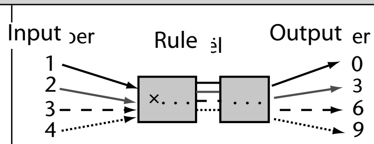
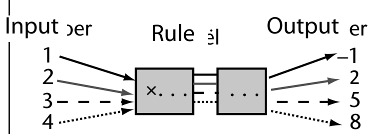
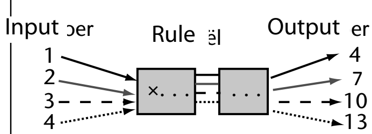
4 **a** $40 \rightarrow \boxed{+20} \rightarrow 60$; $84 \rightarrow \boxed{+20} \rightarrow 104$; $106 \rightarrow \boxed{+20} \rightarrow 126$

b $-15 \rightarrow \boxed{\times 3} \rightarrow \boxed{\div 4} \rightarrow -3$; $25 \rightarrow \boxed{+3} \rightarrow \boxed{\div 4} \rightarrow 7$;
$37 \rightarrow \boxed{+3} \rightarrow \boxed{\div 4} \rightarrow 10$; $69 \rightarrow \boxed{+3} \rightarrow \boxed{\div 4} \rightarrow 18$

c $-8 \rightarrow \boxed{\div 2} \rightarrow \boxed{-1} \rightarrow -5$; $-2 \rightarrow \boxed{\div 2} \rightarrow \boxed{-1} \rightarrow -2$;
$8 \rightarrow \boxed{\div 2} \rightarrow \boxed{-1} \rightarrow 3$; $12 \rightarrow \boxed{\div 2} \rightarrow \boxed{-1} \rightarrow 5$

d $3 \rightarrow \boxed{\times 3} \rightarrow 9$; $9 \rightarrow \boxed{\times 3} \rightarrow 27$; $11 \rightarrow \boxed{\times 3} \rightarrow 33$; $32 \rightarrow \boxed{\times 3} \rightarrow 96$

5

<table>
<tr><th>COLUMN 1</th><th>COLUMN 2</th><th>COLUMN 3</th></tr>
<tr><th>Flow diagram</th><th>Equation</th><th>Table</th></tr>
<tr><td>a </td><td>$y = 3x - 3$</td><td><table><tr><td>x</td><td>–1</td><td>8</td><td>13</td><td>41</td></tr><tr><td>y</td><td>–6</td><td>21</td><td>36</td><td>120</td></tr></table></td></tr>
<tr><td>b </td><td>$y = 3x - 4$</td><td><table><tr><td>x</td><td>–3</td><td>5</td><td>7</td><td>11</td></tr><tr><td>y</td><td>–13</td><td>11</td><td>17</td><td>29</td></tr></table></td></tr>
<tr><td>c </td><td>$y = 3x + 1$</td><td><table><tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>y</td><td>4</td><td>7</td><td>10</td><td>28</td></tr></table></td></tr>
</table>

CHAPTER

8 Algebraic expressions

Chapter overview

Learner's Book pages 205 to 230
Recommended pacing: 4,5 hours

Unit 1:	**Terminology**	45 minutes
	Revision: Algebraic language	
Unit 2:	**Simplifying algebraic expressions**	2,25 hours
	Adding expressions	
	Subtracting polynomials	
	Multiplying expressions	
	Dividing algebraic expressions	
Unit 3:	**Further algebraic manipulations**	1,5 hours
	The product of two binomials	
	Multiplying two identical binomial	
	Multiplying binomials of the form $(a - b)(a + b)$	
	Simplifying expressions	
	Squares, cubes, square roots and cube roots of monomials	
	Substitution	
Chapter 8 Revision		35 minutes

UNIT

1 Terminology

Unit overview

Learner's Book page 206
Recommended pacing: 45 minutes

This unit focuses on the following:
- Revision: Algebraic language
- Variables, constants and coefficients
- Naming algebraic expressions
- Algebraic notation
- Using algebraic language
- Interpreting algebraic language.

Resources: Learner's Book; calculator; exercise book; squared paper/graph paper

Background information

In Grade 8 learners revised:
- recognising and interpreting rules or relationships represented in symbolic form
- identifying variables and constants in given formulae and/or equations.

Learners recognised and identified conventions for writing algebraic expressions and identified and classified like and unlike terms in algebraic expressions. They also recognised and identified coefficients and exponents in algebraic expressions.

In Grade 9, learners will revise the work done in Grade 8. They learn to recognise and distinguish between monomials, binomials and trinomials.

Teaching guidelines

Every subject that we study has a set of terms that we need to be able to use fluently to get the best value from that subject. Mathematics is no different and in addition every topic in Mathematics has some unique terminology. When we learn a new language, we need to practise saying and using the words often so that they become part of our regular vocabulary.

Unit 1 Terminology revises the language that learners have been using in Algebra. It is important to give learners an opportunity to learn and absorb the new terms. In classroom discussions, use every opportunity to use the words mentioned in this unit so that learners become used to them again. Constantly ask questions about the number of terms and whether a polynomial is a mono-, bi-, tri- or polynomial.

It is also useful to have a poster with the terms explained on the wall of the classroom so that learners can refer to it. We should try at all costs not to allow language to prevent learners from reaching their full potential in Mathematics.

Revision: Algebraic language

Terms and factors; Variables, constants and coefficients; Naming algebraic expressions

Activities 1–2 Find terms in expressions; Understand algebraic language

Learner's Book page 206–208

Guidelines for implementing these activities

- Revise all the words that could have been forgotten or perhaps not completely understood before.
- You can ask learners to make a reference section at the end of their exercise books.
- Another way to make sure learners *do* engage with the words and learn them is to ask them to work in groups and make posters for the classroom wall with simple explanations of the words. Check the definitions or explanations before they make the poster though.
- Posters can be made with cardboard from boxes or by gluing recycled A4 sheets together if poster paper or board is not available.
- Learners do the activities as quickly as possible as this is revision.

Remedial and extension

Remedial: Repetition when learning new words is essential. In writing a list of words, learners also consolidate the words. Use the roots of words to make it easier to remember what the words mean, for example, binomial: the bi-prefix means two as in bicycle; similarly trinomial means three as in tricycle, and so on.
Extension: If learners complete the activity quickly, they can be asked to design the poster mentioned above, while the other learners do more practice exercises.

Suggested answers

Activity 1

1 2 **2** 1 **3** 3 **4** 2 **5** 3 **6** 1

Activity 2

1 **a** 4 **b** $4x$ **c** $\frac{6}{5}$
d −2; three x factors; y **e** Degree is 3 **f** −5

2 **a** No, only one term **b** Yes
c Yes **d** No, variable in the denominator
e No, variable in the denominator **f** Yes
g No, variable in the denominator **h** Yes

Algebraic notation

Activity 3 Use algebraic notation and simplify expressions

Learner's Book page 209

Guidelines for implementing this activity

- In this section, learners revise more of the algebraic language that they have already encountered.
- It is very important to spend time on *like* and *unlike* terms, because this is something that learners often find challenging.
- Also emphasise that like terms are only necessary for adding and subtracting. Division and multiplication can take place between any two or more terms.
- Reminders for like terms:
  - Are the letters the same?
  - If they are the same do they have the same exponent?
- Remind learners about the commutative property of multiplication, for example, abc^2 is the same as c^2ab and it is therefore a good idea to always organise the variables according to the algebraic conventions of alphabetical order and descending powers of the first variable.
- Learners do Activity 3 independently.

Remedial and extension

Remedial: One way to emphasise like terms is to get learners to make "play money" or to Photostat some coins and/or notes for them to cut out. Ask learners to simplify the following expression: $3x + 2xy + 5x$. If they give the answer as $10xy$ ask them to do the following:
$3 \times$ R50 + $2 \times$ R100 + $5 \times$ R50. Learners should be able to see that there are $8 \times$ R50 notes + $2 \times$ R100 notes and together they do NOT give us $10 \times$ R150 notes! Give learners a few more examples to try out with money before redoing the activity.
Extension: Give learners a few more examples that are more complex. Include some examples with fractions and decimal coefficients.

Suggested answers

1 a $6p^3 + 7p^3 = 13p^3$ b $5a + 3b$
c $13y$ d $7x^2 - 2x$
e $9ab - 7xy$ f $7r^2t - 10rt$
g $-6xy + 12xy^2$ h $9x^3 - 7y^2$

2 a $-5 + 7x - 2x^2 - x^3 + 3x^4 + 19x^5$ b $7 + 19y + 2y^2 - 18y^3$
c $-18 + 4x + 10x^2$ d $-10 + 5x^2 + 6x^3$
e $2 + 8x - 5x^2 - 3x^3 + 2x^4$

3 a 5 b 3 c 2 d 3 e 4

Using algebraic language; Interpreting algebraic language

Activities 4–6 **Translate into algebraic language; Interpret algebra; Write in algebraic language**

Learner's Book pages 210–211

Guidelines for implementing these activities

- Before starting this section, remind learners that Mathematics is a tool to solve problems and that we need to convert problems that are in words into mathematical symbols and operators so that we can process them.
- This section should be very useful for learners when doing "word problems" or problem-solving.
- The section *Using algebraic language* in the Learner's Book can be made into a poster in the Mathematics classroom.
- Let learners do Activity 4 with a partner. It helps when they verbalize the words aloud, and they can also check each other's understanding.
- Remind learners that the questions in Activity 4 generate expressions and not equations. They would need input values to find output values.
- Activity 5 can also be done in pairs. Recommend that learners use the algebraic language list in the Learner's Book to assist them where necessary.
- Activity 6 can be done independently as learners have had some practice at these already.

Remedial and extension

Remedial: With word sums and word problems it helps for learners to read aloud and discuss their thoughts until they are more confident. Also, if they read aloud to you, you can check their thinking and correct where necessary. You can also ask learners to write the words in the Learner's Book on a "prompt sheet" which they can refer to quickly. When they write the words down, it also consolidates their learning.
Extension: Give learners some Olympiad problems to work through. You can find past papers for the South African Maths Olympiad site at http://www.samf.ac.za/QuestionPapers.aspx?AspxAutoDetectCookieSupport=1
or alternatively contact the South African Maths Foundation at:
South African Mathematics Foundation Private Bag X173 Pretoria 0001
Tel +27 (0)12 392 9372
Fax +27 (0)12 392 9312
Email: info@samf.ac.za
Physical Address:
Didacta Building 211 Skinner Street Pretoria Central 0002
Encourage learners to participate in all Mathematics competitions that are available. Participation encourages enthusiasm about Mathematics and one of the other spin-offs is that learners that do participate are often offered bursaries at universities and colleges.

Suggested answers

Activity 4

1	$x + 5$	2	$x + 3$	3	$x + 8$	4	$x + 3$	5	$x + 7$
6	$x - 5$	7	$2x - 4$	8	$4 - x$	9	$x - 10$	10	$10x$
11	$2x$	12	$\frac{10}{x}$	13	$5x - 11$	14	$x + 2y$	15	$x - 2x$
16	$\frac{1}{2}x - 10$	17	$\frac{1}{2}x + 2y$	18	$4(x + 5)$				

Activity 5

There may be more than one correct answer. Some suggestions are:

1 A number minus three
2 A number multiplied by three is added to three
3 Two minus a number
4 A number added to another number
5 A number added to double another number
6 Half a number or a number divided by two
7 The sum of a third of a number and double the same number
8 Five divided by a number and added to five times another number

Activity 6

1 $6k$ 2 $3n$ 3 $4y$
4 a $2m$ b $3m$ c $4 \times 3m = 12m$
5 a $P = 2(2x^2 + x^2) = 2(3x^2) = 6x^2$
b $P = 2(5x + x + 5x + x + 3x + 8) = 2(15x + 8) = 30x + 16$

UNIT

2 Simplifying algebraic expressions

Unit overview

Learner's Book page 212
Recommended pacing: 2,25 hours

This unit focuses on the following:

- Adding expressions
- Subtracting polynomials
- Multiplying expressions
- Dividing algebraic expressions.

Resources: Learner's Book; calculator; exercise book

Background information

In Grade 8, learners were introduced to expanding and simplifying algebraic expressions. They used the commutative, associative and distributive laws for rational numbers and laws of exponents to:

- add and subtract like terms in algebraic expressions
- multiply integers and monomials by monomials, binomials and trinomials
- divide monomials, binomials and trinomials by integers or monomials.

In Grade 9 learners revise the work done in Grade 8 and do more examples.

Teaching guidelines

The skills used in this section are a revision of work done in Grade 8. Some learners may not have fully grasped the different operations in Grade 8 or may have forgotten them, so it is necessary to revisit these activities in detail. It is also necessary to correct the work done in this section so that you can identify any consistent errors. Although it is time consuming, it is the best investment that can be made in learners being able to continue doing Mathematics to Grade 12 and beyond. Have extra worksheets ready for drill and practice when required in this section.

Make sure that you use the terms that learners revise in Unit 1, often in class. Ask learners constantly to identify, for example, what the order of a polynomial is even if they are doing addition or another simplification on the polynomial; ask how many terms there are in a polynomial.

Adding expressions

Activity 1 **Add polynomials**

Learner's Book page 212

Guidelines for implementing this activity

- Remind learners about *like terms* again. Use the money analogy mentioned in the Remedial section of the previous unit if necessary.
- Remind learners about the commutative and the associative laws in Mathematics.
- Work through the examples in the Learner's Book. Some learners may prefer to use the column method while others may prefer to use the linear method.
- Use the same example and show how it works with both methods. This will give learners an opportunity to evaluate which method they prefer.
- Learners do Activity 1 independently.
- Adding in algebra is such an important basic skill and it would be useful if you corrected the work and gave learners feedback about any consistent errors.

Remedial and extension

Remedial: Provide lots of additional practice at identifying like terms. Once they have identified the like terms, ask them to group them together and add the ones that have like terms. If they are still making some mistakes, ask them to use the "play money" again.

Extension: Give learners a few more complex examples including some word problems to do. Challenge learners to design five challenging examples and solve them. Spend some time giving feedback and checking for errors.

Suggested answers

1 a $7x$ b $4c + 3x + 5y$ c $14y - 10p$
d 0 e $3x^2 - x + 3$ f $20y^3 - 4y^2 - 9$
2 a $11x^2 - 5x + 5$ b $-6x^3y - 2x^2y + 10xy$ c 0
d $11x + 2$ e $5a^2b^2 + 4abc^2 + 7b^2$ f $-18t^2 + 17t$
3 a $P = 4x - 1 + x + 2 + 7x - 3 = 12x - 2$
b $4x^2 - 2x + 1 + 2x^2 + x - 1 + x^2 + 2 + 7x^2 + x - 3$
$= 14x^2 - 1$

Subtracting polynomials

Activity 2 Write additive inverses and subtract polynomials

Learner's Book page 214

Guidelines for implementing this activity

- Remind learners that like addition, we can only subtract if there are like terms.
- Explain what an *additive inverse* is. Give a few examples on the board and have learners call out the answers or write them in their exercise books. They must do these very quickly.
- Example 1 shows how to use the additive inverse to subtract.
- In Example 2 it is necessary to distribute a minus sign to all the terms inside the bracket, or in other words to add the additive inverse of all the terms inside the bracket. This is generally a very widespread mistake; learners will only distribute the minus to the first term and not the rest.
- Emphasise that all terms inside the bracket must change to their additive inverse if there is a negative sign in front of the bracket.
- Learners do Activity 2 independently.
- Like the addition in algebra, this is a very important basic skill. When marking focus particularly on the fact that the minus is distributed to all the terms in the brackets.

Remedial and extension

Remedial: Provide a lot of additional practice at finding like terms as well as giving the additive inverses of terms. Have learners use a highlighter or pencil to make sure that they distribute the minus sign inside the brackets.
Extension: Give learners a few more complex examples. Include some examples with fraction and decimal coefficients.

Suggested answers

1 a $-7x$ b x^2 c $9y^2$
d $-x + 2$ e $-p - q$ f $-x^3 - 5z + 3$
2 a $-4p - 13$ b $5x + 14$ c $-x - 4y$
d $3x^2 - x$ e $-2y^3$ f $-2x^2 - 2x - 33$
3 Difference: $-3x + 8y - 9z$; Sum: $-x + 5y + 2z$; Final difference: $2x - 3y + 11z$.
4 $-23x + 6y - 11$

Multiplying expressions

The product of two monomials; The product of a monomial and a polynomial

Activities 3–4 Find products; Multiply monomials by polynomials

Learner's Book pages 215–216

Guidelines for implementing these activities

- Remind learners from the outset that we can multiply and divide any two or more terms in Algebra; they do not have to be like terms.
- Revise the exponent laws of multiplication: $a^m \times a^n = a^{m+n}$. Make sure that learners remember that this applies to bases that are the same.
- Work through examples 1 and 2 in the Learner's Book.
- Ask learners to follow a pattern when multiplying, for example:
  - first the signs
  - then the number coefficients
  - then the different letter variables one by one.
- Learners do Activity 3 independently.
- Do the next worked examples 1 and 2 in the Learner's Book with learners.
- Use the terms monomials, binomials, trinomials and polynomials often so that learners become confident with their meanings.
- Make sure that learners multiply every term in the brackets by the monomial.
- Learners do Activity 4 independently.
- Because of the importance of both these activities, correct their work and provide feedback.

Remedial and extension

Remedial: Provide a lot of practice with this important basic tool in Algebra. Make sure they multiply the signs, numbers and variables in that order. Check that they are multiplying all the terms inside brackets. Make sure that learners correct any of the questions that they get wrong in these two activities.
Extension: Give learners a few more complex examples such as question **2** of Activity 4.

Suggested answers

Activity 3

1 **a** $60n$ **b** $-10n$ **c** $-24mn$
d $35p^2q$ **e** $-5xy^2z$ **f** $12ab^2c^2$
2 **a** $8x^5$ **b** $22a^4b^2$ **c** $4x^3y^5$
d $24a^3$ **e** $96x^4y$ **f** $-48a$

Activity 4

1 **a** $21x^2y^2 + 15x^2y - 33xy$ **b** $20x^5 - 4x^4 - 12x^3 - 16x^2$
c $10x^2y^2 - 40x^2y + 45xy^2$ **d** $-48a^2 + 96a^3 - 24a$
e $-3x^4 + x^3 - x^2$ **f** $-15y^4 - 21y^3 + 9y^2$
g $4a^7 - a^6 + 2a^5$
2 **a** $12a^2 - 4ab - 15a^2 - 3ab = -3a^2 - 7ab$ **b** $8p - 2pq + 4pq - 7p = p + 2pq$

Dividing algebraic expressions

Dividing binomials and trinomials by monomials

Activities 5–6 **Divide algebraic expressions; Divide polynomials by monomials**

Learner's Book pages 216–218

Guidelines for implementing these activities

- Revise division and emphasise what the division line in a fraction means as seen in Example 2 in the Learner's Book.
- Make sure that learners apply the distributive law and divide all the terms in the numerator if possible.
- Explain how to separate the terms of the numerator as shown in Example 2 in the Learner's Book.
- Make sure that learners do all the questions in Activity 5 and Activity 6.
- As with the other operations, this is one of the basic skills in Algebra and it is best to correct work giving individual feedback in learner's exercise books. Check for any consistent errors that need attention.

Remedial and extension

Remedial: Spend time with learners who find this section challenging. Go through the steps of the worked examples again until you are sure that they understand each step and can do the calculation as required. Ask learners to do a few calculations with a friend so that they can verbalise each step. Alternatively ask them to write in sentences next to each step what they have done. Drill and practice does make a difference, but learners need to have the answers so that they can check their own work or you need to correct them. Computer programs are available that can also give immediate feedback and there are even programs that learners can do on their cell phones if any of these are available. However, good pen and paper exercises and extra worksheets are still the best!
Extension: These algebraic fractions can get quite complex and it is possible to give learners some very challenging examples to do. Questions **9** and **10** of Activity 6 involve a few more algebraic steps and are a good example of the type of exercises you can give to learners. It is also useful to show learners how to correct their work and ask them to assist other learners by working through their answers and pointing out errors.

Suggested answers

Activity 5

1	**a** $5a$	**b** $-4m$	**c** ab	**d** $\frac{-2}{m}$
2	**a** $-3a$	**b** $-2xy$	**c** $5qr$	**d** $-\frac{16}{m}$

Activity 6

1 $x - 3$ **2** $-7x - 2y + 5$

3 $\frac{4}{3}x^2 - 2x$

4 $-4x$

5 $-2x^2 + 3x$

6 $x - 2$

7 $5x^2 - 4x + 2$

8 $\frac{9mn - 15}{2}$

9 $\frac{y^3 - 5y^2 + 7y + y^3}{y} = y^2 - 5y + 7 + y^2 = 2y^2 - 5y + 7$

10 $\frac{14x^3 - 28x^2 - 42x}{7x^2} = \frac{2x^3 - 4x^2 - 6x}{x^2} = 2x - 4 - \frac{6}{x}$

UNIT

3 Further algebraic manipulations

Unit overview

Learner's Book page 219
Recommended pacing: 1,5 hours

This unit focuses on the following:
- The product of two binomials
- Multiplying two identical binomials
- Multiplying binomials of the kind: $(a - b)(a + b)$
- Simplifying expressions
- Squares, cubes, square roots and cube roots of algebraic terms
- Substitution

Resources: Learner's Book; calculator; exercise book

Background information

In Grade 8, learners did the following:
- Simplified algebraic expressions involving the above operations
- Determined the squares, cubes, square roots and cube roots of single algebraic terms or like algebraic terms
- Determined the numerical value of algebraic expressions by substitution.

In Grade 9, revise the work done in Grade 8 and then extend algebraic manipulations to include:
- multiplying integers and monomials by polynomials
- dividing polynomials by integers or monomials
- the product of two binomials
- the square of a binomial.

Teaching guidelines

In this unit, learners will use the basic skills that they have revised in units 1 and 2 and apply them to more and more complex situations. Make sure that the basics are very well grounded before continuing with this section. At the beginning of every lesson, take a few minutes to go through the basics of the four operations in Algebra again. Give learners four or five examples as a warm-up each day so that they hone their skills.

Work through the worked examples carefully making sure that learners understand each step. Remind them that these simplifications are just applications of the ones done previously. Ask learners to verbalise their steps as this will help consolidate their thinking of the processes involved.

The product of two binomials

An easy way to remember how to multiply two binomials; More complex expressions

Activities 1–2 **Multiply two binomials; Multiply more complex expressions**

Learner's Book pages 221–222

Guidelines for implementing these activities

- Spend a few minutes revising all four operations in Algebra. Remind learners about like and unlike terms for addition and subtraction and in multiplication and division to do the signs, number coefficients and variables in order. Write a few examples on the board that learners can do as quickly as possible and check before continuing.
- Work through the examples in the Leaner's Book on multiplying two binomials. There are a number of ways to remind learners how to remember all the terms that need to be multiplied, of which the 'FOIL' method is one. It is important that learners understand that the distributive law is being used and that all terms in the one bracket has to be multiplied by all the terms in the other bracket.
- Let learners work with a partner for the first four or five questions of Activity 1 and then do the rest independently. Make sure that they correct any answers that they do not get correct the first time.
- Work through the example in more complex expressions. Remind learners that this is not new. The order of operations is brackets first, so they need to simplify the brackets before multiplying all the terms by the term in front of the brackets.
- Learners work through Activity 2 independently.

Remedial and extension

Remedial: Mathematics really depends on the basics being well grounded. Revise the basics again before going to the more complex applications. Provide a lot of opportunities to practise.
Extension: Give learners some examples with fractions and decimals and also a few where all the variables are different, for example:
$(3a + 4b)(5c + 3d)$; $\frac{1}{2}a(c - 3d)(p + q)$; $\left(\frac{1}{2}x + \frac{3}{x}\right)\left(\frac{5x}{3} - \frac{1}{3x}\right)$; $0{,}9a\left(\frac{7a^3}{b^3} + 3c^5\right)\left(\frac{7a^3}{b^3} - 3c^5\right)$

Suggested answers

Activity 1

1 $x^2 + 12x + 35$ 2 $x^2 - 10x + 21$ 3 $x^2 + x - 12$
4 $3p^2 + pq - 2q^2$ 5 $x^2 - xy - 6y^2$ 6 $28 + 3x - x^2$
7 $2x^4 + x^2y - y^2$ 8 $4a^2 - 27ab + 18b^2$ 9 $8x^2 - 2xy - 21y^2$
10 $8a^2b - 12a^2 + 2b^2 - 3b$ 11 $8x^2 + 2x^2y - 4y - y^2$ 12 $8x^3 - 6x^2 - 12x + 9$

Activity 2

1 $2(a^2 - ab - 12b^2) = 2a^2 - 2ab - 24b^2$
2 $3(2x^2 - 5xy - 12y^2) = 6x^2 - 15xy - 36y^2$
3 $-5(-x^2 - 5x - 6) = 5x^2 + 25x + 30$
4 $2p(3q^2 - 10q - 8) = 6pq^2 - 20pq - 16p$

Multiplying two identical binomials; Multiplying binomials of the kind: $(a - b)(a + b)$

Activities 3–4 Multiply identical binomials; Multiply special binomials

Learner's Book pages 223–224

Guidelines for implementing these activities

- Make sure that learners can multiply two binomials. Revise this again if necessary, or point learners to the previous worked examples to assist them.
- Remind learners about the expanded form, that is, $x^2 = x \times x$.
- In the same way when a binomial is squared, when it is written in expanded form it becomes for example, $(a + 2b)^2 = (a + 2b) \times (a + 2b) = (a + 2b)(a + 2b)$.
- Now learners can use the FOIL method or any other that they are comfortable with to find the product and write the answers.
- Work through the examples in the Learner's Book.
- Learners do Activity 3 independently.
- Work through the special binomials also known as "the difference of two squares". Let learners discover that the middle terms are additive inverses of each other.
- Emphasise again and again that this is only the case where there is a sum of two terms in one bracket multiplied by the difference of the same terms in the other bracket. It does not apply to binomials with the same sign in both brackets.
- This is a very important simplification and learners should be reminded to constantly look out for the pattern $(a - b)(a + b)$.
- Activity 4 is mixed so learners need to be sure before they start what the difference is between $(a - b)(a + b)$ and $(a - b)^2$.

Remedial and extension

Remedial: Generate a few extra worksheets containing similar questions as in the activity for learners to do as additional practice. Talk them through the fact that these "special cases" are not unique but an application of what they have been doing before.
Extension: Challenge learners to write the answers for binomials squared and difference of squares in one step. Can they write a formula for how they do this one step? Ask them to use a few examples to test if their proposed formula works.

Suggested answers

Activity 3

1 $4x^2 - 12x + 9$

2 $16x^2 - 24x + 9$

3 $9m^2 - 12mq + 4q^2$

4 $4x^2 - 12x + 36$

5 $4x^2 + 20x + 25$

6 $9y^2 - 42y + 49$

7 $25a^2 + 30ab + 9b^2$

8 $36x^2 - 12x + 1$

9 $x^2 - 2x + y^2$

10 $4x^2y^2 - 20xy + 25$

Activity 4

1 $x^2 + 4x + 4$

2 $p^2 + 4pq + 4q^2$

3 $9x^2 - 6x + 1$

4 $4x^2 - 4xy + y^2$

5 $p^2 - q^2$

6 $9x^2 - 4y^2$

7 $a^2 - 9$

8 $9a^2 - 24ab + 16b^2$

9 $16p^2 - 49$

10 $4p - \frac{1}{q^2}$

11 $25a^2 + 20\text{ab} + 4b^2$

12 $64a^2 + 144\text{ac} + 81c^2$

13 $x^4 + 4x^2y^2 + 4y^4$

14 $m^4 - 6m^2n^2 + 9n^4$

Simplifying expressions

Activity 5 Simplify mixed expressions

Learner's Book page 224

Guidelines for implementing this activity

- Mixed examples test all the basic skills that have been practised until now.
- Spend a little time at the beginning of the lesson recapping the operations again. Work through the example and possibly give a few more before learners start on Activity 5.
- Let learners who can continue without the revision start directly on Activity 5.
- Learners can work in pairs on Activity 5 so that they can remind each other of all the skills that need to be applied.
- It is a good idea to mark this activity to check that learners have mastered the necessary skills.

Remedial and extension

Remedial: Sometimes learners find it easier to break the components of such complex polynomials into separate terms. They then simplify the separated terms as far as possible and then add them together at the end. Alternatively, learners can write each term in a different colour and all the work done on that term in the same colour.
Extension: Ask learners to devise four complex expressions for their friends to solve. They must work out the answer before swapping with a friend.

Suggested answers

1 $x^2 + 6xy + 9y^2 - x^2 + 2xy + 15y^2 = 8xy + 24y^2$

2 $1 - a^2 + 2ab - b^2 = -a^2 + 2ab - b^2 + 1$

3 $x^2 - x - 2 + x^2 - 1 = 2x^2 - x - 1$

4 $5x(x^2 - 16) - 5x^3 + x = 5x^3 - 80x - 5x^3 + x = -79x$

5 $25y^2 - \frac{1}{25y^2} - \frac{1}{25y^2} = 25y^2 - \frac{2}{25y^2}$

Squares, cubes, square roots and cube roots of monomials

Squares and cubes of monomials

Activity 6 Find squares and cubes of monomials

Learner's Book page 225

Guidelines for implementing this activity

- Remind learners what the *expanded form* means.
- Revise the exponent laws. Make sure that learners practise a few examples before starting this lesson.
- Work through all the examples in the Learner's Book. Constantly remind learners of the basics that are being applied.
- Work through an example or two with negative monomials being squared and cubed. Clearly show the difference between the negative sign being outside brackets and inside, for example, $(-3mn^3)^2$ and $-(3mn^3)^2$.
- Challenge learners to work through the activity as quickly as possible.

Remedial and extension

Remedial: Make sure that learners understand algebraic multiplication very well. Remind them about multiplying signs, number coefficients and variables in order. Provide additional examples especially where the signs are different inside and outside brackets.
Extension: Questions **7** and **8** are more challenging for learners.

Suggested answers

1 $-8x^9$
2 $9y^6$
3 $3(25x^4) = 75x^4$
4 $225x^4$
5 $a^2b^4 \times a^3b^3 = a^5b^7$
6 1
7 $\frac{x^4}{4y^2}$
8 $\frac{8x^6y^3}{16x^4y^4} = \frac{x^2}{2y}$

Finding the square root and cube root of monomials

Activity 7 Find square and cube roots of monomials

Learner's Book page 227

Guidelines for implementing this activity

- Roots often pose a problem for learners so it is necessary to revise how to take numerical square and cube roots. Remind learners how to do prime factorisation and how this can be used to take roots.
- Remind learners about the order of operations where roots are involved. Explain that terms under a root sign are effectively in a bracket and should be simplified first. Also, roots cannot be taken of the individual terms under a root sign, only factors.

- Learners do Activity 7 independently.

Remedial and extension

Remedial: Make sure that learners are very comfortable with what a square, cube, square root and cube root are at the outset as they may have forgotten. It is useful to revise the laws of exponents again before doing the numerical square roots because the laws can make the process quicker. Give learners extra practice and check their work to see if there are any conceptual errors.
Extension: Give learners a mixed exercise that tests their conceptual understanding of the order of operations.

Suggested answers

1	a	$4x^2$	b	$9x^3y^2$	c	$10m^4$	d	$3y^4$
	e	$6m^3$	f	$4x^2 \times 3y = 12x^2y$	g	$2x + 3$	h	$\frac{x}{10}$
	i	$2a^2$	j	$3x^3$	k	$\frac{x}{4}$	l	$5x^6$
2	a	$4n$	b	$10x^2$	c	$5n^3$	d	$12y^2$
3	a	$3x$	b	$5y^3$				

Substitution

Activity 8 **Substitute values into algebraic expressions**

Learner's Book page 228

Guidelines for implementing this activity

- Substitution is used so often throughout Mathematics.
- Remind learners about input and output values in relationships. Also revise other equivalent representations of expressions such as tables.
- Work through the examples in the Learner's Book.
- Make sure that learners use brackets around the substituted value.
- Once they have written the expression they can use a calculator to find the answer but it is not essential.
- Learners do the activity independently.
- Substitution is a very important skill for Mathematics. Correct the activity and check that learners are using the brackets when substituting.

Remedial and extension

Remedial: Give learners a few easy examples to start with in order to build their confidence. Substitution is used often but needs to be practised often too. Provide some drill and practice and ask learners to be very careful about substituting the correct value in brackets for the appropriate variable.
Extension: Questions **5** and **6** are a good challenge for learners. Again, make use of some Olympiad past papers to stimulate learners interest in doing Mathematic competitions. It is useful to have a few copies of past Olympiad papers available in class so that when learners finish quickly they can work on these. It is quite rewarding for them when they have a friend who is doing something similar and they can discuss their answers. Keep a copy of the answers separately and only distribute them once you are sure that a good attempt has been made.

Suggested answers

1 **a** $-16 - 7 = -23$ **b** $\frac{13}{10}$ **c** $(-28)(-9) = 252$

2

m	1	2	3	4
$m + \mathbf{3}$	4	5	6	7

3

N	–2	0	2	4
L	–1	5	11	17

4

x	–2	–1	0	1	2
y	–3	–1	1	3	5

5 **a** 148,9 °C **b** No, 220 °F = 104,4 °C

6 **a** R135

b $140 - 100 = 40$ minutes $\therefore 40 \times 60 + \text{R}135 = \text{R}24 + \text{R}135 = \text{R}159$

c $0{,}60(x - 100) + 135$

Chapter 8 Revision

Learner's Book page 230

Encourage learners to review the content covered before attempting the revision activities. The revision activities should be used to assess learners' progress thus far, and to assess where remediation may be required.

Suggested answers

1

Polynomial	Degree	Number of terms	Constant	Numerical coefficient of x^2
$4x^3 - 2xy + 7$	3	3	7	0
$3x^2 + 5x^2 - 7x^4 + 8x$	4	3	0	8
$4x + 5$	1	2	5	0
$7x^2 + 3$	2	2	3	7
$-3 + 5x^2$	2	2	-3	5
$24x^4$	4	1	0	0

2 Last column of table above.

3 **a** xy **b** $-8x^2y - 3xy^2$

4 $2(3x + y) + 2(2x + y) = 6x + 2y + 4x + 2y = 10x + 4y$

5 **a** $-8x^3$ **b** $6n$

c $3ab$ **d** $12pq - 8p$

e $2 - 5x + 10y = -5x + 10y + 2$ **f** $x^2 - 2x - 8$

g $4x^2 - 12x + 9$ **h** $1 - 16p^2$

6 **a** $-4 + 18 = 14$ **b** 5

CHAPTER

9 Algebraic equations

Chapter overview

Learner's Book pages 231 to 250
Recommended pacing: 4 hours

This chapter focuses on:

Unit 1:	**Revision: Linear equations**	1 hour
	Equations and expressions	
	Solving equations	
Unit 2:	**More difficult linear equations**	1 hour
	Simplifying equations with brackets and many terms on both sides	
	Checking solutions	
Unit 3:	**Equations with fractions**	1 hour
	Solving equations that contain fractions	
Unit 4:	**Using equations to solve problems**	1 hour
	Creating equations to solve word problems	
	Number problems	
	Problems involving age	
	Problems involving geometry	
	Problems involving formulae	
Chapter 9 Revision		25 minutes

UNIT

1 Revision: Linear equations

Unit overview

Learner's Book page 232
Recommended pacing: 1 hour

This unit focuses on the following:

- Solving equations through inspection
- Solving equations by using inverse operations
- Solving equations with variables

Resources: Learner's Book; calculator; exercise book; squared paper or graph paper

Background information

In Grade 8, learners revised how to:

- set up equations to describe problem situations
- analyse and interpret equations that describe a given situation
- solve equations by inspection
- determine the numerical value of an expression by substitution
- identify variables and constants in given formulae or equations.

Learners also learnt how to use substitution in equations to generate tables of ordered pairs and extend solving equations to include:

- using additive and multiplicative inverses
- using laws of exponents.

In Grade 9, learners revise the work done in Grade 8. Solving equations is extended to include using factorisation and equations of the form: a product of factors = 0.

Teaching guidelines

The work covered in Grades 7 and 8 needs to be revised before continuing with solving equations by factorising and equations where the product of their factors is zero.

Chapter 8 dealt with Algebraic expressions which gave learners the terminology and tools to manipulate algebraic expressions. It is important to explain at the outset that the difference between expressions and equations is what they represent. An expression represents a relationship between variables while an equation shows two expressions that are equal.

Learners have been solving equations informally and formally since Grade 1. In Grades 7 and 8, the methods of solving were formalised by using additive and multiplicative inverses as well as the laws of exponents. Many equations can be solved by inspection and it is important to emphasise this tool when dealing with equations.

When equations become more complex, it is necessary to use inverse operations. This links back to the work done on reversing the order and using the inverse operation dealt with in Chapter 7 Functions and Relationships.

The reason we develop equations is to solve problems, although we start with the mechanics of solving equations. It is important to emphasise to learners that the word problems are representative of real life situations where we could use Mathematics to solve problems.

Equations and expressions; Solving equations

Solving equations by inspection

Activity 1 Solve by inspection

Learner's Book page 234

Guidelines for implementing this activity

- Start the lesson by explaining the difference between expressions and equations. Learners have dealt with expressions in Chapter 8 and should be familiar with them.
- Explain that equations link two equivalent expressions with an 'is equal to' sign. The values of the expressions are equal although they may appear to be different.
- The first example in the Learner's Book starts with input values for an expression. Both the input and output values are put into a table. This links to Chapter 7 where the different equivalent forms of representation of a relationship was discussed.
- Equations can be solved, but expressions cannot.
- Emphasise that solving by inspection is a very handy tool. Ask learners to write in sentences how they do the process by inspection. This makes an important link to solving using inverse operations and reverse flow diagrams in the next section.
- Learners should do Activity 1 as quickly as possible.

Remedial and extension

Remedial: We have been doing Mathematics for many years, but we sometimes forget how confusing the myriad of symbols on a Maths page is for a learner who has not managed to grasp all of them. It is useful to write the equations with squares instead of an x as the variable. This reminds learners that they have been doing this work since they were in the Foundation phase. Once they are more confident, explain that the x represents/replaces the square used in lower grades. Provide more examples for learners to work on.
Extension: Provide learners with a few equations with fractions to solve by inspection.

Suggested answers

1 $x = 2$	**2** $x = 7$	**3** $x = -1$	**4** $x = -1$
5 $x = 3$	**6** $x = 5$	**7** $x = -3$	**8** $x = -3$
9 $x = 6$	**10** $x = 12$		

Solving equations using inverse operations

Activities 2–3 **Write reverse flow diagrams; Solve equations using inverse operations**

Learner's Book pages 235–236

Guidelines for implementing these activities

- Remind learners about the flow diagrams they used in Chapter 7. Give them one or two verbal descriptions and ask them to write the flow diagrams. Also revise inverse operations and reverse flow diagrams.
- Ask learners to solve $3x - 11 = 13$ by inspection. Ask them to write in words and sentences how they solved it by inspection.
- Ask one or two learners to write their explanations on the board.
- Ask learners to write the flow diagram for the equation and below it the reverse flow diagram.
- Learners do Activity 2 on their own.
- Link reverse flow diagrams to solving equations using inverse operations by doing Example 1 in the Learner's Book.
- Work through Examples 2 and 3 which give the learners the mathematical thinking behind each step.

Remedial and extension

Remedial: Remind learners what a variable is; that it is a letter that is used to represent a number that we don't know yet. A variable is generally the value we are trying to find. If necessary use squares instead of variables again until learners are comfortable. Encourage learners to "talk through the steps" either aloud to you or to a classmate. This will give an indication where there are problems with understanding.
Extension: Provide a few problems with decimals and fractions to solve. Provide learners with a few word problems to solve, for example: *There are 800 learners at a new school in an area that is growing quickly. It seems that every day another three new learners are entering the school. The school can take a maximum of 1 200 learners. If this trend continues, in how many days will the school be full?* Challenge learners to think through all the complexities here, for example: three new learners on each *school* day; how many days in a term?; if there were 800 learners on 12 January of a particular year, on which exact day will the school be full? Ask them to add more complexities and to write equations and solve the equations where appropriate.

Suggested answers

Activity 2

1 $30 \rightarrow -12 \rightarrow x$
2 $16 \rightarrow -14 \rightarrow \div 3 \rightarrow x$
3 $y \rightarrow -5 \rightarrow \div 7 \rightarrow x$

Activity 3

1 $x + 4 - 4 = 11 - 4; x = 7$

2 $x - 5 + 5 = 3 + 5; x = 8$

3 $-3x = -12; \frac{-3x}{-3} = \frac{-12}{-3}; x = 4$

4 $5x + 4 = 19; 5x + 4 - 4 = 19 - 4; 5x = 15; x = 3$

5 $3x - 61 = 19; 3x - 61 + 61 = 19 + 61; 3x = 80; x = \frac{80}{3} = 26\frac{2}{3}$

6 $3x + 2 = 29; 3x + 2 - 2 = 29 - 2; 3x = 27; x = 9$

7 $3x + 6 = -33; 3x + 6 - 6 = -33 - 6; 3x = -39; x = -13$

8 $2x - 25 = 17; 2x - 25 + 25 = 17 + 25; 2x = 42; x = 21$

9 $9x - 7 = -11; 9x - 7 + 7 = -11 + 7; 9x = -4; x = -\frac{4}{9}$

10 $4x + 88 = 91; 4x + 88 - 88 = 91 - 88; 4x = 3; x = \frac{3}{4}$

Equations with expressions containing the unknown on both sides

Activity 4 Solve equations with the unknown on both sides

Learner's Book page 236

Guidelines for implementing this activity

- Remind learners about the analogy of an equation being like scales. What we do to the one side of the scale we must do to the other side. Adding, subtracting, multiplying or dividing must be applied to both sides of an equation to keep it balanced.
- Ideally when we are solving equations, we would like the variable to be on one side of the equation and the coefficient of the variable to be 1. This is called *isolating* the variable.
- With that aim in mind, learners use inverse operations to move variables to one side of the equation and constants to the other side.
- Work through the example, making sure that learners understand every step. It may be necessary to do an additional example before learners tackle the activity by themselves.

Remedial and extension

Remedial: Try using an actual balance as follows. You can possibly borrow a scale from the Natural Sciences or Physical Sciences classes. Do a few examples like this before doing the activity.

1 Wrap some weights and call them x.
2 Write the equation $3x + 2 = 11$ on the board. Put three wrapped weights plus two regular weights in one pan of the scale and 11 regular weights in the other pan of the scale.
3 Explain that the wrapped packages represent the variable, x. You put three x weights and two regular weights to balance the 11 weights on the other side.

Remedial and extension continued

4 To solve the equation you only want one wrapped package on one side because that will be what that variable is worth.
5 To keep the scale balanced you need to do the same thing to both sides.
6 You need to start by removing the two regular weights from the pan with the wrapped packages, but then you will have to do the same to the other pan.
7 Underneath the equation on the board, write $3x + 2 - 2 = 11 - 2$
8 To get one wrapped package instead of three, you have to divide by three on both sides. (You can also multiply by $\frac{1}{3}$.)

Extension: Ask learners how they would go about checking their answers. Ask them to check each other answer as quickly as possible.

Suggested answers

1 $5x + 4 = 3x - 10$; $5x + 4 - 4 = 3x - 10 - 4$; $5x = 3x - 14$; $5x - 3x = 3x - 3x - 14$
$2x = -14$; $x = -7$
2 $3n - 5 = 13 - 6n$; $3n - 5 + 5 + 6n = 13 + 5 - 6n + 6n$; $9n = 18$; $n = 2$
3 $9p - 2 = 7 + 6p$; $3p = 9$; $p = 3$
4 $5q - 2 = q + 4$; $4q = 6$; $q = \frac{6}{4} = \frac{3}{2} = 1\frac{1}{2}$
5 $7x - 2 = 2x + 3$; $5x = 5$; $x = 1$
6 $8m - 13 = -6m + 7$; $14m = 20$; $m = \frac{20}{14} = 1\frac{3}{7}$
7 $4a - 61 = 19 + a$; $3a = 80$; $a = \frac{80}{3} = 26\frac{2}{3}$
8 $5a - 3 = 2a + 4$; $3a = 7$; $a = \frac{7}{3}$
9 $-2y + 5 = 2y + 9$; $-4y = 4$; $y = -1$
10 $3x - 4 = 8 - x$; $4x = 12$; $x = 3$
11 $13x + 4 = 5$; $13x + 4 - 4 = 5 - 4$; $13x = 1$; $x = \frac{1}{13}$
12 $6x + 9 = x + 4$; $5x = -5$; $x = -1$

UNIT

2 More difficult linear equations

Unit overview

Learner's Book page 237
Recommended pacing: 1 hour

This unit focuses on the following:

- Solving equations with variables
- Solving equations containing brackets
- Checking an answer to an equation

Resources: Learner's Book; calculator; exercise book; block paper

Background information

In the previous unit, learners revised methods for solving equations which they had learnt in Grade 8. In this unit, learners solve more complex equations that have to be simplified before the movement of variables to one side and constants to the other can begin.

Teaching guidelines

It is important that learners remember that the order of operations in terms of brackets, multiplication and division and lastly addition and subtraction is also important when solving complex linear equations that have several terms and brackets in them. Some revision of the order of operations may be necessary as well as a revision of simplifying from Chapter 8.

It is really important to emphasise sound mathematical thinking as learners move to more complex equations. The Think-Do examples provide the thinking that learners should be doing at each step. Refer learners back to these examples to check their thinking if they find certain questions challenging.

Simplifying equations with brackets and many terms on both sides

Simplifying equations with brackets on both sides

Activities 1–2 **Solve more difficult equations; Solve complex equations**

Learner's Book pages 238–239

Guidelines for implementing these activities

- Revise the order of operations by doing a few quick examples on the board. Alternatively, set a short 10-mark test or worksheet.
- Work through examples 1 and 2 in the Learner's Book. Emphasise that even though there are more terms, the principle of moving the variables to one side and then making its coefficient one, is still the aim. This is achieved by using inverse operations.
- Revise like and unlike terms.
- Learners do Activity 1.
- Check learners' answers to Activity 1 before continuing the lesson.
- Although learners did simplification in Chapter 8, it is useful to provide additional practice with this very important mathematical skill.
- Work through examples 3 and 4 on the board.
- Learners complete Activity 2 independently in their exercise books.
- Check that learners distribute the minus sign to all the terms inside a bracket, for example, Activity 2, questions **2**, **3** and **6**.

Remedial and extension

Remedial: Revise the basics of the inverse operations again. Slowly give learners more and more complex equations to solve, for example:

$x + 3 = 15$	[Just the inverse operation of addition]
$2x + 3 = 15$	[Inverse operation of addition and multiplication]
$2x + 3 = x + 15$	[Split variables; collect on one side]
$2x + 3 - x = 4x + x + 15 - x$	[Multiple split variables; collect on one side]
$2(x + 3) = 22$	[Equations with brackets]

Extension: Give learners equations that have decimals and fractions to solve.

Suggested answers

Activity 1

1. $5x - 4 = 2x + 4x + 2$; $5x - 4 + 4 = 6x + 2 + 4$; $5x - 6x = 6$; $-x = 6$; $x = -6$
2. $8x + 3 = 4x - 12 + x$; $3x = -15$; $x = -5$
3. $3x - 24 + 2x = 2x + 24$; $3x = 48$; $x = 16$
4. $8x - 8 = 5x + 12 - 6x$; $9x = 20$; $x = \frac{20}{9}$
5. $3a - 6 + 7a = -5a - 3$; $15a = 3$; $a = \frac{3}{15} = \frac{1}{5}$
6. $7b - b = -b - 8 + 2b$; $5b = -8$; $b = -\frac{8}{5}$

Activity 2

1. $2x - 8 = 3x - 6$; $-x = 2$; $x = -2$
2. $3x - 3 - x - 3 = 0$; $2x = 6$; $x = 3$
3. $2y - 6 = y - 3y - 6$; $2y + 2y = 0$; $y = 0$
4. $2a - 2 + 2a + 2 = 0$; $4a = 0$; $a = 0$
5. $5y + 10 + 3y - 15 = 9$; $8y = 14$; $y = \frac{7}{4}$
6. $3b + 12 = 6b - 6 - 5b + 10$; $2b = -8$; $b = -4$
7. $12k - 4 = 11k - 3k + 12$; $4k = 16$; $k = 4$
8. $2x - 3x - 3 = 4$; $-x = 7$; $x = -7$
9. $2(3x + 1) = 3(x - 2)$; $6x + 2 = 3x - 6$; $3x = -8$; $x = -\frac{8}{3}$
10. $-4(2x - 5) = -4x + 10$; $-8x + 20 = -4x + 10$; $-4x = -10$; $x = \frac{5}{2}$
11. $9(3x - 2) = 4(6x + 4)$; $27x - 18 = 24x + 16$; $3x = 34$; $x = 11\frac{1}{3}$
12. $2(x + 3) = 2(11 - x)$; $2x + 6 = 22 - 2x$; $4x = 16$; $x = 4$

Checking solutions

Activity 3 Check solutions

Learner's Book page 241

Guidelines for implementing this activity

- Do a quick revision of substitution in equations and solving.

- Remind learners how we check addition by using the inverse operation and likewise division and multiplication can be used to check answers of the other.
- When we check solutions of equations, we substitute the value into the original equation.
- This can be done mentally without writing down the steps.
- When it is done formally learners must use the LHS (Left–Hand Side) and RHS (Right–Hand Side) method shown in Example 1 in the Learner's Book.
- This is a very important tool in Mathematics and you should emphasise as often as possible that learners cannot simply substitute the value into both sides of the equation. The LHS MUST be done SEPARATELY from the RHS and compared at the end.
- Learners do Activity 3. Mark this exercise because of the importance of checking answers and the LHS-RHS method.

Remedial and extension

Remedial: Make sure that learners do questions **1a** to **1g**. Have them repeat any questions that they do not get correct the first time. Checking answers should become routine.
Extension: Provide a few word problems where learners write the equations themselves, solve them and then check their answers showing all their steps. Ask them to work together to see how to use their calculators to check their answers.

Suggested answers

1 a $7a - 3a = 3a - 3a + 20$
$\frac{4a}{4} = \frac{20}{4}$
$a = 5$

Check: LHS $= 7(5) = 35$
RHS $= 3(5) + 20 = 35$
$\therefore$ LHS = RHS, so $a = 5$ is correct

b $7c - 6 = c + 6$
$7c - c = c - c + 6$
$\frac{6c}{6} = \frac{6}{6}$
$c = 1$
Check: LHS $= 7(1) - 6 = 1$
RHS $= 1$
$\therefore$ LHS = RHS, so $c = 1$ is correct

c $1 - 1 + 2x + 5x = 22 - 5x + 5x$
$\frac{7x}{7} = \frac{21}{7}$
$x = 3$

Check: LHS $= 1 + 2(3) = 7$
RHS $= 22 - 5(3) = 7$
$\therefore$ LHS = RHS, so $x = 3$ is correct

d $2k = 3k + 4 - 5k$
$2k + 2k = -2k + 2k + 4$
$\frac{4k}{4} = \frac{4}{4}$
$k = 1$
Check: LHS $= 2(1) = 2$
RHS $= 3(1) + 4 - 5(1) = 2$
$\therefore$ LHS = RHS, so $k = 1$ is correct

e $9x - 5x + 4 - 4 = 5x - 5x + 6 - 4$
$\frac{4x}{4} = \frac{2}{4}$
$x = \frac{1}{2}$
Check: LHS $= \frac{9}{2} + 4 = 8\frac{1}{2}$
RHS $= \frac{5}{2} + 6 = 8\frac{1}{2}$
$\therefore$ LHS = RHS, so $x = \frac{1}{2}$ is correct

f $2y - 3y - 8 + 8 = 3y - 3y - 6 + 8$
$-y = 2$
$y = -2$
Check: LHS $= 2(-2) - 8 = -12$
RHS $= 3(-2) - 6 = -12$
$\therefore$ LHS = RHS, so $y = -2$ is correct

g $2a - 6 = a - 3a - 6$

$2a + 2a - 6 + 6 = -2a + 2a - 6 + 6$

$\frac{4a}{4} = \frac{0}{4}$

$a = 0$

check: LHS $= 2(0) - 6 = -6$

RHS $= 0 - 3(0) - 6 = -6$

$\therefore$ LHS = RHS, so $a = 0$ is correct

2 Check: LHS $= 8 - 1 = 7$

RHS $= 5 - 4(1) + 6 = 7$

$\therefore$ Mseni is correct

3 Check: LHS $= 2(10) = 20$

RHS $= 20 - 3(10) - 10 = -20$

$\therefore$ LHS $\neq$ RHS so her solution is incorrect.

UNIT

3 Equations with fractions

Unit overview

Learner's Book page 242

Recommended pacing: 1 hour

This unit focuses on the following:

- Solving equations that contain fractions.

Resources: Learner's Book; calculator; exercise book

Background information

Learners revised common fractions in Chapter 4 and have been using fractions since the Intermediate Phase. The previous units have covered solving linear equations of varying complexity. At this stage, the equations will be made more complex by including coefficients that are rational numbers.

Teaching guidelines

Because so many learners still find fractions very challenging, it is advisable to revise the four operations using fractions. Make sure that learners are confident about LCDs and how to calculate them numerically before including variables in the numerators.

Once again, emphasise the importance of sound mathematical thinking when solving these equations. Quick methods are sometimes fine at a less complex level, but when there are fractions involved learners should make sure they do a few more steps to ensure accuracy.

Solving equations that contain fractions

Activity 1 Solve equations with fractions

Learner's Book page 243

Guidelines for implementing this activity

- Revise adding, subtracting, multiplying and dividing fractions. Ask learners to redo the revision exercise at the end of Chapter 4 to refresh their memories, and/or provide similar questions for them to do.
- Work through the worked examples on the board. Example 1 starts with adding where the denominators are all the same. Example 2 looks at different denominators including adding whole numbers to fractions. Example 3 shows a slightly more complex fraction.
- Emphasise that once the equations can be written using numerators, the principles for solving the equations are the same as what learners have been practising in the rest of the chapter.
- Learners do Activity 1 independently.
- Provide learners with individual feedback by marking and correcting this activity.

Remedial and extension

Remedial: These activities really test learners' understanding of mathematical skills that must be really well grounded. Make sure that learners write down every step. When you mark the exercises identify basic mistakes and any consistent errors in their basic understanding that need to be corrected.
Extension: Challenge learners to do some of the steps mentally, but they must check their answers to make sure that they have not made jumps that are too big.

Suggested answers

1 $\frac{x}{2} = 4$
$\therefore x = 8$

2 $\frac{x}{2} - 4 = 6$
$\therefore x = 20$

3 $-\frac{x}{3} + 1 = 3$
$\therefore x = -6$

4 $\frac{2}{3}p + 6 = 22$
$\therefore p = \frac{3}{2} \times 16$
$\therefore p = 24$

5 $\frac{2}{5}n - 2 = \frac{2}{5}n + 4$ no value of n will satisfy this equation.

6 $\frac{1}{3}x - \frac{5}{3} = -\frac{2}{3}x + \frac{1}{3}$; $x = 2$

More complex fraction equations

Activity 2 Solve more complex equations with fractions

Learner's Book page 244

Guidelines for implementing this activity

- Remind learners how to find the LCD.
- Work through the step-by-step worked examples in the Learner's Book.
- Make sure that learners understand these examples well before tackling Activity 2.
- If necessary do more examples on the board.
- Let learners work with a friend for the first three or four questions before doing the rest of the questions independently.
- Remind learners to verbalise their steps to their friend while working together. They can check each other's understanding in this way and therefore correct each other if possible, or call for assistance.
- Correct learners' work and let them redo question that they got wrong.

Remedial and extension

Remedial: Make sure learners understand every step of the Think-Do worked examples. Get them to write out their steps next to two or three extra calculations.
Extension: Give learners a few examples with variables in the denominators to test their understanding. Make sure that they check their answers.

Suggested answers

1 $\frac{x}{7} = \frac{5}{3}$

$x = \frac{35}{3} = 11\frac{2}{3}$

2 $\frac{1-x}{2} = \frac{x+2}{3}$

$3 - 3x = 2x + 4$

$x = -\frac{1}{5}$

3 $\frac{x+1}{2} = \frac{x}{3}$

$3x + 3 = 2x$

$x = -3$

4 $\frac{2x-1}{7} = \frac{2x}{5}$

$10x - 5 = 14x$

$-4x = 5$

$x = -\frac{5}{4}$

5 $\frac{x+1}{2} = \frac{2x-3}{3}$

$3x + 3 = 4x - 6$

$-x = -9$

$x = 9$

6 $\frac{1}{2}(3x + 1) + \frac{1}{3}(2x + 10) = 0$

$9x + 3 + 4x + 20 = 0$

$13x = -23$

$x = -\frac{23}{13}$

7 $\frac{3x-4}{4} - \frac{3x-2}{2} = 2$

$3x - 4 - 6x + 4 = 8$

$-3x = 8$

$\mathrm{x} = -\frac{8}{3}$

8 $\frac{x+3}{4} - \frac{x-2}{8} = \frac{x}{2} + 1$

$2x + 6 - x + 2 = 4x + 8$

$-3x = 0$

$x = 0$

UNIT

4 Using equations to solve problems

Unit overview

Learner's Book page 245
Recommended pacing: 1 hour

This unit focuses on the following:

- Number problems
- Problems involving age
- Problems involving geometry
- Problems involving formulae

Resources: Learner's Book; calculator; exercise book

Background information

In Grade 8 learners revised setting up equations to describe problem situations. As this is a very important aspect of Mathematics. Setting up equations to describe problem situations is continued in Grade 9.

Teaching guidelines

Problem-solving has been a principal component of Mathematics since Mathematics education started. A problem is not a problem when it becomes routine to solve – it is then an exercise. With this in mind, this unit on problem-solving takes learners through processes that will facilitate the process of making problem-solving less of a problem and more of an exercise.

Problems are divided into four different types to make it easier to identify the unique features of each type. They are problems involving number; age; geometry and formulae. Several examples are used to show how to create equations to solve word problems.

It is often more difficult for learners to tease out the important information from a word problem. Even learners who are very talented at Mathematics shy away from word problems. Recent studies quote planning and monitoring as essential skills for the problem- solver. Assist learners to plan their progress through a problem. Confidence is built by achieving success and this builds the self-belief that is also a key feature of a good problem-solver.

Creating equations to solve word problems; Number problems

Activity 1 Solve number problems

Learner's Book page 246

Guidelines for implementing this activity

- Work through the steps given to solve word problems in the Learner's Book.
- Ask learners to design a mnemonic of the key words to remind themselves, for example,
  RILES: **R** Read
  I Identify
  L Let the amount be x
  E Equation
  S Solve
- Encourage learners to come up with a better mnemonic if necessary, which is more meaningful to them.
- Work through the examples with the learners.
- Let learners work in pairs to do Activity 1. They should check that they follow the steps and convert the word problem into an equation.
- If there is time, discuss each problem with them.

Remedial and extension

Remedial: Keep the problems simple at first so that learners can get used to using the steps and building confidence.
Extension: Use some past Olympiad exam papers for learners to hone their skills. The address for the South African Maths Foundation is given in Chapter 8.

Suggested answers

1 Let x be the number. $2(x + 10) = 28$; $2x + 20 = 28$; $2x = 8$; $x = 4$
The number is 4.
2 Let x be the number. $2(x - 8) = 4x$; $2x - 16 = 4x$; $-2x = +16$; $x = -8$
The number is -8.
3 Let x be the number. $\frac{1}{4}(7 + x) = 22 - 2x$; $\frac{7}{4} + \frac{x}{4} = 22 - 2x$; $7 + x = 88 - 8x$; $9x = 81$
$x = 9$. The number is 9.

Problems involving age

Activity 2 Solve problems involving age

Learner's Book page 247

Guidelines for implementing this activity

- Work through the examples with learners.
- Remind them to apply their steps (RILES in the mnemonic suggested here) to each example.

- It is a good idea to allocate the variable to the youngest age when doing these problems. This prevents fraction work which is always a little more difficult than using whole numbers.
- Let learners do the activity independently.

Remedial and extension

Remedial: One of the ways to make problem-solving an exercise and not a problem is a lot of practice. Problem-solving should be done on a continual basis throughout the year so that it becomes routine.
Extension: Challenge learners to make up a few problems themselves. Again, encourage them to do as many of the Maths competition-type questions as possible. Often these can be solved without equations, but they are still excellent practice at the process of problem-solving.

Suggested answers

1 Let the number of years $= x$
$32 + x = 3(8 + x)$
$32 + x = 24 + 3x$
$8 = 2x$
$x = 4$
In four years' time.

2 Let the son's age $= x$
$5x - x = 36$
$4x = 36$
$x = 9$
The son is nine years old now.

Problems involving geometry

Activity 3 **Solve problems involving geometry**

Learner's Book page 248

Guidelines for implementing this activity

- Revise perimeter and area and their formulae before starting this problem-solving activity.
- Also briefly remind learners of the sum of the internal angles of triangles. They will not have done this work since Grade 8 and may need a refresher.
- Algebra is not just used in the algebra section of the Mathematics text book, but in all sections. Emphasise this fact when dealing with the geometry problems. Explain to learners how important it is that they master the skills covered in Algebra so that they can excel in all sections of Mathematics.
- Let learners use their problem-solving mnemonic to work through the example and then to do Activity 3 independently.

Remedial and extension

Provide a few additional questions for learners to practise.

Suggested answers

1 $3x + 2x + x + 30° = 180°$
$6x = 150°$
$x = 25°$

2 $2(3x + 1 + 2x) = 22$
$6x + 2 + 4x = 22$
$10x = 20$
$x = 2$
Length of rectangle = $3x + 1 = 3(2) + 1 = 7$ cm
Breadth of rectangle = $2x = 2(2) = 4$ cm

3 $x + x + 3 + 4 + 7 = 26$
$2x + 14 = 26$
$2x = 12$
$x = 6$ cm
The missing sides are 6 cm long.

Problems involving formulae

Activity 4 **Solve problems that involve formulae**

Learner's Book page 249

Guidelines for implementing this activity

- Explain what a formula is compared to an equation as described in the Learner's Book.
- Work through the example on speed, distance and time given in the Learner's Book.
- Encourage learners to use the steps for problem-solving.
- Do an additional example or two if necessary, and in order to ensure that learners have the confidence to tackle the activity on their own.
- Learners should do the activity independently.

Remedial and extension

Remedial: Learners need to be able to translate from words to mathematical language. Encourage learners to write down an equation and not just find an answer by working in their head. They will need this skill in the higher grades. If they struggle to form an equation, ask them to go back to the list that they made in Chapter 8.
Extension: Have a few challenging problems on cards or worksheets. These can be handed out to learners throughout the year so that they can continue honing their problem-solving skills.

Suggested answers

1 $S = 3$ km/h

2 a $F = \frac{9}{5}(72) + 32 = 161{,}6$ °F

b $97 = \frac{9}{5}C + 32$
$485 = 9C + 160$
$9C = 325$
$C = 36{,}1\ °C$

3 a $V = A \times h$
$45{,}54 = 2{,}3 \times 3{,}6 \times h$
$h = 5{,}5$

b $V = A \times h = 8 \times 3{,}5 \times 4 = 112$

4 $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(30)^3 = 113\ 097{,}34\ cm^3$

Chapter 9 Revision

Learner's Book page 250

Encourage learners to review the content covered before attempting the revision activities. The revision activities should be used to assess learners' progress thus far, and to assess where remediation may be required.

Suggested answers

1 a $x - 5 + 5 = 8 + 5$
$x = -3$

b $-\frac{2x}{2} = \frac{6}{-2}$
$x = -3$

c $x \times \frac{3}{3} = 5 \times \frac{3}{1}$
$x = 15$

2 a $2x + 3 - 3 = 9 - 3$
$\frac{2x}{2} = \frac{6}{2}$
$x = 3$

b $3 - 3 - x = -2 - 3$
$\frac{-x}{-1} = \frac{-5}{-1}$
$x = 5$

c $3a - 7a + 5 - 5 = 7a - 10 - 5$
$\frac{-4a}{-4} = \frac{-15}{-4}$
$a = \frac{15}{4}$

d $3b - b + 2 - 2 = b - b + 5 - 2$
$\frac{2b}{2} = \frac{3}{2}$
$b = \frac{3}{2}$

3 a $3x - 4x - 12 + 12 = 4x + 28 + 12$
$-x = 40$
$x = -40$

b $a - a - 5 = 3a + 2$
$-3a - 5 + 5 = 3a - 3a + 2 + 5$
$-3a = 7$
$a = -\frac{7}{3}$

c $27x - 18 = 21x + 18$
$6x = -36$
$x = -6$

4 $-3b - 4b - 2 + 2 = 4b - 4b - 16 + 2$

$\frac{-7b}{-7} = \frac{-14}{-7}$

b = 2

Check: LHS = –3(2) –2 = –8

RHS = 4(2) – 16 = –8

LHS = RHS $\therefore b = 2$ is correct

5 **a** $\frac{3 \times 3}{7 \times 3} = \frac{y \times 7}{3 \times 7}$ (LCD = 21)

$\frac{9}{7} = \frac{7y}{7}$

$y = \frac{9}{7}$

b $3\frac{y - 3}{4 \times 3} = 2\frac{2y + 5}{6 \times 2}$ (LCD = 12)

3y – 4y –9 + 9 = 4y – 4y + 10 + 9

$-y = 19$

$y = -19$

6 Let the smallest number = x

$x + x + 1 + x + 2 = 108$

$3x + 3 = 108$

$3x = 105$

x = 35

The numbers are 35, 36 and 37.

7 **a** Let earnings = y and the number of tables = x

$y = 125 + 25x$

b $425 = 125 + 25x$

$300 = 25x$

$x = 12$

8 Let the breadth = x

$2(x + 5 + x) = 38$

$2(2x + 5) = 38$

$4x + 10 = 38$

$4x = 28$

$x = 7$

Length = $x + 5 = (7) + 5 = 12$ cm

Breadth = $x = 7$ cm

9 Distance = speed × time = 90 × 5 = 450 km

Speed = = 112,5 km/h

TERM 1

PoA | Test I

Answer the following questions. Work neatly.

1 Solve the following problems.

- **a** A car travels at 30 m/*s* constantly. How far will it travel in $1\frac{1}{2}$ h?
- **b** You borrow R800 at 5% p.a. compound interest. How much will you owe after 3 years?

2 Calculate:

- **a** $-24(-20 + 5) \div 5(-3) + 14(-2 + 3)$
- **b** $(-4)2 - (-3 - (-2))$
- **c** $(-50 \div -5) \times 6$
- **d** $\sqrt[3]{216} \div \sqrt{144}$

3

- **a** Calculate: $\frac{1}{2} + \frac{3}{4} - \frac{5}{8}$
- **b** Write $\frac{39}{60}$ as a decimal fraction and a percentage.
- **c** Calculate.
  - **(i)** $\frac{15}{36} \times \frac{6}{5}$
  - **(ii)** $\frac{4}{7} \div \frac{8}{49}$
- **d** Calculate: $1\frac{2}{3} \times 2\frac{3}{4} - 3\frac{5}{8}$

4 Calculate without using a calculator.

- **a** $24{,}7 + 2{,}47$
- **b** $37{,}39 - 3{,}739$
- **c** $19 \times 26{,}3$
- **d** $41 \times 5{,}78$
- **e** $56 \div 3{,}2$
- **f** $7{,}5 \div 1{,}25$
- **g** $\sqrt[3]{0{,}125}$
- **h** $0{,}8(0{,}6 \div 0{,}06)$
- **i** $(0{,}12)^2$

5 Calculate the following.

- **a** $3^2 \times 2^3$
- **b** $5^9 \div 5^7$
- **c** $(2^3)^2$
- **d** $(-4 \times 5^2)^3$
- **e** $(4^5 \times 5^4)^0$
- **f** $(6{,}5 \times 10^{12}) \div (1{,}3 \times 10^6)$

6 Write in scientific notation.

- **a** 4 173 800
- **b** 0,000652

7 For sequence A: 2; 5; 8; 11; … and sequence B: 2; 4; 8; 16; …, find the following.

- **a** The general rule for T_n.
- **b** The value of T_6.
- **c** The position of the term with the value of 512.

8 Look at the table below.

x	–2	–1	0	1	2	79	n
y	–7	–5	–3	–1	1	m	79

- **a** Determine the equation that describes the relationship between x and y.
- **b** Use the equation to determine the values of m and n.

9 Simplify.

- **a** $-4(x^3 - x) + x(1 - x^2)$
- **b** $(x - 6)(x + 7)$
- **c** $(2x - 5)^2$
- **d** $\frac{12x^4 - 15x^3 + 3x(-2x)}{3x^2}$
- **e** $\sqrt{(16x^2 + 9x^2)}$
- **f** $\sqrt[3]{(27x^6)} - \sqrt{(16x^4)} + x^2$

10 Solve for x.

- **a** $2^x = 64$
- **b** $3^{x-4} = 1$
- **c** $3(7x - 2) = 120$
- **d** $7(x + 1) - 2 = 4(x + 5)$
- **e** $\frac{x}{4} + \frac{x}{3} - 5 = \frac{x}{6}$
- **f** $\frac{2(3x - 7)}{5} = 8$

TERM

1 Test 1 Memorandum

1
a 162 km
b R926,10

2
a –10
b –7
c 60
d $\frac{1}{2}$

3
a $\frac{5}{8}$
b 0,65; 65%
c (i) $\frac{1}{2}$
(ii) $\frac{7}{2}$
d $\frac{23}{24}$

4
a 27,17
b 33,651
c 499,7
d 236,98
e 17,5
f 6
g 0,5
h 8
i 0,0144

5
a 72
b 25
c 64
d –1 000 000
e 1
f 5 000 000

6
a $4{,}1738 \times 10^{6}$
b $6{,}52 \times 10^{-4}$

7
Sequence A:
a $T_n = 3n - 1$
b $T_6 = 17$
c $n = 171$
Sequence B:
a $T_n = 2n$
b $T_6 = 64$
c $n = 512$

8
a $y = 2x - 3$
b $m = 155$; $n = 41$

9
a $-5x^3 + 5x$
b $x^2 + x - 42$
c $4x^2 - 20x + 25$
d $4x^2 - 5x - 2$
e $5x$
f 0

10
a $x = 6$
b $x = 4$
c $x = 6$
d $x = 5$
e $x = 12$
f $x = 9$

CHAPTER

10 Construction of geometric figures

Chapter overview

Learner's Book pages 251 to 269
Recommended pacing: 9 hours

This chapter focuses on the following:

Unit 1: Revision: Constructions of lines and angles	4,5 hours
Revising vocabulary of lines and angles	
Constructing lines and angles	
Constructing special angles: 90º, 45º, 60º and 30º	
Unit 2: Revision: Properties of geometric figures	4,5 hours
Revising vocabulary of geometric shapes	
Investigating the properties of triangles	
Investigating the conditions for triangles to be congruent	
Investigating the conditions for triangles to be similar	
Investigating the properties of quadrilaterals	
Investigating the properties of polygons	
Chapter 10 Revision	1 hour 55 minutes

UNIT

1 Revision: Constructions of lines and angles

Unit overview

Learner's Book page 252
Recommended pacing: 4,5 hours

This unit focuses on the following:

- Constructing straight lines
- Measuring lengths of straight lines
- Measuring angles
- Constructing right angles
- Constructing angles of 600, 450 and 300.

Resources: Learner's Book; exercise book; mathematical set (ruler, protractor, eraser, sharp pencil, pen)

Background information

In Grade 8, learners did the following:

- Constructed straight lines using a ruler and a sharp pencil
- Measured line segments from the starting point to the end point in millimetres (mm) or centimetres (cm)
- Used a protractor to measure angles in degrees (°)
- Used a compass for drawing circles, arcs (parts of circles), perpendicular lines and special angles
- Constructed perpendicular lines
- Constructed parallel lines

This unit revises these basic constructions.

Teaching guidelines

Construction of geometric figures is fundamental in establishing natural properties of geometric figures. By constructing geometric figures learners discover mathematical relationships which were discovered centuries ago by earlier mathematicians. Learning through discovery instils an appreciation and love of geometry in learners. It must be pointed out that constructions are not mathematical proofs but a verification that a mathematical relationship exists.

Since learners should have completed most of the constructions in this unit in previous grades, this unit is an important re-introduction and serves the purpose of revising and consolidating these important skills and concepts.

Ensure that all the necessary equipment is available beforehand either for learners to work with individually or in pairs or small groups. Careful planning is necessary to ensure that all learners in a pair or small group do the constructions by rotation.

Revising vocabulary on lines and angles

Activity 1 **Revise vocabulary of lines and angles**

Learner's Book page 252

Guidelines for implementing this activity

- Learners should be familiar with the vocabulary revised in this activity. Ask learners to answer question **1** on their own and then discuss the answers as a class.
- Make deliberate use of the vocabulary in question **1** throughout the construction activities and check learners' understanding. Do they know what *perpendicular* means? When the instruction says "Draw an arc", do they know what to do? When the instruction says "Draw a perpendicular bisector", do they know what is expected?
- Ask learners to answer question **2** on their own and then discuss the answers as a class.
- Make deliberate use of the vocabulary in question **2** throughout this chapter and check learners' understanding.

Remedial and extension

Learners may need some extra explanations for words they do not recall well. Revise these words at the beginning of every lesson to ensure that learners know and understand the vocabulary. Also, as explained above, make deliberate use of the vocabulary in Activity 1 throughout this chapter and check learners' understanding.

Suggested answers

1 a Straight line: the shortest distance between two points). Explain to learners that any other lines between these two points would be curved.
  b Ray: a straight line extending from a point.
  c Line segment: a line that has a start and end point.
  d Length: the measurement along a line or curve
  e Intersecting lines: lines which cross each other. The place where they cross is called the intersection.
  f Perpendicular lines: lines that meet at 90°
  g Parallel lines: lines that are the same distance apart at every point; they can never cross.
  h Transversal: a straight line that intersects a given set of two or more straight lines.
  i Perpendicular: two lines that meet at right angles to each other.
  j Perpendicular bisector: a perpendicular line which cuts a line segment into two equal parts.
  k Bisector of an angle: a line that divides an angle into two equal angles.
  l Arc: part of the circumference of a circle

2 a Angle: an amount of turn measured in degrees.
  b Acute angle: an angle that measures between 0° and 90°.
  c Right angle: an angle that measures 90°.
  d Obtuse angle: an angle that measures between 90° and 180°.
  e Straight angle: an angle that measures 180°.
  f Reflex angle: an angle that measures between 180° and 360°.
  g Revolution: an angle that measures 360°.
  h Complementary angle: two angles that add up to 90°.
  i Supplementary angles: two angles that add up to 180°.
  j Adjacent angles: angles that are side by side.
  k Vertically opposite angles: a pair of non-adjacent angles formed by the intersection of two straight lines.
  l Corresponding angles: angles that lie in the same position on the same side of the transversal.
  m Alternate angles: angles that lie between two parallel lines on opposite sides of the transversal.
  n Co-interior angles: angles that lie between two lines on the same side of the transversal.

Constructing lines and angles

Constructing and measuring a straight line; Measuring a straight angle (angle on a straight line); Measure an angle where two straight lines intersect; Construct perpendicular lines; Construct parallel lines

Activity 2 Construct lines and angles

Learner's Book page 255

Guidelines for implementing this activity

- Learners work through the steps for each construction in the Learner's Book. They should have done these constructions in Grades 7 and 8 and this should therefore be revision of these basic skills. It is, however, important that learners do all the constructions, and that you start with the simple constructions in order to encourage learners of all ability levels to get involved.
- Let learners work in pairs or small groups in order to share construction equipment if necessary, depending on the size of the class and the availability of resources. Ensure that they use the equipment safely, especially the compasses. Walk around the classroom while they are busy with the constructions in order to monitor their safety as well as their progress, and provide support where necessary.
- While learners are busy, make deliberate use of the vocabulary mentioned in the teaching guidelines.

Remedial and extension

Remedial: More explanation and demonstration may be needed as well as additional practice activities, especially with using the construction tools (rulers, compasses and protractors).
Extension: Learners may work in pairs to create similar construction tasks for their partners.

Suggested answers

Check the constructions that the learners have done to make sure that they have understood what is required of them and that they have completed the tasks correctly.

Constructing special angles: 90°, 45°, 60° and 30°

Constructing a right angle (90°) by bisecting a straight angle; Constructing a 45° angle by bisecting a right angle; Constructing an equilateral triangle to create 60° angles; Constructing a 30° angle by bisecting a 60° angle

Activity 3 Construct angles without using a protractor

Learner's Book page 257

Guidelines for implementing this activity

- This activity builds on from the previous activity. Start by demonstrating to learners how to use a compass to draw a circle and an arc, or let learners demonstrate it to the class.
- Revise how to bisect an angle of 180° using a compass. Emphasise the word *bisect*, and ensure that learners understand what this means. Ensure that they master the technique of bisecting a straight line before moving on to bisecting a right angle. Basically the same technique is used to bisect angles.
- The construction of an equilateral triangle using a compass is another task that can

be challenging to learners if they are not properly guided. Do a few demonstrations if necessary to ensure that learners get to grips with the process and the correct use of the equipment in order to ensure accurate constructions. Allow one or two learners to follow with their own demonstrations.

- Allow learners to work in groups or pairs.
- It is very interesting to learners to discover that irrespective of the size of the equilateral triangle they construct, each interior angle is exactly 60°. If there is time, let them do at least two or three practice rounds before doing the formal constructions. The more learners practise constructing angles, the more comfortable and confident they will be doing these constructions. This will help crystallise their understanding of the step-by-step process.

Remedial and extension

Provide assistance to each group as required. When all learners can easily construct an equilateral triangle, they can move on and use angle bisecting techniques developed earlier to construct angles of 30°.

Suggested answers

The purpose of this activity is to develop the skill of constructing angles. Check learners' constructions and provide feedback.

UNIT

2 Revision: Properties of geometric figures

Unit overview

Learner's Book page 258
Recommended pacing: 4,5 hours

This unit focuses on the following:

- Accurately constructing geometric figures appropriately using a compass, ruler and protractor, including bisecting angles of a triangle
- Constructing angles of 45°, 30°, 60°and and their multiples without using a protractor
- Investigating the properties of geometric figures by construction
- Investigating the angles in a triangle, focusing on the relationship between the exterior angle of a triangle and its interior angles
- By construction, exploring the minimum conditions for two triangles to be congruent
- By construction, investigating sides, angles and diagonals in quadrilaterals, focusing on the diagonals of rectangles, squares, parallelograms, rhombi and kites
- By construction, exploring the sum of the interior angles of polygons

Resources: Learner's Book; exercise book; mathematical set (ruler, protractor, eraser, sharp pencil, etc.) pen

Background information

Learners should be familiar with the geometric figures to be covered in this unit. In Grade 9, the focus is on investigating the properties of triangles and other polygons through construction.

Constructions provide a useful context for exploring or consolidating knowledge of angles and shapes.

Construction of special angles without protractors are done by:

- bisecting a right angle to get 45°
- drawing an equilateral triangle to get 60°
- bisecting the angles of an equilateral triangle to get 30°.

Teaching guidelines

The purpose of this unit is to allow learners to explore the geometric figures and discover their properties before formulating the properties formally.

This unit therefore requires the application of the practical construction skills acquired in previous units and activities.

"Discovered" properties will give rise to formal definitions of geometric figures and their properties. It is important that learners are actively involved in the process and that they are allowed to work through the steps in each investigation in this unit in order to explore the properties of figures through practical investigation.

In future chapters, learners will use these formal definitions and properties of quadrilaterals to solve geometry problems.

Ensure that all construction equipment is handy and that learners have adequate access to them (either individually or in small groups/pairs).

Make sure learners are competent and comfortable in the use of a compass and know how to measure and read angle sizes on a protractor. Revise the constructions of angles if necessary, before proceeding with the new constructions. Start with the constructions of lines, so that learners can first explore angle relationships on straight lines.

Revising vocabulary on geometric shapes

Activity 1 Revising vocabulary on geometric shapes

Learner's Book page 258

Guidelines for implementing this activity

Learners are familiar with the vocabulary revised in this activity. Ask learners to answer the questions on their own and then discuss the answers as a class.

Remedial and extension

Learners may need some extra explanations for words they do not recall well. Pair up learners who need revision with learners who know the words well and ask them to test each other.

Suggested answers

1 **a** Polygon: any closed 2D shape with straight sides.
b Triangle: any polygon with three sides. The interior angles of a triangle add up to 180°.
c Equilateral triangle: a triangle with all sides and angles equal.
d Isosceles triangle: a triangle with two sides and two angles equal.
e Right-angled triangle: a triangle with a right angle.
f Scalene triangle: a triangle with no sides and no angles equal.

2 **a** Quadrilateral: a polygon with four sides. The interior angles add up to 360°.
b Parallel lines: lines that are the same distance apart at all points. Parallel lines can never cross.
c Perpendicular lines: lines that meet at right angles.
d Parallelogram: a quadrilateral with opposite sides parallel to each other.
e Rhombus: a quadrilateral with four equal sides and opposite sides parallel. Another word for it is a diamond shape.
f Rectangle: a quadrilateral with four right angles. Opposite sides are equal and parallel.
g Square: a quadrilateral with four right angles and all sides equal.
h Kite: a quadrilateral with two pairs of adjacent sides of the same length and one pair of equal angles.

Investigating the properties of triangles

The interior angles of a triangle; The exterior angle of a triangle

Activity 2 Construct triangles by using their properties

Learner's Book page 259

Guidelines for implementing this activity

- Work through the examples in the Learner's Book with the class. Ensure that learners are actively involved in these practical investigations, and that they use the equipment safely and correctly.
- Go through the first example with the learners and point out the importance of providing a reason for each step in solving geometry problems. At this stage reasons are limited to the results that have been established in the conclusion in each step-by-step construction, for example: 'The sum of the angles in each triangle that I have constructed is 180°'.
- Encourage learners, in pairs, to draw as many triangles of different shapes and sizes as possible, and to measure these and verbalise their results as well as write down their conclusions.
- Let them also measure each other's triangles and verbalise their conclusions.
- Where necessary, assist learners with measuring these angles, as accuracy of measurement is fundamental in establishing the results.

Remedial and extension

Allow learners to work in pairs to complete this activity. Learners must write out their answers individually.

Suggested answers

1 a The interior angles of a triangle add up to 180° so $\hat{A} = 180° - \hat{B} - \hat{C} = 40°$.
b $AB = BC = 6$ cm because it is an isosceles triangle. We know this already because $\hat{B} = \hat{C}$.

2 a All sides are equal making it an equilateral triangle, therefore all of the angles $= \hat{D} = 60°$.
b EFG is a straight line which equals 180 °and $E\hat{F}D = 60°$ therefore $D\hat{F}G = 120°$.

3 a $\hat{K} + \hat{M} = 90°$ because $\hat{L}$ is 90° and the sum of the angles of a triangle equal 180°.
b Learners construct LM. $K\hat{M}N + K\hat{M}L = 180°$ ($\angle$'s on a straight line).

Investigating the conditions for triangles to be congruent

Three corresponding sides (SSS); Two sides and the included angle (SAS); Two angles and one side (AAS); Hypotenuse and one side of a right-angled triangle (90°HS)

Activity 3 **Construct triangles and check whether they are congruent**

Learner's Book page 262

Guidelines for implementing this activity

- The purpose of the activity is to allow learners to explore congruency in terms of the fact that any two triangles are congruent if they can be perfectly placed one on top of the other without any parts of the triangles overlapping.
- Revise the first condition for congruency by working through the introduction with the learners.
- It is important that learners understand that congruency involves two separate triangles.
- Work through the examples in the Learner's Book, ensuring that learners are actively involved with the constructions, since it is through practical investigation that they are required to discover these properties. At the same time, they are also practising the basic construction skills that they have acquired. These skills should be carefully monitored to ensure that they are in place.
- Allow learners to do more than one construction activity per investigation.
- Give learners different dimensions for different examples so that they can see that each case is valid irrespective of the dimensions chosen.
- Encourage learners to state the conditions in their own words, and to give a summary of the four conditions.
- Revise these conditions for congruency orally once all the investigations have been completed.

Remedial and extension

Remedial: Allow learners who are having difficulty to work in pairs. Provide these learners with additional exercises to complete as homework.
Extension: Learners who complete the activity without difficulty may create their own activities and give them to a partner to complete.

Suggested answers

1 No, they are not congruent. $\hat{F}$ is not included like $\hat{B}$.

2 No, they are not congruent. KL and PR are not corresponding sides.
3 Yes, they are congruent. Both are right-angled (90°HS).

Investigating the conditions for triangles to be similar

Corresponding angles are equal (AAA); Corresponding sides are proportional

Activity 4 **Construct triangles**

Learner's Book page 264

Guidelines for implementing this activity

- The guidelines for this activity are similar to those of the previous activity.
- Work through the examples in the Learner's Book as a class. Be sure to give lots of opportunities for learners to ask questions and to do additional constructions as explained in the previous activity.
- Quiz the class about each of the conditions of similarity triangles before they attempt the activity.

Remedial and extension

Remedial: Allow learners who are having difficulty to work in pairs. Provide these learners with additional exercises to complete as homework.
Extension: Learners who complete the activity without difficulty may create their own activities and give them to a partner to complete.

Suggested answers

1 Yes, they are similar as they are equiangular.
2 Yes, they are similar as the sides are proportional.

Investigating the properties of quadrilaterals

The parallelogram; The rhombus; The rectangle; The square; The kite; The definitions of the five quadrilaterals

Activity 5 **Construct quadrilaterals by using their properties**

Learner's Book page 267

Guidelines for implementing this activity

- Learners should have encountered quadrilaterals in the previous grades. In Grade 9 they explore the validity of some of the properties through practical investigation.
- Work through the examples in the Learner's Book with the class, ensuring that learners are actively involved in each practical investigation.
- Each construction also requires a high level of accuracy. This is where the construction and measurement skills learned in the previous activities are applied. Ensure that they do each construction as required, and that they do not merely draw lines. For example, when constructing the parallelogram they are required to construct the parallel lines using the required equipment, and not draw "parallel" lines. This is very important.

- Provide additional examples for learners to attempt in pairs before they attempt the activities. After exploring each quadrilateral in the investigation, give learners the same quadrilateral, but with different dimensions and ask then to repeat the investigation with the new quadrilateral.
- Let learners state their findings as they are before providing them with a summary of the main points.
- Repeat the above process with the construction of a rhombus, a square and a kite.

Remedial and extension

It may be necessary to give learners additional tasks in each activity to ensure that they master the construction skills, particularly those involving construction of equilateral triangles, parallelograms, rhombuses and kites. Spend time on these constructions and do not rush to formal definitions and formulae as these will be dealt with adequately further on. Once learners successfully discover these results, learning in the following chapters will be much quicker.

Suggested answers

Check and moderate the learners' work and make sure that they have understood and followed the steps correctly.

Investigating the properties of polygons

Sum of the interior angles of polygons

Activity 6 **Construct polygons by using their properties**

Learner's Book page 269

Guidelines for implementing this activity

- Work through the example in the table in the Learner's Book with the class and revise the basic properties of the first six polygons.
- Explain that the sum of the interior angles of a polygon with n sides can always be calculated using the equation $n - 2 \times 180°$.
- Start by showing learners the relationship between the prefix of the name of a polygon and the number of its sides. For example, tri- = 3 sides for a triangle, quad- = 4 sides for a quadrilateral, pent- = 5 sides for a pentagon, and so on.
- Lead learners to discover the relationship between the number of sides of a polygon and the number of triangles into which a polygon can be divided. This must be done through practical investigation. Let learners follow the steps and complete the statements in their exercise books.
- Lead learners to discover the relationship between the number of triangles in a polygon and the sum of the interior angles of the polygon.
- Finally, learners should establish the relationship between the number of sides of a polygon and the sum of its interior angles.
- Do a few additional examples on the board to show that this is true.

Remedial and extension

It is very important that learners are involved in these investigations practically, and that they investigate the properties of polygons in this way. Let them work together through the steps in pairs or small groups if necessary, and monitor their progress and understanding very carefully.

Suggested answers

1 a $180° \times 4 = 720°$
  b All the interior angles = 120° as they all add up to 720° and there are 6 of them.
2 The angles add up to 720°. This is because the shape is made up of 3 triangles.
3 To find the sum of all of the interior angles, use the equation:
  $5 - 2 \times 180°$. To find the size of one angle, divide this number by 5 = 108°

Chapter 10 Revision

Learner's Book pages 270–271

Suggested answers

1

ITEM	DESCRIPTION
Equilateral triangle	A polygon made up of three equal straight lines
A parallelogram	A polygon with two pairs of parallel lines
Right angle	Has a size of 900
Hexagon	Divide into two equal parts
Bisecting an angle	A polygon made up of six straight lines

2 a Verify learner constructions
  b Verify learner constructions
3 a a and d (SSS)
  b b and h (SAS)
  c c and f (RHS)
  d e and l (AAS)
4 a Two triangles
  b $180° \times 2 = 360°$
5 Since any quadrilateral can be divided into 2 triangles and the sum of interior angles of any triangle is 180°, the sum of interior angles of any quadrilateral is 360°.
6 a $n = 20$, so the number of triangles = $(n - 2) = 20 - 2 = 18$
  b sum of interior angles = $180° \times 18 = 3\ 240°$
7 a number of Δs = $\frac{1\ 620°}{180°} = 9$
  b number of sides is given by:
  $n - 2 = 9$, where n = number of sides of a triangles
  Solving for n, $n = 9 + 2 = 11$

CHAPTER

11 Geometry of 2D shapes

Chapter overview

Learner's Book pages 272 to 294
Recommended pacing: 9 hours

This chapter focuses on the following:

Unit 1: Properties of triangles	3 hours
Revision: Vocabulary of triangles	
Triangles and their properties	
Problem-solving: Triangles and their properties	
Unit 2: Properties of quadrilaterals	3 hours
Revision: Vocabulary of quadrilaterals	
Quadrilaterals and their properties	
Problem-solving: Quadrilaterals and their properties	
Unit 3: Congruent and similar triangles	3 hours
Revision: Congruence and similarity	
Congruent triangles	
Problem-solving: Congruent triangles	
Similar triangles	
Problem-solving: Similar triangles	
Chapter 11 Revision	1 hour 55 minutes

UNIT

1 Properties of triangles

Unit overview

Learner's Book page 273
Recommended pacing: 3 hours

This unit focuses on the following:
- Types of triangles
- Triangle theorems
- Problem-solving using triangle theorems.

Resources: Learner's Book; exercise book; mathematical set; eraser; sharp pencil; pen

Background information

The exploration of geometric figures in the previous chapter has laid the foundation for formal definitions of different kinds of polygons as stated in terms of their properties.

The next step is to study the unique properties of specific polygons (particularly those expressing relationships between sides and angles), namely triangles and quadrilaterals. A good understanding of these properties will be useful in solving geometry problems. This unit focuses on the properties of triangles.

Teaching guidelines

Learners have worked with the different triangle types in previous grades and it is necessary to revise the types of triangles in Grade 9 as a precursor to triangle *theorems*. The general properties of triangles that express the relationships between the angles and sides of certain types of triangles are summarised as formal mathematical statements called theorems. It is not necessary for learners to prove these results, but to understand them so that they can apply them in solving Geometry problems. Time should be taken to carefully explore these theorems and their application in the worked examples presented here to ensure optimal competency. This foundation is very important for the FET Phase when problems become more complex, and when learners will be required to access the basic knowledge and skill acquired in Grade 9 to help them solve more complex geometric problems. Generally, great care should be taken to ensure that they set out their work neatly and logically at all times, that they always provide the necessary proofs where asked to do so and that they adhere to the basic conventions that apply in Geometry.
Revise the properties of triangles every day at beginning of the lesson.

Revision: Vocabulary of triangles

Activity 1 **Revise concepts on triangles**

Learner's Book page 273

Guidelines for implementing this activity

- Learners should already be familiar with types of triangles and their properties. Prompt learners to give these definitions to the whole class and discuss the answers as a class. Let individual learners draw rough diagrams on the board showing the properties as you work through them.
- Listen to the language that learners use to ensure that it is correct, and take note of the conventions that they apply to ensure that these are appropriate. The conventions and language required when referring to the properties of triangles are all contained in the table and the worked examples in the Learner's Book.
- Use this vocabulary and the conventions as often as possible so that learners become familiar with and comfortable using them.

Remedial and extension

Learners need to know these definitions in order to progress with this section of Mathematics. Find opportunities to use and emphasise the vocabulary in class, and ask questions continually that prompt learners to use the vocabulary.

Suggested answers

1 Triangle: any polygon that has three sides.

2 Equilateral triangle: a triangle with all three sides equal in length.
3 Right-angled triangle: a triangle that contains one right angle. The longest side is called the *hypotenuse*.
4 Isosceles triangle: a triangle with two sides that are equal.
5 Scalene triangle: a triangle with no sides the same length; the sizes of all the angles are different.
6 Interior angle: an angle formed on the inside of a geometric shape.
7 Exterior angle: an angle formed by one side of a triangle and the extension of the adjacent side.
8 Base angles: the angles of a triangle that have the base as one of their sides.

Triangles and their properties; Problem-solving: Triangles and their properties

Activity 2 Solve problems on triangles

Learner's Book page 277

Guidelines for implementing this activity

- This activity is intended for learners to practise problem-solving using their knowledge of triangles. It also further develops their ability to apply this knowledge to solving problems.
- The examples in the Learner's Book give an indication of the types of problems learners are expected to solve using triangle theorems. Work through the examples in the Learner's Book. Carefully explain the reasoning behind each step.
- Prepare one or two additional examples in advance for learners to do in class/on the board/ in pairs in their exercise books before they do Activity 1.
- Emphasise the importance of working neatly and systematically. Ensure that learners set out their work and their solutions correctly and logically at all times. It often happens that learners who do not set out their work neatly are easily confused and may lose track of the process and their own thinking or logic. Getting this right in Grade 9 lays a crucial foundation for the FET phase when geometry problems become more complex.

Remedial and extension

Provide additional homework exercises.
Many learners find Geometry challenging. Ensure the following to assist them.

- Always provide information in a structured way, for example, if a triangle is isosceles, name the equal sides from the top towards the base angles. It helps to create a clearer picture. Example 3 in the Learner's Book demonstrates this.
- Provide information when it is needed. In Example 3 question **2** in the Learner's Book, the information on AF, FC and DF is only applicable to question **2**, therefore this is where explanation should be provided. Often, all the information is given by teachers at the beginning or in the first question where it does not apply. Learners therefore often find questions confusing as a result of this.
- Also, it is always best to avoid using the letters i, I, L o and O in Geometry since learners can be easily confused by the digits 1 and 0.

Work through the summary at the end of this unit and ask learners to provide an example of each summary point.

Suggested answers

1 **a** Isosceles triangle: a triangle with two equal sides.
b Right-angled triangle: a triangle that has one right angle. The longest side is the hypotenuse.
c Equilateral triangle: a triangle with all three sides the same length.
d Scalene triangle: a triangle with no sides the same length. The sizes of all the angles are different.

2 **a** $x = M\hat{K}L + M\hat{L}K$ (x is the exterior angle)
$= 40° + 90°$
$\therefore x = 130°$
b $y = 180° - (40° + 90°)$ (angles of a triangle = 180°)
$= 180° - 130° = 50°$
$x = 130°$ (proven) and $y = 50°$ (proven)
$\therefore x + y = 130° + 50° = 180°$

3 **a** $\hat{H} = 60°$ (interior angle of equilateral triangle)
b $\hat{K} = 60°$ (interior angle of equilateral triangle)
c $H\hat{L}K = 60°$ (interior angle of equilateral triangle)
d $\hat{N} = 180° - (90° + 60°) = 30°$ ($M\hat{L}N = 60°$ vertically opposite angles; sum of interior angles of a triangle)
e $ML^2 = LN^2 - MN^2$ (Pyth.)
$= 52 - 42$
$= 25 - 16$
$= 9$
$ML = \sqrt{9} = 3$

4 $a = 75°$ (base angles of isosceles triangle)
$b = 105°$ (angles on a straight line)
$c = 35°$ (interior angles of a triangle)
$d = 45°$ (interior angles of a triangle)
$e = 90°$ (angles on a straight line)
$f = 35°$ (interior angles of a triangle)

5 **a** $\hat{A}_1 = 60°$ (interior angles of equilateral triangle); $\hat{B}_2 = 60°$ (interior angles of equilateral triangle); $\hat{C}_3 = 120°$ (angles on a straight line); $\hat{D}_4 = 30°$ (base angle of isosceles triangle)
b $A\hat{B}D = \hat{A}_1 + \hat{A}_5 = 60°$ (proven) + 30° (base angle of isosceles triangle ACD) = 90°. Therefore $\triangle ABD$ is a right-angled triangle.
c $\hat{D}_4 = 30°$ (proven); $\therefore \hat{E}_7 = 30°$ (base angles of isosceles $\triangle AED$
d $A\hat{B}E = 120°$ (angle on straight line); $\hat{E}_7 = 30°$ (proven) ; $\therefore \hat{A}_8 = 30°$ (interior angles of triangle); $\therefore \triangle BAE$ is an isosceles triangle (base angles E_7 and A_8 are equal)

6 **a** $180° = 60° + (2p + 10°) + (p + 35°)$ (interior angles of triangle)
$180° = 105° + 3p$
$180° - 105 = 105° + 3p - 105$
$75° = 3p$
$\frac{75°}{3} = \frac{3p}{3}$ therefore $p = 25°$
b $\hat{B} = (2p + 10°) = 60°$ and $\hat{C} = (p + 35°) = 60°$ $\therefore \hat{A} = 60°$ (interior angles of a triangle)
c All lengths are 80 mm (All angles are the same)
d Equilateral triangle

7 **a** $q + 3q - 0{,}4° + 90° = 180°$ (sum of interior angles of triangle = 180°)
$\therefore 4q = 180° - 90° + 0{,}4°$
$\therefore 4q = 90{,}4°$
$\therefore q = 22{,}6°$
$\hat{D} = q = 22{,}6°$

$\hat{E} = 3q - 0{,}4^\circ = 3(22{,}6^\circ) - 0{,}4^\circ = 67{,}4^\circ$

b $EF^2 = DE^2 - DF^2$ (Pyth.)

$= 13\text{ cm}^2 - 12\text{ cm}^2$

$= 169 - 144 = 25$

$\therefore EF = \sqrt{25} = 5\text{ cm}$

UNIT

2 Properties of quadrilaterals

Unit overview

Learner's Book page 279
Recommended pacing: 3 hours

This unit focuses on the following:
- Properties of quadrilaterals.

Resources: Learner's Book; exercise book; mathematical set; eraser; sharp pencil; pen

Background information

In Chapter 10 learners explored properties of quadrilaterals through constructions. These constructions resulted in formal definitions of these polygons. In this chapter learners take a closer look at those properties that express relationships between sides, angles and diagonals of quadrilaterals.

Teaching guidelines

Start this section by referring to the results of the investigations of properties of quadrilaterals from Chapter 10. Classify the properties of each quadrilateral in terms of sides, angles and diagonals.
Revise the properties of quadrilaterals every day at beginning of each lesson.

Revision: Vocabulary of quadrilaterals

Activity 1 Revise concepts on quadrilaterals

Learner's Book page 279

Guidelines for implementing this activity

- Learners should already be familiar with the properties of and vocabulary related to quadrilaterals.
- As done with triangles, prompt learners to give these definitions to the whole class and discuss the answers as a class. Let learners draw rough diagrams on the board showing the properties as you work through them.
- Again, listen to the language that they use to ensure that it is correct, and take note of the conventions that they apply to, and ensure that these are appropriate. The conventions and language required when referring to the properties of quadrilaterals are all contained in the table and the worked examples in the

Learner's Book.
- Use this vocabulary as often as possible so that learners become familiar with it and comfortable with using it.

Remedial and extension

Learners need to know these definitions in order to progress with this section of Mathematics. Find opportunities to use and emphasise the vocabulary in class, and ask questions continually that prompt learners to use the vocabulary.
Work through the summary at the end of this unit and ask learners to provide an example of each summary point.

Suggested answers

1 Quadrilateral: a polygon with four sides.
2 Parallelogram: a 4-sided polygon with opposite sides equal and parallel.
3 Rhombus: a polygon with four equal sides; opposite sides are parallel. All sides are equal. It is the correct name for a diamond shape.
4 Rectangle: a polygon with 4 right angles and equal opposite sides.
5 Square: a polygon with 4 equal sides and 4 right angles.
6 Kite: a 4-sided polygon with 2 pairs of adjacent sides of equal length and one pair of equal angles.
7 Trapezium: a 4-sided polygon with one pair of parallel sides.
8 Opposite sides: sides of a shape that face each other.
11 Parallel sides: sides of a shape that are equidistant along their length.
12 Diagonal: a straight line that joins any two corners of a shape.

Problem-solving: Quadrilaterals and their properties

Trapezium; Parallelogram; Rhombus; Rectangle; Square; Kite

Activity 2 **Solve problems on quadrilaterals**

Learner's Book page 282

Guidelines for implementing this activity

- Ask learners to identify properties that are common to two or more quadrilaterals. Ask them to give alternative definitions of certain quadrilaterals in terms of other quadrilaterals with similar properties. For example: A square is a rectangle with all 4 sides equal in length.
- The example in the Learner's Book is intended to show learners how these properties can be used to solve problems in Geometry.
- Work through the examples in the Learner's Book with learners, making sure that they understand each example and the properties that are being used as reasons in the answers. Refer learners back to the table of properties if they need to.
- Prepare one or two additional examples in advance for learners to do in class/on the board/ in pairs in their exercise books before they do Activity 2.
- Again, emphasise the importance of working neatly and systematically and check learners' work to ensure that they do.
- Learners complete Activity 2.

Remedial and extension

Provide additional examples for learners to practise. Practice improves their understanding of the properties and their application of this understanding to problems, and generally builds their confidence to tackle problems that may otherwise appear daunting at first. Practice is therefore a way in which to demystify what among many learners is perceived to be a "difficult" branch of Mathematics. Allow learners to work in pairs or small groups and to discuss their answers and share their understanding and reasoning. However, learners should write out their answers individually.

Suggested answers

1 **a** False **b** False **c** False
d True **e** True **f** True
g True **h** False

2 **a** $\hat{1} = 75°$ (base angles of isosceles triangle)
$\hat{2} = 30°$ (interior angles of triangle)
$\hat{3} = 75°$ (opp. angles in parallelogram equal)
$\hat{4} = 75°$ (base angles of isosceles triangle)
$\hat{5} = 30°$ (interior angles of triangle)

b $\hat{1} = 35°$ (base angles of isosceles triangle)
$\hat{2} = 110°$ (interior angles of triangle)
$\hat{3} = 110°$ (opp. angles in rhombus equal)
$\hat{4} = 35°$ (base angles of isosceles triangle)
$\hat{5} = 35°$ (base angles of isosceles triangle)

c $\hat{1} = 60°$ (interior angles of equilateral triangle)
$\hat{2} = 120°$ (angles on a straight line)
$\hat{3} = 30°$ (diagonals are equal in length; base angles of isosceles triangle)
$\hat{4} = 60°$ (complementary angles)
$\hat{5} = 30°$ (interior angles of triangle)

3 **a** $\hat{1} = \hat{2} = 45°$ (base angles of isosceles triangle)
$\hat{3} = 45°$ (base angles of isosceles triangle)
$\hat{4} = 90°$ (interior angles of triangle)

b $\hat{1} = 50°$; $\hat{2} = 50°$; $\hat{3} = 75°$; $\hat{4} = 15°$

c $\hat{1} = 90°$; $\hat{2} = 45°$; $\hat{3} = 45°$

4 **a** $p + 4 = 17$ (opp. sides of parallelogram are equal)
$\therefore p = 13$ units
$\therefore AB = 2p - 3 = 2(13) - 3 = 23$ units

b $q + 8 = 12$ (all sides of a rhombus are equal)
$\therefore q = 4$ units
$5r - 8 = 12$ (all sides of a rhombus are equal)
$\therefore 5r = 20$ units
$\therefore r = 4$ units
$\frac{1}{2}s + 9 = 12$
$\therefore \frac{1}{2}s = 3$
$\therefore s = 6$ units

c $JK^2 + JM^2 = KM^2$ (Pyth.)
$\therefore 24^2 + 18^2 = KM^2$
$\therefore 576 + 324 = 900$
$\therefore KM = \sqrt{900} = 30$
$KM = 3t = 30$
$\therefore t = 10$ units

5 **a** parallelogram **b** rhombus **c** kite

6 **a** RT = 8 cm

b POS = 90° (diagonals of a square are perpendicular to each other)

c In ΔSQR, $QS^2 = QR^2 + RS^2$ (Pyth.)

$= (8\text{ cm})^2 + (8\text{ cm})^2$

$= 128\text{ cm}^2$

$\therefore QS = \sqrt{128}\text{ cm}^2 = 11{,}31\text{ cm}$

d In ΔQST, $ST^2 = SR^2 + RT^2$ (Pyth.)

Therefore, $ST = \sqrt{8^2 + 8^2} = \sqrt{128}$

= 11,31cm

e ST = QS = 11,31cm (proven)

∴ ΔQST is isosceles (2 sides equal)

f PR = QS (diagonals of a square are equal)

∴ PR = 11,31cm

But PR = 2OP (diagonals of a square bisect each other)

$\therefore OP = \frac{PR}{2}$

= 5,66cm

7 $\hat{D} = 85°$ (opp. angles of parallelogram are equal)

$\hat{A} = 95°$ (190° ÷ 2; opp. angles of parallelogram are equal)

$\hat{C} = 95°$ (190° ÷ 2; opp. angles of parallelogram are equal)

8 **a** Trapezium (AD = BE, therefore AB||DE; one pair of opp. sides parallel)

b AD = BC, therefore BC = BE

c $\hat{B}_1 = 56°$ (opp. angles of parallelogram are equal)

$\hat{A} = \hat{C}_1 = 124°$ (opp. angles of parallelogram are equal; sum of interior angles = 360°)

d $\hat{C}_2 = \hat{E} = 56°$ ($\hat{C}_2$ is a supplementary angle; △BCE is isosceles, proven that BC = BE)

$\hat{B} = 68°$ (interior angles of triangle)

9 **a** opp. ∠s of parallelogram are equal (If $Q_2 = 30°$ then $Q_1 = 30°$; opp. to $P\hat{S}T$)

b $\hat{T}_1 = 60°$ (interior angles of triangle)

$\hat{T}_3 = 30°$ (base angles of isosceles triangle)

c $\hat{T}_2 = 90°$ (∠s on straight line) ∴△PQT is right-angled

d $\hat{P}_2 = 60°$ (interior angles of triangle)

UNIT 3

Congruent and similar triangles

Unit overview

Learner's Book page 285
Recommended pacing: 3 hours

This unit focuses on the following:

- Congruent triangles
- Similar triangles.

Resources: Learner's Book; exercise book; mathematical set; eraser; sharp pencil; pen

Background information

Congruency conditions were explored by learners in Chapter 10 where the focus was on understanding congruency as an expression of superimposability. In this chapter

the focus is on formalising congruency conditions. Learners also apply knowledge of congruent triangles in solving geometry problems.
The related concept of *similarity* is also introduced. The introduction of similarity conditions for triangles makes congruency a special case of similarity. Remember that similarity does not imply congruency.

Teaching guidelines

Introduce the section on congruent triangles by referring learners to the investigations conducted in Chapter 10. Pay special attention to the concept of correspondence of sides (vertices) of triangles when defining congruency.
Continually emphasise that the order of vertices is very important in congruency. For example, the fact that $\Delta ABC \equiv \Delta PQR$ does not mean that $\Delta ABC \equiv \Delta PRQ$, and check leaners' work to ensure that they apply this rule consistently.

Revision: Congruency and similarity

Activity 1 **Revise congruent and similar triangles**

Learner's Book page 285

Guidelines for implementing this activity

- Learners should already be familiar with the vocabulary related to congruence and similarity.
- Work with the descriptions and conditions in the same way as with Unit 2 Activity 1.

Suggested answers

1 a Congruent triangles: identical in shape and size
  b Similar triangles: the same shape but not necessarily the same size
2 a SSS, SAS, AAS, RHS
  b Corresponding angles are equal and sides are in proportion
  c They have two identical angles; corresponding sides have lengths in the same ratio; two sides have lengths in the same ratio

Remedial and extension

Learners need to know these definitions in order to progress with this section of Mathematics. Find opportunities to use and emphasise the vocabulary in class, and ask questions continually that prompt learners to use the vocabulary

Congruent triangles; Problem-solving: Congruent triangles

Activity 2 **Solve problems on congruent triangles**

Learner's Book page 287

Guidelines for implementing this activity

- Revise the sign for congruence (≡) and ensure that learners know it and how to use it.
- Work through the first condition in the table in the Learner's Book. Pay special attention to the note in the table (how we write a statement of congruence to

show correspondence between equal sides). This is *very* important.
- Emphasise that we *always* follow the pattern of the vertices. In the first condition in the table in Unit 3, first name AB, then BC, then CA.
- Draw two triangles on the board with different labels as well as the necessary markings that indicate congruence and ask learners to write a statement of congruence (with proofs). Let them swap books and check each other's work, focusing on the following:
  - Have they given reasons?
  - Have they followed the pattern of the vertices?
  - Does the statement show correspondence between the sides?
- Repeat with an additional example/s if necessary.
- Work through the second condition in the table in the Learner's Book. Pay special attention to the note in the table (the given or *included* angle must be between the pair of given sides). This is *very* important.
- Focus on the hint given in the table. Again, the equalities must be ordered to match the order in the congruence case.
- Provide additional examples on the board and work through them in the same way as explained in the first example above.
- Repeat the above with the remaining conditions in the table.
- Work through the worked example in the Learner's Book as a means of consolidating the concepts and conventions that have been covered in the table.

Tip

When working with congruence, each equality should always be justified by giving a reason.

Remedial and extension

Remedial: As you work through the steps in the table and the worked example, prompt learners to provide the next step and the reasons for these steps. Verbalising these conditions will help to concretise their knowledge.
Extension: Provide learners with lots of opportunities to practise matching congruent triangles and writing proofs. Moderate their work and monitor their progress to ensure that they adhere to the conventions in the table in the Learner's Book.

Suggested answers

1 a ΔA, ΔG and ΔH
b ΔC and ΔK
c ΔB, ΔD, ΔE, and ΔI
d ΔA and ΔG (SSS); ΔB and ΔI (SAS) (RHS); ΔD and ΔE (RHS);

2 a BA = CA (corresponding sides of isosceles triangle)
AD = AD (common side)
$B\hat{A}D = C\hat{A}D$ (AD bisects $B\hat{A}C$)
$\therefore \Delta BAD \equiv \Delta CAD$ (SAS)

b KM = LM (M is the given midpoint of KL)
HM = HM
KH = LH (corresponding sides of isosceles triangle)
$\therefore \Delta HKM \equiv \Delta HLM$ (SSS)

c PS = PS (common side)
$P\hat{S}Q = P\hat{S}R = 90°$
$\therefore \Delta SPQ \equiv \Delta SPR$ (90°HS)

3 a AB = DC (opposite sides of a parallelogram)
AD = BC (opposite sides of a parallelogram)
AC = AC (common side)
$\Delta ABC \equiv \Delta CDA$ (SSS) This can also be proved congruent with SAS

b HE = EF = FG = GH (sides of a rhombus are =)
$\hat{E} = \hat{G}$ (corresponding angles of congruent triangles)
HF = HF (common side)
$\therefore \Delta HEF \equiv \Delta HGF$ (SAS)
$\therefore \hat{H}_1 = \hat{H}_2 = \hat{F}_1 = \hat{F}_2$

c In ΔNML and ΔNKL:
NL = NL (common side)
NM = NK (short sides of a kite are =)
ML = KL (long sides of a kite are =)
$\therefore \Delta NML \equiv \Delta NKL$ (SSS)
$\hat{N}_1 = \hat{N}_2$ and $\hat{L}_1 = \hat{L}_2$
$\therefore$ LN bisects $\hat{L}$ and $\hat{N}$.

4 a PT = PT (common side)
$P\hat{T}S = P\hat{T}Q$ (90° $\perp$ diagonal of a square)
ST = TQ (bisected diagonal of a square)
$\therefore \Delta PTS \equiv \Delta PTQ$ (SAS)

b PQ = SR (sides of a square)
$S\hat{T}R = P\hat{T}Q = 90°$ (angles of diagonals of a square are perpendicular)
$\therefore \Delta PTQ \equiv \Delta RTS$ (90°HS)

Similar triangles; Problem-solving: Similar triangles

Activity 3 Solve problems on similar triangles

Learner's Book page 292

Guidelines for implementing this activity

- Squares and rectangles are very useful shapes to use when explaining similarity, leading learners to discover the two conditions of similarity.
- Explain that any two equiangular triangles are similar. This is the essential point that learners need to understand and apply in this section of similar triangles. Note, once again, the importance of notation when writing similar triangles.
- Ensure that learners draw similar triangles in such a way that equal angles correspond, as indicated in the note in the Learner's Book.
- Work through the examples with the learners, and do additional examples following the same approach as for congruence if necessary in order to make sure that learners understand each step and the reasoning behind it.

Remedial and extension

Remedial: Provide many additional examples to help learners to master the skills required for working with similarity. Provide additional examples as homework.
Extension: Encourage learners to work in pairs and create similar questions to this activity for the other to do.
Work through the summary at the end of this unit and ask learners to provide an example of each summary point.

Suggested answers

1 **a** Two triangles are similar if their corresponding angles are equal.
b Two triangles are similar if their sides are in proportion.

2 **a** $A\hat{B}C = E\hat{F}D = 90°$
$B\hat{C}A = F\hat{D}E = 60°$
$\therefore B\hat{A}C = F\hat{E}D$ (sum of interior angles of a triangle = 180°)
$\therefore \triangle ABC \,|||\, \triangle EFD$ (AAA)
b-c All of these pairs of triangles are similar because of AAA.

3 $FE = \frac{2}{3}$ of AB
$FD = \frac{2}{3}$ of BC
$DE = \frac{2}{3}$ of CA
$\therefore \triangle ABC \,|||\, \triangle EFD$ (sides proportional)
All of these pairs of triangles are similar due to proportional sides.

4 **a** $\Delta ABC \,|||\, \Delta EFD$; $\Delta HGK \,|||\, \Delta LNM$; $\Delta PQR \,|||\, \Delta TUS$
b $\frac{AB}{EF} = \frac{BC}{FD} = \frac{CA}{DE}$; $\frac{HG}{LN} = \frac{GK}{NM} = \frac{KH}{ML}$; $\frac{PQ}{TU} = \frac{QR}{US} = \frac{RP}{ST}$
c AB = 8; DE = 5; HK = 18; LN = 7; QR = 9; TU = 6

Chapter 11 Revision

Learner's Book page 294

Suggested answers

1 **a** True
b False
c False
d False
e False

2 **a** $x + 53° = 90°$ (ext. ∟ of a Δ, theorem)
Therefore, $x = 90° - 53° = 37°$
b $a^2 = 5^2 - 3^2$ (Pyth.)
$= 25 - 9 = 16$
$a = \sqrt{16} = 4$

3 **a** AD = 8 and AB = 6 (opposite sides of a parallelogram are equal) [insert diagram from LB]
b $\hat{D} = 80°$ (opp. ∟s of a parallelogram are equal)
c In ΔABC and ΔCDA,
AB = DC (opposite sides of a parallelogram)
BC = DA (opposite sides of a parallelogram)
$\hat{B} = \hat{D}$ (opposite angles of a parallelogram)
Therefore, $\Delta ABC \equiv \Delta CDA$ (SAS)

4 **a** 6 cm; 3 cm; 3 cm
b PF = 6,7 cm; QF = 5,8 cm
c 90°; 59°
d (SAS)
e (SSS)
f No; $\frac{ER}{FR} = \frac{3}{3} = 1$, but $\frac{EP}{FQ} = \frac{6,7}{5,8} \neq 1$. The corresponding sides should be in the same ratio.

CHAPTER

12 Geometry of straight lines

Chapter overview

Learner's Book pages 295 to 313
Recommended pacing: 9 hours

This chapter focuses on the following:

Unit 1: Intersecting lines	3 hours
Revision: Vocabulary of intersecting lines and angles	
Pairs of angles formed by intersecting lines	
Problem-solving: Intersecting lines and their pairs of angles	
Unit 2: Parallel lines	3 hours
Revision: Vocabulary of parallel lines and angles	
Pairs of angles formed by parallel lines	
Problem-solving: Parallel lines and pairs of angles	
Unit 3; Mixed geometric problems	3 hours
Revision: Pairs of angles	
Solving mixed geometric problems	
Chapter 12 Revision	1 hour 55 minutes

UNIT

1 Intersecting lines

Unit overview

Learner's Book page 296
Recommended pacing: 3 hours

This unit focuses on the following:

- Adjacent supplementary angles
- Angles on a straight line
- Vertically opposite angles
- Corresponding angles
- Alternate angles
- Problem-solving using angle relationships.

Resources: Learner's Book; exercise book; mathematical set; pen

Background information

In previous grades, learners learnt the following:

- The sum of the angles on a straight line is 180°.
- Perpendicular lines have adjacent supplementary angles that are each equal to 90°.
- Intersecting lines have vertically opposite angles that are equal.
- Parallel lines cut by a transversal, have corresponding angles that are equal.
- Parallel lines cut by a transversal have alternate angles that are equal.
- Parallel lines cut by a transversal have co-interior angles that are supplementary.

In Grade 9, Learners revise and write clear descriptions of angle relationships on straight lines to solve geometric problems. Learners are expected to give reasons to justify their solutions for every written statement.

Teaching guidelines

Work through the examples in the Learner's Book carefully and be sure to recap as often as possible to reinforce learners' knowledge and grasp of this section.
Provide as many additional examples as possible as learners need practice to master this section.
Make sure that learners have access to all the necessary instruments required for this unit and that they work in pairs or groups if necessary. Always ensure safe use of all equipment, especially compasses.
Ensure that learners use sharp pencils in all constructions and that they measure as precisely as possible so that their answers are accurate.
Great care should always be taken to ensure that learners set out their work neatly and logically at all times, that they always provide the necessary proofs where asked to do so and that they adhere to the basic conventions that apply in Geometry.

Note: Throughout this chapter, reasons are sometimes written out in full and sometimes they are abbreviated. This is to demonstrate these two different ways in which learners may be allowed to write their reasons, and to choose the option which they are most comfortable with. Clarity is of the utmost importance.

Revision: Vocabulary of intersecting lines and angles

Activity 1 **Revise concepts on intersecting lines and angles**

Learner's Book page 296

Guidelines for implementing this activity

- Learners should already be familiar with the vocabulary in Activity 1. Prompt learners to give these definitions to the whole class and discuss the answers as a class. Let individual learners draw rough diagrams on the board showing the lines and angles mentioned.
- Use this vocabulary as often as possible so that learners become familiar with and comfortable using them.

Remedial and extension

Learners need to know these definitions in order to progress with this section of Mathematics. Find opportunities to use and emphasise the vocabulary in class, and ask questions continually that prompt learners to use the vocabulary.

Suggested answers

1 Straight line: the shortest distance between two points. (Explain to learners that any other line between these points would be curved.)
2 Ray: a straight line extending from a point.
3 Line segment: a line that has a start and end point.
4 Angles on a straight line: angles on one side of a straight line add up to 180°.
5 Perpendicular lines: lines that are at right angles to each other.
6 Intersecting lines: lines which cross each other. The point at which they cross is called the intersection.
7 Adjacent angles: angles that share a vertex and lie on opposite sides of a common side.
8 Supplementary angles: angles that add up to 180°.
9 Vertically opposite angles: angles that share a vertex and with sides that form two intersecting lines.
10 Acute angle: an angle that measures more than 0°and less than 90°
11 Right angle: an angle that measures 90°
12 Obtuse angle: an angle that measures more than 90° but less than 180°

Pairs of angles formed by intersecting lines; Problem-solving: Intersecting lines and their pairs of angles

Angles on a straight line; Adjacent supplementary angles; Adjacent complementary angles; Vertically opposite angles

Activity 2 **Solve problems on the intersecting lines and their pairs of angles**

Learner's Book page 299

Guidelines for implementing this activity

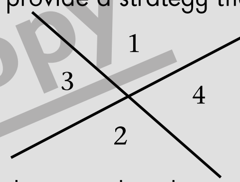
- Highlight the purpose of each task in the activity and emphasise angle relationships arising from this task.
- Work carefully through each construction with the learners.
- Let learners do each construction as you work through them in order to crystallise their understanding not only of the angle relationships, but also of the construction itself.
- Revise words such as *point; perpendicular; common vertex; common side; adjacent angles; supplementary angles; adjacent angles; adjacent supplementary angles; adjacent complementary angles* and *vertically opposite angles* as learners work through the constructions by using these meaningfully in context and in the form of probing questions as explained in the bulleted point that follows.
- Formulate the points in each section *(You should remember the following)* as questions during the course of the construction, for example, the statement "$R\hat{M}B$ and $R\hat{M}P$ share a common vertex M and lie opposite a common side MR, therefore they are adjacent angles" can be formulated as follows: Which is the common side shared by $R\hat{M}B$ and $R\hat{M}P$? What do we call these angles? $R\hat{M}B$ and $R\hat{M}P$ are

complementary angles – what does this mean? And so on.

- Encourage learners to express their findings in their own way before giving them a formal statement of the relationship.
- Once you have worked through all the constructions in this way, revisit the points in each section (*You should remember the following*) and work through them as though they were revision summaries.
- Allow learners to work through the theorems in pairs and report back their understanding of these. Consolidate all feedback received by summarising the theorems.
- Work through the worked example in the Learner's Book systematically. Do not just work through the solution. Instead, question learners about the problem at each step in order to get them to think about the problem and come up with the subsequent steps in the solution by themselves.
- Do an additional example if necessary, or allow learners to do an additional example in pairs or small groups and work through their solutions with the class before they do the activity.
- Allow learners to work in pairs or small groups when doing Activity 2.

Remedial and extension

- Emphasise angle names continually and, where possible, provide a strategy that would make remembering these names easy.
- Learners often misinterpret vertically opposite angles as meaning angles along a vertical line. For example, in the diagram learners may identify $\angle 1$ and $\angle 2$ as vertically opposite angles, but NOT $\angle 3$ and $\angle 4$.



  This misconception can be overcome by explaining to the learners that the term '"vertically opposite" means that the "vertices" of the angles lie "opposite" each other.
- Learners often also assume that corresponding and alternate angles are equal. Emphasise that these pairs of angles are equal only if the two lines are parallel.

Suggested answers

1 a 60° (vertically opposite angles)
  b 120° (supplementary angle on a straight line)
  c 120° (vertically opposite angles)
  d 90° (angle on a straight line)
  e 90° (angle on a straight line)
  f 50° (adjacent angle in a pair that is vertically opposite to 90°)
  g 78° (difference between 180°and known supplementary angles)
  h 78° (vertically opposite angles)
  k 50° (vertically opposite angles)
  m 52° (vertically opposite angles)

2 A Diagram A is correct
  B & C Diagrams B and C are incorrect (angles do not add up to 180°)

3 $x + 2x = 90° \therefore 3x = 90° \therefore x = 30°$
  $\therefore 2x = 60°$

$4y + 3y + 2y = 180° \therefore 9y = 180° \therefore y = 20°$
$\therefore 4y = 80°$; $3y = 60°$ and $2y = 40°$
$7z + 10° + z = 90° \therefore 8z + 10° = 90° \therefore 8z = 80° \therefore z = 10°$
$\therefore 7z + 10 = 80°$; $z = 10°$; $\angle$ RMS $= 80°$; $z = 10°$
($\angle$RMS and z are adjacent complementary angles)

4 **a** 144° (adjacent supplementary angles)
b 144° (vertically opposite angles)
c 36° (vertically opposite angles)
d 126° (exterior angle)
e 126° (vertically opposite angles)
f 125° (adjacent supplementary angles)
g 125° (vertically opposite angles)
h 55° (vertically opposite angles)
j 120° (vertically opposite angles)
k 60° (adjacent supplementary angles)
m 65° (sum of the interior angles of a triangle)
n 115° (adjacent supplementary angles)
p 60° (vertically opposite angles)
q 55° (vertically opposite angles)
r 65° (sum of the interior angles of a triangle)
s 65° (vertically opposite angles)

5 **a** Size of $D\hat{M}F$: $\hat{1} = 90°$ (vertically opp. right angle); $\hat{2} = 90°$ (angles on a straight line)
$\therefore D\hat{M}F = 120°$ [$360° - (90° + 90° + 60°)$; interior angles of quadrilateral AFMD]
Size of $F\hat{M}E$: $\hat{1} = 90°$ (vertically opp. right angle/angles on a straight line); $\hat{2} = 90°$ (vertically opp. right angle)
$\therefore F\hat{M}E = 125°$ [$360° - (90° + 90° + 55°)$]; interior angles of quadrilateral FCEM]
Size of $D\hat{M}E$: $\hat{1} = 90°$ (angles on a straight line); $\hat{2} = 90°$ (angles on a straight line); $D\hat{B}E = 65°$ (interior angle of $\triangle$ ABC)
$\therefore D\hat{M}E = 115°$ [$360° - (90° + 90° + 65°)$; interior angles of quadrilateral MEBD]

b Size of $P\hat{S}Q$: $\hat{1} = 25°$ ($\frac{1}{2}$ of vertically opp. angle);
$\hat{2} = 35°$ ($\frac{1}{2}$ of 70°; angles on a straight line)
$\therefore P\hat{S}Q = 180° - (25° + 35°) = 120°$ (interior angles of triangle)
Size of $P\hat{S}R$: $\hat{2} = 25°$ ($\frac{1}{2}$ of vertically opp. angle); $\hat{1} = 30°$ ($\frac{1}{2}$ of 60°; angles on a straight line)
$\therefore P\hat{S}R = 180° - (25° + 30°) = 125°$ (interior angles of triangle)
Size of $R\hat{S}Q$: $\hat{2} = 30°$ ($\frac{1}{2}$ of 60°; angles on a straight line); $\hat{1} = 35°$ ($\frac{1}{2}$ of 70°; angles on a straight line)
$\therefore R\hat{S}Q = 180° - (35° + 30°) = 115°$

c Smallest angle = 60° (complementary angles; tree at 90° to the ground); largest angle = 120° [(180° – 60° = 120°; angles on XZ (straight line)]

UNIT

2 Parallel lines

Unit overview

Learner's Book page 301
Recommended pacing: 3 hours

This unit focuses on the following:
- Parallel lines
- Pairs of angles

Resources: Learner's book; exercise book; mathematical set; pen

Background information

Learners will apply the same background knowledge of intersecting lines and angle relationships as for Unit 1.
Learners continue to revise and write clear descriptions of angle relationships on straight lines as well as use angle relationships and properties of pairs of angles on parallel lines to solve geometric problems. Learners are expected to give reasons to justify their solutions for every written statement.

Teaching guidelines

Work carefully through the concepts and examples in the Learner's Book as described in the guidelines for each activity. Practical, hands-on revision and consolidation of prior learning as described here will facilitate the effective application of known concepts to problem-solving in the new contexts presented in this unit. "Chalk-and-talk" and merely reading through the concepts and the examples and assuming that learners are taking it in are *not* advised. A practical, step-by-step approach is the best form of consolidation and remediation. With this approach questions that extend learners can also be included.
Learners are expected to give reasons to justify their solutions for every written statement in Grade 9. Provide as many additional examples as possible to help build the confidence that learners need to tackle problems that may otherwise appear daunting at first. Practice is the best way in which to demystify what among many learners is perceived to be a "difficult" branch of Mathematics.
Make sure that learners have access to the necessary instruments unit and that they work in pairs or groups if necessary.
Great care should always be taken to ensure that they set out their work neatly and logically at all times, that they always provide the necessary proofs where asked to do so and that they adhere to the basic conventions that apply in Geometry.

Revision: Vocabulary of parallel lines and angles

Activity 1 Revise concepts on parallel lines and angles

Learner's Book page 301

Guidelines for implementing this activity

- Learners should already be familiar with the vocabulary in Activity 1. Prompt learners to give these definitions to the whole class and discuss the answers as a class. Let individual learners draw rough diagrams on the board to demonstrate their understanding.
- Many opportunities for using these words are presented throughout this unit. Use this vocabulary carefully and deliberately, as often as possible, so that learners become familiar with and comfortable using them.

Remedial and extension

Learners need to know these definitions in order to progress with this section of Mathematics. Use the opportunities presented in this unit to use and emphasise the vocabulary in class, and ask questions continually that prompt learners to use the vocabulary.

Suggested answers

1 Parallel lines: lines which are the same distance apart all along their length. Parallel lines never cross.
2 Transversal: a straight line that cuts across two other straight lines.
3 Adjacent supplementary angles: angles that add up to 180°.
4 Vertically opposite angles: angles that lie between intersecting lines and that share only a vertex.
5 Corresponding angles: angles that lie on the same side of the transversal and on the same side of each of the two lines (their positions correspond).
6 Alternate angles: angles that lie on opposite sides of the transversal and always between the two lines (their positions alter form one side of the transversal to the other).
7 Co-interior angles: angles that lie on the same side of the transversal and between the two lines.

Pairs of angles formed by parallel lines; Problem-solving: Parallel lines and their pairs of angles

Adjacent supplementary angles; Vertically opposite angles; Corresponding angles; Alternate angles; Co-interior angles

Activity 2 Solve problems on parallel lines and angle pairs

Learner's Book page 305

Guidelines for implementing this activity

- The sketches at the beginning of this unit provide a powerful visual of the concepts to be covered. Refer learners to these sketches whenever necessary.
- Work carefully through each concept with the learners.
- Let learners do the actual constructions. Do the construction on the board as well. An alternative to copying the angles from the Learner's Book once they have done the constructions is the following approach:
  - Let the learners close their textbooks after they have drawn the lines and transversals.

- Ask them to fill in a starting angle, for example, angle A by showing them this on the board.
  - Give the instruction: Fill in the adjacent supplementary angle B. Walk around the classroom to check that they have done it correctly.
  - Fill in the adjacent supplementary angle B on your diagram on the board as well.
  - Give the next instruction: Fill in all the adjacent supplementary angles on your diagram. Again, walk around the classroom and check learners' work as they do this.
  - Ask them to count the number of pairs of adjacent supplementary angles that are formed when two non-parallel lines are cut by a transversal (8 pairs). check that they have counted in the adjacent supplementary angles along the transversal (vertical) as well.
  - Ask them to write down the conclusions as shown in the Learner's Book (Learner's Book is still closed), for example, $a + b = 180^\circ$.
  - Repeat the above process with all the concepts.
- Encourage learners to express their findings in their own way before giving them a formal statement of the relationship.
- Once you have worked through all concepts in this way, revisit the points in each section (*You should remember the following*) and work through them as though they were revision summaries.
- Allow learners to work through the theorem in pairs and report back their understanding of it. Consolidate all feedback received by summarising the theorem.
- Let learners work through the examples in pairs or small groups and share and compare their findings with the class in order to get them to think about the problem and come up with the solutions by themselves.
- The activity, as with the steps and worked examples above, allows learner to revise some of the results established in earlier units. It is very important to keep on referring to these results in every Geometry activity so that learners treat Geometry as a whole body of knowledge as opposed to isolated knowledge pieces.

Remedial and extension

Working through the concepts and examples in the manner described above is aimed at ensuring that learners revise and consolidate prior knowledge of angle relationships, and facilitating the acquisition of new knowledge and skills particularly with regards to problem-solving and justifying solutions. "Chalk-and-talk" and merely reading through the concepts and the examples and assuming that learners are taking it in are not advised. A practical, step-by-step approach is the best form of remediation. With this approach questions that extend learners can also be included.

Suggested answers

1 **a** $\hat{1} = 130°$ (adjacent supplementary angles); $\hat{2} = 130°$ (co-interior angles)
  b $\hat{1} = 54°$ (vertically opposite angles); $\hat{2} = 126°$ (co-interior angles)
  c $\hat{1} = 114°$ (adjacent supplementary angles); $\hat{2} = 114°$ (corresponding angles)

2 **a** $\hat{a} = 105°$ (vertically opposite angles); $\hat{b} = 105°$ (corresponding angles); $\hat{c} = 105°$ (alternate angles)

b $\hat{d} = 110°$ (corresponding angles); $\hat{e} = 110°$ (vertically opposite angles); $\hat{f} = 110°$ (corresponding angles)

c $\hat{g} = 40°$ (vertically opposite angles); $\hat{h} = 40°$ (alternate angles); $\hat{k} = 40°$ (vertically opposite angles)

3 **a** $\hat{a} = \hat{b} = \hat{c}$ (corresponding angles) $= \hat{d}$ (alternate angles) $= \hat{e}$ (vertically opposite angles)
$\therefore \hat{a} = \hat{e}$

b $\hat{a} = \hat{b}$ (vertically opposite angles) $= \hat{c}$ (corresponding angles) $= \hat{d}$ (alternate angles) $=$ $\hat{e}$ (alternate angles)
$\therefore \hat{a} = \hat{e}$

c $\hat{a} = \hat{b}$ (alternate angles) $= \hat{c}$ (corresponding angles) $= \hat{d}$ (corresponding angles) $=$ $\hat{e}$ (alternate angles)
$\therefore \hat{a} = \hat{e}$

4 **a** Yes, co-interior angles are supplementary [vertically opp. angle of 77˚ between intersecting lines; co-interior angle of 77˚ is formed; 103˚ + 77˚ = 180˚ therefore the lines are parallel (co-interior angles are supplementary)]

b No, they only add up to 170° (co-interior angles are not supplementary)

c Yes, corresponding angles are equal [interior angle = 139° (angles on a straight line); corresponding angles are therefore equal]

5 **a** $\hat{x} = \hat{3}$ (corresponding angles); $\therefore x + 3x = 180˚$ (angles on straight line)
$\therefore 4x = 180˚ \therefore x = 45˚$

b $4x = \hat{5}$ (vertically opp. angles); $\therefore x + 4x = 180˚$ (co-interior angles)
$\therefore 5x = 180˚$
$\therefore x = 36˚$

c $2x - 30˚ = \hat{4}$ (corresponding angles)
$\therefore (2x - 30˚) + (x - 15˚) = 180˚$ (angles on straight line)
$\therefore 3x - 45˚ = 180˚$
$\therefore 3x = 180˚ + 45˚ = 225˚$
$\therefore x = 75˚$

6 **a** $\hat{1} = \hat{2} = 28°$ (half of supplementary angle 56˚);
$\hat{3} = \hat{4} = 56°$ (vertically opposite angles; alternate angles)

b $\hat{1} = 24°$ (alternate angles); $\hat{2} = 156°$ (co-interior angles);
$\hat{3} = 66°$ (complementary angles); $\hat{4} = 114°$ (co-interior angle to the supplementary angle of 66°); $\hat{5} = 90°$ (corresponding angles)

c $\hat{1} = 50°$ (alternate angles); $\hat{2} = 40°$ (complementary angles); $\hat{2} = 40°$ (complementary)
$\hat{3} = 40°$ (alternate angles); $\hat{4} = 90°$ (alternate angles); $\hat{4} = 90°$ (alternate $\angle$)
$\hat{5} = 56°$ (co-interior angle) supplementary to 124°

7 **a** $\hat{B} = 135°$ (supplementary angles); $\hat{C} = 45°$ (alternate angles) $\hat{D} = 90°$ (given); $\hat{A} = 90°$ (sum of interior angles of quadrilateral = 360°)

b $\hat{N} = 70°$ (given); $\hat{K} = 110°$ (KL||NM; co-interior angles supplementary); sum of interior angles of quadrilateral = 360° $\therefore 4p = 180 \therefore p = 45°$
$\therefore \hat{L} = 135°$ and $\hat{M} = 45°$

c $\hat{S} = 64°$ (given); $\hat{P} = \hat{Q} = 116°$ ($S\hat{P}Q$ co-interior supplementary); $\hat{R} = 64°$ (sum of interior angles of quadrilateral = 360°)

8 **a** $\hat{D} = \hat{B} = 68°$ and $\hat{A} = \hat{C} = 112°$ (opposite angles of parallelogram; sum of interior angles of quadrilateral = 360°)

b $\hat{M}_1 = 42°$ (alternate angles; opp. sides of parallelogram are parallel); $\hat{K}_1 = 64°$ (alternate angles; opp. sides of parallelogram are parallel)
$= \hat{N} = \hat{L} = 74˚$ [opp. angles of parallelogram equal: (360 – 212) ÷ 2]

c $\hat{R}_1 = 90°$ (alternate angles; PQ ∥ SR); $\hat{P}_1 = 36°$ (alternate angles; PQ ∥ SR); $\hat{S} = \hat{Q} = 54˚$ [opp. angles of parallelogram equal: (360 – 252) ÷ 2]

UNIT

3 Mixed geometric problems

Unit overview

Learner's Book page 308
Recommended pacing: 3 hours

This unit focuses on the following:
- Pairs of angles
- Mixed geometric problems.

Resources: Learner's Book; exercise book; mathematical set; pen

Background information

Learners will once again apply the same background knowledge of intersecting lines and angle relationships as for Units 1 and 2.

Learners continue to revise and write clear descriptions of angle relationships of straight lines as well as use angle relationships and properties of pairs of angles on parallel lines to solve geometric problems.

Learners continue to revise and write clear descriptions of angle relationships on straight lines and use angle relationships and properties of triangles, quadrilaterals and other polygons to solve geometric problems. Learners are expected to give reasons to justify their solutions for every written statement.

Teaching guidelines

Work through the examples in the Learner's Book carefully and revise concepts as often as possible to reinforce learners' knowledge and grasp of this section.

Provide as many additional examples as possible.

Solving Geometry problems is about getting practice, achieving small milestones, staying motivated and about having fun. Enthusiasm on the part of the teacher plays a key role and creates the tone and ethos required for a positive, motivated approach to solving Geometry problems.

Revision: Pairs of angles

Activity 1 Revise pairs of angles

Learner's Book page 308

Guidelines for implementing this activity

The vocabulary in Activity 1 should be fresh in learners' memories. Revise the vocabulary and allow learners to draw rough drawings on the board to show the angle relationships referred to here. Do not spend too much time on this.

Suggested answers

1 Adjacent angles: when two lines intersect, four angles are formed; any pair of angles that meet along a straight line are adjacent angles.
2 Complementary angles: a pair of angles that add up to 90°.
3 Adjacent complementary angles: adjacent angles that add up to 90°.
4 Supplementary angles: a pair of angles that add up to 180°.
5 Adjacent supplementary angles: adjacent angles that add up to 180°.
6 Opposite angles: when two lines intersect, four angles are formed. Any pair of these angles that meet at the point of intersection are opposite angles. Opposite angles are equal in size.
7 Vertically opposite angles: another name for opposite angles.
8 Corresponding angles: any pair of angles that lie on the same side of a transversal and on the same relevant sides of the parallel lines. Corresponding angles are equal.
9 Alternate angles: any pair of angles that lie on opposite sides of the transversal and on opposite relevant sides of the parallel lines. Alternate angles are equal in size.
10 Co-interior angles: any pair of angles that lie on the same side of the transversal and between the parallel lines. Co-interior angles add up to 180°.

Solving mixed geometric problems

Activity 2 Solve mixed geometric problems

Learner's Book page 310

Guidelines for implementing this activity

- Work carefully through the examples in the Learner's Book. Draw the diagrams in each example on the board and let learners close their Learner's Books. Ensure optimal learner involvement by working through the examples in the following way:
  - Let learners work through each question in each example in pairs or small groups and take turns to volunteer their solutions. Discuss and work through these solutions with the class, and allow the rest of the learners to compare these with their own solutions.
  - Ensure that pairs or small groups share individual solutions in each example, not all the solutions per example, to ensure that all groups/pairs have a turn to present theirs to the class.
- Iron out all difficulties as you work through the examples with the learners in this way. Ensure that learners always provide reasons to justify their solutions and that they use the correct vocabulary at all times. Also ensure that they understand each step and the reasoning behind the steps.
- Note that these angle relationships allow learners to revise some of the results established in earlier chapters. It is very important to keep on referring to these results in every Geometry activity so that learners can treat geometry as a 'whole' body of knowledge as opposed to isolated knowledge pieces.

Remedial and extension

Provide additional homework examples. The more learners practise these geometric problems the more their confidence will grow, the more motivated they will be to tackle these problems and the better they will become at solving them. Ensure that, if necessary, simple examples are given at first that ensure success, which in turn will inspire a positive attitude and keep learners motivated.

Suggested answers

1 $b = 80^\circ$(adjacent supplementary angles)
$c = b = 80^\circ$ (vertically opposite angles)
$d = 100^\circ$ (vertically opposite angles OR adjacent supplementary)
$e = 100^\circ$ (co-interior angles, $AB \parallel CD$)
$f = b = 80^\circ$ (alternate angles, $AB \parallel CD$ or adjacent supplementary)
$g = b = 80^\circ$ (corresponding angles, $AB \parallel CD$ or adjacent supplementary)
$h = d = 100^\circ$ (corresponding angles, $AB \parallel CD$ or adjacent supplementary)

2 **a** $\angle W_3 = 90^\circ$

b

(i) $\angle O_4 = 80^\circ$ (adj. Supp. $\angle$s)
(ii) $\angle U_1 = \angle T = 50^\circ$ (Alt. $\angle$s, $SU \parallel TW$)
(iii) $\angle W_1 = 180^\circ - (\angle T + \angle O_2)$
But $\angle O_2 = 180^\circ - 100^\circ = 80^\circ$ (angles on a str. line)
$\therefore \angle W_1 = 180^\circ - 50^\circ - 80^\circ$
$= 50^\circ$.
(iv) $\angle W_2 = 90^\circ - \angle W_1$ ($\angle W_1 + \angle W_2 = 90^\circ$)
$= 90^\circ - 50^\circ = 40^\circ$
(v) $\angle V = \angle W_1$ (corr. $\angle$s, $SU \parallel TW$)
$= 50^\circ$
(vi) $\angle S = \angle V$ (opp. $\angle$s of $\parallel$m)
$= 50^\circ$
(vii) $\angle U_3 = 90^\circ - \angle V$ (sum of int. $\angle$s of ΔUWV)
$= 90^\circ - 50^\circ = 40^\circ$

c $\angle U_2 = 180^\circ - (\angle T + \angle W_1 + \angle W_2)$ (sum of int. $\angle$s of ΔUWT)
$= 180^\circ - (50^\circ + 50^\circ + 40^\circ)$
$= 180^\circ - 140^\circ = 40^\circ$
$\therefore \angle U_2 = \angle W_2 = 40^\circ$
But these are base angles of ΔUOW
Hence, ΔUOW is isosceles.

d In ΔSUW and ΔVWU,
$\angle S = \angle V$ (opp. $\angle$s of a $\angle$m)
$\angle VUW = \angle SUW$ (alt. $\angle$s; $SU \parallel T$)
$SW = VU$ (opp. sides of a $\parallel$m)
$\therefore \Delta SUW \equiv \Delta VWU$ (AAS)

e In ΔTOW and ΔTUV
$\angle T$ is common
$\angle W_1 = \angle V$ (corr. $\angle$s; $SU \parallel TW$)
$\therefore \Delta TOW$ and ΔTUV are equiangular
Hence, $\Delta TOW ||| \Delta TUV$

3 **a** $\hat{D}_3 = \hat{C}$ (given); $\hat{B} = \hat{D}_1 = 75°$ (corr. angles; $DE \parallel BC$)
$\therefore \hat{D}_3 = \hat{D}_1 = 75°$ (vertically opp. angles)
$\therefore \hat{C} = 75°$ ($\hat{D}_3 = \hat{C}$; given)
$\therefore \hat{E}_1 = 75°$ (corr. angles; $DE \parallel BC$)
$\therefore \hat{E}_2 = 105°$ (supplementary angles)

b $G\hat{H}L = 41°$ (given); $H\hat{M}N = 122°$ (given)
$\therefore \hat{H}_1 = 58°$ (co-interior angles; $GHK \parallel LN$)
$\therefore \hat{H}_2 = 81°$ (supplementary angles)

c $\hat{Q} = 70°$ (alternate angles; $PQ \parallel RT$); $\hat{S}_1 = 70°$ (interior angles of isosceles triangle)
$\therefore \hat{P} = 40°$ (sum of angles of a triangle is 180°)

4 **a** $\hat{A} = 40°$ (given); $\hat{B} = 70°$ (given)
$\therefore \hat{C}_2 = 70°$ (sum of angles of a triangle is 180°).
$\hat{C}_1 = 40°$ (alternate angles); $\hat{A}_1 = \hat{C}_2 = 70°$ (alternate angles);
$\hat{D} = \hat{B} = 70°$ (opposite angles of a parallelogram are equal) $\therefore \triangle$ CAD is isosceles (base angles are equal)

b $\hat{N}_1 = \hat{N}_2$ (given, LN bisects $K\hat{N}M$); $\hat{L}_1 = \hat{N}_2$ (alternate angles; opp. sides of parallelogram are parallel); $\hat{L}_2 = \hat{N}_1$ (alternate angles)
Since $\hat{N}_1 = \hat{N}_2$ we can say that NL bisects $K\hat{L}M$
$\therefore \hat{N} = \hat{L}$
$\hat{K} = \hat{M}$ (opp. angles of parallelogram are equal)
$\therefore$ KLMN is a rhombus (all opp. angles are equal)

c $\hat{P} = 50°$ (given); $T\hat{Q}S = \hat{S}$ (alt. angles)
$\therefore 2y = y + 25°$
$\therefore y = 25°$
$\therefore 2y = 50°$
$\therefore \triangle$QPS is an isosceles triangle (base angles are equal)

5 **a** DE is a straight line $\therefore \hat{A}_1 + \hat{A}_2 + \hat{A}_3 = 180°$
$\hat{A}_1 = \hat{B}$ (alternate angles) and $\hat{A}_3 = \hat{C}$ (alternate angles)
$\therefore \hat{A}_1 + \hat{A}_2 + \hat{A}_3 = 180° = \hat{A}_2 + \hat{B} + \hat{C}$
$\therefore$ the sum of the interior angles of $\triangle$ ABC is 180°

b $\hat{R}_1 = \hat{R}_1$ (common); $\hat{P} = \hat{R}_2$ (alternate angles); $\hat{Q} = \hat{R}_3$ (corresponding angles)
$\therefore$ the exterior angle of $\triangle$PQR = the sum of the two opposite interior angles

c $\hat{R}_1 + \hat{R}_2 + \hat{R}_3 = 180°$ (supplementary adjacent angles)
Using the proofs from question **5b** we therefore see that the sum of the interior angles of $\triangle$PQR is 180°

6 **a** $\hat{B} = D\hat{E}C$ (corresponding angles); $\hat{C} = \hat{C}$ (common);
$\hat{A} = \hat{D}$ (corresponding angles)
$\therefore \triangle ABC \,|||\, \triangle DEC$ (AAA).

b $DE^2 + EC^2 = DC^2$ (RHS)
$\therefore 3^2 + 4^2 = 25$
$\therefore$ DC = 5 cm

c BE = 8 cm which is $2 \times$ EC (given, EC = 4 cm)
BE + EC = 12 cm
$\therefore \frac{12}{4} = 3$ cm ($ABC \,|||\, \triangle DEC$ proven)
$\therefore$ ratio of similar triangles is 3
$\therefore$ AC = 15 cm ($\frac{15}{5} = 3$: ratio of AC)
$\therefore$ AD = 15 cm – 5 cm = 10 cm

d $AB^2 = AC^2 - BC^2$ (RHS)
$= 15^2 - 12^2 = 225 - 144 = 81$
$\therefore AB = \sqrt{81} = 9$ cm

7 **a** $A\hat{E}B = G\hat{E}F$ (vertically opposite angles); $A\hat{B}E = G\hat{F}E$ (alternate angles); $B\hat{A}E = F\hat{G}E$ (alternate angles)
$\therefore \triangle ABE \,|||\, \triangle GFE$ (AAA).

b CE = EG (given, E bisects CG/midpoint); $E\hat{C}D = E\hat{G}F$ (alternate angles); $C\hat{E}D = G\hat{E}F$ (vertically opposite angles) $\therefore \triangle ECD \,|||\, \triangle GFE$ (ASA).

c CDGF should be a parallelogram because diagonal FD bisects diagonal CG and CD$\|$FG.

8 **a** **To prove that ABGF is a rhombus:**
G is the centre of the circle with radius GB.
Radius of circle around regular hexagon = lengths of sides of hexagon
$\therefore$ AB = BG = GF = FA
$\therefore$ ABGF a rhombus (quadrilateral with four equal sides)

b **To prove that LKPQ is a rectangle:**
R is the centre of the circle with radius RK
$\therefore$ RK = RL = RP = RQ (radii of same circle)
And KP = LQ (diameters of same circle)
$\therefore$ KLPQ is a rectangle (equal diagonals bisect each other)
To prove that HKQR is a trapezium:
Sum of interior angles of octagon = $6 \times 180° = 1\,080°$
$\therefore \angle RHK = 1\,080° \div 8 = 135°$ (angles of regular octagon are equal)
and $\angle HKQ + \angle QKL = 135°$
but $\angle QKL = 90°$ (angle of a rectangle)
$\therefore \angle HKQ = 45°$
$\therefore \angle RHK + \angle HKQ = 135° + 45° = 180°$
but they are co-interior angles
$\therefore$ HR || KQ (co-interior angles supplementary)
$\therefore$ HKQR a trapezium (one pair of parallel sides)

c **To prove that UWXZ is a trapezium:**
Sum of interior angles of heptagon = 5 x 180° = 900°
$\therefore \angle T = 900° \div 7 = 128{,}5714°$
$\therefore \angle TUZ = \frac{1}{2}(180° - 128{,}5714°) = 25{,}7143°$
$\therefore \angle WUZ = 128{,}5714° - 2(25{,}7143°) = 77{,}1428°$ (congruent triangles)
and $\angle UWX = 128{,}5714° - 25{,}7143° = 102{,}8571°$
$\therefore \angle WUZ + \angle UWX = 77{,}1428° + 102{,}8571° = 180°$
$\therefore$ UZ || WX (co-interior angles supplementary)
$\therefore$ UWXZ a trapezium (one pair of parallel sides)
OR USE VARIABLES:
Let $\angle T = a$ and $\angle TUZ = \angle TZU = b$ (base angles of isosceles triangle)
$\therefore a + 2b = 180°$ (angles of triangle)
$\therefore b = \frac{1}{2}(180° - a) = 90° - \frac{1}{2}a$
$\therefore \angle U = \angle V = \angle W = a$ (angles of regular heptagon)
And $\angle ZUW = a - 2b$ and $\angle UWX = a - b$ (base angles of congruent triangles)
$\therefore \angle ZUW + \angle UWX = a - 2b + a - b = 2a - 3b = 2a - 3(90° - \frac{1}{2}a) = 3\frac{1}{2}a - 270°$
But $a = (5 \text{ x } 180°) \div 7 = 900° \div 7$ (angles of regular heptagon are equal)
$\therefore \angle ZUW + \angle UWX = 3\frac{1}{2}(900° \div 7) - 270° = 450° - 270° = 180°$
$\therefore$ UZ || WX (co-interior angles supplementary)
$\therefore$ UWXZ is a trapezium (one pair of parallel sides)

Chapter 12 Revision

Learner's Book page 313

Encourage learners to review the content covered before attempting the revision activities. The revision activities should be used to assess learners' progress thus far, and to assess where remediation may be required.

Suggested answers

1 a (i) $\angle C = 65°$ (int. $\angle$s of an isosceles Δ)
(ii) $\angle A = 50°$ $(180° - 65° - 65°)$(int. $\angle$s of an isosceles Δ)
(iii) $\angle F_2 = \angle C = 65°$ (alt. $\angle$s, EG||BC)
(iv) $\angle E_1 = \angle B = 65°$(corr. $\angle$s, EG||BC)
(v) $\angle F_1 = \angle E_1 + \angle A$ (ext. $\angle$ of a Δ; theorem)

$\angle F_1 = 65° + 65° = 130°$

b In ΔABC and ΔAEF
$\angle A$ is common
$\angle B = \angle E_1$ (proven)
$\therefore$ ΔABC and ΔAEF are equiangular.
Hence, $\Delta ABC \parallel\!| \Delta AEF$

2 **a** **(i)** $HF = 2 \times OF$ (diagonals of a square bisect each other)
$HF = 2 \times 5$ cm $= 10$ cm
(ii) $OE = 5$ cm (diagonals of a square are equal, and they bisect each other)
(iii) $EH^2 = OE^2 + OH^2$ (Pythagoras theorem)
$= (5 \text{ cm})^2 + (5\text{cm})^2 = 25\text{cm}^2$
$EH = \sqrt{50} \text{ cm}^2 = 7{,}07$ cm
(iv) $\angle JFG = \angle FGO$ (alt. $\angle$s, $JK \parallel EG$)
$= 45°$ (given)
(v) $\angle FOG = \angle EOH = 90°$ (vert. opp. $\angle$s)
(vi) $\angle KFL = \angle EOH = 90°$ (corr. $\angle$s, $JK \parallel EG$)
(vii) $\angle OEH = \angle EHI - \angle EOH$ (ext. $\angle$ of a Δ, theorem)
$= 135° - 90°$
$= 45°$
(viii) $\angle OGH = 90° - \angle OGF$ ($\angle FGH = 90°$ and int. $\angle$s of a square $= 90°$)
$= 90° - 45° = 45°$

b In triangles ΔEFG and ΔEOF,
$\angle EFG = 90°$ (int. $\angle$of a square)
But $\angle EOF = 90°$ (diagonals of a square are perpendicular to each other)
$\therefore \angle EFG = \angle EOH$
$\angle OEH = \angle OGF = 45°$
$\therefore$ ΔEFG and ΔEOF are equiangular
Hence $\Delta EFG \equiv \Delta EOF$

3 **a** $R_1 = 86°$ (opp. $\angle$s of a $\parallel$m OPRQ)
b $5x - 1 = 86°$ (corr. $\angle$s, $AB \angle CD$)
$5x = 86° - 1 = 85°$
$x = \frac{85°}{5} = 17°$
c $P_1 = 5x - 1 = 5 \times 17 - 1 = 84°$
$F_2 = 180° - 86°$ (adjacent sup. $\angle$s)
$= 94°$
$R_3 = 5x - 1 = 84°$ (corr. $\angle$s)
$Q_2 = R_3 = 84°$ (corr. $\angle$s, $AB \parallel CD$

CHAPTER

13 Perimeter and area of 2D shapes

Chapter overview

Learner's Book pages 314 to 346
Recommended pacing: 10 hours

This chapter focuses on the following:

Unit 1: The Theorem of Pythagoras	2 hours
Who was Pythagoras?	
Revising the Theorem of Pythagoras	
Applying the Theorem of Pythagoras	
Unit 2: Perimeter of polygons	2 hours
Revising polygons and conversions between SI units of length	
Finding the perimeter of triangles	
Finding the perimeter of quadrilaterals	
Finding the perimeter of other polygons	
Unit 3: Area of polygons	2 hours
Revising conversions between SI units	
Finding the area of triangles	
Finding the area of quadrilaterals	
Finding the area of other polygons	
Unit 4: Circumference and area of circles	2 hours
Revising the circle and its components	
Finding the circumference of circles	
Finding the area of circles	
Unit 5: The effect on perimeter or area if dimensions are doubled	2 hours
Problem-solving	

POA Investigation 3: Theorem of Pythagoras
Chapter 13 Revision

UNIT 1

The Theorem of Pythagoras

Unit overview

Learner's Book page 315
Recommended pacing: 2 hours

This unit focuses on the following:
- Revising the Theorem of Pythagoras
- Applying the Theorem of Pythagoras.

Resources: Learner's Book; exercise book; ruler; calculator; pen

Background information

In Grade 8, learners did the following:
- investigated the relationship between the lengths of the sides of right-angled triangles to develop the Theorem of Pythagoras
- determined whether a triangle is a right-angled triangle when the lengths of the sides of the triangle are known
- used the Theorem of Pythagoras to calculate a missing length in a right-angled triangle.

In Grade 9, the use of the Theorem of Pythagoras to solve problems involving unknown lengths of geometric figures that contain right-angled triangles is new.

Teaching guidelines

It is important that learners consolidate their grasp of the Theorem of Pythagoras because they will use it very often in Mathematics, for example, in Trigonometry, Euclidean geometry and Analytical geometry. This unit aims to revise this theorem in a manner that encourages practical investigation, and it is important that all learners actively participate in these investigations.

Who was Pythagoras?; Revising the Theorem of Pythagoras

Activity 1 Work with the Theorem of Pythagoras

Learner's Book page 317

Guidelines for implementing this activity

- The relationship between the lengths of the sides of triangles is explored in the steps in the Learner's Book. It is important that learners practically engage with steps 2 to 4 by counting the squares in order to consolidate their grasp of the theorem. When they memorise the words of the Theorem of Pythagoras, they need to see these squares in their minds.
- Work through the steps with learners, or let them work through these steps in small groups of three (not more than four).

- Walk around the class to monitor learner progress and understanding.
  - Observe what they are doing: Are they counting the squares? Are they working together? For example, are they taking turns with certain tasks in the step (counting the squares; writing the observations in the table; and so on).
  - Listen to what they are saying and the vocabulary that they are using: Who is doing all the talking? What are they saying? Are they using appropriate vocabulary? Does everyone get a chance to verbalise their thoughts? Are those who are quiet at the beginning building confidence to speak later on as their understanding grows?
- Discuss the points that follow this investigation once learners have completed steps 5 and 6. Do learners see the facts stated in the diagrams? Consolidate these facts by revisiting the diagrams in steps 2, 3 and 4 if necessary.
- Consolidate the theorem stated in symbolic form. Some learners may by now have memorised it from Grade 8, but ensure that they understand what it means through more detailed explanation (based on the investigation they have just completed).
- Include key questions in this section in quizzes at the beginning of every subsequent lesson.

Remedial and extension

Remedial: It is important that learners work through the investigation in a practical and hands-on manner. Provide additional triangles (a combination of right-angled, acute and obtuse-angled triangles with given side lengths). Let learners draw the areas of the sides as shown in the investigation and record the information in a table as shown in order to further explore the Theorem of Pythagoras. Allow learners to verbalise their thinking at all times in order to crystallise their understanding.

Suggested answers

1	Right-angled	2	Acute-angled	3	Acute-angled
4	Obtuse-angled	5	Obtuse-angled	6	Right-angled
7	Right-angled	8	Right-angled	9	Right-angled
10	Right-angled	11	Right-angled	12	Right-angled
13	Right-angled	14	Right-angled	15	Right-angled

Applying the Theorem of Pythagoras

Activity 2 Apply Pythagoras's Theorem

Learner's Book page 319

Guidelines for implementing this activity

- This section focuses on the practical application of the Theorem of Pythagoras. Ensure that learners understand when we use this theorem:
  - to calculate the length of an unknown side of a right-angled triangle
  - to determine whether a triangle is right-angled; acute-angled or obtuse-angled
  - to solve real-life problems.

- The second and third examples reflect real-life problems, and model the type of additional problems that learners can be given to solve if necessary. Work carefully, in a step-by-step manner, through the examples in the Learner's Book. Do the examples on the board, and do additional examples if necessary.
- Always approach the problems by asking, as a first step: What do we know? What information are we given in the problem?
- Write down as many facts as possible in a separate space on the board, before working through the steps. Use these given facts when tackling each step, in this way modelling how a problem should be set out, approached and solved.

Tip

Work in smaller groups with learners if necessary, and/or let learners work in mixed-ability groups.

Suggested answers

1 **a** $a^2 = b^2 + c^2$
$a^2 = 49 + 576$
$a^2 = 625$
$a = 25$

b $b^2 = a^2 - c^2$
$b^2 = 1\,369 - 1\,225$
$b^2 = 144$
$b = 12$

c $c^2 = a^2 - b^2$
$c^2 = 5\,625 - 441$
$c^2 = 5\,184$
$c = 72$

d $d^2 = a^2 - b^2$
$d^2 = 841 - 400$
$d^2 = 441$
$d = 21$

e $e^2 = b^2 + c^2$
$e^2 = 81 + 484$
$e^2 = 565$
$e \approx 23{,}77$

f $a^2 = b^2 + c^2$
$a^2 = (f)^2 + (2f)^2$
$(\sqrt{320})^2 \quad 5f^2$
$[320/5] = f^2$
$64 = f^2$
$8 = f$
$\therefore 2f = 16$

2 **a** $b^2 = a^2 + c^2$
$b^2 = 14\,400 + 8\,100$
$b^2 = 22\,500$
$b = 150$ mm

b $b^2 = a^2 + c^2$
$b^2 = 40\,000 + 22\,500$
$b^2 = 62\,500$
$b = 250$ mm

c $c^2 = b^2 - a^2$
$c^2 = 260\,100 - 57\,600$
$c^2 = 202\,500$
$c \approx 450$

d $a^2 = c^2 - b^2$
$a^2 = 122\,500 - 14\,400$
$a^2 = 108\,100$
$a = 328{,}79$

e $b^2 = a^2 + c^2$
$b^2 = 10\,000 + 67\,600$
$b^2 = 77\,600$
$b \approx 278{,}6$ mm

f $b^2 = a^2 + c^2$
$b^2 = 10\,000 + 67\,600$
$b^2 = 77\,600$
$b \approx 278{,}6$ mm

3 **a** $b^2 = \mathrm{AB}^2 + \mathrm{BD}^2$
$b^2 = 64$ cm $+ 36$ cm
$b^2 = 100$ cm
$b = 10$ cm
$d^2 = \mathrm{AB}^2 + \mathrm{BC}^2$
$d^2 = 64$ cm $+ 225$ cm
$d^2 = 289$ cm
$d = 17$ cm

b $x^2 = \mathrm{PQ}^2 - \mathrm{PS}^2$
$x^2 = 1\,369$ cm $- 144$ cm
$x^2 = 1\,225$
$x \approx 35$
$y^2 = \mathrm{PR}^2 - \mathrm{PS}^2$
$y^2 = 169$ cm $- 144$ cm
$y^2 = 25$ cm
$y = 5$ cm

4 **a** $\mathrm{BD}^2 = 2\mathrm{AB}^2$
$\mathrm{BD}^2 = 2(40 \text{ mm})^2$
$\mathrm{BD}^2 = 2(1\,600)$
$\mathrm{BD}^2 = 3\,200$
$\mathrm{BD} \approx 57$ mm

b $\mathrm{LN}^2 = \mathrm{LM}^2 + \mathrm{MN}^2$
$\mathrm{LN}^2 = 56^2 + 65^2$
$\mathrm{LN}^2 = 3\,136 + 4\,225$
$\mathrm{LN} \approx 86$ mm

c $a^2 = b^2 + c^2$
$a^2 = 25^2 + 50^2$
$a^2 = 625 + 2\,500$
$a \approx 56$ m

5 $a^2 = 2b^2$
$(80 \text{ mm})^2 = 2b^2$
$6\ 400 = 2b^2$
$3\ 200 = b^2$
$56{,}57 \text{ mm} \approx b$

6 $(\text{Distance } 1)^2 = a^2 - b^2$
$(\text{Distance } 1)^2 = 19{,}5^2 - 18^2$
$(\text{Distance } 1)^2 = 380{,}25 - 324$
$(\text{Distance } 1)^2 = 56{,}25$
Distance 1 = 7,5 m
and
$(\text{Distance } 2)^2 = c^2 - d^2$
$(\text{Distance } 2)^2 = 19{,}5^2 - 7{,}5^2$
$(\text{Distance } 2)^2 = 380{,}25 - 56{,}25$
$(\text{Distance } 2)^2 = 324$
Distance 2 = 18 m
$\therefore$ the width of the road is 18 m + 7,5 m = 25,5 m

7 $a^2 = 2b^2$
$a^2 = 2(60 \text{ mm})^2$
$a^2 = 2(3\ 600)$
$a^2 = 7\ 200$
$a \approx 84{,}85 \text{ mm}$

UNIT 2

Perimeter of polygons

Unit overview

Learner's Book page 321
Recommended pacing: 2 hours

This unit focuses on the following:
- Revising polygons and conversions between SI units of length
- Finding the perimeter of triangles
- Finding the perimeter of quadrilaterals
- Finding the perimeter of other polygons

Resources: Learner's Book; exercise book; ruler; calculator; pen

Background information

In Grade 8, learners did the following:
- Revised and investigated the perimeter of polygons
- Formulae for calculating perimeter of polygons such as triangles, quadrilaterals.

In Grade 9, learners revise the perimeter of triangles and quadrilaterals and learn to calculate the perimeter of other polygons.

Teaching guidelines

Learners dealt with perimeter of squares and rectangles in Grades 7 and 8. However, learners often struggle to differentiate between perimeter and area, therefore it is very important that this differentiation is made clear and that learners understand this before perimeter and area formulae are introduced (in the next unit). Do this by revising basic shapes and the difference between area and perimeter before the start of this chapter, and by letting learners do simple perimeter and area calculations at the start of every lesson.

Revising polygons and conversions between SI units of length

Activity 1 Revise polygons and conversions between SI units of length

Learner's Book page 321

Guidelines for implementing this activity

- Spend time revising polygons and conversions by working through Activity 1. Ensure that learners are clear about these descriptions and conversions at the start of every lesson.

Suggested answers

1 **a** Polygon: a flat shape completely enclosed by three or more straight edges.
 b Triangle: a polygon with three edges. Its interior angles add up to 180°.
 c Quadrilateral: a polygon with four edges. Its interior angles add up to 360°.
 d Pentagon: a pentagon with five edges.
 e Hexagon: a polygon with six edges.
 f Heptagon: a polygon with seven edges.
 g Octagon: a polygon with eight edges.
 h Circle: a closed curve. Each point of the curve is equidistant from a fixed point: the centre.
 i Perimeter/Circumference: the total distance around the edge(s) of a 2D shape.

2 **a** km to m = × 1 000; km to cm = × 100 000; km to mm = × 1 000 000
 b m to cm = ÷ 100; m to mm = × 1 000; m to km = ÷ 1 000
 c cm to mm = × 10 ; cm to m = ÷ 100; cm to km = ÷ 100 000
 d mm to cm = ÷ 10; mm to m = ÷ 1 000; mm to km = ÷ 1 000 000

Finding the perimeter of triangles

Activity 2 Determine the perimeter of triangles

Learner's Book page 322

Guidelines for implementing this activity

- Introduce this lesson with a right-angled triangle to show the relationship between the base side and a perpendicular height of a triangle.
- Then show learners that any side of a triangle (including a right-angled triangle) can be a base with a unique height. Whichever side is chosen as a base the area of the triangle remains the same.
- Give learners an opportunity to draw different heights of a triangle by making each of the sides a base. Once learners understand the relationship between the base and the height, they can apply the area formula to solve geometry problems.
- The example provided shows learners the types of geometry problems they are expected to solve using the area of a triangle formula, as well as the Theorem of Pythagoras.
- Go through the example making sure that learners understand each question and the steps involved in getting the answer.

- It may be necessary to do more than one example. Vary the measurements that are given, and the sides for which measurements are given, but always provide the measurements of at least two sides.

Remedial and extension

Provide additional examples similar to the questions in the activities for homework to reinforce the concepts taught in this unit. Ensure that learners progress from solving simple problems, to solving more complex problems. It has been recommended in the teaching guidelines at the beginning of the chapter that learners are given simple problems to solve at the beginning of each lesson as well.

Suggested answers

1
$s_2^2 = s_1^2 - s_3^2$
$s_2^2 = 30^2 - 24^2$
$s_2^2 = 900 - 576$
$s_2^2 = 324$
$s_2 = 18$ cm
$P = s_1 + s_2 + s_3$
$P = 30 + 18 + 24$
$P \triangle ABC = 72$ cm

2
$s_1^2 = s_2^2 + s_3^2$
$s\ s_1^2 = 12^2 + 35^2$
$s_1^2 = 144 + 1\ 225$
$s\ s_1^2 = 1\ 369$
$s_1 = 37$ cm
$P = s_1 + s_2 + s_3$
$P = 37 + 12 + 35$
$P \triangle DEF = 84$ cm

3
$s_2^2 = s_1^2 - s_3^2$
$s_2^2 = 75^2 - 21^2$
$s_2^2 = 5\ 625 - 441$
$s_2^2 = 5\ 184$
$s_2 = 72$ cm
$P = s_1 + s_2 + s_3$
$P = 75 + 21 + 72$
$P \triangle GHK = 168$ cm

4
$s_1^2 = 2s_2^2\ 80^2 = s_2^2 + s_3^2$
$6\ 400 = 2s_2^2$
$3\ 200 = s_2^2$
$56{,}57 \approx s_2$
$P = s_1 + s_2 + s_3$
$P = 80 + 56{,}57 + 56{,}57$
$P \triangle LMN = 193{,}14$ cm

5
$s_1^2 = s_2^2 + s_3^2$
$s\ s_1^2 = 23^2 + 46^2$
$s_1^2 = 529 + 2\ 116$
$s\ s_1^2 = 2\ 645$
$s_1 \approx 51{,}43$ cm
$P = s_1 + s_2 + s_3$
$P = 51{,}43 + 23 + 46$
$P \triangle PQR = 120{,}43$ cm

6
$(\sqrt{432})^2 = x^2 + (2x)^2$
$432 = 5x^2$
$86{,}4 = x^2$
$9{,}3 = x$
$P = s_1 + s_2 + s_3$
$P = 20{,}78 + 9{,}3 + 18{,}6$
$P \triangle TSU = 48{,}68$ cm

Finding the perimeter of quadrilaterals; Finding the perimeter of other polygons

Activities 3–4 **Determine the perimeter of quadrilaterals; Determine the perimeter of polygons**

Learner's Book pages 325–326

Guidelines for implementing this activity

- Show learners how formulae arise out of the definitions of these concepts for each quadrilateral, as shown in the introductory examples in this section. This makes it easier for learners to understand the formulae.
- These introductory example and the worked examples show learners the types of geometry problems that they are expected to solve using the perimeter formulae, and the application of the Theorem of Pythagoras. Go through these examples carefully with learners, ensuring that they understand each step before asking them to do the activities.

Remedial and extension

Activity 4 question **3** is an extension question. Learners work in small groups through one or two questions to assist learners who are having difficulty, but ensure that they do some questions on their own with minimal support.
Let learners re-do questions that they got wrong. However, work through the problem areas with them first before their second attempt. Allow them to verbalise their mistakes as far as possible in order to reduce the risk of repeating them.

Suggested answers

Activity 3

1 In △ AKD
$AD^2 = DK^2 + AK^2$
$AD^2 = 81 + 144$
AD = 15 cm
$P = 2(l + b)$
$P = 2(24 + 15)$
P ABCD = 78 cm

2 In △ FGH
$HG^2 = FH^2 – FG^2$
$HG^2 = 289 – 64$
HG = 15 cm
$P = 2(l + b)$
$P = 2(15 + 8)$
P EFGH = 46 cm

3 In △ JBM
JL ⊥ KM (diagonals of a rhombus are perpendicular)
$JM^2 = JB^2 + MB^2$
$JM^2 = 6^2 + 8^2$
$JM^2 = 100$
KM = 10 cm
$P = 4s$
P JKLM = 40 cm

4 PA = QA (diagonals of a square are equal and bisect each other)
$PQ^2 = 2PA^2$
$PQ^2 = 2(64)$
$PQ^2 = 128$
PQ = 11,3
$P = 4s$
$P = 11,31 \times 4$
P NPQR = 45,24 cm

5 ST = MN = 10 cm (STMN is a square)
TU = SV (corresponding sides of congruent triangles SMV and TNU (SAS))
In △ SMV:
$SV^2 = SM^2 + VM^2$
$SV^2 = 100 + 56,25$
$SV^2 = 156,25$
SV = 12,5 cm
$P = s_1 + s_2 + s_3 + s_4$
$P = 10 + 12,5 + 25 + 12,5$
P STUV = 60 cm

6 In △WXY
$XY^2 = WY^2 – WX^2$
$XY^2 = 210,25 – 110,25$
$XY^2 = 100$
XY = 10 cm
$P = 2(s_1 + s_2)$
$P = 2(10 + 10,5)$
P WXYZ = 41 cm

Activity 4

1 $AB = CD = \frac{1}{2}FE = 45$ mm $= BC = DE$ (given)
And $FE = 90$ mm $= AF$ (given)
P = sum of all sides
$P = 45 + 45 + 45 + 45 + 90 + 90$
P ABCDEF = 360 cm

2 In △ HBK
$HK^2 = HB^2 + BK^2$
$HK^2 = 300 + 100$
$HK^2 = 400$
$HK = 20$
P = sum of all sides (all sides are equal)
$P = 20 \times 6$
$P = 120$ cm

3 In △ QZY
$QY^2 = QZ^2 + YZ^2$
$QY^2 = 25 + 25$
$QY^2 = 50$
$QY \approx 7{,}07$ cm
P = sum of all sides
$P = 7{,}07 \times 8$
$P = 56{,}57$ cm

UNIT 3

Area of polygons

Unit overview

Learner's Book page 327
Recommended pacing: 2 hours

This unit focuses on the following:
- Revising conversions between SI units
- Finding the area of triangles
- Finding the area of quadrilaterals
- Finding the area of other polygons.

Resources: Learner's Book; exercise book; ruler; calculator; pen

Background information

In Grade 8, learners did the following:
- calculated the area of squares, rectangles, triangles and polygons to at least 2 decimal places
- solved problems with or without a calculator.

In Grade 9, learners revise the concepts that they learnt in Grade 8, and apply the Theorem of Pythagoras to solving problems that involve calculating the lengths of sides.

Teaching guidelines

Since learners have worked quite extensively with area and perimeter in Grades 7 and 8, many of the concepts here are revision of the basics. It is important to ensure that these basics are well consolidated and generally, if learners find difficulty with these concepts, ensure that they are given simple examples to practise (Grades 6, 7 and 8 levels) and gradually progress to more challenging examples.

Revising conversions between SI units

Activity 1 Revise conversions between SI units

Learner's Book page 327

Guidelines for implementing this activity

- Introduce this activity by revising conversions.
- Include these conversions in short quizzes at the beginning of every lesson in this unit.

Suggested answers

1 a cm to mm = $\times$ 10; cm² to mm² = $\times$ 100
b m to cm = $\times$ 100; m² to cm² = $\times$ 10 000
c mm to cm = $\div$ 10; mm² to cm² = $\div$ 100
d m to cm = $\div$ 1 000; m² to cm² = $\div$ 10 000

2	a 200 mm²	b 30 000 cm²	c 4 cm²	d 0,05 m²
	e 60 cm²	f 0,7 m²	g 800 cm²	h 9 m²

Finding the area of triangles

Base and height of a triangle; Formula for the area of a triangle

Activity 2 Determine the area and height of triangles

Learner's Book page 330

Guidelines for implementing this activity

- Start this lesson by drawing learners' attention to the fact that any one of the three sides of a triangle can be considered the base. The height is always perpendicular to the base. Let learners close their textbooks. Draw the same triangle three times as per the first example in the Learner's Book, and instruct a learner to draw in the heights (one from each vertex). Repeat a few times if necessary, emphasising the above.
- Repeat the above with right-angled and obtuse-angled triangles, working through these specific ones by pointing out the facts as described in the Learner's Book.
- Learners have already studied the properties of rectangles. Quickly revise these properties if necessary. They also worked with congruent triangles and a short revision of this concept might also be necessary. If they remember these basics, they should understand why the area of a triangle can be seen as half the area of a rectangle. Work through these basics provided in the introductory examples in order to consolidate this knowledge.
- Make sure that learners work correctly with the formula for the area of a triangle: $A = \frac{1}{2} \times b \times h$. It is simplification from left to right and not an application of the Distributive Law as some learners tend to think. This is very important, and should be carefully monitored when they do the activity.
- Work carefully through the examples In the Learner's Book on the board, and do an additional example if necessary.

Suggested answers

1 a Area = 216 cm²
  b Altitude to the hypotenuse = 14,4 cm

2 a 210 cm²
  b 11,35 cm

3 a 756 cm²
  b 20,16 cm

4 a 3 200 cm²
  b 80 cm

5 a 529 cm²
  b 20,57 cm

6 a 86,4 cm²
  b 8,3 cm

Finding the area of quadrilaterals; Finding the area of other polygons

Activities 3–4 Determine the area of quadrilaterals; Determine the area of other polygons

Learner's Book pages 333–334

Guidelines for implementing this activity

- Work through the examples in the Learner's Book to ensure that some of the basic concepts are adequately revised, and that learners know and can apply the formulae correctly.
- Ensure that learners work neatly and set out their steps correctly and logically, using the required proofs.

Remedial and extension

Work with learners through the first question in each activity, providing the required support and guidance by zooming in on specific problem areas and remediating these. Let learners re-do questions that they got wrong.
Refer learners to the table in the summary at the end for the formulae and the interpretation of these formulae if they need to. However, learners should guard against this becoming a "crutch" and should be encouraged as far as possible to memorise the formulae.

Suggested answers

Activity 3

1 Area $= l \times b$
$= 24 \times 12$
$= 288$ cm²

2 Area $= l \times b$
$= 8 \times \sqrt{225}$
$= 120$ cm²

3 Area $= 4(\frac{1}{2}bh)$
$= 4(\frac{1}{2}6 \times 8)$
$= 96$ cm²

4 Area $= s^2$
$= (\sqrt{128}])^2$
$= 128$ cm²

5 Area $= 2(\frac{1}{2}bh) + s^2$
$= 2\frac{1}{2}(7,5 \times 10) + 10 \times 10$
$= 75 + 100$
$= 175$ cm²

6 Area $= 2(\frac{1}{2}bh)$
$= 2(\frac{1}{2} \times 10,5 \times \sqrt{100})$
$= 105$ cm²

Activity 4

1 A = sum of areas of triangles that make up the polygon
ABCDEF is made of 6 congruent triangles.
Area $\triangle ABC = \frac{1}{2}bh$
$= \frac{1}{2}45 \times 45$
$= 1012{,}5 \text{ mm}^2$
Area ABCDEF = $1012{,}5 \text{ mm}^2 \times 6$
$= 6\,075 \text{ mm}^2$

2 HKLMNP is made up of 4 congruent triangles
Area $\triangle HKP = \frac{1}{2}bh$
$= \frac{1}{2} \times 20 \times \sqrt{300}$
$= 173{,}21 \text{ cm}^2$
Area HKLMNP = $173{,}21 \text{ cm}^2 \times 4$
$= 692{,}82 \text{ cm}^2$

3 Area = sum of all polygons that make up the polygon
$= 4(\frac{1}{2}bh)$ (4 triangles) + $2(lb)$ (2 small rectangles) + (lb) (large rectangle)
$= 4(\frac{1}{2}5 \times 5) + 2(5 \times \sqrt{50} + \sqrt{50} \times (10 + \sqrt{50})$
$= 50 + 70{,}7 + 120{,}71$
$= 241{,}41 \text{ cm}^2$

UNIT

4 Circumference and area of circles

Unit overview

Learner's Book page 335
Recommended pacing: 2 hours

This unit focuses on the following:
- Revising the circle and its components
- Finding the circumference of circles
- Finding the area of circles.

Resources: Learner's Book; exercise book; ruler; calculator; pen

Background information

In Grade 8, learners did the following:
- used and described the relationship between the radius, diameter and circumference of a circle in calculations
- used and described the relationship between the radius and area of a circle in calculations
- solved problems with or without a calculator, involving circumference and area of circles
- calculated to at least two decimal places
- used and described the meaning of irrational number Pi (π) in calculations.

In Grade 9, learners now use appropriate formulae and conversions between SI units to solve problems and calculate circumference and area.

Teaching guidelines

The content in this unit should be worked through carefully and systematically and learners' understanding of the differences between circumference and area should be monitored and consolidated at every point.

Revising the circle and its components

Activity 1 Revise the circle and its components

Learner's Book page 335

Guidelines for implementing this activity

- Introduce this activity by revising concepts learnt in previous grades.
- The circle is the only geometric shape whose properties have not been investigated in the previous chapters. Although Grade 9 learners may be familiar with the notion of a circle, start this unit with the explanation of circle terminology.
- Relationships between circle parts could be derived from their definitions, for example, diameter = radius + radius = 2 × radius, and so on.
- Include these types of questions in short quizzes at the beginning of every lesson in this unit.

Suggested answers

1	a	Circumference	b	Centre	c	Radius	d	Chord
	e	Diameter	f	Segment				
2	a	Sector	b	Segment	c	Semicircle	d	Semicircle
3	a	True	b	True	c	True	d	True
	e	True	f	True	g	False	h	False
	i	False	j	False				

Finding the circumference of circles

Activity 2 Find the circumference of a circle

Learner's Book page 337

Guidelines for implementing this activity

- Revise the constant ratio Pi (π) and its approximations, $\frac{22}{7}$ and 3,14. Once these terms are clear, the formulae for calculating the perimeter and area of a circle can be introduced.
- Just as the difference between perimeter and area was explained when dealing with quadrilaterals, the difference between the circumference and the area in a circle can also be explained in a similar manner. In fact, circumference is the 'equivalence' perimeter.
- Examples that follow on from the circle formulae show learners the types of problems they need to know that involve *circle* formulae.

- Work carefully through the examples in the Learner's Book, and do additional examples if possible. Practice is key.
- If calculators are available, learners can use the π button on the calculator. It is preferred that once having been introduced and having calculated the value of π, that learners remember it as: 3,14 or $\frac{22}{7}$.
- Pay attention to the correct rounding off of answers.

Suggested answers

1 **a** 157,1 cm **b** 301,71 mm **c** 19,76 m
2 **a** 157,14 mm **b** 786,71 m **c** 197,55 cm
3 C = 10 cm × r
= 15 × 2π
= 6,286 × 15
= 94,29 cm
Minute hand = 94,29 cm each hour
Hour hand = 5,24 cm in an hour

Finding the area of circles

Activity 3 Find the circumference and area of circles

Learner's Book page 338

Guidelines for implementing this activity

- The introduction includes a powerful visual that aims to further develop learners' understanding of the reasoning behind the formula for the area of a circle.
- Work carefully through the introduction and example in the Learner's Book, ensuring that learners grasp the meaning of the formula.
- Rounding off when working with π has implications for the answer. Advise learners to always round of to two decimal places and when marking their answers, accept small discrepancies that occur as a result of rounding off (or not rounding off). The methodology used is key.

Remedial and extension

Provide as many additional examples for homework as possible, especially after the re-introduction of the area of a circle in the next unit in order to consolidate learners' understanding of the two different concepts (circumference and area of a circle) and their application of the formulae. The skill of calculating circumference and area of circles improves only with practice.

Suggested answers

1 **a** 30 mm **b** 600 mm² **c** 25 mm **d** 157,08 mm
e 1 963,29 mm²
2 **a** 31,83 cm **b** 15,92 cm
c 796,22 cm² (the radius was rounded off to two decimal places)
3 Each semi-circle in questions 3a and 3b is half the perimeter and area of a complete circle.
a 109,9 mm; 1 932,25 mm² **b** 43,96 cm; 307,72 cm²

c The area and circumference of the part of the circle given is $\frac{2}{3}$ the area and circumference of the semi-circle with radius 7 m ($3 \times 60^\circ = 180^\circ$;
one part of the circle = 60° $\therefore$ two parts of the semi-circle = 120° (given)
$\therefore$ Circumference of the semi-circle = $\frac{1}{2}$(Circumference of complete circle)
= $\frac{1}{2}$(43,96) = 21,98.
Circumference of $\frac{2}{3}$ of the semi-circle = 2(21,98 ÷ 3) = 2(7,33) = 14,66 m.
Area of the semi-circle = $\frac{1}{2}$(Area of complete circle) = $\frac{1}{2}$(153,86) = 76,93.
Area of $\frac{2}{3}$ of the semi-circle = 2(76,93 ÷ 3) = 2(25,64) = 51,28 m.

UNIT 5 The effect on perimeter or area if dimensions are doubled

Unit overview

Learner's Book page 339
Recommended pacing: 2 hours

This unit focuses on the following:
- Investigation: the change in perimeter or area of 2d shapes if one or more dimensions change
- Problem-solving

Resources: Learner's Book; exercise book; ruler; calculator; pen

Background information

In Grade 8, learners did the following:
- Investigated area and perimeter of 2D shapes.

In Grade 9, the effect of changed dimensions on area and perimeter is new.

Teaching guidelines

The effects of dimension change are investigated using simple algebraic concepts of substitution, associative and commutative laws. Diagrams are helpful in facilitating learners' understanding of the change effects on the original polygon. It is advisable to provide many additional activities on identifying permitted dimension changes on each polygon and the new dimensions arising from such changes. Examples of quadrilaterals with interesting relationships involving areas and perimeters are squares, rectangles, and kites.

Problem-solving

Activities 1–2 Investigate the change in perimeter or area of 2D shapes if one or more dimension changes; Work with changed dimensions

Learner's Book page 343

Guidelines for implementing this activity

- The investigations focus on the effects of dimension change in a square, rectangle, triangle and circle and model the application of algebraic skills in the methods used. Use a similar approach to investigate the effects of dimension change with other, additional quadrilaterals in order to ensure that learners practise the integration and application of these algebraic skills. Knowing how to use algebra to solve Geometry problems makes working out the answers and problem-solving in general much easier. Practice is vital.
- For triangles, restrict dimension changes to height and base changes as these are changes directly affecting the area.
- For circles, changing any of the circle components (radius, diameter, and circumference) results in changes in other components. For example, if the diameter of a circle is doubled, the radius and the circumference also double.
- The examples provided after the investigations show learners the types of Geometry calculations that require an understanding of the effects of dimension changes on polygons.
- Work through the examples with the learners, referring to the results of the investigation on effects of dimension changes where necessary. It is very helpful to make rough sketches of geometric figures if a diagram is not given in a question.

Tip

The importance of practice cannot be over-emphasised.

Encourage learners at all times to do this.

Suggested answers

1 **a** $A = l \times b$

$\therefore b = \frac{A}{l}$

$= \frac{200 \text{ m}^2}{20 \text{ m}}$

$= 10 \text{ m}$

b $20 \text{ m} \times 3 = 60 \text{ m}$ and $10 \text{ m} \times 3 = 30 \text{ m}$

c P = 2(length) + 2(breadth)

= 2(10 m) + 2(20 m)

= 60 m

d P = 2(length) + 2(breadth) = 180 m

e A = length × breadth

= 30 m × 60 m

= 1 800 m²

2 **a** $A = s^2$

$\therefore s = \sqrt{A}$

$\therefore s = \sqrt{400\ mm^2}$

$= 20$ mm

b $P = 4s$

$= 4 \times 20$ mm

$= 80$ mm

c $A = s^2$

$\therefore s = \sqrt{A}$

$= \sqrt{200\ mm^2}$

$= 14,14$ mm

$= 1,414$ cm

d $P = 4s$

$= 4 \times 1,414$ cm

$= 5,656$ cm

3 **a** $c^2 = a^2 - b^2$

$c^2 = 100$ cm – 64 cm

$c^2 = 36$ cm

$c = 6$ cm

b $A = \frac{1}{2}\, b \times h$

$= \frac{1}{2}\,(8$ cm $\times$ 6 cm)

$= 4$ cm $\times$ 6 cm

$= 24$ cm^2

c $A = \frac{1}{2}\, b \times h$

$= \frac{1}{2}\,(8$ cm $\times$ 12 cm)

$= 4$ cm $\times$ 12 cm

$= 48$ cm^2

4 **a** $C = 2\pi r$

$\therefore r = \frac{C}{2\pi}$

$r = \frac{100\ cm}{6\ 286}$

$= 15,91$ cm

b Radius is halved $\therefore$ (new)r = 15,9 cm ÷ 2 = 7,96 cm

(new)d = (new)$r \times 2$ = 15,9 cm

c $A = \pi r^2$

$= 3,14 \times (15,9$ cm$)^2$

$= 794,58$ cm^2

d $A = \pi r^2$

$= 3,143 \times (7,96$ cm$)^2$

$= 199,15$ cm^2

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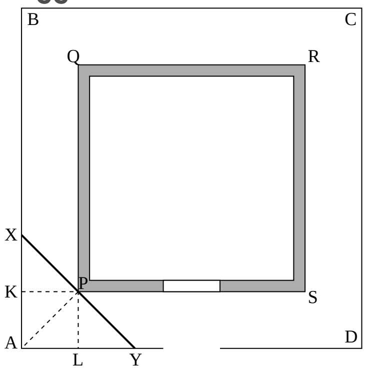
Suggested answers:



$XK = KA = AL = LY = 2$ m (congruence)
$XY^2 = XA^2 + AY^2$
$= 4^2 + 4^2$
$= 32\ m^2$
$\therefore XY = 2R(32\ m^2) \approx 5{,}6$ m (round down to allow for thickness of plate)

Chapter 13 Revision

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Suggested answers

1 **a** $CD^2 = BC^2 - BD^2$ (Pythagoras)
$= (50\ mm)^2 - (40\ mm)^2$
$= 900\ mm^2$
$CD = \sqrt{900}\ mm^2 = 30$ mm

b Area(ΔBCD) $= \frac{1}{2} \times$ base $\times$height
$= \frac{1}{2} \times 40\ mm \times 30\ mm$
$= 600\ mm^2$

c Area(ΔABC) $= \frac{1}{2} \times 30\ mm \times 30\ mm$
$= 450\ mm^2$

d Area of $\Delta ACD = \frac{1}{2} \times 70\ mm \times 30\ mm$
$= 1\ 050\ mm^2$

e $\frac{\text{Area } \Delta BCD}{\text{area } \Delta ABC} = \frac{600\ mm}{450\ mm} = \frac{4}{3}$
$\frac{BD}{AB} = \frac{40\ mm}{30\ mm} = \frac{4}{3}$
The two ratios are equal.

f Area of $\Delta ACD = 1\ 050\ mm^2$ (from (d) above)
Also, area of ΔABC + area ΔBCD
$= 450\ mm^2 + 600\ mm^2$
$= 1\ 050\ mm^2$ = area of ΔACD, as required.

2 Area $= \frac{1}{2} \times$ base $\times$ height $= 100\text{ m}^2$

Height $= 2\,500$ cm $= 25$ m

Therefore, base $= \frac{100\text{ m}^2 \times 2}{25\text{ m}}$

$= \frac{200\text{ m}^2}{25\text{ m}} = 8$ m

3 **a** BG = CH (opposite sides of a parallelogram)

CH = 50 mm (given)

$\therefore$ BG = 50 mm

b $\angle$DHI = $\angle$CDH (alternate angles, AE//FI)

But $\angle$CDH = 90º ($\angle$CDH is right angle)

$\therefore$ $\angle$DHL = 90º

c ΔCDH is a right-angled triangle.

$CD^2 = CH^2 - DH^2$ (Pythagoras)

$= (50\text{ mm})^2 - (40\text{ mm})^2$

$= 2500\text{ mm}^2 - 1600\text{ mm}^2$

$= 900\text{ mm}^2$

$\therefore CD = \sqrt{900\text{ mm}^2}$

$= 30$ mm

d ΔBDH is a right-angled triangle.

$BH^2 = BD^2 + DH^2$ (Pythagoras' theorem)

But, BD = BC + CD, and BC = GH (opposite sides of a parallelogram)

$\therefore$ BC = 40 mm

BD = 40 mm + 30 mm (from 3c above)

= 70 mm

$BH^2 = BD^2 + DH^2$

$= (70\text{ mm})^2 + (40\text{ mm})^2$

$= 6\,500\text{ mm}^2$

$\therefore BH = \sqrt{6\,500\text{ mm}^2}$

$= 80{,}62$ mm

e BCHG is a parallelogram with height DH = 40 mm.

Area BCHG = base $\times$ height = GH $\times$ DH

$= 40\text{ mm} \times 40\text{ mm}$

$= 1\,600\text{ mm}^2$

f BDFH is a trapezium with height DH = 40 mm.

Area BDFH $= \frac{1}{2}$ (sum of parallel sides) $\times$ height

$= \frac{1}{2}(BD + FH) \times DH$

$= \frac{1}{2}(BD + FG + GH) \times DH$ (remember FH = FG + GH)

$= \frac{1}{2}(70\text{ mm} + 15\text{ mm} + 40\text{ mm}) \times 40\text{ mm}$

$= \frac{1}{2}(125\text{ mm}) \times 40\text{ mm}$

$= 2\,500\text{ mm}^2$

4 **a** Perimeter of square $= 4s$

$4s = 60$ mm

$s = \frac{60\text{ mm}}{4}$

$= 15$ mm

b Area of square = s^2
= $(15\text{ mm})^2$
= 225 mm^2
Area in cm^2: 10 mm = 1 cm, so $(10\text{ mm})^2 = (1\text{ cm})^2$
$\therefore 100\text{ mm}^2 = 1\text{ cm}^2$
$225\text{ mm}^2 = 225\text{ mm}^2 \times \frac{1\text{ cm}^2}{100\text{ mm}^2} = 2{,}25\text{ cm}^2$

5 We first convert the given quantities to the same unit. Since the final answer must be in metres, we need to convert cm to m.
1 m = 100 cm
$\therefore$ Length of given diagonal
= $400\text{ cm} \times \frac{1\text{ m}}{100\text{ cm}}$
= 4 m (note the cancellation of *units* (cm) in the conversion)
Area of kite = $\frac{1}{2}(d_1 \times d_2)$
Substitute known values into the formula.
$\frac{1}{2}(4\text{ m} \times d_2) = 6\text{ m}^2$
$4\text{ m} \times d_2 = 12\text{ m}^2$
$d_2 = \frac{12\text{ m}^2}{4\text{ m}} = 3\text{ m}$

6 **a** CH = 3 cm
b $DG^2 = GH^2 - DH^2$ (Pythagoras)
= $(5\text{ cm})^2 - (4\text{ cm})^2$
= $25\text{ cm}^2 - 16\text{ cm}^2 = 9\text{ cm}^2$
$\therefore DG = \sqrt{9\text{ cm}^2} = 3\text{ cm}$
c BD = 2 × DG (point G bisects line BD)
$\therefore$ BD = 2 × 3 cm = 6 cm
d Perimeter = $2(l + w) = 2(7\text{ cm} + 6\text{ cm})$
= 2(13 cm)
= 26 cm
e Area = $l \times b$
= 7 cm × 6 cm
= 42 cm^2
f HIJ = DG = 3 cm
g $\therefore$ GHJ = 900 (diagonals of a kite are perpendicular)
h GJ = 4 cm
i FJ = 3 cm
j $FH^2 = HJ^2 + FJ^2 = (3\text{ cm})^2 + (3\text{ cm})^2$
= 18 cm^2
$\therefore FH \approx 4{,}24\text{ cm}$
k Area(kite) = $\frac{1}{2}$ (diag1 × diag2)
= $\frac{1}{2}$ (FG × EH)
= $\frac{1}{2}$ (7 cm × 6 cm)
= 21 cm^2

7 **a** Area = 10,6 m × 6,5 m
= $68{,}9\text{ m}^2$
b Area (verandah) = 1,6 m × 1,6 m = $2{,}56\text{ m}^2$
c Area (main bedroom)
= (3,4 m × 1,6 m) + (3,2 m × 2,4 m)
= $5{,}44\text{ m}^2 + 7{,}68\text{ m}^2$
= $13{,}12\text{ m}^2$

8 **a** Width $= 2{,}5 \text{ m} \times \frac{100 \text{ cm}}{1 \text{ m}} = 250 \text{ cm}$

b Area $= l \times w = 25\ 000 \text{ cm}^2$, and $b = 250$ cm

Solving for l we get,

$l \times 250 \text{ cm} = 25\ 000 \text{ cm}^2$

$l = \frac{25\ 000 \text{ cm}^2}{250 \text{ cm}} = 100 \text{ cm}$

c Perimeter $= 2(l + b) = 2(250 \text{ cm} + 100 \text{ cm})$

$= 700$ cm

9 **a** DE = 5 units (opp. sides of a rhombus are equal)

b $\angle HEF = \angle EHG = 900$ (alt. $\angle$s, DE$\parallel$BC)

c $GH^2 = EG^2 - EH^2$ (Pythagoras)

But EG = 5 units (BDEG is a rhombus)

$\therefore GH^2 = 52 - 42 = 9$

$GH = \sqrt{9} = 3$ units

d Area(ΔCEG) $= \frac{1}{2} \times 6 \times 4 = 12$ sq. units

e Area (BDEG) $= s \times h = 5 \times 4 = 20$ sq. units

f Area(BDEH) $= \frac{1}{2}(DE + BH) \times EH$

$= \frac{1}{2}(5 + 8) \times 4 = 26$ sq. units

g Area(ΔABC) $= \frac{1}{2} \times BC \times (2 \times EH) = \frac{1}{2} \times 11 \times 8 = 44$ sq.units

h Area(BDEC) $= \frac{1}{2}(DE + BC) \times EH$

$= \frac{1}{2}(5 + 11) \times 4 = 32$sq. units

But area (BCDE) + area ΔGCE = 20 sq. units + 12 sq. units

= 32 sq.units = area of BCDE

i In ΔABC and ΔADE:

$\angle$A is common

$\angle B = \angle ADE$ (corr. $\angle$s, DE$\parallel$BC)

$\therefore$, ΔABC and ΔADE are equiangular

Hence $\Delta ABC \parallel\!| \Delta ADE$

j In ΔADE and ΔCEG:

$\angle ADE = \angle B$ (corr. $\angle$s, DE$\parallel$BC)

But $\angle B = \angle CEG$ (corr. $\angle$s, BD$\parallel$GE)

$\therefore$, $\angle ADE = \angle CEG$

$\angle AED = \angle CEF$ (vert. opposite $\angle$s)

But $\angle CEF = \angle ECG$ (corr. $\angle$s, DF$\parallel$BC

$\therefore$, $\angle AED = \angle CEG$

Also, DE = GE (BDEG is a rhombus)

Therefore, $\Delta ADE \equiv \Delta CEG$ (AAS)

10 **a** Area $= 8 \text{ m} \times 6 \text{ m} = 48 \text{ m}^2$

b Radius $= \frac{1}{2} \times 6 \text{ m} = 3 \text{ m}$

c Circumference $= 2\pi r = 2 \times 3{,}14 \times 3 \text{ m} = 18{,}84 \text{ m}$

d Area(paving) = area (rect. enclosure) – area (pool)

$= 48\text{m}^2 - (3{,}14 \times 9\text{m}^2)$

$= 48\text{m}^2 - 28{,}26 \text{ m}^2 = 19{,}74 \text{ m}^2$

11 **a** $s^2 = 625 \text{ mm}^2$

$s = \sqrt{625 \text{ mm}^2} = 25 \text{ mm} = 2{,}5 \text{ cm}$

Perimeter $= 4 \times s = 4 \times 2{,}5\text{cm} = 10\text{cm}$

b Area (m^2) $= (0{,}1 \text{ m})^2 = 0{,}01 \text{ m}^2$

c Area (new square) $= \frac{1}{4} \times$ (original area)

$= 0{,}0025 \text{ m}^2$

PoA | Exam exemplar 1 (June): Memorandum

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Paper 1 Algebra (Marks: 75)

Question 1

No	N	Z	Q	Q
$-\frac{3}{7}$			✓	
$\sqrt{5}$				✓
$\frac{3}{7}$		✓	✓	
$-\sqrt{16}$		✓	✓	

[6]

Question 2

2.1.1 $4{,}62 \times 10^6$ ✓ (1)
2.1.2 $2{,}37 \times 10^{-4}$ ✓ (1)
2.2 $2{,}982 \times 10^7$ ✓ (1)
2.3 $3{,}75 \times 10^7$ ✓✓ (2)
2.4 $2{,}062 \times 10^7$ ✓✓✓ (3)
[8]

Question 3

3.1 1 4 9 16 25
✓ ✓
$\therefore 7^2 = 49$ ✓ (3)

3.2 1^2 2^2 3^2 4^2 5^2
$\therefore 7^2 = 49$
✓ ✓
$T_n = n^2$ ✓ (3)

3.3

3	7	13	21	31	43	
4	6	8	10	12	14	
57	73	91	111	133		
157	183	211	241	273		
26	28	30	32			
307	343	381	421	463		
34	36	38	40	42		
507	553	601	651			
44	46	48	50		✓✓✓✓	(4)

[9]

Question 4

4.1 $7^2 + 1 = 50$ ✓✓ (2)

4.2 $\frac{5^6}{5^2} = 5^4$ ✓✓ (2)

4.3 $2^2\, x^4 = 4x^4$ ✓✓ (2)

4.4 $-27a^3$✓✓ (2)

4.5 $\frac{p^2r^4}{p^2r^7} = \frac{1}{r^3}$ ✓✓ (2)

4.6 $\frac{2b^2}{3a^2}$ ✓✓✓ (3)

[13]

Question 5

5.1 $2x + 7 = 6x - 9$
$4x = -16$ ✓
$x = 4$ ✓ (2)

5.2 $2x - 6 - 6x + 3 = 17$
$-4x - 3 = 17$ ✓
$-4x = 20$ ✓
$x = -5$ ✓ (3)

5.3 $2(2x + 1) - 3 \times 4 = x - 1$
$4x + 2 - 12 = x - 1$
$3x - 10 = -1$
$3x = 9$
$x = 3$ (5)

[10]

Question 6

6.1 $-0{,}300; -\frac{1}{4}; 0{,}60; \frac{2}{3}; 1\frac{1}{2}; \frac{31}{20}$ ✓✓✓ (3)

6.2.1 $0{,}571428$ ✓
6.2.2 $0{,}3$ ✓
6.2.3 $2{,}1\dot{6}$ ✓
6.2.4 $0{,}952380$ ✓ (4)

6.3.1 $\frac{13}{20}$ ✓ (1)

6.3.2 $1{,}2 = \frac{11}{9} = 1\frac{2}{9}$ ✓✓ (2)

6.3.3 $\frac{54}{99} = \frac{6}{11}$ ✓✓ (2)

[12]

Question 7

7.1

			✓		
x	-2	-1	0	1	**5**
y	**-7**	-5	**-3**	-1	7
	✓		✓		

(3)

7.2 $x \rightarrow \times 2 \rightarrow -3 \rightarrow y$ ✓✓ (2)

7.3 $x = 1$ ✓
$y = -7$ ✓ (2)

[7]

Question 8

8.1 $310 \times 12 \times 3 = \text{R}11\ 160$
$ - \text{R}10\ 000$
Interest $= \text{R}1\ 160$ (2)
✓ ✓

8.2 $A = 3\ 000(1 + 0{,}0945)^4$
$= \text{R}4\ 305{,}11$ ✓ (3)

8.3 $2\ 800 = 2\ 000(1 + i.\frac{18}{12})$ ✓
$\frac{7}{5} = 1 + i.\frac{3}{2}$
$14 = 10 + 5i$
$4 = 15i$
$i = 0{,}2\dot{6}$
ie. 26,6% (4)

[9]

Paper 2 Geometry (Marks: 80)

Question 1

1.1 Learners construct an equilateral triangle with each angle = 60° and each side = 70 mm
✓✓✓ (3)

1.2 Learners bisect all three angles as instructed ✓✓✓ (3)

[6]

Question 2

2.1.1 $\angle ACB = 180° - (\angle A + \angle ABC)$ ✓ (Sum of interior angles of a triangle) ✓
$= 180° - (34° + 90°)$✓
$= 56°$✓ (4)

2.1.2 $\angle ACD = \angle A + \angle ABC$ ✓ (exterior angle of a triangle theorem) ✓
$= 34° + 90°$✓
$\therefore \angle ACB = 124°$✓ (4)

2.1.3 BC = 56 mm ✓✓✓✓✓ (2)

2.1.4 $AB = \sqrt{100}^2 - \sqrt{56}^2$ (Pythagoras) (4)

2.2.1 $x = 105°$✓ (opposite angles of a ||rm) ✓ (2)

2.2.2 In Δ HEF and Δ EGH
EF = GH✓ (opposite sides of a ||rm) ✓
HE = FG✓ (opposite sides of a ||rm) ✓
HF is common✓ (diagonal)
$\therefore \Delta HEF \equiv \Delta FGH$ (SSS) ✓ (6)

2.3.1 10 sides✓ (1)

2.3.2 8 triangles✓✓ (2)

2.3.3 Sum of interior angles $= 180°$✓ $\times 8$✓
$= 1\ 440°$✓✓ (4)

[29]

Question 3

3.1.1 $\angle A = 60°$✓ (Δ ABC is equilateral) ✓ (2)

3.1.2 $\angle ADF = \angle B$ ✓ $= 60°$✓ (corresponding angles DE||BC) ✓ (3)

3.1.3 $FE = \frac{1}{2}DE$✓

But DE = BC = 40 mm ✓ (opposite sides of a ||rm) ✓

$\therefore FE = \frac{1}{2}(40)$ mm

$= 20$ mm ✓ (5)

3.1.4 $\angle EFC = \angle AFD$✓ (vertically opposite angles) ✓

But $\angle AFD = 60°$✓ (sum of interior angles of ΔAFD) ✓

$\therefore \angle EFC = 60°$✓ (5)

3.1.5 In ΔABC and ΔEDF

$\angle A$ is common✓

$\angle ADF = \angle B$✓ (corresponding angles, DE||BC) ✓

$\therefore \Delta ABC\ |||\ \Delta ADF$ ✓ ($\angle\angle\angle$) ✓ (5)

3.1.6 $\Delta AFD\ |||\ \Delta CFE$

Since $\angle A = \angle FCE$✓ (alternate angles, AB||CE) ✓

$\angle CFE = \angle AFD$✓ (vertically opposite angles)

DF = FE (given)

$\therefore$ condition AAS is satisfied✓ (4)

3.2.1

House shape	Dimensions	Perimeter	Area
Square	Side = 7,5 m✓	30 m	$A = s^2$✓ $= (7,5)^2$✓ $= 56,25\ m^2$✓
Rectangle	Length = 10 m Width = 5 m✓✓	30 m	$A = l \times w$✓ $= (10 \times 5)$ ✓ $= 50\ m^2$ ✓
Circle	Radius $= \frac{30\ m✓}{2\pi✓}$ $= 4,78$ m✓✓	30 m	$A = \pi r^2$✓ $= 71,74\ m^2$✓

(15)

3.2.2 Round (circular) shape✓ (1)

3.2.3 It gives the largest area (space) for any given perimeter compared to other shapes✓

(1)

3.2.4 If the circumference doubles, so does the radius and area. ✓

New area = 2(original area) ✓

$= 2(71,74\ m^2)$✓ $= 143,48\ m^2$✓ (4)

[45]

[Total: 80]

Exam exemplar (June): Additional

Marks: 135

Instructions

- Answer all the questions.
- Show all your workings.
- Work neatly and write legibly.

Question 1

1.1 Simplify.

1.1.1 $7x - 3 = 10$ (2)

1.1.2 $\frac{2}{3}x = 6$ (1)

1.1.3 $\frac{2x+1}{5} = 1 + \frac{x+1}{2}$ (3)

1.1.4 $10 \geq 8 - (x - 7) - 3(2x - 5)$ (3)

1.2 Sue and Dean suggest solutions for the equation $3(x - 2) = 4x - (x + 2)$.
Sue says that the answer is 0. Dean says there are no true values.
Decide who is correct and explain why. (3)

1.3 Without solving the equation, show that $x = -4$ is a solution to the equation:
$3 + x = \frac{2(-7 - x)}{6}$ (3)

[15]

Question 2

2.1 Simplify.

2.1.1 $4q^0$ (1)

2.1.2 $a^2 \times a$ (1)

2.1.3 $\frac{a^m}{a^m + 1}$ (1)

2.2.1 Simplify, writing your answer with positive exponents: $\frac{(a^3b^{-2})^3 - ab^8}{a^2b^2}$ (3)

2.2.2 Use scientific notation to simplify: $1{,}2 \times 10^{-3} \times 9{,}3 \times 10^{-2}$ (3)

[9]

Question 3

3.1 The following questions refer to the expression: $-7x^4 + 5x^3 - x^2 + x + 1$

3.1.1 Write down the constant term. (1)

3.1.2 State the degree of the expression. (1)

3.1.3 What is the coefficient of x^2? (1)

3.2 A factory spends R250 000 on training staff in a ratio of 5 : 3 for unskilled to skilled workers. How much do they spend on training for unskilled workers? (2)

3.3 If 23 toys cost R2 573,24, how much do 15 toys cost, to the nearest cent? (2)

3.4 Which two numbers from the list below are rational?
$0{,}333\ldots$; $\sqrt[3]{-8}$; π; $\sqrt{3}$; $\frac{3}{0}$; $\sqrt{-4}$ (2)

3.5 Express the following ratios in their simplest forms.

3.5.1 9 h : 12 h (1)

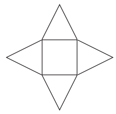
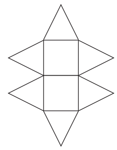
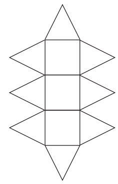
3.5.2 30 g : 2 kg (2)

3.6 Determine x if $13 : 17 = 91 : x$ (2)

[14]

Question 4

The patterns below are made up using sticks.

Pattern number	1	2	3
Pattern			
No. of matches	12	19	26

4.1 Write down the number of sticks needed for the fourth pattern. (1)
4.2 Write down an equation to represent the number of matches in any row. (2)
4.3 Which pattern number has 54 matches? (2)
[5]

Question 5

5.1 Determine the highest common factor (HCF) of 54 and 72. (3)
5.2 Determine the lowest common multiple (LCM) of 24 and 36. (3)
5.3 By first expressing the number as a product of its prime factors, calculate, without using a calculator: $\sqrt[3]{1\,728}$ (3)
5.4 Calculate.
$-\sqrt{9}(\sqrt{49} + \sqrt[3]{8})^2$ (3)

[12]

Question 6

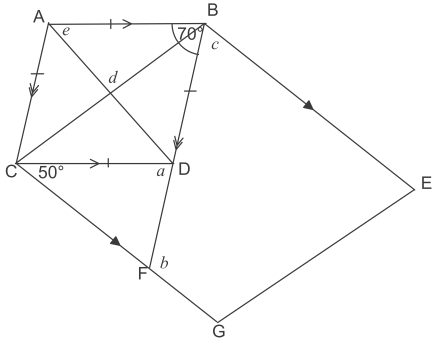
Simplify.

6.1 $(6a - 4) - 3a$ (2)
6.2 $-3(2a^2b^3c)^2$ (3)
6.3 $7a \times 2a^2 - 5 \times 3a^3$ (2)
6.4 $5(a + 4\ ) - 3a(2a + 3a)$ (4)
6.5 $\sqrt{\frac{32x^4}{18y^2}}$ (3)
6.6 $(2x + 3)(x - 4)$ (3)
6.7 $2(x - y)(x + 3y)$ (4)
6.8 $(3p - 4)^2$ (4)
[25]

Question 7

7.1 James and Oliver would each like to borrow R150 000 from their uncle. He offers them both two different options to pay him back.
Option A for James: 12,5% simple interest for 5 years
Option B for Oliver: 9,75% simple interest for $6\frac{1}{2}$ years
Calculate who will pay back the most interest. (5)
7.2 A retailer buys an item for R101,43 and sells it for R95,34.
Calculate his percentage loss. (3)
7.3 Mr van Wyk decides to surprise his family for the holidays and buys a new car. He has to pay for it in 54 equal monthly instalments. The car costs R124 560. Mr van Wyk pays a 20% deposit and is charged 17,25% per annum compound interest. Calculate what his monthly instalment will be. (5)
[13]

Question 8



8.1 Consider the diagram alongside. Determine the sizes of the following angles giving reasons.

8.1.1 a (2)

8.1.2 b (2)

8.1.3 c (2)

8.1.4 d (2)

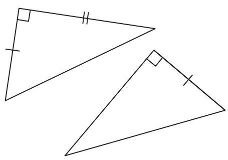
8.1.5 e (2)

[10]

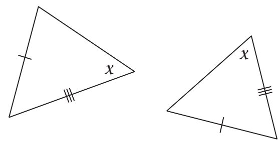
Question 9

Consider the pairs of triangles below. If they are congruent, state the case of congruency. If they are not congruent, explain why they are not congruent.

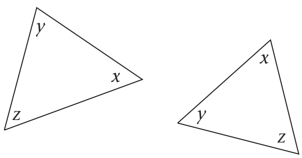
9.1 (2)



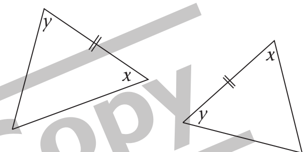
9.2 (2)



9.3 (2)

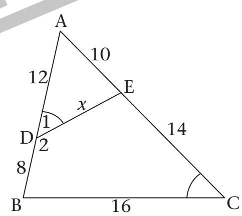


9.4 (2)



[8]

Question 10



10.1 Given triangle ABC with D on AB and E on AC so that $\hat{D}_1 = \hat{C}$.

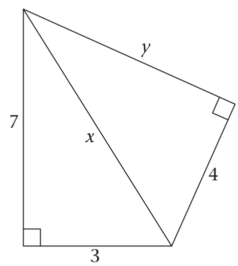
10.1.1 Complete: $\triangle ADE \,|||\, \triangle \ldots (\ldots\ldots)$ (3)

10.1.2 Complete and solve for x: $\frac{x}{16} =$ (2)

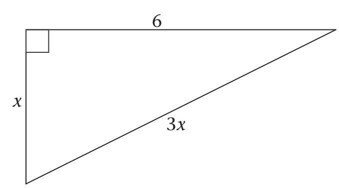
[5]

Question 11

11.1 Find the value of x, then the value of y (leave $\sqrt{\ }$ signs in your answers). (5)



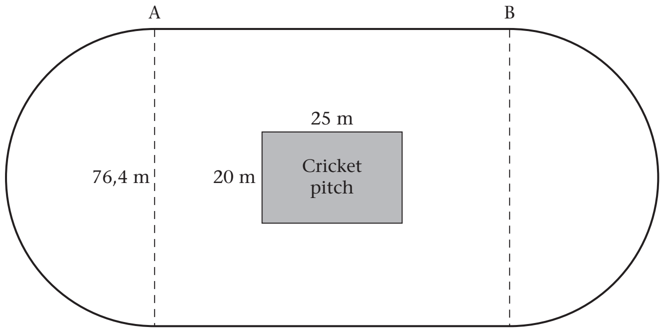
11.2 Find the value of x correct to 2 decimal places. (4)



[9]

Question 12

The diagram represents an athletics track. It comprises a rectangle ABCD and two semi-circles. The length of AD is 76,4 m. The perimeter of the track is 400 m.



12.1 Show that to run anti-clockwise from A to D, the distance is 120 m. (3)

12.2 Hence calculate the length of the straight portion AB. (2)

12.3 The cricket pitch is out of bounds and has been cordoned off. The dimensions of the cordoned off area are 25 m by 20 m. What is the size of the remaining space within the perimeter of the athletic track that is available, i.e. the unshaded area? (6)

[11]

[Total: 135 marks]

Exam exemplar (June): Additional Memorandum

Question 1

1.1.1 $7x - 3 = 10$
$-3x = -3$
$x = 1$ (2)

1.1.2 $\frac{2}{3}x = \frac{6 \times 3}{1 \times 3}$
$\frac{2x}{3} = \frac{18}{2}$
$x = 9$ (1)

1.1.3 $\frac{2(2x + 1)}{5 \times 2} = \frac{1 \times 10}{1 \times 10} + \frac{5(x + 1)}{1 \times 10}$
$4x + 2 = 10 + 5x + 5$
$4x + 2 = 5x + 15$
$-x = 13$
$x = -13$ (3)

1.1.4. $10 \geq 8 - x + 7 - 6x + 15$
$10 \geq 30 - 7x$
$7x \geq 20$
$x = \frac{20}{7}$
$\therefore$ No solution so Dean is correct (3)

1.2 $3(x - 2) = 4x - (x + 2)$
$3x - 6 = 4x - x + 2$
$-6 \neq +2$
$\therefore$ No solution so Dean is correct (3)

1.3. $\text{LK} = 3 - 4 = -1$
$\text{RK} = \frac{2(-7 + 4)}{6} = \frac{2(-3)}{6} = -1$
$\therefore \text{LK} = \text{RK}$
So the answer is correct. (3)
[15]

Question 2

2.1.1 4 (1)
2.1.2 a^3 (1)
2.1.3 $a^m - m - 1 = a^{-1}$
2.2.1 $\frac{a^9b^{-6} \times ab^8}{a^2b^2}$ (1)
$\frac{a^{10}b^2}{a^2b^2} = a^8$ (3)
2.2.2 $0{,}0012 \times 0{,}093$ (5)
$= 1{,}116 \times 10^{-4}$
[9]

Question 3

3.1.1 1 (1)
3.1.2 4 (1)
3.1.3 -1 (1)

3.2 $5 + 3 = 8$ parts (2)

$\frac{250\ 000}{8} = 31\ 250$

unskilled $= 31\ 250 \times 5 =$ R156 250

3.3 1 toy costs R111,88 (2)

$\times\ 15 =$ R1 678,20

3.4 0,333 ... ; $\sqrt[3]{-8}$ (2)

3.5.1 $\frac{3}{4} = 3 : 4$

$\frac{30}{2\ 000} = 3 : 200$ (1)

3.5.2 $\frac{13}{17} = \frac{91}{x}$

$13x = 91 \times 17$

$x = \frac{91 \times 17}{13}$ (2)

3.6 $x = 119$ (2)

Question 4

4.1 33 matches ✓ (1)

4.2 Matches = {[(pattern no. + 2) + 2] × 3} + (pattern no. – 1) ✓✓ (2)

4.3 Pattern 7 ✓✓ (2)

[5]

Question 5

5.1 Prime factors of 54: 2×3^3✓

Prime factors of 72: 2×3^2 ✓

$\therefore$ HCF $= 2 \times 3^2$

$= 18$ ✓ (3)

5.2 Prime factors of $24 = 2^3 \times 3$ ✓

Prime factors of $36 = 2^2 \times 3^2$

$\therefore$ LCM $= 2^3 \times 3^2$

$= 8 \times 9$

$= 72$ ✓ (3)

5.3 Prime factors of $1\ 728 = 26 \times 3^3$ ✓

$\therefore\ \sqrt[3]{1\ 728} = -3(7 + 2)^2$ ✓✓ (3)

5.4 $-\sqrt{9}(\sqrt{49} + \sqrt[3]{8})^2 = -3(7 + 2)^2$ ✓✓

$= -243$ ✓ (3)

[12]

Question 6

6.1 $3a$ ✓ $- 4$ ✓ (2)

6.2 $3(4a^4b6c^2)$ ✓✓ $= -12a^4b^6c^2)$ ✓ (3)

6.3 $14a^3 - 15a^3$ ✓ (2)

6.4 $5a + 20$ ✓ $- 6a^2 - 9a^2$ ✓ $= -15a^2 + 5a + 20$✓✓ (4)

6.5 $\sqrt{\frac{16x^4}{9y^2}}$ ✓✓ $= \frac{4x^2}{3y}$ ✓ (3)

6.6 $2x^2$ ✓ $+ 3x - 8x - 12$ ✓ $= 4x^2 - 5x - 12$ ✓ (3)

6.7 $2(x^2 - xy$ ✓ $+ 3xy - 3y^2)$ ✓ $= 2x^2 + 4xy - 6y^2$ ✓✓ (4)

6.8 $9p^2$ ✓ $- 24p$ ✓✓ $+ 16$ ✓ (4)

[25]

Question 7

7.1 A: James: 12% simple interest for 5 years.
B: Oliver: 9,75% simple interest for 6,5 years.
Interest = Principle × Rate × No. of years
James: R150 000 × 12,5% × 5 ✓
= R93 750,00 ✓
Oliver: R150 000 × 9,75% × 6,5 ✓
= R95 062,50 ✓
Oliver will pay more interest ✓ (5)

7.2 R101,43 – R95,34 = R6,09 ✓
R6,09 ÷ R101,43 = 6,00% loss ✓✓ (3)

7.3 Cost of car = R124 560,00
Deposit = 20%
No. of instalments = 54 months ✓ = 6 years
Compound interest = 17,25%
Deposit = 20% of R124 560,00
= R24 912,00✓
∴ Balance = R99 648,00
∴ Monthly payment ✓✓ (5)

[13]

Question 8

8.1.1 $a = 70°$ (corresponding ∠s) (2)
8.1.2 $b = 120°$ (ext ∠ of Δ) (2)
8.1.3 $c = 60°$ (alt ∠s) (2)
8.1.4 $d = 90°$ (vert opp ∠s) (2)
8.1.5 $e = 55°$ (∠s of Δ) (2)

[10]

Question 9

9.1 Congruent ✓ (SAS) ✓ (2)
9.2 Not congruent, ✓ angle not included. ✓ (2)
9.3 Not congruent, ✓ similar △s ✓ (2)
9.4 Congruent ✓ (SAA) ✓ (2)

[8]

Question 10

10.1 $\Delta ABC \,|||\, \Delta ACB$ (AAA) (3)

$\frac{x}{10} = \frac{10}{20}$ (2)

$x = 8$

[5]

Question 11

11.1 $x^2 = 7^2 + 3^2$ (Pythagoras)
$x^2 = 49 + 9$
$x^2 = 58$
$x = \sqrt{58}$
$y^2 = (\sqrt{58})^2 - 4^2$ (Pythagoras)
$y^2 = 58 - 16$
$y^2 = 42$
$y = \sqrt{42}$ (5)

11.2 $(3x)^2 = x^2 + 6^2$

$9x^2 = x^2 + 36$

$8x^2 = 36$

$x^2 = 4$

$x = \pm 2; x = 2$ (4)

[9]

Question 12

12.1 Circumference $= \frac{\pi(D)}{20}$

$= \frac{\pi^2(76,4)}{20}$

$= 120$ m (3)

12.2 AB + DA = 400 – 2(120)

2AB = 160

AB = 80 mm (2)

12.3 Area of circle $= \pi r^2$

$= \pi\left(\frac{76,1}{2}\right)^2$

$= 4\ 584,35$ m^2

Area of rectangle $= 80 \times 76,4$

$= 6\ 112$ m^2

Area of pitch $= 25 \times 20$

$= 500$ m^2

Unshaded area $= 4\ 584,36 + 6\ 112 - 500$

$= 10\ 196,34$ m^2 (6)

[11]

[Total: 135]

CHAPTER

14 Functions and relationships

Chapter overview

Learner's Book pages 354 to 363
Recommended pacing: 5 hours

This chapter focuses on the following:

Unit 1: Revision: Input and output values	2 hours
Finding output values for equations	
Unit 2: Equivalent forms of a relationship	3 hours
Graphing	
Determining the equivalence of presentations	
Chapter 14 Revision	45 minutes

UNIT

1 Revision: Input and output values

Unit overview

Learner's Book page 355
Recommended pacing: 2 hours

This unit focuses on the following:

- Input and output values
- Finding output values
- Using verbal descriptions
- Using flow diagrams
- Using tables
- Using equations.

Resources: Learner's Book; exercise book; mathematical set (ruler, protractor, eraser, sharp pencil, pen)

Background information

In Grades 7 and 8 learners used verbal descriptions, flow diagrams, tables, formulae and either number sentences or equations to describe relationships. Learners also studied Functions and Relationships in Chapter 7 in Term 1. This unit revises these descriptions before moving on to graphs on a Cartesian plane in Unit 2 of this chapter.

Teaching guidelines

This unit revises work that learners have been doing since Grade 7 and have done in Term 1 and so it should be familiar to them. The same concepts are also used and practised in patterns and sequences and you may find some learners could need a challenge because it is so familiar. Remind learners about how important it is to be able to convert problems that we encounter in everyday life into mathematical terms so that we can solve them.

If necessary, use a good example and work through the equivalent forms on the board, although learners should really be able to revise this section independently. Ask learners to summarise the key features in a mind map as an alternate form of revision.

Input and output values

Finding output values

Activities 1–2 **Use verbal descriptions to find output values; Use flow diagrams to find output values**

Learner's Book page 356

Guidelines for implementing these activities

- Learners work through example 1 in the Learner's Book page by themselves.
- Address any problems that may arise, but encourage learners to work through Activity 1 as quickly as possible.
- Learners have been using flow diagrams in various forms since Grade 1. They can use the equivalent "spider diagrams" and put all the input values on one side and the output values on the other side.
- The flow diagram should be the first step in converting a verbal description to a mathematical relationship.
- Learners work through Activity 2 independently.

Remedial and extension

Remedial: Make flow diagrams, give learners a few simple verbal sentences and ask them to convert these into flow diagrams. There may be a problem with language for some learners, so encourage them to highlight or underline key words to assist them.

Extension: Some learners may get bored, but ask them to design a mind map that they can use as a summary of the section. Learning different study methods is always useful and can be used in other subjects.

Suggested answers

Activity 1

1 **a** $(15 \times 3) - 2 = 43$
  b $(22 \times 3) - 2 = 64$
  c $(38 \times 3) - 2 = 112$
  d $(124 \times 3) - 2 = 370$

2 a 48 ÷ 2 = 24
b 12 ÷ 2 = 6
c 42 ÷ 2 = 21
d 110 ÷ 2 = 55

Activity 2

1 Output values of {7; 12; 17; 22; 27; 32}
2 Output values of {3; 9; 15; 21; 27; 33}
3 Output values of {1; 5; 9; 13; 17; 21}
4 Output values of {3; 8; 13; 18; 23; 28}
5 Output values of {–3; –1; 1; 3; 5; 7}
6 Output values of {2; 7; 12; 17; 22; 27}

Activities 3–4 Use formulae to find output values; Use equations to find output values

Learner's Book page 357

Guidelines for implementing these activities

- Learners should also be very familiar with formulae and tables and should be able to work through the example and Activity 3 independently and as quickly as possible.
- You could show learners how to use their calculators to find the values, but the functions given in Activity 3 are not difficult and will be good mental mathematics. Also learners may have picked up how the output values form a pattern and so would not necessarily have to use the formulae each time.
- Ask learners to work through the example and Activity 4 independently.
- Check that learners write down at least one of the substitutions for each question and use brackets to show where they have substituted.

Remedial and extension

Remedial: In converting from formulae to tables, learners are using substitution. If there is a problem with substitution, let them write each substitution down using brackets to highlight the number that is being substituted each time. Make sure that learners do the calculations using the correct order of operations.
Extension: Give learners a few more complex equations to substitute values into. These could be linear and possibly a few quadratic equations too. Challenge learners to plot the points on a graph and discuss the shapes of the graphs.

Suggested answers

Activity 3

	input value	1	2	3	4	5	6
1	output value	5	9	13	17	21	25
2	output value	9	12	15	18	21	24
3	output value	2	7	12	17	22	27
4	output value	1	4	9	16	25	36

Activity 4

1 Output values of {1; 2; 3; 4; 5; 6; 7}
2 Output values of {–1; 0; 1; 2; 3; 4; 5}
3 Output values of {–9; –5; –1; 3; 7; 11}
4 Output values of {7; 5; 3; 1; –1; –3}

UNIT 2

Equivalent forms of a relationship

Unit overview

Learner's Book page 358
Recommended pacing: 3 hours

This unit focuses on the following:

- Graphing
- Representing a relationship in a graph
- Determining equivalence of representations.

Resources: Learner's Book; calculator; exercise book; block paper

Background information

In Grades 7 and 8 learners used verbal descriptions, flow diagrams, tables, formulae and either number sentences or equations to describe relationships. Learners also studied Functions and Relationships in Chapter 7 in Term 1. Although graphs are not new to Grade 9 learners, plotting relationships onto graphs on a Cartesian plane was not done in Grades 7 or 8.

Teaching guidelines

It is very useful to have graph paper available, preferably the block paper. There are sites on the Internet that have images of block paper, or you can photocopy the sample provided.

Remind learners about the Cartesian plane and coordinate pairs. Show how the coordinate pairs relate to a table of *x*- and *y*-values. Make sure that learners maintain high standards of neatness and plot the points meticulously. They should use sharpened pencils and rulers to draw the points and lines when connecting the points. The relationships that are used here give linear graphs. Make sure that learners are aware of the difference between this type of graph and a broken line graph.

Graphing

Activity 1 Find coordinates

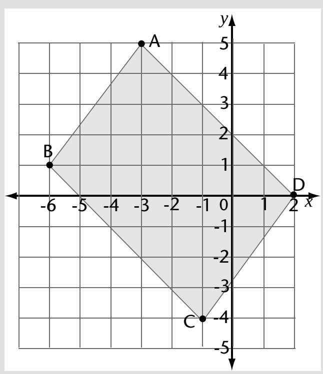
Learner's Book page 358

Guidelines for implementing this activity

- Coordinate pairs should be quite familiar to learners.
- Remind them about the order in the pair: the input (or *x*-value) is first and the output (or *y*-value) is second in the bracketed pair.
- These values can be plotted on the Cartesian or coordinate plane. The first value gives the horizontal distance and the second value in the bracket gives the vertical distance.
- Learners should do Activity 1 as quickly as possible.

Remedial and extension

Remedial: Learners may just need to be reminded of what the coordinate pair means. There are fun games that you can give learners to refresh their memories and they can play these in pairs, for example, give learners a dice and a coin. When they toss the dice, the first toss gives the *x*-value and the second one the *y*-value. They should then toss the coin: heads is positive and tails negative. Let them plot the values on the coordinate plane.
Extension: Ask learners to draw a design on grid paper and then make a list of the coordinate pairs. They should then swap designs with a partner and draw the design using the coordinate pairs listed, for example:



Suggested answers

1	**a**	B (–2; 2)		**b**	E (0; –2)
2	**a**	D		**b**	C

Representing a relationship in a graph

Activity 2 Represent the relationship with a graph

Learner's Book page 359

Guidelines for implementing this activity

- Once learners are comfortable with coordinate pairs again, show them how the coordinate pairs can be plotted in a table.
- Learners then use the table to plot the points onto a graph.
- Make sure learners work very neatly when drawing graphs. Do not accept any untidy work and rather have learners redo the graphs until they are neat.
- The first two relationships in Activity 2 give linear graphs. Any points that are not on the line have been plotted incorrectly. Check that learners are not mixing up the coordinates.
- Question **3** gives the graph and learners read the coordinate pairs off the graph.
- Learners should work independently through Activity 2.

Remedial and extension

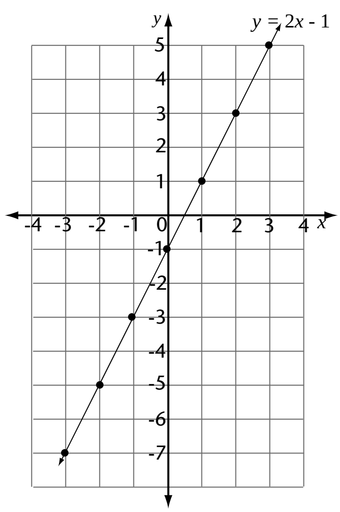
Remedial: The activity provides examples of going from coordinate pairs or a table to a graph and *vice versa*. Make sure that learners can read graphs as well as draw them.
Extension: Give learners a geometric shape like a rhombus or a parallelogram and ask them to draw it on a grid. Ask them to work out the coordinate pairs and equations of all the lines. They should comment on anything that they find significant when looking at the equations that they derive for example, TG 14.2..

Suggested answers

1 a

x	–3	–2	–1	0	1	2	3
$y = 2x - 1$	–7	–5	–3	–1	1	3	5

b and c



d The line is an increasing, straight-line graph.

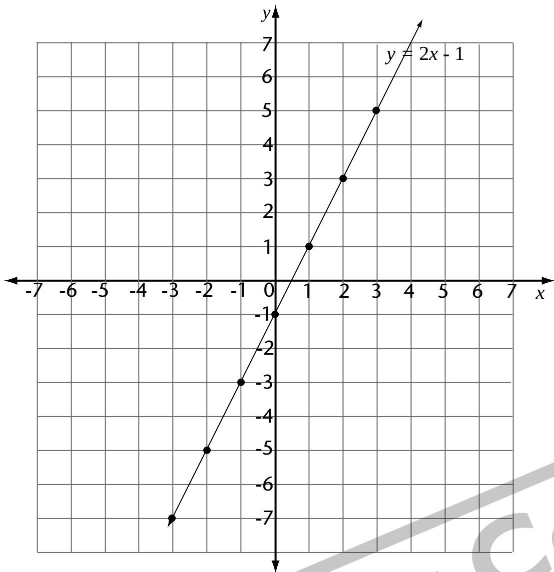
2 **a** $x \rightarrow \times 2 \rightarrow -1 \rightarrow y$

b

x	–6	–5	–4	–3	–2	–1	0	1	2	3	4	5	6
y	–13	–11	–9	–7	–5	–3	–1	1	3	5	7	9	11

c $y = 2x - 1$

d



3 **a**

x	–5	–4	–3	–2	–1	0	1	2	3	4
y	–4	–3	–2	–1	0	1	2	3	4	5

b $x \rightarrow -1 \rightarrow y$

c $y = x + 1$

Determining equivalence of representations

Activity 3 Determine equivalence of representations

Learner's Book page 361

Guidelines for implementing this activity

- In Activity 3 learners are asked to convert from one equivalent form to another.
- Learners should be able to do the activity independently.
- Provide assistance if required.
- The activity requires quite a lot of work and could take most of a lesson for some learners to complete. Graph drawing may still be quite slow at this stage.
- In question **1** Activity 3, learners are asked to check to see if a table is equivalent, and then to correct the values that do not conform to the relationship.

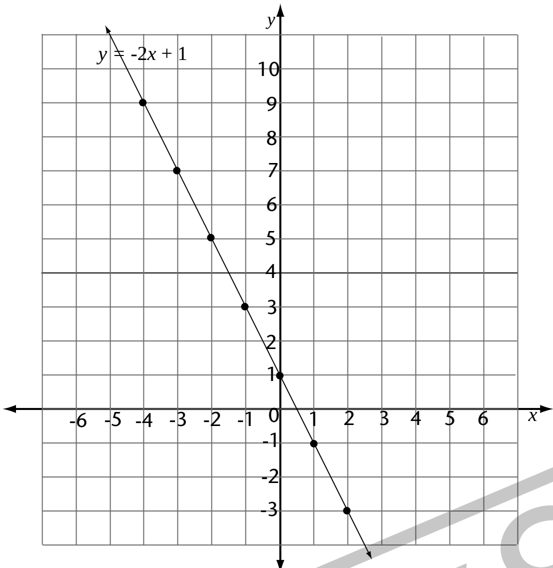
Suggested answers

1 **a** Not entirely.

x	–4	–3	–2	–1	0	1
y	9	7	5	3	1	–1

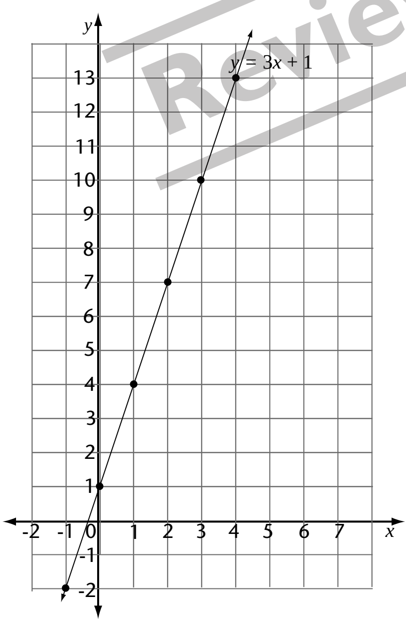
b $y = -2x + 1$

c



2 **a** (–1; –2), (0; 1), (1; 4), (3; 10), (4; 13), (6; 19)

b



3 **a** $y = 11$

b $y = -8$

c $x = 1$

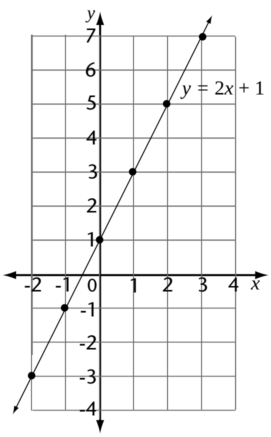
d $x = 2$

4 $x \rightarrow \times -2 \rightarrow +1 \rightarrow y$ Equation: e; Table: d; Graph: d
$x \rightarrow \times 3 \rightarrow y$ Equation: a; Table: c; Graph: a
$x \rightarrow +2 \rightarrow \times 2 \rightarrow y$ Equation: b and c; Table: a; Graph: b

5 **a** The equation in b has been factorised to give the equation in c, or the equation in c has been simplified to give the equation in b. They are equivalent.

b $y = 2x + 1$
$x \rightarrow \times 2 \rightarrow +1 \rightarrow y$

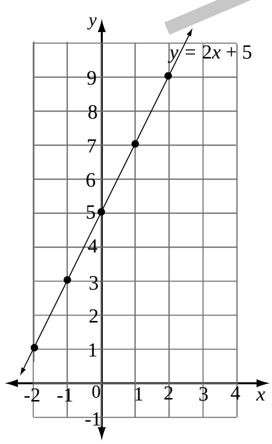
x	–2	–1	0	1	2
y	–3	–1	1	3	5



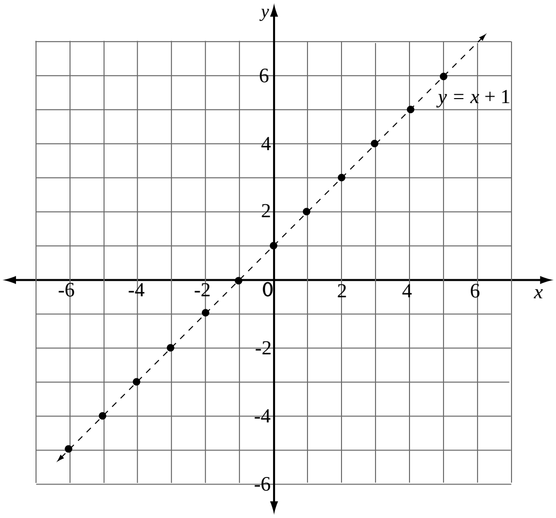
c

x	– 2	– 1	0	1	2
y	1	3	5	7	9

$x \rightarrow \times 2 \rightarrow \times 5$
$y = 2x + 5$



d



x	−5	−4	−3	−2	−1	0	1	2	3	4
y	−4	−3	−2	−1	0	1	2	3	4	5

$x \rightarrow +1 \rightarrow y$

$y = x + 1$

Chapter 14 Revision

Learners' Book page 363

Suggested answers

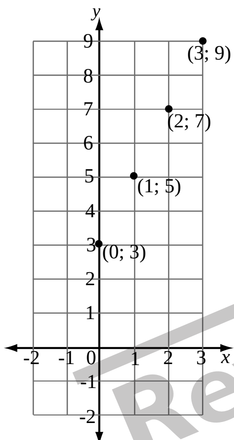
1

x	–3	–2	–1	0	1	2	3
y	–13	–8	–3	2	7	12	17

2

x	–3	0	3	6	9	12	15
y	4	2	0	–2	–4	–6	–8

3 **a**



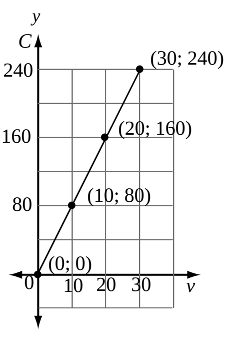
b (4: 11); (5; 13)

4 **a**

Number of visits v	0	10	20	30
Total cost C	0	80	160	240

b $C = 8v$

c

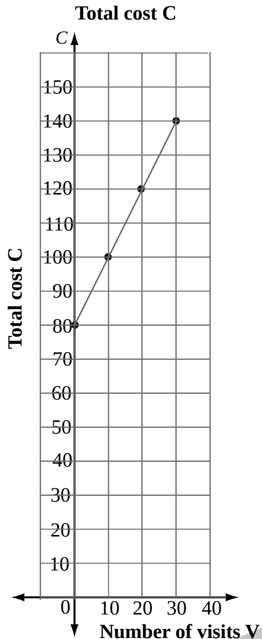


5 **a** $v \rightarrow \times 2 \rightarrow +80$

b

Number of visits v	0	10	20	30
Total cost C	80	100	120	140

c

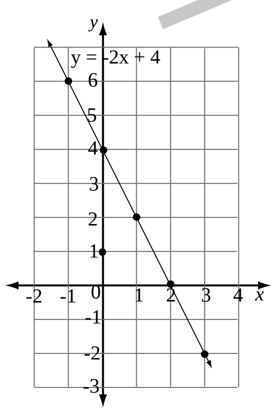


6 **a** $x \rightarrow \times (-2) \rightarrow +4 \rightarrow y$

b

x	–1	0	1	2	3
y	6	4	2	0	–2

c



d A decreasing straight-line graph.

CHAPTER

15 Algebraic expressions

Chapter overview

Learner's Book pages 364 to 384
Recommended pacing: 9 hours

This chapter focuses on the following:

Unit 1: Revision: Algebraic expressions and factors	1 hour
Finding common factors	
Unit 2: Common factors	2 hours
Factorising expressions	
Unit 3: The difference of two squares	2,5 hours
Factorising the difference of two squares	
Unit 4: Factorising trinomials	2,5 hours
Quadratic trinomials	
Factorising trinomials	
Trinomials with common factors	
Unit 5: Simplifying algebraic fractions	1 hour
Revision: Simplifying fractions	
Factorising fractions to simplify them	
Chapter 15 Revision	1 hour

UNIT

1 Revision: Algebraic expressions and factors

Unit overview

Learner's Book page 365
Recommended pacing: 1 hours

This unit focuses on the following:

- Finding common factors

Resources: Learner's Book; calculator; exercise book

Background information

In Grade 8 learners revised finding numerical factors by:

- finding prime factors of numbers to at least 3-digit whole numbers, and
- LCM and HCF of numbers to at least 3-digit whole numbers, by inspection and factorisation.

In Unit 9 learners revised algebraic language and simplifying but this is the first time learners factorise algebraic expressions. Prime factorisation was used to find the HCF and LCM in Unit 1 in Grade 9.

Teaching guidelines

Finding common factors of numbers is not new to learners, but it is advisable to make sure that the work is thoroughly understood before progressing to algebraic common factors. Learner can either use the factor method where they compare the factors of the different numbers with each other or the prime factor method. The prime factor method is a better link to the method used to find algebraic common factors. The examples in the Learner's Book show a step-by-step progression from numbers to the algebraic common factors and learners should work through all the examples very carefully with your guidance.

Finding common factors

Activity 1 **Find factors**

Learner's Book page 366

Guidelines for implementing this activity

- Revise the terms *factors* and *multiples*.
- Work through the first two examples in the Learner's Book with learners. Do more examples on the board until learners are completely confident with factorising and finding the HCF of numbers.
- Use Activity 6 *Find the HCF and LCM using prime factorisaton* in the Learner's Book for extra examples.
- Revise expanded notation and ask learners to expand a few terms, for example, $3x^3y^2$.
- Have learners underline, circle or highlight the common factors. This makes it easier for them to see which occur in all the numbers. These factors should be multiplied to get the HCF.
- Work through the rest of the examples.
- The activity also covers some revision of simplification of algebraic expressions. Remind learners about the Distributive Law and that when simplifying, they should distribute the term or number outside the bracket to all the terms inside the bracket.
- The activity is quite long, but learners should be able to finish most of it in one lesson and the rest can be completed for homework.

Remedial and extension

Remedial: This is quite an important aspect of Algebra and finding the common factor should always be the first step in factorising. It is therefore important to spend time making sure that learners are able to find the HCF. Give learners a few extra exercises to practise and remind them all the way through this chapter about the methods. Make sure that learners use the expanded form until they are quite comfortable with shortcuts.

Extension: Challenge learners to find a quicker method than using the expanded form to find the HCF. Ask them to write a few examples and try their method out before swapping with a partner to check their answers.

Suggested answers

1 a Prime factors of 36: $2 \times 2 \times 3 \times 3$
Prime factors of 54: $2 \times 3 \times 3 \times 3$
Prime factors of 60: $2 \times 2 \times 3 \times 5$
Prime factor of 64: $2 \times 2 \times 2 \times 2 \times 2 \times 2$

b 2

c $2 \times 3 \times 3 = 18$

d $2 \times 2 = 4$

2 a Prime factors of 36: $2 \times 2 \times 3 \times 3$
Prime factors of 48: $2 \times 2 \times 2 \times 2 \times 3$
HCF = $2 \times 2 \times 3 = 12$

b Prime factors of 16: $2 \times 2 \times 2 \times 2$
Prime factors of 36: $2 \times 2 \times 3 \times 3$
HCF = 2×2

c Prime factors of 27 = $3 \times 3 \times 3$
Prime factors of 24 = $2 \times 2 \times 2 \times 3$
HCF = 3

d Factors of $3m^2 = 3 \times m \times m$
Factors of $2m = 2 \times m$
HCF = m

e Factors of $32x^2y$: $2 \times 2 \times 2 \times 2 \times 2 \times x \times x \times y$
Factors of $27xy^2$: $3 \times 3 \times 3 \times x \times y \times y$
HCF = xy

f Factors of x^4y^2: $x \times x \times x \times x \times y \times y$
Factors of x^2y: $x \times x \times y$
HCF = $x \times x \times y = x^2y$

3 a Factors of 12: $2 \times 2 \times 3$
Factors of –204: $(-1) \times 2 \times 2 \times 3 \times 17$
All the factors of 12 are in –204, therefore 12 is a factor.

b Factors of $4x = 2 \times 2 \times x$
Factors of $12x^2 = 2 \times 2 \times 3 \times x \times x$
All the factors of $4x$ are contained in $12x$, therefore $4x$ is a factor.

c Factors of $3xy = 3 \times x \times y$
Factors of $55xy = 5 \times 11 \times x \times y$
Although xy is common to both, $3xy$ is not a factor of $55xy$.

d Factors of $x^2y = x \times x \times y$
Factors of $6x^3y^3 = 2 \times x \times x \times x \times y \times y \times y$
All the factors of x^2y are contained in $6x^3y^3$, therefore x^2y is a factor.

e Factors of $14abc$: $2 \times 7 \times a \times b \times c$
Factors of $-28abcd$: $(-1) \times 2 \times 2 \times 7 \times a \times b \times c \times d$
All the factors of $14abc$ are contained in $-28abcd$, so $14abc$ is a factor.

f Factors of ax^3y^4: $a \times x \times x \times x \times y \times y \times y \times y$
Factors of $8a^2xy^7$: $2 \times 2 \times 2 \times a \times a \times x \times y \times y \times y \times y \times y \times y \times y$
Although axy^4 is common to both, ax^3y^4 is not a factor of $8a^2xy^7$.

4 a $\frac{6x^2}{3x} = 2x$

b $\frac{9x^3y}{3x} = 3x^2y$

c $12m \div 3 = 4m$

d $-\frac{15a^2b}{3ab} = -5a$

5 a $a(a - b) = a^2 - ab$
b $4p(3p + 2) = 12p^2 + 8p$
c $m^2(m^2 + m^4) = m^6 + m^4$
d $-2x(4y - 2x) = 4x^2 - 8xy$

6 a $4a(a + b) + 3ab = 4a^2 + 7ab$
b $-2(x - 3) - (x - 4) = -2x + 6 - x + 4 = -3x + 10$
c $2a(a + b) - 3b(2a - 3c) = 2a^2 - 4ab + 9bc$
d $-2y(x + 4) - 3x(y - 5) = 15x - 5xy - 8y$

7 a $(x + 1)(x - 3) = x^2 - 2x - 3$
b $(2y + 1)(y + 5) = 2y^2 + 11y + 5$
c $(abc + d)(abc - 2d) = a^2b^2c^2 - abcd - 2d^2$
d $(x - 1)(x + 1) = x^2 - 1$
e $(3x + y)(3x - y) = 9x^2 - y^2$
f $(\frac{x}{2} - y)(\frac{x}{2} + y) = \frac{x^2}{2} - y^2$

8 a $(a + 2)^2 = a^2 + 4a + 4$
b $(x + y)^2 = x^2 + 2xy + y^2$
c $(x - 2)^2 = x^2 - 4x + 4$
d $(pq - 2)^2 = p^2q^2 - 4pq + 4$

9 a $2(x + 3)(x - 2) = 2(x^2 + x - 6) = 2x^2 + 2x - 12$
b $-3x(x - 2)(x + 2y) = -3x(x^2 - 2x + 2xy - 4y) = -3x^3 + 6x^2 - 6x^2y + 12xy$

10 a

	Expression	**Values of *x***				
		–2	–1	0	1	2
1	$\frac{3x^2 + 3x}{x + 1}$	–6	undefined	0	3	6
2	$3x$	–6	–3	0	3	6
3	$\frac{x^2 - x - 6}{x - 3}$	0	1	2	3	4
4	$x + 2$	0	1	2	3	4

b They are the same apart from the value for –1.
c They are the same.
d Complete this sentence: $\frac{3x^2 + 3x}{x + 1}$ is equivalent to $3x$ (but $x \neq -1$) and $\frac{x^2 - x - 6}{x - 3}$ is equivalent to $x + 2$.
e It is easier to use the simpler expressions.

UNIT 2 Common factors

Unit overview

Learner's Book page 368
Recommended pacing: 2 hours

This unit focuses on the following:

- Factorising expressions:
  - factorising simple expressions
  - factorisng more complex expressions
  - common factors and the difference of two squares
  - common factors with high powers
  - grouping terms to make a common factor.

Resources: Learner's Book; calculator; exercise book

Background information

Factorising algebraic expressions is new to learners in Grade 9. It is based on the numeric factorising that learners have been doing since Grade 6 so it is important to remind learners that in Algebra we generalise the patterns that we have been using in Arithmetic.

Teaching guidelines

It is important to start from a familiar skill such as simplification. In Chapter 9 Unit 2, learners practised simplification. Once they have revised the simplification explain that factorisation is the inverse process of simplification. Where simplification is about multiplying the factor outside the brackets with all the terms inside the brackets, factorisation is about dividing terms. Emphasise that factorisation is dividing. Also mention that they are going to be using the Distributive Law to factorise various terms. When we find the highest common factor, we divide each of the terms by this factor. The highest common factor is then written on the outside of the bracket and the terms that have been divided on the inside.

It is really important that learners get into the habit right from the outset, of looking for the highest common factors first when they are factorising even if the factorisation involves differences of squares, trinomial or any other factorisation. The first step should always be to look for the HCF. Also encourage learners to take the time to test their answers to make sure that when they simplify they get the original expression.

Factorising expressions

Factorising simple expressions

Activity 1 **Reduce to factors**

Learner's Book page 369

Guidelines for implementing this activity

- Do a quick revision of the exponent laws if necessary.
- Give learners a few examples of simplification to revise.
- Go through one or two step by step, emphasising that simplification means multiplication of the factor outside the bracket with all the terms inside the bracket.
- Explain to learners that factorisation is the inverse of simplification. Revise a few other inverses such as addition is the inverse operation of subtraction, and multiplication and division are inverse operations.
- Use the expanded notation to show how to factorise algebraic expressions as the examples in the Learner's Book. Learners will be finding the HCF of the terms in the expression each time.
- Provide a few extra examples if necessary.
- Learners should finish the activity by themselves in one lesson.
- Make sure that learners check their answers by simplifying.

Remedial and extension

Remedial: Make sure that learners can find the HCF. Provide some extra examples for them. It is sometimes useful to use solid objects to explain the concept. Put different numbers of pieces of coloured chalk into two or three bowls or plastic bags. Ask learners what the maximum number of each is that is common to both bowls/packets.
Extension: Give learners some more complex examples involving three or four terms with number coefficients as well as various variables. Remind them to use their quick method that they developed in the previous unit to find the HCF without the expanded notation. Make sure that learners check their answers by simplifying.

Suggested answers

1 $2a + 6 = 2(a + 3)$
2 $3y + 15 = 3(y + 5)$
3 $8p - 4 = 4(2p - 1)$
4 $12 - 6t = 6(2 - t)$
5 $9a - 3 = 3(3a - 1)$
6 $7b- 14c = 7(b - 2c)$
7 $3w^2 + 3w = 3w(w + 1)$
8 $2z - 2z^2 = 2z(1 - z)$
9 $x^2 + x = x(x + 1)$
10 $4p^2 - 5p = \mathrm{p}(4p - 5)$
11 $7k - 14k^2 = 7k(1 - 2k)$
12 $3n^3 - 2n = n(3n^2 - 2)$
13 $12x^2 - 4xy = 4x(3x - y)$
14 $3x^2y - 6xy = 3xy(x - 2)$

Factorising more complex expressions

Activity 2 Find more complex common factors

Learner's Book page 370

Guidelines for implementing this activity

- This lesson is an extension of the work done in Activity 1.
- Make sure that learners are confident about the type of examples in Activity 1 before moving on.
- Explain that whereas there were only two terms in the examples in Activity 1, here there may be more. The principles remain the same in that they must look at the numbers and then each variable separately and find the HCF. They then divide each term by the HCF.
- In Example 1 in the Learner's Book, the common factor is the expression inside the bracket. The principle is the same and once they have the HCF, they divide each term by it.
- Introduce square brackets. Some learners may find it complicated to remember when to use round (); square [] or curly{} brackets. Generally, the order of nested brackets is { [( )] }
- In Example 2, the complex factor in the bracket is also squared. Do a brief revision of the exponent laws again if necessary. It is useful to use different coloured chalk or markers when dealing with the complex factors. Encourage learners to mark these in their books.
- This is important basic groundwork for Algebra in the FET. Mark learners' books if possible and provide individual feedback. Take special note of any consistent errors and let learners correct them.

Remedial and extension

Remedial: When marking learners' work make notes for them so that they can refer back to any problems that they may have. This will assist them when they do corrections. Encourage learners to check their answers by multiplying out the factors.
Extension: Let learners work up to *Grouping terms to make a common factor* with a partner. Give them a few more complicated examples when they have finished. Challenge learners to find the HCF mentally if possible but if not, to write a few notes on the right-hand side of their page.

Suggested answers

1 $4ab - 6ac + 12ad = 2a(2b - 3c + 6d)$
2 $8ab - 4ab^2 = 4ab(2 - b)$
3 $x^2 + x^3 = x^2(1 + x)$
4 $x^6 + x^5 - x^3 = x^3(x^3 + x^2 - 1)$
5 $24x^3 - 36x^2 + 72x = 12x(2x^2 - 3x + 6)$
6 $10p^6q^2 - 4p^3q^2 + 2p^4q^4 = 2p^3q^2(5p^3 + pq^2 - 2)$
7 $10m^2p + 20m^3p^2 - 30\ m^2p^3 = 10m^2p(m + 20mp - 3p^2)$
8 $9y^4 - 15y^3 + 3y^2 = 3y^2(3y^2 - 5y + 1)$
9 $(a + b)^2 - (a + b) = (a + b)[(a + b) - 1] = (a + b)(a + b - 1)$
10 $3(m + n) + 5p(m + n) = (m + n)(3 + 5p)$
11 $5(r + s)^2 - (r + s) = (r + s)[(5(r + s) - 1] = (r + s)(5r + 5s - 1)$
12 $\mathrm{a}(a + 1) + 2(a + 1) = (a + 1)(a + 2)$

Activities 3–4 Group to find common factors; Analyse and interpret factors

Learner's Book page 371

Guidelines for implementing these activities

- Make sure that learners are able to do the factorising with the grouping before starting this section.
- Work through Example 1 with learners on the board. Use colour chalk or markers to highlight the variables that are the same and could be common. Ask learners to group them by taking out the HCF.
- Example 2 in the Learner's Book is very important and difficult for learners to understand initially. Introduce it slowly and carefully, emphasising that (–1) is the common factor. Revise the signs of integers when dividing (–) ÷ (–1) → (+) and when a (+) ÷ (–1) → (–).
- Let learners try out a few numbers on their calculators and see how that works.
- $-3 \div (-1) = \Delta$; $5 \div (-1) = \Delta$
- Then introduce just the –k – m part of the example to explain the principle.
- Activity 3 questions **2** and **5** practise this skill.
- Learners should do the activities independently. Correct their work if possible and give individual feedback on any issues.

Remedial and extension

Remedial: Make sure that learners are able to do simpler examples before moving on to more complex ones. Have extra worksheets ready for additional practice. Introduce the more complex factors slowly.
Extension: Have a few more complex examples on worksheets or cards to give learners to try. These can be taken from a Grade 10 book as well. Show learners that what they will be doing in FET builds on this work and is a progression of the same principles.

Suggested answers

Activity 3

1 $(a + 1)^2 + a + 1 = (a + 1)[(a + 1) + 1] = (a + 1)(a + 2)$
2 $(p + s)^2 - 3p - 3s = (p + s)^2 - 3(p + s) = (p + s)[(p + s) - 3] = (p + s)(p + s - 3)$
3 $2a + 2b - (a + b)^2 = 2(a + b) - (a + b)^2 = (a + b)[2 - (a + b)] = (a + b)(2 - a - b)$
4 $(a^2 + a) + (2a + 2) = a(a + 1) + 2(a + 1) = (a + 1)(a + 2)$
5 $4x(a - b) - a + b = 4x(a - b) - (a - b) = (a - b)(4x - 1)$
6 $(y^2 + 3y) + 4y + 12 = y(y + 3) + 4(y + 3) = (y + 3)(y + 4)$

Activity 4

1 $4(x + 3) = 4x + 12$
$x(4 + 3x) = 4x + 3x^2$
$3(x - 4) = 3x - 12$
$x(4x - 3) = 4x^2 - 3x$
2 a Only Tawanda has factorised completely.
b Caitlin and Simon did not use the HCF as the common factor.

UNIT 3

The difference of two squares

Unit overview

Learner's Book page 372
Recommended pacing: 2,5 hours

This unit focuses on the following:
- Factorising the difference of two squares
- Factorising complicated differences between two squares
- The sum of two squares
- Common factors and the difference of two squares
- Factorising an expression completely

Resources: Learner's Book; calculator; exercise book

Background information

In Chapter 8, learners expanded two binomials with the same terms but different signs between the terms. These should be revised before starting the factorising.

Teaching guidelines

Revise a few of the polynomials that have two binomials with the same terms but different signs so that learners are familiar with the simplification or expansion. Remind learners that factorising is the inverse operation of multiplying out or simplifying and that they will be finding the inputs from the outputs.
The difference of two squares is a very common form of factorising at school. It is used in Algebra and Trigonometry in the FET phase and learners should constantly be on the lookout for them when factorising. Make sure that learners know how to recognise the two terms that are perfect squares and the minus sign between them.
The difference of two squares can be used to do mental calculations of very large numbers, for example:
$2\ 013^2 - 2\ 012^2 = (2\ 013 - 2\ 012)(2\ 013 + 2\ 012) = (1)(4\ 025) = 4\ 025$
Make sure that you give learners a few of these examples in their activities so that they are constantly reminded that the technique can be used to calculate numbers.

Factorising the difference of two squares

Activity 1 **Factorise the difference of two squares**

Learner's Book page 373

Guidelines for implementing this activity

- Work carefully through Example 1 in the Learner's Book. Example 2 gives a step-by-step Think-Do method for working with these processes.
- Once learners understand how to find the terms in the factorised form, they can do these mentally or use their calculators for the numerical coefficients.
- Learners do Activity 1 independently. Make sure that they get all of them correct and if not they should redo them.

Remedial and extension

Remedial: Make sure that learners understand the term "perfect square" Let them write down five examples of their own. Give them some mixed examples of terms that are and are not perfect squares so that learners can identify them. Revise the concepts of square roots of numbers as well as the exponent laws so that learners are familiar with these processes. Give learners a few more simple examples to practise before proceeding to the next activity.
Extension: Remind learners that the first step for all factorising is looking for the HCF. Once they have taken out the common factor, they should always be on the lookout for perfect squares with a minus sign between them. Challenge learners to do the activity as quickly as possible.

Suggested answers

1 **a** $x^2 - y^2 = (x - y)(x + y)$
  b $36x^2 - 9 = (6x - 3)(6x + 3)$
  c $a^2 - 25 = (a - 5)(a + 5)$

2 a $4x^2 - 1 = (2x - 1)(2x + 1)$
b $25y^2 - 4 = (5y - 2)(5y + 2)$
c $9d^2 - 64 = (3d - 8)(3d + 8)$
d $36z^2 - 49 = (6z - 7)(6z + 7)$
e $49 - 36q^2 = (7 - 6q)(7 + 6q)$
f $81x^2 - 49y^2 = (9x - 7y)(9x + 7y)$

Factorising complicated differences between two squares

Activity 2 Factorise more complicated differences of two squares

Learner's Book page 373

Guidelines for implementing this activity

- Make sure that learners are able to do the simpler examples and provide additional practice exercises if necessary.
- Revise square roots of numbers and the exponent rules.
- Use the method of the empty brackets with a + and – symbol in each as shown in question **1** of Activity 1 in the Learner's Book. Once learners have written this below the original expression, let them take the square roots of the two terms and fill them in the appropriate spaces in the brackets.
- Remind learners to check their answers by multiplying out.
- Learners do the activity independently.
- Once completed, give the full answers on the board and ask learners to check their answers carefully. They should let you know if there is something that they do not understand in the process.
- Remind learners often that this factorisation only works for the difference of two squares and not for the sum of two squares.

Remedial and extension

Remedial: Let learners use the bracket method as mentioned in the guidelines above. This makes it easier for them to analyse what needs to be done to each term in the original binomial. Provide additional practice examples if necessary.
Extension: Give learners some examples that require them to take out a common factor first and also some that have more than one difference of squares to factorise, for example: $12m^4 - 75g^2$
$a^4 - 8$
$a^4b^8 - \frac{c^8}{2}$
Learners should work in pairs and discuss their methods and answers with each other. They check their answers by multiplying out.

Suggested answers

1 $x^4 - 9y^2 = (x^2 - 3y)(x^2 + 3y)$
2 $a^4b^4 - 1 = (a^2b^2 - 1)(a^2b^2 + 1) = (ab - 1)(ab + 1)(a^2b^2 + 1)$
3 $81m^2n^2 - 25k^2 = (9mn - 5k)(9mn + 5k)$

4 $225p^2q^2 - 36x^2y = 9(25p^2q^2 - 4x^2y)$
5 $196x^4 - 49y^2 = 49(4x^2 - y^2) = 49(2x^2 - y)(2x^2 + y)$
6 $(x - 3)^2 - 16 = [(x - 3) - 4][(x - 3) + 4] = (x - 7)(x + 1)$
7 $(2x + 3)^2 - 25 = [(2x + 3) - 5][(2x + 3) + 5] = (2x - 2)(2x + 8) = 4(x - 1)(x + 2)$
8 $(y - 2)^2 - 4 = [(y - 2) - 4][(y - 2) + 4] = (y - 6)(y + 2)$
9 $4(x + 5)^2 - 9 = [2(x + 5) - 3][2(x + 5) + 3] = (2x + 10 - 3)(2x + 10 + 3) = (2x + 7)(2x + 13)$

The sum of two squares; Common factors and the difference of two squares; Factorising an expression completely

Activities 3–4 Factorise in steps; Factorise completely

Learner's Book pages 375–376

Guidelines for implementing these activities

- Remind learners about the HCF again.
- Remind them to always check for common factors first when factorising.
- Once the common factors have been removed, they should focus on the inside of the bracket and look for the difference of squares.
- Work through the examples in the Learner's Book.
- Again remind learners that this factorisation only works for the difference of two squares and not for the sum of two squares. Questions **9** and **10** of Activity 3 will test this.
- Learners should work through Activity 3 as quickly as possible.
- Work through the example on Factorising an expression completely with learners. Remind them that they should constantly be on the lookout for differences (*not* the sums!) of squares even if they think that they have factorised an expression.
- Learners work through Activity 4. Question **5** does not have any further factorisation.

Remedial and extension

Remedial: Make sure that learners can do the following:
1. Prime factorisation
2. Expanded form
3. Finding the HCF of numbers using the expanded form.
4. Finding the HCF of algebraic terms using the expanded form.
5. Understand what squares and square roots are.
6. Understand the exponent laws.
7. Can identify the difference of squares.
8. Know that they cannot factorise the sum of squares.

If there are any gaps, they will find it more and more difficult to cope with the examples. Re-explain and give extra practice where necessary.
Extension: Give learners some numeric examples as well as some examples with larger coefficients to do without the use of a calculator, for example:
$340^2 - 660^2$ $\quad$ $200x^6 - 98y^4$ $\quad$ $7{,}3^2 - 6{,}7^2$
These skills are very useful in Mathematics Olympiads where learners are not allowed to use calculators. Have a few past papers available for them to practise on. See contact details for the South African Maths Foundation in the Learner's Book.

Suggested answers

Activity 3

1 $72p^2 - 50 = 2(36p^2 - 25) = 2(6p - 5)(6p + 5)$
2 $125 - 45t^2 = 5(25 - 9t^2) = 5(5 - 3t^2)(5 + 3t^2)$
3 $9p - 81p^3 = 9p(1 - 9p^2) = 9p(1 - 3p)(1 + 3p)$
4 $27m^2 - 12n^2 = 3(9m^2 - 4n^2) = 3(3m - 2n)(3m + 2n)$
5 $4xy^3 - x^3y = \text{xy}(4y^2 - x^2) = \text{xy}(2y - x)(2y + x)$
6 $a^3 - 25ab = \text{a}(a^2 - 25b)$
7 $81m^2n - 4n = \text{n}(9m^2 - 4) = n(3m - 2)(3m + 2)$
8 $x^2y^2z - 9z^3 = \text{z}(x^2y^2 - 9) = z(xy - 3)(xy + 3)$
9 $9x^3 + x = x(9x^2 + 1)$
10 $4x^2y + 9y^3 = \text{y}(4x^2 + 9y^2)$

Activity 4

1 $x^4 - 4x^2 = x^2(x^2 - 4) = x^2(x - 2)(x + 2)$
2 $a^4b^2 - 9a^2b^2 = a^2b^2(a^2 - 9) = a^2b^2(a - 3)(a + 3)$
3 $a^4b^2 - 81a^2b^2 = a^2b^2(a^2 - 81) = a^2b^2(a - 9)(a + 9)$
4 $y^6z^2 - 144y^4 = y^4(y^2z^2 - 144) = y^4(yz - 12)(yz + 12)$
5 $144a^2 - 1 = (12a - 1)(12a + 1)$
6 $2x^4 - 32x^2 = 2x^2(x^2 - 16) = 2x^2(x - 4)(x + 4)$
7 $x^4 - 1 = (x^2 - 1)(x^2 + 1) = (x - 1)(x + 1)(x^2 + 1)$
8 $48x - 3x^5 = 3x(16 - x^4) = 3x(4 - x^2)(4 + x^2) = 3x(2 - x)(2 + x)(4 + x^2)$

UNIT

4 Factorising trinomials

Unit overview

Learner's Book page 377
Recommended pacing: 2,5 hours

This unit focuses on the following:

- Quadratic trinomials
- Factorising trinomials
- Trinomials with common factors.

Resources: Learner's Book; calculator; exercise book

Background information

This is new work for the Grade 9 learner. Make sure that all terminology is revised so that they are able to deal with the new work.

Teaching guidelines

It is necessary to deal with the terminology again so learners are not stymied by language. Make sure that learners know what the general form looks like and give them a few mixed examples to identify.

The trinomial factorisation is the inverse of a binomial multiplied by a binomial. Remind learners of the FOIL method or the method that they prefer to use when multiplying binomials. Once this is consolidated, introduce the factorising of trinomials as the inverse process.

Quadratic trinomials; Factorising trinomials

Activity 1 Factorise trinomials

Learner's Book page 380

Guidelines for implementing this activity

- Revise the terms monomial, binomial, trinomial, polynomial.
- Revise the degree of polynomials.
- Make sure that learners understand the term quadratic and that it is an expression of the second degree.
- Revise binomial multiplication using the first example on page 377 in the Learner's Book.
- Work through the section Factorising trinomials in the Learner's Book. It is advisable to work through an example and then let learners try a few out themselves before moving on to the next example.
- This section includes Example 1, carefully explaining where each term of the factors comes from. This example has both signs positive.
- Example 2 also has both signs positive and gives a shorter explanation of finding the terms. Learners should do questions **1** to **6** of Activity 1 independently.
- In Example 3, the signs are different in the trinomial, but when learners look at the second sign and it is positive it tells them that both factors have the same sign which is the same as the first sign in the trinomial which in Example 3 is negative. Learners do questions **7** and **8** of Activity 1
- Examples 4 and 5 show the case where the second sign is negative. Because the second sign is negative it means that the signs inside the factors will be different. The numerically larger of the two factors will have the same sign as the first sign in the trinomial. Learners do questions **9** to **12** of Activity 1.
- Questions **13** and **14** are a little more challenging. In both questions, learners must first rearrange the terms in descending order of powers and then factorise.

Remedial and extension

Remedial: There are numerous ways of explaining how to do trinomial factorisation but it is best to keep it as simple as possible. The first term in Grade 9 in general has a coefficient of 1 so is factorised by simply taking the square root. The third term is a product of its factors. When these factors are added together, they should give the middle term's coefficient. Show learners how to check the signs in the trinomial and then to write down empty brackets below the trinomial. They can fill in the appropriate signs into the brackets. Once they have done this, they can factorise the terms as explained above.
Extension: Provide additional practice with some examples that are a little more complicated. It is best not to start factorising with a coefficient other than 1 for either the first or last term yet, but you can give learners some examples where the last term has a coefficient of 1 or –1 to try, for example:
$6x^2 + 5x + 1$
$12x^2 + 2xy - y^2$

Suggested answers

1 $x^2 + 8x + 7 = (x + 7)(x + 1)$
2 $x^2 + 8x + 15 = (x + 5)(x + 3)$
3 $x^2 + 6x + 8 = (x + 4)(x + 2)$
4 $x^2 + 9x + 14 = (x + 7)(x + 2)$
5 $x^2 + 9x + 18 = (x + 6)(x + 3)$
6 $x^2 - 10x + 21 = (x - 7)(x - 3)$
7 $x^2 - 8x + 7 = (x - 7)(x - 1)$
8 $x^2 - 5x + 6 = (x - 3)(x - 2)$
9 $x^2 + 2x - 24 = (x + 6)(x - 4)$
10 $x^2 + x - 90 = (x + 10)(x - 9)$
11 $x^2 - 8x - 20 = (x - 10)(x + 2)$
12 $x^2 + 6x - 7 = (x + 7)(x - 1)$
*13 $35 + 12x + x^2 = (7 + x)(5 + x)$
*14 $72 + x^2 - 17x = x^2 - 17x + 72 = (x - 8)(x - 9)$

Trinomials with common factors

Activity 2 Factorise trinomials with common factors

Learner's Book page 381

Guidelines for implementing this activity

- Remind learners to look for the common factor first when they are factorising.
- Once they have taken out the highest common factor, they work with the terms in the bracket and factorise them.
- Make sure that learners check their answers by multiplying out. Remind learners to multiply the brackets first before they multiply by the factor in front of the brackets.
- Learners do the activity independently.

Remedial and extension

Remedial: Extend the checklist give at the end of Unit 3 to include Trinomial factorising. For example:

1 Prime factorisation
2 Expanded form
3 Finding the HCF of numbers using the expanded form
4 Finding the HCF of algebraic terms using the expanded form
5 Understand what squares and square roots are
6 Understand the exponent laws
7 Can identify the difference of squares
8 Know that they cannot factorise the sum of squares
9 Recognise what a quadratic trinomial is
10 Are able to analyse the signs in the trinomial to decide what signs go into the brackets
11 Are able to factorise terms so that they can add the factors to find the middle term

Extension: Have learners that finish quickly help their peers where necessary. It often crystallizes their own knowledge when they have to explain it to someone else.

Suggested answers

1 $2a^2 + 14a + 24 = 2(a^2 + 7a + 12) = 2(a + 4)(a + 3)$
2 $3x^2 - 12x - 36 = 3(x^2 - 4x - 12) = 3(x - 6)(x + 2)$
3 $4a^2 + 16a - 48 = 4(a^2 + 4a - 12) = 4(a + 6)(x - 2)$
4 $2a^2 + 6a + 4 = 2(a^2 + 3a + 2) = 2(a + 2)(a + 1)$
5 $4x^2 + 12x - 72 = 4(x^2 + 3x - 18) = 4(x + 6)(x - 3)$

6 $2x^3 + 2x^2 - 12x = 2x(x^2 + x - 6) = 2x(x + 3)(x - 2)$
7 $2x^2 + 10x + 12 = 2(x^2 + 5x + 6) = 2(x + 3)(x + 2)$
8 $2x^2 - 12x + 18 = 2(x^2 - 6x + 9) = 2(x - 3)(x - 3) = 2(x - 3)^2$
9 $3x^2 + 21x + 18 = 3(x^2 + 7x + 6) = 3(x + 6)(x + 1)$
10 $5x^2 - 10x - 15 = 5(x^2 - 2x - 3) = 5(x - 3)(x + 1)$

UNIT 5

Simplifying algebraic fractions

Unit overview

Learner's Book page 382
Recommended pacing: 1 hour

This unit focuses on the following:
- Revision: simplifying fractions
- Factorising fractions to simplify.

Resources: Learner's Book; calculator; exercise book

Background information

Learners simplified algebraic fractions that were already factorised in Chapter 9 in the Learner's Book. In this chapter, learners will first factorise and then simplify.

Teaching guidelines

Learners often struggle with fractions even if they are numeric ones. Algebraic fractions are more complex and really test their understanding of fractions. It is important to revise some simpler algebraic fractions first and then move on to the fractions with complex terms.

In addition, learners often forget that they can only cancel factors and not terms. This means that when they are presented with $\frac{x+1}{x+2}$, they will cancel the x in the numerator with the x in the denominator and get $\frac{1}{2}$ as the answer. It is necessary to make the distinction between this example and $\frac{(x+1)}{2(x+2)}$. In this case the $(x + 1)$ is a factor and can be cancelled in its entirety to leave an answer of $\frac{1}{2}$. It is also very important to show the cancelling properly and that learners write the 1 when cancelling. Some learners will assume that it is 0 and get $\frac{0}{2} = 0$ as their answer.

Take a little time to emphasise these differences at every opportunity where these situations arise.

Revision: Simplifying fractions; Factorising fractions to simplify

Activity 1 Factorise to simplify fractions

Learner's Book page 383

Guidelines for implementing this activity

- Revise numeric fractions and the four basic operations.
- Make sure that learners understand the concept of cancelling and that they write the small numbers (*including* 1) above and below the cancelled factors.
- Revise the algebraic simplification done in Chapter 8.
- Remind learners that they can only cancel whole factors and not individual terms of the factors.
- Work through the examples in the Learner's Book.
- Learners do the activity in the Learner's Book. Let learners work in pairs for the first three questions and then do the next three independently.

Remedial and extension

Remedial: Basic exercises on fractions are useful to start with. Have learners explain what they are doing at each step so that you can hear what they are thinking. Gently correct any incorrect thinking. Make sure that they write a note about the correct thinking. When learners make a wrong assumption, let them try an example with numbers and see if it works. They often do not make the connection between the variables and actual numbers.
Extension: Give learners a few more challenging examples. These can get very complicated and so it is best not to over-extend the learners. It is however a great test to see how well they have mastered everything in this chapter. Here are a few extra examples (the third one is very challenging and can be given only to the gifted learner):

$\frac{x^3 + x^2y}{y^3 - y^2x}$ $\frac{4a^2 - 9b^2}{4a^2 - 12ab^2 + 9b^2}$ $\frac{(a-b)^2 - c^2}{a^2 + ab - ac} \times \frac{a^2 - (b-c)^2}{(a-b+c)}$

Suggested answers

1 $\frac{(4x + 12y)}{(x + 3y)} = 4$

2 $\frac{(4m - 16y)}{(8m - 32y)} = \frac{4(m - 4y)}{8(m - 4y)} = \frac{1}{2}$

3 $\frac{2m^2 - 32}{8m - 32} = \frac{2(m^2 - 16)}{8(m - 4)} = \frac{(m - 4)(m + 4)}{4(m - 4)} = \frac{1}{4}(m + 4)$

4 $\frac{x^2 - 36}{5x - 30} = \frac{(x - 6)(x + 6)}{5(x - 6)} = \frac{1}{5}(x + 6)$

5 $\frac{p^2 - 9}{3p + 9} = \frac{(p - 3)(p + 3)}{3(p + 3)} = \frac{1}{3}(p - 3)$

6 $\frac{8x^2 + 32x}{2x + 4} = \frac{8x(x + 4)}{2(x + 2)} = \frac{4x(x + 4)}{(x + 2)}$

Chapter 15 Revision

Learner's Book page 384

Encourage learners to review the content covered before attempting the revision activities. The revision activities should be used to assess learners' progress thus far, and to assess where remediation may be required.

Suggested answers

1 **a** Prime factors of 6: 2×3
Prime factors of 10: 2×5
HCF = 2

b Factors of $-12a$: $(-1) \times 2 \times 2 \times 3 \times a$
Factors of $30ab$: $2 \times 3 \times 5 \times a \times b$
HCF = $2 \times 3 \times a = 6a$

c Factors of $3x^2y^3$: $3 \times x \times x \times y \times y \times y$
Factors of $9x^5y^2$: $3 \times 3 \times x \times x \times x \times x \times x \times y \times y$
Factors of $-18x^3y = (-1) \times 2 \times 3 \times 3 \times x \times x \times x \times y$
HCF = $3 \times x \times x \times y = 3x^2y$

2 **a** $7b - 14c = 7(b - 2c)$
b $8ab - 4ab^2 = 4ab(2 - b)$
c $9y^4 - 15y^3 + 3y^2 = 3y^2(3y^2 - 5y + 1)$
d $10p^6q^2 - 4p^3q^3 + 2p^4q^4 = 2p^3q^2(5p^3 - 2q + pq^2)$

3 **a** $t^2 - s^2 = (t - s)(t + s)$
b $a^2 - 9 = (a - 3)(a + 3)$
c $y^2 - 0{,}25 = (y - 0{,}5)(y + 0{,}5)$
d $25 - 16b^2 = (5 - 4b)(5 + 4b)$
e $x^2 - 16x + 15 = (x - 15)(x - 1)$
f $4x^2 + 4x - 8 = 4(x^2 + x - 2) = 4(x + 2)(x - 1)$
g $x^2 - 7x - 60 = (x - 12)(x + 5)$
h $a^2 - 11a - 12 = (a - 12)(a + 1)$
i $x^2 - 13x + 40 = (x - 8)(x - 5)$
j $x^2y^2 + 2xy + 1 = (xy + 1)(xy + 1) = (xy + 1)^2$

4 **a** $6x^2 - 24 = 6(x^2 - 4) = 6(x - 2)(x + 2)$
b $3 - 3x^2 = 3(1 - x^2) = 3(1 - x)(1 + x)$
c $12x^2 - 3x = 3x(4x - 1)$
d $x^2 - 3xy - 18y^2\ (x - 6y)(x + 3y)$

5 $a^8 - 1 = (a^4 - 1)(a^4 + 1) = (a^2 - 1)(a^2 + 1)(a^4 + 1) = (a - 1)(a + 1)(a^2 + 1)(a^4 + 1)$

6 **a** $101^2 - 99^2 = (101 - 99)(101 + 99) = (2)(200) = 400$
b $56^2 - 44^2 = (56 - 44)(56 + 44) = (12)(100) = 1\ 200$

7 $(x + 2)^2 - 9 = [(x + 2) - 3][(x + 2) + 3] = (x - 1)(x + 5)$

8 **a** $\frac{x^2 + 3x + 2}{x + 2} = \frac{(x + 2)(x + 1)}{x + 2} = x + 1$

b $\frac{x^2 - 16}{x - 4} = \frac{(x - 4)(x + 4)}{x - 4} = x + 4$

c $\frac{x^2 - 4x + 3}{x - 3} = \frac{(x - 3)(x - 1)}{x - 3} = x - 1$

d $\frac{2x^2 + 2}{x + 1} = \frac{2(x^2 + 1)}{x + 1}$

e $\frac{x^2 - x - 6}{x - 3} = \frac{(x - 3)(x + 2)}{(x - 3)} = x + 2$

f $\frac{x^2 - 4x + 3}{x - 3} = \frac{(x - 3)(x - 1)}{x - 3} = x - 1$

g $\frac{x^2 - 2x - 15}{x - 5} = \frac{(x - 5)(x + 3)}{(x - 5)} = x + 3$

h $\frac{x^2 - 9}{3x - 9} = \frac{(x - 3)(x + 3)}{3(x - 3)} = \frac{1}{3}(x + 3)$

CHAPTER 16

Algebraic equations

Chapter overview

Learner's Book pages 385 to 397
Recommended pacing: 9 hours

This chapter focuses on the following:

Unit 1: Exponential equations	3 hours
Powers of prime numbers	
Solving exponential equations	
Unit 2: Quadratic equations	3 hours
The zero product rule	
Using factorisation to solve equations	
Solving quadratic trinomial equations	
Describing situations using equations	
Unit 3: Substitution and ordered pairs	3 hours
Substitution	
Using substitution to generate ordered pairs	
Chapter 16 revision	

UNIT 1

Exponential equations

Unit overview

Learner's Book page 386
Recommended pacing: 3 hours

This unit focuses on the following:

- Powers of prime numbers
- Fractions and negative exponents
- Solve equations with complicated exponents
- Solve exponential equations with RHS = 1

Resources: Learner's Book; calculator; exercise book

Background information

In Grade 8 learners used the laws of exponents. In Chapter 5 learners looked at the laws of exponents more formally.

Teaching guidelines

Thoroughly revise the laws of exponents from Chapter 5. Work through the Revision exercise at the end of the Chapter 5 with learners again. Once learners are familiar with exponents again, discuss solving exponential equations. Learners need to make the bases the same on both sides of the equal sign. Once the bases are the same, the exponents must be equal to each other.
Revise negative exponents and what they mean. Make sure that learners convert a number of these before starting to solve the equations. Solving these equations are based on the same principle of if the bases are equal then the exponents must equal each other in an equation.
Remind learners that a number raised to a power cannot be equal to zero, for example, $3x \neq 0$. But any base raised to the power of 0 equals 1, for example $3^{0} = 1$.

Powers of prime numbers: Solving exponential equations

Activity 1 **Solve simple exponential equations**

Learner's Book page 387

Guidelines for implementing this activity

- Revise the terminology of exponents, for example bases, powers and exponents.
- Explain the difference between the bases that learners have used before and prime bases. This requires prime factorisation. Learners have just completed the chapter on factorisation so should not need too much revision.
- Do revision of the laws of exponents. You can use the revision exercise at the end of Chapter 5 to refresh learners' memories.
- Revise how the LHS and RHS of an equation must be equal.
- Make sure that learners understand this as you work through Example 1 in the Learner's Book.
- Example 2 gives the step-by-step instructions. Make sure that learners understand each step.
- Remind learners to use the lists given at the beginning of the unit so that they get used to recognising the numbers as powers of prime bases.
- Learners do Activity 1 without the use of a calculator if possible.
- Check that learners write their solutions correctly as shown in Example 2. Also make sure that they don't "cancel" the bases. This seems to be a common misconception. Have learners say: Because the bases are equal on both sides of the equation, the exponents must be equal.
- Learners should go back and check their answers using their calculators though.

Remedial and extension

Remedial: Make sure that learners are comfortable with the laws of exponents. Use the expanded notation to help them revise this section. Pay particular attention to the way that they write the solutions down and don't allow any "cancelling" of the bases.
Extension: Give learners a few examples with bigger values. Although they should be able to do these mentally ask them to write three or four solutions down in full.

Suggested answers

1 $2^x = 2^2 \therefore x = 2$
2 $3^x = 3^7 \therefore x = 7$
3 $7^x = 7^3 \therefore x = 3$
4 $5^x = 25 \therefore 5^x = 5^2 \therefore x = 2$
5 $2^x = 32 \therefore 2^x = 2^5 \therefore x = 5$
6 $3^x = 81 \therefore 3^x = 3^4 \therefore x = 4$
7 $2^x = 128 \therefore 2^x = 2^7 \therefore x = 7$
8 $7^x = 49 \therefore 7^x = 7^2 \therefore x = 2$
9 $6^x = 36 \therefore 6^x = 6^2 \therefore x = 2$
10 $10^x = 100 \therefore 10^x = 10^2 \therefore x = 2$

Fractions and negative exponents; Solve equations with complicated exponents

Activities 2–3 Solve equations with negative exponents; Solve exponential equations

Learner's Book pages 387–388

Guidelines for implementing these activities

- Remind learners that $\frac{1}{x} = x^{-1}$ and $a^{-3} = \frac{1}{a^3}$ from the exponent laws.
- Give learners a few examples to convert before starting to solve equations.
- Show learners how they can rewrite fractions that have been converted to their prime factors in exponential form. Give learners a few more examples.
- Work carefully through the examples in the Learner's Book. These give the step-by-step instructions.
- Pay particular attention to the way that learners write their solutions.
- Learners work through the activity independently. Show the solutions on the board and ask learners to raise any concerns that they may have after marking their solutions.

Remedial and extension

Remedial: When the way things are written is important, it is useful to mark the books yourself once learners have checked their answers. This will give you an opportunity to give learners individual feedback and tips.
Extension: Ask learners to work through examples 1 and 2 on their own. Have a few examples of this type of equation ready for them to do with a partner. They must check their answers using substitution.
For example:
$2^{-x} = 32$
$(0,2)^x = 25$
$6^{x-2} = 216$
$9^x = 27^{x-1}$
$2\frac{x}{5}$ Solving quadratic trinomial equations
$\frac{1}{32}$

Suggested answers

Activity 2

1 $5^x = \frac{1}{25}\ 5^x = 5^{-2} \therefore x = -2$

2 $3^x = \frac{1}{27}, 3^x = 3^{-3} \therefore x = -3$

3 $4^x = \frac{1}{64}\ 4^x = 4^{-3} \therefore x = -3$

4 $8^x = \frac{1}{64}, 8^x = 8^{-2} \therefore x = -2$

5 $5^x = \frac{1}{125}\ 5^x = 5^{-3} \therefore x = -3$

6 $2^x = \frac{1}{64}, 2^x = 2^{-6} \therefore x = -6$

Activity 3

1 $2^{x+3} = 2^7 \therefore x + 3 = 7 \therefore x = 4$
2 $3^x - 5 = 3^0 \therefore x - 5 = 0 \therefore x = 5$
3 $3^{x+2} = 243 \therefore 3^{x+2} = 3^5 \therefore x + 2 = 5 \therefore x = 3$
4 $2^{x+1} = 0{,}25 \therefore 2^{x+1} = \therefore 2^{x+1} = 2^{-2} \therefore x + 1 = -2 \therefore x = -3$
5 $2^x.3^x = 36 \therefore 2^x.3^x = 2^2.3^2 \therefore x = 2$
6 $(2 \times 5)^x = 1\,000 \therefore 2^x \times 5^x = 2^3 \times 5^3 \therefore x = 3$
7 $5^{x+1} = 1 \therefore x + 1 = 0 \therefore x = -1$
8 $7^{2x-4} = 1 \therefore 2x - 4 = 0 \therefore x = 2$
9 $3^{2x-3} = 243 \therefore 3^{2x-3} = 3^5 \therefore 2x - 3 = 5 \therefore x = 4$
10 $3 \times 5^x = 75 \therefore 3 \times 5^x = 3 \times 5^2 \therefore x = 2$

UNIT

2 Quadratic equations

Unit overview

Learner's Book page 389
Recommended pacing: 3 hours

This unit focuses on the following:

- The zero product rule
- Using factorisation to solve equations:
  - equations where $c = 0$
  - equations where $b = 0$
- Solving quadratic trinomial equations
- Describing situations using equations.

Resources: Learner's Book; calculator; exercise book

Background information

Learners have just completed the chapter on factorisation which they will use when solving quadratic equations with the zero product rule. The latter is new to Grade 9 learners.

Teaching guidelines

In Unit 2, learners are taught how to solve simple quadratic equations using the zero product rule. This means that if $a \cdot b = 0$ then either $a = 0$ or $b = 0$ or both are 0. Initially, learners are given equations that are already factorised, and then they practise the factorisation skills that they learnt in the previous chapter. Solving quadratic equations is also used to show how these equations can be applied to everyday problems. Make sure that learners are quite comfortable with the terminology. Make sure that you use the correct terminology when working through the examples and encourage learners to use the terminology while they are doing the activities.

The zero product rule

Activity 1 Solve factorised quadratic equations

Learner's Book page 390

Guidelines for implementing this activity

- Remind learners what a quadratic equation is. They factorised quadratic equations in a previous chapter.
- Revise the properties of zero:
  $0 + a = a$
  $a - 0 = a$
  $a \times 0 = 0$
  $\frac{0}{a} = 0$ but $\frac{a}{0}$ is undefined.
- The one that they will be using in the zero product rule is any number multiplied by zero equals zero. If you multiply two numbers together and they give a product of zero, then either one of the two could be zero or they could both be equal to zero.
- To use the zero product rule, the quadratic equation must be factorised *completely* and be made equal to zero if it isn't already.
- Work through the examples in the Learner's Book. Make sure that learners note the way the solutions are written.
- Learners often do not realise the difference between "and" and "or" in Maths. We cannot write $x = 4$ and $x = 10$.x cannot be both at the same time. Ask learners if they can be 15 and 22 at the same time – they may have some clever answers, but learners soon realise that they *must* say that $x = 4$ *or* $x = 10$.
- Learners work through Activity 1 independently. Question **8** is a square number so in that case x will only have one value and not two as the others do.
- Learners should check their answers using substitution. They can use their calculators to do this more quickly.

Remedial and extension

Remedial: When learners tackle a new section it is really important that they either verbalise what they are doing at each step or write down in their own words what they are doing. Spend some time checking this either orally or by taking their books in and marking it. It is very important for learners when they revise that they can return to these notes. Make sure that learners know how to use their calculators to check their answers.

Extension: Give learners a few examples that involve fractions and decimal coefficients and constants. Learners can also assist learners that may be struggling by showing them how to use their calculators to check their answers.

Suggested answers

1. $x(x - 3) = 0$
   $\therefore x = 0$ or $x - 3 = 0$
   $\therefore x = 0$ or $x = 3$
2. $3x(4x - 1) = 0$
   $\therefore 3x = 0$ or $4x - 1 = 0$
   $\therefore x = 0$ or $x = \frac{1}{4}$
3. $(x - 4)(x + 3) = 0 \therefore x = 4$ or $x = -3$
4. $(x + 4)(x + 9) = 0 \therefore x = -4$ or $x = -9$
5. $(5 - x)(2 - 3x) = 0 \therefore x = 5$ or $x = \frac{2}{3}$
6. $(7x - 8)(7x - 8) = 0 \therefore x = \frac{8}{7}$
7. $(6x - 8)(6x + 8) = 0 \therefore x = \frac{4}{3}$ or $x = -\frac{4}{3}$
8. $(2x + 3)^2 = 0 \quad x = -\frac{3}{2}$

Using factorisation to solve equations

Equations where *c* = 0; Equations where *b* = 0

Activity 2 **Solve equations by factorising**

Learner's Book page 391

Guidelines for implementing this activity

- Revise the different forms of factorising. Remind learners to always look for common factors first and then to take out the HCF. Revise differences of squares and trinomial factorisation.
- Ask learners why we need to factorise before we can use the zero product rule.
- A quadratic equation has the general form $ax^2 + bx + c = 0$.
- Work through Example 1 which shows an example where the constant is zero (c = 0).
- Example 2 shows a quadratic equation where the middle term is zero. Remind learners that the difference of squares can be factorised, but that the sum of squares cannot be factorised.
- Learners work through the activity. Take note of the way that they are writing the solutions down. Make sure that learners line up the equal signs underneath each other.
- Learners work through the activity independently.

Remedial and extension

Remedial: Check that learners can factorise correctly and give a few extra examples if necessary. Make sure that learners understand the zero product rule. Give them a number of numerical examples to plug into their calculators to convince themselves that any number multiplied by 0 gives 0 as an answer. Use this to revisit the zero product rule. Give more simple examples until learners are confident with the method and understand what they are doing.
Extension: Give learners a few more complex examples to try.

Suggested answers

1 $2y^2 + 4y = 0 \therefore 2y(y + 2) = 0 \therefore y = 0$ or $y = -2$

2 $3x^2 + x = 0 \therefore x(3x + 1) = 0 \therefore x = 0$ or $x = -\frac{1}{3}$

3 $y^2 - 16 = 0 \therefore (y - 4)(y + 4) = 0 \therefore y = 4$ or $y = -4$

4 $y^2 - 25 = 0 \therefore (y - 5)(y + 5) = 0 \therefore y = 5$ or $y = -5$

5 $x^2 = 9 \therefore x^2 - 9 = 0 \therefore (x - 3)(x + 3) = 0 \therefore x = 3$ or $x = -3$

6 $3y^2 = 75 \therefore 3(y^2 - 25) = 0 \therefore 3(y - 5)(y + 5) \therefore y = 5$ or $y = -5$

7 $2x^2 - 9x = 0 \therefore x(2x - 9) = 0 \therefore x = 0$ or $x = \frac{9}{2}$

8 $5x^3 - 125x = 0 \therefore 5x(x^2 - 25) = 5x(x - 5)(x + 5) = 0 \therefore x = 0$ or $x = 5$ or $x = -5$

9 $4x^2 = 16 \therefore 4x^2 - 16 = 0 \therefore 4(x - 4)(x + 4) = 0 \therefore x = 4$ or $x = -4$

10 $x^4 - 16 = 0 \therefore (x^2 - 4)(x^2 + 4) = 0 \therefore (x - 2)(x + 2)(x^2 + 4) = 0$
$\therefore x^2 + 4 = 0$ or $x = 2$ or $x = -2$ or no solution

Solving quadratic trinomial equations

Activity 3 Solve quadratic equations by factorising

Learner's Book page 392

Guidelines for implementing this activity

- Revise trinomial factorising and give learners a chance to practise a few of these independently.
- Once the answers have been checked remind learners of the zero product rule again. One side of the equation must be completely factorised while the other side must be equal to zero.
- Make sure that learners do not try and solve a quadratic equation when it is not equal to zero. They must take all the terms to one side of the equation making it equal to zero.
- Activity 3 provides some good practice and learners must do all the questions.

Suggested answers

Remedial and extension

Remedial: Ask learners to compile a checklist that they write on the corner of their page. They can develop a mnemonic to make it quicker to write down. The checklist could be something like:
All terms on LHS
RHS = 0
All terms factorised
Use zero product rule
Solve for x (or the variable) in each factor.
Extension: Check that learners can distinguish between the polynomials that can be factorised and those that cannot, for example:
$16x^2 + 40x + 25 = 0$
$4x^2 + 9 = 0$
Let learners work in pairs and challenge them with a complex example such as:
$(2x^2 + 7x)^2 - 4(2x^2 + 7x) - 45 = 0$.

1 Solve these equations. [make two columns]

a $x^2 - 4x + 3 = 0 \therefore (x - 3)(x - 1) = 0 \therefore x = 3$ or $x = 1$

b $x^2 + 3x - 10 = 0 \therefore (x + 5)(x - 2) = 0 \therefore x = -5$ or $x = 2$

c $x^2 - 7x - 8 = 0 \therefore (x - 8)(x + 1) = 0 \therefore x = 8$ or $x = -1$

d $x^2 + 6x + 5 = 0 \therefore (x + 5)(x + 1) = 0 \therefore x = -5$ or $x = -1$

e $x^2 - 7x + 6 = 0 \therefore (x - 6)(x - 1) = 0 \therefore x = 6$ or $x = 1$

f $y^2 + 4y - 5 = 0 \therefore (y + 5)(y - 1) = 0 \therefore y = -5$ or $y = 1$

g $a^2 + 8a = -15 \therefore a^2 + 8a + 15 = 0 \therefore (a + 3)(a + 1) = 0 \therefore a = -3$ or $a = -1$

h $b^2 - 4b = -4 \therefore b^2 - 4b + 4 = 0 \therefore (b - 2)(b - 2) = 0 \therefore b = 2$

*i $2x^2 + 16x + 30 = 0 \therefore 2(x^2 + 8x + 15) = 0 \therefore 2(x + 5)(x + 3) = 0$
$x = -5$ or $x = -3$

*j $5t^2 - 15t + 10 = 0 \therefore 5(t^2 - 3t + 2) = 0 \therefore 5(t - 2)(t - 1) = 0$
$t = 2$ or $t = 1$

2 Solve these equations. Remember to make the RHS = 0.

a
$x^2 + 5x = -6$
$x^2 + 5x + 6 = 0$
$(x + 3)(x + 2) = 0$
$\therefore x = -3$ or $x = -2$

b
$x^2 - 2x = 15$
$x^2 - 2x - 15 + 0$
$(x - 5)(x + 3) = 0$
$\therefore x = 5 or\ x = -3$

c
$x^2 + x = 12$
$x^2 + x - 12 = 0$
$(x + 4)(x - 3) = 0$
$\therefore x = -4$ or $x = 3$

d
$x^2 + 3x = 18$
$x^2 + 3x - 18 = 0$
$(x + 6)(x - 3) = 0$
$\therefore x = -6$ or $x = 3$

e
$(x - 3)(x + 5) = -7$
$x^2 + 2x - 15 + 7 = 0$
$x^2 + 2x - 8 = 0$
$(x + 4)(x - 2) = 0$
$\therefore x = -4$ or $x = 2$

f
$x(3x + 15) = -12$
$3x^2 + 15x + 12 = 0$
$3(x^2 + 5x + 4) = 0$
$3(x + 4)(x + 1) = 0$
$\therefore x = -4$ or $x = -1$

Describing situations using equations

Activity 4 Make equations to describe situations

Learner's Book page 393

Guidelines for implementing this activity

- Mathematics has developed to make sense of the world around us. At all times remind learners that the skills that they are learning in Algebra will be able to be applied to other Mathematics like Geometry and Trigonometry and also problems in everyday life.
- Revise perimeter and area of shapes.
- Revise the list of words and how they translate to mathematical language done in Chapter 8.
- Work through the examples with learners.
- Remind learners to use brackets when they do substitution in question **4**.
- Let learners do the activity in pairs so that they can discuss their methods.

Remedial and extension

Pair learners according to ability, but some pairs will then need your assistance to write the expressions. It is useful to ask learners to assist their peers, but they must not just do the work for them, only guide them. Write a few examples of perimeter of other polygons on the board for learners to practise.

Suggested answers

1 a $A = (3x - 1)(2x + 1)$ b $A = (2x + 5)(x - 1)$

2 a $P = 2[(2x + 8) + (x + 9)] = 2[2x + 8 + x + 9] = 2(3x + 17)$

b $A = (2x + 8)\ (x + 9)$

3 a $A = \frac{1}{2}(5x - 3)(2x + 4)$

b $A = (4x - 3)(2x+ 1) - (x + 2)(x - 2) = 8x^2 - 2x - 3 - x^2 + 4 = 7x^2 - 2x + 1$

c $A = x^2 + 4x^2 + 9x^2 = 14x^2$

4 a If $x = 2$ then $A = \frac{1}{2}(5(2) - 3)(2(2) + 4) = 28$ square units

b If $x = 3$ then $A = 7x^2 - 2x + 1 = 7(3)^2 - 2(3) + 1 = 58$ square units

c If $x = 5$ then $A = 14(5)^2 = 350$ square units

UNIT 3

Substitution and ordered pairs

Unit overview

Learner's Book page 394
Recommended pacing: 3 hours

This unit focuses on the following:

- Substitution
- Using substitution to generate ordered pairs

Resources: Learner's Book; calculator; exercise book; block paper

Background information

In Term 2 learners used substitution to solve linear expressions. In this chapter, learners will use substitution to solve quadratic equations and create ordered pairs.
Learners were reminded of coordinate pairs in an earlier chapter.

Teaching guidelines

Learners are usually quite comfortable with substitution. Make sure that they use brackets when showing the substitution in their solutions. Once this is done, they can use their calculators to solve the problem. They must make sure that they can enter their information directly into their calculators. Scientific calculators *do* observe the order of operations rules, but they need to be more careful with the non-scientific calculators.
Encourage learners to check their answers by substituting the values back into one of the sides of the original equation and checking to see if it gives the same value as the other side.
Remind learners about the Cartesian plane and coordinate pairs. This is a skill that is used in everyday life, but learners need to be reminded about the order in the pair: first the horizontal direction and then vertical direction or $(x; y)$.

Substitution

Activity 1 Determine the value of equations

Learner's Book page 395

Guidelines for implementing this activity

- Revise a few substitutions in linear expressions.
- Make sure that learners use brackets to show where the substitution has taken place in their solutions.
- Work through Examples 1 and 2 in the Learner's Book.
- Learners do the activity independently.

Suggested answers

1 $y = 2x^2 - 3x + 7$ if $x = -2$ then $y = 21$
2 $y = -3x^2 - 3x - 3$ if $x = 3$ then $y = -39$
3 $y = 6x^2 - 5x$ if $x = 0$ then $y = 0$
4 $y = x^2 - 5x - 17$ if $x = -1$ then $y = -11$

Using substitution to generate ordered pairs

Activity 2 Create ordered pairs

Learner's Book page 396

Guidelines for implementing this activity

- Explain that for each value of x there will be a value of y. This is similar to the input and output values that learners have seen in functions and relationships throughout the Senior Phase thus far.
- The input or x-values are the independent values and the output or y-values are the dependent values.
- Every x-value with its corresponding y-value is called an ordered pair. Ordered pairs can be written in a table or in a bracket of the form $(x; y)$.
- Work through the example with learners. The concept is not new, but the type of equation (quadratic) is new.
- Learners do the activity independently.

Suggested answers

Remedial and extension

Remedial: The exercise provides quite a lot of practice of substitution. Make sure learners are able to write the substitution down correctly using brackets and that they can use their calculators to find the actual value.
Extension: Challenge learners to draw the graphs of the relationships shown in the activity. Ask them to comment on the shapes of the graphs.

1 a $y = x^2 - 1$

x	–2	–1	0	1	2	3
y	3	0	–1	0	3	8

b (–2; 3); (–1; 0); (0; –1); (1; 0); (2; 3); (3; 8)

2 a $y = 3x^2 - 6$

x	–10	–7	–3	3	7	10
y	294	141	21	21	141	294

b (–10; 294); (–7; 141); (–3; 21); (3; 21); (7; 141); (10; 294)

3 a $y = -4x^2 + 1$

x	–5	–1	0	1	3	5
y	–99	–3	1	–3	–35	–99

4 a $y = 3x^2 - 2x + 1$

x	–2	–1	0	1	2	3
y	17	6	1	2	9	22

b (–2; 17); (–1; 6); (0; 1); (1; 2); (2; 9); (3; 22)

5 a $y = -2x^2 - x - 1$

x	–3	–1	0	1	3	5
y	–16	–2	–1	–4	–22	–56

b (–3; –16); (–1; –2); (0; –1); (1; –4); (3; –22); (5; –56)

Chapter 16 Revision

Learners' Book page 397

Encourage learners to review the content covered before attempting the revision activities. The revision activities should be used to assess learners' progress thus far, and to assess where remediation may be required.

Suggested answers

1 a $x = 12$ b $x = 4$ c $x = -2$ d $x = 0$

2 a $x = 2$ or $x = -3$ b $x = -5$ or $x = 5$

c $x = -4$ or $x = -7$ d $x = -4$ or $x = -\frac{1}{2}$

e $x = 1$ or $x = -1$ f $(x + 4)(x + 7) = 4$

$x^2 + 11x + 28 - 4 = 0$

$(x + 3)(x + 8) = 0$

$x = -3$ or $x = -8$

3 a $x = 3$ *or* $x = -3$ b $x = \frac{1}{3}$ or $x = -\frac{1}{3}$ c $x = 0$ or $x = \frac{1}{4}$

d $x = \frac{5}{2}$ or $x = -\frac{5}{2}$ e $x = \frac{9}{7}$ or $x = -\frac{9}{7}$ f $x = \frac{3}{2}$ or $x = -\frac{3}{2}$

4 a $x = 6$ or $x = -2$ b $x = -2$ or $x = -3$

c $x = 4$ or $x = -2$ d $x = 7$ or $x = -2$

*5 $3^{x+4} = 3^x \times 3^4 = 34 \times 81 = 2\ 754$

6 a

x	–3	–1	0	1	3	5
y	–26	0	1	–6	–44	–114

b (–3; –26); (–1; 0); (1; –6); (3; –44); (5; –114)

CHAPTER

17 Graphing

Chapter overview

Learner's Book pages 398 to 421
Recommended pacing: 12 hours

This chapter focuses on the following:

Unit 1: Interpreting graphs — 2 hours
- Revision: Graphs describing situations

Unit 2: Drawing graphs — 8 hours
- The Cartesian plane
- Linear graphs
- Properties of linear graphs
- Investigating the gradient m in $y = mx + c$
- Drawing the graph using the function properties
- Finding the equation of the graph

Unit 3: Graphs showing proportion — 2 hours
- Direct proportion
- Indirect proportion

Chapter 17 Revision — 1 hour 30 minutes

UNIT

1 Interpreting graphs

Unit overview

Learner's Book page 399
Recommended pacing: 2 hours

This unit focuses on the following:
- Interpreting graphs
- Distance/time graphs
- Temperature/time graphs.

Resources: Learner's Book; calculator; exercise book; squared paper/graph papern

Background information

In Grade 8 learners revised the following:
Analysing and interpreting global graphs of problem situations with a special focus on the following trends and features:

- linear or non-linear
- constant, increasing or decreasing.

They extended the focus on features of graphs to include:

- maximum or minimum
- discrete or continuous.

In Grade 9 learners revise the work done in Grade 8 and extend the above with a special focus on the following features of linear graphs:

- x-intercept and y-intercept
- gradient.

Teaching guidelines

Learners are given graphs and asked to interpret the meaning of the graphs. Reading graphs is a very important skill and should be practised often throughout the year. Collect interesting graphs from newspapers and magazines and ask learners questions about the graphs. Once they have identified all the salient points on the graph, they can be asked to "tell the story" of the particular situation.

The unit is divided into interpreting distance-time graphs and temperature-time graphs. These are both everyday concepts that learners understand and should not find too abstract.

Revision: Graphs describing situations

Distance /time graphs; Temperature/time graphs

Activities 1–2 **Interpret distance/time graphs; Interpret temperature/time graphs**

Learner's Book pages 400–401

Guidelines for implementing these activities

- Revise the distance-time equations.
- Discuss what a higher speed means in terms of distance covered in a shorter period of time. What would that do to the steepness of the graph? Once learners have a good concept of the fact that a steeper graph means a faster speed, introduce the concept of returning to the starting point and how that affects the way the graph slopes.
- Work through examples 1 and 2 in the Learner's Book.
- Learners work through Activity 1 independently. Check the answers in class.
- Explain what a *continuous graph* means and how this differs from discrete data. Continuous data often involves measurement.
- Learners do Activity 2 independently.

Remedial and extension

Remedial: Many learners struggle with the distance-time graphs because of not understanding that the distance from the starting point is not always the same as the distance travelled. Use objects to demonstrate the difference between the two. Learners will do the distance-time graphs in Physics in the FET phase.

Extension: Ask learners to write a story that can be graphed.

Suggested answers

Activity 1

1 GH – Dudu left home at point G and cycled at a constant speed for 30 minutes to point H.
JK – Dudu arrived at the shopping mall at J and spent 1 hour 15 minutes shopping.
KL – Dudu cycled back from the shopping mall to home in 15 minutes at a constant speed.

2 Speed $= \frac{\text{distance}}{\text{time}} = \frac{4}{0,5} = 8$ km/h

3 08:00

4 Speed $= \frac{\text{distance}}{\text{time}} = \frac{4}{0,5} = 4$ km/h

5 10:15

6 10 km

7 Speed $= \frac{\text{distance}}{\text{time}} = \frac{10}{1,5} = 6,67$ km/h

Activity 2

1 Thursday 16:00
2 20°
3 15:00 on Friday
4 14°
5 Midnight Friday
6 Constant or varying

UNIT

2 Drawing linear graphs

Unit overview

Learner's Book page 402
Recommended pacing: 8 hours

This unit focuses on the following:

- The Cartesian plane
- Linear graphs
- Properties of linear graphs:
  - The value of m
  - The y-intercept
- Investigating the gradient, m in $y = mx + c$
- The gradient of a line
- Linking the value of m and the value of $\Delta y/\Delta x$
- Drawing the graph using the function properties
- Using the intercepts with the axes to draw the graph
- Using the gradient and the y-intercept to draw the line
- Equations of special lines
- Horizontal lines
- Vertical lines
- Parallel lines
- Perpendicular lines
- Finding the equation of a graph

Resources: Learner's Book; calculator; exercise book; block paper

Background information

In Grades 7 and 8, learners drew global graphs from given descriptions of a problem situation, identifying linear or non-linear, constant, increasing or decreasing graphs.

In Grade 8 learners used tables or ordered pairs to plot points and draw graphs on the Cartesian plane.

In Grade 9, the work done in Grade 8 is revised and extended to include a special focus on:

- drawing linear graphs from given equations
- determining equations from given linear graphs.

Reading and interpreting data was covered in the previous unit on Interpreting Graphs.

Teaching guidelines

The following are important to revise:

- What is a graph?
- Useful graphs and plots
- Plotting the coordinate data
- Describing what the graphs look like
- Reading and interpreting data from graphs (Unit 1).

Graphs are a visual representation of number systems and equations. Graphs can show how different parts of data relate to each other. The relationship can then be translated into a useful equation which can be read and interpreted by other people because Maths is a universal language.

Plotting graphs involves a number of simultaneous skills and learners may find it quite difficult to harness all the requirements initially. The key features when constructing the graphs are:

- choosing the appropriate axis for a variable. Remind learners to check the domain and range (the biggest value – smallest value for each).
- choosing an appropriate scale and labelling the axes with numbers. This becomes easier as learners gain experience, but you can give ten situations where learners don't actually need to draw the graph, just say what the appropriate scale would be for a useful graph that fits on half a page.
- plotting the coordinate pairs. Use games that provide a lot of practice in a short space of time and where learners may be less anxious.

A checklist that you can give learners could look like this:

Title	: What is the graph about?
Titles on axes	: What is the independent value (x-axis)?
	: What is the dependent value (y-axis)?
Labels on axes	: What is the range of values?
	: How big should the increments be?
	: How many spaces are there on your piece of graph paper?
Plot the data	: Use the coordinate pairs, for example, (3;5) where the first number is the x-value and the second number is the y-value or the table of values.
Connect the dots	: Use a ruler if the dots are in a row otherwise a smooth curve.
Label the graph	: y = …. or something similar.

Although some learners struggle with the description of the data on a graph, it is the main reason that we plot graphs. Some of the terms that Grade 9 learners will use are:

- linear, non-linear, (perhaps quadratic)
- increasing/decreasing, constant
- discrete/continuous.

The Cartesian plane

Activity 1 Work on the coordinate plane

Learner's Book page 403

Guidelines for implementing this activity

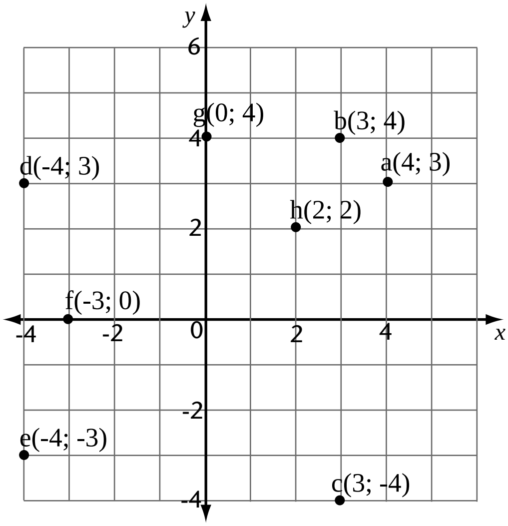
- Make sure that learners know what a Cartesian plane is and can label one. Work through the definitions of the features of the Cartesian plane listed in the Learner's Book with learners.
- This is revision for Grade 9 learners and you should not have to spend too much time on the explanations or the activity.
- Encourage learners to do the activity as quickly as possible.
- Have blocked paper available for learners (there is a grid that you can photocopy in the recources section of this Teacher's Guide).

Remedial and extension

Remedial: Get learners to work in pairs and let each learner give the other learner a few more coordinate pairs to plot and/or to read from a grid.
Extension: Ask learners to choose 20 random points on the Cartesian plane and label them with the most common letters of the alphabet and coordinate pairs. They should then write a secret message and encode it using the coordinate pairs. Challenge learners to do this in five minutes or less!

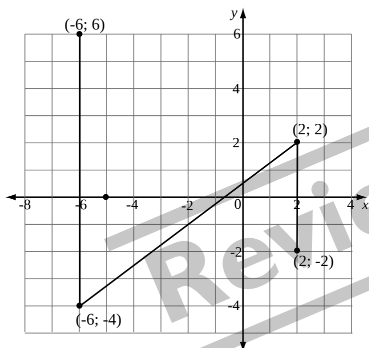
Suggested answers

1 a second
  b third
  c fourth
  d second
  e On the negative part of the y-axis
  f On the negative part of the x-axis
  g On the positive part of the y-axis
  h On the positive part of the x-axis

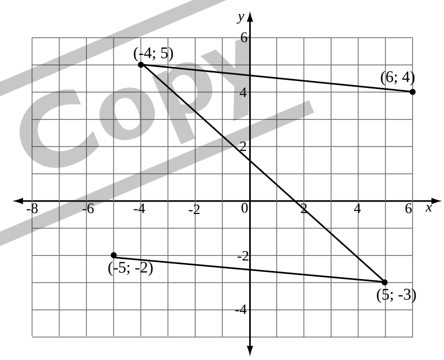
2 

3 A (2; 3); B (6; 4); C (5; 0); D (2; –2); E (0; –2); F (–2; –3); G (–3; 0); H (–3; 1); I (–2; 6); J (0; 7); K (0; 0)

4 a



b



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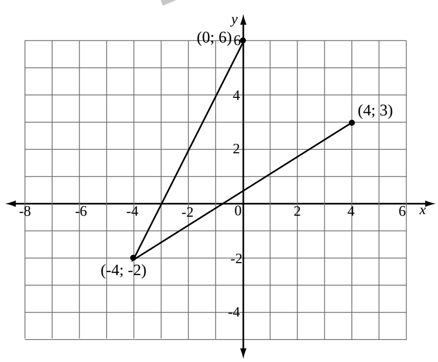

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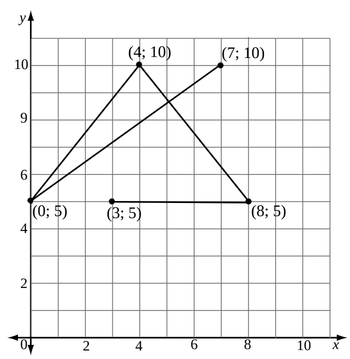


Linear graphs; Properties of linear graphs

Activity 2 Draw linear graphs with *c* constant

Learner's Book page 405

Guidelines for implementing this activity

- Explain the words *domain* and *range*.
- Insist on high standards of drawing and neatness.
- Graphs involve a lot of planning. Learners should get used to asking the questions so that they draw appropriate graphs of about half a page in size. The graph itself should fill up at least two-thirds of the space.
- The activity will take most of a lesson to complete. Some learners may have to complete question **2** for homework.
- It is better for learners to draw questions **1** and **2** on separate axes because it can get quite messy with so many graphs on one set of axes.
- Have learners use different colour pens or pencils to identify the different graphs.
- Learners should find that with c constant as m gets larger, the graph gets steeper. When m is positive, it is an increasing graph (question **1**) and when m is negative, it is a decreasing graph (question **2**).

Remedial and extension

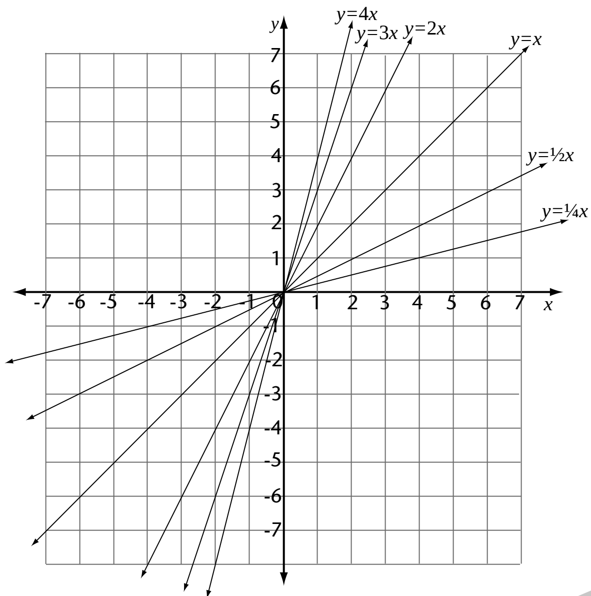
Remedial: It is useful to have a poster in the Maths class of the steps that learners need to follow when drawing a meaningful graph. Have learners work together in pairs to remind each other of all the features that they need to concentrate on. Use the checklist given earlier in the unit or design something similar. It is also useful to have a checklist similar to this when marking graphs.

Extension: If there are computer facilities available with a spread sheet program such as Excel, get learners to draw a few of these graphs on the computer. They could, for example, do question 1 by hand and then do question 2 on a spread sheet program. Drawing graphs on the computer is also a skill that will be used often throughout their lives. A free downloadable graphing program is GeoGebra and learners can use it to plot graphs quickly and effectively.

Suggested answers

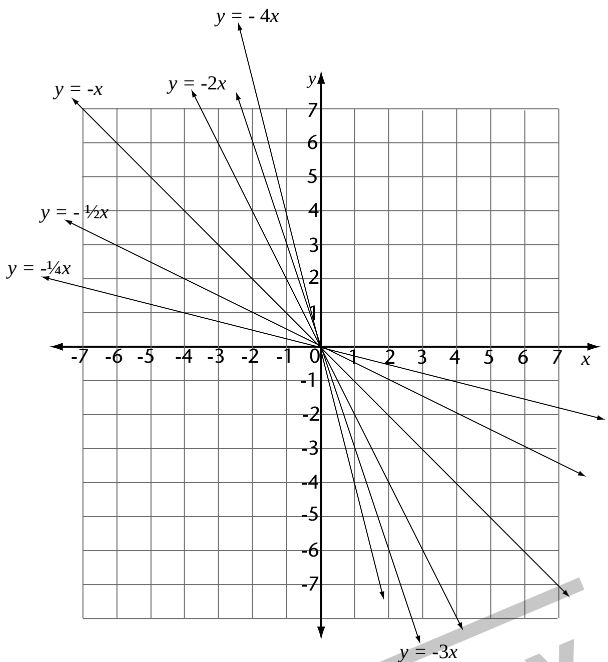
1

	x	–4	–3	–2	–1	0	1	2	3	4
a	$y = x$	–4	–3	–2	–1	0	1	2	3	4
b	$y = 2x$	–8	–6	–4	–2	0	2	4	6	8
c	$y = 3x$	–12	–9	–6	–3	0	3	6	9	12
d	$y = 4x$	–16	–12	–8	–4	0	4	8	12	16
e	$y = \frac{1}{2}$	–2	–1,5	–1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	1,5	2
f	$y = \frac{1}{4}$	–1	$-\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1



2

	x	-4	-3	-2	-1	0	1	2	3	4
a	$y = -x$	4	3	2	1	0	-1	-2	-3	-4
b	$y = -2x$	8	6	4	2	0	-2	-4	-6	-8
c	$y = -3x$	12	9	6	3	0	-3	-6	-9	-12
d	$y = -4x$	16	12	8	4	0	-4	-8	-12	-16
e	$y = -\frac{1}{2}x$	2	$1{,}5$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	$-1{,}5$	-2
f	$y = -\frac{1}{4}$	1	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	0	$-\frac{1}{4}$	$-\frac{1}{2}$	$-\frac{3}{4}$	-1



Activity 3 Draw linear graphs with *m* constant

Learner's Book page 406

Guidelines for implementing this activity

- Follow the hints and tips given for Activity 2.
- Learners should find that with m constant as c gets larger, the graph moves upwards. They should notice that the graphs in question **1** and question **2** are parallel.
- This activity will also take learners most of a lesson and possibly homework time to complete.

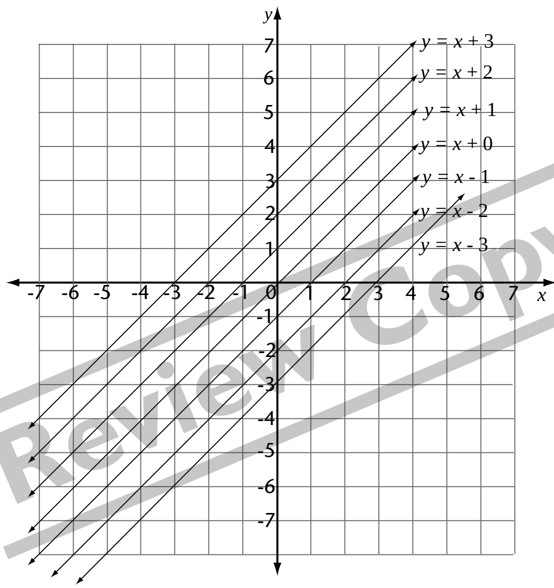
Remedial and extension

Remedial: Continued practice is key. Encourage learners to try and work as quickly as possible without compromising accuracy and neatness. Use the checklist given earlier in the unit or design something similar. It is also useful to have a checklist similar to this when marking graphs.
Extension: Ask learners to investigate the table functions on their scientific calculators if they have one. This is a useful tool and they will use it often in FET Mathematics.

Suggested answers

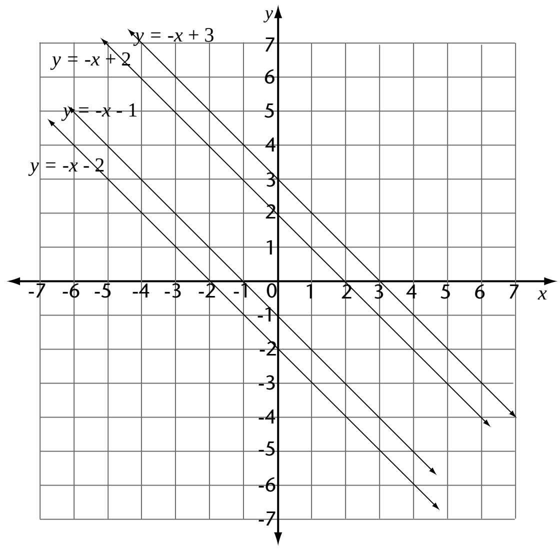
1

	x	–4	–3	–2	–1	0	1	2	3	4	y-int	x-int
a	$y = x + 3$	–1	0	1	2	3	4	5	6	7	3	–3
b	$y = x + 2$	–2	–1	0	1	2	3	4	5	6	2	–2
c	$y = x + 1$	–3	–2	–1	0	1	2	3	4	5	1	–1
d	$y = x + 0$	–4	–3	–2	–1	0	1	2	3	4	0	0
e	$y = x – 1$	–5	–4	–3	–2	–1	0	1	2	3	–1	1
f	$y = x – 2$	–6	–5	–4	–3	–2	–1	0	1	2	–2	2
g	$y = x – 3$	–7	–6	–5	–4	–3	–2	–1	0	1	–3	3



2

	x	–4	–3	–2	–1	0	1	2	3	4	y-int	x-int
a	$y = –x + 3$	7	6	5	4	3	2	1	0	1	3	3
b	$y = –x + 2$	6	5	4	3	2	1	0	1	2	2	2
c	$y =–x – 1$	3	2	1	0	–1	–2	–3	–4	–5	–1	–1
d	$y = –x – 2$	2	1	0	–1	–2	–3	–4	–5	–6	–2	–2



3 Last two columns of tables in questions **1** and **2** above.

Activity 4 Features of linear graphs

Learner's Book page 406

Guidelines for implementing this activity

- This lesson focuses on the x- and y-intercepts and the gradients of the graph.
- These features are very important in describing graphs as well as drawing them.
- Graphically the x- and y-intercepts are simply the place where the graph cuts the x-axis and y-axis respectively.
- Algebraically, the x-intercept is a point on the graph where y is zero and the coordinate pair will be of the form (x; 0)
- The y-intercept is a point on the graph where x is zero and the coordinate pair will be of the form (0; y). It is the c in the standard form of the linear equation $y = mx + c$.
- The gradient or slope is the number m in the standard form of the linear equation $y = mx + c$. It is a measure of how steep a graph is.
- Make sure that learners understand each term clearly.
- Do one or two examples on the board.
- Make sure that learners can write the coordinate pairs of the x- and y-intercepts.
-

Remedial and extension

Remedial: Learners often struggle with this. Do Example 3 first with them reminding them that they have been writing equations for so many different situations. Also give them a numeric or geometric pattern and ask them to find the formula. Once they have established that this is just another way of finding the equation, it may not be that foreign. If they still prefer the table method, they can plot some points from the graph into a table, but they should see that it is going to be very time consuming.
Extension: Ask learners to investigate what it means if two graphs cut each other. What does the point of intersection mean in terms of solutions?

Remedial and extension

Remedial: Make sure that learners can identify these three very important features of linear graphs. Give exercises where learners just identify one at a time. Calculating the *x*-intercept involves substituting the value of 0 for the *y*-value and solving for *x*.
Extension: Ask learners to complete the activity as quickly as possible and to plot the equations in question 1 on a system of axes. Encourage them to use electronic media if these are available.

Suggested answers

1

		y-intercept	x-intercept	gradient	increasing/decreasing
a	$y = x - 4$	−4	4	1	increasing
b	$y = 4x - 3$	−3	$\frac{3}{4}$	4	increasing
c	$y = 3x + 5$	5	$-\frac{5}{3}$	3	increasing
d	$y = -2x + 1$	1	$\frac{1}{2}$	−2	decreasing

2 **a** (0; −4) **b** (0; −34) **c** (0; 0) **d** (0; 1)

3 **a** m = 1 Increasing because the gradient is positive.
b m = −1 Decreasing because the gradient is negative.
c m = −3 Decreasing because the gradient is negative.
d m = −2 Decreasing because the gradient is negative.
e m = 2 Increasing because the gradient is positive.
f m = −12 Decreasing because the gradient is negative.

Investigating the gradient, *m* in *y* = *mx* + *c*

Vertical and horizontal change between two points; The gradient of a line

Activity 5 Investigate the meaning of 'gradient'

Learner's Book page 408

Guidelines for implementing this activity

- Introduce the topic of gradients (or slope) and what it means in real life. Then remind learners what they have discovered in Activity 4 about the signs of m. When $m > 0$ (positive), the graph is increasing and when $m < 0$ (negative), the graph is decreasing.
- The activity links the value of m in the equation to the value of $\frac{\Delta y}{\Delta x}$ which is the ratio of the graphical change in the *x*- and *y*-values.
- Activity 5 can take most of a lesson to complete. Make sure that learners use the correct notation.

Suggested answers

1 Graph A: $\Delta y(PQ) = 2$
Graph B: $\Delta y(PQ) = 2$
Graph C: $\Delta y(PQ) = -3$
Graph D: $\Delta y(PQ) = -2$

2 Graph A: $\Delta x(PQ) = 1$
Graph B: $\Delta x(PQ) = 4$
Graph C: $\Delta x(PQ) = 1$
Graph D: $\Delta x(PQ) = 2$

3 Graph A: $= \frac{\Delta y}{\Delta x} = 2$
Graph B: $= \frac{\Delta y}{\Delta x} = \frac{2}{4} = \frac{1}{2}$
Graph C: $= \frac{\Delta y}{\Delta x} = -3$
Graph D: $= \frac{\Delta y}{\Delta x} = -1$

4 Graph A: $\Delta y(QR) = 6$
Graph B: $\Delta y(QR) = 1$
Graph C: $\Delta y(QR) = -3$
Graph D: $\Delta y(QR) = -2$
Graph A: $\Delta x(QR) = 3$
Graph B: $\Delta x(QR) = 2$
Graph C: $\Delta x(QR) = 1$
Graph D: $\Delta x(QR) = 2$
Graph A: $\frac{\Delta y}{\Delta x} = \frac{6}{3} = 2$
Graph B: $\frac{\Delta y}{\Delta x} = \frac{1}{2}$
Graph C: $\frac{\Delta y}{\Delta x} = \frac{-3}{1} = -3$
Graph D: $\frac{\Delta y}{\Delta x} = \frac{-2}{2} = -1$

5 Graph A: Δy(PR) = 8
Graph B: Δy(PR) = 3
Graph C: Δy(PR) = –6
Graph D: Δy(PR) = –4
Graph A: Δx(PR) = 4
Graph B: Δx(PR) = 6
Graph C: Δx(PR) = 2
Graph D: Δx(PR) = 4
Graph A: $= \frac{8}{4} = 2$
Graph B: $= \frac{3}{6} = \frac{1}{2}$
Graph C: $= \frac{-6}{2} = -3$
Graph D: $= \frac{-4}{4} = -1$

6 Graph A: $y = 2x - 1 \therefore m = 2 = \frac{\Delta y}{\Delta x}$
Graph B: $y = x + 1 \therefore m = 1 = \frac{\Delta y}{\Delta x}$
Graph C: $y = -3x \therefore m = -3 = \frac{\Delta y}{\Delta x}$
Graph D: $y = -x + 2 \therefore m = -1 = \frac{\Delta y}{\Delta x}$

Drawing the graph using the function properties

Using the intercepts with the axes to draw the graph; Using the gradient and the *y*-intercept to draw the line

Activity 6 **Draw graphs using methods other than tables**

Learner's Book page 410

Guidelines for implementing this activity

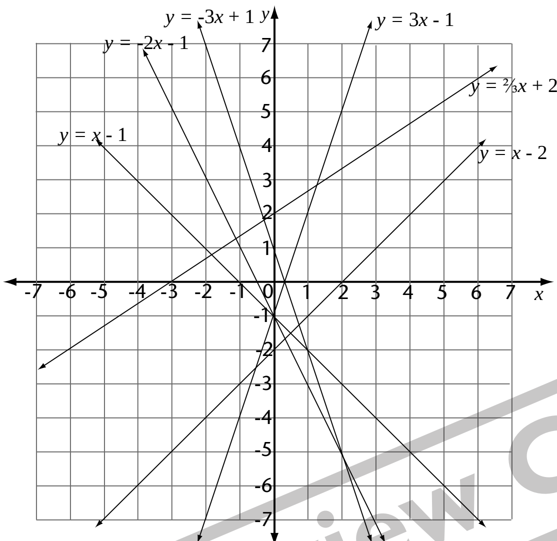
- Two methods of drawing the graph apart from tables and using coordinate pairs are shown in the Learner's Book.
- Both methods depend on the fact that you only need two points to draw a straight line.
- The first one uses the *x*- and *y*-intercepts which learners worked with earlier in this unit. This method may require calculation of the *x*-intercept if the equation is given.
- The second method uses the gradient and *y*-intercepts as the two points necessary to draw the graph. This method is really useful when the equation is given in the standard form because both values are directly available in the equation without further calculation.
- This is very important groundwork for FET Mathematics.
- In this activity, learners are asked to use a specific method in each case, but learners should practise both methods and decide which one they prefer.

Remedial and extension

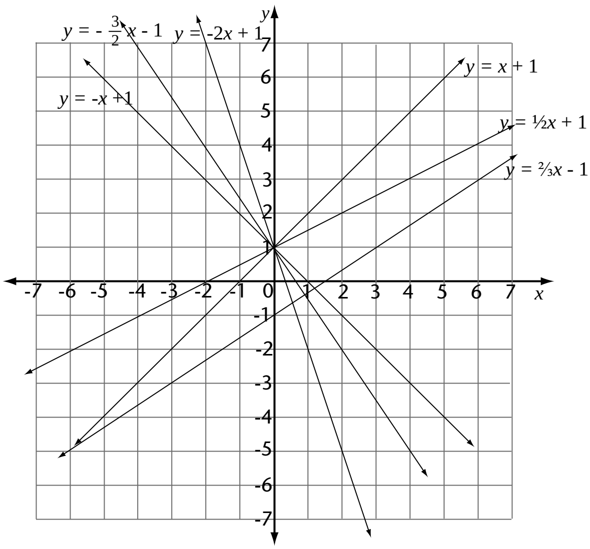
Remedial: Give more examples and making sure learners are coming to terms with the features. They can draw quite a few graphs on one set of axes, but it can get messy and confusing. Make sure there is spare grid paper or ask them to draw their own grids in their books (although this will waste potential practising time).
Extension: Ask learners to assist the learners who are struggling by listening to them as they work through the steps of drawing the graphs using these two methods. They should correct any conceptual errors if there are any and give tips on how to draw the graphs more quickly and effectively.

Suggested answers

1 a $y = -3x + 1$ y-intercept: (0; 1); x-intercept: ($\frac{1}{3}$; 0)
b $y = -2x - 1$ y-intercept: (0; –1); x-intercept: ($-\frac{1}{2}$; 0)
c $y = \frac{2}{3}x + 2$ y-intercept: (0; 2); x-intercept: (–3; 0)
d $y = x - 2$ y-intercept: (0; –2); x-intercept: (2; 0)
e $y = 3x - 1$ y-intercept: (0; –1); x-intercept: ($\frac{1}{3}$; 0)
f $y = -x - 1$ y-intercept: (0; –1); x-intercept: (–1; 0)



2 a $y = \frac{2}{3}x - 1$ $m = \frac{2}{3}$; y-intercept: (0; –1); x-intercept: ($\frac{3}{2}$; 0)
b $y = \frac{1}{2}x + 1$ $m = \frac{1}{2}$; y-intercept: (0; 1); x-intercept: (–2; 0)
c $y = x + 1$ $m = 1$; y-intercept: (0; 1); x-intercept: (–1; 0)
d $y = -x + 1$ $m = -1$; y-intercept: (0; 1); x-intercept: (1; 0)
e $y = -2x + 1$ $m = -2$; y-intercept: (0; 1); x-intercept: ($\frac{1}{2}$; 0)
f $y = -\frac{3}{2}x + 1$ $m = -\frac{3}{2}$; y-intercept: (0; 1); x-intercept: ($\frac{2}{3}$; 0)



Equations of special lines

Activities 7–8 **Parallel lines; Perpendicular lines**

Learner's Book pages 411–412

Guidelines for implementing these activities

- This is also a very important lesson and learners should be asked to give the equations for various horizontal and vertical lines often. These lines are the asymptotes in exponential, logarithmic and hyperbolic graphs that learners will do in FET Maths.
- Draw a system of axes on the board and draw several vertical lines and ask learners to write down the equations. (Include the y-axis.)
- Do the same for horizontal lines, including the x-axis this time.
- Activity 7 and Activity 8 investigate the property of lines that make them parallel ($m_1 = m_2$) and what makes them perpendicular ($m_1 \times m_2 = 1$).

Remedial and extension

Remedial: Drill: If the graph is vertical, the equation is x = something; if the graph is horizontal, the equation is y = something.

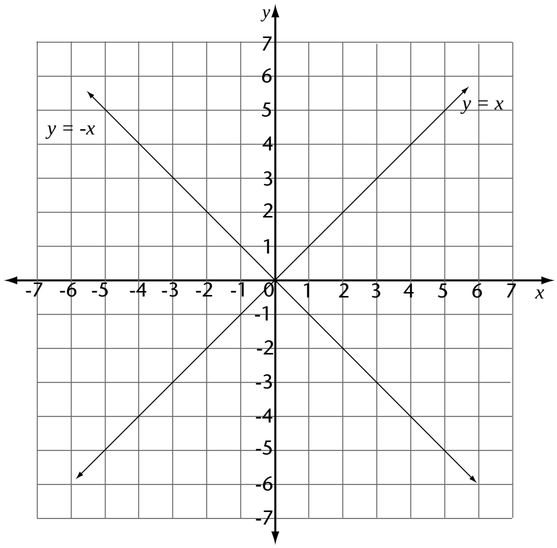
Suggested answers

Activity 7

1. $m(\text{AB}) = 2$; $m(\text{CD}) = 2$; $m(\text{EF}) = 2$
2. y-intercept (AB) = 2; y-intercept (CD) = –1; y-intercept (EF) = –2
3. The gradients of all the lines are equal.
4. The y-intercepts of all the lines differ.
5. If lines have the same gradient they are parallel.

Activity 8

1 a



b $y = x$: Gradient $m = 1$; $y = -x$: Gradient $m = -1$

c 90°   d $1 \times -1 = -1$

2 a $m(\text{AC}) = \frac{1}{2}$ b $m(\text{PR}) = -2$ c 90°   d $\frac{1}{2} \times -2 = -1$

e If two lines are perpendicular the product of their gradients is equal to –1.

Finding the equation of a graph

Activity 9 Finding equations of graphs

Learner's Book page 414

Guidelines for implementing this activity

- Reading information from graphs and finding equations is also very important groundwork for FET Mathematics.
- Work through examples 1 and 2 which use the y-intercepts and gradient to find the equation.
- Example 3 uses a table to find the equation and learners should be quite familiar with this. They have practised it in Functions and Relationships, Patterns, Algebraic expressions and Algebraic equations.
- Although the activity appears to be long, there are no graphs to be drawn, so learners should easily complete it in a lesson.

Suggested answers

1 a $y = 2x + 2$ b $y = -3x - 1$ c $y = -2x + 5$

2 a $y = \frac{5}{8}x + 2{,}5$ b $y = -1{,}5x + 3{,}5$ c $y = 2x + 2$

3 A $y = 1{,}5x + 3$ B $y = 1{,}5x - 0{,}5$ C $y = 1{,}5x - 6$

4 a Graphs of the type: $y = 3x - c$ where c can be any real number.

b Graphs of the type: $y = -x + c$ where c can be any real number.

5 a $y = 2x - 1$ b $y = x$ c $y = -x$

6 a $y = x - 1$ b $y = x + 2$ c $y = \frac{1}{2}x + 1$

UNIT

3 Graphs showing proportion

Unit overview

Learner's Book page 416
Recommended pacing: 2 hours

This unit focuses on the following:
- Direct proportion
- Indirect proportion.

Resources: Learner's Book; calculator; exercise book; block paper

Background information

Learners studied direct and indirect proportion in Chapter 1 Whole numbers. It is important to look at these two concepts again because of their importance in other science subjects.

Teaching guidelines

- Direct and indirect proportionality is quite important in Physical Sciences when the formulae are developed from data collected. This unit will also give learners more practise in the skills learnt in Unit 2.
- When two variables are directly proportional to each other, the relationship that they form will have an equation of the form: $y = kx$
- Where k is a constant and x and y are variables. (Relate the k here to the m used in the standard format of the linear equation).
- When we draw the graph of this equation, we find that it is linear. On the graph we find that when y is increased, it forces x to increase too. Likewise when y decreases, x decreases.
- When two variables are indirectly proportional to each other, the equation is: $xy = k$ which can be written as $y = \frac{k}{x}$. In this form it is easier to see that as y increases, x decreases and vice versa. When the graph is drawn it has a curve with two asymptotes: the x- and y-axes.

Direct proportion; Indirect proportion

Activities 1–2 Draw direct proportion graphs ; Indirectly proportional graphs

Learner's Book page 417–420

Guidelines for implementing these activities

- Explain the term *directly proportional* or *direct proportion*.
- Work through the example in the Learner's Book with learners.
- Activity 1 will take most of a lesson and may leave some homework for some learners.
- Work through the next example in the Learner's Book. Learners may find this more challenging, but once they pick up the pattern it will become easier.
- The first question of Activity 2 says that the relationship is indirectly proportional so learners know that it will be of the format $xy = k$.
- This activity should not take learners very long to complete.
- They should work on the chapter revision when they have completed the activity.

Remedial and extension

Remedial: Give learners time to work through the examples again and assist them where necessary. They should also work through the revision exercise and raise any issues that they still have with graphing.
Extension: When learners complete this activity, they should move to the chapter revision and complete it as quickly as possible. Encourage learners to take the time that they have left at the end to check their answers.

Suggested answers

Activity 1

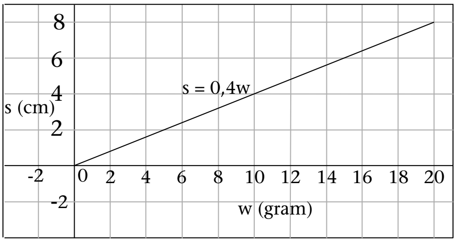
1 **a**

w (gram)	0	1	2	3	4	5	6	7	8	10	20
s (cm)	0	0,4	0,8	1,2	1,6	2,0	2,4	2,8	3,2	4	8

b w is the independent variable because the stretch of the spring is dependent on the weight.

c $m = \frac{\Delta s}{\Delta w} = \frac{0{,}8 - 0{,}4}{2 - 1}$

d



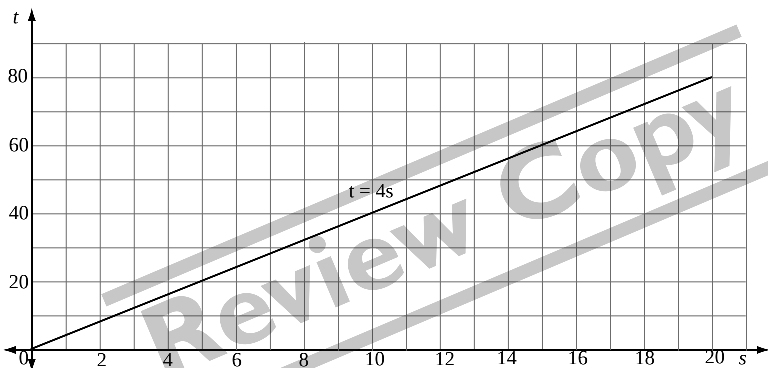
2 **a** $t = 4s$

s	1	4	9	12	18
t	4	16	36	48	72

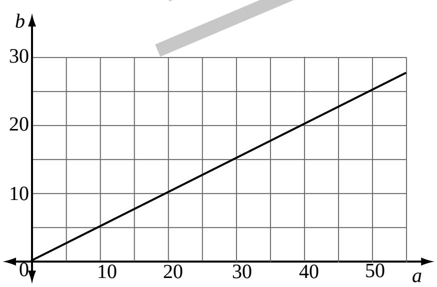
b $b = \frac{1}{2}a$

a	12	14	15	22	28	46	50
b	6	7	7,5	11	14	23	25

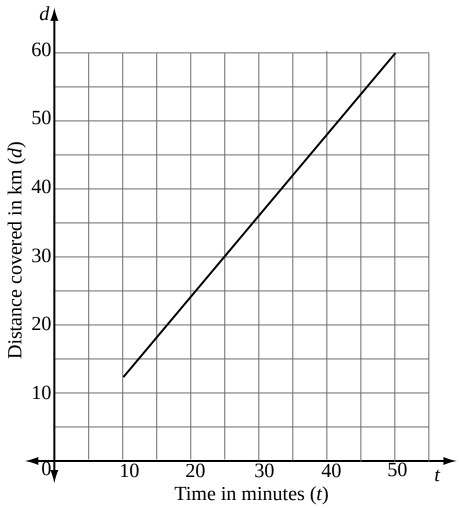
3 **a**



b



4 **a – c**



d Gradient $= \frac{\Delta d}{\Delta t} = \frac{18 - 12}{15 - 10} = \frac{6}{5}$ km/min $= \frac{6 \times 60}{5} = 72$ km/h

e $d = 72t$ in km/h or $d = \frac{6}{5}\,t$ in km/min

f A = 12,5 km

g B = 45,8 minutes but values between 44 and 47 acceptable.

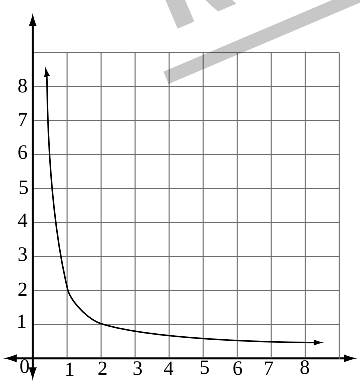
Activity 2

1 $xy = k \therefore k = 6 \times \frac{1}{3} = 2$

2

x	6	4	2	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$
y	$\frac{1}{3}$	$\frac{1}{2}$	1	3	4	6	8

3



4 $y = 1$

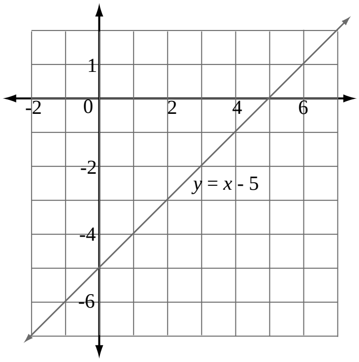
Chapter 17 Revision

Learner's Book page 421

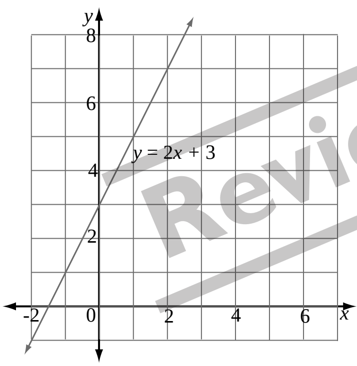
Encourage learners to review the content covered before attempting the revision activities. The revision activities should be used to assess learners' progress thus far, and to assess where remediation may be required.

Suggested answers

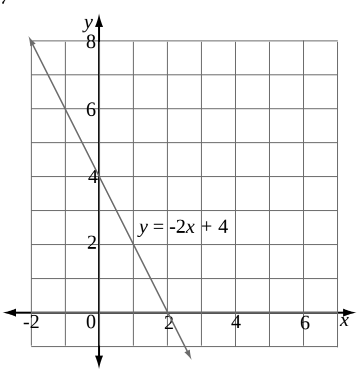
1 a $y = x - 5$



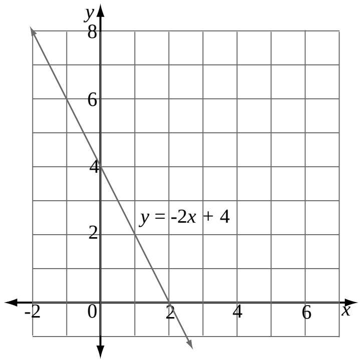
b $y = 2x + 3$



c $y = -2x + 4$



d $y = -2x + 4$

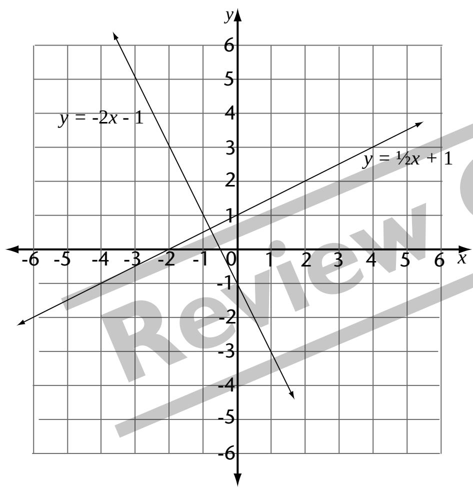


2 **a** $y = \frac{1}{2}x + 5$ **b** $y = 7$ **c** $x = 4$

d $y = -x$ **e** $y = -x + 3$ **f** $y = x$

3 **a** $y = \frac{1}{2}x + 1$ $m = \frac{1}{2}$; y-intercept = 1

b $y = -2x - 1$ y-intercept = –1; x-intercept = $-\frac{1}{2}$



c They are perpendicular. The product of their gradients = –1.

4 **a** $y = -\frac{3}{4}x$ **b** $y = \frac{3}{4}x + 1$ **c** $y = \frac{3}{4}x + 5$ **d** $y = 0$

5 **a** $s_{AB} = \frac{3}{1} = 3$ km/h

b $s_{BC} = \frac{3}{0,5} = 6$ km/h

c 30 minutes or half an hour

d $s_{DE} = \frac{6}{\frac{3}{4}} = \frac{6 \times 4}{3} = 8$ km/h

e $2\frac{3}{4}$ hours after he left home.

CHAPTER

18 Surface area and volume of 3D objects

Chapter overview

Learner's Book pages 423 to 443
Recommended pacing: 5 hours

This chapter focuses on the following:

Unit 1: Surface area of 3D objects 1,5 hours
- Revising concepts and skills to be used
- 3D objects, their nets and types of faces
- Finding the surface area of 3D objects

Unit 2: Volume and capacity of 3D objects 1,5 hours
- Revising conversions between cubic units and litres
- Finding the volume and capacity of 3D objects

Unit 3: The effect on surface area or volume if dimensions are doubled 2 hours
- Changing the dimensions of a cube
- Problem-solving

POA Investigation: Changing dimensions
PoA Project 1: Volume and surface area
Chapter 18 revision

UNIT

1 Surface area of 3D objects

Unit overview

Learner's Book page 424
Recommended pacing: 1,5 hours

This unit focuses on the following:
- Types of 3D objects, their nets and types of faces
- Surface area of 3D for example, dice, a rectangular prism, right triangular prism, and a cylinder, wall charts with nets of 3D objects

Resources: Learner's Book; exercise book; mathematical set; pen

Background information

In Grades 7 and 8 learners did the following:

- Nets of 3D objects such as cubes, rectangular prisms and triangular prisms
- Surface area of 3D objects such as cubes, rectangular prisms and triangular prisms.

The surface area of a cylinder is new in Grade 9.

Teaching guidelines

The content covered in this unit is largely revision of the knowledge and skills covered in Grades 7 and 8. Should learners be able to work through examples and activities on their own, they should be encouraged to do so. Time can then be spent with learners who require individual attention.
Nets of objects provide a useful way of deriving formulae for calculating surface area of 3D objects. Nets can then be folded to form the actual objects when the volume concept is introduced.
Conversion between length, area and volume units is another important aspect of this chapter. Learners should be able to convert between Standard International (SI) units as required.

Revising concepts and skills to be used

Activity 1 **Revise 2D shapes and formulae for their area**

Learner's Book page 424

Guidelines for implementing this activity

- Learners should be familiar with the vocabulary revised in this activity. Ask learners to answer the questions on their own and then to discuss the answers as a class.
- Let learners draw the shapes, especially the height of a triangle, on the board (to show that they understand that the height of a triangle is measured from any of three bases to a vertex.
- When drawing the shapes on the board, let learners also practise explaining their properties and use the markings learnt throughout the Senior Phase thus far to show these properties, for example, showing that all the angles are right angles using the small square; using the marking to indicate parallel sides; and so on.

Remedial and extension

Learners may need some extra explanation for words they do not recall well. Pair up learners who need revision with learners who know the words well and ask them to test each other.

Suggested answers

1 a Square: quadrilateral with all sides and angles equal
  b Rectangle: quadrilateral with four right angles and two pairs of opposite sides equal and parallel
  c Triangle: polygon with three sides and interior angles that add up to 180°.
  d Circle: 2D shape that is completely round.

e Height of a triangle: the length of the perpendicular line from any base to the vertex on the opposite side.

2 a Area of a square: s^2 (the length of one side, squared)
b Area of a rectangle: $l \times b$ (length multiplied by breadth)
c Area of a triangle: $\frac{1}{2}\ b \times h$ (half the base multiplied by the height)
d Area of a circle: πr^2 (the radius squared multiplied by Pi)

3 a cm^2 times 100
b mm^2 divide by 100
c m^2 times 10 000
d cm^2 divide by 10 000
e 1 km^2 times 1 000 000
f 1 m^2 divide by 1 000 000

4 a 50 000 cm^2
b 10 000 000 m^2
c 1 cm^2
d 1 m^2
e 50 000 mm^2
f 0,01 km^2

3D objects, their nets and types of faces

Activity 2 Analyse faces of and define 3D objects

Learner's Book page 425

Guidelines for implementing this activity

- Learners should already be familiar with cubes, triangular prisms and rectangular prisms. Bring examples of these 3D objects together with their nets to the classroom or ask learners ahead of time to make these and bring them to the class.
- Allow learners to explore the relationship between a 3D object and its net using these objects as follows:
  - Count the numbers of faces.
  - Identify the shape of each face.
  - Identify the faces that are the bases. Emphasise the fact that each 3D object has two bases. The top face is also a base face as the top and the bottom of a 3D object are easily interchangeable (and are congruent). In addition to the two base faces, each 3D object has lateral faces. (The lateral faces are the faces around the base on the sides.)
- The first three objects in the Learner's Book are prisms.
- Remind learners that a prism is a geometric shape whose bases are congruent polygons that are joined by rectangular lateral faces. The number of lateral faces of a prism is the same as the number of sides of its base faces.
- The only shape that is not a prism is the cylinder.
- Learners find it difficult to understand that the lateral faces of a prism are rectangles. The use of nets will help clarify this difficulty.
- Also use nets to show the relationship between the circumference of the base of a cylinder and the length of its rectangular lateral surface.

Remedial and extension

Physically handling objects and deconstructing them to show their nets, then re-constructing them to form the 3D objects are an invaluable learning experience that bridges the divide between the concrete and the abstract in this context. However, the aim is to progress learners towards an abstract understanding and their reliance on concrete materials should therefore be managed carefully in order to help them achieve this.

Suggested answers

1 a 6 faces
  b 2 square faces and four rectangular faces
  c The base and the top face are congruent squares
  d The lateral faces are congruent rectangles
  e Square-based prism
2 a six faces
  b two small rectangles and four big rectangles
  c The base and top face are congruent rectangles
  d The lateral faces are two sets of congruent rectangles
  e Rectangular prism
3 a 5 faces
  b 2 triangular faces and three rectangular faces.
  c The base and top faces are congruent triangles
  d The lateral faces are rectangles
  e Triangular prism
4 a 2 faces and a curved surface
  b 2 circular faces and a curved surface
  c The base and the top face are congruent circles
  d The lateral surface when drawn as a net is a rectangle
  e Cylinder

Finding the surface area of 3D objects

Surface area of a cube; Surface area of a square-based prism; Surface area of a rectangular prism; Surface area of a triangular prism; Surface area of a cylinder

Activity 3 **Calculate the surface area of 3D objects**

Learner's Book page 430

Guidelines for implementing this activity

- Use nets and ask learners to identify the shape of the base faces and lateral faces for each 3D object.
- Revise the formulae for calculating the area of a square and the area of a rectangle.
- Ask key questions: If I want to calculate the area of the entire net, how would I go about it? What does *surface area* mean?
- Lead learners to the understanding that surface area means the total area of all the faces of the net combined. Once learners grasp this, they can start calculating the surface area. **Note:** Learners have done surface area in previous grades and should be familiar with this

process. However, now is the time to crystallise their understanding.

- Let learners work through each example on their own or in pairs or small groups if necessary. Let them calculate the area of one face at a time at first.
- Note that learners are not expected to be able to derive surface area formulae for tests and examinations. However, the process of how formulae are derived is a very important mathematical skill that they need to learn.
- Refer learners to the summary to check on the formulae if they need to, but encourage them to think of the formulae as far as possible.

Remedial and extension

Let learners work in mixed-ability groups when doing the examples (not the activity). Walk around the classroom to monitor their progress and address difficulties they may have. This should be an adequate precursor to doing the activity on their own and if they still find certain areas challenging, re-arrange the groups appropriately into smaller working groups and work through one or two questions with them. Beyond this, learners should be pointed to the examples if they need to, but they must attempt at least half of the questions on their own.

Suggested answers

1 a Surface area (cylinder)
$= 2\pi r(r + h)$
$= 2 \times 3{,}14 \times 10 \text{ cm } (10 \text{ cm} + 30 \text{ cm})$
$= 2\ 512 \text{ cm}^2$
Note: $r = \frac{1}{2}d$ (diameter) and 300 mm = 30 cm

b Surface area (rectangular prism)
$= 2(55 \text{ mm} \times 45 \text{ mm}) + 2(55 \text{ mm} \times 36 \text{ mm}) + 2(45\text{mm} \times 36 \text{ mm})$
$= 4\ 950 \text{ mm}^2 + 3\ 960 \text{ mm}^2 + 3\ 240 \text{ mm}^2$
$= 12\ 150 \text{ mm}^2$ or $121{,}5\text{cm}^2$

2 The two cubes form a square-based prism.
Surface area of square-based prism
$= 2 \times$ Base area $+ 4 \times$ Area of one lateral face
$= 2s^2 + 4(s \times h)$
$= 2(14 \text{ mm})^2 + 4(14 \text{ mm} \times 28 \text{ mm})$
$= 2(196 \text{ mm}^2) + 4(392 \text{ mm})$
$= 1\ 960 \text{ mm}^2$

3 a Surface area of cube $= 6s^2$
$= 6(7{,}1 \text{ cm})^2$
$= 6(50{,}41 \text{ cm})^2$
$= 302{,}46 \text{ cm}^2 \approx (302 \text{ cm}^2)$

b Hypotenuse of the base of one of the triangular prisms
$= \sqrt{(71 \text{ mm})^2 + (71 \text{ mm})^2} = \sqrt{10\,082} \approx 100$ mm (rounded off)

c Surface area of a triangular prism
$= 2 \times$ Base area + Sum of areas of three lateral faces
$= 2 \times \frac{1}{2}(b \times h) + s^1 \times h + s^2 \times h + s^3 \times h$
$= 2 \times \frac{1}{2}(7{,}1 \text{ cm} \times 7{,}1 \text{ cm}) + (7{,}1 \text{ cm} \times 7{,}1 \text{ cm}) + (7{,}1 \text{ cm} \times 7{,}1 \text{ cm}) + (7{,}1 \text{ cm} \times 10 \text{ cm})$
$= 2 \times \frac{1}{2}(50{,}41) + (50{,}41) + (50{,}41) + 71 \text{ cm}$
$= 50{,}41 + 50{,}41 + 50{,}41 + 71 \text{ cm} = 222{,}23 \text{ cm}^2$

4 four times

UNIT

2 Volume and capacity of 3D objects

Unit overview

Learner's Book page 432
Recommended pacing: 1,5 hours

This unit focuses on the following:
- Revising conversions between cubic units and litres
- Finding the volume and (for example, a dice), a rectangular prism, right triangular prism, and a cylinder, wall charts with nets of 3D objects.

Resources: Learner's Book; exercise book; mathematical set; pen

Background information

In Grades 7 and 8 learners did the following:
- volume and capacity of 3D objects such as cubes, rectangular prisms and triangular prisms.

The surface area of a cylinder is new in Grade 9.

Teaching guidelines

The content covered in this unit is largely revision of the knowledge and skills covered in Grades 7 and 8. Should learners be able to work through examples and activities on their own, they should be encouraged to do so. Time can then be spent with learners who require individual attention.

Revising conversions between cubic units and litres

Activity 1 Convert between cubic units and litres

Learner's Book page 432

Guidelines for implementing this activity

Revise the conversions in the introduction to this unit, which have been done in previous grades. Test learners on these conversions at the beginning of each lesson.

Suggested answers

1 a 0,1 ml; 2 ml; 30 ml
  b 4 ml; 50 ml; 600 ml
  c 7 000 000 ml; 80 000 000 ml; 900 000 000 ml
2 a 100 cm^3; 2 000 cm^3; 30 000 cm^3
  b 4 000 cm^3; 50 000 cm^3; 600 000 cm^3
  c 7 000 000 cm^3; 80 000 000 cm^3; 900 000 000 cm^3
3 a 0,5 ℓ
  b 500 000 m^3
  c 0,5 kl
  d 500 000 m^3

Finding the volume and capacity of 3D objects

Volume of a cube; Volume of a rectangular prism; Volume of a triangular prism; Volume of a cylinder

Activity 2 Volume of 3D objects

Learner's Book page 435

Guidelines for implementing this activity

- The difference between the surface area and volume concepts is similar to that between perimeter and area. The area concept has been dealt with in a previous chapter. Build from this concept by emphasising the difference in measurement units for these two quantities.
- Explain that *volume* is a 3D quantity while *area* is a 2D quantity.
- Common units for measuring volume such as millilitres and litres should also be revised together with the introduction of their relationship with cubic length units arising from volume formulae.
- Conversion between SI units for measuring volume should also be given a lot of attention. The most important conversion factors are given in the Learner's Book.
- As in the case of surface area, learners are not expected to derive volume formulae for examination purposes, but will benefit a lot from understanding how the formulae are arrived at. Quantities in formulae and their measurement units provide a good platform for explaining volume units.
- An example showing the application of the volume area is provided for each object. Go through the examples with learners making sure that they understand each step, and particularly the conversions.

Suggested answers

1 a We first convert cm to mm, since we required to give the answer in mm^2.
Base length = 80 mm, height = 200 mm
[therefore]
Surface area $= 2(80 \text{ mm})^2 + 4(80 \text{ mm} \times 200 \text{ mm})$
$= 12\,800 \text{ mm}^2 + 64\,000 \text{ mm}^2$
$= 76\,800 \text{ mm}^2$
B Volume (prism) $= 8 \text{ cm} \times 8 \text{ cm} \times 20 \text{ cm}$
$= 1\,280 \text{ cm}^3$

c Capacity (prism) = 1 280 ml

2 a Surface area = surface area(top) + surface area(bottom)
$= 2\pi r(r + h) + 6s^2$
$= 2 \times 3{,}14 \times 5 \text{ mm}(5 \text{ mm} + 25 \text{ mm}) + 6 \times (40 \text{ mm})^2$
$= 942 \text{ mm}^2 + 9\,600 \text{ mm}^2$
$= 10\,542 \text{ mm}^2$

b Volume(cylinder) $= \pi r^2 h$
$= 3{,}14 \times (5 \text{ mm})^2 \times 25 \text{ mm}$
$= 1962{,}5 \text{ mm}^3$ OR $1{,}9625 \text{ cm}^3$

c Volume(container) = volume (cylindrical top) + volume (cubic bottom)
$= 1{,}9625 \text{ cm}^3 + (4 \text{ cm})^3$
$= 1{,}9625 \text{ cm}^3 + 64 \text{ cm}^3$

$= 65{,}9625\ \text{cm}^3$
$\approx 66\ \text{cm}^3$

d Capacity ≈ 66 ml of perfume

3 a Volume(in cm^3) $= 750\ \text{cm}^3$

b $\frac{1}{2} \times \text{base} \times h \times h = \text{volume} = 750\ \text{cm}^3$
$h = 750\ \text{cm}^3/(3{,}5\ \text{cm} \times 10\ \text{cm})$
$\approx 21{,}43\ \text{cm}$

c Surface area $= 2(\frac{1}{2} \times 7\ \text{cm} \times 21{,}43\ \text{cm}) + 3(7\ \text{cm} \times 10\ \text{cm})$
$= 150{,}01\ \text{cm}^2 + 210\ \text{cm}^2$
$= 360{,}01\ \text{cm}^2$

4 a 24,64 kl

b 64 000 cm^3

c 385 boxes

d $10 \times 7 = 70$ boxes

e 5 boxes

f $70 \times 5 = 350$ boxes

g Gap of 20 cm between boxes in top layer and ceiling

5 a Compare only areas of bases; A: 204 cm^2; B: 201 cm^2; C: 201,1 cm^2; Answer: B

b Compare only perimeter of bases; A: 70,41 cm; B: 60,2 cm; C: 50,24 cm; Answer: C

UNIT 3 The effect on surface area or volume if dimensions are doubled

Unit overview

Learner's Book page 437
Recommended pacing: 2 hours

This unit focuses on the following:
- The effects of dimension change on surface area and volume.

Resources: Learner's Book; exercise book; pencil, ruler, 3D objects

Background information

In Grades 7 and 8, learners did the following:
- used formulae to calculate surface area, volume and capacity
- described the interrelationship between surface area and volume of 3D objects
- solved problems involving surface area, volume and capacity.

In Grade 9, the effect on surface area and volume if dimensions are doubled is a new concept.

Teaching guidelines

In this unit the effects of dimension change are investigated using simple algebraic concepts of substitution, associative and commutative rules.
The examples show different kinds of problems that learners can solve using dimension change.
These examples are useful for learners to refer to when they find certain aspects difficult to understand.

Changing the dimensions of a cube; Problem-solving

Changing the dimensions of a rectangular prism; Changing the dimensions of a triangular prism; Changing the dimensions of a cylinder

Activities 1–2 Investigate changing dimensions; Work with changed dimensions

Learner's Book page 440

Guidelines for implementing this activity

- Diagrams are helpful in aiding learners' understanding of the effects of dimension change.
- The investigations focus on the effects of dimension change on the surface area and volume of cubes, rectangular prisms, triangular prisms and cylinders.
  Work carefully through the investigations and problem-solving worked examples on the board with learners. It is important to take time working through these examples one step at a time.
- Refer learners back to these investigations if necessary, should they have particular difficulties.
- Emphasise the relationship between dimension change/s and the effect of such change/s on surface area and volume. Knowledge of such a relationship simplifies mathematical problems which would otherwise take a long time to solve.

Remedial and extension

Learners might require a lot of practice doing these calculations and problems. Work through the problems and investigations with learners in small groups again if necessary before they attempt the activity and in so doing iron out as many of the difficulties as possible.

Suggested answers

1 a Volume(prism) = 8 cm × 8 cm × 10 cm
640 cm^3 = capacity of 640ml = 0,64 ℓ
b surface area = 2(8 cm × 8 cm) + 4(8 cm × 10 cm)
= 128 cm^2 + 320 cm^2
= 448 cm^2
c New base area = 2 × 64 cm^2 = 128 cm^2
Base length = $\sqrt{128\text{cm}^2}$
≈ 11,31 cm
d Volume = 128 cm^2 × 10 cm = 1 280 cm^3
= 1 280ml = 1,28 ℓ
e Surface area = 2 × 128 cm^2 + 4(11,31 cm × 10 cm)
= 256 cm^2 + 452,40 cm^2
= 708,40 cm^2

2 a 69,282 mm
Surface area = $2(\frac{1}{2}$ × 80 mm× 69,282 mm) + 3(80 mm × 90 mm)
= 2(2 771,28) mm^2 + 3(7 200) mm^2
= 2 714,56 mm^2
Volume = $\frac{1}{2}$ × 80 mm × 69,282 mm × 90 mm
= 249 415,2 mm^3 = 249,4 cm^3 =
249 ml

Learner's Book page 440

Suggested answers

L	B	H	V	2LB	2LH	2BH	SA
1	2	120	240	4	240	480	724
2	3	40	240	12	160	240	412
3	4	20	240	24	120	160	304
4	5	12	240	40	96	120	256

L	B	H	V	2LB	2LH	2BH	SA
1	10	10	100	20	20	200	240
2	2	29	116	8	116	116	240
4	4	13	208	32	104	104	240
6	6	7	252	72	84	84	240

Chapter 18 Revision

Learner's Book page 442

Encourage learners to review the content covered before attempting the revision activities. The revision activities should be used to assess learners' progress thus far, and to assess where remediation may be required.

Suggested answers

1
- **a** Circumference = $\pi d = 3{,}14 \times 4{,}2 \approx 13{,}19$ m
- **b** Radius = $\frac{d}{2} = 2{,}1$ m
- **c** Surface area = $\pi r^2 + (\pi d \times h)$
  $= 3{,}14 \times (2{,}1\text{ m})^2 + (13{,}19\text{ m} \times 3\text{ m})$
  $= 13{,}8474\text{ m}^2 + 39{,}57\text{ m}^2$
  $= 53{,}4174\text{ m}^2$
- **d** Volume = $\pi r^2 h = 3{,}14 \times (2{,}1\text{ m})^2 \times 1{,}5$ m
  $\approx 20{,}77\text{ m}^3 \therefore$ Capacity $\approx 20\ 770\ \ell$
- **e** New volume = $2 \times 20{,}77\text{ m}^3 = 41{,}54\text{ m}^3$
  $\pi r^2 h = 41{,}54\text{ m}^3$
  $h = \frac{41{,}54\text{ m}^3}{3{,}14 \times (2{,}1\text{ m})^2}$
  ≈ 3 m

2
- **a** Volume = $31\text{ cm} \times 18\text{ cm} \times 10\text{ cm} = 5\ 580\text{ cm}^3$
- **b** Surface area = $2(31\text{ cm} \times 18\text{ cm}) + 2(10\text{ cm} \times 18\text{ cm}) + 2(31\text{ cm} \times 10\text{ cm})$
  $= 1\ 116\text{ cm}^2 + 360\text{ cm}^2 + 620\text{ cm}^2$
  $= 2\ 096\text{ cm}^2$
- **c** Volume = $\frac{1}{4} \times$ original volume
  $= \frac{1}{4} \times 5\ 580\text{ cm}^3$
  $= 1\ 395\text{ cm}^3$
- **d** $\frac{\text{Volume (new box)}}{\text{volume (original box)}}$
  $= \frac{1\ 395\text{cm}^3}{5\ 580} = \frac{1}{4}$

3
- **a** volume (litres) = $\frac{1}{2} \times 36\text{ mm} \times 24\text{ mm} \times 45\text{ mm}$
  $= 19\ 440\text{ mm}^3$
- **b** Surface area = $2(\frac{1}{2} \times 36\text{ mm} \times 24\text{ mm}) + 3(36\text{ mm} \times 45\text{ mm}) = 864\text{ mm}^2 + 4\ 860\text{ mm}^2$
  $= 2\ 052\text{ mm}^2$
- **c** Volume = $\frac{1}{2} \times 36\text{ mm} \times 24\text{ mm} \times 90\text{ mm}$
  $= 38\ 880\text{ mm}^3$
- **d** Surface area = $864\text{ mm}^2 + 9\ 720\text{ mm}^2$
  $= 10\ 584\text{ mm}^2$
- **e** $\frac{\text{Volume (new)}}{\text{volume (original)}}$
  $= \frac{38\ 880\text{mm}^3}{19\ 440\text{mm}^3} = 2$

PoA | Test

1 Factorise the following expressions.

a $mn - pn + qn$
b $3x^2 + 6$
c $mx - nx + 3m - 3n$
d $3(a - 2b) - a(2b - a)$
e $x^2 - 9$
f $2 - 8b^2$
g $25x^2 - 16y^2$
h $\frac{-3(x + a)}{3x + 3a}$
i $x^2 + 3x + 2$
j $x^2 - 3x - 28$

2 Solve these equations.

a $3x + 6 = -33$
b $3{,}8x = 26{,}6$
c $3(2x + 5) = 27$
d $\frac{1}{3}(2x + 1) + \frac{1}{2}(3x + 1) = 0$
e $(x - 3)(x + 4) = (x - 6)(x - 6)$
f $x^2 - 5x = 0$
g $x^2 - 14 = 0$
h $x^2 + 4x - 21 = 0$

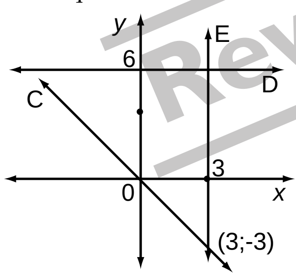
3 Draw the graphs of the following on the same system of axes, using the table method.

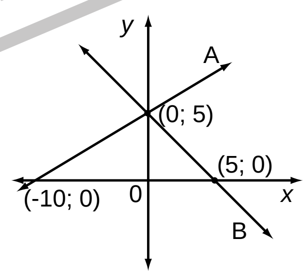
a $y = 2x + 2$
b $y = -\frac{1}{2}x + 2$

4 For each of the graphs drawn in question **3**, give the following:

a the intercepts with the axes
b the gradient
c is it increasing or decreasing

5 Find the equations of the following graphs.

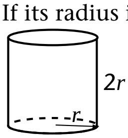




a The line parallel to the line in question **3b** that goes through the origin.
b lines A and B
c lines C, D and E

6 The height of a cylinder is the same as its diameter.

a Express its volume in terms of its radius.
b Express its surface area in terms of its radius.
c If its radius is 5 cm, calculate the surface area and the volume of the cylinder.



TERM

3 Test Memorandum

1 **a** $n(m - n + q)$ **b** $3x^2 + 6 = 3(x^2 + 2)$

c $mx - nx + 3m - 3n = (m - n)(x - 3)$ **d** $(a - 2b)(3 + a)$

e $(x + 3)(x - 3)$ **f** $2(1 - 2b)(1 + 2b)$

g $(5x - 4y)(5x + 4y)$ **h** -1

i $(x + 2)(x + 1)$ **j** $(x + 4)(x - 7)$

2 **a** $x = -13$ **b** $x = 7$ **c** $x = 2$

d $x = -\frac{5}{13}$ **e** $x = \frac{48}{13}$ **f** $x = 0$ or $x = 5$

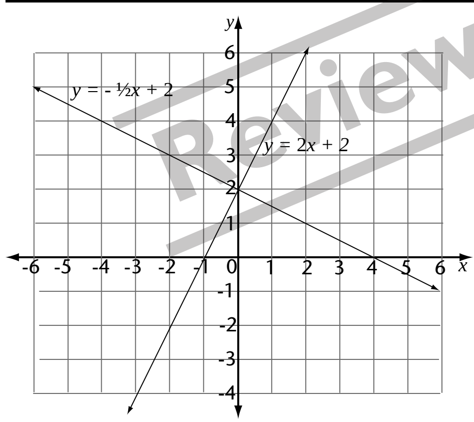
g $x^2 = \pm\sqrt{14}$ **h** $x = 3$ of $x = -7$

3 **a** $y = 2x + 2$

x	-2	-1	0	1	2
y	-2	0	2	4	6

b $y = -\frac{1}{2}x + 2$

x	-2	-1	0	1	2
y	3	$2\frac{1}{2}$	2	$1\frac{1}{2}$	1



4 **a** **i** x-intercept is $(-1; 0)$; y-intercept is $(-1; 0)$

ii x-intercept is $(4; 0)$; y-intercept is $(0; 2)$

b **i** Gradient $= 2$

ii Gradient $= -\frac{1}{2}$

c **i** increasing

ii decreasing

5 **a** $y = -\frac{1}{2}x$

b A: $y = \frac{1}{2}x + 5$; B: $y = -x + 5$

c C: $y = -x$; D: $y = 6$; E: $x = 3$

6 **a** $2\pi r^3$ **b** $6\pi r^2$ **c** $150\pi\ cm^3$

CHAPTER 19

Transformation geometry

Chapter overview

Learner's Book pages 443 to 466
Recommended pacing: 9 hours

This chapter focuses on the following:

Unit 1: Transformations with points on a coordinate plane — 3 hours
- Revising terminology of transformations
- Working on the coordinate plane
- Transforming points on the coordinate plane
- Translating points on the coordinate plane
- Reflecting points on the coordinate plane

Unit 2: Transformations with line segments and geometric figures on a coordinate plane — 3 hours
- Translating and reflecting line segments and geometric figures
- Rotating geometric figures about the origin

Unit 3: Enlargements and reductions — 3 hours
- Revising geometric figures, perimeter and area
- Enlargements and reductions
- Enlarging and reducing triangles
- Enlarging and reducing quadrilaterals

Chapter 19 Revision — 3 hours 10 minutes

UNIT 1

Transformations with points on a coordinate plane

Unit overview

Learner's Book page 444
Recommended pacing: 3 hours

This unit focuses on the following:
- Types of transformations
- Transformations with points
- Transformations with lines and simple geometric figures

Resources: Learner's Book; exercise book; graph book; pencil; ruler

Background information

Transformation geometry plays a useful role in developing learners' understanding of similarity and congruence. In transformation geometry, 2D shapes are placed on a Cartesian plane or $(x;y)$-plane and their vertices are assigned coordinates. This results in a mathematical relationship between geometry and algebra. As vertices are given $(x; y)$ coordinates, learners consolidate their knowledge of plotting points on a Cartesian plane as well as the coordinates of points in each of the FOUR quadrants of the plane.

In transformation geometry 2D shapes such as triangles and quadrilaterals are transformed forming images.

There are TWO main kinds of transformations, namely:

Position change: *translation, reflection and rotation,* and

Size change: *enlargement and reduction*

Learners have already dealt with these types of transformations in the previous grades. The focus of Grade 8 transformations was on translating, reflecting and rotating actual points and shapes on a plane. In Grade 9 leaners build from this knowledge of transforming actual figures to the formation of general transformation formulae.

Teaching guidelines

These lessons should be as practical as possible to consolidate the basic concepts and crystallise learners' understanding of transformations on coordinate planes. Games such as Battleship™ and using graph paper and tracing paper for transforming points are included below. Learners are often quite ingenuous when it comes to creating games. When doing each transformation, ask them how they would make this into a game (other than Battleship ™), and what the rules would be. For example, a grid can also be a maze in which something is moving around to try and hide (for example a monster), or in which ancient treasure is buried. Partners try and catch each other (one being the monster) by guessing the other's coordinates, or find the treasure hidden by their partner. Always try and make it fun for learners, and let them "develop" their game with each different type of transformation.

Revising terminology of transformations

Activity 1 **Revise terminology on transformations**

Learner's Book page 444

Guidelines for implementing this activity

Learners should be familiar with the vocabulary revised in this activity. Ask learners to answer the questions on their own and then discuss the answers as a class. Make deliberate use of these terms throughout the course of this unit.

Suggested answers

1. Coordinate plane: the 2D space defined by Cartesian coordinates $(x; y)$
2. Coordinates: two numbers or letters that describe a position on maps, charts and graphs. The horizontal coordinate is always written first and the vertical coordinate is written second
3. Transformation: a way of moving a shape or object, for example, *rotation*, *translation* and *reflection*.

4 *Translation:* sliding a shape into a different position without turning it
5 *Reflection:* a transformation such that any two corresponding points in the object and its image are both the same distance from a fixed straight line
6 *Rotation:* a transformation about a fixed point. It is described by giving the angle and direction of the turn and the position of the fixed point about which the turn is made.

Working on the coordinate plane

Activity 2 Work on the coordinate plane

Learner's Book page 445

Guidelines for implementing this activity

- Prepare a coordinate plane on grid paper ahead of the class as shown in the Learner's Book (without the x- and y-values), make a few copies and cut them out. (Keep the sets of four together.) Have separate squares of paper with the x- and y-values. Put each set of items in an envelope.
- Start this lesson by giving groups of learners an envelope and instruct them to arrange the quadrants and position the x- and y- values. Discuss their findings using the bulleted points in the Learner's Book as a guide. Ask key questions: What do we call each quarter on the coordinate plane? Which axis is the horizontal axis? What are the coordinates of the origin? Why is $x > 0$ in Quadrant I? And so on.
- Work through the example in the Learner's Book.

Remedial and extension

The coordinate plane can be used to play a game called Battleship™ in which pairs plot the position of their "ships" using coordinates, and take turns to "target" the other's ships by providing possible coordinates.

Suggested answers

1 a A: quadrant 1; B: quadrant 2; C: quadrant 3; D: quadrant 4
  b E: on the y axis between quadrants 1 and 2; F: on the x axis between quadrants 2 and 3; G: on the y axis between quadrants 3 and 4; H: on the x axis between quadrants 1 and 4.
2 a C and A
  b F and G
  c K and N
  d R and T
  e Q
  f H
  g B or M
  h E
3 A(3; 1); B(4; 4); C(1; 3); D($\frac{1}{2}$; $1\frac{1}{2}$); E(–1; 1); F(–2; 3); G(–3; 2); H(–2; 0); K(–4; $\frac{1}{2}$); L(–1; –2); M($-2\frac{1}{2}$; $-2\frac{1}{2}$); N($-1\frac{1}{2}$; –4); P(0; $-3\frac{1}{2}$); Q(1; –1); R(3; –1); S(2; –2); T(1; –3); U(4; –4)

Transforming points on the coordinate plane; Translating points on the coordinate plane

Translation of a point in one direction; Translation of a point in two directions

Activity 3 Translate points on a coordinate plane

Learner's Book page 450

Guidelines for implementing this activity

- The first two examples demonstrate the translation of points in the different directions in the same quadrant: vertical and horizontal. Work carefully through these examples.
- The next example demonstrates the translation of points in different directions across quadrants: vertical and horizontal. Work carefully through these examples.
- Emphasise the effects on the *x*- and *y*- coordinates of transforming a point between different quadrants.
- The last example demonstrates the translation of points in two directions within the same quadrant and across quadrants.
- Work carefully through these examples.
- Use additional, different points in each example to consolidate learners' understanding.
- Emphasise that the horizontal translation is always performed before the vertical translation.
- Spend time consolidating the correct notation that is used when performing translations. Ensure that learners are completely comfortable with this before they start the activity.

Tip

Provide learners with graph paper and tracing paper. Let them work in pairs or small groups. Instruct them to draw a point on the tracing paper, provide a starting position and call out different instructions, such as: Translate the point 3 units to the right. Translate the point 2 units to the right and 3 units up. And so on. Let learners do the translations and write down the new coordinates. Compare and discuss their findings as a class.

Suggested answers

1 **a** (–3; 2)
b (1; 1)
c (–3; –2)
d (–9; 1)
e (–5; –5)
f (2; 5)
g (–6; 6)
h (1; –1)

2 **a** A was translated 2 units to the right.

- **b** B was translated 4 units down.
- **c** C was translated 3 units to the left.
- **d** D was translated 4 units up.
- **e** E was translated 3 units down.
- **f** F was translated 9 units to the left.
- **g** G was translated 6 units up.
- **h** H was translated 8 units to the left.

Reflecting points on a coordinate plane

Reflection in the *y*-axis or *x*-axis; Reflection in the line $y = x$

Activity 4 Reflect points on a coordinate plane

Learner's Book page 453

Guidelines for implementing this activity

- Work through the examples in exactly the same manner as for Activity 3.
- Ensure that learners understand that the perpendicular distance between points and their images remain the same along the x- or y-axis (reflection along the y-axis or x-axis) and the line along $y = x$ (reflection in the line $y = x$).

Remedial and extension

Repeat the graph and tracing paper activity using specific instructions that provide practice with reflections in the y-axis or x-axis and the line $y = x$.

Suggested answers

1 **a** Q(–4; 1)
b Q'(4; 1)
c Q''(1; 4)
d R(–2; 1); quadrant 2

2 **a** Translated 2 units right
b Translated 3 units left, 2 units up
c Reflected on the x-axis
d Reflected on the line $y = x$
e Translated 9 units left, 6 units up

3 **a** Learners represent Q on a coordinate plane in their exercise books.
b (2; –4)
c (–5; –3)
d (–3; 5); quadrant 2

4 **a** Reflected in the line $y = x$
b Reflected in the x-axis
c Translated 3 units left
d Translated 6 units down
e Translated 1 unit right and 6 units down

UNIT

2 Transformations with line segments and geometric figures on a coordinate plane

Unit overview

Learner's Book page 454
Recommended pacing: 3 hours

This unit focuses on the following:
- Translating and reflecting line segments and geometric figures
- Rotating geometric figures about the origins.

Resources: Learner's Book; exercise book; graph book; pencil; ruler; prepared grid paper (with axes drawn on them and photocopied); small cut-out polygons

Background information

This unit explores transformations that involve position change, namely: *translation*, *reflection* and *rotation*.
Learners have already dealt with these types of transformations in the previous grades. The focus of Grade 8 transformations was on translating, reflecting and rotating actual points and shapes on a plane. In Grade 9 leaners build from this knowledge of transforming actual figures to the formation of general transformation formulae.

Teaching guidelines

The emphasis in this unit is on practical engagement with the concepts of translating, reflecting and rotating lines and polygons. Ensure that graph paper with axes are prepared ahead of time and photocopied, and that learners prepare cut-out polygons using soft cardboard and store them in a safe place to use during these lessons.

Translating and reflecting line segments and geometric figures; Rotating geometric figures about the origin

Translation and reflection of line segments; Translation and reflection of geometric figures

Activities 1–2 **Translate and reflect line segments and geometric figures; Rotate triangles through 90° about the origin**

Learner's Book pages 456–458

Guidelines for implementing these activities

- In this section, learners start by transforming a line, then move on to transforming polygons. Work through the translation in the first example in the Learner's Book. Provide additional examples of translations of lines by letting learners translate lines on prepared grid paper in pairs. They take turns to do the translation and to write the new coordinates. Provide them with two types of instructions. For example: 1. Translate line AB 3 units to the right. Write down the coordinates of

the points of the image. 2. Translate any line MN and provide the new coordinates. Let *y*our partner explain how *y*ou have translated the line.
- Work through the remaining examples in the same way, ensuring that learners are actively involved. Let learners use cut-out polygons to translate, reflect and rotate in the subsequent examples.

Remedial and extension

The more actively learners are involved, the better.
Provide extension questions that require that learners do the reverse: Draw or position the image on the grid paper with an explanation as to how the polygon was translated, reflected and/or rotated. Let learners provide the original coordinates.

Suggested answers

Activity 1

1 **a** K(–4; 5); L(–4; 1); M(–1; 1)
b K(–1; 5); L(–1; 1); M(2; 1)
c K(–4; –5); L(–4; –1); M(–1; –1)
d K(5; –4); L(1; –4); M(1; –1)
e A(2; 5); B(1; 2)
f A(5; 2); B(2; 1)

2 **a** Reflection over the line $y = –1$
b Reflection over the line $y = x$
c Reflection over the x-axis
d Reflection over the y-axis

Activity 2

1 For ΔGHK: G(1; 4) → G'(4; –1); H(1; 1) → H'(1; –1); K(4; 1) → K'(1; –4)
For ΔLMN: L(–4; 4) → L'(4; 4); M(–4; 1) → M'(1; 4); N(–1; 4) → N'(4; 1)
For ΔPQR: P(–1; –1) → P'(–1; 1); Q(–4; –3) → Q'(–3; 4); R(–3; –4) → R'(–4; 3)
For ΔSTU: S(2; –2) → S'(–2; –2); T(0; –3) → T'(–3; 0); U(4; –4) → U'(–4; –4)

2 For ΔGHK: G(1; 4) → G'(–4; 1); H(1; 1) → H'(–1; 1); K(4; 1) → K'(–1; 4)
For ΔLMN: L(–4; 4) → L'(–4; –4); M(–4; 1) → M'(–1; –4); N(–1; 4) → N'(–4; –1)
For ΔPQR: P(–1; –1) → P'(1; –1); Q(–4; –3) → Q'(3; –4); R(–3; –4) → R'(4; –3)
For ΔSTU: S(2; –2) → S'(2; 2); T(0; –3) → T'(3; 0); U(4; –4) → U'(4; 4)

3 For ΔGHK: G(1; 4) → G'(–1; –4); H(1; 1) → H'(–1; –1); K(4; 1) → K'(–4; –1)
For ΔLMN: L(–4; 4) → L'(4; –4); M(–4; 1) → M'(4; –1); N(–1; 4) → N'(1; –4)
For ΔPQR: P(–1; –1) → P'(1; 1); Q(–4; –3) → Q'(4; 3); R(–3; –4) → R'(3; 4)
For ΔSTU: S(2; –2) → S'(–2; 2); T(0; –3) → T'(0; 3); U(4; –4) → U'(–4; 4)

4 Same answers as for question **3**.

UNIT

3 Enlargements and reductions

Unit overview

Learner's Book page 459
Recommended pacing: 3 hours

This unit focuses on the following:
- Revising geometric figures, perimeter and area
- Enlargements and reductions
- Enlarging and reducing triangles
- Enlarging and reducing quadrilaterals.

Resources: Learner's Book; exercise book; graph book; pencil; ruler; prepared grid paper (with axes drawn on them and photocopied); small cut-out polygons

Background information

This unit focuses on enlargements and reductions. As the names imply these transformations change the size of the original object to produce images that are *similar* to the original object but *larger* (enlargement) or *smaller* (reduction) in size, compared to the afore-going transformations that produced *congruent* images.
It is also important for learners to see that enlarging or reducing a geometric shape will affect its area and perimeter.

Teaching guidelines

The examples are aimed at showing learners the process of enlargement and reduction, the point of enlargement (or reduction) being one of the vertices of a triangle. This simplifies the enlargement (or reduction) to that of two lines – the third line is automatically enlarged (or reduced).
It is important to consolidate the following:
- *All sides of a geometric figure must be enlarged (or reduced) by the same amount, called the enlargement (or reduction) factor.*
- *Images produced by enlargements (or reduction) are similar.*

The emphasis is again on practical engagement with the concepts of enlargement and reduction. Ensure that graph paper with axes are prepared ahead of time and photocopied, and that learners prepare cut-out polygons using soft cardboard and store them in a safe place to use during these lessons.

Revising geometric figures, perimeter and area

Activity 1 Revise geometric figures, perimeter and area

Learner's Book page 459

Guidelines for implementing this activity

Learners should familiar with the vocabulary revised in this activity. This activity therefore revises the important vocabulary that learners are expected to know.

Suggested answers

1 a Vertex: a point at which two or more edges meet.
 b Triangle: a polygon which has three edges. Its interior angles add up to 180°.
 c Right-angled triangle: a triangle with one vertex equal to 90°.
 d Height of a triangle: a line from a vertex to the opposite edge and at right angles to that edge.
 e Base of a triangle: any edge chosen to suit the purpose: usually the edge at the 'bottom' when in a given position.
 f Quadrilateral: a polygon with four edges. Its interior angles add up to 360°.
 g Square: a quadrilateral with all edges of equal length and all interior angles equal to 90°.
 h Rectangle: a quadrilateral with interior angles of 90° and two sets of opposite edges that are equal.

2 a Area of a triangle = $\frac{1}{2}b \times h$
 b Area of a square = s^2
 c Area of a rectangle = $l \times b$
 d Perimeter of a rectangle = $2l + 2b$ or $2(l + b)$

Enlargements and reductions; Enlarging and reducing triangles; Enlarging and reducing quadrilaterals;

Enlargement of a triangle; Reduction of a triangle

Activities 2–3 Work with enlargements and reductions of triangles; Work with enlargements and reductions of quadrilaterals

Learner's Book pages 463–465

Guidelines for implementing these activities

- Once again as with the preceding units, practical engagement with the examples is key to the understanding of the concepts covered in this unit.
- Work through the examples with learners, providing additional examples at all times. Let learners do the actual examples on graph paper and practically engage with the application of prior knowledge of scale factor and similarity in this context.
- Consolidate the following for triangles. If the coordinates of the vertices of a triangle are doubled:
  - the lengths of the new sides are twice the length of the original sides
  - the new perimeter is twice the original perimeter
  - the new area is four times the original area.

- If the coordinates of the vertices of a triangle are halved:
  - the lengths of the new sides are half the length of the original sides
  - the new perimeter is half the original perimeter
  - the new area is one-quarter the original area.

Remedial and extension

The more actively learners are involved, the better.

Suggested answers

Activity 2

1 a PQ = 6; QR = 6; PR = 8,49
b $P = s_1 + s_2 + s_3$
$= 6 + 6 + 8{,}49$
$= 20{,}49$
c $A = \frac{1}{2} \times b \times h$
$= \frac{1}{2} \times 6 \times 6$
= 18 square units

2 a $PQ^2 + QR^2 = PR^2$
$144 + 144 = PR^2$
$\sqrt{288} = PR$
$16{,}97 \approx PR$
b $P = s_1 + s_2 + s_3$
$= 12 + 12 + 16{,}97$
$= 40{,}97$
c $A = \frac{1}{2} \times b \times h$
$= \frac{1}{2} \times 12 \times 12$
= 72 square units

3 a AB = AC = 21; BC = 15
b AB = BC = 4,2; BC = 3

Activity 3

1 a *P* ABCD = sum of all sides
AD = 2
DC = 3
BC = 3
$AB^2 = 3^2 + 1^2$
$AB^2 = 10$
$AB \approx 3{,}16$
$\therefore P = 2 + 3 + 3 + 3{,}16$
= 11, 16
P PQRS = sum of all sides
PS = 6
RS = 9
QR = 9
$PQ^2 = 9^2 + 3^2$
$PQ^2 = 90$
$PQ \approx 9{,}49$
$\therefore P = 6 + 9 + 9 + 9{,}49$
$= 33{,}49$

b A ABCD $= \frac{1}{2}(b \times h) + (l \times b)$
$= \frac{1}{2}(3 \times 1) + (3 \times 2)$
$= 1,5 + 6$
$= 7,5$ square units
A PQRS $= \frac{1}{2}(b \times h) + (l \times b)$
$= \frac{1}{2}(9 \times 3) + (9 \times 6)$
$= 13,5 + 54$
$= 67,5$ square units

c ABCD to PQRS has a scale factor of 3.

d PQRS to ABCD has a scale factor of $\frac{1}{3}$.

2 **a** The perimeter of ABCD is three times smaller than the perimeter of its image PQRS.
b The perimeter of PQRS is nine times bigger than the perimeter of its image ABCD.
c The area of ABCD is nine times smaller than the area of its image PQRS.
d The area of PQRS is nine times bigger than the area of its image ABCD.

3 **a** The perimeter of PQRS is three times bigger than the area of ABCD.
b The area of PQRS is nine times bigger than the area of ABCD.

Chapter 19 Revision

Learner's Book page 466

Encourage learners to review the content covered before attempting the revision activities. The revision activities should be used to assess learners' progress thus far, and to assess where remediation may be required.

Suggested answers

1. X(2; –1); Y(1; –5); Z(3; –5)
2. YZ = 2 units
3. 10,24 units
4. MX = 4 units
5. 4 units^2
6. YZ = 2 × Area ÷ base
   = 8 units^2 ÷ $\sqrt{17}$ units
   = 1,94 units
7. X(–1; –1); Y(–2; –5); Z(0; –5)
8. X(2; 1); Y(1; 5); Z(3; 5)
9. X(–1; 2); Y(–5; 1); Z(–5; 3)
10. X(3; 3); Y(1; –5); Z(5; –5) or any coordinates that reflect this size triangle.

CHAPTER 20

Geometry of 3D objects

Chapter overview

Learner's Book pages 467 to 482
Recommended pacing: 9 hours

This chapter focuses on the following:

Unit 1: Properties and definitions of the Platonic solids	3 hours
Revising vocabulary on Platonic solids	
Types of Platonic solids and their shapes	
Faces, edges and vertices of Platonic solids	
Unit 2: Properties of spheres and cylinders	3 hours
Surface area and volume of a sphere	
Surface area and volume of a cylinder	
Unit 3: Models of cubes, prisms, pyramids and cylinders	3 hours
Linking nets with polyhedrons	
Building models of cubes, prisms, pyramids and cylinders	
Chapter 20 Revision	3 hours 10 minutes

UNIT 1

Properties and definitions of the Platonic solids

Unit overview

Learner's Book page 468
Recommended pacing: 3 hours

This unit focuses on the following:

- Revising vocabulary on platonic solids
- Types of platonic solids and their shapes
- Faces, edges and vertices of platonic solids.

Resources: Learner's Book; exercise book; pencil; ruler; protractor; compass; real models of prisms, cylinders and pyramids; wall charts with nets of prisms, cylinders and pyramids; Plasticine; toothpicks; jelly tots; straws

Background information

In Grade 8 learners were exposed to 5 Platonic solids. Platonic solids are different from prisms which were discussed in a previous chapter.
A prism has two base faces which are polygons and all the lateral faces joining the bases are rectangles.
Platonic solids are 3D objects made up of polygons joined together by straight lines called *edges*. These polygons meet at corners of the shape at points called *vertices*. Because the 3D objects are made up of a number of polygons joined together, they are called *polyhedrons* or *polyhedra* (*poly* means "many").
If all the polygons making up a polyhedron are regular, the polyhedron is regular. Regular polyhedra were first discovered by Plato and are called Platonic solids.
Finally, there is another group of polyhedra called *pyramids*. Pyramids are made up of *one* base face and triangular lateral faces meeting at one point called the vertex. The base of a pyramid can be any polygon but the lateral faces are always triangles. The simplest pyramid has a triangular base and if all four triangular faces are equilateral triangles, the pyramid is a tetrahedron - making it a Platonic solid.

Teaching guidelines

This chapter provides learners with an opportunity to explore 3D objects and identify interesting characteristics they possess.
Nets of 3D objects are particularly useful in demonstrating to learners the number and shape of the faces that make up a solid.
Ensure all the materials are available ahead of time.

Revising vocabulary on Platonic solids

Activity 1 **Revise vocabulary on the Platonic solids**

Learner's Book page 468

Guidelines for implementing this activity

- Revise the vocabulary of Platonic solids with learners.
- Ensure that the vocabulary is used correctly throughout the duration of this chapter.

Suggested answers

1 **a** Polygon: a flat shape completely enclosed by three or more straight sides.
b Side of a polygon: the edge of a polygon.
c Vertex of a polygon: a point at which two or more edges of a shape meet.
d Regular polygon: a polygon which is both equilateral and equiangular.
e Polyhedron: a 3D shape whose faces are all polygons.
f Edge of a polyhedron: a straight line formed where two faces of a polyhedron meet.
g Vertex of a polyhedron: the angular point where three or more edges of a polyhedron meet.
h Regular polyhedron: a polyhedron with all faces identical and the same number of edges meeting at each vertex.

2 a Tetrahedron: a 3D figure with four triangular faces, four vertices and six edges. A regular tetrahedron has equilateral triangles as faces so all the edges have the same length.
   b Hexahedron: a solid figure with six faces. The regular hexahedron is a cube.
   c Octahedron: a figure with eight faces, six vertices and twelve edges. A regular octahedron has equilateral triangles as faces.
   d Dodecahedron: a figure with twelve faces, twenty vertices and thirty edges. A regular dodecahedron has faces that are regular pentagons.
   e Icosahedron: a figure with twenty faces, twelve vertices and thirty edges. A regular icosahedron has equilateral triangles as faces.
3 They are all made up of regular polygons.
4 They have different numbers of faces, edges and vertices.

Types of Platonic solids and their shapes; Faces, edges and vertices of Platonic solids

The tetrahedron; The hexahedron (cube); The octahedron; The dodecahedron; The icosahedron; Find the number of faces of the Platonic solids; Find the number of edges of the Platonic solids; Find the number of vertices of the Platonic solids

Activities 2–3 **Build models of the 5 Platonic solids; Work with Platonic solids**

Learner's Book pages 470–472

Guidelines for implementing these activities

- Building models is a good starting point from which to explore the Platonic solids in terms of their number of faces, number of edges and number of vertices. Let learners build their models. Let them also experiment with using compasses to create their nets.
- Walk around the class and observe what learners are doing.
- Work through the different shapes and nets of Platonic solids with the learners, pointing out the relationship between the prefix of the name of a solid and the number of faces it possesses.
- Let learners use their models to explore these aspects of the Platonic solids practically while verbalising their findings, as well as jotting them down informally as they work through each.
- Lead learners to discovering Euler's theorem (formula) on the relationship between the number of vertices, faces and edges of Platonic solids, then state the formal theorem. Remember that any variable in the Euler formula can be made the subject of the formula.
- Give a summary of Euler's formula as it applies to each Platonic solid. This summary is also provided in the Learner's Book.
- Let learners work though the definitions using their models as the point of discovery and to confirm their answers (Activity 3).
- Revise the different Platonic solids at the beginning of every subsequent lesson.

Remedial and extension

Get learners to construct models using different materials other than cardboard nets, using a combination of the following: drinking straws and Prestik™/Plasticine or toothpicks and Prestik™/jelly tots.

Suggested answers

Activity 2

Learners build models of the five Platonic solids and complete the table.

Activity 3

1 a Polygon: Polygon: a flat shape completely enclosed by three or more straight edges.
b Polyhedron: a 3D shape whose faces are all polygons.
c Platonic solid: a regular polyhedron with all faces as equal, regular polygons and all vertices alike.
d Edge of a Platonic solid: a straight line formed where two faces of a Platonic solid meet; these edges are all equal.
e Vertex of a Platonic solid: the angular point where three or more edges of a Platonic solid meet; these vertices are equal.

2 a Octahedron
b $F + V = E + 2$
$8 + 6 = E + 2$
$14 = E + 2$
$12 = E$

3 a A: triangle-based prism; B: regular hexahedron/cube; C: hexahedron/square-based prism; D: hexahedron/rectangle-based prism; E pentagon-based prism; F: hexagon-based prism; G: heptagon-based prism; H: octagon-based prism
b They are all polyhedrons because they all have at least four faces. These faces are all regular polygons.
c They are irregular polyhedrons because their faces and vertices are not identical.
d These are all prisms because they are all solid figures with matching bases and their cross-sections are congruent with their bases.
e $F + V = E + 2$
A: $5 + 6 = E + 2$ [therefore] $9 = E$ (yes)
B: $6 + 8 = E + 2$ [therefore] $12 = E$ (yes)
C: $6 + 8 = E + 2$ [therefore] $12 = E$ (yes)
D: $6 + 8 = E + 2$ [therefore] $12 = E$ (yes)
E: $7 + 10 = E + 2$ [therefore] $15 = E$ (yes)
F: $8 + 12 = E + 2$ [therefore] $18 = E$ (yes)
G: $9 + 14 = E + 2$ [therefore] $21 = E$ (yes)
H: $10 + 24 = E + 2$ [therefore] $32 = E$ (yes)

UNIT

2 Properties of spheres and cylinders

Unit overview

Learner's Book page 474
Recommended pacing: 3 hours

This unit focuses on the following:
- Surface area and volume of a sphere
- Surface area and volume of a cylinder.

Resources: Learner's Book; exercise book

Background information

The surface area and volume of a cylinder was covered in a previous chapter. The surface area and volume of a sphere is new.

Teaching guidelines

Deriving the formulae for surface area and volume of a sphere is complex for Grade 9s. It suffices to give them the formulae to apply. However, always ensure that learners are given many opportunities to practise applying the formulae they learn.

Surface area and volume of a sphere; Surface area and volume of a cylinder

Activities 1–2 **Find the surface area and volume of spheres; Calculate the surface area and volume of spheres and cylinders**

Learner's Book pages 475–476

Guidelines for implementing these activities

- Revise what a sphere and a cylinder is by working through the introduction to each in the Learner's Book.
- Revise what *surface area* and *volume* mean.
- Allow learners to manipulate actual spheres and cylinders when revising surface area and volume.
- Working with these two objects at the same time helps learners identify some similarities between the two solids. For example, both the sphere and the cylinder have a diameter and a radius. They also both have circumference and hence the constant π (pi) applies to both.
- Deriving the formulae for *surface area* and *volume* of a sphere is complex for Grade 9s.
  It suffices to give them the formulae to apply.

- Work through the examples in the Learner's Book, ensuring that learners practise using the formulae using additional examples before they do the activity.
- Work through the summary as a form of revision.

Remedial and extension

Practice of the application of the formulae is important.

Suggested answers

Activity 1

1 2 m

2 Surface area $= 4\pi r^2$
$= 4 \times 3{,}14 \times 4$
$= 50{,}24 \text{ m}^2$
$= 502\ 400 \text{ cm}^2$

3 Volume $= \frac{4}{3}\pi r^2$
$= 1{,}33 \times 3{,}14 \times 4$
$\approx 16{,}7 \text{ m}^3$
$= 16\ 700\ 000 \text{ cm}^3$

Activity 2

1 **a** Surface area $= 2\pi r(r + h)$
$= 2 \times 3{,}14 \times 8(8 + 15)$
$= 6{,}28 \times 184$
$= 1\ 155{,}52 \text{ cm}^2$

b $V = \pi r^2 h$
$= 3{,}14 \times 8^2 \times 15$
$= 3\ 014{,}4 \text{ cm}^3$

c $x = 9$ cm

2 Capacity of cube = 74 088 cm^3; Volume of cylinder = 58 189 cm^3; Volume of air = 15 899 cm^3

3 Edge of cube = Height of cylinder = 7 cm; Capacity of cylinder = 539 cm^3; Volume of cube = 343 cm^3; Volume of air = 166 cm^3

UNIT 3

Models of cubes, prisms, pyramids and cylinders

Unit overview

Learner's Book page 477
Recommended pacing: 3 hours

This unit focuses on the following:
- Linking nets with polyhedrons
- Building models of cubes, prisms, pyramids and cylinders

Resources: Learner's Book; exercise book

Background information

At this stage, leaners are familiar with shapes of prisms, cubes, pyramids and cylinders. This unit is intended to provide learners with an opportunity to design models of these solids using their nets. Construction skills developed in Chapter 10 will be used in designing nets of solids of different sizes.

Teaching guidelines

When building nets it is always a good idea to start with the simplest net and give learners the dimensions of the faces of the net they have to construct. Always encourage learners to be creative with their models once they have built them by adding colour and designs to them.

Linking nets with polyhedrons

Activity 1 Link nets with polyhedrons

Learner's Book page 477

Guidelines for implementing this activity

- Learners should be able to complete this activity on their own.

Remedial and extension

Learners can work in mixed-ability pairs if they require support.

Suggested answers

A: triangle-based pyramid
B: square-based pyramid
C: hexagon-based prism
D: pentagon-based pyramid
E: heptagon-based pyramid
F: rectangle-based prism

G: triangle-based pyramid
H: triangle-based prism
K: pentagon-based prism
L: rectangle-based pyramid
M: octagon-based prism
N: octagon-based pyramid
P: hexagon-based pyramid
Q: square-based prism
R: heptagon-based prism
S: cube/regular hexahedron

Building models of cubes, prisms, pyramids and cylinders

Building models of cubes; Building models of prisms; Building models of pyramids; Building models of cylinders

Activities 2–6 **Build models of cubes; Build models of prisms: Build models of pyramids; Build models of cylinders**

Learner's Book page 477–481

Guidelines for implementing these activities

- Let learners produce small rough but neat sketches of the nets, and construct them nets using their nets. Learners use these nets to identify nets hat will not form a cube.
- Learners complete Activity 2.
- Activities 3 to 4 test learners' ability to design their own nets using the given specifications. Learners may work in pairs, but they must produce their own models.
- The activity can be repeated any number of times with one shape, until learners understand the process. These activities can also be used as projects.

Remedial and extension

Allow learners to work in pairs or small groups, though learners must produce their own models.

Suggested answers

Activity 2

1 H; M; N; P and Q cannot be used as nets of cubes.
2 a Learners design a net.
  b Learners design a net (edges should be 49,5 mm long).
  c Learners design a net (edges should be 50 cm long).
3 Learners build their own models.

Activity 3

1 to 2 Learners design nets for prisms.
3 Learners build a model of a prism.

Activity 4

1 to 3 Learners design nets for pyramids.
4 Learners build a model of a pyramid.

Activity 5

1 Learners design nets for cylinders.
2 Learners build a model of a cylinder.

Chapter 20 Revision

Learner's Book page 482

Encourage learners to review the content covered before attempting the revision activities. The revision activities should be used to assess learners' progress thus far, and to assess where remediation may be required.

Suggested answers

1
- **a** Tetrahedron, hexahedron, octahedron, dodecahedron and icosahedron
- **b** Four faces
- **c** Icosahedron
- **d** V = 2 – F + E
- **e** V = 2 – V + E
  = 2 – 12 + 30 = 20 faces
- **f** Icosahedron

2
- **a** Diameter $= \frac{711 \text{ mm}}{3{,}14} \approx 226{,}43$ mm
- **b** Radius $= \frac{226{,}43 \text{ mm}}{2} = 113{,}22$ mm
- **c** Surface area $= 4 \times 3{,}14 \times (11{,}322 \text{ cm})^2$
  $= 1\ 609{,}75 \text{ cm}^2$
- **d** Volume $= \frac{4}{3} \times 3{,}14 \times (11{,}321)^3$
  $= 6\ 076{,}28 \text{ cm}^3$

3
- **a** $r^2 = \frac{\text{volume}}{(\pi \times h)}$
  $= \frac{500 \text{ cm}^3}{(3{,}14 \times 13 \text{ cm})}$
  $\approx 12{,}25 \text{ cm}^2$
  So, r = 3,50 cm OR 35 mm
- **b** Circumference $= 2 \times 3{,}14 \times 3{,}50$ cm
  = 21,98 cm
- **c** Surface area $= 2\pi r(r + h)$
  $= 2 \times 3{,}14 \times 3{,}5 \text{ cm}(3{,}5 \text{ cm} + 13 \text{ cm})$
  $= 362{,}67 \text{ cm}^2$

CHAPTER

21 Data handling and Probability

Chapter overview

Learner's Book pages 463 to 528
Recommended pacing: 10,5 hours + 4,5 hours

This chapter focuses on the following:

Unit 1: Collecting and recording data	2 hours
Revising terminology on data handling	
What is data handling?	
Collecting data using different methods and instruments	
Unit 2: Organising, ordering and summarising data	2,5 hours
Revising more terminology on data handling	
Organising data	
Ordering data	
Summarising data using statistical measures	
Unit 3: Representing data	3 hours
Revising representations of data	
Representing data in pie charts	
Representing data in bar graphs and double bar graphs	
Representing data in histograms	
Representing data in broken-line graphs	
Representing data in scatter plots	
Unit 4: Analysing, interpreting and reporting data	3 hours
Identifying sources of bias and error	
Comparing different representations of the same data	
Comparing and choosing between mode, median, mean and range	
Comparing graphs with different scales	
Analysing and interpreting misleading representations	
Unit 5: Probability	4,5 hours
Revising terminology on probability	
Finding probabilities of simple events	
Comparing probability and relative frequency	
Finding probabilities of compound events	
Chapter 21 Revision	5 hours 40 minutes

UNIT

1 Collecting and recording data

Unit overview

Learner's Book page 484
Recommended pacing: 2 hours

This unit focuses on the following:

- The data cycle, stages of the cycle and the processes involved in each stage
- The meaning of data and different types of data
- Posing different kinds of problems and questions whose solutions require data collection
- Sampling
- Data collection methods
- Data collection instruments.

Resources: Learner's Book; exercise book; pencil; coloured pencils, ruler; graph book

Background information

Learners were introduced to data handling in the Foundation Phase. The important aspect of the topic at this level is its usefulness in providing a basis for solutions to many problems and questions.

Problems may arise in various contexts. In this chapter learners' knowledge of the stages of the data cycle is extended. Learners are expected to be able to perform various processes involved in each stage of the cycle.

With an understanding of these processes learners should be able to do a complete task where they follow the stages of the data cycle. Their knowledge of probability is extended to include probabilities of compound events. This is done by a discussion of tree diagrams and two-way tables.

Teaching guidelines

Learners should be familiar with all the concepts covered in this section since they have encountered these in previous grades. Care should, however, still be taken to ensure that they know these concepts and that they are revised thoroughly.

Revising terminology on data handling

Activity 1 **Revise terminology on data handling**

Learner's Book page 484

Guidelines for implementing this activity

- Learners should be familiar with the vocabulary revised in this activity. Ask learners to answer the questions on their own and then discuss the answers as a class.

Suggested answers

1 **a-c** Answers provided
d-h Any appropriate answer from learners.

2 **a** Data: information or facts we find out or are given. Data can be numbers, words or a mixture of both.
b Population: the total number of people who live in a particular area, city or country.
c Bias: a strong feeling in favour of or against one group of people or one side of an argument.
d Numerical data: data in the form of numbers.
e Categorical data: factual, rather than numerical, data.
f Sample: a number of people or things taken from a larger group and used in tests to provide information about the group.
g Questionnaire: a written list of questions that are answered by a number of people so that information can be collected from the answers.
h Survey: an investigation of the opinions, behaviour, etc. of a particular group of people, which is usually done by asking them questions.

What is data handling?

The data cycle and the processes involved in each stage

Activity 2 Identify stages of the data cycle

Learner's Book page 486

Guidelines for implementing this activity

- The diagram serves both as revision of the stages as well as the relationship between them. It would be very helpful to use a simple problem like finding the most popular cellphone name in explaining the five stages.
- Ensure that learners know these stages.
- Make a wall chart of the diagram and keep it on the wall during the teaching of this chapter.
- Activity 2 is intended to test learners' understanding of the stages of the data cycle and the processes involved in each stage.

Suggested answers

Column A	Column B
Problem posing	How did Grade 8 learners perform in the March test?
Data collection	Interview all Grade 9 learners in a school about top four favourite subjects
Data organisation	Calculate measures of central tendency
Data representation	Draw a pie chart
Analysis and interpretation	Identify trends on a bar graph

Important concepts that are used in the data cycle

Activity 3 Work with data, populations, samples and sampling

Learner's Book page 489

Guidelines for implementing this activity

- This section is intended to show learners the sampling process. Work through all the terms in this section, paying special attention to the following: *sample size, sample representativity* and *bias*. Ensure that learners understand the basic terminology and are able to use them with ease.
- Revise the meaning of *discrete* and *continuous* data.
- Work carefully through the notes with learners ensuring that they understand the effects of *sample size, sample representativity* and *bias* on the outcome.
- The example provided shows one aspect of bias in sampling. There are many examples of bias in sampling depending on the desired outcome of the data analysis process.
- Encourage learners to share their own examples.

Suggested answers

1 **Numerical** data e.g. ages of babies in a crèche = {2; 4; 3; 5; 3…}
Categorical data e.g. these are different type of sports at Olympic Games = {soccer; boxing; swimming; gymnastics; jumping; throwing ;…}

2 a Population is the entire group of objects from which data is collected
b Sampling is the process of selecting a small group of objects from a population for data collection purposes
c A biased sample is a sample chosen in such a way that it influences the results of the investigation.

3 a Entire population
b Entire population
c Sample
d Sample

Collecting data using different methods and instruments

Activity 4 Use methods and instruments to collect data

Learner's Book page 490

Guidelines for implementing this activity

Discuss the method and instruments of data collection in the Learner's Book. Ensure that learners understand these and can explain what they are and when they are used. Work through the summary as a form of revision.

Suggested answers

1 a Interview
b Research
c Observation
d Interview
e Research
f Interview

2 a Survey
b Database
c Measuring instrument at the beginning and the end of the week.
d Survey
e Database or Internet
f Interviews and questionnaire

UNIT 2

Organising, ordering and summarising data

Unit overview

Learner's Book page 491
Recommended pacing: 2,5 hours

This unit focuses on the following:
- Revising more terminology on Data Handling
- Organising data
- Ordering data
- Summarising data using statistical measures.

Resources: Learner's Book; exercise book; pencil; coloured pencils, ruler; graph book

Background information

Learners were introduced to the content in this unit in the Intermediate Phase, and continued working with these statistical measures in Grades 7 and 8. This section therefore serves as revision.

Teaching guidelines

Once data has been collected it needs to be organised in a meaningful way so as to facilitate analysis and formulation of conclusions. Data can be sorted according to categories or class intervals using tally or frequency tables. Learners can be given sorting criteria first and then asked to sort data by choosing their own categories. Examples in the Learner's Book are intended to show this process. They also provide some examples of questions that can be answered by analysing sorted data.

Revising more terminology on data handling

Activity 2 **Revise more terminology on data handling**

Learner's Book page 491

Guidelines for implementing this activity

- Learners should be familiar with the vocabulary revised in this activity. The vocabulary in Activity 1 is important since it will be used throughout this section and beyond, and care should be taken to ensure that learners know it and understand its use.

Suggested answers

1 Ascending order: putting numbers into a sequence that increases in size
2 Descending order: putting numbers into a sequence that decreases in size
3 Alphabetical order: putting words into a sequences according to the letters of the alphabet
4 Frequency: the number of times a piece of data is found.
5 Tally tables: a table made up of marks showing how often something has happened
6 Frequency table: a table showing the number of times a piece of data occurs.
7 Raw data: the data as it was originally collected, before any processing has been done.
8 Ordered data: data that has been arranged to make it easier to use.
9 Grouped data: data that has been arranged into intervals.
10 Measures of central tendency: any values about which the distribution of the data may be considered to be roughly balanced.
11 Measures of dispersion: a measure of the way in which the data is spread out.
12 Mode: a type of average that is calculated by finding the quantity that occurs most often in a set.
13 Median: a type of average that is calculated by writing all quantities in order. The median is the quantity that is in the exact centre.
14 Mean: a type of average that is calculated by adding all of the quantities and then dividing by the number of quantities.
15 Range: the difference between the smallest value and the largest value in a set.

Organising data

Sorting categorical data by categories

Activity 2 Sort data by category

Learner's Book page 492

Guidelines for implementing this activity

- Work through the example in the Learner's Book with the class ensuring that learners follow the entire process.
- Ask questions, such as: What are some examples of data that can be sorted by category? Which category can be used for this data? (*alphabetically; numerically; chronologically; natural order*)
- When doing activities such as the one in Activity 2, always work through the information with learners and have a pertinent discussion about the information presented first before learners attempt the activity, such as: What question/s do you think he asked learners when collecting the data? How would the results of a survey such as this one be useful? Who was his sample? And so on.

Suggested answers

1 42 learners
2 6 flavours
3 Tally table

Flavour	Tally	Frequency
Cola	𝍸 𝍸 |	11
Lemon	𝍸 |||	8
Grenadilla	𝍸	5
Orange	𝍸 |	6
Grape	||||	4
Ginger-ale	𝍸 |||	8
Total		42

4 Cola
5 Grape
6 % (Ginger-ale) = $\frac{8}{42} \times 100\% = 19{,}05\%$
7 Number of cases to order

Flavour	Calculation	Number
Cola	$\frac{11}{42} \times 50 = 13$	13 + 1(Cola is preferred by many, so we can add 1 case that resulted from rounding)
Lemon	$\frac{8}{42} \times 50 = 9$	9
Grenadilla	$\frac{5}{42} \times 50 = 6$	6
Orange	$\frac{6}{42} \times 50 = 7$	7
Grape	$\frac{4}{42} \times 50 = 5$	5
Ginger-ale	$\frac{8}{42} \times 50 = 9$	9
Total		50

Sorting numerical data into groups or class intervals

Activity 3 Group data into intervals

Learner's Book page 494

Guidelines for implementing this activity

- Explain to learners that at times data needs to be grouped into intervals so that it is easier to manage.
- Let learners work through the example in pairs as a precursor to the activity, and compare their results with those of the rest of the class. You may even photocopy the example (without the solutions) beforehand and hand these out. Their progress should reflect their understanding, and those areas that they find challenging should then be addressed as you work through their solutions and challenges with them.
- Ask learners whether they can think of additional examples, and discuss how these examples are helpful to have as intervals.

Suggested answers

1 Tally/frequency table
2 1,90 metres
3 Difference = (1,90 m – 1,33 m) = 0,57 m
4 1,66 m

Class interval	Tally	Frequency
(1,30–1,39)	II	2
(1,40–1,49)	III	3
(1,50–1,59)	IIII	4
(1,60–1,69)	~~IIII~~	5
(1,70–1,79)	II	2
(1,80–1,89)	III	3
(1,90–1,99)	I	1
Total		20

Ordering data

Activity 4 Order numerical data

Learner's Book page 496

Guidelines for implementing this activity

- There are only two concepts regarding data ordering that learners must know, ascending and descending order.
- It will also be useful to tell learners that non-numerical data such as names can be ordered alphabetically. Here we are mainly concerned with numerical data.
- Let learners work through the example and the activity on their own.

Suggested answers

1 18,65 19,02 22,09 22,18 22,19 22,19 22,27 23,01 23,45
23,91 24,09 24,27 24,38 25,36 26,07 26,12
2 Difference = 26,12 cm – 18,65 cm = 7,47 cm
3 Length of 26,12 = 26,12 cm × (1 inch/2,540 cm) = 10,28 in
4 26,07cm = 26,07cm × (1 inch/2,540 cm) = 10,26. The person will wear a size slightly bigger than 8.

Summarising data using statistical measures

Measures of central tendency; Measures of dispersion

Activities 5–6 Calculate measures of central tendency; Determine the range

Learner's Book pages 498–499

Guidelines for implementing these activities

- Many learners enjoy this section and should be familiar with these concepts since they have covered them extensively in previous grades.
- Ensure that they know that they should first arrange data in ascending order before calculating measures of central tendency, and that they should always start by counting the number of items in a data set before they calculate the median. An example for calculating each of these measures is provided in the Learner's Book.
- Provide learners with additional examples of data sets, especially those whose items are decimals so that they get an opportunity to revise ordering decimals as well.

Suggested answers

Activity 5

1 Data in ascending order

6,5 7 7 7 7,5 8 8 8 8,5 9 10

Median = 8

Mode = 7

Mean = 86,5/11 = 7,86

2 Data in ascending order

129 218 238 248 265 275 326 386 415

Median = 265

Mode = no mode

Mean = $\frac{2\ 500}{9}$ = 277,8

3 Data in ascending order

523 900	562 400	562 400	596 600	602 800
613 900	626 200	628 600	634 100	645 700

Median = (602 800 + 613 900) ÷ 2 = 608 350

Mode = 562 400

Mean = 599 660

Activity 6

1 Data in ascending order

129 218 238 248 265 275 326 386 415

Range = 415 – 129

= 286

2 Data in ascending order

523 900	562 400	562 400	596 600	602 800
613 900	626 200	628 600	634 100	645 700

Range = 645 700 – 523 900 = 121 800

UNIT

3 Representing data

Unit overview

Learner's Book page 500
Recommended pacing: 3 hours

This unit focuses on the following:

- Pie charts
- Bar and double bar graphs
- Histograms
- Broken-line graphs
- Scatter plots
- Pie charts.

Resources: Learner's Book; exercise book; pencil; coloured pencils, ruler; graph book

Background information

Learners were introduced to the content in this unit in the Intermediate Phase, and continued working with these data representations in Grades 7 and 8. This section therefore serves as revision. Scatter plots are new in Grade 9.

Teaching guidelines

This section should be treated as revision and its teaching should be approached in this way. This means that instead of working through the information in the Learner's Book on each type of representation as if it was new, learners should be expected to describe and give information about each type of representation, and the points in the Learner's Book used to tie it all together. They should also be encouraged to do the examples on their own or in pairs.

Revising representations of data; Representing data in pie charts; Representing data in bar graphs and double bar graphs; Representing data in histograms; Representing data in broken-line graphs

Activities 1–5 **Revise representations of data; Work with a pie chart; Work with bar graphs and double bar graphs; Work with histograms; Work with broken-line graphs**

Learner's Book pages 500–511

Guidelines for implementing these activities

- Work through the information about these different representations in the manner described in the teaching guidelines above.
- Learners should work as independently as possible, and be allowed to work in pairs only if they require this type of support.

Suggested answers

Activity 1

1–5 Learners draw rough sketches to show the difference between pairs of graphs.

Activity 2

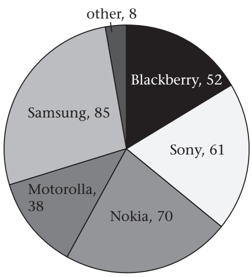
Given the following data:

Blackberry	Sony	Nokia	Motorola	Samsung	Other
52	61	70	38	85	8

1 314 people

2 22,29% prefer Nokia.

3 **Cellphone brand preferences**



4 Samsung

Activity 3

1 Number of schools = no. of primary schools + no. of high schools.
= 1 098 + 696 = 1 794

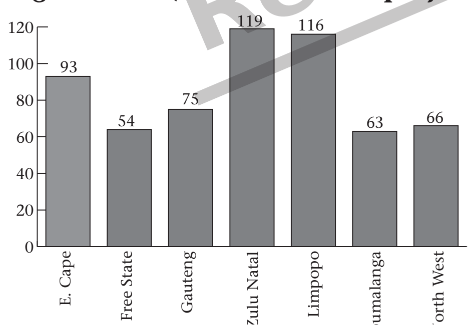
2 %Eastern Cape = $\frac{245}{1\ 794} \times 100\% = 13{,}6\%$

3 Bar graph

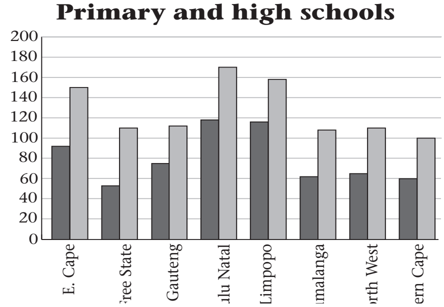
4 Double bar graph

5 In all 9 provinces more primary schools participated in the project than high schools.

High schools (Census @ School project)



Primary and high schools

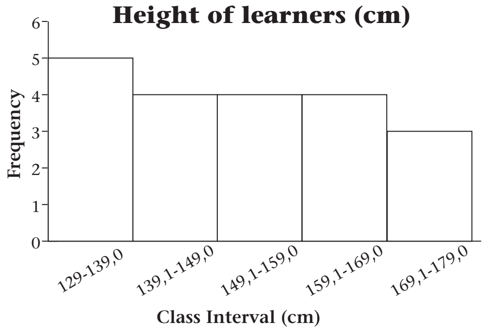


Activity 4

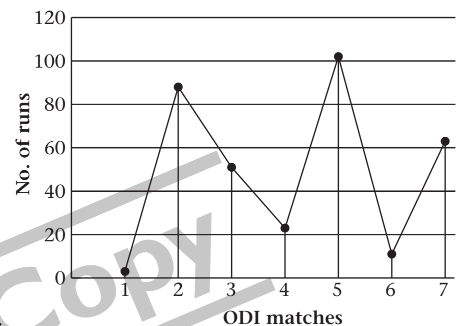
1 Frequency table

Class interval	129–139,0	139,1–149,0	149,1–159,0	159,1–169,0	169,1–179,0
Frequency	5	4	4	4	3

2 Histogram



3 129–139
4 7 learners
5 9 learners



Activity 5

1 Kallis's match scores

Matches	1	2	3	4	5	6	7
Score	3	88	51	23	102	11	63

2 Kallis's performance has been fluctuating and unpredictable.

Representing data in scatter plots

Activity 6 Work with scatter plots

Learner's Book page 513

Guidelines for implementing this activity

- Draw some examples on the board to explain to learners that scatter plots are used to represent two sets of data. The purpose is to determine if the values of each data set have any relationship with the values in the other data set. If there is a relationship between the two sets of data, the points on an $(x; y)$-plane will be scattered along a defined path. If there is no relationship between the two data sets, the points will be scattered all over with no observable pattern.
- Work carefully through the introduction and the example in the Learner's Book with the learners.
- Work through the summary as a form of revision and consolidation.

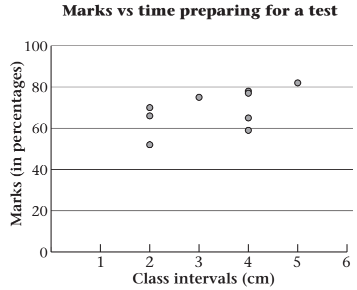
Suggested answers

1 Marks obtained versus time spent in preparation for tests

Test	Test 1	Test 2	Test 3	Test 4	Test 5	Test 6	Test 7	Test 8	Test 9
Time(hours)	3	2	5	4	4	2	2	4	4
Mark (%)	75	70	82	59	77	52	66	78	65

a Scatter plot

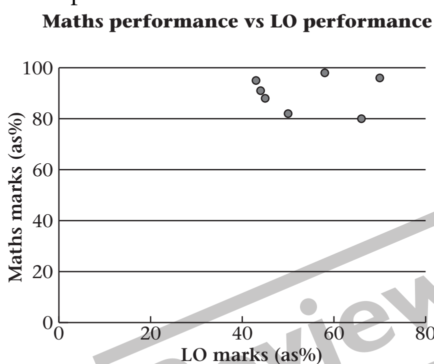
b The more the number of hours spent preparing for a test the better the marks obtained



2 Mathematics marks versus Life Orientation marks

LO	80	88	82	91	96	98	95
Maths	66	45	50	44	70	58	43

a Scatter plot



b There is no relationship between performance in Maths and performance in Life Orientation

UNIT 4

Analysing, interpreting and reporting data

Unit overview

Learner's Book page 514
Recommended pacing: 3 hours

This unit focuses on the following:
- Identifying sources of bias and error
- Comparing different representations of the same data
- Comparing and choosing between mode, median, mean and range
- Comparing graphs with different scales
- Analysing and interpreting misleading representations.

Resources: Learner's Book; exercise book; pencil; coloured pencils, ruler; graph book

Background information

Throughout this chapter learners have been exposed to analysis, interpretation and reporting based on data collected. This 5th stage of the data cycle can take place after completion of any or all of the other stages. The questions that are asked based on frequency tables, bar graphs, pie charts, measures of central tendency and spread, line and broken-line graphs and scatter plots all constitute analysis, interpretation and reporting data.

Teaching guidelines

In this section there is more emphasis on critical analysis of the processes of data cycle stages. Learners must develop the awareness that data presented in various forms to justify different points of view may be biased.

Identifying sources of bias and error; Comparing different representations of the same data

Activity 1 Compare different representations of the same data

Learner's Book page 516

Guidelines for implementing this activity

- Work through the introduction and the table with learners.
- Copy the example ahead of time (without the solutions) and let learners work through the example in pairs and compare their findings to the ones presented in the Learner's Book.
- Learners should work through the activity independently.

Suggested answers

1. 0–4 days and 10–14 days
2. 12 learners
3. Learners' own answers
4. This cannot be answered.
5. There is no data on learners who were not absent.
6. The size of the entire class could have been specified.
7. It can sometimes be difficult to tell exact amounts of data unless these are specified.

Comparing and choosing between mode, median, mean and range; Comparing graphs with different scales; Analysing and interpreting misleading representations

Outliers in a set of data; Outliers in a representation of data

Activities 2–4 **Compare and choose between mode, median, mean and range; Compare graphs with different scales; Analyse and interpret misleading representations**

Learner's Book pages 518–521

Guidelines for implementing these activities

- Copy the examples ahead of time (without the solutions) and let learners work through the examples in pairs and compare their findings to the ones presented in the Learner's Book.
- Learners should work through the activities independently.

Suggested answers

Activity 2

1 a Grade 12A Mathematics results: June exam (ascending order)

10	18	20	23	28	33	35	41	41	50	56	88

Mean = [sum of scores/number of scores]
$= \frac{443}{12} = 36{,}91\%$
Median = $\frac{33 + 35}{2}$
= 34%
Mode = 41%
Range = 88 – 10 = 78%

b Mean = $\frac{494 + x}{9} = 64$
$\therefore x = (64 \times 9) - 494$
= 576 – 494
= 82%

c Do not choose the mean, as the highest mark of 88% is an outlier. This would push the mean up and give the wrong impression. Rather choose the median of 34%, which is not influenced by the outlier, 88%.

d Range of Grade 12B's marks = 82 – 44 = 38. It is true that Grade 12A's mark range is higher. However, Mr Mpunga's statement is not correct, as he is using an inappropriate measure. Range only tells us how widely spread the scores are, and does not tell us anything about the data values between these two scores.

2 a Marks in Maths excessively low
b Marks in English excessively low
c No; another learner scored 99% in the English test
d No; another learner scored 100% in the Mathematics test
e There is a very good chance (about 6 out of 7, which is about 86%)

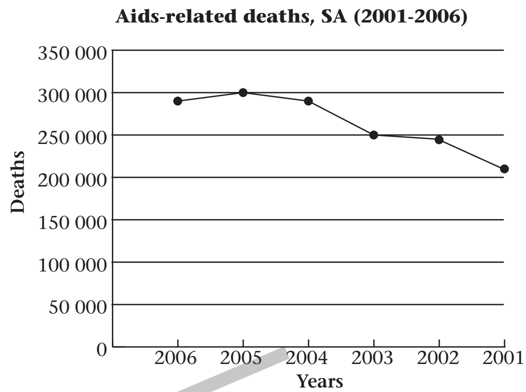
Activity 3

1 They all show profit increasing annually.
2 The profit is shown in different increments.
3 The third histogram.
4 The first two histograms are misleading because they represent round numbers annually and this is highly unlikely.
5 The trends of the first two histograms are identical.

Activity 4

Line graph

1 300 000
2 In 2001
3 Years are plotted in reverse order from 2006 t0 2001 giving the impression that AIDS deaths were decreasing during this period.



UNIT 5

Probability

Unit overview

Learner's Book page 522
Recommended pacing: 4,5 hours

This unit focuses on the following:

- revising terminology on probability
- finding probabilities of simple events
- comparing probability and relative frequency
- finding probabilities of compound events

Resources: Learner's Book; exercise book; pencil; coloured pencils, ruler; graph book

Background information

Learners have already been introduced to the probability concept in Grades 7 and 8. This section is intended to revise their knowledge so that they can use this knowledge to deal with the probability of compound events.

Teaching guidelines

Probability is the theory of chance. Although the curriculum document places it together with Data Handling, Probability is a completely different field and has its own terminology. Probability plays a very important role in our lives. For example, climatologists forecast weather conditions in terms probability, game results are predicted using probability, natural disasters are predicted using probability, and so on.

Revising terminology on probability

Activity 1 Revise terminology on probability

Learner's Book page 522

Guidelines for implementing this activity

- Learners may have come across these words previously but it is best to introduce these words to learners in an easy-to-understand way again.

Suggested answers

1 Event: something that may or may not happen, e.g. tossing a coin
2 Outcome: the result or effect of an action or event
3 Possible outcome: a result that can happen when an event happens
4 Favourable outcome: the outcome that is most preferred
5 Probability: the likelihood of something happening or not happening
6 Relative frequency: the ratio of the actual number of positive events to the total possible number of events
7 Trial: an event with more than one outcome
8 Experiment: a series of more than one trial

Finding probabilities of simple events

Activity 2 Find the probability of simple events

Learner's Book page 523

Guidelines for implementing this activity

- Revise the mathematical definition of probability paying special attention to probability scales and theoretical values in the scale.
- Ask learners to give examples of certain, impossible and possible events to show their understanding of probability scale.

Suggested answers

1 **a** 0 **b** 0 **c** $\frac{1}{2}$ **d** 1 **e** 0
2 **a** $\frac{1}{2}$ **b** $\frac{1}{2}$ **c** $\frac{1}{4}$ **d** $\frac{1}{13}$ **e** $\frac{3}{13}$
f $\frac{10}{13}$ **g** $\frac{4}{13}$

Comparing probability and relative frequency

Probability; Relative frequency; Probability versus relative frequency

Activity 3 Work with probability and relative frequency

Learner's Book page 525

Guidelines for implementing this activity

- Explore the meaning of theoretical probability and relative frequency as described in the Learner's Book. Work carefully through the introduction to ensure that learners understand what these terms, and that the relative frequency measures the experimental probability of an outcome.
- It is important that learners understand the difference between these terms.

Suggested answers

1 Results of rolling a dice 30 times

2	5	5	3	4	1	3	3	1	2
3	5	6	5	2	5	3	4	1	2
1	3	2	5	3	5	3	5	2	6

a Set of possible outcomes = {1; 2; 3; 4; 5; 6}

b $P(5) = \frac{4}{15}$

c '5' $= \frac{8}{30} = \frac{4}{15} = 0{,}2667$

2 **a** B; G; R; Y

b $P(\text{Red}) = \frac{5}{21}$

c $P(\text{Red}) = \frac{4}{20}$

Finding probabilities of compound events

Using a two-way table; Using a tree diagram

Activity 4 Find probabilities of compound events

Learner's Book page 527

Guidelines for implementing this activity

- In this section, learners' knowledge of the probability concept is expanded to determining probabilities of compound events. There are two types of compound events, namely, *simultaneous* and *successive* events. Tossing two coins at the same time is an example of a *simultaneous* compound event. Tossing a single coin twice is an example of a *successive* compound event.
- Tree diagrams and two-way tables are very useful techniques in determining probabilities of compound events. Theoretically, both the tree diagrams and two-way tables can be used to determine probabilities of compound events but it makes more sense to say that tree diagrams are suitable for successive compound events while two-way tables are suitable for simultaneous compound events.
- Work through the examples in the Learner's Book that show how these two techniques are used.

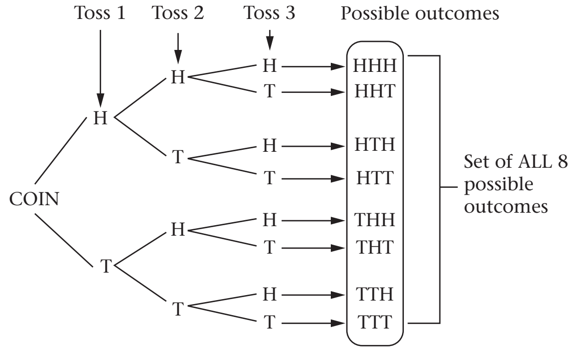
Suggested answers

1 Tree diagram

a $\frac{1}{8}$

b 0

c $\frac{3}{8}$



2 a

Jeans		Blue	Black	Brown
T-shirt	**White**	Blue jeans; White T-shirt	Black jeans; White T-shirt	Brown jeans; White T-shirt
	Black	Blue jeans; Black T-shirt	Black jeans; Black T-shirt	Brown jeans; Black T-shirt
	Red	Blue jeans; Red T-shirt	Black jeans; Red T-shirt	Brown jeans; Red T-shirt
	Pink	Blue jeans; Pink T-shirt	Black jeans; Pink T-shirt	Brown jeans; Pink T-shirt

b P(black jeans; black T-shirt) = $\frac{1}{12}$

c P(blue jeans; red T-shirt or pink T-shirt) = $\frac{2}{12}$

Chapter 21 Revision

Learner's Book page 528

Encourage learners to review the content covered before attempting the revision activities. The revision activities should be used to assess learners' progress thus far, and to assess where remediation may be required.

Suggested answers

1 a Problem/question posing, data collection, data organisation, data representation and data analysis, interpretation and reporting.

b i A population is the entire group of objects under investigation while a sample is a subgroup of a population chosen for data collection.

ii numerical data is data whose items are numbers while categorical data is data whose items are not numbers.

iii Biased sample is a sample chosen in such a way that it influences the results of an investigation while an unbiased sample is a sample representative of its population.

c Questionnaires, tables, interviews

2 a 72

b Categorical

Sportswear brand	Tally	Frequency
Adidas	卌 卌 I	11
Kappa	卌 IIII	9
Reebok	卌 卌 III	13
Nike	卌 卌 卌 卌 III	23
Puma	卌 卌 卌 I	16
Total		72

d Nike

e Kappa

3 a 1 161

b KZN, 922

c Gauteng, KZN and W. Cape

d Eastern Cape, Free State, Mpumalanga, North West, Limpopo, Northern Cape

e total number killed = passengers killed + pedestrians killed

= 5 206 + 4 614

= 9 820

f difference = 5 206 – 4 614 = 592

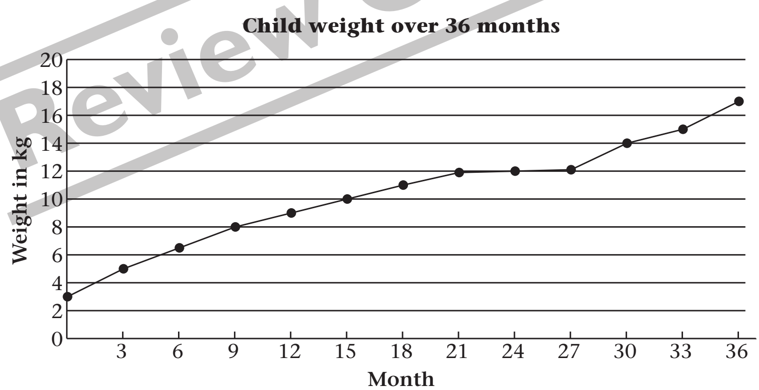
4 a Mean = $\frac{134,20}{13}$ = 10,32 kg

Median = 10,80

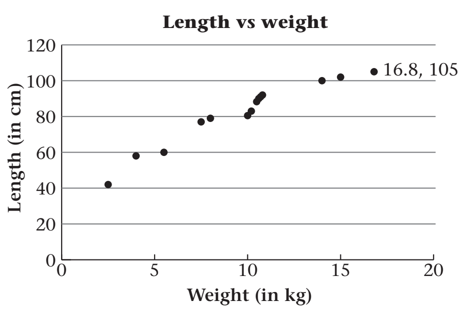
Mode = no mode

b 58 cm

c Line graph



d Scatter plot



e As the length of the baby increases, its weight increases

f BMI $= \frac{\text{weight in kg}}{(\text{length in metres})^2}$
$= \frac{12{,}2 \text{ kg}}{(0{,}92 \text{ m})^2}$
$= \frac{12{,}2 \text{ kg}}{0{,}8464 \text{ m}^2}$
$= 14{,}41 \text{ kg/m}^2$

5 **a** Probability $= \frac{1}{49}$ **b** Probability $= \frac{9}{49}$
c Probability $= \frac{4}{49}$ **d** Probability $= \frac{18}{49}$

6 **a** Relative frequency is the actual occurrence of an outcome in an experiment while probability is the mathematical possibility that an outcome will occur.
b $\frac{9}{49}$
c Draw 1: $\frac{1}{6}$; Draw 2: $\frac{2}{6}$; Draw 3: $\frac{0}{6}$; Draw 4: $\frac{3}{6}$; Draw 5: $\frac{2}{6}$; Draw 6: $\frac{3}{6}$
d 6b: theoretical probability; 6c: experimental probability
e 13

7 **a** 146 learners
b Probability(Hindu) $= \frac{41}{146}$
c Probability(Christian girl) $= \frac{46}{146}$
d Probability(Christian boy) $= \frac{34}{69}$, and probability(Muslim learner) = 1
So, the probability of choosing a Christian boy from a group of all boys is lower than the probability of choosing a Muslim learner from a group of Muslim learners.

Exam exemplar 2 (November) Memorandum

Learner's Book page 332

Paper 1 Algebra (Marks: 100)

Question 1

No.	N		Q	Q'
$\sqrt{11}$				
$\frac{3}{4}$				
0				
$3{,}\dot{2}\dot{5}$				
$\sqrt{-6}$				

[8]

Question 2

2.1.1 2,136

2.1.2 $\frac{49}{4}$

2.1.3 –3,875

2.1.4 2,6

2.2.1 $\frac{322}{25}$

2.2.2 $\frac{212}{99}$ **[6]**

Question 3

3.1.1 12

3.1.2 $3(2a)^2 - 2(2a) + 7$

$= 12a^2 - 4a + 7$

3.2 $3a + 2b - 3c$

$= 3b + 3a - 3c - (2a + 3b - 4c) = a + c$

3.3 5 : 9 = 1 218 learners

14 units = 1 218

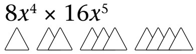
$\therefore$ 1 unit = 87

Boys = 9 × 87 = 783 **[10]**

Question 4

4.1.1 –10; –14

4.1.2 $8x^4 \times 16x^5$

4.2.1  (6)

4.2.2

Sketch	1	2	3	4	5	25
Matches	6	10	14	18	22	102

(4)

4.2.3 $T_n = 4n + 2$

4.2.4 $4n + 2 = 402$

$(4n = 400;\ n = 100)$ **[14]**

Question 5

5.1.1 $2x^2 + 6x + x + 3$ (2)

$= 2x^2 + 7x + 3$

5.1.2 $(2x - 3y)(2x - 3y)$ (3)

$4x^2 - 6xy - 6xy + 9y^2 = 4x^2 - 12xy + 9y^2)$

5.2 $(2x + 3)^2 - (x - 1)^2$

$(2x + 3)(2x + 3) - (x - 1)(x - 1) = 4x^2 + 6x + 6x + 9 - (x^2 - x - x + 1)$ (8)

$= 4x^2 + 12x + 9 - x^2 + 2x - 1 = 3x^2 + 14x = 8$

[13]

Question 6

6.1 $3xy(x - 2y^3)$ (2)

6.2 $2(x^4 - 16)$ (3)

$2(x^2 - 4)(x^2 + 4) = 2(x - 2)(x + 2)(x^2 + 4)$ (2)

6.3 $(x + 9)\ (x - 2)$ (3)

6.4 $2x\ (x^2 + 3x - 10)$

$2x(x + 5)(x - 2)$

[10]

Question 7

7.1 $\dfrac{2x(x-5)}{2(x^2-25)} = \dfrac{2x(x-5)}{2(x-5)(x+5)}$ (4)

$= \dfrac{x}{x+6}$

7.2 $\dfrac{4(4p+1) - 5(3p-2)}{20}$ (4)

$\dfrac{416p + 4 - 15p + 10}{20} = \dfrac{p+14}{20}$ **[8]**

Question 8

8.1.1 y^{10} (1)

8.1.2 $\dfrac{2n^2}{4n} = \dfrac{n}{2}$ (2)

8.1.3 $8x^{32}$ (2)

8.2.1 $\dfrac{9x^2y^6}{18x^2y^4} = \dfrac{y^2}{2}$ (3)

8.2.2 $\dfrac{4}{wv^2} \times \dfrac{y^2}{6wv^3} = \dfrac{4}{3v^4w^3}$ (3)

8.3 $b = 15$ (1)

8.4 $\sqrt{(-1)^2 - (-2)^3}$ (3)

$= \sqrt{1+8} = \sqrt{9} = 3$

8.5 $4{,}6 \times 10^{-6}$ (1)

8.6 $\dfrac{3 \times 10^8}{1000} = 3 \times 10^5$ km/second (2)

$3 \times 10^5 \times 60 \times 60 \times 24 \times 365$

$= 9{,}4608 \times 10^{12}$

[18]

Question 9

9.1.1 $-6x = -30$ (2)

$x = 5$

9.1.2 $2^x = 2^6$ (1)

$x = 6$

9.1.3 $3(x + 3) - 2(x - 6) = 6(x - 4)$

$3x + 9 - 2x + 12 = 6x - 24$

$-5x + 21 = -24$

$-5x = -45$

$x = 9$ (4)

9.1.4 $x = a$ or $x = b$ *(3)*

9.2

	Now	**Future**
John	$x + 5$	$x + 12$
Mary	$= \frac{3}{4}$	John
$x + 7$	$= \frac{3}{4}(x + 12)$	
$4x + 28$	$= 3x + 36$	
x	$= 8$	

(3)

[13]

Paper 2 Geometry [Marks: 135]

Question 1

1.1.1 $\angle A = 60°$ ($\triangle ABC$ is equilateral) (2)

1.1.2 $GC^2 = EC^2 + GE^2$

$= (4 \text{ cm})^2 + (3 \text{ cm})^2$

$= 16 \text{ cm}^2 + 9 \text{ cm}^2$

$\therefore GC = 5$ cm (4)

1.1.3 DB = 4 cm (opposite sides of a rectangle) (2)

1.1.4 In $\triangle ABC$ and $\triangle AFG$

$\angle A$ = common

$\angle ABC = \angle AFG$ (corresponding angles, DC||BC)

$\therefore \triangle ABC \ ||| \ \triangle AFG$ (equiangular triangles) (5)

1.1.5 $\frac{FG}{BC} = \frac{AG}{AC}$

$FG = BC \times \frac{AG}{AC}$

But BC = AC = 4 cm + 5 cm = 9 cm

$\therefore FG = 9 \text{ cm} \times \frac{4 \text{ cm}}{9 \text{ cm}}$

= 4 cm

or

Since $\triangle ABC ||| \triangle AFG$ (from 1.1.4 above)

$\triangle AFG$ is an equilateral triangle

$\therefore$ FG = 4 cm (4)

1.1.6 $\angle GXE = 180° - (\angle E = \angle EC)$ (sum of interior angles of a $\triangle$)

But $\angle EGC = \angle AGF$ (vertically opposite angles)

$\angle AGF = 60°$ ($\triangle AFG$ is equilateral)

$\therefore \angle EGC = 60°$

$\therefore \angle GCE = 180° - 90° - 60°$

$= 30°$ (5)

1.1.7 In $\triangle EGC$ and $\triangle DFB$
$\angle E = 90° = \angle D$ (DECB is a rectangle)
$GC = BF = 5$ cm
$DB = EC$ (opposite sides of a rectangle)
$\therefore \triangle EGC \equiv \triangle DFB$ (RHS) (5)

1.2.1 $180°(n - 2)$ (2)

1.2.2 $180°(10 - 2)$
$= 180° \times 8$
$= 1\ 440°$ (3)

[32]

Question 2

2.1.1 $\angle KRS = \angle K + \angle KQR$ (external angle of a triangle theorem)
$\therefore \angle KRS = 50° + 40° = 90°$ (3)

2.1.2 In $\triangle KLM$ and $\triangle KQR$
$\angle K$ is common
$\angle KLM = \angle KQR$ (corresponding angles, $PS || LN$)
$\therefore \triangle KLM \ ||| \ \triangle KQR$ (equiangular triangles) (5)

2.1.3 $KM = KR + RM$
$= 20$ mm $+ 40$ mm
$= 60$ mm (2)

2.1.4 $\frac{KQ}{KL} = \frac{KR}{KM}$
$KQ = 100 \text{ mm} \times \frac{20 \text{ mm}}{60 \text{ mm}}$
$= 33{,}3$ mm (4)

2.1.5 $\angle KRS = 90°$ (from 2.1.1 above)
$\angle KRS = \angle KMN$ (corresponding angles, $PS || LN$)
$\therefore \angle KMN = 90°$
$\angle KML = 180° - 90° = 90°$
$\therefore \triangle KML$ is a right-angled triangle (4)

2.1.6 $LM^2 = KL^2 - KM^2$ (Pythagoras' theorem)
$= (100 \text{ mm})^2 - (60 \text{ mm})^2$
$= 10\ 000 \text{ mm}^2 - 3\ 600 \text{ mm}^2$
$= 6\ 400$ (4)
$\therefore LM = \sqrt{6\ 400} = 80$ mm

2.1.7 Area $\triangle$ KLM $= \frac{1}{2}$base $\times$ height
$= \frac{1}{2} \times 80 \text{ mm} \times 60 \text{ mm}$
$= 2\ 400 \text{ mm}^2$
$= 24 \text{ cm}^2$ (4)

2.1.8 Area of trapezium QRML $= \frac{1}{2}(QR + LM) \times RM$
$= \frac{1}{2}(26{,}66 + 80 \text{ mm}) \times 40$ mm
$= 2\ 133{,}33$
Area of $\triangle KLM = 2\ 400$ mm (from 2.1.7 above)
Area of $\triangle KQR = \frac{1}{2} QR \times KR$
$= \frac{1}{2} \times 26{,}66 \text{ mm} \times 20$ mm
$= 266{,}6 \text{ mm}^2$
$\therefore$ area $\triangle KLM$ – area $\triangle KQR$
$= 2\ 400 \text{ mm}^2 - 266{,}6 \text{ mm}^2$
$= 2\ 133{,}3 =$ area of trapezium QRML as required (6)

2.2.1 F(2;–1); E(1;–3); G(4;–3); H(3;–4); I(2;–3) (5)

2.2.2 Translation by 3 units up (2)

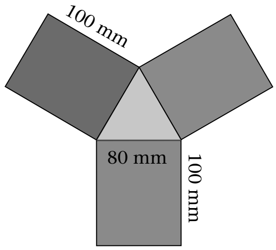
2.2.3 $F_1(-1;2)$; $E_1(-3;1)$; G(–3;4); H(–4;3) and I(–3;2) (5)

[44]

Question 3

3.1.1 100 mm (1)

3.1.2 (3)



3.1.3 Surface area $= 3 \times (100\text{ mm})^2 + 2(\frac{1}{2} \times 100\text{ mm} \times 80\text{ mm})$
$= 30\ 000\text{ mm} + 2\ 000\text{ mm}$
$= 32\ 000\text{ mm}^2$ (4)

3.1.4 Volume(prism) $= \frac{1}{2}$base × height × height
$= \frac{1}{2}(100\text{ mm} \times 80\text{ mm}) \times 100\text{ mm}$
$= 400\ 000\text{ mm}^3$ (4)

3.2.1 Hexahedron or cube (2)

3.2.2 6 faces (1)

3.2.3 $V - E + F = 2$ (Euler's theorem)
$V = 2 + E - F$
$= 2 + 12 - 6$
$= 8$ (3)

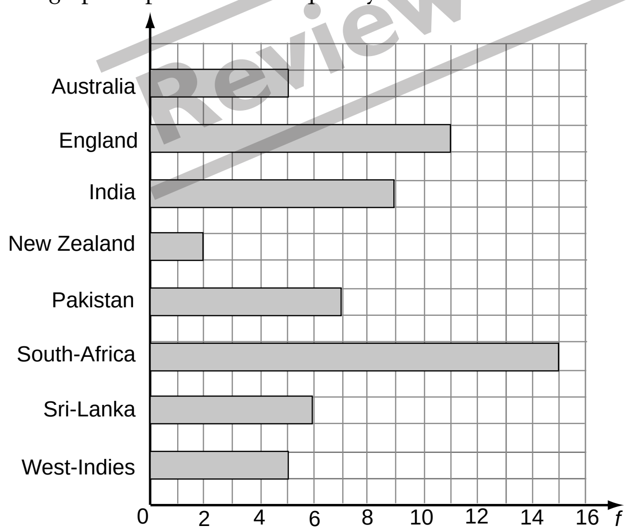
[18]

Question 4

4.1.1 60 people (2)

4.1.2 Bar graph or pie chart or frequency table (2)

4.1.3 (5)

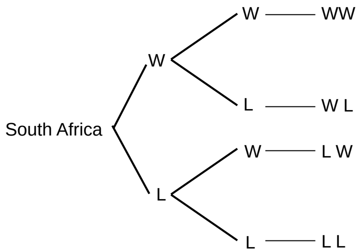


Result of survey to determine the favourite to win the T20-cricket tournament in 2012.

4.1.4.1 SA (1)

4.1.4.2 Kiwis (New Zealand) (1)

4.2.1 (6)



4.2.2 P(at least W) = $\frac{3}{4}$ (4)

4.2.3 P(WW) = $\frac{1}{4}$ (3)

4.3.1 Range = 98 – 61 = 37 (3)

4.3.2 61........65........67........72........75........80........86........90........98........98

Median = $\left[75 + \frac{80}{2}\right]$

= 77,5 (3)

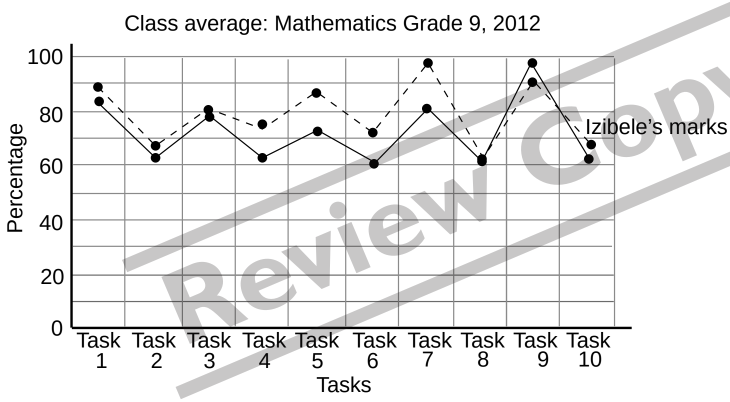
4.3.3 Mean = [sum/number of marks]

= $\frac{792}{10}$

= 79,2 (3)

4.4

4.4.1 (6)



4.4.2 Izibele's performance is above the class average except for test 9.
Izibele will pass Maths with a good score. (2)

[41]

Exam exemplar (November): Additional

Paper 1 Algebra (Marks: 120)

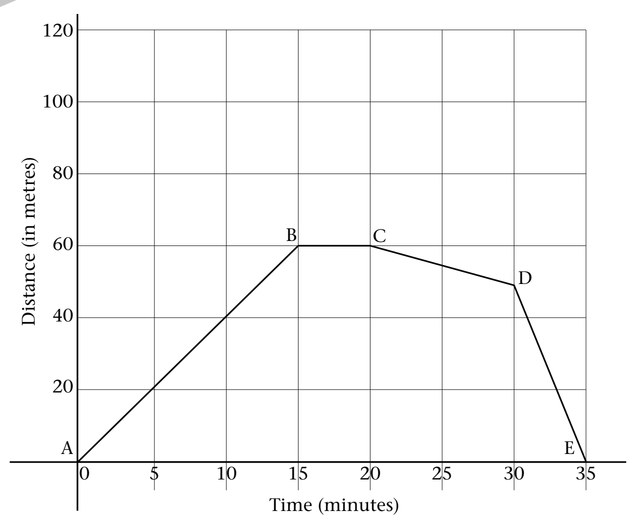
Instructions
Answer all questions.
Show all your workings.
Work neatly and write legibly.

Question 1

1.1 a and b are integers not equal to zero. Answer the following questions:
1.1.1 Write two ways of expressing the rational number, n, in terms of a and b. (4)
1.1.2 Show that any integer a can be written as a rational number. (2)
1.1.3 Write 0,45 as a rational number in its simplest form. (3)
1.1.4 Write $0,\overline{12}$ in the form $\frac{a}{b}$, where $a, b \in \mathbb{Z}$ en $b \neq 0$ (4)
1.2 Simplify the following expressions:
1.2.1 $(1\frac{3}{7} + 2) \div \frac{8}{3}$ (3)
1.2.2 $(1{,}82 \times 10^{-3}) \times (1{,}15 \times 10^{3})$ (2)
1.3 Terms of a number pattern are connected by the following rule:
$T_n = \frac{1}{n^2}$, where n is a natural number.
1.3.1 Calculate the first term of the pattern. (2)
1.3.2 Write down the first four terms of the pattern. (3)
1.3.3 Which term of the pattern is $\frac{201}{100}$? (2)
[25]

Question 2

1 Bella goes to the shop to buy milk and bread every Saturday morning. The graph alongside shows a typical trip from Bella's home to the shop and back. Use the graph to answer the following questions:
2.1.1 How far is the shop from Bella's home? (2)
2.1.2 Calculate Bella's average speed from home to the shop. (4)
2.1.3 Describe briefly what might be happening to Bella between points B and C. (2)



2.1.4 How many hours did Bella take to complete her trip? (3)

2.1.5 Between which two consecutive points was Bella's speed the fastest? (2)

2.2 Kamve and Vuyo each decided to invest R2 000 with different banks for a period of 24 months. Kamve's bank offers an interest rate of 8% compounded annually. Vuyo's bank offers simple interest values only. They both have the same amount in their accounts at the end of the investment period

2.2.1 Calculate the amount in Kamve's account after 24 months. (4)

2.2.2 Determine the simple interest rate that Vuyo received from his bank. (4)

[21]

Question 3

3.1 Consider the following polynomial:

$4px^5 + 10x^3 - qx^2 - (r - s)x^6 + 3x - 2$

3.1.1 How many terms does the polynomial have? (2)

3.1.2 Write down the degree of the polynomial. (2)

3.1.3 Write down the numerical coefficient of x^6. (2)

3.2 Simplify the following expressions and write all terms with positive exponents:

3.2.1 $\frac{-tp^4q^3r^2}{4p^2q^{-2}r^3}$ (3)

3.2.2 $4x(x - y) - 5x(2x - y)$ (3)

3.2.3 $2x^2(xy - 3y + 4x) - (x^2y + 2x^3 - x^3y) + 4x^2y$ (4)

3.2.4 $(5p + 2q)(-2q + 5p)$ (3)

3.2.5 $(4x - 3y)^2$ (4)

3.2.6 $\frac{2b + c}{3} + \frac{5b - c}{4}$ (4)

[27]

Question 4

4.1 Factorise the following expressions:

4.1.1 $(a - b) + 3(b - a)$ (2)

4.1.2 $27x^2 - 12$ (4)

4.1.3 $a^2 - 5a + 6$ (3)

4.2 Simplify: $\frac{x^2 - 9}{x^2 - x - 6}$ (4)

4.3 Solve:

4.3.1 $2x + 9 = 5 - 2x$ (3)

4.3.2 $4(3k - 1) + 3(k - 4) = -11k$ (4)

4.3.3 $(2x - 3)(x + 1) = 0$ (3)

4.3.4 $\frac{x + 1}{2} = \frac{2x - 3}{3}$ (4)

4.3.5 $3^x - \frac{1}{27} = 0$ (3)

[30]

Question 5

5.1 The incomplete table below shows output values for certain input values for f and g.

x	–3	–2	–1	0	1	2	3	4
$f(x) = 2x - 3$	–9	…	…	–3	…	…	…	5
$g(x) = -2x + 1$	…	–3	…	…	–1	…	–5	…

5.1.1 Copy and complete the table in your answer book. (5)

5.1.2 On the same set of axes draw graphs of f and g. (6)

5.1.3 On your graphs mark the point of intersection with X and write down the x-coordinate of point X. (2)

5.1.4 Solve for x in the equation: $2x - 3 = -2x + 1$ (3)

5.1.5 How do your answers to **5.1.3** and **5.1.4** compare? (1)

[17]

[Total: 120]

Paper 2 Geometry (Marks: 100)

Instructions
Answer all the questions.
Show all your working.
Work neatly and write legibly.
Use π = 3,14 where appropriate and round all answers to two decimal places.

Question 1

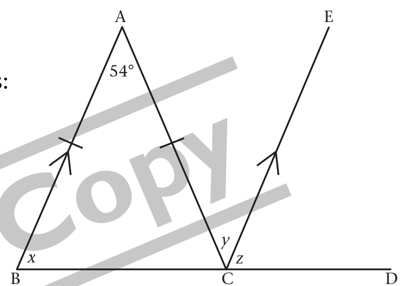
1 State whether the following statements are true or false. If a statement is false give an example to justify.

1.1 Any two equilateral triangles are similar. (2)
1.2 Right-angled triangles have one internal angle equal to 900. (2)
1.3 A trapezium is a parallelogram. (2)
1.4 The sum of interior angles of a hexagon is 5400. (2)
1.5 All polyhedrons are prisms. (2)

[10]

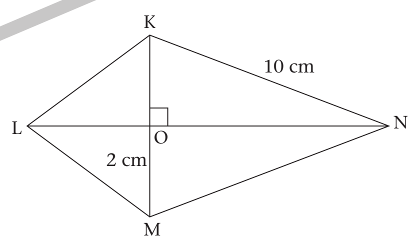
Question 2

2.1 In the diagram alongside ΔABC is isosceles with ∠A = 540. AB||EC. Calculate, with reasons:



2.1.1 the value of x (3)
2.1.2 the value of y (3)
2.1.3 the value of z. (3)
2.1.4 Show that: $y + z = \angle A + x$ (3)

2.2 In the figure alongside, KLMN is a kite. The diagonals LN and KM intersect at O.

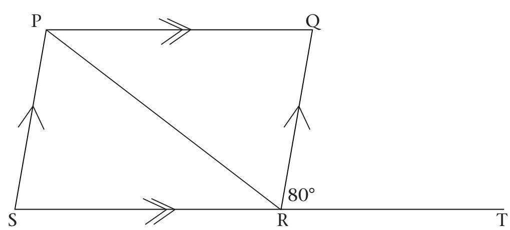


2.2.1 Write down the length of KO. (2)
2.2.2 Calculate the length of ON. (5)
2.2.3 Prove that △LKN≡ΔLMN. (7)

[26]

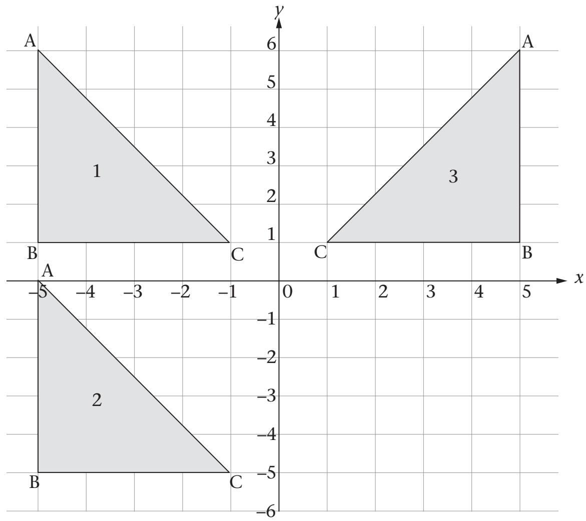
Question 3

3.1 In the diagram alongside, PQRS is a parallelogram. SR is produced to T such that PQ||ST and ∠QRT = 800.



3.1.1 Calculate, with reasons, the size of ∠Q. (3)
3.1.2 Write down the size of ∠S. (2)
3.1.3 Determine the size of ∠SPQ. (4)
3.1.4 Show that △PQR|||△RSP. (4)

3.2 The diagram alongside shows transformation of ΔABC on a Cartesian plane. Answer the following questions.



3.2.1 Write down the coordinates of the vertices A, B and C. (3)

3.2.2 Describe the transformation that produced triangle 2 from triangle 1. (2)

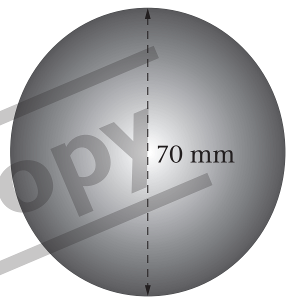
3.2.3 Describe the transformation that produced triangle 3 from triangle 1. (2)

3.2.4 Write down the coordinates of the vertices of triangle 3. (3)

[23]

Question 4

4.1 The sphere alongside has a diameter of 70 mm.



4.1.1 Calculate the circumference of the sphere in cm. (4)

4.1.2 Determine the surface area of the sphere in mm^2. (4)

4.1.3 Determine the volume of the sphere to the nearest litre. (5)

4.2 A certain Platonic shape is a pyramid.

4.2.1 Write down the name of the shape. (2)

4.2.2 How many faces does the shape have? (2)

4.2.3 Determine the number of vertices of the shape. (2)

[19]

Question 5

5.1 The following are final Mathematics results for the Grade 9 class of 2011 at Sitholeni Junior School.

34	45	30	55	70	45	40	44	38	66
50	58	63	80	64	77	66	85	50	52
66	13	28	37	86	38	24	49	55	57

5.1.1 How many Grade 9 learners were registered at the school in 2011? (2)

5.1.2 Arrange the Grade 9 learner marks in ascending order. (3)

5.1.3 Determine the range of the scores. (2)

5.1.4 Calculate the mean, median and mode for the learner results. (5)

5.2 Learners whose final marks in Maths are less than 40% are advised to take Maths Literacy in Grade 10. To advise parents on subject choices, Mr Scotch, the Maths teacher prepared the following frequency table, but couldn't finish it.

Code	Interval	Tally	Frequency
1	0 – 29	|||	3
2	30 – 39	…	…
3	40 – 49	~~||||~~	5
4	50 – 59	…	…
5	60 – 69	…	…
6	70 – 79	…	…
7	80 – 100		

5.2.1 Copy and complete Mr Scotch's frequency table (6)

5.2.2 How many learners will be advised to take Maths Literacy in Grade 10? (1)

5.2.3 Determine the probability of randomly choosing a learner with a score of 80% and above in the Grade 9 class. (3)

[22]

[Total: 100]

Exam exemplar (November) Additional Memorandum

Paper 1: Algebra [Marks: 120]

Question 1

1.1.1 $n = \frac{a}{b}$ or $n = \frac{b}{a}$ (4)

1.1.2 $a = \frac{a}{1}$ which is rational (2)

1.1.3 $0,45 = \frac{45}{100}$

$= \frac{9}{20}$ (3)

1.1.4 Let $r = 0,12121212\ldots$

Then $100r = 12,121212\ldots$

So $100r - r = 12,121212\ldots - 0,121212$

$99r = 12$

$r = \frac{12}{99} = \frac{4}{33}$ (4)

1.2.1 $(1\frac{3}{7} + 2) \div \frac{8}{3}$

$= (\frac{10}{7} + 2) \div \frac{8}{3}$

$= \frac{24}{7} \times \frac{3}{8}$

$= \frac{9}{7}$ (3)

1.2.2 $(1,82 \times 10–3) \times (1,15 \times 10^3)$

$= 1,82 \times 1,15$

$= 2,093$ (2)

1.3 $T_n = \frac{1}{n^2} + 2$

1.3.1 $T_1 = \frac{1}{1^2} + 2 = 1 + 2 = 3$ (2)

1.3.2 $3;\ \frac{1}{2^2} + 2;\ \frac{1}{3^2} + 2;\ \frac{1}{4^2} + 2;\ 3;\ \frac{9}{4};\ \frac{19}{9};\ \frac{33}{16}$ (3)

1.3.3 $\frac{201}{100} = \frac{1}{n^2} + 2$

$\frac{1}{n^2} = \frac{201}{100} - 2$

$\frac{1}{n^2} = \frac{1}{100}$

$n^2 = 100$

$\therefore n = +10$

10^{th} term, or by inspection. (2)

[25]

Question 2

2.1.1 60 metres (2)

2.1.2 Average speed = $\frac{60 \text{ m}}{15 \text{ m}}$
$= 4$ m/min (4)

2.1.3 She is at the shop buying milk and bread (2)

2.1.4 Bella took 35 min
$= 35 \text{ min} \times [1 \text{ h}/60 \text{ min}]$
$= 0{,}58$ h (3)

2.1.5 Between D and E (2)

2.2.1 Amount $\text{P}(1 + 8)n$
$= \text{R2 000}(1 + \frac{8}{10})^2$
$= \text{R2 332,80}$ (4)

2.2.2 For Vuyo, P = R2 000; A = R2 332,80; $n = 2$
Using scruple interest formula
$1 + nr = \frac{\text{A}}{\text{P}}$
$nr = \frac{\text{A}}{\text{P}} - 1$
$= \frac{\text{R2 332,80}}{\text{R2 000}} - 1$
$= \text{R2 332,80} - \frac{\text{R2 000}}{\text{R2 000}}$
$r = 0{,}0832$
$= 8{,}32\%$ (4)

[21]

Question 3

3.1.1 6 terms (2)

3.1.2 6 (2)

3.1.3 $(r - s)$ (2)

3.2.1 $\frac{-\cancel{4}p^4q^3r^2}{\cancel{4}p^2q^{-2}r^3}$
$= \frac{-p^2q^5}{r}$ **(3)**

3.2.2 $4x(x - y) - 5x(2x - y)$
$= 4x^2 - 4xy - 10x^2 + 5xy$
$= -6x^2 + xy$ (3)

3.2.3 $2x^2(xy - 3y + 4x) - (x^2y + 2x^3 - x^3y) + 4x^2y$
$= 2x^3y - 6x^2y + 8x^3 - x^2y - 2x^3 + x^3y + 4x^2y$
$= 3x^3y - 3x^2y + 6x^3$ (4)

3.2.4 $(5p + 2q)(-2q + 5p)$
$= -10pq + 25p^2 - 4q^2 + 10pq$
$= 25p^2 - 4q^2$ (3)

3.2.5 $(4x - 3y)^2 = 4x - 3y)(4x - 3y)$
$= 16x^2 - 12xy - 12xy + 9y^2$
$= 16x^2 - 24xy + 9y^2$ (4)

3.2.6 $\frac{2b + c)}{3} + \frac{5b - c)}{4}$
$= \frac{4(2b + c) + 3(5b - c)}{12}$
$= \frac{8b + 4c + 15b - 3c}{12}$
$= \frac{23b + 7c)}{12}$ (4)

[27]

Question 4

4.1.1 $(a - b) + 3(b - a)$

$= (a - b) - 3(a - b)$

$= (a - b)(1 - 3)$

$= -2(a - b)$ (2)

4.1.2 $27x^2 - 12$

$= 3(9x^2 - 4)$

$= 3[(3x)^2 - (2)^2]$

$= 3[(3x - 2)(3x + 2)]$

$= 3(3x - 2)(3x + 2)$ (4)

4.1.3 $a^2 - 5a + 6$

$= (a - 2)(a - 3)$ (3)

4.2 $\frac{x^2 - 9}{x^2 - x - 6}$

$= \frac{\cancel{(x - 3)}(x + 3)}{\cancel{(x - 3)}(x + 2)}$

$= \frac{(x + 3)}{(x + 2)}$ (4)

4.3.1 $2x + 9 = 5 - 2x$

$2x + 2x = -4$

$x = -1$ (3)

4.3.2 $4(3k - 1) + 3(k - 4) = -11k$

$12k - 4 + 3k - 12 = -11k$

$12k + 11k + 3k = 12 + 4$

$26k = 16$

$\mathrm{k} = \frac{16}{26}$

$\mathrm{k} = \frac{8}{13}$ (4)

4.3.3 $(2x - 3)(x - 1) = 0$

Either $(2x - 3) = 0$ or $x + 1 = 0$

$2x = 3$ or $x = -1$

$\therefore x = \frac{3}{2}$ or $x = -1$ (3)

4.3.4 $\frac{(6)x + 1}{2} = \frac{(6)2x - 3}{3}$

$3x + 3 = 4x - 6$

$3x - 4x = -6 - 3$

$-x = -9$

$\therefore x = 9$ (4)

4.3.5 $3x - \frac{1}{27} = 0$

$3x = \frac{1}{27}$

$3x = 3{-3}$

$\therefore x = -3$ (3)

[30]

Question 5

5.1.1

x	–3	–2	–1	0	1	2	3	4
$f(x) = 2x - 3$	–9	–7	–5	–3	–1	1	3	5
$g(x) = -2x + 1$	7	5	3	1	–1	–3	–5	–7

(5)

5.1.2 [insert graph] (6)

5.1.3 $x = 1$ (2)

5.1.4 $2x - 3 = -2x + 1$

$4x = 1 + 3$

$4x = 4$

$x = 1$ (3)

5.1.5 The values are the answers. (1)

[17]

[Total: 120]

Paper 2 Geometry (Marks: 100)

Question 1

1.1 True (2)

1.2 True (2)

1.3 False. Only one pair of opposite sides of a trapezium is parallel; a parallelogram has both sides parallel. (2)

1.4 False. A hexagon has six sides and so can be divided into four triangles. Its interior angles add up to 720°. (2)

1.5 False. A tetrahedron is a polyhedron but not a prism. (2)

[10]

Question 2

2.1.1 $x + A\hat{C}B = 180° - 54°$ (sum of interior angles of $\triangle$ABC)

But $x = A\hat{C}B$ (base angles of isosceles $\triangle$s ABC)

$\therefore x = \frac{126°}{2}$

$x = 63°$ (5)

2.1.2 $y = \hat{A}$ (alternate angles, AB||EC)

$\therefore y = 54°$ (3)

2.1.3 $z = \hat{B} = x$ (corresponding angles< AB||EC)

$\therefore z = 63°$ (3)

2.1.4 $y + z = 54° + 63° = 117°$

$\hat{A} + x = 54° + 63° = 117°$

$\therefore y + z = \hat{A} + x$ as required (3)

2.2.1 KO = 2 cm (2)

2.2.2 $ON^2 = KN^2 - KO^2$ (Pythagoras theorem)

$= (10 \text{ cm})^2 - (2 \text{ cm})^2$

$= 100 \text{ cm}^2 - 4 \text{ cm}^2$

$= 96 \text{ cm}^2$

$\therefore$ ON = 9,78 cm (5)

2.2.3 In $\triangle$LKN en $\triangle$LMN

KN = MN (adjacent sides of a kite)

LK = LM (adjacent sides of a kite)

LN is common

$\therefore \triangle LKN \equiv \triangle LMN$ (SSS) (5)

[26]

Question 3

3.1.1 $Q\hat{R}T = \hat{Q}$ (alternate angles, PQ||ST)
$\therefore \hat{Q} = 80°$ (3)

3.1.2 $\hat{S} = Q\hat{R}T = 80°$ (corresponding angles, PS||QR) (2)

3.1.3 $S\hat{P}Q = S\hat{R}Q$ (opposite angles of a ||rm)
Maar $S\hat{R}Q = 180° - 80° = 100°$ (angles on a straight line)
$\therefore S\hat{P}Q = 100°$ (4)

3.1.4 In $\triangle PQR$ en $\triangle RSP$
$\hat{S} = \hat{Q}$ (opposite angles of a ||rm)
$S\hat{P}R = P\hat{R}Q$ (alternate angles, PS||QR)
$\therefore \triangle PQR \equiv \triangle RSP$ (4)

3.2.1 A(–5; 6); B(–5; 1); C(–1; 1) (3)

3.2.2 Translation 6 units down. (2)

3.2.3 Reflection on the *y*-axis. (2)

3.2.4 A(5; 6); B(5; 1); C(1; 1) (3)

[23]

Question 4

4.1.1 Circumference $= \pi d$
$= 3,14 \times 70$ mm
$= 219,8$ mm
$= 21,98$ cm (4)

4.1.2 $SA = 4\pi r^2$
$= 4(3,14) \times (35 \text{ mm})^2$
$= 15\ 386 \text{ mm}^2$ (4)

4.1.3 Volume $= \frac{4}{3} \pi r^3$
$= \frac{4}{3} (3,14)(35 \text{ mm})^3$
$= 179\ 503,33 \text{ mm}^3$
≈ 180 ml (5)

4.2.1 Tetrahedron (2)

4.2.2 4 (2)

4.2.3 4 (2)

[19]

Question 5

5.1.1 30 learners (2)

5.1.2 13; 24; 28; 30; 34; 37; 38; 38; 40; 44; 45; 45; 49; 50; 52; 55; 55; 57; 58; ; 63; 64; 66; 66; 66;
70; 77; 80; 85; 86 (3)

5.1.3 Range = 86 – 13 = 73 (2)

5.1.4 Mean $= \frac{\text{sum of scores}}{30}$

$= \frac{1\ 520}{30}$

$= 50,66$

Median $= \frac{50 + 52}{2}$

$= 51$

Mode = 66 (5)

5.2.1

Code	**Interval**	**Tally**	**Frequency**
1	0-29	𝍷𝍷𝍷	3
2	30-39	𝍸	5
3	40-49	𝍸	5
4	50-59	𝍸 𝍷𝍷	7
5	60-69	𝍸	5
6	0-79	𝍷𝍷	2
7	80-100	𝍷𝍷𝍷	3
TOTAL			30

(6)

5.2.2 8 (1)

5.2.3 $\frac{3}{80}$ (3)

[22]

[Total: 100]

Resources

Question words: Verbs to introduce questions/activities

Verbs to address skills to be assessed at the various levels, from the lower to the higher cognitive levels. Below is a list of such verbs.

Recall of knowledge:

Choose	Identify	Provide
Define	Indicate	Select
Describe	Label	State
Fill in	Mention	Supply
Give	Name	Write notes on

Comprehension/insight:

Arrange	Estimate	Interpret
Change	Expand	Justify
Compare	Explain	Predict
Deduce	Generalise	Rewrite
Distinguish between	Indicate	Tabulate

Application:

Apply	Deduce	Formulate
Calculate	Differentiate	Integrate
Change	Discover	Solve
Construct	Discuss critically	Suggest

Analysis:

Break down	Discriminate	Indicate
Deduce	Distinguish between	Select
Differentiate	Illustrate	Separate

Synthesis:

Categorise	Compose	Form
Classify	Create	Rewrite
Combine	Design	Sum up
Compile	Estimate	

Evaluation:

Compare	Distinguish between	Support
Contrast	Interpret	Value
Criticise	Justify	Verify
Deduce	Sum up	

1 cm-Square paper

0.5 cm-Square paper