OXFORD

Successful

Mathematics

TEACHER'S GUIDE

GRADE 8

WITH EXAM INFO AND EXEMPLAR PAPERS

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How this course works

This series meets the requirements of the National Curriculum and Assessment Policy Statement (CAPS) for the Senior Phase.
In Grade 8, this series consists of two core components: a Learner's Book and a Teacher's Guide.

The Learner's Book

The Learner's Book provides content and subject knowledge as well as activities for learners to develop, practise and consolidate their mathematical knowledge and skills. Written texts are supported by diagrams and illustrations that help to explain content. All examples, exercises and illustrations are representative of all cultural groups. Exercises steadily become more challenging so that learners progressively develop their understanding of concepts.

The Teacher's Guide

The Teacher's Guide provides you, the teacher, with all the planning, teaching and assessment tools. Teachers receive guidance on how to teach important concepts and advice on how to teach each exercise.

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How to use this Teacher's Guide

The Teacher's Guide covers all Mathematics content and provides rich resources to ensure complete curriculum coverage and the successful development of mathematical concepts and skills in Grade 8.
The Teacher's Guide supports you by providing support and information on how to teach the subject. Some of the features you will find in the Teacher's Guide include the following:

The CAPS for Mathematics

Each CAPS document provides:
- an overview of topics and content areas for its subject (see below)
- the weighting prescribed for each content area (see below)
- a teaching plan for the subject (see Section C – Planning and assessment).

The following content areas comprise the Senior Phase Mathematics curriculum:
- Numbers, Operations and Relationships
- Patterns, Functions and Algebra
- Space and Shape (Geometry)
- Measurement
- Data Handling.

Each content area has a prescribed weighting to ensure complete curriculum coverage.

Content Area	Grade 7	Grade 8	Grade 9
Numbers, Operations and Relationships	30%	25%	15%
Patterns, Functions and Algebra	25%	30%	35%
Space and Shape (Geometry)	25%	25%	30%
Measurement	10%	10%	10%
Data Handling (Statistics)	10%	10%	10%
Total	100%	100%	100%

Defining the subject, CAPS and teaching terminology for the teacher.

SECTION 4 Teaching and learning Mathematics

Chapter 20 Programme of Assessment

This chapter provides all the resources you need to ensure your learners meet the requirements for promotion.
- Two options for each of the required formal programme of assessment tasks.
- Exemplar exams for learners to use to practise for their exams. These exemplars are in the Learner's book with the memoranda supplied in this section of the Teacher's Guide.
- Control tests for you to use as part of the POA, along with the memoranda.
- A June and a December exam paper, with memoranda.

The following table lays out the programme for you. The shaded cells are only in the Teacher's Guide to ensure the tests and exams are unseen by the learners.

Term	Task		Learner's Book page	Teacher's Guide page
1	Assignment	Option 1 Ratio, rate and integers	431	333
		Option 2 Exponents and algebra	432	335
	Control Test 1			337
2	Investigation	Option 1 Constructions	147, 150, 151	341
		Option 2 Geometry	156	342
	Control Test 2			343
	Exemplar June exam	Exemplar paper for revision purposes	434	352
	June Exam			347

Providing Formal Assessment Tasks as required by the CAPS.

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Unit 3 Calculations with whole numbers

Learner's book page 17

Unit focus:
- how to check answers using the inverse operation
- estimation
- rounding off and compensating
- adding, subtracting and multiplying in columns
- long division
- using a calculator for larger numbers

Background information on calculations with whole numbers
Learners have performed these operations in depth in the intermediate phase. In Grade 8 the emphasis continues to be less on performing the actual calculation itself (learners increasingly use calculators to do this), but on judging the reasonableness of the solution they get on a calculator. However, in order to be able to understand the operation and calculation, learners need to revise these calculations at the beginning of the year to ensure they can perform them correctly.

In order to judge the reasonableness of a solution, learners need to be able to estimate effectively, and round off numbers. Learners have also covered this in depth in earlier grades; however these skills are still in great demand, and need to be practised and sharpened in Grade 8.

Structuring the course into units, with advice on pacing content according to the CAPS.

The National Curriculum and Assessment Policy Statements

This series is based on the National Curriculum Statement Grades R–12 (NCS, January 2012), which is the policy document for learning and teaching in South Africa. The NCS consists of three documents, namely:

- Curriculum and Assessment Policy Statements (CAPS) for all approved subjects for Grades R–12
- National Policy pertaining to the Programme and Promotion Requirements of the National Curriculum Statement Grades R–12
- National Protocol for Assessment Grades R–12 (January 2012).

Each CAPS document has four sections:

- Section 1: Introduction to the Curriculum and Assessment Policy Statements for the specific subject
- Section 2: The specific subject's aims, time allocations and requirements to offer it as a subject
- Section 3: Overview of topics and teaching plan for the specific subject
- Section 4: Assessment in the specific subject.

Sections 2, 3 and 4 of the CAPS documents, together with the National Policy pertaining to the Programme and Promotion Requirements of the NCS, represent the norms and standards of the National Curriculum Statement Grades R–12. Together these documents are the basis for determining minimum outcomes, processes and procedures for the assessment of learner achievement in public and independent schools.

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Instructional time allocation

The instructional time in the Senior Phase is as follows:

Subject	Teaching hours per week	Total hours per term
Home Language	5	50
First Additional Language	4	40
Mathematics	4,5	45
Natural Science	3	30
Social Sciences	3	30
Technology	2	20
Economic Management Sciences	2	20
Life Orientation	2	20
Creative Arts	2	20
Total	27,5	275

The CAPS for Mathematics

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Each CAPS document provides:
- an overview of topics and content areas for its subject (see below)
- the weighting prescribed for each content area (see below)
- a teaching plan for the subject (see Section C – Planning and assessment).

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- Space and Shape (Geometry)
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- Data Handling.

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Measurement	10%	10%	10%
Data Handling (Statistics)	10%	10%	10%
Total	100%	100%	100%

Topic overview

	Grade 7	Grade 8	Grade 9
Term 1	• Mental calculations • Order and compare whole numbers (9 digits) • Properties of whole numbers • Calculations with whole numbers • Addition and subtraction (6 digits) • Multiplication and division (4-digit by 2-digit) • Multiples and factors (of 2- and 3-digit whole numbers) • Prime factors • LCM and HCF (3-digit whole numbers) • Solve problems (ratio and rate; percentages, decimal fractions; financial context) • Exponents • Measure angles • Construct geometric figures • Classify 2D shapes • Similar and congruent 2D shapes • Solve problems	• Order and compare whole numbers (prime numbers to 100) • Properties of whole numbers • Calculations with whole numbers • Multiples and factors • Solve problems (ratio and rate; percentages, decimal fractions; financial context) • Count, order and compare integers • Calculations with integers • Properties of integers • Solve problems • Represent numbers in exponential form • Calculations in exponential form • Laws of exponents • Numeric and geometric patterns • Input and output values or rules for patterns and relationships • Equivalent forms • Algebraic language • Expand and simplify algebraic expressions • Set up equations and solve by inspection	• Properties of whole numbers • Calculations with whole numbers • Multiples and factors • Solve problems (ratio and rate; direct and indirect proportion; percentages, decimal fractions; financial context) • Calculations with integers • Properties of integers • Solve problems • Common fractions • Decimal fractions • Exponents • Calculations in exponential form • Solve problems • Numeric and geometric patterns • Input and output values or rules for patterns and relationships • Equivalent forms • Algebraic language • Expand and simplify algebraic expressions • Equations (use factorisation of the form where a product of factors = 0)
Term 2	• Common fractions • Percentages • Decimal fractions • Equivalent forms • Solve problems • Input and output values for patterns and relationships • Equivalent forms (verbal, flow diagrams, tables, formulae, number sentences) • Area and perimeter of 2D shapes • Convert SI units • Surface area and volume of 3D objects	• Algebraic language • Expand and simplify algebraic expressions • Set up equations and solve by using additive and multiplicative inverses • Construct and investigate geometric figures • Classify 2D shapes • Similar and congruent triangles • Angle relationships • Solve problems	• Investigate properties of geometric figures by construction • Classify 2D shapes • Similar and congruent triangles • Solve problems • Angle relationships • Use the Theorem of Pythagoras • Area and perimeter of 2D shapes (polygons and circles)

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Term 3	• Numeric and geometric patterns • Input and output values for patterns and relationships • Equivalent forms • Algebraic language • Number sentences • Interpret and draw graphs • Transformations • Classify 3D objects • Build 3D models	• Common fractions • Percentages • Decimal fractions • The Theorem of Pythagoras • Area and perimeter of 2D shapes • Surface area and volume of 3D objects • Solve problems • Data handling	• Input and output values or rules for patterns and relationships • Algebraic language • Expand and simplify algebraic expressions • Factorise algebraic expressions • Equations • Draw and interpret graphs • Draw linear graphs from given equations • Surface area and volume of 3D objects (include cylinders)
Term 4	• Integers • Numeric and geometric patterns • Input and output values for patterns and relationships • Algebraic language • Number sentences • Data handling • Probability	• Input and output values or rules for patterns and relationships • Equivalent forms • Solve algebraic equations • Interpret and draw graphs • Transformations • Enlargements and reductions • Classify 3D objects • Build 3D models • Probability	• Transformations • Enlargements and reductions • Classify 3D objects • Build 3D models • Data handling • Probability

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Planning

Types of planning tools

The following planning tools are provided:
- a teaching plan
- a sample lesson plan

Teaching plan for Mathematics Grade 8

This teaching plan shows:
- the pacing of the topics for the course by term
- where to find the relevant content and activities in the Learner's Book
- when Formal Assessment takes place, cross-referenced to suitable activities in the Learner's Book.

This teaching plan follows the time allocations as set out in the CAPS for Mathematics. It assumes four and a half hours of teaching per week.

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Term	Content/topics (as per CAPS)	Learner's Book	LB page	TG page	Time allocation	Assessment
1	Whole numbers	Chapter 1	9	28	6 hrs	Informal in class assessment OR Option 1 Assignment PoA
	Integers	Chapter 2	36	52	9 hrs	
	Exponents	Chapter 3	58	71	9 hrs	Informal in class assessment OR Option 2 Assignment PoA
	Numeric and geometric patterns	Chapter 4 Units 1 - 2	75-82	85	4,5 hrs	Informal in class assessment
	Functions and relationships	Chapter 4 Units 3 - 4	83-88	93	3 hrs	Informal in class assessment
	Algebraic expressions	Chapter 5 Units 1 – 4	92-105	101	4,5 hrs	Informal in class assessment OR Option 2 Assignment PoA
	Algebraic equations	Chapter 5 Unit 5	106	117	3hrs	
	Revision				4 hrs	Informal in class assessment
	End of term test			337	1 hr	Formal assessment

Term	Content/topics (as per CAPS)	Learner's Book	LB page	TG page	Time allocation	Assessment
2	Algebraic expressions	Chapter 6 Unit 1 - 5	112-127	117	9 hrs	Informal in class assessment
	Algebraic equations	Chapter 6 Unit 6 – 7	128-132	130	3 hrs	Informal in class assessment
	Constructions	Chapter 7	135	135	8 hrs	Informal in class assessment OR Option 1 Investigation PoA
	Geometry of 2D shapes	Chapter 8	160	149	8 hrs	Informal in class assessment OR Option 2 Investigation PoA
	Test			343	1 hr	Formal assessment Test 2
	Geometry of straight lines	Chapter 9	182	163	9 hrs	Informal in class assessment OR Option 2 Investigation PoA
	Revision				5 hrs	Informal in class assessment
	Mid Year Exam			347	2 hrs	Formal assessment

Term	Content/topics (as per CAPS)	Learner's Book	LB page	TG page	Time Allocation	Assessment
3	Common fractions	Chapter 10	202	179	7 hrs	Informal in class assessment OR Option 1 Assignment PoA
	Decimal fractions	Chapter 11	224	202	6 hrs	
	Theorem of Pythagoras	Chapter 12	247	219	5 hrs	Informal in class assessment OR Option 2 Assignment PoA
	Area and perimeter	Chapter 13 Units 1- 2	258-271	229	5 hrs	Informal in class assessment OR Option 1 Project PoA
	Volume and surface area	Chapter 13 Units 3 - 5	272-286	236	5 hrs	
	Collect, organize and summarise data	Chapter 14 Units 1 - 3	289-314	245	4 hrs	Informal in class assessment OR Option 2 Project PoA
	Represent data	Chapter 14 Unit 3	304	255	3 hrs	
	Analyse, interpret and report data	Chapter 14 Unit 5	318	259	3,5 hrs	
	Revision				4,5 hrs	Informal in class assessment
	End of term test			367	1 hr	Formal assessment Test 3

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Term	Content/topics (as per CAPS)	Learner's Book	LB page	TG page	Time Allocation	Assessment
4	Functions and relationships	Chapter 15	325	264	6 hrs	Informal in class assessment
	Algebraic equations	Chapter 16 Units 1 - 3	343-356	275	3 hrs	Informal in class assessment OR Option 1 Assignment PoA
	Graphs	Chapter 16 Units 4 – 6	357-382	287	9 hrs	Informal in class assessment OR Option 2 Assignment PoA
	Transformation geometry	Chapter 17	387	303	6 hrs	Informal in class assessment OR Option 1 Investigation PoA
	Geometry of 3D objects	Chapter 18	404	316	7 hrs	Informal in class assessment OR Investigation option 2 for POA
	Probability	Chapter 19	418	324	4,5 hrs	Informal in class assessment
	Revision				7,5 hrs	Informal in class assessment
	End of year Exam			380	2 hrs	Formal assessment

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Example of a lesson plan

Some may find daily lesson plans useful, although these are not a formal policy requirement. An example of how to complete a lesson plan is below.

Date:	**Grade:** 8	**Term:** 1
Chapter: 1	**Unit:** 1	**Contact time:** 1 hour
Content/concept: Content/concept: Ordering and comparing whole numbers	**Exercise:** 1 and 2	**Resources required:** n/a
Exercise 1		
Links with previous knowledge or exercise: Learners should already be able to: • do mental calculations with whole numbers including multiplication facts up to 12 × 12 • round off numbers to the nearest 10 • understand basic place value up to 9 digits • be able to recognize and represent prime numbers • know about multiples and factors		
Links with next activity: This unit helps learners revise mental calculations and the necessary concepts of ordering and comparing numbers in preparation for working with the properties of whole numbers and calculating with whole numbers.		
Teaching plan • Spend time assessing learner's knowledge of the basic skills, as listed above, before doing the exercises in this unit. Learners may struggle if this revision is not done. • Revise multiplication tables by dividing the class into groups and having time trials. • Revise ordering numbers up to 9 digits. Do examples together as a class, by having learners come up to the board to fill in the correct signs. • Prescribe exercise 1. Learners complete the exercises on their own. • Mark the work in class, by writing the answers on the board, and learners mark their own work. • Revise the various terms used in this unit. Such as: prime, whole numbers, counting numbers, and factors. • Ensure learners can distinguish between the different kinds of numbers. • Revise the concept of prime numbers with the class. Discuss what learners know about prime numbers. Discuss methods for finding prime numbers. • Prescribe exercise 2. • Walk round the class and observe how learners coped with the exercise. • Discuss why learners can use divisibility rules and multiples to eliminate for prime numbers.		
Assessment: Informal self-assessment, with teacher supervision.		
Teacher reflection:		

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Assessment

Assessment is the planned process of identifying, gathering and interpreting information about learners' performance on an on going basis. Assessment should be both informal and formal, and a variety of assessment tasks should be used. Learners should timeously receive feedback on both informal and formal assessment.

The four steps of assessment

1 Generating and collecting evidence of achievement.
2 Evaluating the evidence.
3 Recording the findings.
4 Using the findings to guide future learning and teaching.

Types of assessment

Type of assessment	Description
Baseline assessment	Establishes whether learners meet basic skills and knowledge level required. Helps teacher plan for the year and for each learner. Is administered at the beginning of the year and before a particular topic. Results are used as a guide for teaching and not for promotion purposes.
Diagnostic assessment	Informs the teacher about certain specific problem areas that may hinder performance. May help determine whether a learner's problems are content or psycho-social based. Appropriate interventions should follow on from diagnostic assessment. Results should inform interventions and not be used for promotion purposes.
Formative assessment	Used to aid the learning process and not for promotion purposes. Usually informal, to provide the teacher and learner with a more frequent account of where the learner is at. Teachers can use this form of assessment to modify and adapt their own teaching.
Summative evaluation	Carried out after completion of a topic or cluster of topics. Is an assessment of learning that has taken place. Recorded and used for promotion. This is usually formal assessment, making up the Formal Programme of Assessment.

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Informal or daily assessment

Informal assessment is a daily monitoring of learners' progress. This is done through observations, discussions, practical demonstrations, learner-teacher conferences, and informal classroom interactions. All daily exercises completed in the mathematics classroom can be used for informal assessment.
CAPS tells us that informal assessment should be used to provide feedback to the learners and to inform planning for teaching, but need not be recorded or taken into account for promotion. It should not be seen as separate from learning activities taking place in the classroom. Learners or teachers can mark these assessment tasks.

Formal assessment

Certain tasks make up the formal programme of assessment for the year. Formal assessment tasks are marked and formally recorded by the teacher for progression and certification purposes. All formal assessment tasks are subject to moderation for the purpose of quality assurance and to ensure that appropriate standards are maintained. Formal assessment provides teachers with a systematic way of evaluating how well learners are progressing in a grade and in a particular subject. Examples of formal assessments include tests, examinations, projects, assignments and investigations. Formal assessment tasks form part of a year-long formal programme of assessment in each grade and subject.

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Formal Programme of Assessment

Formal assessment requirements of Mathematics

The forms of assessment used should be appropriate for the learners' ages and developmental levels. Learners must complete formal assessments each term. Formal assessments include formally assessed tasks, along with projects and examinations. The Formal Programme of Assessment as prescribed by the CAPS is shown below. This Programme of Assessment is generic across the three grades in the Senior Phase, and lists the types of formal assessments required each term.

Minimum requirements for formal assessment

	Forms of assessment	Minimum requirements per term				Number of tasks per year	Weighting
		Term 1	Term 2	Term 3	Term 4		
SBA (School-based Assessment)	Tests	1	1	1		3	40%
	Examination		1			1	
	Assignment	1		1	1	3	
	Investigation		1		1	2	
	Project			1		1	
	Total	2	3	3	2	10*	
Final examination		End of year				1	60%

* To be completed before the final examination at the end of the year

Types of formal assessment for Mathematics

Tests and examinations

These are individual assessment tasks. Tests and examinations for formal assessment should cover a substantial amount of content. Tests and examinations must be completed under strictly controlled conditions.
Each test and examination must cater for a range of cognitive levels in the correct allocation (see the table below).

Cognitive level	Description of skill to be demonstrated
Knowledge ≈ 25%	• Estimation and appropriate rounding of numbers • Straight recall • Identification and direct use of correct formula • Use of mathematical facts • Appropriate use of mathematical vocabulary
Routine procedures ≈ 45%	• Performing of well-known procedures • Simple applications and calculations which might involve many steps • Derivation from given information may be involved • Identification and use (after changing the subject) of correct formulae generally similar to those encountered in class
Complex procedures ≈ 20%	• Problems involving complex calculations and/or higher-order reasoning • Investigate elementary axioms to generalise them into proofs for straight line geometry, congruence and similarity • No obvious route to the solution • Problems not necessarily based on real-world contexts • Making significant connections between different representations • Require conceptual understanding
Problem-solving ≈ 10%	• Unseen non-routine problems (which are not necessarily difficult) • Higher-order understanding and processes are often involved • Might require the ability to break the problem down into its constituent parts

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Projects

Learners complete *one* project in Mathematics in each grade. Projects can be used to test a range of skills and competencies. It is prescribed that learners complete a project in Term 3 of each grade. Projects must provide learners with the ability to demonstrate their understanding of a mathematical concept and apply it to a real-life situation. Be wary of prescribing projects that are beyond the cognitive level of the learners, or that will simply involve duplicated facts and data from reference material.

Assignments

An assignment is also an individual task, similar to tests and examinations. However, the assignment should be an extended piece of work with a focus on more demanding work than that covered in class. *Three a*ssignments per year are required by the CAPS. The assignment can include past questions, but should also include more challenging aspects encouraging the learner to use additional material to help them. The assignment can be completed at home.

Investigations

An investigation should be used to discover rules or concepts. It is recommended that learners should conduct investigations in class as much as possible, and that the final written task definitely be done in class. Rubrics are used to assess investigations. *Two* investigations per year are required by the CAPS.
The skills involved in investigations include:

- organising and recording ideas and discoveries in tables and diagrams
- explaining ideas in appropriate forms
- showing clear understanding of concepts and procedures through calculations
- generalising and drawing conclusions.

Guidelines for Assessment Tasks

Tasks should be designed to cover the content and concepts of the subject and include a variety of activities selected to assess the identified aims and skills.
Before handing out an assessment task to learners, teachers should ensure that they are able to answer all the questions themselves. When teachers set an assessment task, they should draw up a memorandum of answers and/or a rubric for the assessment. Refer to the seven-point rating code or scale of achievement when constructing a rubric.
Feedback should acknowledge strengths and identify areas of weakness for learner's developmental needs. Action plans on how learners will be supported should accompany this feedback. It is important that the feedback provided to learners encourages them to do better, and builds their self-confidence.

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Planning for assessment

We have provided a full assessment plan for you to use.

Programme of Assessment

Term	Task	Topics	Learner's Book page	Teacher's Guide page
1	Assignment	Option 1: Ratio, rate and integers	431	334
		Option 2: Exponents and algebra	432	336
	Test	Chapters 1 – 5		338
2	Investigation	Option 1: Constructions	147, 150, 152	139, 142, 143, 341
		Option 2: Geometry of 2D shapes	156	145, 342
	Test	Chapters 6 – 8		343
	Exam	Chapters 1 – 9		347
3	Assignment	Option 1: Fractions	437	360
		Option 2: Solving triangles	438	361
	Project	Option 1: Measurement	439	362
		Option 2: Data	441	364
	Test	Chapters 10 - 14		365
4	Assignment	Option 1: Algebra	443	369
		Option 2: Graphs	444	371
	Investigation	Option 1: Transformation	399	312, 372
		Option 2: Geometry of 3D objects	407	319, 373
	Exam	Chapters 1 - 19		378

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Inclusive assessment

Teachers need to develop adaptive and alternative methods to assess learners with barriers to learning, so that learners are given opportunities to demonstrate competence in ways that suit their needs. Here are some examples of how to assess these learners while still maintaining the validity of the assessment.

- Some learners may need concrete apparatus for a longer time than their peers.
- Assessments tasks, especially written tasks, may have to be broken up into smaller sections for learners who cannot concentrate or work for a long time, or they may be given short breaks during the tasks. Learners can also be given extra time to complete tasks.
- Some learners may need to do their assessment tasks in a separate venue to limit distractions.
- A variety of assessment instruments should be used, as a learner may find that a particular assessment instrument does not allow him/her to show what they can do.

- Learners who cannot read can have tasks read to them and they can orally dictate answers. Assessment can also include a practical component in which learners can demonstrate their competence without having to use language.
- A sign language interpreter can be used.
- Assessment tasks could be available in Braille or enlarged with bolded text.
- Assessment can include the use of Dictaphones or computers with voice synthesisers.
- The forms of assessment used should be age and developmental level appropriate. The design of these tasks should cover the content of the subject and include a variety of tasks designed to achieve the objectives of the subject.

Recording and reporting assessment

- Recording: Recording documents the level of a learner's performance in a specific assessment task. It indicates learner progress towards the achievement of the knowledge as prescribed in the curriculum. Records of learner performance should be used to verify the progress made by teachers and learners in the teaching and learning process.
- Reporting: Learners' performance can be reported in a number of ways. These include report cards, parents' meetings, school visitation days, parent-teacher conferences, phone calls, letters, class or school newsletters, etc. Teachers in all grades report in percentages against the subject. The various achievement levels and their corresponding percentage bands are as shown in the table below.

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Rating code	Description of competence	Marks
7	Outstanding achievement	80–100
6	Meritorious achievement	70–79
5	Substantial achievement	60–69
4	Adequate achievement	50–59
3	Moderate achievement	40–49
2	Elementary achievement	30–39
1	Not achieved	20–29

Metacognitive Strategies

What are metacognitive strategies and how can I use them?

Metacognition is the process of thinking about how you think. Adults often do this automatically. Before taking on something new, we may ask ourselves: What do I already know about this? What will help me understand it better? How is it structured? As we engage with a text or action, we may ask ourselves: Did I understand that? Why do I think that? How does this connect with what I already know? How could I apply this in my life? Then we evaluate what we have learnt or done by asking questions like: Did I understand that well? What strategies helped and

what strategies didn't help? What should I do the next time I take on a task like this? Learners, however, are often unaware of how they think and engage with learning material. You help learners to learn independently by explicitly guiding them to plan, monitor, and evaluate their reading and learning strategies. This is particularly effective for those learning in English as a second language and for learners who are struggling. It can dramatically improve their performance.

You teach metacognitive skills by asking learners to explain what they are thinking and what strategies they are using to understand material. This is best done in small groups. You can also use 'think aloud' strategies when engaging with texts and images. 'Think-alouds' are often effective when reading texts to learners, and during small-group and pair reading exercises. Here is an example of how to teach metacognitive strategies using a 'think aloud':

1. Choose a short piece of text and note where you will stop during reading to model your thought processes.
2. Things to include in this planning stage could be:
   - reading the text title and the table of contents
   - looking at the images and predicting what the text may be about
   - skim-reading the text looking for headings, words in bold, and summaries. As you skim read, think about what you already know about the subject and what more you would like to know.
3. In class, explain what you will be doing to the learners. Start by explaining how you planned before reading the text.
4. To monitor understanding during reading, you can explain where you stopped to ask yourself whether you understood the content. If the text has a long or complex sentence, describe how you divided it up to understand it. Find places where you could ask questions such as:
   - Why would this ....?
   - Is this similar to ...?
   - How can I figure out what this new word means?
   - What does the writer want me to know?
   - What do I think will happen next? Why do I think that?
   - Do I need to re-read this for detailed information?
5. Now show learners how to evaluate their metacognitive strategies by asking and answering questions such as:
   - Did I read and understand this well?
   - What helped me to understand? What didn't help?
   - What should I do next time I read about this topic?
   - What will help me remember what I read?

By engaging with how learners think, you can better prepare them for their lives and learning in the future.

Review Copy

What is Mathematics?

Mathematics is a language. It uses symbols and notations to describe relationships. Mathematics is a human activity that involves observing, representing and investigating patterns and relationships in both the physical and social dimensions. Mathematics helps develop key mental processes such as logical and critical thinking, accuracy and problem-solving. All of these processes contribute to a learner's decision-making ability.

The specific aims of Mathematics

The aims of Mathematics are to develop:

- a critical awareness of mathematical relationships
- confidence and competence in Mathematics without fear of the subject
- curiosity and a love of Mathematics
- appreciation for the beauty and elegance of Mathematics
- recognition of the subject as a creative art
- the deep conceptual understanding required to understand Mathematics
- the acquisition of specific skills in order to apply Mathematics
- the study of related subject matter and to study further in Mathematics.

Review Copy

Specific skills for Mathematics

In order to develop the essential skills, the learner should:

- develop correct mathematical language use
- develop number vocabulary, number concept and application skills
- learn to listen, communicate, think, reason logically and apply the knowledge gained
- learn to investigate, analyse, represent and interpret information
- learn to pose and solve problems
- be aware that Mathematics plays a key role in real-life situations.

Content area focus in the Senior Phase

Content area	General focus	Specific focus
Numbers, Operations and Relationships	Meaning of different kinds of numbers Relationships between different kinds of numbers Relative sizes of different numbers Representations of numbers in various ways Operations with numbers Estimation and checking solutions	Represent numbers in a variety of ways and move flexibly between representations Recognise and use properties of operations with different number systems Solve a variety of problems, use an increased range of numbers and are able to perform multiple operations correctly and fluently
Patterns, Functions and Algebra	Achieve efficient manipulation skills which will carry over into other domains of the subject Describe patterns and relationships through the use of symbolic expressions, graphs and tables Identify and analyse regularities and change in patterns Make predictions and solve problems	Investigate numerical and geometric patterns to establish the relationships between the variables Express rules governing patterns in algebraic language or symbols Develop algebraic manipulative skills to recognise the equivalence between different representations of the same relationship Analyse situations in a variety of contexts Use of different and equivalent representations – algebraic language, formulae, expressions, equations and graphs
Space and Shape (Geometry)	Properties of shapes and objects Relationships between these properties Orientations, positions and transformations of two-dimensional shapes and three-dimensional objects	Draw and construct a wide range of geometric figures and solids using appropriate instruments Appreciate the use of constructions to investigate the properties of geometric figures and solids Develop clear and precise descriptions and classification categories of geometric figures and solids Solve geometric problems drawing on known properties of geometric figures and solids
Measurement	Select and use appropriate units, instruments and formulae Make sensible estimates Be aware of sensibleness and reasonableness of measurement and results	Use formulae for measuring area, perimeter, surface area and volume of geometric figures and solids Select and convert between units of measurement Use the Theorem of Pythagoras to solve problems involving right-angled triangles
Data Handling	Ask questions and find answers in order to describe events and the social, technological and economic environment Collect, organise, represent, analyse, interpret and report data Enabled to make informed predictions by the study of probability Describe randomness and uncertainty	Pose questions for investigation Collect, summarise, represent and critically analyse data Interpret, report and make predictions about situations Probability – include single and compound events and their relative frequency in simple experiments

Review Copy

Teaching Mathematics in the Senior Phase

The Senior Phase can be a difficult phase to teach. Each grade brings with it an added challenge. In Grade 7 learners are in the final year of Primary School and preparing for the transition to High School. In Grade 8 learners are adjusting to a new school, new peers and usually a new way of teaching and learning. In Grade 9 learners need to select their subjects for matric, and ultimately start preparing for their career and a life outside of school. Each of these periods add additional pressure and stress to learners in this phase.

Senior Phase learners do Mathematics every day. In this phase learners move from mathematics with a primarily arithmetic focus to that of a more formal and abstract approach. Essentially learners start doing 'real' maths. In order to cope with this shift, learners must be challenged to think abstractly and critically, and not to merely copy formulae and do substitutions.

The writing of formal tests and exams becomes even more important. The Mathematics teacher must spend time developing exam techniques which include unpacking terminology used in exams, such as: determine, identify, deduce, predict, present, summarise, expand, suggest, illustrate, and so on. The Learner's Book provides many built-in opportunities for learners to engage with these. Presenting answers, time management, and exam stress management are all important areas in which learners must receive constant coaching. Mathematics teachers should work very closely with Life Orientation teachers in order to support learners with these issues.

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The volume and depth of material that learners are expected to engage with in this phase are higher. Learners are expected to start marking their own work (from the board) and this is new to many Grade 8 learners. They will need support and instruction as they learn how to manage this form of responsibility.

Grade 9 is a crucial year in the teaching of Mathematics: learners are required to make a choice between Mathematics and Mathematical Literacy in Grade 10. This choice will be based on their experience and level of success achieved in Grade 9. For learners who have some idea of their future career, the choice between these two might be somewhat more straightforward.

It is of the utmost importance that teachers lay a good foundation for basic algebra and geometry in Grades 8 and 9 in order to facilitate learners who wish to pursue further Mathematics in Grade 10.

Inclusive teaching

What is inclusive teaching?

In the Senior Phase, it is crucial that learners find themselves in an environment where they can develop an interest in learning and the belief that they can learn. Inclusive Education is defined as a learning environment that promotes the full personal, academic and professional development of all learners irrespective of race, class, gender, disability, religion, culture, sexual preference, learning styles and language.

Inclusion is about acknowledging and respecting:
- that all children have the right to learn
- that all children can learn
- that all learners need support
- that all learners are unique and have different, but equally valued, learning needs
- that all learners need the opportunity to build on their own unique strengths
- that the learner is the centre of the teaching and learning process
- that there are differences in learners, such as age, gender, language, culture, learning styles, disabilities, HIV status and so on.

Inclusion is also about:
- enabling educational structures, systems and learning methodologies to meet the needs of all learners
- more than just formal schooling – it embraces learning that occurs in the home, community and so on
- changing attitudes, behaviour, methodologies and environments to meet the needs of all learners
- ensuring maximum participation of all learners in the culture and curriculum of all educational institutions
- identifying and minimising barriers to learning that can occur at any level of the system.

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Some of the learners in your class may already suffer from exclusion or think negatively about education. There is no reason for their exclusion from class activities. It is the responsibility of the teacher to ensure the inclusion of these learners. This means adapting activities to suit their needs and capabilities. It is equally important that the class is not divided because of this. Rather, learners with these challenges should be accepted and helped where possible by their peers. Learners should at all times be discouraged from teasing, bullying or ignoring learners with special needs. When these attitudes are directed towards a learner they create in that learner a barrier to learning.

Practical guidelines for inclusive teaching

- Have a true understanding of each learner's background, strengths, unique abilities, needs and barriers. Then use this information to inform your planning and give a clearer focus.
- Remember that the teacher is a facilitator of learning.
- Keep the content and material as relevant as possible.
- Break down learning into small, manageable and logical steps. Keep instructions clear and short (plan beforehand).
- Grade activities according to the different levels and abilities of learners. Try to ensure that learners remain challenged enough without undue stress.
- Develop a balance between individual, peer tutoring, cooperative learning and whole class teaching.
- Use learners to help one another in the form of group types, peer assisted learning, buddy systems and so on. Ensure that learners feel included and supported in the classroom by both the teacher and their peers.
- Set up pairs and groups of learners where members can have different tasks according to strengths and abilities. Promote self-management skills and responsibility through group roles and the types of tasks you set.
- Motivate learners and affirm their efforts and individual progress. Build confidence. Encourage questioning, reasoning, experimentation with ideas and risking opinions.
- Determine the learner's Zone of Proximal Development (ZPD) and use it for effective teaching and learning. Vygotsky described the ZPD as the distance between what the learner already knows and understands and what he/she can understand with adult support. Learning is thus a social interaction as the teacher mediates and supports the learner as he/she understands a new concept.
- Spend time on consolidating new learning. Use different ways to do this until all learners understand the concept. Make time to go back to tasks so that learners can learn from their own and others' experiences and methods.
- Use and develop effective language skills (expressive and receptive, verbal and non-verbal).
- Experiment with a variety of teaching methods and strategies to keep learners interested and to cater for and develop different learning styles. Use games, co-operative group work, brainstorming, problem-solving, debates, presentations, and so on.

Review Copy

Learners with barriers to learning

A barrier to learning is anything that prevents a learner from participating fully and learning effectively. This includes learners who were formerly disadvantaged and excluded from education because of the historical, political, cultural and health challenges facing South Africans. Some other examples of barriers to learning may be learners who are visually or hearing impaired; learners who are left handed or learners who are intellectually challenged. Barriers to learning cover a wide range of possibilities and learners may often experience more than one barrier. Some barriers, therefore, require more than one adaptation in the classroom and varying types and levels of support.

These learners may require and should be granted more time for:

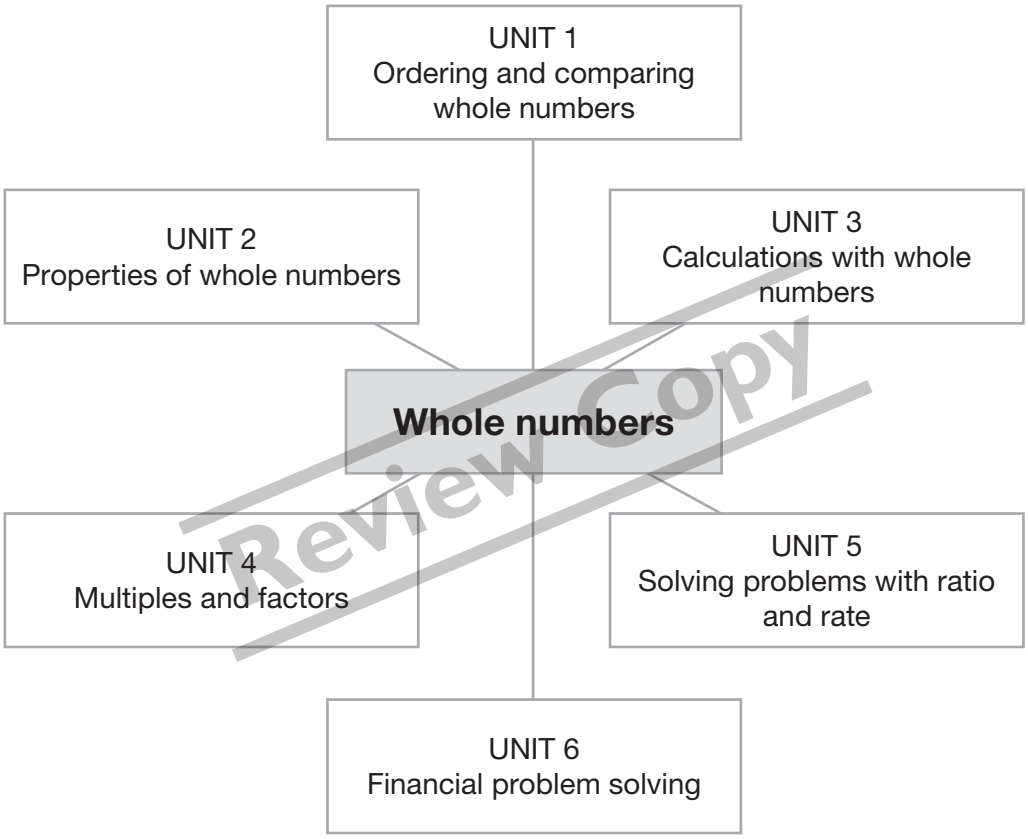
- completing tasks
- acquiring thinking skills (own strategies)
- assessment activities.

Teachers need to adapt the number of activities to be completed without interfering with the learners gaining the required language skills.

Review Copy

Chapter 1 Whole numbers

Overview of concepts



Review Copy

Content		Time allocations	LB page
Unit 1	Ordering and comparing whole numbers	1 hour	10
Unit 2	Properties of whole numbers	1 hour	13
Unit 3	Calculations with whole numbers	1 hour	17
Unit 4	Multiples and factors	1 hour	21
Unit 5	Solving problems with ratio and rate	1 hour	25
Unit 6	Financial problem solving	1 hour	29

Background information on whole numbers

Ensure that learners understand the basics of whole numbers and working with them. It is not possible to learn prime numbers, multiples or any other more complex sequences of numbers without understanding how to count or how to use the basic operations with whole numbers. Revise terminology such as:

- Addend plus addend equals sum.
- Multiplier times multiplicand = product.
- Dividend divided by divisor = quotient.
- Subtrahend minus minuend = difference.
- Whole numbers are all positive numbers including 0. For example 0; 1; 2; 3; …
- Natural numbers are generally regarded as all positive numbers excluding 0 (0 is neither positive nor negative). For example 1; 2; 3; 4; …
- Integers are all whole numbers including 0 as well as all negative numbers. For example … –3; –2; –1; 0; 1; 2; 3… The further right you go the bigger the value of the number. The further left you go the smaller the value of the number.

Generic teaching guidelines for teaching whole numbers

When introducing whole numbers have learners count in 1s, 2s, 3s and 4s as far as possible and revise multiplication tables up to 12 × 12. Play games and have class competitions to encourage learners to participate and remember these core concepts. Set the class up in teams and reward correct answers with points. Learners can also play ball games and skipping games outside. They have to see how many bounces or skips they can make while counting or repeating the multiplication tables.

Revise the four basic operations of addition, subtraction, multiplication and division. Do examples together as a class on the board. Have learners come up to the board and complete the sums. Revise the inverse operations between addition and subtraction, and multiplication and division. Show how the inverse operations work.

Revise multiplication facts for units and tens by: multiples of 10, multiples of 100, multiples of 1 000, and multiples of 10 000. Revise division facts as well.

Review Copy

Resources

An HTU Chart showing place value up to 9 digits; a calculator for each learner; blocks or sweets to use to demonstrate ratios.

Unit 1 Ordering and comparing whole numbers

Learner's Book page 10

Unit focus:

- revise multiplication tables up to 12 × 12
- order and compare numbers up to nine digits
- revise prime numbers.

Background information on ordering and comparing whole numbers

Learners should have a thorough sense of the size of whole numbers by now. Have an HTU chart that goes up to ten million available for learners to refer to. Ordering and comparing numbers is a core skill that learners have to master.

Exercise 1

Learner's Book page 11

Guidelines on how to implement this activity

As a class, revise multiplication of whole numbers based on 10; for example × 10, × 100, × 1 000 and discuss the effect on place value (where the digit is in the number).

Do examples of arranging whole numbers in order from biggest to smallest and from smallest to biggest, as a class. Ensure that learners know that the position of the digit is more important than the value of the digit. For example, although 9 has a greater value than 1 and than 0,9 it is less than 10. The 9 is in the Units column. The 1 is in the Tens column and the 0 is simply a place holder. Revise the (=) equals, (>) greater than, and (<) less than signs. Have learners come up to the board and fill in the correct sign between numbers. Learners should do this exercise on their own. You can use learners' results to assess their understanding of whole numbers.

Suggested answers

1.1	45 008 < 45 016	1.2	780 < 7 080
1.3	897 654 > 879 006	1.4	900 000 001 > 899 999 998
1.5	765 990 < 786 990	1.6	445 908 < 445 980

2.1 543 786; 564 987; 654 987; 890 765; 908 765
2.2 769 805 436; 785 609 643; 789 605 436; 796 508 643
3.1 998 000 000; 990 000 980; 990 000 890; 980 000 098
3.2 987 456; 908 321; 665 489; 321 990; 310 226

Review Copy

Remedial

If learners experience difficulty doing this exercise some remediation of whole numbers using Grade 6 and 7 material may be required. Prescribe additional practice for these learners. Encourage these learners to use an HTU grid when comparing numbers and to fill in the numbers into the grid. This helps clarify the place value and makes comparisons easier.

Exercise 2

Learner's Book page 12

Guidelines on how to implement this activity

This exercise can be done as a class or in small groups. Use a 100 square to find the prime numbers less than 100. Remind learners that prime numbers are numbers with only two different factors, 1 and itself. 1 × 2 = 2. 1 and 2 are factors of 2. Two is a prime number. Using the 100 square, first cross off 1, which is not a prime number. Then cross out all the multiples of 2, excluding 2 itself. This means all the even numbers > 2.

They are all divisible by 1 and 2, as well as themselves. For example, the factors of 4 are 1, 2 and 4. It has three factors, so is not a prime number. Then cross out all multiples of 3 except for 3 and its multiples which are even numbers; for example, 6, 12, 18 will already be crossed out. Next do the same with 5. Leave 5 and cross out its multiples which have not been crossed out previously. Repeat the process with 7, 11, 13, 17, 19. Check any numbers greater than 19 which have not been crossed out. Find the factors of these numbers. If they have more than two factors they are not prime

numbers. For example: 39 has factors 1; 3; 13; 39. It has more than two factors so is not a prime number.

41 has factors 1; 41. It has two factors, so is a prime number.

There are 25 prime numbers less than 100.

Suggested answers

1.1–1.3

1	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	~~49~~	~~50~~
~~51~~	~~52~~	53	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	59	~~60~~
61	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	67	~~68~~	~~69~~	~~70~~
71	~~72~~	73	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	79	~~80~~
~~81~~	~~82~~	83	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	89	~~90~~
~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	97	~~98~~	~~99~~	~~100~~

1.4 2; 3; 5; 7; 11; 13; 17; 19; 23; 29; 31; 37; 41; 43; 47; 53; 59; 61; 67; 71; 73; 79; 83; 89; 97

Review Copy

Remedial

Learners should not have had any problems with this guided activity.

Extension

Learners who are confident with prime numbers can complete this exercise on their own.

Unit 2 Properties of whole numbers

Learner's Book page 13

Unit focus:

- revise the commutative property of whole numbers
- revise the associative property of whole numbers
- revise the distributive property of whole numbers
- revise the additive properties of 0
- revise the multiplicative property of 1
- learn about the division property of 0
- learn about identity elements of whole numbers.

Background information on properties of whole numbers

Knowledge of the properties of numbers is basic to doing calculations. In order to manage a range of techniques in calculating whole numbers, learners must be familiar with the commutative, distributive and associative properties.

The commutative property applies to addition and multiplication. It does not apply to subtraction or division. The associative property applies to addition and multiplication. It does not apply to subtraction or division. By using brackets, multiplication and division can be distributed (spread) over addition and subtraction. The properties of 1 and 0 are also very important.

Exercise 1

Learner's Book page 13

Guidelines on how to implement this activity

Revise the properties of whole numbers. Discuss the commutative, associative and distributive properties. Do examples of each property on the board. Remind learners which properties apply to which operations. The associative and commutative properties only work for addition and multiplication, but not for division and subtraction. The distributive law works with brackets. Ensure learners can identify he property when they see it.

Review Copy

Suggested answers

1.1

$$\begin{array}{r} {}^{1}3^{1}9\ 2 \\ +\quad 7\ 7\ 8 \\ \hline 1\ 1\ 7\ 0 \end{array} \qquad \begin{array}{r} {}^{1}7^{1}7\ 8 \\ +\quad 3\ 9\ 2 \\ \hline 1\ 1\ 7\ 0 \end{array}$$

The commutative property works because the answer is the same even though the order of addition is different.

1.2

$$\begin{array}{r} 329 \\ \times\quad 271 \\ \hline 329 \\ 23\ 030 \\ 65\ 800 \\ \hline 89\ 159 \end{array} \qquad \begin{array}{r} 271 \\ \times\quad 329 \\ \hline 2\ 439 \\ 5\ 420 \\ 81\ 300 \\ \hline 89\ 159 \end{array}$$

The commutative property works because the answer is the same even though the order of multiplication is different.

1.3

$$\begin{array}{r} {}^{2}2^{2}7^{2}8{,}6 \\ 4\ 4\ 8\ 9{,}9 \\ +\quad 6\ 5{,}7 \\ \hline 4\ 8\ 3\ 4{,}2 \end{array} \qquad \begin{array}{r} {}^{2}2^{2}7^{2}8{,}6 \\ 6\ 5{,}7 \\ +\quad 4\ 4\ 8\ 9{,}9 \\ \hline 4\ 8\ 3\ 4{,}2 \end{array}$$

The commutative property works because the answer is the same even though the order of addition is different.

1.4 $\frac{2}{3}+\frac{3}{4}+\frac{5}{6}=\frac{8}{12}+\frac{9}{12}+\frac{10}{12}=\frac{27}{12}$ $\qquad$ $\frac{2}{3}+\frac{5}{6}+\frac{3}{4}=\frac{8}{12}+\frac{10}{12}+\frac{9}{12}=\frac{27}{12}$

The commutative property works because the answer is the same even though the order of addition is different.

1.5

$$\begin{array}{r} 3\,964 \\ \times \quad 519 \\ \hline 35\,676 \\ 39\,640 \\ 1\,982\,000 \\ \hline 2\,057,316 \end{array} \qquad \begin{array}{r} 519 \\ \times \quad 3\,964 \\ \hline 2\,076 \\ 31\,140 \\ 467\,100 \\ 1\,557\,000 \\ \hline 2\,057,316 \end{array}$$

The commutative property works because the answer is the same even though the order of multiplication is different.

1.6 $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$ $\qquad \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$

The commutative property works because the answer is the same even though the order of multiplication is different.

1.7 $\frac{3}{4} \times \frac{16}{21} = \frac{48}{44} = 1\frac{1}{11}$ $\qquad \frac{16}{21} \times \frac{3}{4} = \frac{48}{44} = 6\frac{1}{11}$

The commutative property works because the answer is the same even though the order of multiplication is different.

1.8

$$\begin{array}{r} {}^{8}9\,{}^{12}3\,{}^{13}4\,{}^{1}2 \\ -\quad 5\;6\;7\;8 \\ \hline 3\;6\;6\;4 \end{array} \qquad \begin{array}{r} 5\;6\;7\;8 \\ -\quad 9\;3\;4\;2 \\ \hline -3\;6\;6\;4 \end{array}$$

The commutative property does not work because the answer is not the same when the order of subtraction is different

1.9 $\frac{7}{8} - \frac{3}{4} = \frac{7}{8} - \frac{6}{8} = \frac{1}{8}$ $\qquad \frac{3}{4} - \frac{7}{8} = \frac{6}{8} - \frac{7}{8} = -\frac{1}{8}$

The commutative property does not work because the answer is not the same when the order of subtraction is different.

1.10

$$\begin{array}{r} 357 \\ 36\overline{)12852} \\ \underline{108} \\ 205 \\ \underline{180} \\ 252 \\ \underline{252} \\ 0 \end{array} \qquad \frac{36}{12852} = \frac{1}{357}$$

The commutative property does not work because the answer is not the same when the order of division is different.

1.11 $\frac{5}{9} \div \frac{1}{3} = \frac{5}{9} \times \frac{3}{1} = \frac{15}{9} = \frac{5}{3}$ $\qquad \frac{1}{3} \div \frac{5}{9} = \frac{1}{3} \times \frac{9}{5} = \frac{9}{15} = \frac{3}{5}$

The commutative property does not work because the answer is not the same when the order of subtraction is different.

1.12

$$\begin{array}{r} 657 \\ \times \quad 123 \\ \hline 1\,971 \\ 13\,140 \\ 65\,700 \\ \hline 80\,811 \end{array} \qquad \begin{array}{r} 123 \\ \times \quad 657 \\ \hline 861 \\ 6\,150 \\ 73\,800 \\ \hline 80\,811 \end{array}$$

Review Copy

The commutative property works because the answer is the same even though the order of multiplication is different.

Remedial

Learners may need to go through the process of proving the properties of operations for themselves so that they grasp them thoroughly. You may need to outline why certain properties only work for certain operations. If necessary, work through this exercise with learners who experience any problems, and provide additional material for learners to practise at home.

Exercise 2

Learner's Book page 14

Guidelines on how to implement this activity

Learners need to understand how the properties of numbers can help us when working with whole numbers. Provide examples of using each of the properties in calculations to make the sum easier to solve. Do these examples together as a class. Although the focus is on whole numbers, show learners that these properties of numbers work with fractions and decimals as well. Learners should manage this exercise on their own.

Review Copy

Suggested answers

1.1 $(23 + 14) + 35 = 37 + 35 = 72$
$23 + (14 + 35) = 23 + 49 = 72$

1.2 $(12 \times 10) \times 14 = 120 \times 14 = 1\ 680$
$12 \times (10 \times 14) = 12 \times 140 = 1\ 680$

1.3 $(3{,}5 + 4{,}6) + 3{,}9 = 8{,}1 + 3{,}9 = 12$
$3{,}5 + (4{,}6 + 3{,}9) = 3{,}5 + 8{,}5 = 12$

1.4 $(1{,}3 \times 8{,}9) \times 5{,}4 = 11{,}57 \times 5{,}4 = 62{,}478$
$1{,}3 \times (8{,}9 \times 5{,}4) = 1{,}3 \times 48{,}06 = 62{,}478$

1.5 $\left(\frac{1}{2} + \frac{3}{4}\right) + \frac{5}{6} = \left(\frac{2}{4} + \frac{3}{4}\right) + \frac{5}{6} = \frac{5}{4} + \frac{5}{6} = \frac{15}{12} + \frac{10}{12} = \frac{25}{12}$

$\frac{1}{2} + \left(\frac{3}{4} + \frac{5}{6}\right) = \frac{1}{2} + \left(\frac{9}{12} + \frac{10}{12}\right) = \frac{1}{2} + \frac{19}{12} = \frac{6}{12} + \frac{19}{12} = \frac{25}{12}$

1.6 $\left(\frac{5}{9} \times \frac{2}{3}\right) \times \frac{1}{4} = \frac{10}{27} \times \frac{1}{4} = \frac{10}{108} = \frac{5}{54}$

$\frac{5}{9} \times \left(\frac{2}{3} \times \frac{1}{4}\right) = \frac{5}{9} \times \frac{2}{12} = \frac{10}{108} = \frac{5}{54}$

1.7 $(234 + 432) + 324 = 666 + 324 = 990$
$234 + (432 + 324) = 234 + 756 = 990$

1.8 $(23 \times 34) \times 45 = 782 \times 45 = 35\ 190$
$23 \times (34 \times 45) = 23 \times 1\ 530 = 35\ 190$

2.1 $276{,}4 + (89{,}5 + 887{,}62) = (276{,}4 + 89{,}5) + 887{,}62 = 1\ 253{,}52$

2.2 $\left(\frac{2}{3} + \frac{7}{9}\right) + \frac{1}{6} = \frac{2}{3} + \left(\frac{7}{9} + \frac{1}{6}\right) = \frac{29}{18}$

2.3 $(24{,}2 \times 4{,}4) \times 3{,}23 = 24{,}2 \times (4{,}4 \times 3{,}23) = 343{,}9304$

2.4 $\frac{7}{8} \times \left(\frac{2}{3} + \frac{1}{2}\right) = \left(\frac{7}{8} \times \frac{2}{3}\right) \times \frac{1}{2} = \frac{7}{24}$

3.1 $9 - (7 - 2) = 9 - 5 = 4$
$(9 - 7) - 2 = 2 - 2 = 0$
So $9 - (7 - 2) \neq (9 - 7) - 2$

3.2 $20 \div (10 \div 2) = 20 \div 5 = 4$
$(20 \div 10) \div 2 = 2 \div 2 = 1$
So $20 \div (10 \div 2) \neq (20 \div 10) \div 2$

3.3 The associative property does not work with subtraction and division.

Exercise 3

Learner's Book page 15

Guidelines on how to implement this activity

This exercise extends the work that learners did in the previous exercise. If necessary do additional examples as a class to revise how the properties can be used to solve problems and in calculations.

Suggested answers

1.1 $3 \times 5 + 3 \times 7 = 15 + 21 = 36$

1.2 $6 \div 2 - 4 \div 2 = 3 - 2 = 1$

1.3 $7 \div 7 + 14 \div 7 = 1 + 2 = 3$

1.4 $9 \times 15 + 9 \times 5 = 135 + 45 = 180$

1.5 $24 \times 67 + 24 \times 89 = 1\ 608 + 2\ 136 = 3\ 744$

1.6 $24 \times 89 - 24 \times 67 = 2\ 136 - 1\ 608 = 528$

2.1 $1{,}2 \times (3{,}4 + 0{,}32) = (1{,}2 \times 3{,}4) + (1{,}2 \times 0{,}32) = 4{,}08 + 0{,}3264 = 4{,}4064$

2.2 $\frac{4}{5} \times \left(\frac{1}{2} + \frac{3}{4}\right) = \left(\frac{4}{5} \times \frac{1}{2}\right) + \left(\frac{4}{5} \times \frac{3}{4}\right) = \frac{4}{10} + \frac{12}{20} = \frac{8}{20} + \frac{12}{20} = \frac{20}{20} = 1$

2.3 $(8{,}4 + 2{,}4) \div 1{,}2 = (8{,}4 \div 1{,}2) + (2{,}4 \div 1{,}2) = 7 + 2 = 9$

2.4 $\left(\frac{5}{6} + \frac{2}{3}\right) \div \frac{1}{2} = \left(\frac{5}{6} \div \frac{1}{2}\right) + \left(\frac{2}{3} \div \frac{1}{2}\right) = \left(\frac{5}{6} \times \frac{2}{1}\right) + \left(\frac{2}{3} \times \frac{2}{1}\right) = \frac{10}{6} + \frac{4}{3} = \frac{10}{6} + \frac{8}{6} = \frac{18}{6} = 3$

3.1 $40 \times 8 = (20 + 20) \times 8 = (20 \times 8) + (20 \times 8) = 160 + 160 = 320$

3.2 $88 \times 3 = (80 + 8) \times 3 = (80 \times 3) + (8 \times 3) = 240 + 24 = 264$

3.3 $130 \times 6 = (100 + 30) \times 6 = (100 \times 6) + (30 \times 6) = 600 + 180 = 780$

Review Copy

Remedial

Learners experiencing problems with this exercise need to revise the previous exercise to help them manage the concepts covered here.

Exercise 4

Learner's Book page 16

Guidelines on how to implement this activity

Discuss the properties of 0 and 1. Be sure to show learners the difference between adding 0 and 1, and multiplying by 0 and 1. Discuss what an identity element is, and which numbers are the identity elements of addition and of multiplication
Do examples together as a class. Learners should do this exercise on their own.

Suggested answers

1.1	4	1.2	35	1.3	1 000
1.4	450	1.5	0	1.6	undefined

1.7	67	1.8	56 000	1.9	0
2.1	associative	2.2	identity element for addition		
2.3	distributive rule	2.4	identity element for multiplication		
2.5	division by zero	2.6	associative		
2.7	commutative	2.8	identity element for addition		
2.9	associative	2.10	distributive		

Remedial

Learners may need to be reminded that when a number is divided by 0 it is undefined and not 0. This means a denominator cannot be 0.

Unit 3 Calculations with whole numbers

Learner's Book page 17

Unit focus:

- how to check answers using the inverse operation
- estimation
- rounding off and compensating
- adding, subtracting and multiplying in columns
- long division
- using a calculator for larger numbers.

Review Copy

Background information on calculations with whole numbers

Learners have performed these operations in depth in the Intermediate phase. In Grade 8 the emphasis continues to be less on performing the actual calculation itself (learners increasingly use calculators to do this), but on judging the reasonableness of the solution they get on a calculator. However, in order to be able to understand the operation and calculation, learners need to revise these calculations at the beginning of the year to ensure they can perform them correctly.

In order to judge the reasonableness of a solution, learners need to be able to estimate effectively, and round off numbers. Learners have also covered this in depth in earlier grades; however these skills are still in great demand, and need to be practised and sharpened in Grade 8.

Exercise 1

Learner's Book page 20

Guidelines on how to implement this activity

Revise with learners the use of rounding off and compensating as a method of doing addition and subtraction. Encourage learners to come up to the board and perform examples of this method to demonstrate the method to the class. Discuss the column method of addition, subtraction and multiplication. Ask learners to state the advantages of using the column method. Do examples using the column method together as a class. Discuss the concept of division and the method of long division. Ask learners to contribute any tips they may have for working with long division. Do examples together as a class. Learners should complete this exercise on their own.

Suggested answers

1.1 5 786 – 2 376 ≈ 5 800 – 2 400 = 3 400
3 400 + (24 – 14) = 3 400 + 10 = 3 410
∴ 5786 – 2376 = 3410

1.2 4 599 + 1 265 ≈ 4 500 + 1 200 = 5 700
5 700 + 99 + 65 = 5 864
∴ 4 599 + 1 265 = 5 864

1.3 33 876 + 21 110 ≈ 33 800 + 21 100 + 54 900
54 900 + 76 + 10 = 54 986
∴ 4 599 + 1 265 = 5 864

1.4 76 550 – 43 889 ≈ 76 600 – 43 900 = 32 700
32 700 – (50 – 11) = 32 700 –39 = 32 661
∴ 76 550 – 43 889 = 32 661

1.5 12 008 + 67 998 ≈ 12 000 + 67 000 = 89 000
89 000 + 8 + 998 = 90 006
∴ 12 008 + 67 998 = 90 006

1.6 657 876 – 234 006 ≈ 658 000 – 234 100 = 423 900
423 900 – (124 – 94) = 423 900 – 30 = 423 870
∴ 65 7876 – 23 4006 = 423 870

2.1 7 234 + 1 432 ≈ 7 000 + 1 000 = 8 000

Calculate:

$$\begin{array}{rr} & 7\,234 \\ + & 1\,432 \\ \hline & 8\,666 \\ \hline\hline \end{array}$$

Check:

$$\begin{array}{rr} & 8\,666 \\ - & 7\,234 \\ \hline & 1\,432 \\ \hline\hline \end{array}$$

2.2 5 678 + 3 678 ≈ 5 600 + 3 600 = 9 200

Calculate:

$$\begin{array}{rr} & 5\,678 \\ + & 3\,678 \\ \hline & 9\,356 \\ \hline\hline \end{array}$$

Check:

$$\begin{array}{rr} & 9\,356 \\ - & 5\,678 \\ \hline & 3\,678 \\ \hline\hline \end{array}$$

2.3 48 954 + 45 478 ≈ 48 900 + 45 400 = 94 300

Calculate:

$$\begin{array}{rr} & 48\,954 \\ + & 45\,478 \\ \hline & 94\,432 \\ \hline\hline \end{array}$$

Check:

$$\begin{array}{rr} & 94\,432 \\ - & 48\,954 \\ \hline & 45\,478 \\ \hline\hline \end{array}$$

2.4 7 600 – 1 456 ≈ 7 600 – 1 500 = 6 100

Calculate:

$$\begin{array}{rr} & 7\,600 \\ - & 1\,456 \\ \hline & 6\,144 \\ \hline\hline \end{array}$$

Check:

$$\begin{array}{rr} & 6\,144 \\ + & 1\,456 \\ \hline & 7\,600 \\ \hline\hline \end{array}$$

2.5 89 887 – 45 990 ≈ 89 900 – 46 000 = 43 900

Calculate:

$$\begin{array}{rr} & 89\,887 \\ - & 45\,990 \\ \hline & 43\,897 \\ \hline\hline \end{array}$$

Check:

$$\begin{array}{rr} & 43\,897 \\ + & 45\,990 \\ \hline & 89\,887 \\ \hline\hline \end{array}$$

Review Copy

2.6 654 009 – 543 017 ≈ 654 100 – 543 100 = 111 000

Calculate:

$$\begin{array}{r} 654\,009 \\ -\ 543\,017 \\ \hline 110\,992 \end{array}$$

Check:

$$\begin{array}{r} 110\,992 \\ +\ 543\,017 \\ \hline 654\,009 \end{array}$$

2.7 567 × 17 ≈ 500 × 20 = 10 000

Calculate:

$$\begin{array}{r} 567 \\ \times\ 17 \\ \hline 3\,969 \\ 5\,670 \\ \hline 9\,639 \end{array}$$

Check:

$$\begin{array}{r|l} & 567 \\ \hline 17 & 9\,639 \\ & 85 \\ \hline & 113 \\ & 102 \\ \hline & 119 \\ & 119 \\ \hline & 0 \end{array}$$

2.8 4 579 × 32 ≈ 5 000 × 30 = 150 000

Calculate:

$$\begin{array}{r} 4\,579 \\ \times\ 32 \\ \hline 9\,158 \\ 137\,370 \\ \hline 146\,528 \end{array}$$

Check:

$$\begin{array}{r|l} & 4579 \\ \hline 32 & 14\,6528 \\ & 128 \\ \hline & 185 \\ & 160 \\ \hline & 252 \\ & 224 \\ \hline & 288 \\ & 288 \\ \hline & 0 \end{array}$$

Review Copy

2.9 7 890 × 238 ≈ 7 900 × 200 = 1 580 000

Calculate:

$$\begin{array}{r} 7\,890 \\ \times\ 238 \\ \hline 63\,120 \\ 236\,700 \\ 1\,578\,000 \\ \hline 1\,877\,820 \end{array}$$

Check:

$$\begin{array}{r|l} & 7890 \\ \hline 238 & 1877820 \\ & 1666 \\ \hline & 2118 \\ & 1904 \\ \hline & 2142 \\ & 2142 \\ \hline & 00 \\ & 00 \\ \hline & 0 \end{array}$$

2.10 782 ÷ 23 ≈ 800 ÷ 20 = 40

Calculate:

$$\begin{array}{r|l} & 34 \\ \hline 23 & 782 \\ & 69 \\ \hline & 92 \\ & 92 \\ \hline & 0 \end{array}$$

Check:

$$\begin{array}{r} 34 \\ \times\ 23 \\ \hline 102 \\ 680 \\ \hline 782 \end{array}$$

2.11 6 004 ÷ 76 ≈ 6 000 ÷ 80 = 75

Calculate:

```
      79
76 | 6004
     532
      684
      684
        0
```

Check:

```
      79
×     76
     474
   5 530
   6 004
```

2.12 77 982 ÷ 123 ≈ 78 000 ÷ 100 = 780

Calculate:

```
       634
123 | 77 982
     738
       418
       369
        492
        492
          0
```

Check:

```
       634
×      123
     1 902
    12 680
    63 400
    77 982
```

Remedial

Use this exercise to assess learners' ability in calculating. As learners have come from different Junior schools, this exercise provides some insight into their understanding and mastery of performing calculations. Provide remediation for learners who may struggle and experience any difficulty performing the calculations in this exercise. Provide additional problems for them to take home and practise.

Review Copy

Extension

Learners who managed easily with this exercise can be given more complex numbers and larger number ranges to work with when performing calculations.

Exercise 2

Learner's Book page 20

Guidelines on how to implement this activity

This exercise consolidates two vital steps required in calculations, namely estimating answers by rounding off, and checking the final answer by performing the inverse operation. Learners must appreciate the importance of these steps and use them throughout any calculations required of them, including calculator work. Do a few examples together as a class. Learners should complete this exercise on their own.

Suggested answers

2.1 460 000 + 123 000 = 583 000
458 765 + 123 654 = 582 419
582 419 – 458 765 = 123 654

2.2 879 000 – 168 000 = 711 000
879 199 – 167 888 = 711 311
711 311 + 167 888 = 879 199

2.3 335 000 + 112 000 = 447 000
334 665 + 111 999 = 446 664
446 664 – 334 665 = 334 665

2.4 35 000 000 + 440 000 = 35 440 000
34 990 776 + 443 887 = 35 434 663
35 434 663 – 34 990 776 = 443 887

2.5 89 000 000 – 14 000 000 = 75 000 000
89 000 654 – 14 113 009 = 74 887 645
74 887 645 + 14 113 009 = 89 000 654

2.6 4 567 × 456 ≈ 4 600 × 500 = 2 300 000
4 567 × 546 = 2 493 582
2 493 582 ÷ 546 = 4 567

2.7 87 990 ÷ 769 ≈ 88 000 ÷ 800 = 110
87 990 ÷ 769 = 114,4213264
769 × 114,4213264 = 87 990

2.8 65 009 × 123 ≈ 65 000 × 100 = 6 500 000
65 009 × 123 = 7 996 107
7 996 107 ÷ 123 = 65 009

Remedial

Learners must be able to use their calculators appropriately. Ensure learners know how to input calculations correctly into their calculators. Provide additional calculator practice exercises for learners if required.

Unit 4 Multiples and factors

Learner's Book page 21

Review Copy

Unit focus:

- find factors and multiples of whole numbers
- find prime factors
- find the lowest common multiple (LCM)
- find the highest common factor (HCF).

Background information on multiples and factors

Learners have worked with multiples and factors in earlier grades. Learners should be able to recognise multiples and factors, as well as determine them. In Grade 7 learners extended their knowledge to include: finding prime factors, finding the highest common factor (HCF) and the lowest common multiple (LCM), and finding factors of larger numbers. Grade 8 extends this work even further by focusing on finding prime factors of larger numbers and using prime number factorisation to find the square and cube roots of numbers without using a calculator. Learners will need their times-table skills here in order to quickly determine multiples and factors.

Exercise 1

Learner's Book page 21

Guidelines on how to implement this activity

This exercise revises finding factors and multiples. Discuss these concepts and ensure that learners understand what a factor is and what a multiple is. Do a few examples on the board as a class. Encourage learners to supply the answers and to come up to board to fill them in. Learners should complete this exercise on their own.

Suggested answers

1 7 → 7; 14; 21; 28; 35; 42; 49; 56; 63; 70; 77; 84
8 → 8; 16; 24; 32; 40; 48; 56; 64; 72; 80; 88; 96
9 → 9; 18; 27; 36; 45; 54; 63; 72; 81; 90; 99; 108

2 70 → 70; 140; 210; 280; 350; 420; 490; 560; 630; 700; 770; 840
80 → 80; 160; 240; 320; 400; 480; 560; 640; 720; 800; 880; 960
90 → 90; 180; 270; 360; 450; 540; 630; 720; 810; 900; 990; 1 080

3 36; 40; 44; 48; 52; 56; 60; 64; 68; 72; 76; 80; 84

4 1; 2; 4; 7; 8; 14; 28; 56

5 12; 24; 36; 48; 60; 72; 84; 96; 108; 120; 132; 144

6 1; 2; 3; 4; 6; 8; 9; 12; 16; 18; 24; 36; 48; 72; 144

7 120; 240; 360; 480; 600; 720; 840; 960

8 550; 1 100; 1 650; 2 200; 2 750; 3 300; 3 850; 4 400; 4 950; 5 500; 6 050; 6 600; 7 150 ; 7 700; 8 250; 8 800; 9 350; 9 900

9 100 → 1; 2; 4; 5; 10; 20; 25; 50; 100
144 → 1; 2; 3; 4; 6; 8; 9; 12; 16; 18; 24; 36; 48; 72; 144
625 → 1; 5; 25; 125; 625

Remedial

Learners need a strong sense of their multiplication tables in order manage this exercise. Learners experiencing difficulty should be supplied with a times-table chart and encouraged to memorise their tables. Test learners on their tables often and encourage learners to keep revising the tables.

Extension

If some learners are "sailing" effortlessly through the activity, provide larger numbers with more factors for these learners to work with.

Review Copy

Exercise 2

Learner's Book page 23

Guidelines on how to implement this activity

Writing numbers in terms of their prime factors is an important skill that learners will use when they work with exponents in FET mathematics. It is important that learners understand that every number can be written as a product of prime numbers. Learners can call out numbers and, as a class you can find the prime factors. Work through the worked examples in the learner's book and ensure learners know to always start with the smallest possible prime number, and work their way up. Learners may also need revising as to how to write numbers in exponential form. Learners should attempt this exercise on their won, but they may work in pairs if necessary.

Suggested answers

1

168	2
84	2
42	2
21	3
7	7
	1

240	2
120	2
60	2
30	2
15	3
5	5
1	

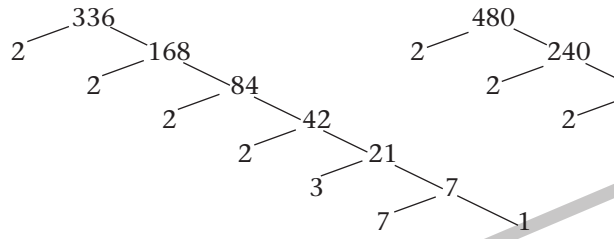
$168 = 2 \times 2 \times 2 \times 3 \times 7$
$\quad = 2^3 \times 3 \times 7$

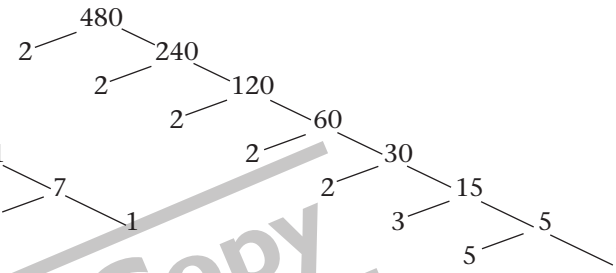
$240 = 2 \times 2 \times 2 \times 3 \times 5$
$\quad = 2^4 \times 3 \times 5$

360	22
180	22
90	22
45	23
15	23
5	25
1	

$360 = 2 \times 2 \times 2 \times 3 \times 3 \times 5$
$\quad = 2^3 \times 3^2 \times 5$

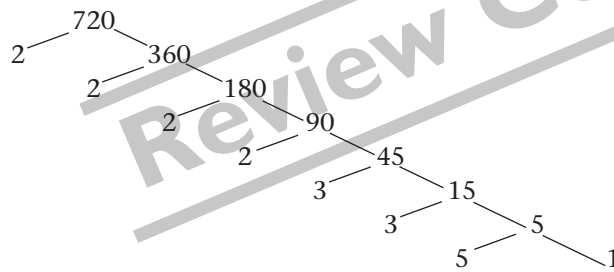
2





$336 = 2 \times 2 \times 2 \times 2 \times 3 \times 7$
$\quad = 2^4 \times 3 \times 5$

$480 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 5$
$\quad = 2^5 \times 3 \times 5$

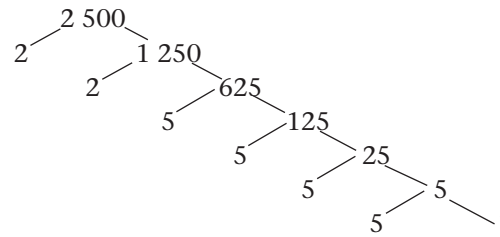


$720 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5$
$\quad = 2^4 \times 3^2 \times 5$

3 The factors in questions 1 and 2 are similar except that they differ by a factor of 2.

4

1 000	2
500	2
250	2
125	5
25	5
5	5
1	1



$1\,000 = 2 \times 2 \times 2 \times 5 \times 5 \times 5$
$\quad = 2^3 \times 5^3$

$2\,500 = 2 \times 2 \times 5 \times 5 \times 5 \times 5$
$\quad = 2^2 \times 5^4$

Review Copy

3 600	2
1 800	2
900	2
450	3
225	3
75	5
25	5
5	5
1	1

$360 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5$
$= 2^4 \times 3^2 \times 5$

5 $100 \rightarrow 2; 5$
$144 \rightarrow 2; 3$
$625 \rightarrow 5$

6.1 $210 \times 126 = 26\ 460$

6.2

26 460	2
13 230	2
6 615	3
2 205	3
735	3
245	5
49	7
7	7
1	1

$26\ 460 = 2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 7 \times 7$
$= 2^2 \times 3^3 \times 5 \times 7^2$

Review Copy

Remedial

Learners may experience difficulty with how to represent the primes, having divided the number. Show learners how to write the prime numbers in expanded form and then as powers.

Extension

Encourage learners to challenge each other to find the prime factors of larger and larger numbers.

Exercise 3

Learner's Book page 24

Guidelines on how to implement this activity

Discuss the concepts of a highest common factor and a lowest common multiple. Learners have worked with these concepts before, so it should be mainly revision. However it may be useful to ensure that all learners are clear on the concepts. Work through some examples of finding the HCF and LCM of numbers. Show learners that once you have found the LCM, there is no need to continue finding multiples. Learners should manage this exercise on their own.

Suggested answers

1 104 → 1; 2; 4; 8; 13; 26; 52; 104
143 → 1; 11; 13; 143
HCF = 13

2 90 → 1; 2; 3; 5; 6; 9; 10; 15; 18; 30; 45; 90
160 → 1; 2; 3; 4; 5; 8; 10; 16; 20; 32; 40; 90; 80; 160
HCF = 90

3 8 → 8; 16; 32; 40; 48
12 → 12; 24; 36; 48
16 → 16; 32; 48
LCM = 48

4 126 → 1; 2; 3; 6; 7; 9; 14; 18; 21; 42; 63; 126
210 → 1; 2; 3; 5; 6; 7; 10; 21; 30; 35; 42; 70; 105; 210
HCF = 42

5 156 → 1; 2; 3; 4; 6; 12; 13; 26; 39; 52; 78; 156
180 → 1; 2; 3; 4; 5; 6; 9; 10; 18; 20; 30; 36; 45; 60; 90; 180
Both numbers are multiples of 2; 3; 4 or 6

6 6

7 $\frac{156}{6} = 26$; $\frac{180}{6} = 30$

8 81 → 1; 3; 9; 27; 81
144 → 1; 2; 3; 4; 6; 8; 9; 12; 16; 18; 24; 36; 48; 72; 144
162 → 1; 2; 3; 6; 9; 18; 27; 54; 81; 162
HCF = 9

9 12 → 12; 24; 36; 48; 60; 72; 84; 96; 108
18 → 18; 36; 54; 72; 90; 108
27 → 27; 54; 81; 108
LCM = 108

Review Copy

Unit 5 Solving problems with ratio and rate

Learner's Book page 25

Unit focus:

- revise what ratio means
- solve problems with ratio
- revise what rate means
- solve problems with rates.

Background information on solving problems with ratio and rate

Learners have worked with ratio and rate in earlier grades. Ratio involves comparing two quantities of the same kind, while rate involves comparing two quantities of different kinds. Learners must know that rate occurs in everyday life, for example, as speed (km/h) and Rands per kilogram. It is important that learners can identify the use of ratio and rate in everyday life.

Exercise 1

Learner's Book page 26

Guidelines on how to implement this activity

Introduce the concept of ratio. Learners must understand that ratio is a comparison of two quantities that have the same units. Show learners that this comparison can be represented using a common fraction or using a colon. Discuss how ratio can be used to solve problems. Work through the example with the learners. Do additional examples such as sharing money or goods in a set ratio between two or more learners and increasing or decreasing amounts by ratios. If possible have concrete apparatus to use, such as sweets or blocks, to make the initial examples more realistic for learners.

Suggested answers

1.1 3 1.2 4 1.3 Thabo gets $\frac{3}{7}$ Nandi gets $\frac{4}{7}$

2.1 $1\,200 \times \frac{3}{8} = \frac{3\,600}{8} = 450$ boys 2.2 $1\,200 \times \frac{5}{8} = \frac{6\,000}{8} = 750$ girls

2.2 $\frac{3}{8}$ 2.4 $\frac{5}{8}$

3.1 450 : 300 = 9 : 6 = 3 : 2 3.2 450 + 300 = R750

3.3 $\frac{450}{750} = \frac{9}{15} = \frac{3}{5}$ 3.4 $\frac{300}{750} = \frac{6}{15} = \frac{2}{5}$

4.1 $33\,640 \times \frac{3}{8} = \frac{100\,920}{8} = 12\,615$ tickets

4.2 $33\,640 \times \frac{5}{8} = \frac{168\,200}{8} = 21\,025$ tickets

4.3 $\frac{3}{8}$

4.4 33 640 + 880 = 34 520 tickets
$34\,520 \times \frac{3}{8} = \frac{103\,560}{8} = 12\,945$ tickets

4.5 $34\,520 \times \frac{5}{8} = \frac{172\,600}{8} = 21\,575$ tickets

5.1 Reach for a dream: $132\,000 \times \frac{3}{12} = \frac{396\,000}{12} = R33\,000$
NSPCA: $132\,000 \times \frac{4}{12} = \frac{660\,000}{12} = R44\,000$
Smile: $132\,000 \times \frac{5}{12} = \frac{660\,000}{12} = R55\,000$

5.2 $\frac{44\,000}{132\,000} \times 100 = 33{,}33\%$

6. 1 x : 4 500 = 6 : 5
x = 5 400

6.2 y : 230 = 9 : 4
y = 517,5

6.3 z : 24 000 = 11 : 7
z = 37 714,29

7.1 x : 560 = 6 : 9
x = 373,333

7.2 y : 4 050 = 5 : 6
y = 3 375

7.3 z : 72 000 = 13 : 15
z = 62 400

Review Copy

8. Let the number of vegetables she eats now be x.
   x : 300 = 5 : 2
   x = 750
   Nombuso now eats 750 grams of vegetables per day.
9. Let the amount of water James consumes now be y.
   y : 560 = 5 : 7
   y = 400

Remedial

Learners who have problems with fractions, may struggle with ratios. If necessary have some fraction problems at hand to help learners revise how to work with fractions.

Extension

Provide more complex examples for stronger learners. Let the problems involve decimals.

Exercise 2

Learner's Book page 28

Guidelines on how to implement this activity

Rate has a strong presence in everyday life and it is important for learners to observe and note this. If possible, have real life examples for learners to work from. Discuss the concept of rate and show learners its use in everyday life – show learners cellphone rate charts, electricity rate charts, and pricing charts from a supermarket for fruit and vegetables sold by weight. Do a few examples together with the learners calculating rate. Encourage learners to try this exercise on their own.

Review Copy

Suggested answers

1.1 $\frac{18}{6}$ = R3 per orange
$\therefore 5 \times 3$ = R15 for 5 oranges

1.2 15×3 = R45

2 Ratio because the units are the same.

3 Rate because the units are different.

4.1 $70 \times 3 = 210$ km
$80 \times 4 = 320$ km
Total distance = 210 + 320 = 530 km

4.2 Not including rest stop = 7 hours
(including rest stop = 8 hours)

5.1 15 000 ÷ 7,89 = $1 901,14

5.2 15 000 ÷ 12,08 = £1 241,72

5.3 $15 \times 1\,901{,}14$ = $28 517,10

5.4 $10 \times 1\,241{,}72$ = £12 417,20

5.5 $25 \times 15\,000$ = R375 000

6 $150 \times 10{,}96$ = R1 644

7.1 240 ÷ 1,5 = 160 km

7.2 $160 \times 5 = 800$ km

7.3 $160 \times 0{,}5 = 80$ km

Remedial

If learners have experienced problems with this exercise, they may require additional assistance to work effectively with ratio and rate. Provide additional problems for learners to work with at home. Provide Grade 6 and 7 examples of working with ratio and rate, and then gradually increase the complexity until learners can cope with the prescribed level.

Unit 6 Financial problem solving

Learner's Book page 29

Unit focus:

- profit, loss, discount and VAT
- budgets
- accounts
- loans
- simple interest
- hire purchase
- exchange rates.

Background information on financial problem solving

Learners have been doing problem solving since the foundation phase. However, as soon as problems are presented in word form, many learners struggle. It is important that learners revise the steps for solving problems effectively. Learners should follow the following steps:

1 Read the question carefully.
2 Identify what they are being asked to find.
3 Identify what information they have been given.
4 Develop (work through) a number sentence to find what is being asked.
5 Substitute in the correct numbers into the number sentence.
6 Solve the number sentence.
7 Check the reasonableness of their solution.

Review Copy

Financial problems work best with a relevant context for learners. Learners are aware of the world around them, and financial mathematics helps them make sense of a lot of the information they have seen around them. This involves discount, tax, hire purchase, accounts and loans. Keep the information as relevant to learners as possible. Help them to relate it to events they are familiar with.

Learners will need to use percentage and fractions, so some additional revision of these concepts may be necessary for learners to cope with the exercises in this unit.

Exercise 1

Learner's Book page 31

Guidelines on how to implement this activity

Explain the concept of percentage increase and decrease. Work through the examples showing how to calculate percentage increase and decrease. Discuss the concept of profit and loss to learners. Explain that when companies, or individuals make more money than they spend, in other words, their income exceeds their expenses, they have made a profit.

Introduce the concept of selling price and cost price, and profit percentage. When companies, or individuals, spend more than they earn – which happens when their expenses are larger than their income, or their cost price is more than their selling price – they make a loss. Show learners how to determine profit or loss and then to write this as a percentage.

Discuss budgets and how important it is to budget. Show learners how we can use percentages to help us budget. For example, Mrs Mbali spends 10% of her salary on petrol and 20% of her salary on rent. Introduce learners to the concept of hire purchase. Discuss the pros and cons of hire purchase and for what goods it may be necessary.

Introduce the concept of a loan. Loans are given by banks. Banks make money from loans by charging interest. Discuss what the interest rate is. Show learners examples of bank advertisements showing the interest rate. Show learners how to work out interest using the simple interest formula. Remind learners that the amount we pay back to the bank in full is the total loan amount plus the total interest amount. Work through the example with learners. Do some additional examples if you feel they are necessary. Encourage learners to do this exercise on their own.

Suggested answers

1 $R450 \times \frac{20}{100} = R90$

$\therefore$ Selling price = 450 – 90 = R360

2.1 100 – 25 – 20 – 15 – 15 – 10 = 15%

2.2

Expenses	Percentage of salary	Money	Totals
Rent	31,65%	R5 000,00	R5 000,00
Food	25%	R2 700,00	R7 700,00
Car	20%	R2 160,00	R9 860,00
Water etc	15%	R1 620,00	R11 480,00
Clothes	15%	R1 620,00	R13 100,00
Entertainment	10%	R1 080,00	R14 180,00
Savings	15%	R1 620,00	R15 800,00
		R15 800,00	R15 800,00

Review Copy

3.1 $\frac{1\ 044\ 000}{12} = R87\ 000$

3.2 Profit = Income $\times \frac{33}{100} = R1\ 044\ 000$

$\therefore$ Income $= \frac{1\ 044\ 000}{33} \times 100 = R3\ 163\ 636{,}36$

4.1 504 + 70 + 1 200 + 400 + 200 = R2 374

4.2 2 240 – 2 374 = – 134 $\therefore$ loss

4.3 difference = R134

$\frac{134}{2\ 240} \times 100 = 5{,}98\%$

4.4 Answers will vary. Any answer that includes ideas to improve income or to decrease costs.

5.1 12 × 300 = R3 600

5.2 4 560,79 – 3600 = R960,79

5.3 $960{,}79 \times \frac{10}{100} \times 1 = R96{,}08$

6 Hire Purchase: $4\ 200 \times \frac{12}{100} \times 1 = R504$

Total costs = 4 200 + 504 = R4 704

$\therefore$ cheaper to pay cash.

7 Interest = $654\ 000 \times \frac{11{,}3}{100} \times 20$ = R1 478 040

$\therefore$ Total = 654 000 + 1 478 040 = R2 132 040

7.2 $\frac{21\ 32\ 040}{20 \times 12} = \frac{21\ 32\ 040}{240}$ = R8 883,50

8.1.1 $199{,}87 - \left(199{,}87 \times \frac{25}{100}\right) = 199{,}87 - 49{,}97$ = R149,90

8.1.2 $476{,}96 - \left(476{,}96 \times \frac{25}{100}\right) = 476{,}96 - 119{,}24$ = R357,72

8.1.3 $1\ 345{,}95 - \left(1\ 345{,}95 \times \frac{25}{100}\right) = 1\ 345{,}95 - 336{,}49$ = R1 009,46

8.1.4 $399{,}85 - \left(399{,}85 \times \frac{25}{100}\right) = 399{,}85 - 99{,}96$ = R299,89

8.1.5 $149{,}65 - \left(149{,}65 \times \frac{25}{100}\right) = 149{,}65 - 37{,}41$ = R112,24

8.2.1 $149{,}90 \times \frac{14}{100}$ = R20,97

8.2.2 $357{,}72 \times \frac{14}{100}$ = R50,08

8.2.3 $1\ 009{,}46 \times \frac{14}{100}$ = R141,31

8.2.4 $299{,}89 \times \frac{14}{100}$ = R41,98

8.2.5 $112{,}24 \times \frac{14}{100}$ = R15,71

Review Copy

Remedial

Learners are encouraged to use calculators when working with financial problems. Ensure all learners can input the information into their calculators correctly. Encourage learners to write down the steps in the calculation, even if they are doing the working out on the calculator. This will help them remember the steps involved and help them to revise later on.

Extension

Have learners investigate different payment options on large items. For example, a car or furniture. Learners can compare taking a loan, buying through hire purchase, or paying cash. Learners must identify the initial cost, monthly cost and total cost. Use real life examples to contextualise the activity.

Consolidation

Learner's Book page 35

Before doing this consolidation exercise, encourage learners to review the work covered in this chapter. Advise learners to use the summary and to revise their work. This exercise can be used as an informal assessment task for you to track how learners are coping with the chapter and the concepts covered. The mark allocation provides guidelines on how to assess learners.

Suggested answers

1.1 1; 2; 3; 4; 5; 6; 7; 8; 9; 10

1.2 0; 1; 2; 3; 4; 5; 6; 7; 8; 9

1.3 9; 18; 27; 36; 45; 54; 63; 72; 81; 90

1.4 50; 100; 150; 200; 250; 300; 350; 400; 450; 500

1.5 1; 2; 4; 7; 14; 28

1.6 2; 7 (6)

2 799 678 892; 799 687 892; 799 688 892; 799 689 879 (1)

3 135 882 689; 134 786 786; 134 780 798; 125 768 768 (1)

4 (675 + 756) + 576 = 1 431 + 576 = 2 007
675 + (756 + 576) = 675 + 1 332 = 2 007
(13 × 31) × 15 = 403 × 15 = 6 045
13 × (31 × 15) = 13 × 465 = 6 045 (2)

5 14 × 98 + 14 × 82 = 1 372 + 1 148 = 2 520
14 × 98 – 14 × 82 = 1 372 – 1 148 = 224 (2)

6

862	156
+ 156	+ 862
1 018	1 018

156	862
× 862	× 152
312	5 172
9 360	43 100
124 800	86 200
134 472	134 472

(2)

7.1

	$^{1}45$
	$^{2}463$
	498
+	2 123
	3 129

(3)

7.2

	$^{8}9\ ^{11}2\ ^{13}$
–	2 5 8
	6 6 5

(2)

7.3

	325
×	13
	975
	3 250
	4 225

(3)

8

	4 576
×	2
	9 152

	4 576
×	42
	9 152
	183 040
	192 192

	4 576
×	342
	9 152
	183 040
	1 37 2800
	1 564 992

(9)

9 24 → 24; 48; 72; 96; 120; 144; 168; 192; 216; **240**; 264; 288; 312; 336
20 → 20; 40; 60; 80; 100; 120; 140; 160; 180; 200; 220; **240**; 260; 280
16 → 16; 32; 48; 64; 80; 96; 112; 128; 144; 160; 176; 192; 208; 224; **240** (2)

10 240 (2)

11.1 385 → 1; 5; 7; 55; 77; 385
231 → 1; 3; 7; 33; 77; 231 (3)

11.2. 385 → 5; 7 (2)
231 → 3; 7

Review Copy

12 $\frac{3\ 600}{1} \times \frac{95}{100} = \frac{36}{1} \times \frac{95}{1} = \text{R3 420}$

OR $\frac{3\ 600}{1} \times \frac{5}{100} = \frac{36}{1} \times \frac{5}{1} = \text{R180}$

New Salary = R3 600 – R180 = R3 420 (2)

13.1 $\frac{160}{1} \times \frac{5}{100} = \frac{16}{1} \times \frac{5}{10} = \frac{80}{10} = \text{R8,00}$

13.2 $\frac{20}{1} \times \frac{16}{100} = \frac{2}{1} \times \frac{16}{10} = \frac{32}{10} = \text{R3,20}$

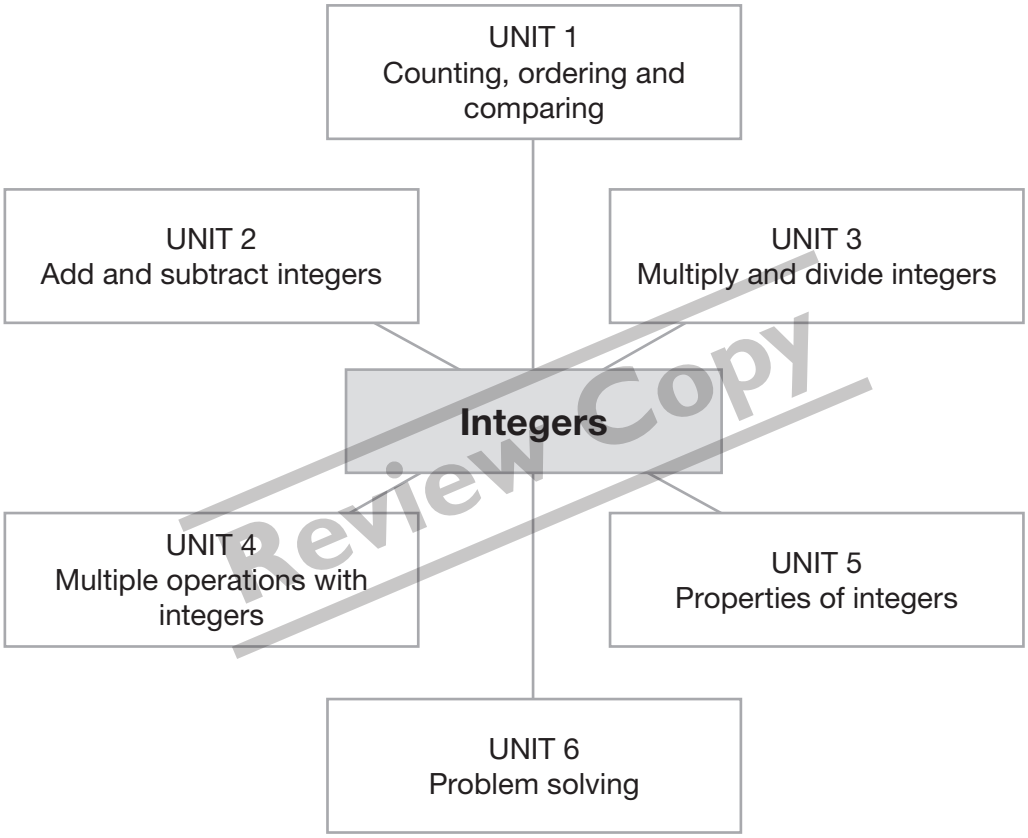
13.3 $\frac{3\ 600}{1} \times \frac{75}{100} = \frac{36}{1} \times \frac{75}{1} = \text{R2 700}$ (3)

Total marks [45]

Review Copy

Chapter 2 Integers

Overview of concepts



Review Copy

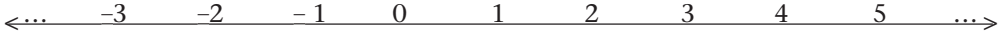
Content		Time allocations	LB page
Unit 1	Counting, ordering and comparing	1 hour	37
Unit 2	Add and subtract integers	2 hours	40
Unit 3	Multiply and divide integers	2 hours	44
Unit 4	Multiple operations with integers	1 hour	47
Unit 5	Properties of integers	1 hour	49
Unit 6	Problem solving	1 hour	53

Background information on integers

Learners were introduced to integers in Grade 7; however the concept of negative numbers is still relatively new and may require additional revision for learners to be completely comfortable with the concept.

Generic teaching guidelines for teaching integers

When working with integers it is important to always have a number line displayed in the class. This helps learners to visualise the numbers they are working with. Try have the number line as long as possible, but at least from –10 to 10. Here is a short number line:



Use a number line to compare integers, to skip-count forwards and backwards in integers, and even when doing addition and subtraction. Look at the line above. To go from –1 to 1 you move 2 spaces. So the difference between –1 and 1 is 2. The difference between – 2 and 1 is 3. Allow learners to practise this on a number line on a chart or on the board.

Have learners complete number lines by filling in missing integers.

Ensure that learners know that negative numbers always have lower values than positive numbers. For example +1 (usually written without the plus sign) is greater than – 9 000 (always written with the minus sign).

Besides using a number line, addition and subtraction of integers can also be taught using diagrams or cards. Learners cut out rectangles to represent positive (plus) numbers and triangles to represent negative (minus) numbers. Or they sketch the representative shapes.

- If the calculation is 5 + 3 this means 5 plus numbers are added to 3 plus numbers so the answer is 8 plus numbers. Arrange plus cards (or sketched rectangles) like this:
  □□□□□ + □□□ = □□□□□□□□ 5 + 3 = 8
- If the calculation is –5 + (–3) arrange minus cards like this:
  ΔΔΔΔΔ + ΔΔΔ = ΔΔΔΔΔΔΔΔ – 5 + (–3) = –8
- If the calculation is 5 + (–3) arrange the cards like this:
  ⊟⊟⊟□□ + ~~ΔΔΔ~~ = □□ strike through the 3 minus as well as 3 plus cards. The result is 2 plus cards are left. 5 + (–3) = 2
- If the calculation is –5 + 3 arrange the cards like this:
  ΔΔ~~ΔΔΔ~~ + ⊟⊟⊟ = ΔΔ strike through the 3 minus as well as 3 plus signs. The result is 2 minus cards are left. –5 + 3 = –2

Resources

Blank number lines; cardboard and colour pens to create number lines; cardboard to make flash cards, plus and minus cards, and comparison cards (for example with the greater than, less than and equal to symbols); calculators.

Review Copy

Unit 1 Counting, ordering and comparing

Learner's Book page 37

Unit focus:

- revise the definition of integers
- count forwards and backwards in integers
- recognise, order and compare integers.

Background information on counting, ordering and comparing integers

Ensure learners can count, order and compare whole numbers. Learners should be able to count in varied number ranges, both forwards and backwards in integers. Revise the signs for comparing, namely equal to (=), greater than (>) and less than (<). Show them that the signs have a thin end next to the smaller number, and a thick end next to the larger number, for example 5 > 3 and 3 < 5. Learners should know the difference between ascending order and descending order.

Exercise 1

Learner's Book page 38

Guidelines on how to implement this activity

Discuss a regular number line and what might come before 0. Learners should remember from Grade 7 that there are negative numbers to the left of 0. Create a new number line that extends below zero. Discuss the order these negative numbers will be in. Show learners how we can count in different intervals; just as we do with positive numbers, so we can do with integers. Do a few examples as a class counting in intervals of 2s, 3s, 4s and 5s. In each instance count forwards and backwards. Draw a variety of number lines on the board with numbers missing. Have learners come up to the board to fill in the missing numbers. Learners should then complete this exercise on their own.

Review Copy

Suggested answers

1.1 –1; 0; 1; 2; 3; 4

1.2 –9; –8; –7; –6; –5; –4; –3; –2; –1; 0.; 1

2.1 –15 –14 –13 –12 –11 –10 –9 –8

2.2 –1 0 1 2 3 4 5 6 7

2.3 –3 –2 –1 0 1 2 3

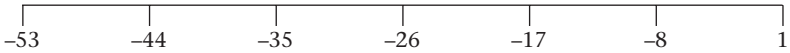
3.1 –54 –50 –46 –42 –38 –34 –30 –26 –22 –18 –14 –10 –6 –2

3.2 100 85 70 55 40 25 10 –5 –20 –35 –50

3.3 –17 –12 –7 –2 3 8 13 18 23 28 33 38 43

4 Fill in the missing values:

4.1 –65 –58 –51 –44 –37 –30 –23 –16

4.2 

This was solved by working out the difference between –26 and –8 and dividing by 2 to see what size the jumps are. 26 – 8 = 18 and because there is a blank between, the jump size is $\frac{18}{2} = 9$

Remedial

Learners experiencing problems may require additional time to understand the concepts of negative numbers. Provide context in terms of temperature of a negative bank balance to make the idea of negative numbers more real and understandable.

Extension

Learners that managed easily with the work can extend their number ranges to the negative 100s and 1 000s.

Exercise 2

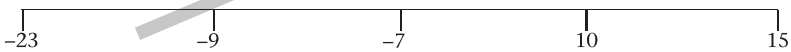
Learner's Book page 39

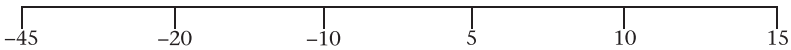
Guidelines on how to implement this activity

This exercise focuses on ordering numbers on a number line. Have class trials, where you call out five numbers (including negative numbers) and learners arrange them in order on a number line. The fastest learner wins a point. Remind learners of ascending and descending order. Write numbers on the board, for learners to write in either ascending or descending order. Do as many examples as necessary for learners to become adept at this. Revise the greater than (>) and less (<) than signs and encourage learners to compare two numbers without using a number line. Learners should complete this exercise on their own.

Review Copy

Suggested answers

1.1 

1.2 

1.3

1.4

1.5

2 Numbers in ascending order:

2.1	–215; –34; –21; 0; 2	2.2	–67; –46; –1; 46; 67
2.3	–100; –65; 54; 76; 99	2.4	–2 9875; –800; –5; 1 675
2.5	–9 900; –7 543; –1 456; 1 000; 7 898		

3 Numbers in descending order:

3.1	3; 0; –3; –6; –21	3.2	89; 75; 21; –199; –222
3.3	89; 76; 0; –77; –90	3.4	900; 650; 456; –700; –876
3.5	9 765; 8 789; –5 432; – 7 865; –9 990		
4.1	56 = + 56	4.2	–1 > –11

4.3 $-4 < 4$ 4.4 $5 > -6$ 4.5 $-7 > -71$
5.1 He climbed + 2 + 4 + 6 + 5 + 3 = 20 steps altogether.
5.2 He went down (–3 – 7 – 1 – 3 – 2 = –16) 16 steps altogether.
5.3 He went up more steps than down.
5.4 The difference is 4.

Remedial
If learners really struggle, allow them to use and refer to a number line while completing the exercise.

Unit 2 Add and subtract integers

Learner's Book page 40

Unit focus:
- revise adding and subtracting integers
- learn more about adding and subtracting negative numbers.

Background information on adding and subtracting integers
Learners have done some basic work on adding and subtracting of integers in Grade 7. It can be a confusing and problematic concept, so additional time may need to be scheduled to provide the necessary practice required.

Learners should use number lines or cards to help them. Appropriate practical apparatus has shown to have great benefit to a learner's understanding of complex concepts.

It is necessary to show to learners that a positive number and the same number with a negative sign are additive inverses, for example 25 + (–25) = 0 and –76 + 76 = 0.

Review Copy

Exercise 1
Learner's Book page 41

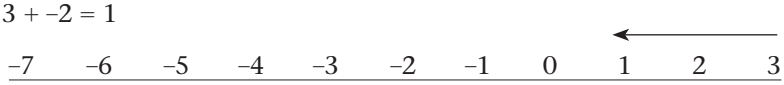
Guidelines on how to implement this activity
Ask learners how you can add negative numbers. Use a large number line on the board and show learners, using positive numbers, that when we add we move to the right, and when we subtract we move to the left. Start with simple numbers and add and subtract using negative numbers as the starting number. Then ask learners what we do when we add a negative number.

For example, 4 + (–3). Adding a negative number is the same as subtracting the additive inverse of the number. In other words, take the inverse operation with the additive inverse of the number. Then show learners what happens when we subtract a negative number – it is the same as adding that number. Do a few examples on the board until learners are able to manage alone. Allow learners to work in pairs to complete this exercise.

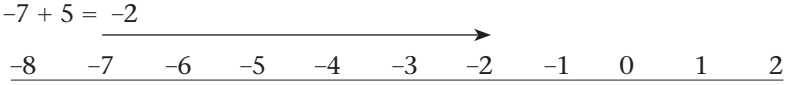
Suggested answers
1.1 34 + 25 = 59 1.2 34 + (–25) = 34 – 25 = 9
1.3 –34 + (–25) = –34 – 25 = –59
2.1 34 – 25 = 9 2.2 34 – (–25) = 59
2.3 –34 – (–25) = –9

3 Copy the number line and calculate using arrows to show your calculation:

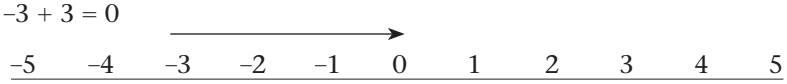
3.1 3 + –2 = 1



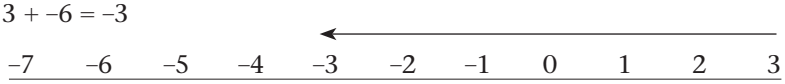
3.2 –7 + 5 = –2



3.3 –3 + 3 = 0

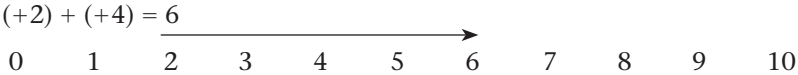


3.4 3 + –6 = –3

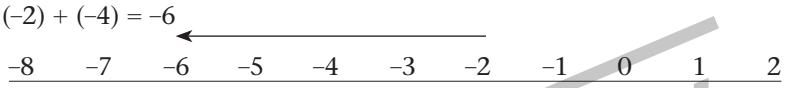


4 Draw a number line and use it work out the following:

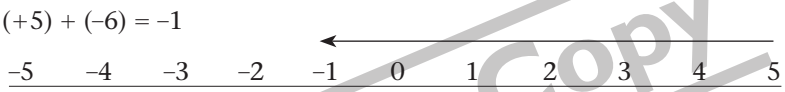
4.1 (+2) + (+4) = 6



4.2 (–2) + (–4) = –6

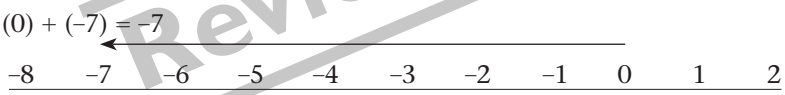


4.3 (+5) + (–6) = –1

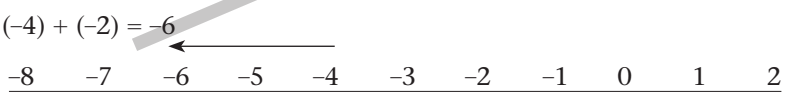


4.4 (–3) + (+8) = 5

4.5 (0) + (–7) = –7

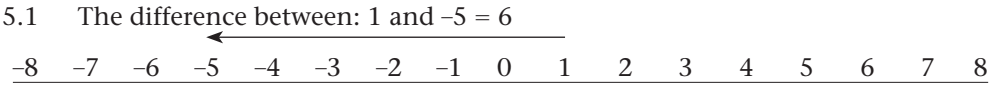


4.6 (–4) + (–2) = –6

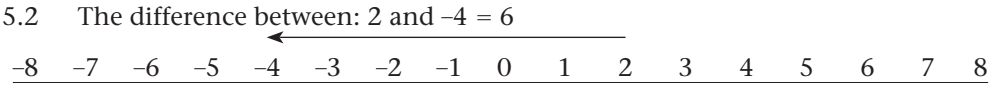


5 Make a number line from –8 to 8.

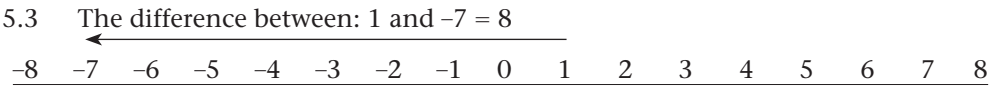
5.1 The difference between: 1 and –5 = 6



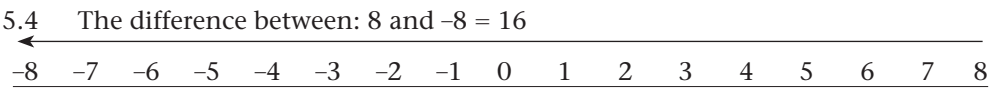
5.2 The difference between: 2 and –4 = 6



5.3 The difference between: 1 and –7 = 8



5.4 The difference between: 8 and –8 = 16



5.5 8 is the biggest number on the number line.

5.6 –8 is the smallest number on the number line.

6.1 (–5) + (–7) = –12

6.2 (–12) + (–15) = –37

Review Copy

6.3 (–4) + (–5) + (–9) = –18

6.4 (–20) + (–80) = –100

6.5 (–5) + (+7) = 2

6.6 (+12) + (–15) = –3

6.7 (–4) + (+5) + (–9) = –8

6.8 (–20) + (+80) = 60

7.1 (+5) + (–5) = 0

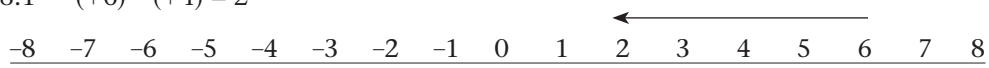
7.2 (–9) + (9) = 0

7.3 (10) + (–10) = 0

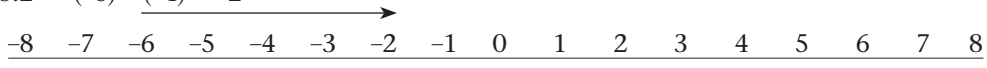
7.4 23 + (–23) = 0

8 Draw a number line and show the jumps you would use to work out the following differences:

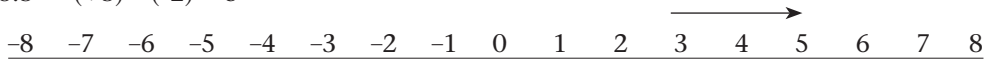
8.1 (+6) – (+4) = 2



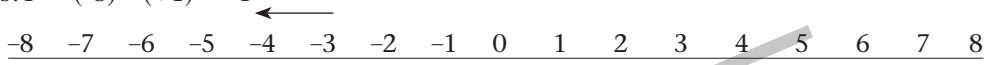
8.2 (–6) – (–4) = –2



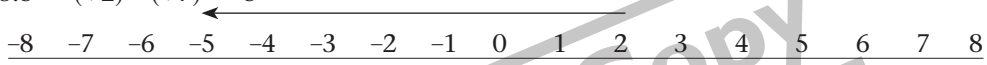
8.3 (+3) – (–2) = 5



8.4 (–3) – (+1) = –4



8.5 (+2) – (+7) = –5



8.6 0 – (–7) = 7

9.1 (+7) + (–3) = –4

9.2 (+3) + (–7) = –4

9.3 (–5) + (+2) = –3

9.4 (+1) + (+1) = 2

10.1 20 + 10 = 30

10.2 (–13) + 5 = –8

10.3 19 – 11 = 8

10.4 11 – 19 = –8

10.5 (–123) + 23 = –100

10.6 (–78) + 78 = 0

Review Copy

Remedial

Although this exercise is quite comprehensive some learners may still need additional assistance. Use the number cards to help them. Encourage learners to use practical apparatus whenever working with integers.

Extension

Extend the number range for stronger learners. Encourage them to work with thousands.

Exercise 2

Learner's Book page 43

Guidelines on how to implement this activity

Explain to learners that just as in positive integers, where there are fractional and decimal numbers lying in-between the whole numbers, so are there negative fractional and decimal numbers between the negative integers. Show learners how to fill in negative fractions and decimals on a number line. Then show learners how we use the laws of adding and subtracting integers to add and subtract negative fractions

and decimals. Do some examples on the board together as a class. Learners can do this exercise in pairs.

Suggested answers

1 $-\frac{5}{6} - -\frac{3}{4} = -\frac{10}{12} + \frac{9}{12} = -\frac{1}{12}$

2 $\frac{1}{5} + -\frac{4}{10} = \frac{2}{10} - \frac{4}{10} = -\frac{2}{10}$

3 $-\frac{4}{5} - -\frac{9}{10} = -\frac{8}{10} + \frac{9}{10} = \frac{1}{10}$

4 $-\frac{2}{7} - -\frac{1}{14} = -\frac{4}{14} + \frac{1}{14} = -\frac{3}{14}$

5 (–34,5) + (–19,25) = –53,75

6 (–721,8) – (–96,9) = –721,8 + 96,9 = –624,9

7 (+0,625) + (–0,625) = 0

8 (+201,45) + (–98,69) – (–344) =201,45 – 98,69 + 344 = 446,76

Remedial

Learners who can't manage adding and subtracting negative numbers will really struggle in this exercise. Learners who have trouble with fractions will also experience difficulty. Try to find out where the problem area is for the student, and then provide additional practice on integers or fractions as required.

Extension

Encourage stronger learners to convert between negative fractions and decimals. Also encourage these learners to work with larger numbers and mixed number fractions.

Review Copy

Unit 3 Multiply and divide integers

Learner's Book page 44

Unit focus:

- learn how to multiply integers
- learn how to divide integers

Background information on multiplying and dividing integers

Multiplying and dividing integers is a new concept introduced in Grade 8. It is important that it is introduced correctly as it is a skill that will be required throughout the learner's mathematical career. Learners already know about additive inverses. Learners should also have an understanding of multiplicative inverses. Learners need to learn the foundational principle of multiplying integers, which is what happens to the signs when you multiply or divide.

When multiplying or dividing integers, remember that:

- if the signs of the numbers to be multiplied or divided are the same, the answer is positive. For example: (+2) × (+12) = +24 and (–2) × (–12) = +24 or (+24) ÷ (+12) = +2 and (–24) ÷ (–12) = +2
- if the signs of the numbers to be multiplied or divided differ, the answer is negative. For example: (+2) × (–12) = –24 and (–2) × (+12) = –24 or (+24) ÷ (–12) = –2 and (–24) ÷ (+12) = –2.

Learners also need to know their multiplication tables fluently in order to manage this unit.

Exercise 1

Learner's Book page 45

Guidelines on how to implement this activity

Revise multiplication as repeated addition with whole numbers. Discuss with learners what this repeated addition would look like with negative numbers. Use number lines to help learners visualise the operation. Work through the examples as a class. Show learners the different forms of writing multiplication:

- with a multiplication sign
- with brackets and a multiplication sign
- with brackets only

Ensure that learners know that brackets written next to each other means multiply. Learners can use the repeated addition method, but must also be able to multiply the numbers and apply the sign. Ensure learners know how the signs work when you multiply them together. Learners should do this exercise on their own, but allow learners to discuss their answers in pairs.

Review Copy

Suggested answers

1.1 $3 \times (-2) = (-2) + (-2) + (-2) = -6$
1.2 $4 \times (-1) = (-1) + (-1) + (-1) + (-1) = -4$
1.3 $7 \times (-5) = (-5) + (-5) + (-5) + (-5) + (-5) + (-5) + (-5) = -35$
1.4 $(-3) \times 4 = (-3) + (-3) + (-3) + (-3) = -12$
1.5 $(-2) \times 9 = (-2) + (-2) + (-2) + (-2) + (-2) + (-2) + (-2) + (-2) + (-2) = -18$
1.6 $(-8) \times 3 = (-8) + (-8) + (-8) = -24$
1.7 $(-5)(5) = (-5) + (-5) + (-5) + (-5) + (-5) = -25$
1.8 $(-2)(8) = (-2) + (-2) + (-2) + (-2) + (-2) + (-2) + (-2) + (-2) = -16$
1.9 $(4)(-6) = (-6) + (-6) + (-6) + (-6) = -24$
1.10 $(1)(-6) = (-6) = -6$
1.11 $(8)(-7) = (-7) + (-7) + (-7) + (-7) + (-7) + (-7) + (-7) + (-7) = -56$
1.12 $(20)(-4) = (-4) + (-4) + (-4) + (-4) + (-4) + (-4) + (-4) + (-4) + (-4) + (-4) + (-4) + (-4) + (-4) + (-4) + (-4) + (-4) + (-4) + (-4) + (-4) = -80$

2.1 $(-4) \times (-2) = 8$
2.2 $(-3) \times (-7) = 21$
2.3 $(-2) \times (-5) = 10$
2.4 $(-9) \times (-10) = 90$
2.5 $(-7) \times (-12) = 84$
2.6 $(-6) \times (-3) = 18$
2.7 $(-6)(-9) = 54$
2.8 $(-15)(-6) = 90$
2.9 $(-9)(-11) = 99$
2.10 $(-5)(-18) = 90$
2.11 $(-100)(-90) = 9\ 000$
2.12 $(-76)(-2) = 152$
3.1 $(-12) \times (+5) = -60$
3.2 $(+7)(-8) = -56$
3.3 $(-10) \times (0{,}75) = -7{,}5$
3.4 $(-15)(-7) = 105$
3.5 $(-6)(-5)(-2) = -60$
3.6 $12 \times \left(-\frac{3}{4}\right) = -9$
3.7 $-1\ 000 \times (-1) = 1\ 000$
3.8 $(-3)(-4)(+6) = 72$
3.9 $(-8)(-6)(-2) = -96$

Remedial

Learners may need to revise their multiplication tables. Have a times-table test before the start of this lesson to prepare learners.

Extension

Encourage learners to multiply up to four numbers together at a time. Remind learners to pay attention to the signs.

Exercise 2

Learner's Book page 46

Guidelines on how to implement this activity

Explain to learners that the same rules of signs apply to division as to multiplication. Learners must know that:

- a positive number divided by a positive number is positive number
- a negative number divided by a negative number is positive number
- a negative number divided by a positive number is negative number
- a positive number divided by a negative number is negative number

Other than the signs, the division is the same as for whole numbers. Dividing by zero is undefined.

Do examples together as a class, and learners should do this exercise on their own.

Suggested answers

1.1 $(-49) \div 7 = \frac{-49}{7} = -7$

1.2 $54 \div (-6) = \frac{54}{-6} = -9$

1.3 $(-96) \div (-12) = \frac{-96}{-12} = 8$

1.4 $(-10 \times 1) \div (-5) = \frac{-10}{-5} = 2$

1.5 $\frac{36}{-9} = -4$

1.6 $\frac{-132}{-11} = 12$

2.1 $(-7) \times \frac{(-15)}{(-35)} = -7 \times \frac{-15}{5} = -3$

2.2 $108 + \frac{(-27)}{(-2)(9)} = 108 + \frac{(3)}{(2)} = 109\frac{1}{2}$

2.3 $(-10) \div (-5)(+5) = \frac{-10}{(-5)(5)} = \frac{-10}{-25} = \frac{2}{5}$

2.4 $\frac{(-12)}{(-4)(-1)} + \frac{(-2)}{(-6)} = \frac{-12}{4} + \frac{1}{3} = \frac{-3}{1} + \frac{1}{3} = -3\frac{1}{3}$

2.5 $\frac{(-27)(-3)}{(-9)} - \frac{(+3)}{(-6)} = \frac{(-27)(1)}{(3)} - \frac{(1)}{(-2)} = -9 - \frac{-1}{2} = -8\frac{(1)}{(2)}$

Remedial

Division cam be problematic for learners. The concept seems to put learners off so they don't try. Provide additional encouragement and support. Provide more time for learners to feel less pressured to complete the activity. Reserve 2.4 and 2.5 for stronger learners.

Extension

Encourage stronger learners to work on their own and answer all questions in the exercise.

Review Copy

Exercise 3

Learner's Book page 46

Guidelines on how to implement this activity

Demonstrate to learners how to apply the rules for multiplication and division of integers to the multiplication and division of negative fractions and decimals. Learners need to consolidate their knowledge, so provide adequate revision of multiplication and division of fractions and decimals. Do some examples with negative numbers together before tackling examples with positive numbers. Allow learners to work on this exercise in pairs.

Suggested answers

1 $\left(\frac{-4}{5}\right) \times \left(\frac{1}{2}\right) = \frac{(-4)}{(5)} \times \frac{1}{2} = \frac{-2}{5}$

2 $\left(\frac{-3}{4}\right) \times \left(\frac{2}{3}\right) = \frac{-3}{4} \times \frac{-2}{3} = \frac{1}{2}$

3 $\left(\frac{-9}{10}\right) \times \left(\frac{-8}{12}\right) = \frac{(-9)}{10} \times \frac{-8}{12} = \frac{3}{5}$

4 $(0{,}6) \times (-0{,}2) = -0{,}12$

5 $(-2{,}5) \times (-0{,}25) = +1{,}25$

6 $(-10 \times 0{,}75) \div 1{,}5 = \frac{-7{,}5}{1{,}5} = -5$

Remedial

Learners may need additional revision of multiplying and dividing fractions and decimals. Historically these are known problem areas in mathematics. Provide Grade 7 exercises on these content areas for learners to practise before doing this exercise.

Review Copy

Unit 4 Multiple operations with integers

Learner's Book page 47

Unit focus:

- perform mixed operation with integers
- extend learners' knowledge of integers to include:
  - squares of numbers
  - cubes of numbers
  - square roots
  - cube roots.

Background information on multiple operations with integers

So far learners have seen that the four operations, addition, subtraction, multiplication and division can all be applied to integers. Adding or multiplying positive integers is the same as adding or multiplying whole numbers or natural numbers. For example: $5 + 7 = 12$ and $5 \times 7 = 35$.

When adding negative numbers, this is what happens: $-5 + (-7) = -12$ and when multiplying negative numbers this is what happens: $-5 \times (-7) = +35$.

This unit shows how we can integrate all four operations into an exercise, with each example having two or more operations to be performed.

Revision

Learner's Book page 47

Guidelines on how to implement this activity

This exercise serves as a means of checking on the knowledge learners should have gained through working through this chapter. Use this exercise as an informal assessment. All learners should cope easily and those that do not require immediate remediation.

Suggested answers

1	$(+22) \times (+2) = 44$	2	$(-34) \times (-3) = 102$	3	$(+43) \times (-4) = -172$
4	$(-16) \times (+6) = -96$	5	$(+10) \div (+2) = 5$	6	$(-8) \div (-4) = 2$
7	$(+22) \div (-2) = -11$	8	$(-12) \div (+4) = -3$		

Remedial

Learners who experience difficulty with this exercise must work through the relevant unit of this chapter again.

Exercise 1

Learner's Book page 47

Guidelines on how to implement this activity

Learners have to work with more than one operation in a calculation. Remind learners of BODMAS and the order of operations. Instruct learners to approach each sum by identifying the BODMAS components. This then acts like a "to do" list of when and what to do. Do a few examples together and then let learners do this activity on their own.

Review Copy

Suggested answers

1 $(-45) \div [(-3) \times (-3)] = \frac{-45}{(-3)(-3)} = \frac{-45}{9} = -5$

2 $\frac{(-98)}{(-7)} + (-5) = 14 - 5 = 9$

3 $78 - (-3)(-27) = 78 - (81) = -3$

4 $(-90) \div [8 - (-1)] = \frac{-90}{9} = -10$

5 $\frac{96}{(-6)} - (-10) = -16 + 10 = -6$

6 $(-98) + \frac{32}{-4} - (-10) = (-98) + (-8) + 10 = -96$

7 $-89 - [(-87) \times 2] = -89 - (-174) = -89 + 174 = 85$

8 $\left[\frac{-78}{-26} - (-45)\right] \div 2 = \frac{3 + 45}{2} = \frac{48}{2} = 24$

9 $(-9)(-37) + (-56)(-3) = +333 + (+168) = 501$

10 $\frac{-144}{4} - (-45) = -36 + 45 = 9$

Remedial

If learners continue to have problems doing this exercise, make them write out BODMAS on card and keep it alongside them as they do the exercise. Learners can colour code each step, and highlight the parts of the sum in the necessary colour as guidance.

Extension

Provide additional fraction and decimal examples to provide more of a challenge to learners who have coped easily with the work so far.

Exercise 2

Learner's Book page 48

Guidelines on how to implement this activity

Revise squares and cubes of numbers, and square roots and cube roots of numbers. Discuss writing numbers in factor form.

Explain to learners that we cannot find the square root of a negative number because there is no negative number that one can square to give a negative answer. Any negative number multiplied by itself gives a positive number, and any positive number multiplied by itself also gives a positive number.

But we can get the cube root of negative numbers. Demonstrate this to the learners. Do a few examples finding square roots and cube roots of numbers. Do a few examples of squaring and cubing negative numbers. Learners can work in pairs to complete this exercise.

Suggested answers

1.1 $\sqrt{4 \times 4} = 4$ 1.2 $\sqrt{5 \times 5} = 5$ 1.3 $\sqrt{8 \times 8} = 8$

1.4 $\sqrt{10 \times 10} = 10$ 1.5 $\sqrt{100 \times 100} = 100$

1.6 $\sqrt{\frac{1}{2} \times \frac{1}{2}} = \frac{1}{4}$ 1.7 $\sqrt{\frac{2}{3} \times \frac{2}{3}} = \frac{2}{3}$ 1.8 $\sqrt{12 \times 12} = 12$

2.1 $\sqrt[3]{1 \times 1 \times 1} = 1$ 2.2 $\sqrt[3]{3 \times 3 \times 3} = 3$ 2.3 $\sqrt[3]{5 \times 5 \times 5} = 5$

2.4 $\sqrt[3]{10 \times 10 \times 10} = 10$ 2.5 $\sqrt[3]{100 \times 100 \times 100} = 100$

2.6 $\sqrt[3]{-3 \times (-3) \times (-3)} = -3$ 2.7 $\sqrt[3]{-1 \times (-1) \times (-1)} = -1$

2.8 $\sqrt[3]{-\frac{1}{2} \times \left(-\frac{1}{2}\right) \times \left(-\frac{1}{2}\right)} = -\frac{1}{2}$

3.1 $\sqrt{3} = 1{,}73$ 3.2 $\sqrt{10} = 3{,}16$ 3.3 $\sqrt{15} = 3{,}87$

3.4 $\sqrt{169} = 13$ 3.5 $\sqrt{1\,000} = 31{,}62$ 3.6 $\sqrt{225} = 15$

3.7 $\sqrt{100} = 10$ 3.8 $\sqrt{-1}$ = not a real number.

4.1 $5^2 + (-7)^2 - 6^2 = 25 + 49 - 36 = 38$

4.2 $(-3)^2 \times 9^2 - 12^2 = (9 \times 81) - 144 = 585$

4.3 $(-5)^3 \div 5^2 + 4^3 = \frac{-125}{25} + 64 = -5 + 64 = 59$

5.1 $(-4)^2 + (-2)^3 - (-3)^3 = 6 + (-8) - (-27) = 6 - 8 + 27 = 25$

5.2 $(-2)^2 \times \sqrt{64} \div (-4)^3 = 4 \times 8 \div (-64) = \frac{4 \times 8}{-64} = \frac{32}{-64} = -\frac{1}{2}$

5.3 $\sqrt[3]{216} + \sqrt{16} = 6 + 4 = 10$

5.4 $\sqrt[3]{-125} \times (-2)^2 = -5 \times 4 = -20$

5.5 $(-6)^3 \div \sqrt[3]{64} + (-1) = -216 \div 4 - 1 = \frac{-216}{4} + 1 = -54 + 1 = -53$

5.6 $\sqrt[3]{-8} + (-7)^2 \times (-2) = -2 + 49 \times (-2) = -2 + (-98) = -100$

6

−1	+5	−4
−3	0	+3
+4	−5	+1

Review Copy

Remedial

For learners experiencing difficulty with this exercise, have them revise the squares and cubes chart. Learners need to write out all the square numbers up to 12^2 and all the cube numbers up to 6^3. Learners need to know these by rote in order to manage well with the material at hand.

Extension

Learners can try their prime factorisation skills to find cube roots of large negative numbers.

Unit 5 Properties of integers

Learner's Book page 49

Unit focus:

- revise properties of whole numbers
- apply the commutative property to integers
- apply the associative property to integers
- apply the distributive property to integers
- apply the inverse operations to integers.

Background information on properties of integers

The commutative property, the associative property and the distributive property can be applied to integers. The commutative property, which says that the order of terms doesn't matter, can be applied in addition and multiplication; for example, $2 + 3 = 3 + 2$ and $2 \times 3 = 3 \times 2$. This also applies to negative numbers; for example $(-2) + (-3) = (-3) + (-2)$ and $(-2) \times (-3) = (-3) \times (-2)$. It cannot be applied to subtraction or to division.

The associative property, which has to do with grouping, can be applied to addition and multiplication; for example $(2 + 3) + 4 = 2 + (3 + 4)$ and $(2 \times 3) \times 4 = 2 \times (3 \times 4)$. This also applies to negative numbers; for example, $[(-2) + (-3)] + (-4) = (-2) + [(-3) + (-4)]$ and $[(-2) \times (-3)] \times 4 = (-2) \times [(-3) \times (-4)]$.

The distributive property has to do with spreading multiplication or division over addition. For example: $2(6 + 4) = 2 \times 6 + 2 \times 4$

Review Copy

Revision

Learner's Book page 49

Guidelines on how to implement this activity

This exercise helps assess learners' understanding of the properties of whole numbers. Use this exercise to assess their prior knowledge and at what level you need to present the remaining lesson in this unit. If any learners struggle, provide adequate revision of the properties of whole numbers before continuing with the unit.

Suggested answers

Calculate and then check the answer by changing the order:

1 $243 + 587 = 830$ or $587 + 243 = 830$

2 $(321 + 435) + 459 = 667 + 435 = 1215$ or
$321 + (435 + 459) = 321 + 894 = 1215$

3 $23 \times 24 = 552$ or $24 \times 23 = 552$
4 $57 \times 71 = 4\,047$ or $71 \times 57 = 4\,047$
5 $3(14 + 23) = (3 \times 14) + (3 \times 23) = 42 + 69 = 111$
6 $5(24 - 15) = (5 \times 24) - (5 \times 15) = 120 - 75 = 45$
7 $23 + 34 + 45 = (23 + 34) + 45 = 57 + 45 = 102$ or
$23 + 34 + 45 = 23 + (34 + 45) = 23 + 79 = 102$
8 $7 \times 8 \times 9 = (7 \times 8) \times 9 = 56 \times 9 = 504$ or
$7 \times 8 \times 9 = 7 \times (8 \times 9) = 7 \times 72 = 504$

Remedial

Refer learners back to Chapter 1 to revise the properties of whole numbers.

Exercise 1

Learner's Book page 50

Guidelines on how to implement this activity

Demonstrate by means of examples on the board how the commutative and associative properties of whole numbers apply to integers. Examples are kept simple in the exercise to help learners grasp the concept and not to be distracted by complex calculations. Learners can use number lines and calculators to check their work. Learners should do this activity on their own.

Review Copy

Suggested answers

1.1 $5 + 7 = 7 + 5 = 12$
1.2 $(-5) + (-7) = -7 + -5 = -12$
1.3 $(-5) + 7 = 7 + -5 = 2$
1.4 $5 + (-7) = -7 + 5 = -2$
1.5 The commutative property was applied.
2.1 $67 + 49 + 52 = 52 + 49 + 67 = 168$
2.2 $(-56) + (-54) + (-10) = (-10) + (-54) + (-56) = -120$
2.3 $(-40) + 89 + (-49) = (-49) + 89 + (-40) = 0$
3.1 $5 \times 7 = 7 \times 5 = 35$
3.2 $(-9) \times (-12) = (-12) \times (-9) = 108$
3.3 $(-16) \times 7 = 7 \times (-16) = -112$
3.4 $45 \times (-10) = (-10) \times 45 = -450$
3.5 The commutative property was applied.

Remedial

If learners struggle with the calculation they need extra help and should revise their times tables and multiplication methods. Provide Grade 7 material to help them reach the required Grade 8 level.

Exercise 2

Learner's Book page 51

Guidelines on how to implement this activity

This exercise extends learners' knowledge and skills from the previous activity. Learners should be able to manage this activity with limited additional introduction.

Suggested answers

1.1 $42 + 75 + 98 = (42 + 75) + 98 = 117 + 98 = 215$ or
$42 + (75 + 98) = 42 + 173 = 215$

1.2 $(-42) + (-75) + (-98) = (-117) - 98 = -215$ or
$-42 + (-173) = -215$
1.3 $42 + (-75) + 98 = (42 - 75) + 98 = (-33) + 98 = 65$ or
$42 + (-75 + 98) = 42 + (23) = 65$
1.4 $(-42) + 75 + 98 = (-42 + 75) + 98 = 33 + 98 = 131$ or
$(-42) + (75 + 98) = (-42) + 173 = 131$
2.1 $9 + 5 + 7 = (9 + 5) + 7 = 14 + 7 = 21$
$= 9 + (5 + 7) = 9 + 12 = 21$
2.2 $-43 + (-21) + (-20) = (-43 + -21) + -20 = -64 + -20 = -84$
$-43 + (-21 + -20) = -43 + -41 = -84$
2.3 $65 + (-21) + (-43) = (65 + -21) + -43 = 44 + -43 = 1$
$65 + (-21) + (-43) = 65 + (-21 + -43) = 65 + -64 = 1$
2.4 The associative property was applied.

Exercise 3

Learner's Book page 51

Guidelines on how to implement this activity

Discuss the distributive property and how learners have used it in the past. How does it work? Discuss how each number in the bracket has to be multiplied by the number in front of the bracket. Do examples together as a class on the board. Keep the initial examples simple and slowly increase the complexity. Learners should do this exercise on their own.

Review Copy

Suggested answers

1.1 $2(3 + 4) = (2 \times 3) + (2 \times 4) = 6 + 8 = 14$
1.2 $3(5 - 4) = (3 \times 5) - (3 \times 4) = 15 + (-12) = 3$
1.3 $-7[18 + (-2)] = [(-7) \times 18] + [(-7) \times (-2)) = -126 + 14 = -112$
1.4 The distributive property was applied.
2.1 $(-9)[(-1) + (-3)] = -9(-1) + [-9 \times -3)] = 9 + 27 = 36$
2.2 $-7[9 - (-2)] = -7(9) - [-7 \times (-2)] = -63 - 14 = -77$
2.3 $3[(-4) + 9] = 3(-4) + 3(9) = -12 + 27 = 15$
2.4 $(-2)[(-2) - (8)] = -2(-2) - [-2 \times (-8)] = 4 - 16 = -12$
2.5 $(+8)[9 - (+4)] = 8(9) - 8(4) = 72 - 32 = 40$
2.6 $(-1)[(-7) - (-3)] = -1(-7) - [-1 \times (-3)] = 7 - 3 = 4$

Remedial

Learners having problems with this exercise should colour code the numbers in the bracket and ensure they get multiplied by the number in front of the bracket by drawing the matching colour arrow to the front of the bracket.

Extension

Increase the number range for learners experiencing little or no difficulty in completing the exercise.

Exercise 4

Learner's Book page 52

Guidelines on how to implement this activity

Learners need to recognise that the properties of numbers apply to negative numbers as well. This includes the operation inverses.

- Is addition the inverse of subtraction for integers?
- Is multiplication the inverse of division for integers?

Learners need to do calculations and then perform the inverse operation using their answer to check if the inverse operations are valid with working with integers. Explain the process and have learners do the investigating themselves. Allow learners to work in small groups to complete this exercise.

Suggested answers

1.1 $(-6) + 16 = 16 - 6 = 10$

1.2 $(-5) - (-19) = -5 + 19 = 14$

1.3 $(-6) + (+8) = 8 - 6 = 2$

1.4 $(-24) - (-75) = 75 - 24 = 51$

1.5 $90 - (-87) = 90 + 87 = 177$

1.6 $(-24) + (-67) = -24 - 67 = -91$

1.7 The inverse operation of addition and subtraction can be applied to integers.

2 Test whether multiplication and division hold as inverse operations of integers in the following examples.

2.1 $(-8) \times (+6) = 6 \times (-8) = -48$

2.2 $(+7) \times (-4) = -4 \times 7 = -28$

2.3 $(-48) \div (-6) = \frac{48}{6} = 8$

2.4 $(96) \div (-4) = -\frac{96}{4} = -24$

2.5 $(-1\,200) \div (-50) = \frac{1\,200}{50} = 24$

2.6 $(-100) \times (90) = -900$

2.7 Multiplication can be used to test division problems when working with integers.

Review Copy

Remedial

Ensure that learners reach the conclusion that the inverse operations are valid for the negative integers.

Unit 6 Problem solving

Learner's Book page 53

Unit focus:

- revise learners' knowledge of integers
- use learners' knowledge of working with integers to solve problems involving negative numbers.

Background information on problem solving

Learners have been doing problem solving since the foundation phase. However, as soon as problems are presented in word form, many learners struggle. It is important that learners revise the steps for solving problems effectively. Learners should follow the following steps:

1 Read the question carefully.
2 Identify what they are being asked to find.
3 Identify what information they have been given.

4 Develop a number sentence to find what is being asked.
5 Substitute in the correct numbers into the number sentence.
6 Solve the number sentence.
7 Check the reasonableness of their solution.

Revision

Learner's Book page 53

Guidelines on how to implement this activity

Learners should do this exercise on their own. Asses how each learner coped with the work in this exercise. Use this informal assessment to determine how learners have coped with the work covered in this chapter so far. If learners experience any problems, provide the necessary help they need.

Suggested answers

1 –76 –54 –1 5 8
2 99 4 0 –1 –3
3 24 + (–38) + (–765) = 24 – 38 – 765 = –779
4 46 × (–41) × 32 = –60 352
5 85 – (–94) + (–65) = 85 + 94 – 65 = 179 – 65 = 114
6.1 +100 – 107,50 = –7,50
6.2 She has –R7,50 (that means she owes R7,50).

Review Copy

Exercise 1

Learner's Book page 54

Guidelines on how to implement this activity

Revise the steps for problem solving. Work through a few examples with learners. Pay special attention to creating the number sentence or equation. Learners can usually manage the calculation themselves but struggle to formulate the correct equation. Encourage learners to work on their own, but allow them to work in pairs if they wish.

Suggested answers

1.1 +500 – 465,75 + 75,50 1.2 R109,75
2 R49,67 – R100 = –R50,33 Maria still needs R50,33.
3.1 4 × 12 = 48 eggs 3.2 48 – 54 = –6 He will be short of 6 eggs.
4.1 –10 – (–4) = –6 4.2 15 – (–4) = 19
4.3 –10 – (–1) = –10+1 = –9 4.4 –3 –(14) = –17
5.1 Each room is 5 × 5 = 25 m^2. 5.2 All the rooms are 65 × 25 = 1 625 m2 .
6.1 45 + (45 + 3) + (2 × 45) + (90 – 7) + (45 – 6) = 45 + 48 + 90 + 83 + 39 = 305 kg
6.2 200 – 305 = –105 kg
The total mass is 105 kg heavier than the allowed mass.

Remedial

Provide additional support by helping learners create the necessary number sentences.

Extension

Learners can investigate integers around them. Encourage learners to find negative numbers in daily use. Refer them to temperatures around the world, scientific experiments, stock market fluctuations and overdraft facilities at banks. Try not to provide too much guidance as it is important for learners to be able to find their own information.

Consolidation

Learner's Book page 57

Before doing this consolidation exercise, encourage learners to review the work covered in this chapter. Advise learners to use the summary and to revise their work. This exercise can be used as an informal assessment task for you to track how learners are coping with the chapter and the concepts covered. The mark allocation provides guidelines on how to assess learners.

Suggested answers

1 Counting forwards and backwards:

1.1 –7 –6 –5 –4 –3 –2 –1 0 1 2 3 (1)

1.2 –8 –7 –6 –5 –4 –3 –2 –1 0 1 2 (1)

2.1 10 – (–10) = 10 + 10 = 20 (1)

2.2 12 – (–2) = 12 + 2 = 14 (1)

2.3 3 – (–2) = 3 + 2 = 5 (1)

3.1 (+9) – (+5) = 9 – 5 = 4

3.2 (–17) – (–3) = –17 + 3 = –14

3.3 (–81) – (+50) = –81 – 50 = –131 (4)

3.4 (+79) – (–51) = 79 + 51 = 130

4.1 52 + 63 + 108 = (52 + 63) + 108 = 115 + 108 = 223

4.2 (–52) + (–63) + (–108) = (–52 – 63) – 108 = –115 – 108 = –223

4.3 52 + (–63) + 108 = (52 – 63) + 108 = (–11) + 108 = 97

4.4 (–52) + 63 + 108 = (–52 + 63) + 108 = 11 + 108 = 119 (4)

5.1 46 + 17 = 17 + 46 = 63

5.2 (–19) + (–39) = (–39) + (–19) = –58

5.3 (–45) + 29 = 29 + (–45) = –16

5.4 205 + (–76) = (–76) + 205 = 129

5.5 The commutative property was applied. (5)

6 $\sqrt[3]{27\,000} = 30$ (3)

7.1 The coldest temperature was –8 °C. (1)

7.2 The warmest temperature was 35 °C. (1)

7.3 The difference between the two temperatures is 35 – (–8) = 43 °C. (1)

7.4 Differences between normal and record minimum temperatures

15 – 7 = 8

4 – (–8) = 12

11 – 0 = 11

Differences between normal and record maximum temperatures

26 – 35 = –9

18 – 23 = –5

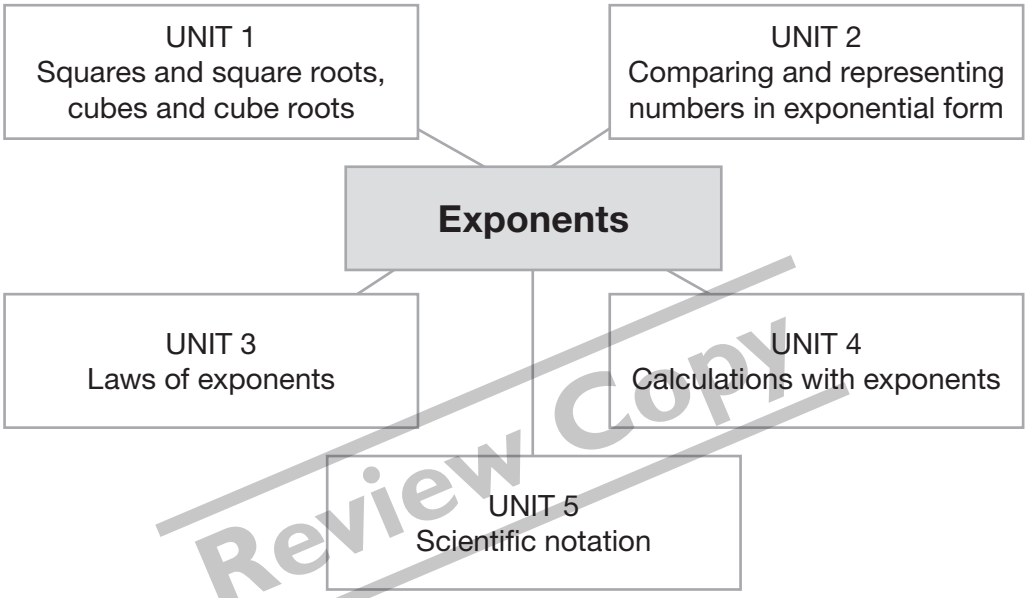
24 – 32 = –8 (2)

Total marks: [25]

Review Copy

Chapter 3 Exponents

Overview of concepts



Review Copy

Content		Time allocations	LB page
Unit 1	Squares and square roots, cubes and cube roots	1,5 hours	59
Unit 2	Comparing and representing numbers in exponential form	1,5 hours	61
Unit 3	Laws of exponents	3 hours	64
Unit 4	Calculations with exponents	2 hours	66
Unit 5	Scientific notation	1 hour	68

Background information on exponents

Exponents are also known as indices. When any number is multiplied by itself, for example 3×3, we can write it in exponential form as 3^2. The small 2 is the exponent or index. Any number can be an exponent. For example $4 \times 4 \times 4 = 4^3$ and $5 \times 5 \times 5 \times 5 = 5^4$. The idea of exponents goes back as far as the ancient Greeks. Euclid used the term "power" to show the number of times that a number was to be multiplied by itself. The 14th century scholar, Nicole Oresme, used numbers to show the use of powers. Raised numbers to show exponents goes back to the 17th century when Rene Descartes used positive exponents written in the modern way.

Generic teaching guidelines for teaching exponents

- Discuss repeated multiplication with learners;
  for example $2 \times 2 = 4$ and $2 \times 2 \times 2 = 8$ and $2 \times 2 \times 2 \times 2 = 16$.
  $10 \times 10 = 100$ and $10 \times 10 \times 10 = 1\,000$.
- Revise that instead of writing $10 \times 10 \times 10 \times 10 \times 10$ we can write 10^5 and it is equal to 100 000. Ask learners to write $3 \times 3 \times 3$ as a power (3^3) and to write the product (27). Do the same with $4 \times 4 \times 4 \times 4$ and $2 \times 2 \times 2 \times 2 \times 2$.
- Revise that when a number is written in exponential notation it has 2 parts. The big number is called the base, and the small number is called the index or the exponent. The entire base and exponent is called a power. For example 5^3 is five to the power of three, 5 is the base number, and 3 is the exponent.

Resources

Cardboard and colour pens to create flash cards and comparison cards (for example with the greater than, less than and equal to symbols); calculators; old newspaper and magazines to create a collage.

Unit 1 Squares and square roots, cubes and cube roots

Learner's Book page 59

Review Copy

Unit focus:

- revise squares and cubes
- revise square roots and cube roots.

Background information on squares and square roots, cubes and cube roots

A square is a polygon with four equal sides and four right angles.

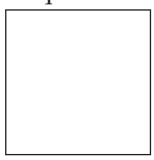


Figure 1 A square

If the sides of the square are each 3 units long, the area is 3×3 or 3^2 which equals 9. If the square is 9 then the square root is 3. So $\sqrt{9} = 3$.

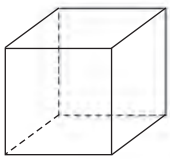


Figure 2 A cube

If the sides of the cube are each 3 units long, the volume is $3 \times 3 \times 3$ or 3^3 which equals 27. If the cube is 27 then the cube root is 3.
$\sqrt[3]{27} = 3$.

Exercise 1

Learner's Book page 59

Guidelines on how to implement this activity

Ask learners to draw a square and label it 2×2. $2 \times 2 = 4$. 4 is a square number, also called a perfect square because it is the square of an integer. Write 2×2 in exponential notation, for example 2^2.

$2^2 = 4$. Ask them to do the same with squares 3 by 3 and 5 by 5. By doing this, learners will have a better understanding of why we call 9 and 25 square numbers. Ask learners to list square numbers up to 144. Learners should provide the following numbers 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144. If 1 is a square number, what is the square root of 1? And what are $\sqrt{4}$ and $\sqrt{9}$ and $\sqrt{16}$?

Ask learners to draw a cube and label it $2 \times 2 \times 2$. We know $2 \times 2 \times 2 = 8$, so 8 is a cube number. Write $2 \times 2 \times 2$ in exponential notation, which is 2^3. Now $2^3 = 8$. Ask them to do the same with cubes 3 by 3 by 3 and 5 by 5 by 5. 27 and 125 are also cube numbers. Ask them to list the first 6 cube numbers. Learners should provide the following numbers: 1, 8, 27, 64, 125, 216. If 1 is a cube number, what is the cube root of 1? And $\sqrt[3]{8}$ and $\sqrt[3]{27}$ and $\sqrt[3]{64}$?

Learners should memorise squares up to 144 and cubes up to 216. Learners should complete this exercise on their own.

Review Copy

Suggested answers

1.1	2^2	1.2	2^3		
2.1	3^2	2.2	9^2	2.3	3^3
3.1	$3 \times 3 = 9$	3.2	$4 \times 4 = 16$	3.3	$5 \times 5 \times 5 = 125$
4.1	$12 \times 12 = 144$	4.2	$9 \times 9 = 81$	4.3	$7 \times 7 = 49$
4.4	$11 \times 11 = 121$	4.5	$6 \times 6 = 36$	4.6	$8 \times 8 = 64$
4.7	$10 \times 10 = 100$	4.8	$5 \times 5 = 25$		
5.1	$1 \times 1 \times 1 = 1$	5.2	$5 \times 5 \times 5 = 125$	5.3	$3 \times 3 \times 3 = 27$
5.4	$6 \times 6 \times 6 = 216$	5.5	$2 \times 2 \times 2 = 8$	5.6	$4 \times 4 \times 4 = 64$

6

Squares	Cubes
$2^2 = 2 \times 2 = 4$	$2^3 = 2 \times 2 \times 2 = 8$
$3^2 = 3 \times 3 = 9$	$3^3 = 3 \times 3 \times 3 = 27$
$4^2 = 4 \times 4 = 16$	$4^3 = 4 \times 4 \times 4 = 64$
$5^2 = 5 \times 5 = 25$	$5^3 = 5 \times 5 \times 5 = 125$

Remedial

Have learners create a chart showing cubes (up to 216) and squares (up to 144). Have learners memorise these, and test them every day until they are proficient.

Extension

Give learners old magazines and newspapers and ask them to cut out squares of a variety of sizes. If the square is 2×2 they must write 2^2 on it in very bold script. Each square must have an exponential number on it. Next they must make a collage of the squares on a sheet of plain paper, trying, where possible to take colour and shade into account.

Exercise 2

Learner's Book page 60

Guidelines on how to implement this activity

Using the square and cube numbers learners learnt in the previous section, demonstrate how to find the square root and cube root. Learners should find this reasonably easy to do. Learners should complete this exercise on their own.

Suggested answers

1.1	$\sqrt{4} = 2$	1.2	$\sqrt{25} = 5$	1.3	$\sqrt{9} = 3$		
1.4	$\sqrt{81} = 9$	1.5	$\sqrt{121} = 11$	1.6	$\sqrt{64} = 8$		
2.1	$\sqrt[3]{125} = 5$	2.2	$\sqrt[3]{216} = 6$	2.3	$\sqrt[3]{8} = 2$	2.4	$\sqrt[3]{64} = 4$
3.1	$\sqrt[2]{36} = 6$	3.2	$\sqrt[2]{16} = 4$	3.3	$\sqrt[2]{144} = 12$	3.4	$\sqrt[2]{169} = 13$
4.1	$\sqrt[3]{-1} = -1$	4.2	$\sqrt[3]{1\,000} = 10$	4.3	$\sqrt[3]{27} = 3$	4.4	$\sqrt[3]{-125} = -5$
5.1	5^2	5.2	10^3				
6.1	$\sqrt[2]{16} = 4$	6.2	$\sqrt{49} = 7$	6.3	$\sqrt[3]{8} = 2$	6.4	$\sqrt[3]{27} = 3$

Extension

Ask learners to list a variety of square and cube numbers. Then tell them to find the square roots and cube roots of these numbers.

Review Copy

Unit 2 Comparing and representing numbers in exponential form

Learner's Book page 61

Unit focus:

- represent whole numbers in exponential form
- compare whole numbers in exponential form
- compare and represent integers in exponential form

Background information on comparing and representing numbers in exponential form

Learners have worked with representing whole numbers exponentially in Grade 7. What is new in Grade 8 is representing negative numbers in exponential form. Focus on this in this unit. Learners have done multiplication of integers in Chapter 2, and thus understand how the signs are derived when multiplying with integers. To be sure they understand, revise this with learners before demonstrating how we can represent negative numbers.

To compare and represent whole numbers in exponential form, start with a perfect square. For example, 81.

$81 = 9 \times 9 = 9^2 = 9 \times 9 = 81$ (81 is a square number).

Consider 9. $9 = 3 \times 3 = 3^2 = 9$.

So: $81 = 3 \times 3 \times 3 \times 3 = 3^4 = 9^2$

To compare and represent integers in exponential form start with a number. For example, start with 81 again but use a negative integer:

$81 = -9 \times (-9) = (-9)^2 = -9 \times (-9) = 81$

Continue: $9 = -3 \times (-3) = (-3)^2 = 9$

So: $81 = -3 \times (-3) \times (-3) \times (-3) = (-3)^4 = (-9)^2 = 81$

Exercise 1

Learner's Book page 61

Guidelines on how to implement this activity

Start with whole numbers, and revise how to represent whole numbers in exponential form, by providing the following examples: $2 \times 2 \times 2 \times 2 \times 2$ and $5 \times 5 \times 5$ and $10 \times 10 \times 10 \times 10 \times 10 \times 10$. Have learners come up to the board to fill in the answer.

Introduce integers and negative numbers. Write the following on the board: $-3 \times (-3)$ and $-1 \times (-1) \times (-1) \times (-1)$ and $-2 \times (-2) \times (-2)$, and again encourage learners to come up and write the answers. Discuss as a class what happens to the signs when we multiply.

Lastly discuss variables, and how would they write the following terms (which contain variables) in exponential form. Provide the following examples on the board: $b \times b \times b \times b$ and $s \times s \times s$ and $5a \times b \times a \times b$ ($5a^2b^2$). Work through the answers together as a class.

Now ask learners to work backwards. How would they write the following in expanded form? Provide the following examples: 6^2; 8^6; $(2a)^5$; b^7. Learners should complete this exercise on their own.

Suggested answers

1.1	4^5	1.2	13^3	1.3	10^5	1.4	18^4
1.5	75^3	1.6	6^8	1.7	9^5	1.8	2^6

2.1 $4 \times 4 \times 4$

2.2 $12 \times 12 \times 12 \times 12 \times 12 \times 12 \times 12$

2.3 $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$

2.4 15×15

2.5 $50 \times 50 \times 50 \times 50 \times 50 \times 50 \times 50 \times 50$

2.6 $7 \times 7 \times 7 \times 7 \times 7 \times 7$

2.7 $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

2.8 1

Review Copy

Remedial

Provide the following remedial activity if learners struggle with this exercise:
Ask learners to write the following in exponential form: 1×1 and 2×2 and 3×3.
What number is the base number in each of the above?

All the above have the same exponent. Why? What is it? What is the exponent in $1 \times 1 \times 1$?

Ask learners to list five numbers with the exponent 2 and then work out their values. These numbers are called perfect squares or square numbers.
Ask learners to list five numbers with the exponent 3 and then work out their values. These numbers are called perfect cubes or cube numbers.

Extension

Ask learners who are confident to try to work with negative integers; for example, "What do you think about 2^{-1}?"
What does it mean that $2^0 = 1$ and $2^1 = 2$?

Exercise 2

Learner's Book page 62

Guidelines on how to implement this activity

Revise comparing numbers and the greater-than and less-than sign. Discuss methods for comparing numbers. Show learners that in order to compare number we can look at the bases. If the bases are the same, we can compare the numbers by simply looking at the exponents. The bigger number is the one with the bigger exponent.

If the exponents are the same, we can compare numbers by looking at the bases. The bigger number is the number with the larger base. If neither the base nor the exponent is the same, then we need to work out the value of the power in order to compare. Do examples of each type of comparison as a class.

Revise ascending and descending order. Ensure learners know that ascending is from smallest to biggest. When arranging numbers in order, learners can use the methods used above for comparing two numbers, and then arrange the numbers correctly. Learners should complete this exercise on their own.

Suggested answers

1.1 $4 \times 4 = 16 < 3 \times 3 \times 3 = 27$ 1.2 $4 \times 4 \times 4 = 64 < 3 \times 3 \times 3 \times 3 \times 3 = 243$

1.3 $5 \times 5 = 25 < 3 \times 3 \times 3 \times 3 = 81$ 1.4 $5 \times 5 \times 5 = 125 < 3 \times 3 \times 3 \times 3 \times 3 = 243$

1.5 $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 2\,187 > 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$

1.6 $4 \times 4 \times 4 \times 4 = 256 > 5 \times 5 \times 5 = 125$

1.7 $2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64 > 2 \times 2 \times 2 = 8$

1.8 $7 \times 7 \times 7 = 343 < 7 \times 7 \times 7 \times 7 \times 7 \times 7 = 117\,649$

1.9 $17 \times 17 = 289 < 15 \times 15 \times 15 = 3\,375$

1.10 $56 \times 56 = 3\,136 < 23 \times 23 \times 23 = 12\,167$

2.1 $5^0 = 1$; $16^1 = 16$; $3^3 = 27$; $6^2 = 36$; $4^5 = 537\,824$

2.2 $1\,555^0 = 1$; $16^2 = 256$; $320^1 = 320$; $15^3 = 3375$; $14^5 = 537\,824$

2.3 $16^0 = 1$; $13^3 = 2\,197$ $106^2 = 11\,236$; $12^8 = 429\,981\,696$; $16^{10} = 1\,099\,511\,627\,776$

2.4 $12^3 = 1\,728$; $6^5 = 7\,776$; $89^2 = 7\,921$; $3^9 = 19\,683$; $17^5 = 1\,419\,857$

2.5 $305^0 = 1$; 116; $22^2 = 484$; $10^3 = 1\,000$; $16^4 = 65\,536$

2.6 $11\,115^0 = 1$; $12^2 = 144$; $7^3 = 343$; $7^6 = 117\,649$; $104^3 = 1\,124\,864$

Review Copy

Remedial

Learners can use their calculators to determine the value of powers with larger numbers. Provide scrap paper for learners to write the numbers on and then allow them to arrange the pieces of paper in order before writing down the answers in their books.

Exercise 3

Learner's Book page 63

Guidelines on how to implement this activity

Revise an example of representing a whole number with a power in expanded form. Ask learners what they think a negative number to a power will look like in expanded form.

Allow learners to come up to the board and draw what they think it will look like. Ensure that you show the correct way after learners have had a chance to demonstrate.

Make certain learners know why the brackets are kept around the integers.

Do a few examples of writing integers in expanded notation and in exponential notation. Ask learners how they would compare negative integers. Discuss methods they can use to compare. Warn learners always to check on their signs when multiplying integers.

Suggested answers

1.1 $(-2)(-2)(-2) = -8$
1.2 $(-9)(-9)(-9)(-9)(-9)(-9) = 531\ 441$
1.3 $(-21)(-21)(-21)(-21)(-21) = -4\ 084\ 101$
1.4 $(-4)(-4)(-4)(-4) \times (-3)(-3)(-3) = (256)(-27) = -6\ 912$
1.5 $(-8)(-8) \times (-2)(-2) \times (1) = 64 \times 4 \times 1 = 256$
1.6 $(-1)(-1)(-1)(-1)(-1)(-1)(-1) = -1$
1.7 $(-4)(-4)(-4)(-4)(-4)(-4)(-4)(-4) \times (-1)(-1)(-1) = 65\ 536 \times (-1) = -65\ 536$
1.8 $(-2)(-2)(-2)(-2)(-2)(-2)(-2) \times (-1)(-1)(-1)(-1)(-1) \times (-5)(-5)(-5)(-5)(-5)(-5)$
$= (-128) \times (-1) \times (15\ 625) = 2\ 000\ 000$

2.1	$(-5)^4$	2.2	$(-3)^3$	2.3	$(-15)^3$
2.4	$(-10)^4$	2.5	$(-4)^3 \times (-5)^3 \times (-7)^1$	2.6	$(-5)^2 \times (-10)^2 \times (-15)^3$
2.7	$(-3)^3 \times (-1)^2 \times (-6)^2$	2.8	$(-1)^5 \times (-4)^2$		

3.1	$(-3)(-3)$	>	-9
3.2	$(-5) \times (-5) \times (-5)$	=	-125
3.3	$(-6) \times (-6) \times (-6) \times (-6)$	=	6^4
3.4	$(-675)^0$	<	$(-4)^6$
3.5	$(-10)^4$	>	$(-10\ 000)^{-1}$
3.6	$(-4)^{10}$	=	$(-2)^{20}$

Review Copy

Remedial

Some learners may struggle with the concept of negative numbers. If learners are experiencing difficulty here, revise Chapter 2 with them. Give them activities from the chapter to revise and familiarise themselves with negative numbers.

Unit 3 Laws of exponents

Learner's Book page 64

Unit focus:

- learn about the laws of exponents
- learn to recognize and use the laws of exponents
- use the laws to operations applied to exponents and square and cube roots

Background information on laws of exponents

Although learners have worked with exponents before, in terms of squares and cubes, and being able to write numbers in exponential form, this is their first formal introduction to the laws of exponents. The laws of exponents provide the foundation for further work in algebra and it is crucial that learners understand and can apply them appropriately. Initially learners will find them challenging, but the more

practice they have working with the laws, the easier they will become.
Ensure learners know the following laws:

- $a^0 = 1$
- $(a \times t)^n = a^n \times t^n$
- $(a^m)^n = a^m \times {}^n = a^{mn}$
- $a^m \div a^n = a^{m-n}$
- $a^m \times a^n = a^{m+n}$

Exercise 1

Learner's Book page 65

Guidelines on how to implement this activity

Discuss the idea of laws and what they mean. Learners should understand that these are rules that have to be applied and cannot be broken. Work through each of the four laws one at a time. Discuss the law and provide examples. Have learners write the laws, with an example for each, in the exercise books. Learners can then refer to these laws as they complete the exercise. Prescribe each part of the exercise as you complete the relevant exponent law. Learners should attempt this exercise on their own. Encourage learners to write the answers in expanded notation at first, until they are sure of the laws.

Review Copy

Suggested answers

1.1 $3^2 \times 3^3 = 3^{2+3} = 3^5 = 3 \times 3 \times 3 \times 3 \times 3 = 243$
1.2 $3^4 \times 3^2 = 3^{4+2} = 3^6 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 729$
1.3 $(3^2)^3 = 3^6 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 729$
1.4 $(3 \times 4)^2 = (12)^2 = 144$
1.5 $4^3 \times 4^3 = (4)^{3+3} = (4)^6 = 4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4\ 096$
1.6 $2^2 \times 2^3 = 2^{2+3} = 2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$
1.7 $5^4 \times 5^2 = 5^{4+2} = 5^6 = 5 \times 5 \times 5 \times 5 \times 5 \times 5 = 15\ 625$
1.8 $(4^2)^2 = 4^4 = 4 \times 4 \times 4 \times 4 = 256$
2.1 $3^5 \div 3^3 = 3^{5-3} = 3^2 = 3 \times 3 = 9$
2.2 $3^4 \div 3^2 = 3^{4-2} = 3^2 = 3 \times 3 = 9$
2.3 $3^4 \div 3^3 = 3^{4-3} = 3^1 = 3$
2.4 $4^2 \div 4^1 = 4^{2-1} = 4^1 = 4$
2.5 $4^5 \div 4^3 = 4^{5-3} = 4^2 = 4 \times 4 = 16$
2.6 $2^6 \div 2^3 = 2^{6-3} = 2^3 = 2 \times 2 \times 2 = 8$
2.7 $5^4 \div 5^2 = 5^{4-2} = 5^2 = 5 \times 5 = 25$
2.8 $4^4 \div 4^2 = 4^{4-2} = 4^2 = 4 \times 4 = 16$
3.1 $2^3 \times 3^3 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 8 \times 27 = 216$
3.2 $3^4 \times 2^4 = 3 \times 3 \times 3 \times 3 \times 2 \times 2 \times 2 \times 2 = 81 \times 16 = 1\ 296$
3.3 $3^3 \times 2^3 = 3 \times 3 \times 3 \times 2 \times 2 \times 2 = 27 \times 8 = 216$
3.4 $4^2 \times 3^2 = 4 \times 4 \times 3 \times 3 = 16 \times 9 = 144$
3.5 $4^3 \times 2^3 = 4 \times 4 \times 4 \times 2 \times 2 \times 2 = 64 \times 8 = 512$
3.6 $2^3 \times 5^3 = 2 \times 2 \times 2 \times 5 \times 5 \times 5 = 8 \times 125 = 1\ 000$
3.7 $5^2 \times 6^2 = 5 \times 5 \times 6 \times 6 = 25 \times 36 = 900$
3.8 $4^2 \times 5^2 = 4 \times 4 \times 5 \times 5 = 16 \times 25 = 400$
4.1 Zero or 0
There are no (0) zeros in the number 1.
10 to the power (0) zero = 1 or $10^0 = 1$.
4.2 $10^6 \times 100 = 10^8$
$10^3 \times 100 = 10^5$
$10^{15} \times 10^0 = 10^{15}$
5.1 False $x^m \times y^n = (xy)^{m+n}$
5.2 True $a^m \div a^n = a^{m-n}$
5.3 False $3^5 \times 3^7 = 9^{12}$
5.4 True $(x \times y)^m = x^m \times y^m$
5.5 False $x^m + x^n = x^{m+n}$
5.6 True $(a + t)^m = a^m + t^m$

Remedial

Learners can make cards with the abbreviated laws on them. Learners can keep these next to them as they complete the exercise. This card acts as a reference to help learners' identify when they can use one of the laws.

Unit 4 Calculations with exponents

Learner's Book page 66

Unit focus:

- use the laws of exponents to add, subtract, multiply and divide integers
- use the laws of exponents to add, subtract, multiply and divide squares, cubes, square roots and cube roots
- use the laws of exponents to add, subtract, multiply and divide rational numbers.

Background information on calculations with exponents

In this unit learners extend their knowledge of working with the laws of exponents to using mixed operations with numbers that have exponents. This can be confusing for learners, and you should revise BODMAS and remind learners to break down the calculation into parts that they can tackle it step by step. It is also important that learners remember how to work with negative numbers and exponents.

Learners are also required to use mixed operations with square roots and cube roots. There is no need for complicated surd work at this level, and learners should also determine the value of the root when performing mixed operations. If learners have memorised their square and cube numbers this should be simple and straightforward.

Review Copy

Exercise 1

Learner's Book page 66

Guidelines on how to implement this activity

Remind learners that to add or subtract exponential numbers you first have to expand them, and then simplify them:

$(-2)^2 + (3)^3 - (-2)^3 = [-2 \times (-2)] + [3 \times 3 \times 3] - [-2 \times (-2) \times (-2)]$

$= 4 + 27 - (-8)$

$= 31 + 8 = 39$

To multiply or divide exponential numbers you can use the laws of exponents:

$3^2 \times 3^3 = 3^{2+3} = 3^5 = 3 \times 3 \times 3 \times 3 \times 3 = 243$

$3^4 \div 3^3 = 3^{4-3} = 3^1 = 3$

Do additional examples, involving fractions, and negative numbers raised to powers.

Suggested answers

1.1	$2^2 \times 2^3 = 2^{2+3} = 2^5 = 32$	1.2	$4^4 \times 4^2 = 4^{4+2} = 4^6 = 4\ 096$
1.3	$(5^2)^3 = 5^{2\times3} = 5^6 = 15\ 625$	1.4	$(2 \times 4)^2 = 8^2 = 64$
2.1	$2^5 \div 2^3 = 2^{5-3} = 2^2 = 4$	2.2	$4^4 \div 4^2 = 4^{4-2} = 4^2 = 16$
2.3	$3^4 \div 3^3 = 3^{4-3} = 3^1 = 3$	2.4	$5^2 \div 5^1 = 5^{2-1} = 5^1 = 5$
3.1	$(-5)^4 \times (-4)^4 = 625 \times 246 = 160\ 000$		

3.2 $\frac{(3+4)(-4)^3}{(4)} + \frac{216}{(-6)^2} = \frac{(7)(-64)}{(4)} + \frac{216}{(36)} = \frac{(-448)}{(4)} + \frac{216}{(36)} = -112 + 6 = -106$

3.3 $\frac{(-4)^2}{(4)(-3)^2} - (7)^3 = \frac{16}{(4)9} - 343 = \frac{4}{9} - 343 = -342\frac{5}{9}$

3.4 $[(-7)(-3)]^2 \div 7 = \frac{[21]^2}{7} = \frac{441}{7} = 63$

Remedial

Learners must realise that exponential numbers with different bases can be added and subtracted by working out each term. For example:
$3^2 + 2^3 = (3 \times 3) + (2 \times 2 \times 2) = 9 + 8 = 17$.
But if necessary, remind learners that variables with different bases cannot be added and subtracted unless you first work out their values. Allow learners to do a number of similar examples of numbers with exponents.

Extension

Encourage learners to try to extend their knowledge of exponents by visiting the library and reading up as much as they can. Ask learners to find square and cube roots of larger numbers, for example 900: $900 = (3 \times 2 \times 5) \times (3 \times 2 \times 5) = 30 \times 30$, so $\sqrt{900} = 30$.

Review Copy

Exercise 2

Learner's Book page 67

Guidelines on how to implement this activity

Show learners examples of adding and subtracting with square and cube roots. Show learners how to manage the calculation in each root before trying to find the root. Once the roots have been found learners can perform the desired operation.

Suggested answers

1.1 $\sqrt{144} + \sqrt{25} = 12 + 5 = 17$
1.2 $\sqrt{100} + \sqrt{36} = 10 + 6 = 16$
1.3 $\sqrt{81} - \sqrt{16} = 9 - 4 = 5$
1.4 $\sqrt{36} - \sqrt{36} = 6 - 6 = 0$
2.1 $\sqrt[3]{27} + \sqrt[3]{8} = 3 + 2 = 5$
2.2 $\sqrt[3]{64} + \sqrt[3]{125} = 4 + 5 = 9$
2.3 $\sqrt[3]{216} - \sqrt[3]{1} = 6 - 1 = 5$
2.4 $\sqrt[3]{343} - \sqrt[3]{216} = 7 - 6 = 1$
3.1 $\sqrt{132} + \sqrt[3]{126} = 11{,}49 + 6 = 17{,}49$
3.2 $\sqrt{16} - \sqrt[3]{2} = 4 - 1{,}26 = 2{,}74$
3.3 $\sqrt[3]{216} \times \sqrt{100} = 6 \times 10 = 60$
4.1 $\sqrt{154 - 10} \div \sqrt{132} = \sqrt{144} \div \sqrt{132} = 12 \div 11{,}49 = 1{,}05$
4.2 $\sqrt{78 + 22} \times \sqrt{36} = \sqrt{100} \times \sqrt{36} = 10 \times 6 = 60$
4.3 $\sqrt[3]{220 - 4} \div \sqrt[3]{125} = \sqrt[3]{216} \div \sqrt[3]{125} = 6 \div 5 = 1{,}2$
4.4 $\sqrt[3]{200 + 16} + 16 \times \sqrt[3]{27} = \sqrt[3]{216} + 16 \times \sqrt[3]{27} = 6 + 16 \times 3 = 54$

Remedial

The initial sight of roots and calculations may scare certain learners, but by showing them how the calculation can simply be broken down should help dispel some of those fears.

Exercise 3

Learner's Book page 67

Guidelines on how to implement this activity

This exercise extends learners' knowledge of exponents and roots by applying it to rational numbers and fractions. Many learners are insecure about fractions, so provide much additional support by doing many examples as a class. This will help to show learners how to break down the problem to make it more manageable. All the rules of BODMAS, exponents and fractions apply. Learners should attempt this exercise on their own; however be available to assist learners should they experience any difficulties.

Suggested answers

1 $[(-\frac{1}{2}) \times (-\frac{1}{2}) \times (-\frac{1}{2})] \times \frac{1}{4} = [-\frac{1}{2} \times (-\frac{1}{2}) \times (-\frac{1}{2})] \times \frac{1}{4} = -\frac{1}{8} \times \frac{1}{4} = -\frac{1}{32} = 0{,}031$

2 $(0{,}4)^5 \div 2^5 = (0{,}4 \div 2)^5 = (0{,}2)^5 = 0{,}00032$

3 $(-\frac{1}{2})^2 \times \sqrt{\frac{64}{9}} \div (-\frac{1}{2})^3 = \frac{1}{4} \times \frac{8}{3} \div (-\frac{1}{8}) = \frac{1}{4} \times \frac{8}{3} \times (-\frac{8}{1}) = -\frac{64}{12} = -5{,}33$

4 $\sqrt[3]{\frac{-64}{9}} + [(\frac{4}{5}) - (\frac{2}{3})]^3 = \frac{-4}{2} + [(\frac{4}{5}) - (\frac{2}{3})]^3 = -2 + [\frac{12-10}{15}]^3 = -2 + \frac{8}{3375}$
$= 2 + 0{,}0024 = 1{,}998$

5 $\sqrt{(-5)^2 + (\frac{2}{-3})^2} = \sqrt{25 + \frac{4}{9}} = \sqrt{\frac{229}{9}} = 5{,}04$

6 $[(\frac{3}{4})^3 - (\frac{1}{2})^4] \div 2 = [\frac{27}{64} - \frac{1}{16}] \div 2 = [\frac{27}{64} - \frac{4}{64}] \div 2 = [\frac{23}{64}] \times \frac{1}{2}$
$= 0{,}36 \times \frac{1}{2} = 0{,}18$

Review Copy

Remedial

Provide revision exercises on adding, subtracting, multiplying and dividing fractions. Remind learners when finding the root of fractions, we need to find the root of the numerator and the denominator separately.

Unit 5 Scientific notation

Learner's Book page 68

Unit focus:

- scientific notation
- how to represent large numbers using scientific notation
- solving problems using scientific notation

Background information on scientific notation

Some numbers that scientists use every day are so large or so small that it is impractical to use them in integer or decimal form on a daily basis. For example, distances related to the Solar System are enormous, and sizes of atomic particles, or quantities related to some medical measurements are tiny. Scientific notation is used to work with these numbers.

For example: $2\,300 = 2{,}3 \times 1\,000 = 2{,}3 \times 10^3$

The estimated distance from the Earth to the sun is about 149 600 000 km. This is an awkward number to work with, so we use scientific notation. This is an exponential number worked on a base of 10, and is $1{,}496 \times 10^8$ km in this case.

Scientific notation makes it easier to work with very large and very small numbers. In Grade 8 learners only focus on writing large numbers in scientific notation.

Exercise 1

Learner's Book page 68

Guidelines on how to implement this activity

Demonstrate to the learners how a large number can be made more manageable in scientific notation. For example: the number 7 896,2.

In scientific notation it is required to have only 1 non-zero digit to the left of the decimal point. In this example the decimal point must be moved 3 spaces to the left, therefore you must divide by 1 000. This means that: $7\,896{,}2 \div 1\,000 = 7896{,}2 \div 10^3 = 7{,}8962$.

But 7,8962 is not equivalent to the original number 7 896,2. We make them equivalent by multiplying 7,8962 by 1 000 or 10^3

$7{,}8962 \times 10^3 = 7\,896{,}2$

Do additional examples as a class writing large numbers in scientific notation. Learners also need to know how to convert from scientific notation to large numbers. Do examples with learners demonstrating how to do this. Do as many examples as necessary for learners to become adept at this skill. Learners should complete this exercise on their own.

Review Copy

Suggested answers

1.1	$3{,}2\,4989 \times 10^4$	1.2	$6{,}7829 \times 10^3$	1.3	$9{,}3169 \times 10^3$
1.4	$4{,}659 \times 10^2$	1.5	$9{,}7532 \times 10^2$	1.6	$6{,}9845 \times 10^2$
1.7	$7{,}7777 \times 10^2$	1.8	$5{,}9601 \times 10^2$	1.9	$7{,}89648 \times 10^3$
1.10	$9{,}12378 \times 10^3$	1.11	$8{,}67863 \times 10^3$	1.12	$1{,}99999 \times 10^3$
2.1	$4{,}35 \times 10^6$ m^2	2.2	$5{,}7 \times 10^4$ m^2	2.3	$2{,}3 \times 10^7$ m^2
3.1	659 930	3.2	587,93	3.3	39 898 900
3.4	5 980,76	3.5	3 000 890	3.6	61 900 000

4.1	$4{,}65 \times 10^6$	>	$4{,}650 \times 10^5$
4.2	$3{,}2 \times 10^1$	=	32
4.3	55 888 999	>	$5{,}56 \times 10^7$
4.4	$8{,}8 \times 10^2$	<	8 800

5.1 5 009; $6{,}768 \times 10^2$; 0, 675; – 65 000

5.2 $1{,}654 \times 10^7$; $1{,}6780 \times 10^5$; 165 889; 16 009

5.3 123 890 009; 879 999; $8{,}760001 \times 10^5$; 879

Remedial

For learners experiencing difficulty with scientific notation, give them a selection of numbers to be made smaller by division by 10 or a power of 10. For example, numbers such as 200, 3 000 and 40 000. Ask learners to write an equivalent number for these numbers. Example: $600 \div 100 = 6$ or 6,0 and $100 = 10 \times 10 = 10^2$. This means to get 600 they must multiply 6,0 by 10^2:

$6{,}0 \times 10^2 = 6{,}0 \times 100 = 600$

Now give the learners a slightly more challenging selection of numbers to be made smaller by division by 10 or a power of 10, for example numbers such as: 3 400; 776; 8 967

Extension

Encourage learners to find out as much as they can about *Curiosity* and the NASA Mars landing. Encourage learners to report on distances, speeds and costs. Let learners make a wall chart with all their information and pictures, and write the numbers they found about distances, speeds and costs in scientific notation.

Exercise 2

Learner's Book page 69

Guidelines on how to implement this activity

Discuss how we could use scientific notation in problem solving. Revise the steps for problem solving:

- identifying what we are being asked
- identifying what we are give
- creating a number sentence or equation
- performing the calculation
- checking the reasonableness of our answer

Do an example together as a class that uses problem solving with scientific notation. Learners can work together in pairs or small groups to complete this exercise.

Review Copy

Suggested answers

1 The distance from:
1.1 Mercury to the Sun = $5{,}8 \times 10^7$ km
1.2 Venus to the Sun = $1{,}082 \times 10^8$ km
1.3 Earth to the sun = $1{,}497 \times 10^8$ km
1.4 Mars to the sun = $2{,}278 \times 10^8$ km
2.1 Mercury is closest to the Sun.
2.2 Mercury is 50 200 000 km closer to the Sun, than Venus.
2.3 Venus is 41 500 000 km closer to the Sun, than the Earth.
2.4 Mars is 78 100 000 km further from the Sun than the Earth.
2.5 $5{,}02 \times 10^7$ km; $4{,}15 \times 10^7$ km; $7{,}81 \times 10^7$ km
3.1 The average distance from the Earth to the moon is $3{,}84 \times 10^5$ km.
3.2 The population of South Africa is $4{,}25 \times 10^7$.
3.3 The population of China is about $8{,}5 \times 10^8$.
4.1 $8{,}236 \times 10^2$ 4.2 $2{,}4374 \times 10^3$
4.3 $6{,}7598 \times 10^3$ 4.4 $1{,}80035 \times 10^4$
4.5 $1{,}25764 \times 10^4$
5.1 The area of South Africa is $1{,}22 \times 10^6$ km^2.
5.2 The area of Swaziland is $1{,}74 \times 10^4$ km^2.
5.3 The area of Botswana is $5{,}82 \times 10^5$ km^2.
5.4 There are about $6{,}00 \times 10^9$ people on earth.
5.5 The Russian Federation is $1{,}71 \times 10^7$ km^2.
5.6 5.2; 5.3; 5.1; 5.5; 5.4
6.1 William travels 400 + 343 + 125 = 868 km.
6.2 His friend travels 868 – 81 = 787 km.

6.3 The two travel in total 1 655 km.
6.4 The difference between the two is 81 km.

Consolidation

Learner's Book page 72

Before doing this consolidation exercise, encourage learners to review the work covered in this chapter. Advise learners to use the summary and to revise their work. This exercise can be used as an informal assessment task for you to keep track of how learners are coping with the chapter and the concepts covered. The mark allocation provides guidelines on how to assess learners.

Suggested answers

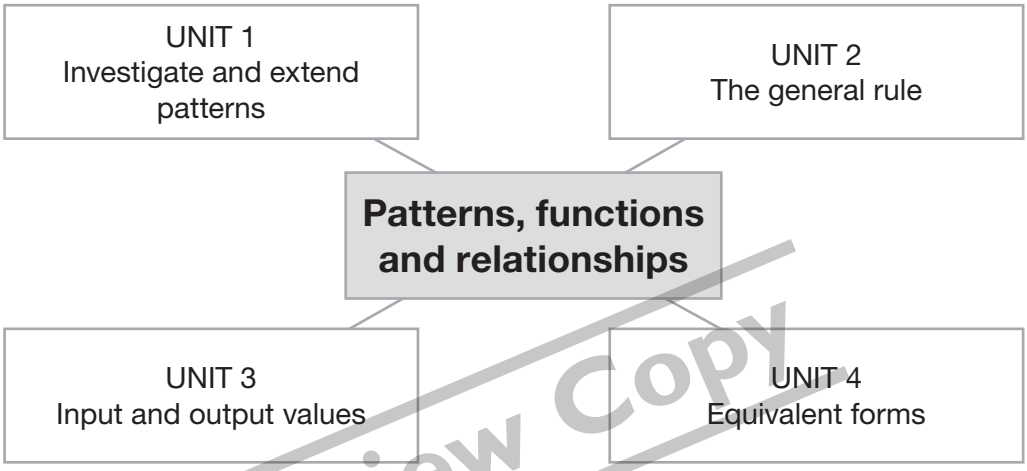
1.1	3	1.2	4	1.3	7	1.4	8	(4)
2.1	6	2.2	3	2.3	10	2.4	7	(4)
3.1	13 + 2 = 15	3.2	100 + 7 = 17	3.3	11 – 5 = 6			(6)
4.1	3 + 2 = 5	4.2	5 + 5 =10	4.3	7 – 1 = 6			(6)
5.1	$4{,}5\ 6789 \times 10^4$			5.2	$7{,}2829 \times 10^3$			
5.3	$4{,}4569 \times 10^3$			5.4	$9{,}879 \times 10^2$			(4)
6.1	$7{,}85 \times 10^6$ m^2			6.2	$4{,}38 \times 10^4$ m^2			
6.3	$6{,}7\ 5 \times 10^7$ m^2							(6)
7.1	$4^3 \times 3^3 = 64 \times 27 = 1\ 728$			7.2	$3^4 \times 1^4 = 81 \times 1 = 81$			
7.3	$3^3 \times 2^3 = 27 \times 8 = 216$			7.4	$5^2 \times 3^2 = 25 \times 9 = 225$			(4)
8	$\sqrt{6} = 2{,}45$ – it is between 3 and 2 which are $\sqrt{9}$ and $\sqrt{4}$							(3)
9	On Monday Thandi made $x = \sqrt{144} = 12$ pancakes.							(3)
	She made 12 + 144 = 156 pancakes altogether.							

Total marks [40]

Review copy

Chapter 4 Patterns, functions and relationships

Chapter overview



Review Copy

Content		Time allocations	LB page
Unit 1	Investigate and extend patterns	2,5 hours	74
Unit 2	The general rule	2 hours	78
Unit 3	Input and output values	1,5 hours	83
Unit 4	Equivalent forms	1,5 hours	87

Background information on patterns, functions and relationships

Patterns can take many forms; for example they can be represented in physical or diagram form. Most commonly in mathematics, we use number patterns.

A number pattern can be described by the rule which determines its order. To find the rule, look carefully at the pattern to see the relationship between successive numbers. Multiplication tables are simple number patterns with consistent or constant differences between successive terms. For example: 5; 10; 15; 20; …

In this pattern the rule is + 5. It is a constant, regular rule. This means the difference between any two successive numbers in the pattern is the same.

Mathematics has been described as the language of patterns.

Generic teaching guidelines for teaching patterns, functions and relationships

Start by asking learners to complete a series of number sequences, for example:
10, 20, 30, …, …, …, …, …., …, 100
Now describe the difference between any two numbers in the sequence, or the rule that governs the sequence. In the above case the rule is + 10.

Flow diagrams show clearly what the input and output values are and help to discover the rule, for example:

Input		Rule		Output
7	→	$+a$	→	14

Figure 1 The rule of a simple number pattern

Some number patterns are irregular and have more complicated rules. Ask learners to complete a series of irregular number sequences, for example:
1; 4; 9; …; …; …; …; …; …; 100

Now describe the difference between any two numbers in the sequence, or the rule that governs the sequence. The numbers above are perfect squares: $1 \times 1 = 1^2 = 1$ and $2 \times 2 = 2^2 = 4$, etc.

Resources

Matchsticks, counters, blocks and other objects to create geometric patterns. Paper, glue, colour pens and pencils to draw and create patterns. Each learner should have a calculator.

Review Copy

Unit 1 Investigate and extend patterns

Learner's Book page 74

Unit focus:

- investigating and extending numeric patterns
- investigating and extending geometric patterns
- looking for relationships in patterns.

Background information on investigating and extending patterns

Learners must be able to describe and extend patterns in their own words. To do this they must recognise the rules and relationships that apply. They can use flow diagrams, tables, number sentences and equations, diagrams, or any method to discover the relationship between numbers. Once the rule has been discovered it is simple to extend the pattern.

Exercise 1

Learner's Book page 76

Guidelines on how to implement this activity

Encourage learners to look for patterns in nature and in man-made things. Allow them to describe in words what makes the thing they have identified a pattern, for example, the two wings of a butterfly have a mirror image pattern.

Next, ask them to look for patterns in mathematics. Point out that counting numbers are governed by a rule (+1), so we get 1; 2; 3; … etc. The pattern 9; 8; 7; 6; … is governed by the rule (–1). Mathematics is a way of describing and using patterns.

Ask learners to use pebbles or dried beans to arrange in number patterns, for example: 1 bean (+1) = 2; 2 beans (+3) = 5; 5 beans (+5) = 10

So far the number sequence is 1; 2; 5; 10; … The rule is: sequential odd numbers have been added to each number to get the next one. Show this on a flow diagram:

Input		Rule		Output
1	→	+ 1	→	2
2	→	+ 3	→	*a*
5	→	+ 5	→	*b*
10	→	+ 7	→	c

Figure 2 Extending a pattern by applying the rule

Discuss with learners how to extend patterns. Let different members of the class explain how they determine how to extend a pattern. Discuss each of these ways and trial them on different examples. Work through the examples in the Learner's Book on extending patterns. Discuss patterns that extend with a constant difference and those that work without a constant difference. Tell the learners that some patterns can have very complicated rules that govern them. Learners can do this exercise in pairs or small groups.

Review Copy

Suggested answers

1.1 4 – 2 = 2
6 – 4 = 2
8 – 6 = 2

1.2 3 – 1 = 2
5 – 3 = 2
7 – 5 = 2

1.3 10 – 5 = 5
15 – 10 = 5
20 – 15 = 5

2.1 4 – 1 = 3
9 – 4 = 5
16 – 9 = 7 increasing odd numbers

2.2 3 – 1 = 2
6 – 3 = 3
10 – 6 = 4 increasing natural numbers

2.3 8 – 1 = 7
27 – 8 = 19
64 – 27 = 37 differences are prime numbers

3.1 1 ; 3 ; 5 ; 7 ; <u>9</u> ; <u>11</u> ; <u>13</u> ; <u>15</u> ; <u>17</u>

3.2 5 ; 10 ; 15 ; <u>20</u> ; <u>25</u> ; <u>30</u> ; <u>35</u> ; <u>40</u>

3.3 100 ; 90 ; 80 ; <u>70</u> ; <u>60</u> ; <u>50</u> ; <u>40</u> ; <u>30</u>

4.1 –36 ; –30 ; –24 ; <u>–18</u> ; <u>–12</u> ; <u>–6</u> ; <u>0</u> ; <u>6</u> ; <u>12</u> ; <u>18</u>

4.2 25 ; 20 ; 15 ; <u>10</u> ; <u>5</u> ; <u>0</u> ; <u>–5</u> ; <u>–10</u> ; <u>–15</u> ; <u>–20</u>

4.3 <u>12</u> ; <u>10</u> ; 8 ; <u>6</u> ; 4 ; <u>2</u> ; <u>0</u> ; –2 ; <u>–4</u> ; <u>–6</u> ; –8

5.1–5.3 Learners complete the patterns in the Learner's Book.

6 Learners create own patterns and draw them in their exercise books.

7.1

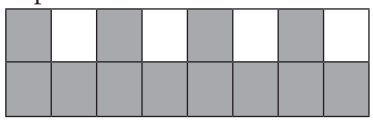
Number of friends	1	2	3	4	5	10	20	25	30
Number of SMSs	1	**2**	**3**	**4**	**5**	**10**	**20**	**25**	**30**

7.2

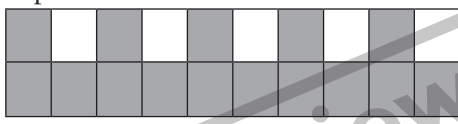
Number of friends	1	2	3	4	5	10	20	25	30
Number of SMSs	3	**6**	**9**	**12**	**15**	**30**	**60**	**75**	**90**

8.1 Repeat 3:

Repeat 4:



Repeat 5:



Review Copy

8.2 18 shaded tiles.

8.3 Each repeat adds four more tiles, three of which are shaded. Therefore each repeat adds three shaded tiles (4 – 1 = 3). So the number of shaded tiles equals three times the repeat number.

8.4 $s = r(4 - 1) = 15(4 - 1) = 15(3) = 45$ shaded tiles

9.1

$1 \times 99 = 99$	$9 + 9 = 18$	$1 + 8 = 9$
$2 \times 99 = 198$	$1 + 9 + 8 = 18$	$1 + 8 = 9$
$3 \times 99 = 297$	$2 + 9 + 7 = 18$	$1 + 8 = 9$
$4 \times 99 = 396$	$3 + 9 + 6 = 18$	$1 + 8 = 9$
$5 \times 99 = 495$	$4 + 9 + 5 = 18$	$1 + 8 = 9$
$6 \times 99 = 594$	$5 + 9 + 4 = 18$	$1 + 8 = 9$
$7 \times 99 = 693$	$6 + 9 + 3 = 18$	$1 + 8 = 9$
$8 \times 99 = 792$	$7 + 9 + 2 = 18$	$1 + 8 = 9$
$9 \times 99 = 891$	$8 + 9 + 1 = 18$	$1 + 8 = 9$
$10 \times 99 = 990$	$9 + 9 + 0 = 18$	$1 + 8 = 9$

9.2 Yes

9.3

$11 \times 99 = 1\,089$	$1 + 0 + 8 + 9 = 18$	$1 + 8 = 9$
$12 \times 99 = 1\,188$	$1 + 1 + 8 + 8 = 18$	$1 + 8 = 9$
$13 \times 99 = 1\,287$	$1 + 2 + 8 + 7 = 18$	$1 + 8 = 9$
$14 \times 99 = 1\,386$	$1 + 3 + 8 + 6 = 18$	$1 + 8 = 9$
$15 \times 99 = 1\,485$	$1 + 4 + 8 + 5 = 18$	$1 + 8 = 9$

The pattern still holds.

10.1

9	8	7	6	5	4	3	2	1	1	2	3	4	5	6	7	8	9
	8	7	6	5	4	3	2	1	1	2	3	4	5	6	7	8	
		7	6	5	4	3	2	1	1	2	3	4	5	6	7		
			6	5	4	3	2	1	1	2	3	4	5	6			
				5	4	3	2	1	1	2	3	4	5				
					4	3	2	1	1	2	3	4					
						3	2	1	1	2	3						
							2	1	1	2							
								1	1								
9	16	21	24	25	24	21	16	9	9	16	21	24	25	24	21	16	9

10.2 The pattern starts at 9 goes up to 25, then descends to 9 in a mirror image.
10.3 Learners compare and discuss answers.

Remedial

Allow learners who have trouble grasping these concepts to complete a number of simple number sequences, for example 2; 4; 6, 8;… and 3; 6; 9; 12;… Next ask them to describe the difference between the numbers in each sequence. So 4 is 2 bigger than 2, 6 is 2 bigger than 4. Therefore the difference is + 2. Next ask them to make up a number sequence of their own.
They might like to use a table to do this. For example:

Number of bowls	1	2	3	4	5	6
Number of cherries in a bowl	15	30	45	60	75	90

Alternatively, they could use a flow diagram. For example:

Input		**Rules**		**Output**
2	→	× 3 – 1	→	5
4	→	× 3 – 1	→	11
6	→	× 3 – 1	→	17
8	→	× 3 – 1	→	23

Extension

Ask learners to fold a piece paper in four and cut out a number of fish shapes. Colour the shapes and glue them to a sheet of paper to form a pattern. When they are all glued in place, use bright coloured pencils to draw a pattern over the fish.

Search in the library to find patterns used on pottery in Africa. Try to copy some of them onto an empty tin. Paint the patterns.

Unit 2 The general rule

Learner's Book page 78

Unit focus:

- learning more about the rules that govern patterns
- describing and justifying rules of patterns.

Review Copy

Background information on the general rule

It is possible to show the correspondence between a flow diagram, a table, a graph and a word sentence by identifying the rule. The rule is a fixed procedure for solving a problem. Word sentences can be rewritten as number sentences and used to solve problems. To do this it is sometimes necessary to use and identify variables in equations or in other mathematical expressions.

A finite sequence is made up of n terms. For example 1; 2; 3; … ; n where n is a positive integer and the length of the sequence. To find the relationship between the numbers it is necessary to describe the general rule.

Exercise 1

Learner's Book page 79

Guidelines on how to implement this activity

Revise working with flow diagrams. Work through examples of flow diagrams, identifying input, output and rule. Use flow diagrams to help learners to identify the rule between numbers. For example:

Input		Rule				Output
7	→	+ 5	→	× 2	→	y
14	→	+ 5	→	× 2	→	z
x	→	+ 5	→	× 2	→	52

Ask them to transfer the information from the flow diagram to a table.

Input	7	14	x
Rules	+ 5	+ 5	+ 5
Rules	× 2	× 2	× 2
Output	y	z	52

Review Copy

Next rewrite the information in a word sentence which can then be translated to a formula or equation. For example: Add five to a multiple of seven and then multiply the sum by two. This can be written: $(7 + 5) \times 2 = x$

Discuss how the different forms reveal the general rule. Explain the steps learners can follow to identify the general rule. Do examples of determining the general rule of patterns as a class. Learners should complete the exercise on their own, but allow them to discuss their answers in small groups.

Suggested answers

1.1 3; 6; 9; <u>12</u>; <u>15</u>; <u>18</u>
1.2 99 ; 88 ; <u>77</u>; <u>66</u>; <u>55</u>; <u>44</u>
1.3 –2; <u>–1</u>; 0; 1; 2 ; 3
1.4 4; 3; 2; <u>1</u>; <u>0</u>; <u>–1</u>; <u>–2</u>
2.1 Multiples of 3.
2.2 Decreasing multiples of 11.
2.3 Increasing integers with a difference of 1.
2.4 Decreasing integers with a difference of 1.

3

A		B		C
4	×	4	=	16
4	×	3	=	12
4	×	2	=	8
4	×	1	=	4
4	×	0	=	0
4	×	(–1)	=	–4
4	×	(–2)	=	–8
4	×	(–3)	=	–12

3.1 Constant term (positive).
3.2 Decreasing positive to negative integers.
3.3 Decreasing positive to negative integers.
3.4 Multiply by positive and negative integers.
4. Patterns of learners' own creations.
Check that learners have created five of each type of pattern and that the pattern can be determined using the general rule they have written down.

5.1

Number of visitors	1	2	3	4	10	…	18	…	**27**
Entry fee	R6	R12	**R18**	**R24**	**R60**	…	**R108**	…	162

5.2 Total entry fee is equal to the product of 6 and the number of entries.
5.3 Total fee = 6 × 59 = R354.

6.1

Distance travelled	1	5	10	12	18	60	130
Cost	R5,70	R28,50	R57,00	R68,40	R102,60	R342,00	R741,00

6.2 $C = 5{,}70 \times d$
6.3 $C = 5{,}70 \times 98 = R558{,}60$
7.1 $C = 3{,}20 \times d + 20$
7.2 $C = 3{,}20 \times 60 + 20 = 192 + 20 = R212$
8.1 Dhansay: Cost = 5,70 × 6 = R34,20
Rubin's: Cost = 3,20 × 6 + 20 = R39,20
Dhansay is cheaper
8.2 Dhansay: Cost = 5,70 × 50 = R285
Rubin's: Cost = 3,20 × 50 + R20 = R160
Rubin's is cheaper.

8.3

	6	8	10
Dhansay	R34,20	R45,60	R57
Rubin's	R39,20	R45,60	R52

At 8 km they cost the same amount.

9.1

Week	1	2	3	4	5	…	10
Option 1	R50	R52	**R54**	**R56**	**R58**		**R70**
Option 2	R2	R4	**R8**	**R16**	**R32**		**R1 024**

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9.2 There is a constant difference of R2
9.3 If you subtract the consecutive terms you get different answers.
9.4 $A = 2w + 50$
9.5 Each week is multiplied by 2.
$2^1 = 2$; $2^2 = 4$; $2^3 = 8$
9.6 Option 1 because it increases faster initially.
9.7 Option 2 because $2^7 = 128$ and $2 \times 7 + 50 = 64$.
10.1 $13\frac{1}{2}$

10.2

Repeat	1	2	3	4	5	6
Total number of squares	$3^1 = 3$	$3^2 = 9$	$\mathbf{3^3 = 27}$	$\mathbf{3^4 = 81}$	$\mathbf{3^5 = 243}$	$\mathbf{3^6 = 729}$
Number of shaded squares	$1\frac{1}{2}$	$4\frac{1}{2}$	$\mathbf{13\frac{1}{2}}$	$\mathbf{40\frac{1}{2}}$	$\mathbf{121\frac{1}{2}}$	$\mathbf{364\frac{1}{2}}$

10.3 $s = 3^r$

10.4 $t = \frac{3^r}{2}$

10.5 $t = \frac{3^r}{2} = \frac{3^8}{2} = \frac{6\,561}{2} = 3\,280\frac{1}{2}$

10.6 No, because 8 is not a multiple of 3.

Remedial

Allow learners to revise patterns by doing a number of simple flow diagrams.
For example:

Input		**Rules**		**Output**
2	→	+ 5	→	y
x	→	+ 5	→	9
6	→	+ z	→	11

Transfer the information to a table. Like this:

Input	2	x	6
Rules	+ 5	+ 5	+ z
Output	y	9	11

Next rewrite the information in a word sentence which can then be translated to a formula or equation. For example: Add five to a multiple of two. This can be written: $2x + 5 = y$.

Extension

Learners who are confident may be asked to create their own patterns to give to one another to extend and find the general rule of.

Exercise 2

Learner's Book page 81

Guidelines on how to implement this activity

This exercise extends learners' skills in working with number patterns. It focuses on providing real life contexts for number patterns. Learners should be able to manage

Review Copy

with the explanation provided before the previous exercise. However, if they require you to do additional examples, work through these as a class. Learners can complete this exercise in pairs.

Suggested answers

1.1

Number of magazines	1	2	3	4	5	6	7	8	9	10
Cost	R15,40	R30,80	**R46,20**	**R61,60**	**R77**	**R92,40**	**R107,80**	**R123,20**	**R138,60**	**R154**

1.2 R77

1.3 R107,80 – R46,20 = R61,60

2.1

No. of months	2	3	4	5	6	7	8	9	10	11	12
Cost	R114,30	R171,45	R228,60	R285,75	R342,90	R400,05	R457,20	R514,35	R571,50	R628,65	R685,80

2.2

No. of years	1	2	3	4	5
Cost	R685,80	R1 371,60	R2 057,40	R2 743,20	R3429

3.1

Time spent on calls (minutes) (t)	25	50	75	100	125	…	200	…	225
Total Cost (C)	R94,25	R110,50	R126,75	**R143**	**R159,25**	…	**R208**	…	R224,25

3.2 R94,25 – R78 = R16.25

3.3 $\frac{\text{R}16{,}25}{25}$ = R0,65 per minute

3.4 Cost = 78 + 0,65 × 312 = R280,80

3.5 Cost = 78 + 0,65 × 128 = R161,20
Not billed correctly.

3.6 $C = 78 + 0{,}65t$

4.1

Length	2 m	3 m	5 m	7 m	x m
Floor area	4 m^2	9 m^2	25 m^2	49 m^2	x^2 m^2
Volume	8 m^3	27 m^3	125 m^3	343 m^3	x^3 m^3

4.2

Floor area	4 m^2	9 m^2	25 m^2	49 m^2	x^2 m^2
Cost	R6 000	R13 500	R37 500	R73 500	R1 500 x^2

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Extension

Allow learners to work in pairs to make up a game based on patterns and rules. Mention that all games are based on certain rules and on patterns. For example, think of chess, tennis and monopoly.

Unit 3 Input and output values

Learner's Book page 83

Unit focus:

- finding input and output values using given equations in flow diagrams
- including multiplication and division of integers in finding input and output values
- including numbers in exponential form in finding input and output values
- finding input and output values using formulae.

Background information on Input and output values

By substituting values for variables in an expression you can calculate the value of the whole expression. For example: If $z = 1$, then $5 + z = 6$. If $z = 2$, then $5 + z = 7$. If $z = 3$, then $5 + z = 8$.

The values you put in the place of z are the input values. The value of the answer is the output value.

Exercise 1

Learner's Book page 85

Guidelines on how to implement this activity

Ask learners to look at a flow diagram on the board.

$$x \longrightarrow +3 \longrightarrow 4$$
$$y \longrightarrow +3 \longrightarrow 7$$

Discuss with the class:

- Which are the input values? (x and y)
- Which are the output values? (4 and 7). What is the rule? (+ 3)

Now do a similar exercise where the variable is z. If the value of z is in the first column, put this number in the place of z to find the value of the output.

$$1 \longrightarrow 3z + 3 \longrightarrow a$$
$$2 \longrightarrow 3z + 3 \longrightarrow b$$

Discuss with the class:

- Which are the input values?
- Which are the output values?
- What is the rule? ($3z + 3$).

Show learners how to translate the input, output and rule into an equation. Demonstrate how, given an input value you can find the output, and given the output you can find the input by working with the rule in reverse. Discuss the use of variables in the rule to represent the input values. Work through a few examples as a class and then allow learners to complete the exercise.

Review Copy

Suggested answers

1 $a = (484 + 16) \div 25 = 500 \div 25 = 20$
$b = (609 + 16) \div 25 = 625 \div 25 = 25$
$c = 5 \times 25 - 16 = 125 - 16 = 109$
$d = 10 \times 25 - 16 = 250 - 16 = 234$
$e = 15 \times 25 - 16 = 375 - 16 = 359$
$g = 30 \times 25 - 16 = 750 - 16 = 734$

2.1 $a = 10 - 10 = 0$ $b = 21 - 10 = 11$
$x = 7 + 10 = 17$ $y = -5 + 10 = 5$

2.2

Input	7	10	21	17	5
Output	–3	0	17	7	–5

3 $a = 2^2 - 4 = 4 - 4 = 0$
$b = 5^2 - 4 = 25 - 4 = 21$
$c = 9^2 - 4 = 81 - 4 = 77$

$d = 12^2 - 4 = 144 - 4 = 140$
$e = 15^2 - 4 = 225 - 4 = 221$

4.1 $a = (5 + 1) \div 3 = 6 \div 3 = 2$
$b = (44 + 1) \div 3 = 45 \div 3 = 15$
$c = 3 \times 5 - 1 = 15 - 1 = 14$
$d = 3 \times 8 - 1 = 24 - 1 = 23$

4.2 $a = (5 - 3^2) \times (-1) = (5 - 9) \times (-1) = (-4) \times (-1) = 4$
$b = (0 - 3^2) \times (-1) = (0 - 9) \times (-1) = (-9) \times (-1) = 9$
$c = -(3) + 3^2 = -3 + 9 = 6$
$d = -(12) + 3^2 = -12 + 9 = -3$

4.3 $t = -(3 \times -2) \div 2 = -(-6) \div 2 = 6 \div 2 = 3$
$r = -(3 \times -8) \div 2 = -(-24) \div 2 = 24 \div 2 = 12$
$s = -(3 \times 10) \div 2 = -(30) \div 2 = -30 \div 2 = -15$
$p = -(3 \times 17) \div 2 = -(-51) \div 2 = -51 \div 2 = -\frac{51}{2} = -25{,}5$

4.4 $u = \sqrt{25} - 1 = 5 - 1 = 4$
$w = \sqrt{144} - 1 = 12 - 1 = 13$
$x = (6 + 1)^2 = 7^2 = 49$
$y = (9 + 1)^2 = 10^2 = 100$

4.5 $a = \frac{-27}{-3} \times \frac{2}{4} = \frac{9}{2}$

$b = \frac{-150}{-3} \times \frac{2}{4} = 25$

$c = \frac{4 \times 1}{2} \times (-3) = \frac{4}{2} \times \frac{-3}{1} = \frac{-12}{2} = -6$

$d = \frac{4 \times 13}{2} \times (-3) = \frac{52}{2} \times \frac{-3}{1} = \frac{-156}{2} = -78$

4.6 $a = (3 - 4)^3 = (-1)^3 = -1$
$b = (3 - 4)^5 = (-1)^5 = -1$
$c = (3 - 4)^{10} = (-1)^{10} = -1$
$d = (3 - 4)^{15} = (-1)^{15} = -1$

4.7 $p = ((-2)^3)^2 = (-8)^2 = 64$
$s = ((-5)^3)^2 = (-125)^2 = 15\ 625$
$q = \sqrt[3]{\sqrt{262\ 144}} = \sqrt[3]{512} = 8$

$r = \sqrt[3]{\sqrt{1\ 000\ 000}} = \sqrt[3]{1\ 000} = 10$

$t = ((-12)^3)^2 = (-1\ 728)^2 = 2\ 985\ 984$

Remedial

For learners experiencing problems with this exercise, revise simple flow diagrams, and transferring the information to tables. Ask learners to find the value of variables in simple number sentences, for example:

$2a = 6$
$2 + a = 4$
$a - 1 = 5$

Review Copy

Extension

Use geometric shapes in patterns based on the work of artists like Mondrian. Describe the rules behind the patterns in the paintings. For example,if the painting is 6 tessellated rectangles of different sizes, the rules could be: Rectangle 1 is twice the size of rectangle 2 which is twice the size of … and so forth.

Exercise 2

Learner's Book page 86

Guidelines on how to implement this activity

Ask learners about formulae they have learnt in mathematics. Discuss what a formula is. What makes a formula different from an equation? Identify inputs and outputs in various formulae. Practise substituting values into a formula as a class. Learners should complete this exercise on their own.

Suggested answers

1.1 $A = \ell \times b$

$12 = 4 \times b$

$b = \frac{12}{4} = 3$ m

1.2 $A = \ell \times b$

1.3 $p = 2\ell + 2b$

1.4 $p = 2\ell + 2b = 2 \times 4 + 2 \times 3 = 8 + 6 = 14$ m

2.1 $A = \ell \times b = 5 \times 3 = 15$ cm²

2.2 $A = s^2 = (7{,}5)^2 = 56{,}25$ cm²

3.1 $V = \ell \times b \times h = 5{,}6^3 = 175{,}616$ m³

3.2 $V = \ell \times b \times h = 5{,}2 \times 3{,}1 \times 4{,}8 = 77{,}376$ cm³

4.1 distance = time × speed

$d = 60x$

4.2 $d = 15t$

5.1 $d = t \times s$

$60 = t \times 120$

$t = \frac{60}{120} = 0{,}5$ hours

5.2 $d = t \times s$

$60 = t \times 30$

$t = \frac{60}{30} = 2$ hours

5.3 $d = t \times s$

$60 = 0{,}75 \times s$

$s = \frac{60}{0{,}75} = 80$ km per hour

Review Copy

Remedial

Remind learners of the formulae for area and perimeter of rectangles and circles. Discuss with them how they worked with the formula in these contexts. Providing context is helpful for learners, instead of dealing only with pure theory. Learners

know how to work with formulae but often find them problematic when they are removed from context. Do simple examples determining area and perimeter of shapes, and substituting in values. Link this to the exercise and have learners try again with the material.

Unit 4 Equivalent forms

Learner's Book page 87

Unit focus:

- determining and understanding different ways of representing functions and relationships
- translating information from one form to another.

Background information on equivalent forms

Equivalent forms refer to the fact that we can represent the same relationship or rule in several different ways. In Grade 8 these ways are:

- verbally
- in flow diagrams
- in tables
- by formulae
- by equations

Learners must be able to recognise equivalent forms and be able to represent a given relationship in each of the different ways.

Review Copy

Exercise 1

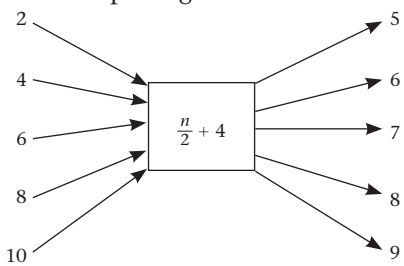
Learner's Book page 88

Guidelines on how to implement this activity

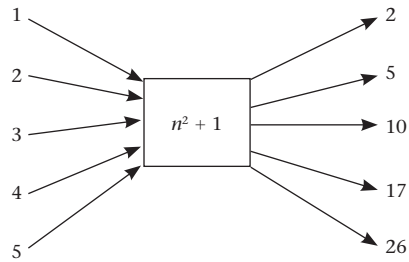
Discuss the different forms of representing relationships. Revise verbal descriptions, flow diagrams, tables, equations and formulae. Do examples involving converting between the different forms as a class. Discuss the process of translating (converting), and remind learners that the relationship does not change, but only the representation. Learners should complete the exercise on their own, but encourage learners to check their answers in small groups and discuss their thinking and the methods they used.

Suggested answers

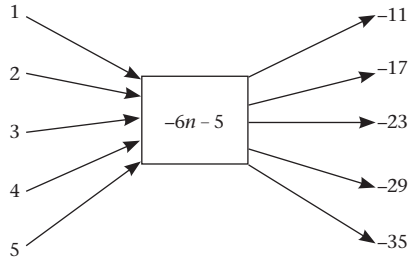
1.1 Answers will vary between learners. An example is given.



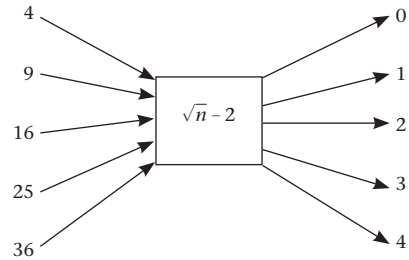
1.2



1.3



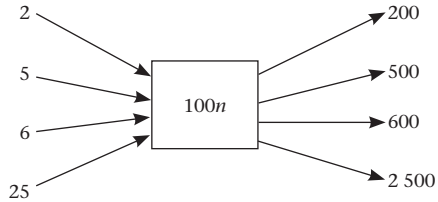
1.4



2.1

Depth of coal desposit in metres	2 m	5 m	**6 m**	25 m
Conversion	× 100	× 100	× 100	× 100
Depth of coal deposit in centimetres	**200 cm**	**500 cm**	600 cm	**2 500 cm**

2.2



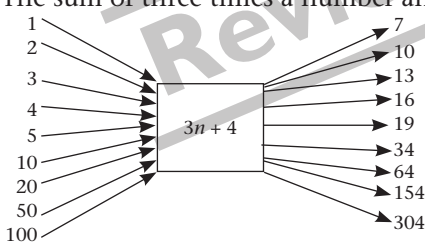
2.3 depth in cm = 100 × depth in m

3.1

n	1	2	3	4	5	10	20	50	100
$3n + 4$	7	10	13	**16**	**19**	**34**	**64**	**154**	**304**

3.2 The sum of three times a number and four.

3.3

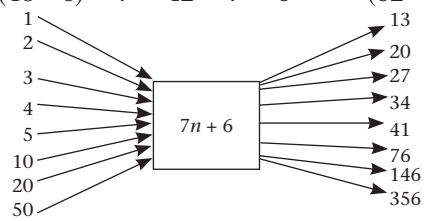


4.1

Input	1	2	3	4	5	10	15	20
Output	13	20	27	34	41	76	111	146

4.2 (48 – 6) ÷ 7 = 42 ÷ 7 = 6 (62 – 6) ÷ 7 = 56 ÷ 7 = 8

4.3



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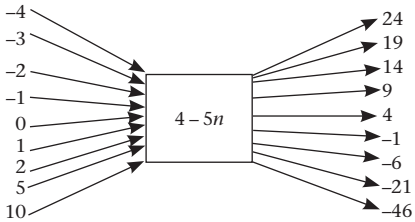
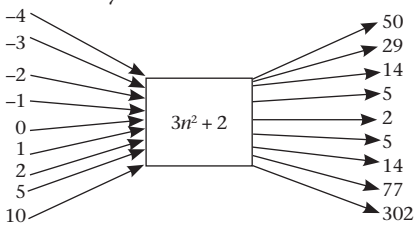
5.1



n	–4	–3	–2	–1	0	1	2	5	10
$4 - 5n$	24	19	14	9	4	–1	–6	–21	–46

Formula: $y = 4 - 5x$

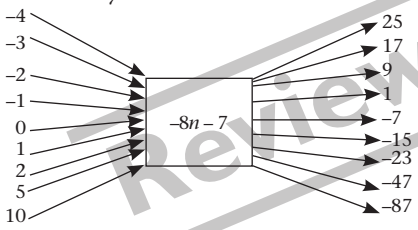
5.2



n	–4	–3	–2	–1	0	1	2	5	10
$3n^2 + 2$	50	29	14	**5**	**2**	**5**	**14**	**77**	**302**

Formula: $y = 3x^2 + 2$

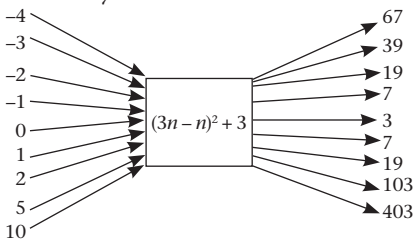
5.3



n	–4	–3	–2	–1	0	1	2	5	10
$-8n - 7$	25	17	9	**1**	**–7**	**–15**	**–23**	**–47**	**–87**

Formula: $y = -8x - 7$

5.4



n	–4	–3	–2	–1	0	1	2	5	10
$(3n - n)^2 + 3$	67	39	19	**7**	**3**	**7**	**19**	**103**	**403**

Formula: $y = (3x - x)^2 + 3$
$= (2x)^2 + 3$
$= 4x^2 + 3$

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Remedial

If learners struggle to convert between the forms, provide simpler examples for learners to practise with. Once learners are adept at translating simple patterns from one form to another, increase the complexity until they can manage the prescribed exercise.

Extension

Learners can refer back to the patterns in Unit 3. Encourage learners to translate the representations in the exercise to different forms.

Consolidation

Learner's Book page 90

Before doing this consolidation exercise, encourage learners to review the work covered in this chapter. Advise learners to use the summary and to revise their work. This exercise can be used as an informal assessment task for you to track how learners are coping with the chapter and the concepts covered. The mark allocation provides guidelines on how to assess learners.

Suggested answers

1.1 5; 10; 15; 20; 25; 30 1.2 89; 83; 77; 71; 65; 59

1.3 –5; –4; –3; –2; –1; 0; 1; 2 1.4 8; 6; 4; 2; 0; –2; –4 (5)

2.1 increasing by the constant value of 5.

2.2 decreasing by the constant value of 6.

2.3 increasing by the constant value of 1.

2.4 decreasing by the constant value of 2. (5)

3 $n \rightarrow 3n - 8 \rightarrow 7$

$n = 5$ so $3n - 8 = 7$ (2)

4. 1

Input	–3	–2	–1	0	1	2	3
Output: $7n - 2$	–23	–16	–9	–2	5	12	19

i)

ii) $y = 7x - 2$ (4)

4. 2

Input	–3	–2	–1	0	1	2	3
Output: $-5n - n + 2$	20	14	8	2	–4	–10	–16

i)

ii) $y = -5x - x + 2$ (4)

4.3

Input	–3	–2	–1	0	1	2	3
Output: $2n^2 - 5n + 1$	–2	3	4	1	–2	–1	4

i)

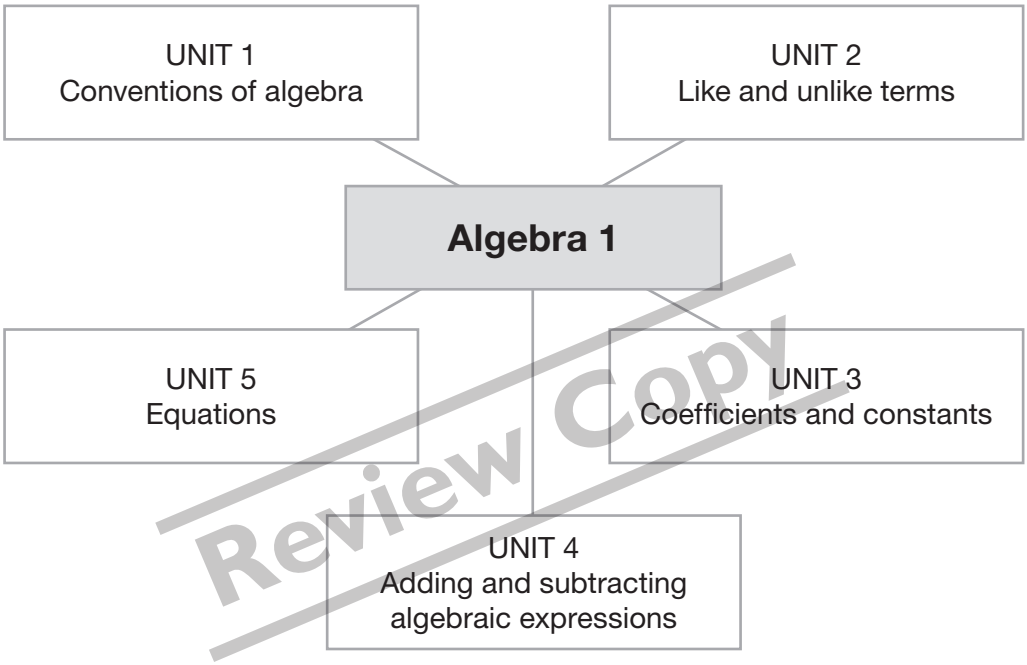
ii) $y = 2x^2 - 5x + 1$ (5)

Total marks [25]

Review Copy

Chapter 5 Algebra 1

Chapter overview



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Content		Time allocations	LB page
Unit 1	Conventions of algebra	1 hour	92
Unit 2	Like and unlike terms	1 hour	96
Unit 3	Coefficients and constants	1 hour	99
Unit 4	Adding and subtracting algebraic expressions	1,5 hours	102
Unit 5	Equations	3 hours	106

Background information on algebra

Algebra is the language for investigating and communicating most of Mathematics and can be extended to the study of functions and other relationships between variables.

In Grade 7 the learners did not learn the conventions of algebraic language and manipulating algebraic expressions. In Grade 8 they will learn about:

- recognising and identifying conventions for writing algebraic expressions
- like and unlike terms

- coefficients and constants
- adding and subtracting with algebra
- equations.

The main objectives in this chapter are:
- interpreting algebraic expressions
- introducing learners to conventions of algebraic language through adding and subtracting like terms
- algebraic equations – inspection only.

Teaching guidelines for teaching algebra

Revise the Grade 7 work as introduction for this chapter. The most important concept to bring across in Algebra 1 is the conventions of the algebraic language. All the important rules of algebra and terminology should be taught and consolidated. Consolidate the knowledge and application of the associative, commutative and distributive laws for real numbers. Use learners' previous knowledge of equations to consolidate solving equations using inspection.

Resources

Each learner should have a calculator and blank paper to assist with their working out.

Review Copy

Unit 1 Conventions of algebra

Learner's Book page 92

Unit focus:
- revising rules and relationships
- revising variables and constants
- learning basic algebraic conventions
- learning how to write in algebraic language.

Background information on the conventions of algebra

In Grade 7 the learners didn't cover conventions of algebra; however their knowledge of functions and patterns, and writing equations with a variable can serve as a starting point for this chapter. Learners have worked in depth on writing number sentences to describe problem situations, and by substituting in variables learners are shown how to write problems in algebraic language.

In algebra letters are used in number sentences to represent unknown values. The conventions of algebra are the agreements shared by all users of mathematics. Here misconceptions can be dealt with, for example: $x + x = 2x$ and not x^2, $(-2x^2)^3 = -8x^6$, and not $-6x^5$.

Exercise 1

Learner's Book page 92

Guidelines on how to implement this activity

Revise using a formula to describe the general rule for a pattern. Discuss what the rule

means and what the variable in the rule means. Explain the concepts of a variable and a constant in a formula and equation. Do a few examples together as a class, writing patterns in terms of the general rule and substituting in values to find the outputs. Encourage learners to complete this exercise on their own to develop their understanding of algebra.

Suggested answers

1.1 n is the position of a number in the sequence. The sequence is a list of the multiples of 3.
3 ; 6 ; 9 ; 12 ; <u>15</u> ; <u>18</u> ; <u>21</u>
$3 \times 50 = 150$ so the 50th term is 150

1.2 $y = 4x + 3$

x	0	2	4	6	8	10
y	3	11	19	27	35	43

1.3 $y = 3x + 3$
1.4 $35 + x = 35 + 14 = 49$ years
2.1.1 8
2.2.1 $2n - 1$
2.2.2 a; b
2.3.1 $a = 13 + 5 - 2 = 16$
$b = (30 + 2) - 5 = 27$
$c = 9 + 5 - 2 = 12$
2.3.2 5; –2

Review Copy

Remedial

If learners struggle to identify the pattern in terms of a general rule, revise this section from Grade 7 with the learners.

Exercise 2

Learner's Book page 94

Guidelines on how to implement this activity

Learners have worked with exponents before, and should know how to write numbers in expanded form. Use the learners' knowledge of exponents and extend this to include variables. Work through the algebraic conventions as listed in the table in the Learner's Book, and explain what a convention is. Do a few examples as a class on the board. Learners should do this exercise on their own.

Suggested answers

1	$6x$	2	a^4	3	x
4	$3b$	5	$-yz^2$	6	$a \times b \times 5 = 5ab$

Remedial

Learners can complete more exercises to get confident with the conventions and algebraic notations. For example, give learners the following algebraic expressions to simplify:

1 $a + a + a + a + a$ 2 $4a - a$

3	$x \times x \times x \times x \times x \times x$	4	$6 \times r \times 2 \times s$
5	$3y + y$	6	$5x \times 5y$
7	$7 \times b \times b \times b \times 2 \times b \times b$	8	$a \times b \times c\ (4 + 3)$

Extension

Learners can make wall charts to display the various algebraic conventions.

Exercise 3

Learner's Book page 94

Guidelines on how to implement this activity

As an introduction ask a learner to think of a number, but the number must be represented by a letter. Explain that in algebra we use letters as variables, which can represent different numbers. Show the steps for performing operations on the variable, and how these can be represented in algebraic form. Use the example below, and then work through two or three more, on the board, to discuss as a whole class. Encourage learners to make deductions from the workings on the board. They must take note how the number sentences are written, as shown below:

Think of a number:	a
Add 7 to the number:	$a + 7$
Treble your answer:	$3(a + 7) = 3a + 21$
Subtract 6:	$3a + 21 - 6 = 3a + 15$
Divide by 3:	$a + 5$

Review Copy

Take note:

- We write the number first.
- We write $a + 7$ and not $7 + a$.
- We don't write the multiplication sign $\times$, $3a$ and not 3×7.
- We don't write the division sign $\div$.
- If there is more than one letter in the term, we write them alphabetically.

Suggested answers

1.1	$x + 2$	1.2	$\frac{12a}{2} = 6a$	1.3	3 x 2y = 6y
1.4	$4b + (6 + c)$	1.5	p^2	1.6	$2d + 4e$
1.7	$2z - 5$	1.8	$7n + 16$	2.1	$7x$
2.2	$25 - x$	2.3	$2x \times 4 = 8x$	2.4	$3x \times 4y = 12xy$
2.5	$y + x$	2.6	$3x + 3x + 4y + 4y = 6x + 8y$		

Remedial

Learners can complete more exercises to get confident with the conventions. Give learners the following statements to write as algebraic expressions:

1 The sum of two numbers is divided by 3.
2 The product of 12 and a number is subtracted from 4 and a number.
3 A number decreased by 3 divided by 6.
4 Double a number, increase it by 5.
5 The difference between 10 and a number is multiplied by 3.
6 The quotient of 6 and a number is increased by 8.

Extension

Learners in this group work on writing a message to the class using the letters of the alphabet as a code. (A=1, B=2, C=3, D=4,…) They make use of / to indicate the start of a new letter and // to show the end of a word.
For example: $14 + 6 \times 3 \times 4 \div 8$/ $\sqrt{81}$// $100 - 99$/ $144 \div 12$/ $48 - (9 \times 4)$//
The encoded message above reads: "Hi all"

Unit 2 Like and unlike terms

Learner's Book page 96

Unit focus:

- learning about like and unlike terms
- learning to identify and classify like and unlike terms.

Background information on like and unlike terms

Terms are like terms if they contain exactly the same variables and exactly the same indices or powers of the variables. We can say that $5a$ and $7a$ are like terms, they share the same variable, namely a, and the same power of a, a^1. In the same way $12c^2d$ and $9c^2d$ are like terms.

On the other hand, we can see that $3b$ and 8 are unlike terms; they do not share the same variables. Other examples of unlike terms are $2xy$ and $10ab$, (variables not the same) or $4a^3b$ and $16ab^3$ (powers of the variables not the same).

Review Copy

Exercise 1

Learner's Book page 97

Guidelines on how to implement this activity

Discuss the concept of a term. Provide the definition. Show learners that an algebraic expression is an expression made up of any number of terms separated from each other by + or – signs, for example $2x - 1$; $6y^2 + 12$. Show learners examples of expressions and how to identify the number of terms by looking at the separation of + and – signs. Ensure learners know that addition and subtraction separate terms, and that multiplication, brackets and division unite terms. Discuss how to name expressions using the number of terms. Ensure learners know the names of monomials, binomials and trinomials.

Suggested answers

1.1	2 ; binomial	1. 2	1 ; monomial	1. 3	1 ; monomial
1. 4	4 ; polynomial	1. 5	2 ; binomial	1. 6	3 ; trinomial
1. 7	3 ; trinomial	1. 8	3 ; trinomial	1. 9	2 ; binomial
1. 10	2 ; binomial				

Remedial

Provide additional practice for identifying terms in an expression.

Extension

Learners can pair up with another learner that struggles and explain the concepts to them.

Exercise 2

Learner's Book page 97

Guidelines on how to implement this activity

Write this expression on the board: $2x^2 + 3y + 5x^2 + y^3$. Now use it to explain like terms (terms with the same variables and same index) and unlike terms (terms that contain different variables or indices). This is learners' first introduction to the concepts of like and unlike terms; therefore use as many examples as necessary to demonstrate. Show learners that in the expression $8y^2 - 5y^2$, there are two terms, with the same variable y, and the same power of y, y^2. These are called like terms. Like terms have the same variable(s) and the same power of the variable.

Suggested answers

1.1 unlike ; different variables.
1.2 like ; same variable & same exponent.
1.3 unlike ; same variable but different exponents.
1.4 like ; same variable & same exponent.
1.5 unlike ; different variables.
1.6 unlike ; $\frac{cd}{cd} = 1$ and so 2cd + 5 which are different variables.
1.7 unlike ; same variable but different exponents.
1.8 unlike ; same variable but different exponents.
1.9 unlike ; same variable but different exponents.
1.10 $4c^5d$ and c^5d are like terms but the rest are not.
1.11 unlike ; different variables.
1.12 unlike ; different variables.

Review Copy

Remedial

Give learners more practice identifying like and unlike terms. Concentrate on terms without exponents to start with, and then increase the examples in complexity until learners can cope with the prescribed exercise.

Extension

Learners can pair up and write up expressions that the other must classify.

Exercise 3

Learner's Book page 98

Guidelines on how to implement this activity

Inform learners that we can only add and subtract like terms. By adding and subtracting the terms we simplify the expression. We cannot add or subtract unlike terms. To add or subtract terms we need to identify the like terms. Show learners on the board, using colour chalk, how to group the like terms together. Once the like term shave been grouped, you add and subtract them to simplify the expression. Do a few examples on the board together as a class. Allow learners to come up to the board and identify the like terms. Learners should complete this exercise on their own.

Suggested answers

1	$2x + 5x + 3y + y = 7x + 4y$	2	$2x + 7x + 3x = 12x$
3	$2a + 3b + 5c$	4	0
5	$6a - 2a + 10b - 5b = 4a + 5b$	6	$5x + 3y$
7	$10a^2 + 2a^2 + 5a + 6a = 12a^2 + 11a$	8	$-2y^3 - y^3 + x - x = -3y^3 + 0 = -3y^3$
9	$-a + a + 6b - 6b^2 = 0 + 6b - 6b^2 = 6b - 6b^2$		
10	$5y + x - a^2 - 5y^2$		

Remedial

Encourage learners to use colour pencils in their books to identify like terms. For example, circle all x^2 terms in blue and all ax terms in red. This system helps learners to be able to group the terms correctly.

Extension

Learners can complete more exercises to practise more challenging problems.
For example:

1 Remove parentheses and add like terms.

1.1 $(5x - 2y + z) + (3x - 2y + z)$

1.2 $(a - b) - (b - ab - a) - (2a - 4ab - 2b)$

1.3 $(3xy - 2x + 4y - 1) - (2xy - 3x - 3y + 6)$

1.4 $(2a + 3b + 2c - 6d) - (5a - 9b - c - 3d)$

1.5 $(6a^3 + 4a^2 - 2a - 6) - (2a^3 - 8a^3 + a + 2)$

2 Add and subtract the like terms.

2.1 Subtract $3x - 2$ from $8x - 5$

2.2 Add $3mn - 4mno + 3m$ to $8mn + 5mno - 9m$

2.3 Subtract $8a - 3b + 3$ from $4a + 8b - 5$

2.4 Add $x^2 - x - 1$ to $3x^2 + 3x + 2$

2.5 Subtract $y^3 - 9y + 13$ from $3y^3 - 2y - 9$

Review Copy

Unit 3 Coefficients and constants

Learner's Book page 99

Unit focus:

- learning more about algebraic expressions
- learning about coefficients
- recognising and identifying coefficients, constants and exponents in algebraic expressions.

Background information on coefficients and constants

An algebraic expression consists of a term or terms. Terms contain a variable or variables that form the product of a constant and a variable. The constant which forms the product of the variable is called the coefficient. A term which has only a number (not multiplied by a variable) has a fixed value and is called the constant. Learners must know these definitions and concepts in order to work effectively with algebra.

Exercise 1

Learner's Book page 100

Guidelines on how to implement this activity

Use revision of terms as an introduction to this unit. Write various algebraic expressions on the board. Involve the learners in counting the terms, and naming the expression (monomial, binomial and trinomial).

Use the expressions on the board to identify and label the coefficients and the constants. Demonstrate by doing two expressions, and thereafter ask the learners to verbally direct you to label and identify the remaining expressions.

Expression	Coefficient of x	Constant
$3x^2 - x + 3$	-1	3

The use of a table can be included to consolidate the knowledge. Draw it on a flip chart, filled with expressions; learners come to front of the class to complete the other two columns.

Suggested answers

1	Expression	Coefficient	Constant
1.1	$8x + 2x^2 - 11$	8	-11
1.2	$6xy - 3x + 2$	-3	$+2$
1.3	$x + yx - 1$	1	-1
1.4	$7x^2 + 12x - 4$	12	-4
1.5	$\frac{-x - y}{2}$	-1	None
1.6	$2(5x + x)$	5 ; 1	None
1.7	$(x^2 - y)(6x - y^2)$	6	None
1.8	$\frac{-y + 5x}{2}$	$\frac{5}{2}$	None
1.9	$4x + 3xy + 2y$	4	None
1.10	$4x^3 - x - 8 + x^3$	-1	-8

Review Copy

2.1	$4x$	2.2	$2a$	2.3	0	2.4	$5x$
2.5	$8a^2$	2.6	$4x^2$	2.7	$13xy$	2.8	$-2y$
3.1.1	$2x + 2x$	3.1.2	$a + a$	3.1.3	$8y - 8y$	3.1.4	$4x + 2x - x$
3.1.5	$3a^2 + 5a^2$	3.1.6	$7x^2 - 3x^2$	3.1.7	$12xy + xy$	3.1.8	$-y - y$
3.2.1	$\underline{2x} + \underline{2x}$	3.2.2	$\underline{a} + \underline{a}$	3.2.3	$\underline{8y} - \underline{8y}$	3.2.4	$\underline{4x} + \underline{2x} - \underline{x}$
3.2.5	$\underline{3a^2} + \underline{5a^2}$	3.2.6	$\underline{7x^2} - \underline{3x^2}$	3.2.7	$\underline{12xy} + \underline{xy}$	3.2.8	$-\underline{y} - \underline{y}$

3.3 All the variables within each question are the same. They are like terms.

Remedial

Give learners more practice on identifying the terms in the expressions. Once this is understood it will be easier for them to consider the parts of the expression. The learners can use different colour highlighters or crayonsto identify the different parts of the expressions.

Extension

Stronger learners can draw up exercise sheets to practise identifying coefficients and constants, including an answer sheet. They can work on each other's "papers".

Exercise 2

Learner's Book page 101

Guidelines on how to implement this activity

This exercise consolidates the work done in this chapter so far. Ensure learners can recognise and identify constants, variables, coefficients and the number of terms in an expression. Learners also need to be able to recognise the highest power in an expression. Explain that this called the degree of the expression. Learners should be able to recognise exponents, but if necessary revise identifying exponents in expressions.

Suggested answers

1.1 ^{2} belongs to b
1.2 1st term: ^{3} belongs to y
2nd term: ^{3} belongs to x
1.3 1st term: ^{2} belongs to p
2nd term: ^{5} belongs to q
3rd term: p has an exponent of 1
1.4 1st term: ^{5} belongs to a
2nd term: ^{3} belongs to b
1.5 1st term: ^{3} belongs to n
2nd term: ^{6} belongs to m

2.1.1	–5	2.1.2	3	2.1.3	trinomial	2.1.4	1
2.2.1.	1	2.2.2	–6	2.2.3	2	2.2.4	4
2.2.5	polynomial	2.3.1	n	2.3.2	$6n^5$	2.3.3	8
2.3.4	–2	2.3.5	4	2.3.6	polynomial	2.4.1	2
2.4.2	binomial	2.4.3	$15b$	2.4.4	$15b$	2.5.1	c
2.5.2	–3	2.5.3	3	2.5.4	trinomial	2.5.5	12

Review Copy

Remedial

This exercise should provide a good indication as to how learners are coping with the concepts of algebra. If learners are struggling, provide additional teaching and practice exercises on the problem area.

Unit 4 Adding and subtracting with algebra

Learner's Book page 102

Unit focus:

- revision of the commutative, associative and distributive laws
- applying these laws to algebraic expressions
- adding and subtracting like terms
- using the commutative, associative, distributive laws and laws of exponents to add and subtract with algebraic expressions

Background information on adding and subtracting with algebra

In this unit learners will add and subtract like terms in algebraic expressions using the commutative, associative and distributive laws for real numbers and the laws of exponents.

It is important that learners know:

- Associative Law of Addition: $(a + b) + c = a + (b + c)$
- Commutative Law of Addition: $a + b = b + a$
- Distributive Law: $a(b + c) = ab + ac$
- The laws of exponents:
  - $x^a \times x^b = x^{a+b}$
  - $\frac{x^m}{x^n} = x^{m-n}$
  - $(x^m)^n = x^{mn}$

Exercise 1

Learner's Book page 103

Guidelines on how to implement this activity

Use learners' previous knowledge of like terms as an introduction to this unit. Display a few expressions on the board to revise that like terms must have the same exponents and variables. Involve learners in identifying the like terms and unlike terms. Illustrate that unlike terms cannot be simplified. Stress the fact that we can only add and subtract like terms by simplifying the expression.

Use expressions with like terms to illustrate the process of addition and subtraction. Revise the associative, commutative and distributive properties and how they apply to expressions. Work through the worked examples in the Learner's Book. Have learners come up to the board and do some of the working out themselves.

Make sure to show the following:

- We group like terms using the commutative law.
- We remove brackets with the distributive law.
- We distribute the sign to every term inside the brackets when using the distributive law.
- We add or subtract the coefficients of like terms.
- That the order of operations applies also to terms within brackets.

Review Copy

Suggested answers

1 $3a + 10a + 8 + 5 = 13a + 13$

2 $5x^2 + 14x^2 + 12x + 8x + 7 - 3 = 19x^2 + 20x + 4$

3 $-11x + 18 + 14x = -11x + 14x + 18 = 3x + 18$

4 $12b + 20c$

5 $6x^2 - 15x^2 + 8 = -9x^2 + 8$

6 $10x + 16 - 2x - 3 = 10x - 2x + 16 - 3 = 8x - 13$

7 $2x^2 - 2 + (-9x^2 + 12x) = 2x^2 - 9x^2 - 2 + 12x = -7x^2 + 12x - 2$

8 $3a^3 + 2a^3 + 4a^3 + 5a^2 + 5a^2 + 2a^2 + 6a + 9 - 15 = 9a^3 + 12a^2 + 6a - 6$

Remedial

Show learners how to use colours to identify like terms, and how to use arrows to help them multiply out brackets. Provide additional examples for learners to practise.

Spend time revising what happens to the signs of negative numbers when we multiply them into (with the contents of) brackets.

Extension

Encourage stronger learners to set up a question paper on adding and subtracting in algebra, with an answer sheet. Learners swap with one another to take the test, swap and mark each other's tests.

Exercise 2

Learner's Book page 104

Guidelines on how to implement this activity

Demonstrate to learners using either the examples in the Learner's Book, or your own example, how the associative rule can be used to help with addition of terms. It may be necessary to revise how to apply the associative rule with whole numbers before introducing the rule with terms. Discuss why the associative rule works. Learners can complete this exercise on their own. Remind learners that they can only add and subtract like terms.

Suggested answers

1 $2a + 4a + 3 + 2 = 6a + 5$
2 $3x^2 + 4x^2 + 3y + 7y = 7x^2 + 10y$
3 $7b^5 + b^5 + 3 = 8b^5 + 3$
4 $2y^2 + 3 + x + 4x = 2y^2 + 5x + 3$
5 $6a^2 + 3a$
6 $5b^2 + 9b + 8 - 4b^2 + 7b + 7 = 5b^2 - 4b^2 + 9b + 7b + 8 + 7 = b^2 + 16b + 15$
7 $4c + c + 3 + 2d^3 = 5c + 2d^3 + 3$
8 $8z + 2z + 8 + 2 = 10z + 10$

Review Copy

Remedial

Spend additional time revising the associative property with whole numbers. Do as many examples as required until learners are able to work effectively with the property. Then introduce variables. Do additional examples until learners can manage the prescribed activity on their own.

Extension

Write out adding and subtracting calculations on chart paper, showing each step. This can be displayed in classroom.

Exercise 3

Learner's Book page 105

Guidelines on how to implement this activity

When working with algebraic expressions and multiplying out brackets, learners need to use the laws of exponents. Explain to learners that this is because when we multiply out brackets using the distributive law, we need to remember that multiplying powers with the same bases means adding exponents. Revise this law and other exponent laws using examples on the board. Do as many examples as necessary

until learners are comfortable with the subject material. Once learners have multiplied out brackets they still need to simplify the like terms. This exercise ties together the work covered in this unit.

Suggested answers

1.1 $-2x^4 + 25x^6$

1.2 $2a^2b^2 + 3a^2b^2 + ab - 3 + 2 = 5a^2b^2 + ab - 1$

1.3 $4x^3y^2 + 5x^2y^2$

1.4 $3n^2 - 6n + 16m^4 + 7$

1.5 $7x^5 + x^3 + 5x^2 - 2x^2 + y^2 + 4y^2 = 7x^5 + x^3 + 3x^2 + 5y^2$

1.6 $3r^4s^2 + 4r^4s^2 = 7r^4s^2$

1.7 $16x^4y^2 + 5 - x^2y^2$

2.1 $5a + 7b - 4c + (-2a + b - 8c) + (2a - 5b + 6c)$
$= 5a + 7b - 4c - 2a + b - 8c + 2a - 5b + 6c$
$= 5a - 2a + 2a + 7b + b - 5b - 4c - 8c + 6c$
$= 5a + 3b - 6c$

2.2 $c^3 - 1 - (4a^2 + 4a - 2)$
$= c^3 - 1 - 4a^2 - 4a + 2$
$= c^3 - 4a^2 - 4a + 2 - 1$
$= c^3 - 4a^2 - 4a + 1$

2.3 $3x - 2x^2 - (2x - 3x^2)$
$= 3x - 2x^2 - 2x + 3x^2$
$= 2x^2 + 3x^2 + 3x - 2x = x^2 + x$

2.4 $a^2 + 5 + 2ab + 3b^2 + a^3 + 5 + 3ab - 7b^2$
$= a^3 + a^2 + 2ab + 3ab + 3b^2 - 7b^2 + 5 + 5$
$= a^3 + a^2 + 5ab - 4b^2 + 10$

2.5 $[2x - 5y - 3z + (-3x + y + 4z)] - [x - y + 3z + (-2y - 2z)]$
$= (2x - 5y - 3z - 3x + y + 4z) - (x - y + 3z - 2y - 2z)$
$= 2x - 5y - 3z - 3x + y + 4z - x + y - 3z + 2y + 2z$
$= 2x - 3x - x - 5y + y + y + 2y - 3z + 4z - 3z + 2z$
$= -2x - y$

2.6 $6a - 2b - (4b + a) - (a - 8b)$
$= 6a - 2b - 4b - a - a + 8b$
$= 6a - a - a - 2b - 4b + 8b$
$= 4a + 2b$

2.7 $4b^3 + 2b^2 - 5b + 7 + 3b^3 + 6b^2 - 2$
$= 4b^3 + 3b^3 + 2b^2 + 6b^2 - 5b + 7 - 2$
$= 7b^3 + 8b^2 - 5b + 5$

Review Copy

Remedial

Learners must take time to go over the laws for rational numbers and exponents, and know how to apply these to collecting the terms. Give them additional material to do, such as the following:

1 Apply the distributive law.

1.1 $5(a + 8)$ 1.2 $6x(4 - y)$ 1.3 $2a(a + 2ab - 3)$

1.4 $3(c + 7) - 3c + 7$ 1.5 $-3(g + 5 - 5h) + 5h - g$ 1.6 $8(d + 4 - 4e) + 4e - d$

2 Apply the first law of exponents.

2.1 $a^4 \times b^6$ 2.2 $x^9 \times x$ 2.3 $(a^{18})(a^{12})$

2.4 $4xy^2 \times 6x^4y$ 2.5 $c^4(d.d^8)$ 2.6 $3e^6.2ef - e^3f \times 8e^6$

3 Apply the second law of exponents.

3.1 $\frac{x^{18}}{y^{12}}$ 3.2 $\frac{24abc^7}{8abc}$ 3.3 $\frac{mnop}{mno}$

3.4 $\frac{4^3c^3}{5c}$ 3.5 $\frac{ab^9d}{adb^5}$ 3.6 $\frac{2^5 3^4 5^3}{2^3 3^3 5}$

4 Apply the third law of exponents.

4.1 $(x^3)^2$ 4.2 $(y^4)^3$ 4.3 $(r^4s^2)^5$

4.4 $(4a^2bc^5)^2$ 4.5 $2(2^3b^9c^8)^4$ 4.6 $10(e^{10})^3 3$

Learners can also complete more exercises on simplification of like terms, until the pattern of operations is grasped.

Unit 5 Equations

Learner's Book page 106

Unit focus:

- revising using equations to describe problems
- revising the analysis and interpretation of equations
- solving equations by inspection
- determining the value of an expression by substitution.

Background information on equations

In Grade 7 learners solved equations by inspection only. In this unit they will revise most of the Grade 7 work. They will write and solve equations and they will write general rules to describe relationships between numbers in a number pattern. They will also determine the numerical value of an expression.

Exercise 1

Learner's Book page 107

Review Copy

Guidelines on how to implement this activity

As an introduction, write a problem in words on the board. Involve the learners by asking them to find a way to express the problem using algebraic symbols. Write the correct symbol form on the board.

Ensure learners know how to interpret the problem; can determine what information is given and how to use it; can identify what they are trying to find; and that unless provided with a variable, to assume that the unknown value is x. Demonstrate a few more examples to the whole class, with learner participation. Learners can complete this exercise on their own.

Suggested answers

Problem in words	Equation (symbols)
A number times 7 subtract 15 is 20	$7x - 15 = 20$
Charmaine is x years old and in 14 years she will be twenty-five years old.	$x + 14 = 25$
A number that is halved is equal to 4 more than 20.	$\frac{x}{2} = 20 + 4$
A large pizza with 16 slices is shared among p friends so that they can each have two slices.	$\frac{16}{p} = 2$
3 is twenty-one, divided by the sum of a number plus 5.	$3 = \frac{21}{x + 5}$
A metre of rope costs R2,30 a metre. Peter wants to buy three and half metres. What will it cost Peter?	$x = 2{,}30 \times 3{,}5 = \text{R}8{,}05$

Remedial

If any learners have difficulty with this exercise, give them more exercises of this kind for homework. The repetition of the same calculating and thinking process will help to consolidate the work covered.

Extension

Design and make wall charts showing equations along with their meanings in words.

Exercise 2

Learner's Book page 108

Guidelines on how to implement this activity

Write a few equations on the board to illustrate the analysis of equations. Have learners look at the equations and decide if the information it shows is true. Remind learners what the "equal to" sign represents. Encourage different responses from learners, and ask learners to give the reasoning behind their thinking. Learners must take note that equations are used to relate numbers and variables. These equations can be true or false.

Suggested answers

1	True	2	False: $1 \div 3 = \frac{1}{3}$
3	True	4	True
5	True	6	False: $-y(-y - x^2) = y^2 + x^2y$

Review Copy

Remedial

Show learners how to break down an equation into a LHS and a RHS. Learners should simplify each side as far as possible and then compare. This provides an effective tool for learners struggling to analyse the equations.

Exercise 3

Learner's Book page 108

Guidelines on how to implement this activity

Revise solving equations by means of inspection. Write three or four equations on the board and select four capable learners to complete the examples on the board. Work through each of the examples and confirm the thinking behind each step. Ensure that learners understand that when we solve an equation it means we determine the value of the variable (unknown) that will prove the equation true. To solve by inspection means using mental mathematics. Learners should complete this exercise on their own.

Suggested answers

1	$y = 12$	2	$x = 54$	3	$b = -3$	4	$a = 10$
5	$n = 5$	6	$c = -16$	7	$m = 54$	8	$d = -12$
9	$z = 8$	10	$e = 2$	11	$a = 1$	12	$c = 1$

Remedial

If learners experience any problems, help learners to break down the equation by saying what value will satisfy the expression. Teach learners to use trial and error to find the correct answer.

Exercise 4

Learner's Book page 109

Guidelines on how to implement this activity

Learners need to know that the value of the variable determines the value of the expression.

Introduce learners to substitution. Discuss methods of substituting in variables. Advise learners to use brackets as this helps when working with the signs. Do examples on the board, including at least one with fractions. Remind learners to watch out for the signs at all times. If necessary revise multiplication of integers and what happens to the signs. Learners should complete this activity on their own.

Suggested answers

1 $(4) + 7 = 11$

2 $3(2) - 4(-3) = 6 + (12) = 18$

3 $(-4) + \frac{-1}{5} = \frac{-4}{1} - \frac{1}{5} = -\frac{20}{5} - \frac{1}{5} = -\frac{21}{5}$

4 $5(2)^2 - 3(2) + 12 = 5(4) - 6 + 12 = 20 - 6 + 12 = 26$

5 $8(4) - 5(7) + 12 = 32 - 35 + 12 = 9$

6 $2[(3) + (4)] + 3(4) - 2(3) + 5(3)$
$= 2(7) + 12 - 6 + 15 = 14 + 12 - 6 + 15 = 35$

7 $2(-2)(-3) - 4(-2) + (-3) = 12 + 8 - 3 = 17$

8 $4(-1)^2 - 2[2(2)]^3 + (-1)(2)$
$= 4(1) - 2(4)^3 - 2 = 4 - 2(64) - 2 = 4 - 128 - 2 = -126$

9 $\frac{4(2)^5}{(2)^3} + 4(-4) = \frac{4(32)}{8} - 16 = \frac{128}{8} - 16 = 16 - 16 = 0$

10 $\frac{2(5) - 4(-4) + 1}{(5 + (-4))} - (5)^2 = \frac{10 + 16 + 1}{5 - 4} - 25 = \frac{17}{1} - 25 = -8$

Review Copy

Remedial

Learners may struggle to work with fractions in this exercise. Revise equivalent forms of fractions and simplifying fractions with these learners. Provide additional problems with fractions for these learners to practise at home.

Extension

Learners can be given equations to solve that require other methods than just inspection. Provide them with an exercise to practise the other methods.

Consolidation

Learner's Book page 111

Before doing this consolidation exercise, encourage learners to review the work covered in this chapter. Advise learners to use the summary and to revise their work. This exercise can be used as an informal assessment task for you to track how learners are coping with the chapter and the concepts covered. The mark allocation provides guidelines on how to assess learners.

Suggested answers

1.1

$3x - 2$							
x	-1	0	1	2	3	-5	$\frac{1}{2}$
	$3(-1) - 2$ $= -3 - 2$ $= -5$	$3(0) - 2$ $= 0 - 2$ $= -2$	$3(1) - 2$ $= 3 - 2$ $= 1$	$3(2) - 2$ $= 6 - 2$ $= 4$	$3(3) - 2$ $= 9 - 2$ $= 7$	$3(-5) - 2$ $= -15 - 2$ $= -17$	$3\left(\frac{1}{2}\right) - 2$ $= \frac{3}{2} - \frac{4}{2}$ $= -\frac{1}{2}$

(5)

1.2

$4x^2 + 3$							
x	-1	-2	0	1	3	4	5
	$4(-1)^2 + 3$ $= 4(1) + 3$ $= 4 + 3$ $= 7$	$4(-2)^2 + 3$ $= 4(4) + 3$ $= 16 + 3$ $= 19$	$4(0)^2 + 3$ $= 4(0) + 3$ $= 0 + 3$ $= 3$	$4(1)^2 + 3$ $= 4(1) + 3$ $= 4 + 3$ $= 7$	$4(3)^2 + 3$ $= 4(9) + 3$ $= 36 + 3$ $= 39$	$4(4)^2 + 3$ $= 4(16) + 3$ $= 64 + 3$ $= 67$	$4(5)^2 + 3$ $= 4(25) + 3$ $= 100 + 3$ $= 103$

(5)

Review Copy

2.1 2; binomial 2.2 2; binomial 2.3 1; monomial
2.4 3; trinomial 2.5 3; trinomial 2.6 3; trinomial
2.7 3; trinomial 2.8 2; binomial (8)

3.1 $x + 3 - x - 1 = x - x + 3 - 1 = 0 + 2 = 2$ (2)

3.2 $6a^2 + 3a$ (1)

3.3 $6c + 7 + 6c + 6 = 6c + 6c + 7 + 6 = 12c + 13$ (2)

3.4 $5y + 3 - y - 8 = 5y - y + 3 - 8 = 4y - 5$ (2)

3.5 $8z + 8 + 2z + 2 = 8z + 2z + 8 + 2 = 10z + 10$ (2)

3.6 $3e^2 + 3e + 6 + 3e^2 + 8 = 3e^2 + 3e^2 + 3e + 6 + 8 = 6e^2 + 3e + 14$ (3)

3.7 $9d^2 + 8d + 4 - 6d^2 + 3d = 9d^2 - 6d^2 + 8d + 3d + 4 = 3d^2 + 11d + 4$ (3)

3.8 $5b^2 + 9b + 8 - 4b^2 - 7b - 7 = 5b^2 - 4b^2 + 9b - 7b + 8 - 7 = b^2 + 2b^2 + 1$ (3)

4.1

Input	2	3	5	10	12	15	21	28
Value of x	6	9	**15**	**30**	**36**	45	**63**	**84**

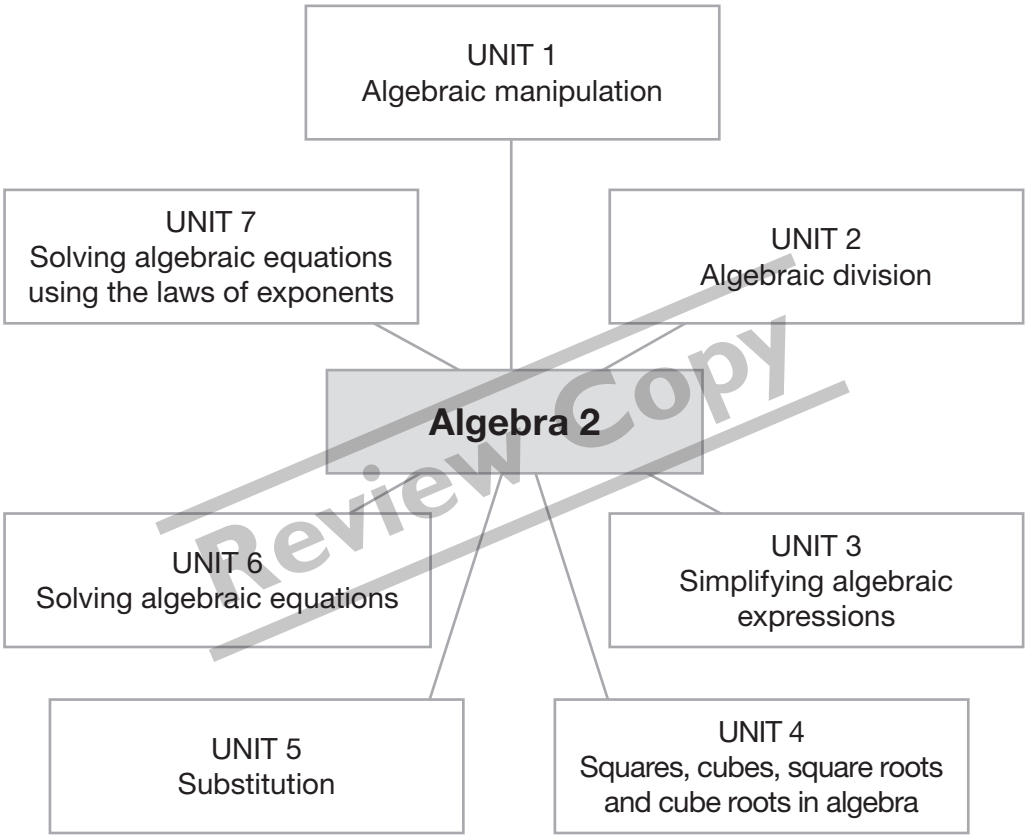
(2)

4.2 $3n$ (where n is the input value) (2)

Total marks [40]

Chapter 6 Algebra 2

Chapter overview



Review Copy

Content		Time allocations	LB page
Unit 1	Algebraic manipulation	1,5 hours	113
Unit 2	Algebraic division	1,5 hours	117
Unit 3	Simplifying algebraic expressions	3 hours	121
Unit 4	Squares, cubes, square roots and cube roots in algebra	1,5 hours	123
Unit 5	Substitution	1,5 hours	126
Unit 6	Solving algebraic equations	2 hours	128
Unit 7	Solving algebraic equations using the laws of exponents	1 hour	131

Background information on Algebra 2

In Grade 7 learners did not manipulate algebraic expressions at all. They did however, work with addition and subtraction of algebraic expressions in Algebra 1. This will offer a way to introduce the work of Algebra 2. In this chapter of algebra the focus is on expanding and simplifying algebraic expressions. Laying a strong foundation in algebra is crucial to ensuring learners' future success in mathematics. Algebraic manipulation is used in geometry and trigonometry and is necessary for all mathematics work at FET level.

Teaching guidelines for Algebra 2

This chapter is about taking the concepts and skills of Algebra 1 further. As an introduction to this chapter, revise the content of Algebra 1. Revise addition and subtraction, and then move on to multiplication and division. As with the previous algebra chapter, it is vital to ensure that core concepts are conveyed to learners, so be sure to draw their attention to these concepts in the Learner's Book.

Before starting this chapter take time to revise the laws for real numbers, the laws of exponents, and the rules for algebraic operations. It may also be necessary to revise finding the squares, cubes, square roots and cube roots of whole numbers before learners work with these in algebra.

Resources

Each learner should have a calculator and blank paper to assist with their working out.

Review Copy

Unit 1 Algebraic manipulation

Learner's Book page 113

Unit focus:

- Multiplying with
  - monomials
  - binomials
  - trinomials.

Background information on multiplying with algebra

In algebra the × symbol can be replaced by a dot or brackets. For example 2 × 3, 2.3, or (2)(3) are all ways of expressing 2 multiplied by 3. In order to multiply algebraic expressions, learners need to know the associative, commutative and distributive properties of mathematical operations, be familiar with the laws of exponents, and have a strong sense of the order in which operations must be applied. Ensure learners remember the following when multiplying:

- $+ \times + = +$ $\quad + \times - = -$ $\quad - \times + = -$ $\quad - \times - = +$
  The product of two numbers with the same sign is positive and the product of two numbers with opposite signs is negative.
- Remember BODMAS gives the order of operations: Brackets, Of, Division, Multiplication, Addition and Subtraction.
- Remember that terms in the numerator or denominator of a fraction, or within a

square or cube root sign, must be handled as if they are in brackets.

Exercise 1

Learner's Book page 114

Guidelines on how to implement this activity

Discuss multiplying negative numbers and the rules for multiplying negative numbers. Next review the 'Important notes' in the Learner's Book. Work through each of the examples and ensure your learners are familiar with each of these concepts. Introduce multiplying monomials to the learners. Show them that there is an order to multiplying – learners should always determine the sign of the answer, then multiply the numbers or coefficients, then the variables, using the laws of exponents. While learning to multiply with algebra, learners should write the calculation in expanded notation to help them.

Suggested answers

1.1	12	1.2	16	1.3	$3 \times 3 = 9$
1.4	-60	1.5	16	1.6	$(8)^2 = 64$
2.1	a^3	2.2	b^2	2.3	$-xy$
2.4	$-ab$	2.5	a^4	2.6	x^3
3.1	$2 \times a \times (-3) \times a = 2 \times (-3) \times a \times a = -6a^2$			3.2	$(-4)^5 = -1\ 024$
3.3	$-3 \times x^2 \times (-1) \times x^2 = -3 \times (-1) \times x^2 = 3x^4$			3.4	$5^8 = 390\ 625$
3.5	$4 \times a^2 \times b^2 \times (-3) \times a \times b = 4 \times (-3) \times a^2 \times a \times b^2 \times b = -12a^3b^3$				
3.6	$-343 \times (-6) = 2\ 058$				

Review Copy

Remedial

If learners are experiencing difficulty multiplying with brackets, revise multiplying integers using brackets around the integer.

Extension

Encourage stronger learners to try the multiplication without writing in expanded notation, and to use the laws.

Exercise 2

Learner's Book page 115

Guidelines on how to implement this activity

Revise the distributive property of whole numbers. Do a few examples of this together as a class. Ensure that learners know to multiply the factor in front of the bracket with each term in the bracket. Provide another example this time with variables – ensure you have only two terms inside the bracket.

Remind learners that an expression with two terms, such as the expression in the bracket, is called a binomial. Ensure that learners know that we are multiplying a monomial and a binomial. Using the distributive property, demonstrate how to multiply algebraic terms together. Remind learners to determine the sign of the answer, then multiply the numbers, then variables and lastly exponents. Learners should complete the exercise on their own.

Suggested answers

1 $4y \times 3x + 4y \times 9 = 12xy + 36y$

2 $x \times 2x + x \times x^2 = 2x^2 + x^3$

3 $5ab \times a^2 + 2ab \times 5ab = 5a^3b + 10a^2b^2$

4 $-6fg^2 \times 4f - 6fg^2 \times 2g^4 = -24f^2g^2 + 12fg^6$

5 $-a^3 \times (-x) - a^3 \times (-ax) = a^3x + a^4x$

6 $-3xy \times (-4x^2) + (-3xy) \times 3xy^2 = 12x^3y - 9x^2y^3$

7 $5a^2 \times (-b^2) + 5a^2 \times 3ab = -5a^2b^2 + 15a^3b$

8 $-6x^3 \times 3a^2b + (-6x^3) \times (-6) = -18a^2bx^3 + 36x^3$

9 $-\frac{1}{2}ab \times \frac{2}{3}a^4 + \left(-\frac{1}{2}ab\right) \times -b = -\frac{1}{3}a^5b + \frac{1}{2}ab^2$

10 $\frac{3x}{4} \times 2x + \frac{3x}{4} \times (-4x^2) = \frac{3}{4} \times \frac{2}{1} \times x \times x + \frac{3}{4} \times -\frac{4}{1} \times x^2 = \frac{3}{2}x^2 - 3x^3$

Remedial

If learners experience problems with questions 9 and 10, do some simple revision of multiplying fractions. Remind learners that when we multiply fractions, we multiply both the numerator and the denominator, and that a whole number or variable can be written as a fraction with a denominator 1, for example, $x = \frac{x}{1}$.

Exercise 3

Learner's Book page 116

Review Copy

Guidelines on how to implement this activity

In the previous activity learners multiplied monomials by binomials; in this exercise they multiply monomials by trinomials. Write a trinomial on the board. Ask learners what type of expression it is. Explain that we will now multiply the monomial by a trinomial. Do an example on the board. Show learners how we now multiply the monomial by the three terms in the bracket, one by one. Remind learners that they must first find the sign of the answer, then multiply numbers, then variables, using the laws of exponents. Learners should complete the exercise on their own.

Suggested answers

1 $(5a \times 7a) + (5a \times 2b) + [5a \times (5a \times (-3c)] = 35a^2 + 10ab - 15ac$

2 $(-2x \times x) + (-2x \times x^3) + (-2x \times x^3) = 2x^2 - 2x^4 + 2x^4 = -2x^2$

3 $(y^2 \times 2y) + (y^2 - y) + (y^2 \times 6p) = 2y^3 - y^3 + 6py^2 = y^3 + 6py^2$

4 $(-2x^2 \times 3x) + (-2x^2 \times 5xy) + [-2x^2 \times (-2)] = 6x^3 - 10x^3y + 4x^2$

5 $[6a^2b \times (-3ab)] + [6a^2b \times (2ac)] + [6a^2b \times (-4bc)]$
$= -18a^3b^2 + 12a^3bc - 24a^2b^2c$

6 $(-n^2m^3 \times -3n^2) + (-n^2m^3 \times -4nm) + (-n^2m^3 \times 5m^2)$
$= 3m^3n^4 + 4m^4n^3 - 5m^5n^2$

7 $(-4rs^2t \times 2rs) + (-4rs^2t \times 4r^4) + (-4rs^2t \times -3s^2)$
$= -8r^2s^3t - 16r^5s^2t + 12rs^4t$

8 $(4xy^4 \times 6xyz^3) + (4xy^4 \times -2xy^3) + (4xy^4 \times -z^3)$
$= 24x^2y^5z^3 - 8x^2y^7 + 4xy^4z^3$

9 $(-9a^5b^5c^5 \times 3a^2) + (-9a^5b^5c^5 \times -5ab) + (-9a^5b^5c^5 \times b^2)$
$= -27a^7b^5c^5 + 45a^6b^6c^5 - 9a^5b^7c^5$

10 $\left(\frac{1}{2}a^2b^3 \times abc\right) + \left(\frac{1}{2}a^2b^3 \times -ac^3\right) + \left(\frac{1}{2}a^2b^3 \times -\frac{1}{4}b^2\right)$

$= \frac{1}{2}a^3b^4c - \frac{1}{2}a^3b^3c^3 - \frac{1}{8}a^2b^5$

Remedial

Allow learners who are still experiencing problems with multiplying algebra to write each term in expanded form to help them.

Unit 2 Algebraic division

Learner's Book page 117

Unit focus:

Divide:

- monomials
- binomials
- trinomials

by monomials

Background information on division with algebra

In division in algebra the ÷ symbol can be replaced by a division line (fraction form): $x \div y = \frac{x}{y}$.

Division is the opposite or inverse operation of multiplication. When we divide we first write the variables in the numerator and denominator as products of the factors that include the variables. Then divide any numbers. To simplify we use the property of 1, the quotient of any two numbers or factors that are the same is 1 (cancelling), in the numerator and denominator, and subtract the exponents of the powers that have the same bases.

- Remember the following when dividing:
  the quotient of two numbers with the same sign is positive and the quotient of two numbers with opposite signs is negative.
- Remember BODMAS gives the order of operations: Brackets, Of, Division, Multiplication, Addition, and Subtraction.

Review Copy

Exercise 1

Learner's Book page 118

Guidelines on how to implement this activity

Remind learners about the signs when we multiply, and that these rules also apply when we divide. Do a few examples of dividing integers to remind learners how to work with division and signs.

Revise the law of exponents for dividing bases with the same exponent. Demonstrate by means of an example, using expanded notation that we subtract the exponents if the bases are the same when we divide.

Start with a simple example containing a single variable, and increase the examples in complexity until you have a monomial divided by a monomial, each a product of integers and powers of variables. Explain to learners that we follow the same process

as with multiplication and first find the sign of the answer. Then divide the numerical coefficient, then the variables, subtracting the exponents of powers with the same bases. Learners should complete this exercise on their own.

Suggested answers

1.1 5 1.2 –2 1.3 –16

1.4 1 1.5 –7 1.6 $9 \div 9 = 1$

2

	16	12	8	–4	–12	–16
(÷)(–2)	–8	–6	–4	2	6	8

3.1 $\frac{4xy}{2y} = \frac{4 \times x \times y}{2 \times y} = 2x$

3.2 $\frac{-10y}{5} = \frac{-10 \times y}{5} = -2y$

3.3 $\frac{6x^4}{-2x^2} = \frac{6 \times x \times x \times x \times x}{-2 \times x \times x} = -3x^2$

3.4 $\frac{12x^5y^2}{-3x^2y^6} = \frac{12 \times x \times x \times x \times x \times x \times y \times y}{-3 \times x \times x \times y \times y \times y \times y \times y \times y} = \frac{-4x^3}{y^4}$

3.5 $\frac{3xy^2}{xy^4} = \frac{3 \times x \times y \times y}{x \times y \times y \times y \times y} = \frac{3}{y^2}$

3.6 $\frac{24p^5s}{-8p^9s^4} = \frac{24 \times p \times p \times p \times p \times p \times s}{-8 \times p \times p \times p \times p \times p \times p \times p \times p \times p \times s \times s \times s \times s} = -\frac{3}{p^4s^3}$

Remedial

Encourage learners to write each monomial in expanded form and to cancel by way of using the property of 1. Allow learners to use this form in division until they feel comfortable enough using the laws.

Extension

For learners who are coping easily with the prescribed material, encourage them to work the examples out without expanding the monomials.

Review Copy

Exercise 2

Learner's Book page 119

Guidelines on how to implement this activity

Demonstrate by means of an example how to divide a binomial by a monomial. Make each term of the binomial in the numerator, a separate fraction with the denominator. You now divide each monomial term by the monomial denominator. Do a few examples, starting with simple variables and expanding to complex monomials that are products of integers and powers of variables. Remind learners to watch out for the signs of the answers.

Suggested answers

1 $\frac{p^3 - 6p}{p} = \frac{p^3}{p} - \frac{6p}{p} = p^2 - 6$

2 $\frac{12x - 4x}{2} = \frac{8x}{2} = 4x$

3 $\frac{18a^2b^3 - 12a^3b^2}{3ab^2} = \frac{18a^2b^3}{3ab^2} - \frac{12a^3b^2}{3ab^2} = 6ab - 4a^2$

4 $\frac{8n(4n-3m)}{8n} = 4n - 3m$

5 $\frac{r^2 - rs}{r} = \frac{r^2}{r} - \frac{rs}{r} = r - s$

6 $\frac{5d^2 - 3de}{15d} = \frac{5d^2}{15d} - \frac{3de}{15d} = \frac{d}{3} - \frac{e}{5}$

7 $\frac{4x+8}{8} + \frac{5x+2}{6} = \frac{4x}{8} + \frac{8}{8} + \frac{5x}{6} + \frac{2}{6} = \frac{x}{2} + 1 + \frac{5x}{6} + \frac{1}{3} = \frac{x}{2} + \frac{5x}{6} + 1 + \frac{1}{3}$

Remedial

Allow learners to work in expanded form once when they have a monomial over a monomial.

Exercise 3

Learner's Book page 120

Guidelines on how to implement this activity

Extend what learners did with binomials divided by monomials by including a trinomial divided by a monomial. Explain to learners that we break up the trinomial into three terms, each over the monomial denominator. Do an example as a class. Learners should complete this exercise on their own.

Suggested answers

1 $\frac{12xy + 8x^2 + 16x}{4x} = \frac{12xy}{4x} + \frac{8x^2}{4x} - \frac{16x}{4x} = 3y + 2x - 4$

2 $\frac{2xy + 8x^2 - 16x}{4x} = \frac{2xy}{4x} + \frac{8x^2}{4x} - \frac{16x}{4x} = \frac{y}{2} + 2x - 4$

3 $\frac{6a^3b^2 + 10a^2b^4 - 4ab}{2ab} = \frac{6a^3b^2}{2ab} + \frac{10a^2b^4}{2ab} - \frac{4ab}{2ab} = 3a^2b + 5ab^3 - 2$

4 $\frac{4r^2s - 8rs + rs^2}{4rs^2} = \frac{4r^2s}{4rs^2} - \frac{8rs}{4rs^2} + \frac{rs^2}{4rs^2} = \frac{r}{s} - \frac{2}{s} + 1$

5 $\frac{b^3 + 18b^2 - 15b}{3b} = \frac{b^3}{3b} + \frac{18b^2}{3b} - \frac{15b}{3b} = \frac{b^2}{3b} + 6b - 5$

6 $\frac{7c + 14c^4 + 21c^3}{7c} - \frac{20c - 5c^2 + 15c}{5c^2} = \frac{7c}{7c} + \frac{14c^4}{7c} + \frac{21c^3}{7c} - \left(\frac{20c}{5c^2} - \frac{5c^2}{5c^2} + \frac{15c}{5c^2}\right)$

$= 1 + 2c^3 + 3c^2 - \frac{4}{c} - 1 + \frac{3}{c} = 1 + 2c^3 + 3c^2 - \frac{4}{c} + 1 - \frac{3}{c} = 2 + 2c^3 + 3c^2 - \frac{7}{c}$

7 $\frac{-5m^2 - 3m^3 + 10}{5m} = -\frac{5m^2}{5m} - \frac{3m^3}{5m} + \frac{10}{5m} = -m - \frac{3m^2}{5} + \frac{2}{m}$

8 $\frac{15e + 4f - 18g + 12 - 3e}{6} = \frac{12e + 4f - 18g + 12}{6} = \frac{12e}{6} + \frac{4f}{6} - \frac{18g}{6} + \frac{12}{6} = 2e + \frac{2f}{3} - 3g + 2$

Remedial

Encourage learners to rely less on the expanded method and more on the laws of exponents. Provide addition examples for learners to practise at home.

Extension

Provide additional examples, that are similar to question 7, as it involves simplifying before dividing the terms.

Review Copy

Unit 3 Simplifying algebraic expressions

Learner's Book page 121

Unit focus:

- how to simplify algebraic expressions.

Background information on simplifying algebraic expressions

Simplifying algebraic expressions means to change the expression into a more convenient form. The simplified form of the expression is an equivalent expression. Simplifying expressions involves combining the terms. When $10xy - 2xy$ is simplified, the solution is $8xy$. The two terms combine to one term. We can say they have been simplified to a more convenient, shorter form. In some cases simplifying is needed before terms can be combined. In this situation we make use of the distributive law to eliminate the brackets.

For example, $3(6a - 2) - (4a + 5)$

$= 18a - 6 - 4a - 5$

$= 14a - 11$

Removing or eliminating brackets is key! Ensure that learners know that not all expressions can be simplified. It is imperative that learners grasp the skills of simplification, as it is an important skill that permeates their mathematics education.

Review Copy

Exercise 1

Learner's Book page 122

Guidelines on how to implement this activity

Discuss with learners what simplifying an algebraic expression means. Record learners' responses. Learners should cope with the following; and if they do not, guide them to these competencies:

- removing brackets using the distributive law
- adding the like terms
- applying BODMAS
- applying the laws of exponents
- using the property of 1 to cancel factors when dividing.

Revise each of the above means of simplifying using an example. Use the Worked examples in the Learner's Book or provide your own additional examples. Ensure learners understand each method of simplifying and can recognise it and perform it, before they start on the following exercise. Learners should do this exercise on their own.

Suggested answers

1.1 x^6y^6

1.2 $4^3a^{2\times3}y^3 = 64a^6y^3$

1.3 $4^3a^3 \times 2a = 64 \times 2 \times a^3 \times a = 128a^{3+1} = 128a^4$

1.4 $8x \times (-2)^3 \times x^3 = 8 \times (-8) \times x \times x^3 = -64x^{1+3} = -64x^4$

1.5 $(p^2 \times 3p) + (p^2 \times (-p)) + (p^2 \times 6s) = 3p^3 + (-p^3) + 6p^2s = 2p^3 + 6p^2s$

1.6 $(-x^2 \times 2x) + (-x^2 \times 10xy) + (-x^2 \times -1) = -2x^3 - 10x^3y + x^2$

1.7 $(6x \times y) + (6x \times -12x) = 6xy - 72x^2$

2.1 $(3 \times 5x) + (3 \times 2) - 5x = 15x + 6 - 5x = 15x - 5x + 6 = 10x + 6$

2.2 $4a + 5b + (6 \times (-3b)) = 4a + 5b + 12a + (-18b) = 16a - 13b$

2.3 $4c^2 + (2c \times 3c) + (2c \times -7) = 4c^2 + 6c^2 + (-14c) = 10c^2 - 14c$

2.4 $(5 \times 2n^4) + (5 \times (-7)) + (3 \times n^4) + (3 \times 4) = 10n^4 + (-35) + 3n^4 + 12$
$= 10n^4 + 3n^4 + 12 - 35 = 13n^4 - 23$

2.5 $(3a^2b^5 \times 5a) + (3a^2b^5 \times (-8b)) + (3a^2b^5 \times 7) + (ab^5 \times 5a^2) + (ab^5 \times 10ab)$
$+ (ab^5 \times (-7a))$
$= 15a^3b^5 + (-24a^2b^6) + 21a^2b^5 + 5a^3b^5 + 10a^2b^6 + (-7a^2b^5)$
$= 15a^3b^5 + 5a^3b^5 - 24a^2b^6 + 10a^2b^6 + 21a^2b^5 - 7a^2b^5$
$= 20a^3b^5 - 14a^2b^6 + 14a^2b^5$

2.6 $(-4 \times 5y^3) + (-4 \times (-2x^2y)) + (-y \times 5x^2) + (-y \times 3y^2)$
$= (-20y^3) + 8x^2y + (-5x^2y) + (-3y^3)$
$= 8x^2y - 5x^2y - 20y^3 - 3y^3$
$= 3x^2y - 23y^3$

2.7 $12f^{10} + (-4f^7 \times 2g) + (-4f^7 \times 3f^3)$
$= 12f^{10} + (-8f^7g) + (-12f^{10})$
$= 12f^{10} - 12f^{10} - 8f^7g$
$= -8f^7g$

2.8 $\frac{15a^6}{7a^2} + \frac{a^6}{7a^2} - (2^2 \times a^2) \times a^2 = \frac{15a^4}{7} + \frac{a^4}{7} - 4a^4 = \frac{15a^4}{7} + \frac{a^6}{7} - \frac{28a^4}{7} = -\frac{12a^4}{7}$

3.1 $3(4x + 2y) \times \frac{2}{1} = (2 \times 3 \times 4x) + (2 \times 3 \times 2y) = 24x + 12y$

3.2 $\frac{2^3c^3d^3}{4c^4d} = \frac{8c^3d^3}{4c^4d} = \frac{2d^2}{c}$

3.3 $6xy + 4x - 6xy + 12y = 6xy - 6xy + 4x + 12y = 4x + 12y$

3.4 $3s^3$

3.5 $2x - y + 2y\,(2x + 1) = 2x - y + (2y \times 2x) + (2y \times 1)$
$= 2x - y + 4xy + 2y = 2x + 4xy + y$

3.6 $\frac{5y^2}{2y} = \frac{5y}{2}$

3.7 $\frac{\sqrt[3]{8}\,(6ab + b)}{2b} = \frac{2(6ab + b)}{2b} = \frac{2 \times 6ab + 2 \times b)}{2b} = \frac{12ab}{2b} + \frac{2b}{2b} = 6a + 1$

3.8 $\frac{3eg}{f^2}$

3.9 $\frac{8m}{4} + \frac{4n}{2} = 2m + 2n$

4.1 False: $a + a = 2a$

4.2 False: $(-4)^4b^{2 \times 4} = 256b^8$

4.3 False: $y^2 + y^2 = 2y^2$

4.4 True

4.5 False $\frac{6x^2}{x^2} + \frac{1}{x^2} = 6 + \frac{1}{x^2}$

Remedial

Provide additional practice by means of additional material. It is important that learners are able to simplify algebraic expressions as this forms an important part of their mathematics education all the way to matric.

Review Copy

Unit 4 Squares, cubes, square roots and cube roots in algebra

Learner's Book page 123

Unit focus:

- how to determine squares and square roots of algebraic terms
- how to determine cubes and cube roots of algebraic terms.

Background information on using squares, cubes, square roots and cube roots in algebra

Learners should be familiar with square and cube numbers; however their use in algebra is new to learners. When we multiply a number by itself the product is the square of the number. Examples:

$5^2 = 5 \times 5 = 25$ $\quad$ $12^2 = 12 \times 12 = 144$

$x^2 = x \times x = x^2$ $\quad$ $a^6 = a^3 \times a^3 = a^6$

When we multiply a number by itself three times the product of that number is called a cube. Examples:

$5^3 = 5 \times 5 \times 5 = 125$ $\quad$ $13^3 = 13 \times 13 = 2\ 197$

$y^3 = y \times y \times y = y^3$ $\quad$ $b^6 = b^2 \times b^2 \times b^2 = b^6$

The square root of a number is that number which when squared will give you the original number. Examples:

$\sqrt{16y^2} = 4y$ $\quad$ $\sqrt{256a^6b^{12}} = 16a^3b^6$

The cube root of a number is the number which, when cubed will give you the original number. Examples:

$\sqrt[3]{27a^3} = 3a$ $\quad$ $\sqrt[3]{1000x^{27}y^{12}} = 10x^9y^4$

Review Copy

Exercise 1

Learner's Book page 124

Guidelines on how to implement this activity

Discuss squared numbers. Ask learners to provide the perfect squares (squared integers) up to 144.

Revise the exponent law $(a^m)^n = a^{m \times n} = a^{mn}$ and show learners how this applies to squaring numbers. Show learners that each factor in the bracket must be squared. Do a few examples together as a class.

Discuss square roots and do some examples of finding square roots of whole numbers. Show learners the following exponent law $\sqrt{a^m} = a^{m \div 2} = a^{m/2}$. Explain that when we find the square root, we divide the exponent by 2. Do a few examples finding the square root of terms with powers of variables.

Ensure that learners understand that they only find the square roots of powers with even exponents. Learners can work in pairs to complete this exercise.

Suggested answers

1 $\quad$ 1; 4; 16; 25; 36; 49; 64; 81; 100; 121; 144

2.1 $\quad$ $(8ab^2)^2 = 8^{1\times2}a^{1\times2}b^{2\times2} = 64a^2b^4$

2.2 $\quad$ $(3x^2y^3)^2 = 3^2x^{2\times2}y^{3\times2} = 9x^4y^6$

2.3 $\quad$ $(10c^2d^7)^2 = 10^{1\times2}c^{2\times2}d^{7\times2} = 100c^4d^{14}$

2.4 $(9y^6z^3)^2 = 9^{1\times2}y^{6\times2}z^{3\times2} = 81y^{12}z^6$

2.5 $((-7)a^2b^2c^9)^2 = (-7)^{1\times2}a^{2\times2}b^{2\times2}c^{9\times2} = 49a^4b^4c^{18}$

2.6 $(-5r^3s^6t^0)^2 = (-5)^{1\times2}r^{3\times2}s^{6\times2}t^{0\times2} = 25r^6s^{12}t^0 = 25r^6s^{12}$

3.1 $y^{\frac{36}{2}}z^{\frac{4}{2}} = y^{18}z^2$

3.2 $2a^{\frac{6}{2}}b^{\frac{8}{2}} = 2a^3b^4$

3.3 $9c^{\frac{12}{2}}d^{\frac{10}{2}}e^{\frac{2}{2}} = 9c^6d^5e$

3.4 $7r^{\frac{22}{2}}s^{\frac{16}{2}} \times 11r^{\frac{6}{2}}s^{\frac{8}{2}} = 7r^{11}s^8 \times 11r^3s^4 = 77r^{11}+{}^3s^8+{}^4 = 77r^{14}s^{12}$

3.5 $\dfrac{\sqrt{100x^6}}{\sqrt{25x^2}} = \dfrac{10x^{\frac{6}{2}}}{5x^{\frac{2}{2}}} = \dfrac{10x^3}{5x} = 2x^2$

3.6 $8m^{\frac{32}{2}}n^{\frac{40}{2}}p^{\frac{20}{2}} \times 7m^{\frac{16}{2}}n^{\frac{24}{2}}p^{\frac{14}{2}} = 8m^{16}n^{20}p^{10} \times 7m^8n^{12}p^7$

$= 56m^{16+8}n^{20+12}p^{10+7} = 56m^{24}n^{32}p^{17}$

Remedial

Having fractional exponents may confuse learners; however remind learners to work step by step and that the only exponent fractions they will come across have here will have 2 as the denominator. Allow learners to simply divide the exponent by 2, if they find having the fractional exponent is too distracting or confusing.

Extension

Provide more complex examples such as 3.5 and 3.6 for learners to practise with.

Review Copy

Exercise 2

Learner's Book page 125

Guidelines on how to implement this activity

Discuss cubed numbers. Ask learners to provide all the perfect cubes (cubed integers) up to 216. Revise the exponent law $(a^m)^n = a^{m\times n} = a^{mn}$ and show learners how this applies to cubing numbers.

Show learners that if a bracket (containing one term) is cubed, each factor in the bracket must be cubed. Do a few examples together as a class.

Discuss cube roots and do some examples of finding cube roots of whole numbers.

Show learners the following exponent definition: $\sqrt[3]{a^m} = a^{m\div3} = a^{\frac{m}{3}}$.

Explain that when we find the cube root, we divide the exponent by 3. Do a few examples finding the cube root of terms with variables and exponents.

Ensure that learners understand that all variables with exponents that are divisible by 3, have cube roots. Learners can work in pairs to complete this exercise.

Suggested answers

1 1; 8; 27; 64; 125; 216

2.1 $(4a^2b^2)^3 = 4^{1\times3}a^{2\times3}b^{2\times3} = 64a^6b^6$

2.2 $(2x^6y^4)^3 = 2^{1\times3}x^{6\times3}y^{4\times3} = 8x^{18}y^{12}$

2.3 $(5c^2d^7)^3 = 5^{1\times3}c^{2\times3}d^{7\times3} = 125c^6d^{21}$

2.4 $(9y^6z^3)^3 = 9^{1\times3}y^{6\times3}z^{3\times3} = 729y^{18}z^9$

2.5 $(-7a^2b^2c^9)^3 = (-7)^{1\times3}a^{2\times3}b^{2\times3}c^{9\times3} = -343a^6b^6c^{27}$

2.6 $(-5r^3s^6t^0)^3 = (-5)^{1\times3}r^{3\times3}s^{6\times3}t^{0\times3} = -125r^9s^{18}t^0 = -125r^9s^{18}$

3.1 $5n^{\frac{27}{3}} = 5n^9$

3.2 $2a^{\frac{18}{3}}b^{\frac{24}{3}} = 2a^6b^8$

3.3 $\sqrt[3]{27x^9} = 3x^{\frac{9}{3}} = 3x^3$

3.4 $3y^{\frac{12}{3}} + 7y^{\frac{12}{3}} = 3y^4 + 7y^4 = 10y^4$

3.5 $20k^{\frac{9}{3}} = 20k^3$

3.6 $a^{\frac{9}{3}}b^{\frac{12}{3}} = a^3b^4$

3.7 $0{,}2m^{\frac{3}{3}}n^{\frac{6}{3}} = 0{,}32mn^2$

4.1 $50a^2$

4.2 $3b^9$

4.3 $\sqrt{25x^6} = 5x^3$

4.4 $2 \times 4e^9f^8g^7 = 8e^9f^8g^7$

4.5 $\frac{7b^4}{9b^5} = \frac{7}{9b}$

4.6 $3 \times 3c^6d^7 = 9c^6d^7$

4.7 $36y^2$

4.8 $\sqrt[3]{8p^6} = 2p^2$

Remedial

Learners who struggle here may need to revise their multiplication tables. Sound knowledge of times-tables will greatly increase learners speed and accuracy with working with squaring and cubing terms with variables and exponents.

Unit 5 Substitution

Learner's Book page 126

Unit focus:

- how to use substitution to determine the numerical value of an algebraic expression.

Background information on substitution

In Grade 7 the learners learnt substitution, and it was also revised in Algebra 1. In Algebra 2 the skill acquired will be developed and applied in ways other than previously practised. We use substitution to determine the numerical value of an algebraic expression. The variable is replaced by a numerical value and the exact value of the expression is then determined. We can also use substitution to check our answers when we solve equations.

Review Copy

Exercise 1

Learner's Book page 127

Guidelines on how to implement this activity

Discuss the concept of substitution. What does substitution mean? We replace an unknown value with a known value. Revise with learners, previous experiences with substitution. Do a few simple monomial examples of substituting values. Remind learners to watch out for the signs of the answers.

Increase the complexity of examples you do as a class until you have substituted into a complex trinomial expression with powers and even fractions. Encourage learners to do numbers 1 to 4 on their own, and allow them to do number 5 in pairs or small groups.

Suggested answers

1.1 $3(3) = 9$

1.2 $2(3) + 4 = 6 + 4 = 10$

1.3 $4(3) - 6 = 12 - 6 = 6$

1.4 $6(3) + 3(3) - 12 = 18 + 9 - 12 = 27 - 12 = 15$

1.5 $3(3) + 4 - 2(3) = 9 + 4 - 6 = 13 - 6 = 7$

1.6 $8(3) - 5(3) + 9 = 24 - 15 + 9 = 9 + 9 = 18$

1.7 $(3) + 5 - 10 - (3) + 3 = -2$

1.8 $[4(3) + 6] - [3(3) \times 3] = (12 + 6) - (27) = 18 - 27 = -9$

2.1 $4[(3) + (5)] = 4(8) = 32$

2.2 $[(5) - (3)][(3) - (5)] = (2)(-2) = -4$

2.3 $3[(5) - (3) + 8] = 3(2 + 8) = 3(10) = 30$

2.4 $8(3) - 10(5) \div 5 - (3) = 24 - 50 \div 5 - 3 = 24 - 10 - 3 = 11$

2.5 $3[(5) + (3)] + 12 = 3(8) + 12 = 24 + 12 = 36$

2.6 $\frac{5(3)}{4(5)} = \frac{15}{20} = \frac{3}{4}$

2.7 $\frac{y-x}{4x} = \frac{(5)-(3)}{4(3)} = \frac{2}{12} = \frac{1}{6}$

2.8 $[2(3) + 4]5 - [2(5)4] + (3)(5) = (6 + 4)5 - 40 + 15 = (10)5 - 40 + 15$
$= 50 - 40 + 15 = 10 + 15 = 25$

3

x	-2	5	-1
y	4	-3	2
3.1 $x + y$	$(-2) + (4) = 2$	$(5) + (-3) = 2$	$(-1) + (2) = 1$
3.2 $3(x - y)$	$3[(-2) - (4)]$ $= 3(-6) = -18$	$3[(5) - (-3)]$ $= 3(8) = 24$	$3[(-1) - (2)]$ $= 3(-3) = -9$
3.3 $7 + x - y$	$7+(-2) - (4)$ $= 7 - 2 - 4 = 1$	$7 + (5) - (-3)$ $= 7 + 5 + 3 = 15$	$7 + (-1) - (2)$ $= 7 - 1 - 2 = 4$
3.4 $(2y)(4x)+5$	$[2(4)][4(-2)] + 5$ $= (8)(-8) + 5$ $= -64 + 5 = -59$	$[2(-3)][4(5)] + 5$ $= (-6)(20) + 5$ $= -120 + 5$ $= -115$	$[2(2)][4(-1)] + 5$ $= (4)(-4) + 5$ $= -16 + 5 = -11$
3.5 $(y - x)(x + y)$	$[(4) - (-2)][(-2) + (4)]$ $= (6)(2) = 12$	$[(-3) - (5)][(5) - 3]$ $= (-8)(8) = -64$	$[(2) - 1)][(-1) - (2)]$ $= (3)(-3) = -9$
3.6 $3x + 5y - 4$	$3(-2) + 5(4) - 4$ $= -6 + 20 - 4 = 13$	$3(5) + 5(-3) - 4$ $= 15 - 15 - 4$ $= -4$	$3(-1)+ 5(2) - 4$ $= -3 + 10 - 4 = 3$

Review Copy

4.1 $y = 5(2) - 7 = 10 - 7 = 3$

4.2 $y = 3(-4) + 2 = -12 + 2 = -10$

4.3 $y = (1) + 10 = 11$

4.4 $y = 2(4) - 3 = 8 - 3 = 5$
$z = 4(5) - 8 = 20 - 8 = 12$

4.5 $b = 2(4) = 8$
$c = 2(8) - 2(4) = 16 - 8 = 8$

5.1 $p = x + 3 + x + 3 + x + 1 + x + 1 = 4x + 8 = 4(4) + 8 = 16 + 8 = 24$

5.2 $p = x + 3 + x + 3 + x + 3 = 3x + 9 = 3(4) + 9 = 12 + 9 = 21$

5.3 $p = 4(x + 2) = 4x + 8 = 4(4) + 8 = 16 + 8 = 24$

5.4 $p = 5x + 5x + 7x = 17x = 17(4) = 68$

Remedial

If learners experience any problems with the more complex expressions, provide additional simple monomial and binomial expressions for learners to practise on before progressing to the more complex expressions.

Extension

For learners who enjoy more of a challenge, provide examples similar to question 5 for learners to try.

Unit 6 Solving algebraic equations

Learner's Book page 128

Unit focus:

- revising setting up equations to solve problems
- revising analysis and interpretation equations
- revising solving equations by inspection
- revising determination of numerical value using substitution
- solving equations using additive and multiplicative inverses.

Background information on solving algebraic equations

In Grade 7 learners solved algebraic equations using the method of inspection only. In Grade 8 they will learn to solve algebraic equations by using the inverses of addition and multiplication.

To master this method of solving equations it would be useful to revise and consolidate addition and multiplication with algebra, substitution, as well as solving equations, from Algebra 1.

Review Copy

Revision

Learner's Book page 128

Guidelines on how to implement this activity

Before progressing to equations, have learners complete this revision exercise to assess how they coped with the material covered in this chapter. Learners must complete this exercise on their own.

Suggested answers

1.1 $6x + 6 = 108$

$6x + 6 - 6 = 108 - 6$

$6x = 102$

$\frac{6x}{6} = \frac{102}{6}$

$x = 17$

1.2 $25x = \frac{141 + 159}{2}$

$25x = \frac{300}{2}$

$25x = 150$

$\frac{25x}{25} = \frac{150}{25}$

$x = 6$

1.3 $x - 32 = 6 \times 8$
$x - 32 = 48$
$x - 32 + 32 = 48 + 32$
$x = 80$

2.1 true 2.2 true

2.3 false: a positive number divided by a negative number gives a negative answer.

3.1 $a = -5$ 3.2 $b = 24$ 3.3 $c = 9$

3.4 $a = 75$ 3.5 $d = 6$

4.1 $[(-2) + (3)][(3) - (4)] = (1)(-1) = -1$ 4.2 $(-2)^2(3)^2 = 4 \times 9 = 36$

4.3 $\frac{2(4)}{-2} = \frac{8}{-2} = -4$

Remedial

Identify any problem areas and refer learners back to that unit to revise the necessary concepts. Ensure learners remediate the necessary material before starting equations.

Exercise 1

Learner's Book page 130

Guidelines on how to implement this activity

Provide learners with a basic equation on the board, such as $x + 5 = 16$. Ask learners how they would solve for x. Learners should provide the answer of 11. Ask learners how they obtained this answer. Show learners how we can use the additive inverse, which is subtraction. We solve the equation by subtracting 5 from the LHS and RHS, following the golden rule of doing to the one side what we do the other.

Show learners how this provides us with an answer of 11. Repeat with another example, $3x = 9$. Demonstrate the use of division as the multiplicative inverse to solve the equation. Do another example, this time with both additive and multiplicative inverses. Do as many examples as necessary, ensuring learners know and follow the golden rule of equations and always do the same operation on both sides.

Review Copy

Suggested answers

1
$6a - 5 = 23 - 4$
$6a - 5 = 19$
$6a - 5 + 5 = 19 + 5$
$6a = 24$
$\frac{6a}{6} = \frac{24}{6}$
$a = 4$

2
$\frac{x}{9} - 12 = 8$
$\frac{x}{9} - 12 + 12 = 8 + 12$
$\frac{x}{9} = 20$
$\frac{x}{9} \times 9 = 20 \times 9$
$x = 180$

3
$4b + 7 = 23$
$4b + 7 - 7 = 23 - 7$
$4b = 16$
$\frac{4b}{4} = \frac{16}{4}$
$b = 4$

4
$y - 34 = 28 + 7$
$y - 34 = 35$
$y - 34 + 34 = 35 + 34$
$y = 69$

5 $8c - 2 = 6 - 8$
$8c - 2 = -2$
$8c - 2 + 2 = -2 + 2$
$8c = 0$
$\frac{8c}{8} = \frac{0}{8}$
$c = 0$

6 $9p = 39 + 6$
$9p = 45$
$\frac{9p}{9} = \frac{45}{9}$
$p = 5$

7 $\frac{7}{z} = 9 - 4$
$\frac{7}{z} = 5$
$\frac{7}{z} \times z = 5 \times z$
$7 = 5z$
$\frac{7}{5} = z$

8 $4x - 1 = 0$
$4x - 1 + 1 = 0 + 1$
$4x = 1$
$x = \frac{1}{4}$

9 $\frac{3x}{4} + 3 = 7$
$\frac{3x}{4} + 3 - 3 = 7 - 3$
$\frac{3x}{4} = 4$
$3x = 16$
$x = \frac{16}{3} = 5\frac{1}{3}$

Review Copy

Revision

Learners have to be able to work with equations, so for those learners struggling to grasp the concept, provide as many addition exercises as possible. It can seem confusing at first, but the more practice learners have the better they will do.

Unit 7 Solving algebraic equations using the laws of exponents

Learner's Book page 131

Unit focus:

- how to solve equations using laws of exponents.

Background information on solving algebraic equations using the laws of exponents

In Grade 7 learners solved algebraic equations using the method of inspection only. In this section of Grade 8 they learn how to solve equations with exponents. In exponential equations the variable can be in the exponent, or in the base.

$2^x = 4$ The unknown or variable is the exponent.
$x^3 = 8$ The unknown is the base.

Exercise 1

Learner's Book page 132

Guidelines on how to implement this activity

Revise the exponent laws with learners. Do a few examples together as a class using the laws, and ensure learners are able to work confidently with these laws. Introduce an exponential equation to learners.

Show them the two types – one with the variable in the exponent, and the other with the variable in the base. Show learners how to solve exponential equations by equating either the exponents or the bases. Do as many examples together as a class until learners are able to manage on their own.

Suggested answers

Solve for x:

1 $x^5 + 4 = 36$
$x^5 + 4 - 4 = 36 - 4$
$x^5 = 32$
$x^5 = 2^5$
$x = 2$

2 $x^3 - 15 + 15 = 110 + 15$
$x^3 = 125$
$x^3 = 5^3$
$x = 5$

3 $x^2 - 12 = 24$
$x^2 - 12 + 12 = 24 + 12$
$x^2 = 36$
$x^2 = 6^2$
$x = 6$

4 $x^4 + 8 - 8 = 3^4 + 8 - 8$
$x^4 = 3^4$
$x = 3$

5 $15 + x^6 = 79$
$15 + x^6 - 15 = 79 - 15$
$x^6 = 64$
$x^6 = 2^6$
$x = 2$

6 $x^2 + 4 - 4 = 20 - 4$
$x^2 = 16$
$x^2 = 4^2$
$x = 4$

7 $x^2 + 1 - 1 = 226 - 1$
$x^2 = 225$
$x^2 = 15^2$
$x = 15$

8 $x^3 - 9 = 18$
$x^3 - 9 + 9 = 18 + 9$
$x^3 = 27$
$x^3 = 3^3$
$x = 3$

9 $9x^2 - 1 = 80$
$9x^2 - 1 + 1 = 80 + 1$
$9x^2 = 81$
$9x^2 = 81$
$\frac{9x^2}{9} = \frac{81}{9}$
$x^2 = 9$
$x^2 = 3^2$
$x = 3$

10 $2a^{12} - a^{12} = 1$
$a^{12} = 1$
$a^{12} = 1^{12}$
$a = 1$

Review Copy

Consolidation

Learner's Book page 134

Before doing this consolidation exercise, encourage learners to review the work covered in this chapter. Advise learners to use the summary and to revise their work. This exercise can be used as an informal assessment task for you to track how learners are coping with the chapter and the concepts covered. The mark allocation provides guidelines on how to assess learners.

Suggested answers

1.1 $\frac{6}{y2}$ (1) 1.2 $-5x^2$ (1)

1.3 $9y^2 - \frac{x}{2y}$ (1) 1.4 $-10zs - \frac{4}{s^2}$ (1)

1.5 $3 + \frac{xy^3}{2}$ (2)

2.1

$3x - 2$							
x	-1	0	1	2	3	-5	$\frac{1}{2}$
	$3(-1) - 2$ $= -3 - 2$ $= -5$	$3(0) - 2$ $= 0 - 2$ $= -2$	$3(1) - 2$ $= 3 - 2$ $= 1$	$3(2) - 2$ $= 6 - 2$ $= 4$	$3(3) - 2$ $= 9 - 2$ $= 7$	$3(-5) - 2$ $= -15 - 2$ $= -17$	$3\left(\frac{1}{2}\right) - 2$ $= \frac{3}{2} - \frac{4}{2}$ $= -\frac{1}{2}$

(3)

2.2

$4x^2 + 3$							
x	-1	-2	0	1	3	4	5
	$4(-1)^2 + 3$ $= 4(1)$ $+ 3$ $= 4 + 3$ $= 7$	$4(-2)^2 + 3$ $= 4(4)$ $+ 3$ $= 16 + 3$ $= 19$	$4(0)^2 + 3$ $= 4(0)$ $+ 3$ $= 0 + 3$ $= 3$	$4(1)^2 + 3$ $= 4(1)$ $+ 3$ $= 4 + 3$ $= 7$	$4(3)^2 + 3$ $= 4(9)$ $+ 3$ $= 36 + 3$ $= 39$	$4(4)^2 + 3$ $= 4(16) + 3$ $= 64 + 3$ $= 67$	$4(5)^2 + 3$ $= 4(25)$ $+ 3$ $= 100 + 3$ $= 103$

(3)

Review Copy

3.1 $4a^{17}b^7$ (1) 3.2 $3x^{11}y$ (1)

3.3 $84m^{16}n^{23}p^9 \times 7m^8n^{11}p^6 = 56m^{24}n^{34}p^{15}$ (2)

3.4 $\sqrt{144a^{45}b^{36}c^{27}} \times \sqrt[3]{125a^{12}b^3c^{15}}$

$= 12a^{\frac{45}{2}}b^{18}c^{\frac{27}{2}} \times 5a^4b^1c^5$

$= 60a^{\frac{53}{2}}b^{19}c^{\frac{37}{2}}$ (2)

3.5 $\sqrt{\sqrt{81r^{16}}} = \sqrt{9r^8} = 3r^4$

4.1 $2x = 46$

$x = 23$ (2)

4.2 $3x^3 = 3^3$

$x = 3$ (1)

4.3 $x + \frac{1}{2} = \frac{21}{4}$

$x = \frac{19}{4}$

$x = 4\frac{3}{4}$ (2)

4.4 $4^x = 4^2$

$x = 2$ (1)

4.5 $3x + 6 = 12$

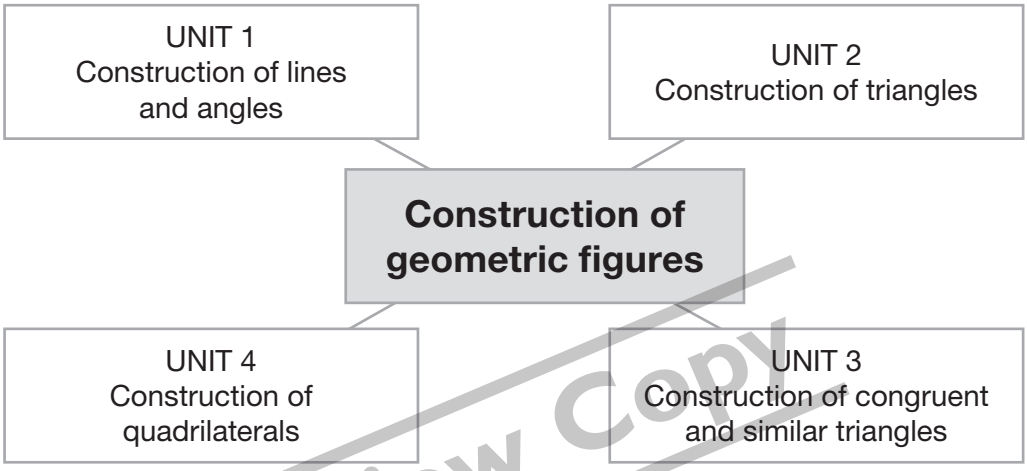
$3x = 6$

$x = 2$ (2)

Total marks [30]

Chapter 7 Construction of geometric figures

Chapter overview



Review Copy

Content		Time allocations	LB page
Unit 1	Construction of lines and angles	2 hours	136
Unit 2	Construction of triangles	2 hours	145
Unit 3	Construction of congruent and similar triangles	2 hours	149
Unit 4	Construction of quadrilaterals	2 hours	153

Background information on constructions

- Drawing instruments have been in use since ancient times. For example, excavations in Pompeii in Italy have found a bronze tool kit used by the ancient Romans, which contained triangle rulers and compasses. Since about the 13th century protractors were used to measure and draw angles and arcs of a circle accurately.
- Drawing instruments used at school level include pairs of compasses, protractors, rulers and set squares. Each instrument provides unique opportunities to explore or consolidate knowledge on lines, angles and 2D shapes.

Teaching guidelines

Before starting with this chapter make sure learners are competent and comfortable using a pair of compasses, and know how to measure and read angle sizes on a protractor. It is also a good idea to revise the concepts of a circle, centre, radius, diameter and arc of a circle, as well as the construction of angles, before proceeding with the new constructions.

Resources

Learner's Book; workbook; a pair of compasses, protractor, metre stick and set squares to illustrate constructions on the blackboard; cut-outs of equilateral, isosceles, scalene (skew), acute-angled, right-angled and obtuse-angled triangles; cut-outs of congruent and similar triangles; cut-outs of various types of quadrilaterals, including trapeziums, parallelograms, rectangles, rhombuses, squares, kites and arrowheads.

Unit 1 Construction of lines and angles

Learner's Book page 136

Unit focus:

- using a pair of compasses to bisect lines and angles
- using a pair of compasses to construct perpendicular and parallel lines
- using a protractor to measure and construct different types of angles
- using a pair of compasses to construct multiples of 45°, 60° and 30°.

Background information on construction of lines and angles

Learners should have constructed lines and angles in Grade 7. They should be familiar with the tools in a geometry set and be able to manipulate them correctly. It is advisable to assess learners' skills by having them draw standard prescribed shapes using a ruler, a pair of compasses and a pencil.

Review Copy

Exercise 1

Learner's Book page 140

Guidelines on how to implement this activity

- Discuss the concepts of point, line, line segment, ray and angle as described in the Learner's Book. Illustrate how to use a pair of compasses to bisect a line segment and an angle. Work through the examples in the Learner's Book.
- Discuss the concepts of intersecting lines, perpendicular lines, perpendiculars and parallel lines as described in the Learner's Book. Illustrate how to use a pair of compasses to construct a perpendicular at any point on a line, as well as from any point towards a line. Work through the examples in the Learner's Book.
- Discuss how to use a pair of compasses to construct parallel lines. Point out that some more advanced constructions will be covered in the next chapter. Work through the example in the Learner's Book.
- Learners answer questions from Exercise 1.

Suggested answers

1.1 It is an angle.
1.2 It is the vertex of $A\hat{B}C$.
1.3 It is part of the circle with B as centre.
1.4 They are parts of identical circles with centres at the points where the first arc cuts rays BA and BC.
1.5 It joins the vertex of $A\hat{B}C$ and the intersection of the two arcs at D.
1.6 The two angles are equal in size.

1.7 BD is the bisector of AC.
2.1 It is a line segment.
2.2 They are parts of identical circles with K and L as centres.
2.3 PQ joins the points of intersection of the two arcs.
2.4 M is the point of intersection of KL and PQ.
2.5 KM = LM; therefore PQ bisects KL.
2.6 All four angles are right angles; therefore PQ⊥KL.
2.7 PQ cuts KL in half at an angle of 90°.
3.1 It is a line.
3.2 It is part of a circle with P as centre.
3.3 They are parts of identical circles with centres at the points where the first arc cuts line XY.
3.4 It joins point P and the intersection of the arcs at Q.
3.5 M is the point of intersection of PQ and XY.
3.6 All four angles are right angles; therefore PQ⊥XY.
3.7 MP is a perpendicular drawn from point P to line XY.
4.1 – 4.4 The perpendicular bisectors of the sides of a triangle meet in one point (they are concurrent).

Remedial

Allow slower learners to use the classroom drawing instruments to practise the relevant constructions on the blackboard.

Extension

Learners can create beautiful artwork using shapes and figures they construct using their geometry tools. Encourage the use of colour and pattern.

Review Copy

Exercise 2

Learner's Book page 144

Guidelines on how to implement this activity

- Discuss the features of a protractor (centre point, gradation lines on the curved edge, outer scale and inner scale), and the concepts of angle, size of an angle and types of angles as described in the Learner's Book.
- Illustrate how to use a protractor to construct an angle of any size. Pay special attention to the construction of reflex angles. Work through the examples in the Learner's Book.
- Illustrate how to use a pair of compasses to construct multiples of 45° (by bisecting a right angle), 60° (by drawing an equilateral triangle), and 30° (by bisecting any angle of an equilateral triangle). Work through the examples in the Learner's Book.
- Learners answer questions from Exercise 2.

Suggested answers

1.1 $\frac{1}{2}$ of a revolution = 180°; $\frac{1}{3}$ of a revolution = 120°; $\frac{1}{4}$ of a revolution = 90°;

$\frac{1}{5}$ of a revolution = 72°; $\frac{1}{6}$ of a revolution = 60°;

1.2 A protractor covers 180°; $\frac{1}{8}$ of a protractor = $22\frac{1}{2}$°; $\frac{1}{9}$ of a protractor = 20°;

$\frac{1}{10}$ of a protractor = 18°; $\frac{1}{12}$ of a protractor = 15°

1.3 360° ÷ 30° = 12 wedges; 360° ÷ 18° = 20 wedges; 360° ÷ 15° = 24 wedges

1.4 360° ÷ 12 = 30° each; 360° ÷ 40 = 9° each; 360° ÷ 150 = 2,4° each

1.5 360° ÷ 22,5° = 16 wedges, yes; 360° ÷ 75° ≈ 5 wedges, no; 360° ÷ 65° ≈ 6 wedges, no

2.1 90° 2.2 80° 2.3 120°

2.4 90° (because base angles are 45° each)

3.1 The two angles together form a right angle.

3.2 The two angles together form a straight angle.

3.3 The two angles together form a revolution.

4 Learners construct an angle of 30° and it multiples up to 330° without using a protractor.

Remedial

Allow slower learners to use the classroom drawing instruments to practise the relevant constructions on the blackboard.

Extension

Learners draw various triangles and use construction to comment on the following:

- the bisectors of the interior angles of each triangle
- the perpendicular bisectors of the sides of each triangle
- the perpendiculars drawn from the vertices of each triangle to the opposite sides.

In all these cases the three construction lines should meet in one point (they are concurrent).

Review Copy

Unit 2 Construction of triangles

Learner's Book page 145

Unit focus:

- using a pair of compasses, ruler and protractor to construct triangles
- investigating by construction of triangles:
  - the sum of the interior angles of triangles
  - size of angles in an equilateral triangle
  - length of the sides and base angles of an isosceles triangle
  - size of angles of right-angled triangles
  - bisectors of the interior angles of a triangle
  - the exterior angle of a triangle.

Background information on construction of triangles

Accuracy and neatness are crucial for construction. In order for the sides of triangles to meet at the correct vertices, they have to be drawn precisely. The pyramids built by the ancient Egyptians were huge structures, but they were built with such precision and accuracy that it continues to astound many modern day scientists and builders.

Exercise 1

Learner's Book page 146

Guidelines on how to implement this activity

Discuss the concept of triangles and how triangles are labelled in the Learner's Book. Use cut-outs of triangles to illustrate that they are named, described and classified in terms of the lengths of their sides (equilateral, isosceles or scalene) or the sizes of their angles (acute-angled, right-angled or obtuse-angled).

Point out that it is impossible to construct equilateral triangles which are also right-angled or obtuse-angled. Learners answer some questions from Exercise 1.

Suggested answers

1.1 ΔABC: AB = 40 mm; BC = 40 mm; CA = 57 mm; $\hat{A} = 45°$; $\hat{B} = 90°$; $\hat{C} = 45°$
ΔDEF: DE = 58 mm; EF = 50 mm; FD = 72 mm; $\hat{D} = 44°$; $\hat{E} = 83°$; $\hat{F} = 53°$
ΔKLM: KL = LM = MK = 46 mm; $\hat{K} = \hat{L} = \hat{M} = 60°$
ΔPQR: PQ = 43 mm; QR = 14 mm; RP = 50 mm; $\hat{P} = 15°$; $\hat{Q} = 112°$; $\hat{R} = 53°$
ΔXYZ: XY = 50 mm; YZ = 40 mm; ZX = 30 mm; $\hat{X} = 53°$; $\hat{Y} = 37°$; $\hat{Z} = 90°$

1.2 equilateral: ΔKLM; isosceles: ΔABC; scalene: ΔDEF, ΔPQR, ΔXYZ

1.3 acute-angled: Δ DEF, ΔKLM; right-angled: ΔABC, ΔXYZ; obtuse-angled: ΔPQR

2.1 equilateral triangles: Δ ABC

2.2 isosceles triangles: Δ ACD, Δ ADE

2.3 scalene triangles: Δ ABD, Δ ABE, Δ ACE

2.4 acute-angled triangles: Δ ABC

2.5 right-angled triangles: Δ ABD

2.6 obtuse-angled triangles: Δ ACD, Δ ACE, Δ ADE, Δ ABE

3 Learners discuss how to construct each triangle in Table 1 of the Learner's Book.

Review Copy

Remedial

Provide copies of the triangles in the Learner's Book on paper, which will be easier for learners to measure and work with.

PoA | Investigation

Learner's Book page 147

(This investigation is compulsory. The teacher can decide to include this investigation as part of the formal PoA or simply as an in class task activity.)

Guidelines on how to implement this activity

Ensure learners have the correct instruments they need to complete the investigation. Encourage learners to perform the constructions on their own, but to compare their thinking with other learners. Discuss the steps before learners start the investigation, and ensure all learners know what is expected of them.

Suggested answers

There is no formal answer for this investigation, and each learner's responses will differ. However, at the end of this investigation learners should be aware of the following properties of triangles:

- The sum of the interior angles of any triangle = 180°.
- An equilateral triangle has all sides equal and all interior angles = 60°.

- An isosceles triangle has at least two equal sides and its base angles are equal.
- A right-angled triangle has one angle that is a right angle.
- The side opposite the right angle in a right-angled triangle is called the hypotenuse.
- Any exterior angle of a triangle = the sum of the opposite two interior angles.
- The bisectors of the interior angles of any triangle meet in the same point.

If using this investigation as part of the formal programme of assessment, please refer to the other possible investigation options in the Leaner's Book. For marking purposes please refer to the rubric in the POA at the back of this Teacher's Guide.

Remedial

Allow slower learners to work in groups during the investigation.

Exercise 2

Learner's Book page 148

Guidelines on how to implement this activity

- Constructions serve as a useful context for exploring properties of triangles.
- Point out that the Theorem of Pythagoras will be addressed in Chapter 12.
- When constructing triangles learners should draw on known properties and the construction of circles.
- Discuss the concept of exterior angles of a triangle as described in the Learner's Book.
- Learners answer questions from Exercise 2.

Review Copy

Suggested answers

1.1 The sum of the interior angles of a triangle is equal to 180°.
1.2 An equilateral triangle has three equal sides and each interior angle equal to 60°.
1.3 An isosceles triangle has two equal sides and its base angles are equal.
1.4 A right-angled triangle has one interior angle of 90° and two acute interior angles which also add up to 90°.
1.5 In a right-angled triangle the side opposite the right angle is called the hypotenuse.
1.6 The bisectors of the interior angles of a triangle meet in the same point.
1.7 An exterior angle of a triangle is equal to the sum of the two opposite interior angles.
2 Learners construct and classify some triangles.
Δ ABC is an isosceles acute-angled triangle.
Δ DEF is an equilateral triangle.
Δ GHK is a scalene right-angled triangle.
Δ LMN is a scalene obtuse-angled triangle.
Δ PQR is an isosceles right-angled triangle.
Δ XYZ is an isosceles right-angled triangle.

Extension

Learners draw various triangles and use construction to prove the following properties of triangles:

- The bisectors of the interior angles of a triangle meet in the same point.

- The perpendicular bisectors of the sides of a triangle meet in the same point.
- The altitudes drawn from the vertices of a triangle to the opposite sides meet in the same point.

Unit 3 Construction of congruent and similar triangles

Learner's Book page 149

Unit focus:

- constructing congruent triangles
- constructing similar triangles.

Background information on the construction of congruent and similar triangles

In Grade 7 learners learnt about congruent and similar figures for the first time. They were able to identify when figures were congruent or similar. This unit focuses on triangles, and uses construction to help learners determine for themselves the required minimum conditions for congruency and similarity.

Exercise 1

Learner's Book page 149

Review Copy

Guidelines on how to implement this activity

- Discuss the concept of congruent triangles (having the same shape and size).
- Point out that the drawings in the Learner's Book show how congruent triangular tiles can be used to make tiling patterns.
- Learners answer questions from Exercise 1.

Suggested answers

1.1 congruent to triangle I: VI
1.2 congruent to triangle II: XI
1.3 congruent to triangle III: XIV
1.4 congruent to triangle IV: X
1.5 congruent to triangle V: VIII
1.6 congruent to triangle VII: XIII
2 No; triangles XII and XIV differ in size.

Remedial

Use doubling and halving of the sides to show slower learners that the angles of a triangle stays the same when the lengths of its sides are changed proportionally.

Extension

Challenge faster learners to use construction and prove the following statements:

- If the corresponding angles of two triangles are equal, their corresponding sides are proportional.
- If the corresponding sides of two triangles are proportional, their corresponding angles are equal.

PoA | Investigation Construction of congruent triangles Learner's Book page 150

(This investigation is compulsory. The teacher can decide to include this investigation as part of the formal PoA or simply as an in class task activity.)

Guidelines on how to implement this activity

Constructions are a useful context for establishing the minimum conditions for two triangles to be congruent.

At the end of this investigation learners should be aware of the following conditions for two triangles to be congruent (refer to the drawings in the Learner's Book):

- three corresponding sides are equal (SSS): III
- two corresponding sides and the included angle are equal (SAS): IX
- two corresponding angles and a corresponding side are equal (AAS): VII and VIII
- a right angle, hypotenuse and one other corresponding side are equal (RHS): XII

Discuss the difference between sufficient, insufficient, ambiguous and superfluous sets of information as described in the Learner's Book.

Learners work in groups and find the answers to Questions 1 and 2 by comparing their drawings.

Suggested answers

1 Triangle I: one side is given; triangles not congruent; insufficient information
Triangle II: two sides are given; triangles not congruent; insufficient information
Triangle III: three sides are given; all triangles congruent; *sufficient information*
Triangle IV: one angle is given; triangles not congruent; insufficient information
Triangle V: two angles are given; triangles not congruent (but all have the same shape, therefore they are similar); insufficient information
Triangle VI: three angles are given; triangles not congruent (but all have the same shape, therefore they are similar); insufficient information
Triangle VII: two angles and the corresponding side are given; all triangles congruent; *sufficient information*
Triangle VIII: two angles and the corresponding side are given; all triangles congruent (to complete the construction, first find $\hat{A}$); *sufficient information*
Triangle IX: two sides and the included angle are given; all triangles congruent; *sufficient information*
Triangle X: two sides and one angle (not included) are given; triangles not congruent (third side too short to reach the bottom side); insufficient information
Triangle XI: two sides and one angle (not included) are given; triangles not congruent (can be either acute-angled or obtuse-angled); ambiguous information
Triangle XII: right angle, hypotenuse and one side are given; all triangles congruent; *sufficient information*

2 Three corresponding sides are equal (S; S; S).
Two corresponding sides and the included angle are equal (S; A; S).
Two corresponding angles and a corresponding side are equal (A; A; S).
A right angle, hypotenuse and one more side are equal (R; H; S).

If using this investigation as part of the formal programme of assessment, please use it in conjunction with the investigation of similar triangles in the Leaner's Book. For

Review Copy

marking purposes please refer to the rubric in the POA at the back of this Teacher's Guide.

Exercise 2

Learner's Book page 151

Guidelines on how to implement this activity

Discuss the concept of similar triangles (having the same shape but different sizes). Point out that the drawings in the Learner's Book show how congruent triangular tiles can be used to make tiling patterns which represent similar triangles. Revise the concepts of corresponding angles and corresponding sides of similar triangles as shown in the Learner's Book. Learners answer questions from Exercise 2.

Suggested answers

1 Triangle III (or XIV) and triangle XII are similar.

2 Sets V (two angles) and VI (three angles) produced similar triangles.

PoA | Investigation Construction of similar triangles

Learner's Book page 151

(This investigation is compulsory. The teacher can decide to include this investigation as part of the formal POA or simply as in class task activity.)

Review Copy

Guidelines on how to implement this activity

Constructions are a useful context for establishing the minimum conditions for two triangles to be similar. At the end of this investigation learners should be aware of the following condition for two triangles to be similar (refer to the drawings in the Learner's Book):

- corresponding angles are equal and corresponding sides are proportional (AAA). Learners answer questions 1 and 2 of the investigation.

Suggested answers

1.1 For all triangles: $\hat{A} \approx 53°$; $\hat{B} \approx 37°$; $\hat{C} = 90°$
corresponding angles are equal

1.2 triangle I: AB : BC : CA = 10 : 8 : 6 = 5 : 4 : 3
triangle II: AB : BC : CA = 20 : 16 : 12 = 5 : 4 : 3
triangle III: AB : BC : CA = 30 : 24 : 18 = 5 : 4 : 3
triangle IV: AB : BC : CA = 40 : 32 : 24 = 5 : 4 : 3
triangle V: AB : BC : CA = 50 : 40 : 30 = 5 : 4 : 3
corresponding sides are proportional (if one is doubled, all are doubled)

1.3 Triangles will be similar if their corresponding angles are equal and their corresponding sides are proportional.

2.2 The corresponding angles are equal.

2.2 $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD} = \frac{1}{2}$

3.1 $\frac{KL}{PQ} = \frac{LM}{QR} = \frac{MK}{RP} = \frac{2}{3}$

3.2 The corresponding angles are equal.

4 If two triangles are similar their corresponding angles are equal and their corresponding sides are proportional.

If using this investigation as part of the formal programme of assessment, please use it in conjunction with the investigation of congruent triangles on page 150 of the Leaner's Book. For marking purposes please refer to the rubric in the POA at the back of this Teacher's Guide.

Unit 4 Construction of quadrilaterals

Learner's Book page 153

Unit focus:

- constructing different types of quadrilaterals
- investigating sides and angles of different types of quadrilaterals
- investigating the diagonals of different types of quadrilaterals.

Background information on the construction of quadrilaterals

A quadrilateral is any four-sided figure. It can also be referred to as a tetragon, where tetra also means four, or a quadrangle. Learners have learnt about quadrilaterals in Grade 7, where they noted the properties of the parallel and perpendicular sides of certain parallelograms. In Grade 8 this knowledge is extended to include kites, as well examining some of the other quadrilaterals in more detail.

Review Copy

Exercise 1

Learner's Book page 155

Guidelines on how to implement this activity

Display a variety of quadrilaterals and discuss the concept of quadrilaterals (closed 2D shape with four straight sides only), opposite sides of a quadrilateral (having no vertices in common) and adjacent sides of a quadrilateral (sharing a vertex) as described in the Learner's Book.

Point out opposite sides which are parallel, some which are equal in length and some which are parallel and equal in length. Refer to the drawing in the Learner's Book that illustrates how to use a ruler and a set square to construct opposite sides which are parallel.

Point out adjacent sides can be equal in length and that they form acute, right, obtuse or reflex angles at their common vertex. Refer to the drawing in the Learner's Book that illustrates how to use a pair of compasses to construct adjacent sides which are equal in length.

Introduce the learners to trapeziums, parallelograms, rectangles, rhombuses, squares, kites and arrowheads. Challenge them to name unique properties of each shape and to use such properties to define each shape. Work through the examples in the Learner's Book.

Suggested answers

1 Learners construct some quadrilaterals. No protractor should be used.

1.1 Recall that opposite sides of a rhombus are parallel and all four sides are equal in length.

1.2 Recall that all sides of a square are equal and all interior angles are 90°.

1.3 Recall that a trapezium has only one pair of opposite sides parallel, which implies that the parallel sides cannot be equal in length.

1.4 Recall that a kite has two pairs of equal adjacent sides with no interior angle greater than 180°.

1.5 Recall that an arrowhead has two pairs of equal adjacent sides with one interior angle greater than 180°.

2 Learners construct some quadrilaterals by using a ruler, set square, protractor and/or a pair of compasses.

PoA | Investigation Properties of quadrilaterals

Learner's Book page 156

(This investigation is compulsory. The teacher can decide to include this investigation as part of the formal POA or simply as in class task activity.)

Guidelines on how to implement this activity

Constructions serve as a useful context for exploring properties of quadrilaterals.

At the end of this investigation learners should be aware of the following properties of quadrilaterals:

- The sum of the interior angles of a quadrilateral = 360°.
- The opposite sides of a parallelogram are parallel and equal.
- The opposite angles of a parallelogram are equal.
- The opposite sides of a rhombus are parallel and equal.
- The opposite angles of a rhombus are equal.
- The angles of rectangles and squares are 90°.
- A trapezium has one pair of opposite sides parallel.
- A kite has two pairs of adjacent sides equal.
- The diagonals of a square, rectangle, parallelogram and rhombus bisect each other.
- The diagonals of a square, rhombus and kite are perpendicular.

Discuss the concept of diagonals of a quadrilateral as described in the Learner's Book.

Review Copy

Suggested answers

Learners copy Tables 2, 3 and 4 and complete them by measuring the drawings in the Learner's Book and ticking the appropriate cells.

Table 2: The sides of quadrilaterals

	ABCD	EFGH	JKLM	NPQR	STUV	WXYZ
Only one pair of opposite sides is parallel.	✓					
Both pairs of opposite sides are parallel.		✓	✓	✓	✓	
Two pairs of opposite sides are equal.		✓	✓	✓	✓	
Two pairs of adjacent sides are equal.						✓
All sides are equal.				✓	✓	

Table 3: The angles of quadrilaterals

	ABCD	EFGH	JKLM	NPQR	STUV	WXYZ
One pair of opposite angles is equal.						✓
Two pairs of opposite angles are equal.		✓	✓	✓	✓	
All angles are right angles.			✓		✓	

Table 4: The diagonals of quadrilaterals

	ABCD	EFGH	JKLM	NPQR	STUV	WXYZ
The two diagonals are equal in length.			✓		✓	
The two diagonals bisect each other.		✓	✓	✓	✓	
The two diagonals are perpendicular.				✓	✓	✓

If using this investigation as part of the formal programme of assessment, ensure learners adhere to the guidelines for formal assessment. For marking purposes please refer to the rubric in the PoA at the back of this Teacher's Guide.

Exercise 2

Learner's Book page 157

Review Copy

Guidelines on how to implement this activity

By comparing the properties listed in the tables learners should realise that:

- rectangles and rhombuses are special kinds of parallelograms
- a square is a special kind of rectangle and rhombus

Learners answer questions from Exercise 2.

Suggested answers

1 Definitions provided by learners may differ from those listed below.

- For drawing ABCD: It is a trapezium. The sum of its interior angles is 360°. It has one pair of opposite sides parallel. A trapezium is a quadrilateral with one pair of opposite sides parallel.
- For drawing EFGH: It is a parallelogram. The sum of its interior angles is 360°. It has two pairs of opposite sides parallel and equal, two pairs of opposite angles equal, and its diagonals bisect each other. A parallelogram is a quadrilateral with two pairs of opposite sides parallel.
- For drawing JKLM: It is a rectangle. The sum of its interior angles is 360°. It has two pairs of opposite sides parallel and equal, four interior angles which are right angles, and diagonals which are equal in length and bisect each other. A rectangle is a parallelogram with all its interior angles right angles.
- For drawing NPQR: It is a rhombus. The sum of its interior angles is 360°. It has two pairs of opposite sides parallel, four equal sides, two pairs of opposite angles equal, and its diagonals bisect each other perpendicularly. A rhombus is a parallelogram with all its sides equal in length.
- For drawing STUV: It is a square. The sum of its interior angles is 360°. It has two pairs of opposite sides parallel, four equal sides, four interior angles which are right angles, and diagonals which are equal in length and bisect each other perpendicularly. A square is either a rectangle with four equal sides or a rhombus with each interior angle a right angle.

- For drawing WXYZ: It is a kite. The sum of its interior angles is 360°. It has two pairs of equal adjacent sides, one pair of opposite angles equal, and diagonals which bisect each other perpendicularly. A kite is a quadrilateral with two pairs of adjacent sides equal.

2.1 A rectangle is a parallelogram with each interior angle a right angle.
2.2 A rhombus is a parallelogram with all four sides equal in length.
2.3 A square is a rectangle with all four sides equal in length or a rhombus with each interior angle a right angle.

Remedial

Assist learners in simplifying their definitions of a rectangle, rhombus and square by substituting part of the definition with a single word, for example:

- A rectangle is a (quadrilateral with two pairs of opposite sides parallel) and with each interior angle a right angle, therefore a rectangle is a (parallelogram) with each interior angle a right angle.
- A rhombus is a (quadrilateral with two pairs of opposite sides parallel) and all sides equal in length, therefore a rhombus is a (parallelogram) with all sides equal in length.
- A square is a (quadrilateral with each interior angle a right angle) and all sides equal in length, therefore a square is a (rectangle) with all sides equal in length.
- A square is a (quadrilateral with all sides equal in length) and each interior angle a right angle, therefore a square is a (rhombus) with each interior angle a right angle.

Extension

Challenge learners to define parallelograms, rectangles, rhombuses and squares by referring to their diagonals only.

Review Copy

Consolidation

Learner's Book page 159

Before doing this consolidation exercise, encourage learners to review the work covered in this chapter. Advise learners to use the summary and to revise their work. This exercise can be used as an informal assessment task for you to track how learners are coping with the chapter and the concepts covered. The mark allocation provides guidelines on how to assess learners. The mark allocation provides guidelines on how to assess learners.

Suggested answers

1 Learners use a pair of compasses to:
1.1 bisect a line segment of 80 mm: use equal radii bigger than 40 mm to construct the two arcs
1.2 bisect an angle of 60°: construct an equilateral triangle and bisect any of its interior angles
1.3 construct an angle of 135°: 135° = 90° + 45°; bisect a straight angle, then bisect one of the right angles
1.4 construct an angle of 300°: 300° = 360° – 60°; construct an equilateral triangle and mark the angle outside any of its interior angles. (8)
2.1 AB = AC = 60 mm; all sides are equal in length. (3)
2.2 Δ ABC is an equilateral triangle. (1)

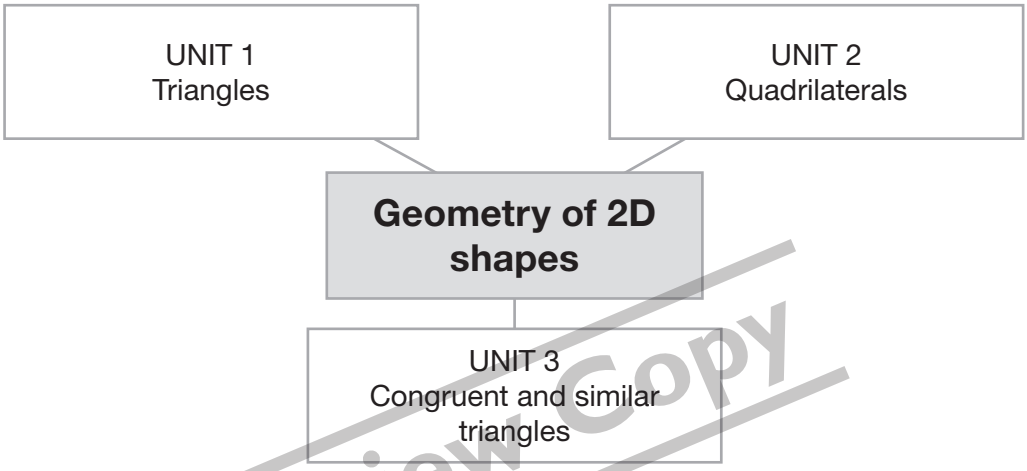
2.3	$\hat{E} = \hat{F} = 70°$; the base angles are equal.	(3)
2.4	ΔDEF is an isosceles triangle.	(1)
2.5	$GH^2 + HK^2 = 80^2 + 60^2 = 6\ 400 + 3\ 600 = 10\ 000$	(3)
2.6	The square root of 10 000 = 100, which is the same as the length of the hypotenuse.	(2)
2.7	ΔGHK is a right-angled triangle (because ^H measures 90°).	(1)
3.1	DC = 120 mm	(3)
3.2	ABCD is a trapezium; it has only one pair of opposite sides parallel.	(1)
3.3	HF = 80 mm	(3)
3.4	EFGH is a parallelogram; it has two pairs of opposite sides parallel.	(1)
3.5	$\hat{M} = \hat{K} = 90°$	(3)
3.6	JKLM is a rectangle; all interior angles are right angles.	(1)
4	Answers may differ from those provided below.	
4.1	A trapezium is a quadrilateral with only one pair of opposite sides parallel.	(1)
4.2	A parallelogram is a quadrilateral with two pairs of opposite sides parallel.	(1)
4.3	A rectangle is a quadrilateral with two pairs of opposite sides parallel and all interior angles are right angles.	(1)
4.4	A rhombus is a quadrilateral with two pairs of opposite sides parallel and all sides are equal in length.	(1)
4.5	A square is a quadrilateral with all sides equal in length and all interior angles are right angles.	(1)
4.6	A kite is a quadrilateral with two pairs of adjacent sides equal.	(1)

Total marks [40]

Review Copy

Chapter 8 Geometry of 2D shapes

Overview of concepts



Review Copy

Content		Time allocations	LB page
Unit 1	Triangles	2,5 hours	161
Unit 2	Quadrilaterals	2,5 hours	168
Unit 3	Congruent and similar triangles	3 hours	174

Background information on geometry of 2D shapes

Euclidean geometry at school level is the study of points and lines and the shapes they form in a plane. It is based on definitions and axioms (self-evident truths or assumptions) and was developed by the Alexandrian Greek mathematician Euclid (330 BC). He used these definitions and axioms to prove many facts on the geometry of his time and organised the results into a system where a fact is proven by using other facts that were already proven; this is a *deductive* system.

Drawing instruments such as compasses, protractors, rulers and set squares are used to explore or consolidate knowledge on lines, angles and 2D shapes (refer to Chapter 7).

Teaching guidelines for teaching geometry of 2D shapes

Revise the results of the investigations that learners performed in Unit 2 of Chapter 7. Ensure that learners can define each type of triangle by its sides and by its angles.

Resources

- Learner's Book; workbook
- drawing instruments to confirm properties of triangles and quadrilaterals, as well as conditions for congruent and similar triangles investigated in Chapter 7
- all other resources used in Chapter 7.

Unit 1 Triangles

Learner's Book page 161

Unit focus:

- revision of the properties and definitions of triangles in terms of their sides and angles
- identification and writing clear definitions of triangles in terms of their sides and angles
- using the properties of triangles to solve problems.

Exercise 1

Learner's Book page 162

Guidelines on how to implement this activity

Discuss how we can classify triangles according to their sides and angles. Refer learners to Table 1 and Table 2 in the Learner's Book which summarise the properties of triangle types in terms of their sides and angles, respectively. Encourage learners to memorise these properties. Discuss with learners the properties that all triangles have in common. Ask them how we could go about proving that these properties are common. Discuss construction and the techniques involved. Learners should do the following exercise on their own, but can compare their answers in pairs.

Review Copy

Suggested answers

1 Learners use a protractor to prove that the sum of the interior angles of a triangle is equal to 180°.

2 Learners use a protractor to prove that an exterior angle of a triangle is equal to the sum of the two opposite interior angles.

3 If each of the three sides of a triangle is extended, two equal exterior angles are formed at each vertex. Learners must indicate which are equal pairs.

4 An equilateral triangle is a triangle with three equal sides (it will automatically have three equal angles).
An isosceles triangle is a triangle with at least two equal sides (its base angles will automatically be equal).
A right-angled triangle is a triangle with one interior angle that is a right angle.
A scalene triangle is a triangle of which all three sides differ in length.

Remedial

Observe learners as they perform the construction. Ensure they are holding their instruments correctly to draw and measure. Remind learners that accuracy and neatness are paramount when constructing.

Exercise 2

Learner's Book page 163

Guidelines on how to implement this activity

In this exercise learners solve geometric problems to find unknown sides and angles of triangles. Encourage the learners to give reasons and justify their solutions for every written statement. Note that solving geometric problems is an opportunity to practise solving equations. Remember that the Theorem of Pythagoras will only be addressed in Chapter 11. Point out that the symbol Δ means *triangle* and the symbol ∠ means *angle*.

Suggested answers

1 $a = 45°$ (∠s of Δ = 180°); scalene
$b = 71°$ (∠s of Δ = 180°); isosceles
$c = 90°$ (∠s of Δ = 180°); right-angled
$6d = 180°$ (∠s of Δ = 180°); $d = 30°$; right-angled

2 a = 74° (isosceles Δ); $b = 32°$ (∠s of Δ)
c = d = 45° (isosceles Δ; ∠s of Δ)
$e = f = 21°$ (isosceles Δ; ∠s of Δ)
$2g + 10° = g + 35°$ (isosceles Δ); $g = 25°$; $h = 60°$ (∠s of Δ); $k = 70$ mm (equilateral Δ)

3 $a = 65°$ (exterior ∠ of Δ)
$b = 45°$ (exterior ∠ of Δ)
$c = 42°$ (exterior ∠ of Δ)

4 $a = 70°$ (isosceles Δ); $b = 40°$ (∠s of Δ); $c = 110°$ (exterior ∠ of Δ)
$d = 55°$ (isosceles Δ); $e = 110°$ (exterior ∠ of Δ); $f = 70°$ (∠s of Δ)
$g = 55°$ (isosceles Δ); $h = 70°$ (∠s of Δ); $k = 125°$ (exterior ∠ of Δ)

Review Copy

Remedial

Revise the notation and layout of working with geometric proofs. Remind learners when finding values, to handle them in the order requested, as often the information acquired in finding one value is required to determine later values. Learners may require help to construct the number sentences; be available to provide assistance.

Investigation

Learner's Book page 164

Guidelines on how to implement this activity

Encourage learners to work through this investigation in pairs, although each learner must construct their own lines, and record their findings. Ensure all learners reach the conclusion that angles on a straight line add up to 180°.

Exercise 3

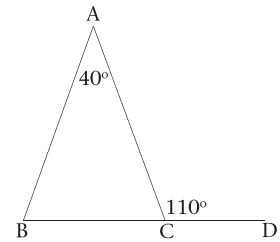
Learner's Book page 166

Guidelines on how to implement this activity

- From the investigation in the Learner's Book, learners should deduce that:
  - the sum of angles on a straight line = 180°
  - if two angles on either side of a common vertex and a common ray add up to

180°, the two rays on the outside should have opposite directions; therefore they should form a straight line.

- Learners solve geometric problems to find unknown sides and angles of triangles, using known definitions and properties of triangles as well as angles on a straight line.
- Reversed reasoning (thinking backwards from what they have to prove) helps learners identify what they should really prove to answer the question. For example, in the Learner's Book (Worked example 2) it is required to prove that ΔABC is isosceles. To do this, we first have to prove that $\hat{B} = A\hat{C}B$.



Suggested answers

1.1 $a = 72°$ (base ∠s of isosceles Δ); $b = 108°$ (∠s on straight line); $c = 30°$ (exterior ∠ of Δ)

1.2 $d = 90°$ (∠s on straight line); $e = f = 45°$ (base ∠s of isosceles Δ)

1.3 $g = h = 50°$ (base ∠s of isosceles Δ); $k = 32°$ (exterior ∠ of Δ)

1.4 $m = 33°$ (base ∠s of isosceles Δ); $n = 66°$ (ext ∠ of Δ) $p = 66°$ (base ∠s of isosceles Δ); $q = 48°$ (∠s of Δ)

1.5 $r = s$ (∠s of equilateral Δ); $t = 120°$ (∠s on straight line); $u = v = 30°$ (base ∠s of isosceles Δ)

1.6 $w = 47°$ (∠s of Δ); $x = y = 69°$ (base ∠s of isosceles Δ); $z = 63°$ (∠s on straight line)

2.1 $a = 85°$ (base ∠s of isosceles Δ); $b = 19°$ (exterior ∠ of Δ); $c = 10°$ (∠s of Δ)

2.2 $d = 68°$ (base ∠s of isosceles Δ); $e = 44°$ (∠s of Δ); $f = 90°$ (∠s on straight line); $g = h = 45°$ (base ∠s of isosceles Δs)

2.3 $p = 72°$ (base ∠s of isosceles Δ); $q = 45°$ (base ∠of isosceles Δ); $r = 27°$ (exterior ∠ of Δ); $s = 108°$ (exterior ∠ of Δ)

2.4 $t = 78°$ (base ∠s of isosceles Δ); $u = 102°$ (exterior ∠ of Δ); $v = 39°$ (base ∠s of isosceles Δ); $w = 63°$ (∠s on straight line); $x = 54°$ (∠s of isosceles Δ)

3 $C\hat{D}A = 77°$ (CA = CD)
$\therefore A\hat{C}D = 26°$ (∠s of ΔACD)
$\therefore C\hat{A}B = C\hat{B}A$ (CA = CB)
$= \frac{1}{2}(26°) = 13°$ (exterior ∠ of ΔABC)
$\therefore B\hat{A}D = 13° + 77° = 90°$ (proven)
∴ ΔABD is right-angled (interior ∠ of 90°)

4 $\hat{L} = 60°$ (∠s of ΔKLN)
$\therefore M\hat{K}N = 30°$ (MK = MN)
$\therefore L\hat{M}K = 60°$ (exterior ∠ of ΔKMN);
$\therefore L\hat{K}M = 60°$ (∠s of ΔKLM)
∴ ΔKLM is equilateral (all interior ∠s = 60°)

5 $P\hat{R}S = 80°$ (exterior ∠ of ΔPQR)
$S\hat{P}T = 40°$ (SP = ST)
$\therefore P\hat{S}R = 80°$ (exterior ∠ of ΔPST)
∴ ΔPRS is isosceles (base ∠s are equal)

6.1 $W\hat{Z}V = 68°$ (WV = WZ)
$\therefore V\hat{W}Z = 180° - 2 \times 68° = 44°$ (∠s of ΔWVZ)

Review Copy

6.2 $W\hat{Y}X = 67°$ (WX = WY)
$\therefore X\hat{W}Y = 180° - 2 \times 67° = 46°$ ($\angle$s of ΔWXY)
6.3 $Z\hat{W}Y = 180° - (44° + 46°) = 90°$ ($\angle$s on straight line)
$\therefore \Delta$WYZ is right-angled (interior $\angle$ of 90°)

Remedial
Start with simple problems where only one property of triangles is involved.

Extension
Provide faster learners with more examples similar to those in Exercise 3.

Unit 2 Quadrilaterals

Learner's Book page 168

Unit focus:
- revision of the properties and definitions of different types of quadrilaterals
- identifying and writing clear definitions of different types of quadrilaterals
- using properties of quadrilaterals to solve problems.

Background information on quadrilaterals
Revise the results of the investigations that learners performed in Unit 4 of Chapter 7.

Review Copy

Exercise 1
Learner's Book page 170

Guidelines on how to implement this activity
Point out that different definitions of quadrilaterals can be formulated by using different features of a quadrilateral. Show learners a few examples, such as:
- A parallelogram is a quadrilateral with two pairs of opposite *sides* parallel.
- A parallelogram is a quadrilateral with two pairs of opposite *angles* equal.
- A parallelogram is a quadrilateral with *diagonals* that bisect each other.

Discuss other quadrilaterals in this way. Assign the learners into groups and have them determine different definitions for each of the quadrilaterals covered in Grade 8. Learners should complete this exercise on their own, and can use the properties table to help them.

Suggested answers
1.1 A rectangle is a parallelogram with right angles: True
1.2 A rhombus is a parallelogram with equal sides: True
1.3 False: Two pairs of adjacent sides of a kite are equal.
1.4 A square is a rhombus with sides that are perpendicular: True.
1.5 A parallelogram with equal sides is called a rhombus: True.
1.6 False: A trapezium has no opposite angles equal.
1.7 A square is a rectangle with equal sides: True.
1.8 The opposite sides of a rectangle are parallel and equal: True.

1.9 The interior angles of a trapezium add up to 360°: True.
1.10 False: A kite has two pairs of adjacent sides equal.
1.11 False: The diagonals of a rectangle bisect each other but not at 90°.
1.12 The diagonals of a kite are perpendicular: True.
1.13 The diagonals of a rhombus bisect each other at 90°: True.
1.14 A rectangle is a special parallelogram: True.
1.15 False: A square is a special rhombus.
2

Property	ABCD	EFGH	JKLM	NPQR	STUV	WXYZ
All angles are right angels (90° each)				✓	✓	
All sides are equal		✓			✓	
Two pairs of opposite sides are equal	✓	✓		✓	✓	
Two pairs of opposite sides are parallel	✓	✓		✓	✓	
Both pairs of opposite angles are equal	✓	✓		✓	✓	
One pair of opposite angles is equal						✓
Diagonals bisect each other	✓	✓		✓	✓	
One diagonal is cut in half						✓
Diagonals intersect at 90° (are perpendicular)		✓			✓	✓
Diagonals are equal in length				✓	✓	
Opposite angles add up to 180°				✓	✓	

3.1 A rhombus is a quadrilateral of which the diagonals bisect each other perpendicularly (at 90°).
3.2 A rectangle is a quadrilateral of which the diagonals are equal in length and bisect each other.
3.3 A square is a quadrilateral of which the diagonals are equal in length and bisect each other perpendicularly (at 90°).
3.4 A kite is a quadrilateral of which only one diagonal bisects the other.
4.1 Rectangles are special parallelograms because they are parallelograms with equal diagonals.
4.2 Rhombuses are special parallelograms because they are parallelograms of which the diagonals are perpendicular.
4.3 Squares are special parallelograms because they are parallelograms with equal diagonals which are perpendicular.
4.4 Squares are special rectangles because they are rectangles of which the diagonals are perpendicular.
4.5 Squares are special rhombuses because they are rhombuses with equal diagonals.

Review Copy

Remedial

Allow slower learners to work in groups and to check their answers with one another.

Extension

Create posters of each quadrilateral and all its properties to put up in the classroom.

Exercise 2

Learner's Book page 172

Guidelines on how to implement this activity

Remind learners how they used the properties of triangles to solve for missing sides and angles in Unit 1. In this unit learners solve geometric problems requiring them to find unknown sides and angles of quadrilaterals. Encourage the learners to always give reasons and justify their solutions for every written statement. Note that solving geometric problems is an opportunity to practise solving equations. Remember that the Theorem of Pythagoras will only be addressed in Chapter 11. Point out that the symbol $||^m$ means *parallelogram* and the symbol ∠s means *angles*. Learners should complete this exercise on their own.

Suggested answers

1.1 $a = 90°$ (∠ of a square); $b = 50$ mm (sides of a square are equal)
1.2 $c = 90°$ (∠ of a square); $d = 45°$ (base ∠s of isosceles Δ)
1.3 $e = 90°$ (diagonals of a square are perpendicular); $f = 45$ mm (equal diagonals of a square bisect each other)
1.4 $g = 2 \times 36$ mm $= 72$ mm (equal diagonals of a square bisect each other)
2.1 $a = 50$ mm (opposite sides of a rectangle are equal); $b = 70$ mm (opposite sides of a rectangle are equal); c = 90° (∠ of a rectangle)
2.2 $d = 90°$ (∠ of a rectangle); $e = 51°$ (∠s of Δ = 180°); $f + 39° = 90°$, so $f = 51°$ (∠ of a rectangle)
2.3 $g + 51° = 90°$ so $g = 39°$ (∠ of a rectangle); $h = 51°$ (base ∠s of isosceles triangle; equal diagonals bisect each other); $k = h + 51° = 102°$ (exterior angle of Δ)
2.4 $m + 41° = 90°$, so $m = 49°$ (∠ of a rectangle); $n = 2 \times m = 98°$ (exterior angle of an isosceles triangle; equal diagonals bisect each other); $p + 41° + 90° = 180°$, so $p = 49°$ (∠ of a rectangle; ∠s of Δ)
3.1 $a - 20° = 100°$, so $a = 120°$ (opposite ∠s of a ||m); $b + c + 100° + 100° = 360°$, so $b = c = 80°$ (interior ∠s of a ||m = 360°; opposite ∠s are equal); $d = 45$ mm (opposite sides of ||m are equal); $e + 40$ mm $= 55$ mm, so $e = 15$ mm (opposite sides of ||m are equal)
3.2 $f = 95° - 65° = 30°$ (exterior ∠ of Δ); $g = f + 30° = 60°$ (opposite ∠s of ||m are equal); $2h + 2g = 360°$, so $h = 120°$ (interior ∠s of ||m = 360°); $k + 65° = 120°$, so $k = 55°$ (opposite ∠s of ||m are equal)
3.3 $m + 2(30°) = 180°$, so $m = 120°$ (isosceles Δ; ∠s of Δ); $n = m = 120°$ (opposite ∠s of rhombus are equal); $2p + n = 180°$, so $p = 30°$ (base ∠s of isosceles Δ are equal); $q = 63$ mm (all sides of rhombus are equal)
3.4 $r = 90°$ (diagonals of rhombus cut at 90°); $s + r + 72° = 180°$, so $s = 18°$ (∠s of Δ); $t = 57$ mm (diagonals of rhombus bisect each other)
3.5 $a = 70°$ (opposite ∠s of a kite are equal); $b + a + 75° + 70° = 360°$, $b = 145°$ (interior ∠s of a kite = 360°); $c = 80$ mm (pair of adjacent sides are equal)
3.6 $d = 90°$ (diagonals of kite cut at 90°); $e = 25$ mm (main diagonal bisects the other)
3.7 $4f + 5f + 90° + 90° = 360°$, so $f = 20°$ (interior ∠s of a trapezium = 360°);
3.8 $g + 2g + 90° = 180°$, so $g = 30°$ (∠s of Δ); $h + g + 30° = 180°$, so $h = 120°$ (∠s of Δ)

Remedial

Pair weaker learners with stronger learners who can work through the examples with them and provide guidance as they complete the exercise.

Review Copy

Exercise 3

Learner's Book page 173

Guidelines on how to implement this activity

This section contains combinations of quadrilaterals and/or triangles. Learners need to find unknown sides and angles of triangles and quadrilaterals, using the known definitions and properties of triangles and quadrilaterals as well as angles on a straight line.

Reversed reasoning helps learners identify what they need to prove to answer a question. For example, in Worked example 2 in the Learner's book it is required to prove quadrilateral ABCD is a parallelogram; therefore it has to be proven that both pairs of opposite sides are equal. Work through the worked examples as a class and allow learners to work together in pairs.

Suggested answers

1 Prove that each interior angle is a right angle.
In ΔPLM:
$P\hat{M}L = 45°$ (base ∠s; PL = PM)
$\therefore \hat{P} = 180° - 2(45°) = 90°$ (∠s of ΔPLM)
In quadrilateral KPMN:
$\hat{K} + \hat{P} + P\hat{M}N + \hat{N} = 360°$ (∠s of a quadrilateral = 360°)
$\therefore 90° + 90° + P\hat{M}N + 90° = 360°$ (proven)
$\therefore P\hat{M}N = 90°$
∴ KPMN is a rectangle (all interior ∠s = 90°)

2 Prove that $Q\hat{T}R = 60°$.
In quadrilateral PQTS:
PQ || ST (opposite sides of trapezium PQRS)
PS || QT (given)
∴ PQTS is a parallelogram (opposite sides are parallel)
$\therefore \hat{P} = Q\hat{T}S = 120°$ (opposite ∠s of ||m are equal)
but $Q\hat{T}S + Q\hat{T}R = 180°$ (∠s on a straight line)
$\therefore Q\hat{T}R = 180° - 120° = 60°$
In ΔQTR:
$T\hat{Q}R + Q\hat{T}R + Q\hat{R}T = 180°$ (∠s of ΔQTR)
$\therefore T\hat{Q}R + 60° + 60° = 180°$ (proven)
$\therefore T\hat{Q}R = 60°$
∴ ΔQTR is equilateral (all interior ∠s = 60°)

3 Prove that $F\hat{B}D = 90°$ and that BF = BD.
In square ABEF:
$B\hat{E}F = 90°$ (∠ of a square)
$\therefore E\hat{B}F + E\hat{F}B + 90° = 180°$ (∠s of ΔEBF)
$\therefore E\hat{B}F + E\hat{F}B = 90°$
but EB = EF (sides of a square)
$\therefore E\hat{B}F = E\hat{F}B = 90° \div 2 = 45°$
In square BCDE it can be proven in the same way that $E\hat{B}D = E\hat{D}B = 45°$.
In ΔBFD:
$F\hat{B}E + D\hat{B}E = 45° + 45°$ (proven) = 90°
$B\hat{F}D = B\hat{D}F = 45°$ (proven)
∴ BF = BD (base ∠s of Δ BFD are equal)
∴ ΔBFD is an isosceles right-angled triangle (BF = BD; $F\hat{B}D = 90°$)

Review Copy

4.1 In ΔABD: $\hat{A} + A\hat{B}D + B\hat{D}A = 180°$ ($\angle$s of Δ)
In Δ CBD: $\hat{C} + C\hat{B}D + B\hat{D}C = 180°$ ($\angle$s of Δ)
$\therefore \hat{A} + A\hat{B}D + B\hat{D}A + \hat{C} + C\hat{B}D + B\hat{D}C = 180° + 180° = 360°$
$\therefore \hat{A} + \hat{B} + \hat{C} + \hat{D} = 360°$

4.2 Proof similar as above

4.3 Proof similar as above

5.1 In quadrilateral ABCD:
BA = BC (radii of circle B are equal)
DC = DA (radii of circle D are equal)
but AB < AD (smaller circle)
$\therefore$ ABCD is a kite (two pairs of equal adjacent sides)
$\therefore$ AC$\perp$BD (diagonals of a kite cut at 90°)

5.2 In quadrilateral KLMN:
XK = XM (radii of the smaller circle are equal)
XN = XL (radii of the larger circle are equal)
KM$\perp$NL (given)
$\therefore$ KLMN is a rhombus (diagonals bisect each other at 90°)
$\therefore$ KL = LM (sides of a rhombus are equal)

5.3 In quadrilateral PQRS:
YP = YR (radii of the smaller circle are equal)
YQ = YS (radii of the larger circle are equal)
$\therefore$ PQRS is a parallelogram (diagonals bisect each other)
$\therefore$ PQ = SR (opposite sides of a ||m are equal)

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Remedial

If learners are experiencing problems, work with these learners in a small group working through the examples and unpacking what is being asked; formulate strategies for tackling the problem and steps learners should take to solve it. Allow learners to do the actual calculations themselves.

Extension

Provide additional complex examples that involve combinations of triangles and quadrilaterals for learners to gain more practice.

Unit 3 Congruent and similar triangles

Learner's Book page 174

Unit focus:

- identifying and describing the properties of congruent and similar shapes
- establishing the minimum conditions for congruency
- solving geometric problems using properties of congruent triangles
- establishing the minimum conditions for similarity
- solving geometric problems using properties of similar triangles.

Background information on congruent and similar triangles

Learners have worked with congruency and similarity in Grade 7. They only needed to be able to identify if a shape was congruent or similar. The minimum conditions are new in Grade 8, and were unpacked in Chapter 7.

Exercise 1

Learner's Book page 175

Guidelines on how to implement this activity

Revise the results of the investigations that learners performed in Unit 3 of Chapter 7. Discuss the minimum conditions for congruency. Ensure learners understand and can identify congruency in triangles. Learners should complete this exercise on their own.

Suggested answers

1 $\Delta ABC \equiv \Delta DFE$ (S;S;S)
2 They are not congruent, but $\Delta KLM \parallel\parallel \Delta PRQ$ (A;A;A)
3 $\Delta ABC \equiv \Delta DFE$ (S;A;S)
4 $\Delta KLM \equiv \Delta QRP$ (R;H;S)
5 $\Delta ABC \equiv \Delta EFD$ (A;A;S)
6 Not congruent: In ΔKLM the side of 70 mm lies opposite the angle of 30°, but in ΔPQR the side of 70 mm lies opposite the angle of 50°.

Remedial

If learners are experiencing difficulty with identifying triangles as congruent, have them revisit the investigation and construction exercises in Chapter 7 Unit 3. Work through the unit again, before retrying this exercise.

Review Copy

Exercise 2

Learner's Book page 177

Guidelines on how to implement this activity

Demonstrate by means of the worked examples how we can use congruency to find unknown sides and angles. First identify the case of congruence you are working with, for example, (R;H;S). Start by writing down the three statements of equality in the same order as the symbols in brackets: (1) equal right angles (with reason); (2) equal hypotenuses (with reason); (3) equal sides (with reason). This method will assist learners in writing down the names of the two congruent triangles in the correct order. Point out that the symbol ≡ means *congruent*.

Remember that, if $\Delta ABC \equiv \Delta DEF$ we can use the order of the letters in the names of the two triangles to deduce that $\hat{A} = \hat{D}$, $\hat{B} = \hat{E}$, $\hat{C} = \hat{F}$, AB = DE, BC = EF and AC = DF. Learners should complete this exercise on their own.

Suggested answers

1.1 In ΔABD and ΔACD:
(1) AB = AC (given)
(2) BD = CD (given)
(3) AD = AD (common side)
$\therefore \Delta ABD \equiv \Delta ACD$ (S;S;S)

1.2 $A\hat{D}B + A\hat{D}C = 180°$ ($\angle$s on a straight line)
$\therefore A\hat{D}B = A\hat{D}C = 90°$ (corresponding $\angle$s of congruent Δs)
$\therefore AD \perp BC$
$\therefore$ AD bisects AC perpendicularly (BD = DC)

2.1 In ΔEHG and ΔEFG:
(1) EH = EF (adjacent sides of a square)
(2) $\hat{H} = \hat{F}$ ($\angle$s of square = 90°)
(3) HG = FG (adjacent sides of square)
$\therefore \Delta EHG \equiv \Delta EFG$ (S;A;S)

2.2 In ΔEHG:
$H\hat{E}G + H\hat{G}E + 90° = 180°$ ($\angle$s of ΔHEG)
$\therefore H\hat{E}G + H\hat{G}E = 90°$
but $H\hat{E}G = H\hat{G}E$ (HE = HG; sides of a square)
$\therefore H\hat{E}G = 90° \div 2 = 45°$
In ΔEFG it can be proven in a similar way that $F\hat{E}G = 45°$
$\therefore H\hat{E}G = F\hat{E}G$ (proven)
$\therefore$ EG is the bisector of $H\hat{E}F$.

3.1 In ΔKLM and ΔMNK:
(1) $\hat{K}_2 = \hat{M}_2$ (given)
(2) $\hat{M}_1 = \hat{K}_1$ (given)
(3) KM = MK (common side)
$\therefore \Delta KLM \equiv \Delta MNK$ (A;A;S)

3.2 In quadrilateral KLMN:
$\hat{K}_1 + \hat{K}_2 = \hat{M}_1 + \hat{M}_2$ (given)
$\hat{L} = \hat{N}$ (corresponding $\angle$s of congruent Δs)
$\therefore$ KLMN is a parallelogram (two pairs of opposite $\angle$s are equal).

4.1 In ΔPQS and ΔRSQ:
(1) $\hat{P} = \hat{R}$ (given)
(2) $\hat{Q}_1 = \hat{S}_2$ (given)
(3) QS = SQ (common side)
$\therefore \Delta PQS \equiv \Delta RSQ$ (A;A;S)

4.2 In quadrilateral PQRS:
PQ = RS (corresponding sides of congruent triangles)
PS = RQ (given)
$\therefore$ PQRS is a parallelogram (pairs of opposite sides are equal).

Remedial

Name congruent and similar triangles in the correct order until learners feel confident enough to do it by themselves.

Exercise 3

Learner's Book page 178

Guidelines on how to implement this activity

Discuss the conditions for similarity. For polygons other than triangles to be similar, their corresponding angles must be equal AND their corresponding sides must proportional (refer to a kite and an arrowhead with the same lengths of sides). For triangles to be similar EITHER their corresponding angles must be equal OR their

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corresponding sides must be proportional. Satisfying one of these conditions results in similar triangles, because one condition can be proven if the other is given. Point out that the symbol ||| means *similar*. Learners can complete this exercise on their own.

Suggested answers

1 If $\Delta KLM \parallel\parallel\parallel \Delta XYZ$ then $\hat{K} = \hat{X}$; $\hat{L} = \hat{Y}$; $\hat{M} = \hat{Z}$; $\frac{KL}{XY} = \frac{LM}{YZ} = \frac{MK}{ZX}$

2 If $\hat{X} = \hat{M}$, $\hat{Y} = \hat{L}$ and $\hat{Z} = \hat{K}$ then $\Delta XYZ \parallel\parallel\parallel \Delta MLK$.

3 If $\frac{PT}{GF} = \frac{TR}{FH} = \frac{RP}{HG}$ then $\Delta PTR \parallel\parallel\parallel \Delta GFH$.

Remedial

If learners experience any problems with this exercise, refer them back to the construction of similar shapes in Chapter 7 for them to revise.

Exercise 4

Learner's Book page 179

Guidelines on how to implement this activity

Discuss how we could use the properties of similarity to find unknown sides and angles of triangles.

- If two triangles have two pairs of equal angles, the third pair will also be equal, because the sum of the interior angles of each triangle is 180°.
- If the corresponding angles of two triangles are equal, write down three statements of equality and use the letters on each side to name the similar triangles in the correct order.
- If the sides of two triangles are proportional, write down the names of the triangles by arranging the sides of each triangle from the shortest to the longest.
- Point out that, in ΔABC, the side (BC) opposite $\hat{B}$ is labelled *a*, the side (CA) opposite $\hat{A}$ is labelled *b* and the side (AB) opposite $\hat{C}$ is labelled *c*. Learners should complete this exercise on their own.

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Suggested answers

1.1 Check that the triangles are named in the correct order.
In ΔABC and ΔDEF:
(1) $\hat{A} = \hat{D}$ (given)
(2) $\hat{B} = \hat{E}$ (given)
(3) $\hat{C} = \hat{F}$ (sum of ∠s of Δs = 180° each)
$\therefore \Delta ABC \parallel\parallel\parallel \Delta DEF$ (A;A;A)

1.2 $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$ (corresponding sides of similar Δs)

$\therefore \frac{c}{10} = \frac{24}{12} = \frac{30}{e} = 2$ (given)
$\therefore c = 2 \times 10 = 20$ units and $e = 30 \div 2 = 15$ units

2.1 First name the triangles in the correct order.
In ΔPRQ and ΔUTS:
(1) $\hat{P} = \hat{U} = 90°$ (given)
(2) $\hat{R} = \hat{T}$ (given)

(3) $\hat{Q} = \hat{S}$ (sum of ∠s of Δs = 180°)
∴ ΔPRQ ||| ΔUTS (A;A;A)

2.2 $\frac{PR}{UT} = \frac{RQ}{TS} = \frac{QP}{SU}$ (corresponding sides of similar Δs)

∴ $\frac{4}{s} = \frac{5}{u} = \frac{3}{12} = \frac{1}{4}$ (given)
∴ s = 4 ÷ $\frac{1}{4}$ = 16 units and u = 5 ÷ $\frac{1}{4}$ = 20 units

Remedial

Allow weaker learners to pair with stronger learners to help demonstrate how to tackle such geometry problems.

Extension

Provide more advanced problems, for example, where two congruent or similar triangles share a common angle or side (one triangle lies partially on top of the other) for learners to extend their knowledge of working with triangles.

Consolidation

Learner's Book page 181

Before doing this consolidation exercise, encourage learners to review the work covered in this chapter. Advise learners to use the summary and to revise their work. This exercise can be used as an informal assessment task for you to track how learners are coping with the chapter and the concepts covered. The mark allocation provides guidelines on how to assess learners.

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Suggested answers

1.1 The sum of the interior angles of a triangle is 180°. (1)
1.2 The exterior angle of a triangle is equal to the sum of the two opposite interior angles. (1)
1.3 A rectangle is a parallelogram with right angles. (1)
1.4 A rhombus is a parallelogram with all sides equal. (1)
1.5 A square is a rectangle with all sides equal or a rhombus with all interior angles right angles. (1)
2.1 a = 60° (∠s on a straight line); $2b$ = 120°, so b = 60° (exterior ∠ of an isosceles Δ); c = 45 mm (sides of an equilateral Δ are equal).
2.2 d = 78° (opposite ∠s of a rhombus are equal); $e + f + 2(78°) = 360°$, so $e = f$ = 102° (sum of ∠s of rhombus = 360°; opposite ∠s of rhombus are equal).
2.3 $2g + 56° = 180°$, so g = 62° (∠s of an isosceles Δ); $2h + 90° = 180°$, so h = 45° (∠s of an isosceles Δ); $k = g + h$ = 107° (pair of opposite ∠s of kite are equal). (2)
3.1 $p + 30° = 70°$, so p = 40° (base ∠s are equal; LH = LK)
$q + 70° + 70° = 180°$, so q = 40° (∠s of ΔLHK)
$q + r + 20° + 2r - 30° = 180°$, so r = 50° (∠s on a straight line)
$2s = 2r - 30°$, so s = 35° (base ∠s are equal; NL = NM)
$t + 2r - 30° + 2s = 180°$, so t = 40° (∠s of ΔNLM) (5)
3.2 In ΔLHK:
$\hat{L}$ = 40° (proven)
$\hat{H}$ = 70° (given)

$\hat{K} = p + 30° = 40° + 30° = 70°$ (proven)
In ΔNLM:
$\hat{N} = 40°$ (proven)
$N\hat{L}M = 2r - 30° = 2(50°) - 30° = 70°$ (proven)
$\hat{M} = 2s = 2(35°) = 70°$ (proven)
In ΔLHK and ΔNLM:
(1) $\hat{L} = \hat{N} = 40°$ (proven)
(2) $\hat{H} = \hat{L} = 70°$ (proven)
(3) $\hat{K} = \hat{M} = 70°$(proven)
$\angle\ \Delta LHK \parallel\parallel\parallel \Delta NLM$ (A;A;A) (2)

4 Prove that each interior $\angle = 90°$
In ΔQTR:
$T\hat{Q}R = 45°$ (base $\angle$s; TQ = TR)
$\therefore Q\hat{T}S = 45° + 45° = 90°$ (exterior $\angle$ of ΔQTR)
In quadrilateral PQTS:
$\hat{P} = 90°$ ($SP \perp PQ$)
$\hat{T} = 90°$ (proven)
$\hat{S} = 90°$ ($PS \perp ST$)
$\therefore 90° + \hat{Q} + 90° + 90° = 360°$ ($\angle$s of a quadrilateral = 360°)
$\therefore \hat{Q} = 90°$
$\therefore$ PQTS is a rectangle (all interior $\angle$s = 90°) (4)

5.1 In $\Delta DEG \equiv \Delta DFG$:
(1) DE = DF (given)
(2) $\hat{D}_1 = \hat{D}_2$ (bisector DG)
(3) DG = DG (common side)
$\therefore \Delta DEG \equiv \Delta DFG$ (S;A;S) (3)

5.2 $\hat{G}_1 = \hat{G}_2$ (corresponding $\angle$s of congruent Δs)
$= 180° \div 2 = 90°$ ($\angle$s on a straight line)
EG = FG (corresponding sides of congruent Δs)
$\therefore$ DG is the perpendicular bisector of EF (3)

6.1 In ΔABC and ΔDCB:
(1) $\hat{A} = \hat{D}$ (given)
(2) $\hat{C}_2 = \hat{B}_2$ (given)
(3) BC = CB (common side)
$\therefore \Delta ABC \equiv \Delta DCB$ (A;A;S) (3)

6.2 $A\hat{B}C = D\hat{C}B$ (corresponding $\angle$s of congruent Δs)
$\therefore \hat{B}_1 + \hat{B}_2 = \hat{C}_1 + \hat{C}_2$
but $\hat{B}_2 = \hat{C}_2$ (given)
$\therefore \hat{B}_1 = \hat{C}_1$ (3)

Total marks [30]

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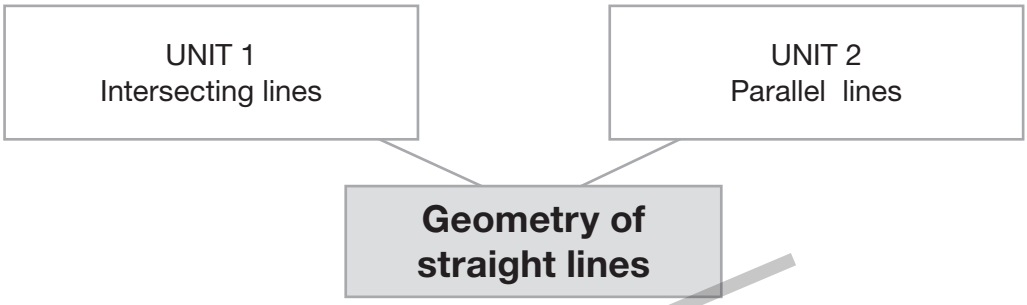
Chapter 9 Geometry of straight lines

Overview of concepts



Content		Time allocations	LB page
Unit 1	Intersecting lines	4 hours	183
Unit 2	Parallel lines	4 hours	191

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Background information on geometry of the geometry of straight lines

Euclid formulated the following postulates (assumptions) in his book *Elements*:

1 A straight line segment can be drawn joining any two points.
2 Any straight line segment can be extended indefinitely in a straight line.
3 Given any straight line segment, a circle can be drawn having the segment as radius and one endpoint as centre.
4 All right angles are congruent.
5 If two lines are drawn which intersect a third in such a way that the sum of the inner angles on one side is less than two right angles, then the two lines inevitably must intersect each other on that side if extended far enough.

Euclid's fifth postulate is equivalent to what is known as the *parallel postulate* and cannot be proven as a theorem.

Read more about these postulates on the Internet.

An important note

Point out in this chapter that drawings are not perfectly accurate constructions; therefore measurement is not an option to solve problems.

Generic guidelines for teaching the geometry of straight lines

Before starting this chapter, revise the concepts of point, line, ray and angle and the construction of perpendicular lines (Chapter 7 Unit 1). Wherever possible, learners

should perform their own constructions and measurements to test the theorems of geometry. Even if not prescribed, encourage learners to do this in order to help them form a concrete idea of the concepts involved when working with straight lines.

Resources

Learner's Book; workbook; poster that illustrates the various types of angle pairs formed by perpendicular, intersecting and parallel lines; poster that illustrates the four steps of a problem-solving strategy.

Unit 1 Intersecting lines

Learner's Book page 183

Unit focus:

- describing the relationship between angles formed by intersecting lines
- solving problems using the relationship between pairs of angles formed by intersecting and perpendicular lines.

Background information on intersecting lines

Learners have worked with intersecting lines in Grade 7, and should know that angles are formed when lines intersect. Learners should also know that if the angle formed is 90°, then the lines are perpendicular. Use perpendicular lines as a starting point for Exercise 1.

Review Copy

Exercise 1

Learner's Book page 184

Guidelines on how to implement this activity

Discuss complementary angles. Ask learners to provide examples of complementary angle pairs. Show learners how to use algebraic equations to find unknown angles. Note that solving geometric problems is an opportunity to practise solving equations. Learners should complete this exercise on their own.

Suggested answers

1 Subtract the given angle from 90°.

1.1 30°: $90° - 30° = 60°$; 45°: $90° - 45° = 45°$; 60°: $90° - 60° = 30°$

1.2 20°: $90° - 20° = 70°$; 50°: $90° - 50° = 40°$; 70°: $90° - 70° = 20°$

1.3 5°: $90° - 5° = 85°$; 55°: $90° - 55° = 35°$; 85°: $90° - 85° = 5°$

1.4 x: $90° - x$; $2x$: $90° - 2x$; $\frac{1}{2}x$: $90° - \frac{1}{2}x$

1.5 $90° - x$: $90° - (90° - x) = 90° - 90° + x = x$

1.6 $x + 45°$: $90° - (x + 45°) = 90° - x - 45° = 45° - x$

1.7 $x - 63°$: $90° - (x - 63°) = 90° - x + 63° = 153° - x$

1.8 $2x + 15°$: $90° - (2x + 15°) = 90° - 2x - 15° = 75° - 2x$

1.9 $(\frac{1}{4}x - 25)°$: $90° - (\frac{1}{4}x - 25°) = 90° - \frac{1}{4}x + 25° = 115° - \frac{1}{4}x$

2.1 True 2.2 True; 50° is larger than 40°

2.3 True; 40° is smaller than 50° 2.4 True; 30° is two times smaller than 60°
2.5 True; 75° is five times larger than 15°
2.6 True; 1° is 89 times smaller than 89°
3.1 $x = 2 \times 44° = 88°$; $y = 90° - 88° = 2°$
3.2 $x = 44° \div 2 = 22°$; $y = 90° - 22° = 68°$
3.3 $x = 44° + 30° = 74°$; $y - 90° - 74° = 16°$
3.4 $x = 44° - 18° = 26°$; $y = 90° - 26° = 64°$
4.1 $2x + 20° + x + 10° = 90°$; $3x + 30° = 90°$; $3x = 60°$; $x = 20°$
4.2 $4x - 15° + 3x + 7° = 90°$; $7x - 8° = 90°$; $7x = 98°$; $x = 14°$
4.3 $\frac{1}{2}x + 35° + \frac{1}{3}x + 25° = 90°$; $\frac{5}{6}x + 60° = 90°$; $\frac{5}{6}x = 30°$; $x = 36°$
4.4 $9x - 21° + 48° - 2x = 90°$; $7x + 27° = 90°$; $7x = 63°$; $x = 9°$
5.1 $x + x = 90°$; $2x = 90°$; $x = 45°$
5.2 $x + x + 20° = 90°$; $2x = 70°$; $x = 35°$ (20° smaller than 55°)
5.3 $x + x - 30° = 90°$; $2x = 120°$; $x = 60°$ (30° larger than 30°)
5.4 $x + \frac{1}{3}x = 90°$; $\frac{4}{3}x = 90°$; $x = 67{,}5°$ (three times 22,5°)
5.5 $x + 4x = 90°$; $5x = 90°$; $x = 18°$ (one quarter of 72°)
6.1 $\hat{1} + 40° = 90°$ (complementary ∠s); $\hat{1} = 90° - 40° = 50°$
6.2 $90° + \hat{2} + 65° = 180°$ (∠s on a straight line); $\hat{2} = 180° - 155° = 25°$
6.3 $\hat{3} + 32° = 180°$ (∠s on a straight line); $\hat{3} = 180° - 32° = 148°$
$\hat{4} + 32° = 90°$ (complementary ∠s); $\hat{4} = 90° - 32° = 58°$
$\hat{5} + \hat{4} + \hat{3} + 32° = 360°$ (a revolution); $\hat{5} = 122°$

Remedial

For learners experiencing any problems establishing and solving equations, revise how to solve equations.

Review Copy

Exercise 2

Learner's Book page 185

Guidelines on how to implement this activity

- Revise angles on a straight line (Chapter 8 Unit 1). Discuss supplementary angles and supplementary angle pairs.
- Show learners how we can use the knowledge of supplementary angles to solve problems.

Learners should complete this exercise on their own.

Suggested answers

1 Subtract the given angle from 180°.
1.1 30°: $180° - 30° = 150°$; 45°: $180° - 45° = 135°$; 60°: $180° - 60° = 120°$
1.2 120°: $180° - 120° = 60°$; 150°: $180° - 150° = 30°$; 170°: $180° - 170° = 10°$
1.3 5°: $180° - 5° = 175°$; 95°: $180° - 95° = 85°$; 175°: $180° - 175° = 5°$
1.4 x: $180° - x$; $2x$: $180° - 2x$; $\frac{1}{2}x$: $180° - \frac{1}{2}x$
1.5 $90° - x$: $180° - (90° - x) = 180° - 90° + x = 90° + x$
1.6 $x + 45°$: $180° - (x + 45°) = 180° - x - 45° = 135° - x$
1.7 $x - 63°$: $180° - (x - 63°) = 180° - x + 63° = 243° - x$
1.8 $2x + 105°$: $180° - (2x + 105°) = 180° - 2x - 105° = 75° - 2x$

1.9 $(\frac{1}{4}x - 125°)$: $180° - (\frac{1}{4}x - 125°) = 180° - \frac{1}{4}x + 125° = 305° - \frac{1}{4}x$

2.1 False.

2.2 True; 100° is larger than 80°.

2.3 True; 80° is smaller than 100°.

2.4 True; 150° is five times larger than 30°.

2.5 True; 45° is three times smaller than 135°.

2.6 True; 178° is 89 times larger than 2°.

3.1 $x = 2 \times 66° = 132°$; $y = 180° - 132° = 48°$

3.2 $x = 66° \div 2 = 33°$; $y = 180° - 33° = 147°$

3.3 $x = 66° + 36° = 102°$; $y = 180° - 102° = 78°$

3.4 $x = 66° - 48° = 18°$; $y = 180° - 18° = 162°$

4.1 $2x + 20° + x + 10° = 180°$; $3x + 30° = 180°$; $3x = 150°$; $x = 50°$

4.2 $4x - 16° + 3x + 7° = 180°$; $7x - 9° = 180°$; $7x = 189°$; $x = 27°$

4.3 $\frac{1}{2}x + 25° + \frac{1}{3}x + 35° = 180°$; $\frac{5}{6}x + 60° = 180°$; $\frac{5}{6}x = 120°$; $x = 144°$

4.4 $13x - 41° + 133° - 5x = 180°$; $8x + 92° = 180°$; $8x = 88°$; $x = 11°$

5.1 $x + x = 180°$; $2x = 180°$; $x = 90°$

5.2 $x + x + 70° = 180°$; $2x = 110°$; $x = 55°$ (70° smaller than 125°)

5.3 $x + x - 44° = 180°$; $2x = 224°$; $x = 112°$ (44° larger than 68°)

5.4 $\frac{4}{5}x = 180°$; $x = 144°$ (four times 36°)

5.5 $x + 5x = 180°$; $6x = 180°$; $x = 30°$ (one fifth of 150°)

6.1 $\hat{1} + 65° = 180°$ (supplementary ∠s); $\hat{1} = 180° - 65° = 115°$

6.2 $p + 90° + p = 180°$ (supplementary ∠s); $2p = 90°$; $p = 45°$; $\hat{2} = 45°$; $\hat{3} = 45°$

6.3 $2q + 90° + q = 180°$ (supplementary ∠s); $3q = 90°$; $q = 30°$; $\hat{4} = 60°$; $\hat{5} = 30°$

Review Copy

Exercise 3

Learner's Book page 187

Guidelines on how to implement this activity

Revise complementary and supplementary angles with learners.

Introduce the concept of adjacent angles. Ensure learners know that an adjacent angle shares an arm (a ray) and a vertex with another. Ensure that learners can write a clear description of the relationship between adjacent supplementary angles (a pair of adjacent angles which add together to make 180°).

Learners also need to be able to use adjacent angles to solve geometric problems, and understand how adjacent supplementary angles and angles on a straight line can be used in this way. Work through the worked examples as a class. Learners can complete this exercise in pairs.

Suggested answers

Note the different reasons used for pairs of adjacent supplementary angles and for three angles on a straight line.

1 $\hat{1} + 115° = 180°$ (adjacent supplementary ∠s); $\hat{1} = 180° - 115° = 65°$
$\hat{1} + \hat{2} = 180°$ (adjacent supplementary ∠s); $\hat{2} = 180° - 65° = 115°$
$115° + \hat{3} = 180°$ (adjacent supplementary ∠s); $\hat{3} = 180° - 115° = 65°$

2 $90° + \hat{4} = 180°$ (adjacent supplementary ∠s); $\hat{4} = 180° - 90° = 90°$
$\hat{4} + 28° + \hat{5} = 180°$ (∠s on a straight line); $\hat{5} = 180° - 90° - 28° = 62°$

3 $34° + \hat{6} + 68° = 180°$ ($\angle$s on a straight line); $\hat{6} = 180° - 102° = 78°$
$\hat{7} + 34° + \hat{6} = 180°$ ($\angle$s on a straight line); $\hat{7} = 180° - 34° - 78° = 68°$
$\hat{6} + 68° + \hat{8} = 180°$ ($\angle$s on a straight line); $\hat{8} = 180° - 78° - 68° = 34°$
$34° + \hat{7} + \hat{9} = 180°$ ($\angle$s on a straight line); $\hat{9} = 180° - 34° - 68° = 78°$

4 $m + 90° + m = 180°$ ($\angle$s on a straight line); $2m = 90°$; $m = 45°$; $\hat{10} = 45°$
$\hat{10} + \hat{11} = 180°$ (adjacent supplementary $\angle$s); $\hat{11} = 180° - 45° = 135°$

5 $2n + 90° + n = 180°$ ($\angle$s on a straight line); $3n = 90°$; $n = 30°$
$\hat{12} + n = 180°$ (adjacent supplementary $\angle$s); $\hat{12} = 180° - 30° = 150°$
$\hat{12} + \hat{13} = 180°$ (adjacent supplementary $\angle$s); $\hat{13} = 180° - 150° = 30°$

6 $4p + 90° + 9p + 25° = 180°$ (adjacent supplementary $\angle$s); $13p = 65°$; $p = 5°$
$\hat{14} + 4(5°) + 90° = 180°$ (adjacent supplementary $\angle$s); $\hat{14} = 180° - 110° = 70°$
$\hat{14} + \hat{15} = 180°$ (adjacent supplementary $\angle$s); $\hat{15} = 180° - 70° = 110°$

Remedial

Show learners how to set out the work appropriately and ensure learners can use and understand the concepts of adjacent supplementary angles and angles on a straight line as reasons for steps in the process for finding unknown sides of polygons and angles.

Investigation: Vertically opposite angles

Learner's Book page 187

Guidelines on how to implement this activity

When doing the investigation learners should work on their own.
Learners should discover the following:

- When two lines intersect, each pair of non-adjacent angles formed by the two lines is always equal in size, therefore $a = c$ (vertically opposite $\angle$s) and $b = d$ (vertically opposite $\angle$s).
- For two equal angles to be vertically opposite the four rays that make the angles must form two intersecting lines.

Review Copy

Exercise 4

Learner's Book page 188

Guidelines on how to implement this activity

- Revise the concept of adjacent supplementary angles, and integrate this with the conclusions learners have drawn from the investigation. Learners should be able to use both sets of information to find angles.
- Do a few examples together as a class.

Encourage learners to complete this exercise on their own.

Suggested answers

1

$A\hat{M}D$	$D\hat{M}B$	$A\hat{M}C$	$C\hat{M}B$
120°	60°	60°	120°
136°	44°	44°	136°
29°	151°	151°	29°
x	$180° - x$	$180° - x$	x

2.1 $\hat{1} = 115°$ (vertically opposite ∠s)
2.2 $\hat{2} = 28°$ (vertically opposite ∠s)
$\hat{3} + 90° + 28° = 180°$ (∠s on a straight line); $\hat{3} = 180° - 118° = 62°$
$\hat{4} = \hat{3} + 90° = 62° + 90° = 152°$ (vertically opposite ∠s)
2.3 $\hat{5} = 34°$ (vertically opposite ∠s)
$\hat{6} = 68°$ (vertically opposite ∠s)
$\hat{6} + \hat{7} + \hat{5} = 180°$ (∠s on a straight line); $\hat{7} = 180° - 68° - 34° = 78°$
2.4 $m + 90° + m = 180°$ (∠s on a straight line); $2m = 90°$; $m = 45°$; $\hat{8} = 45°$
$\hat{9} = m + 90° = 45° + 90° = 135°$ (vertically opposite ∠s)
2.5 $\hat{10} = n = 27°$ (vertically opposite ∠s)
$\hat{11} = 2n = 2(27°) = 54°$ (given)
$\hat{12} = \hat{11} = 54°$ (vertically opposite ∠s)
2.6 $4p + 90° = 9p + 55°$ (vertically opposite ∠s); $5p - 35°$; $p = 7°$
$\hat{13} = 4(7°) + 90° = 118°$
$\hat{14} = 9(7°) + 55° = 118°$
$\hat{14} + \hat{15} = 180°$ (adjacent supplementary ∠s); $\hat{15} = 180° - 118° = 62$

Remedial

Pair learners experiencing problems with stronger learners to help show them the strategies they employ to tackle geometric problems.

Exercise 5

Learner's Book page 190

Review Copy

Guidelines on how to implement this activity

In this section drawings include triangles.

- Learners need to integrate the knowledge they have acquired about lines and angles to include triangles. Revise all properties of the different types of triangles (Chapter 8 Unit 1).
- Learners solve geometric problems to find unknown angles using properties of the relevant triangles together with the angle relationships between pairs of complementary angles, adjacent supplementary angles and vertically opposite angles, as well as angles on a straight line.
- Learners should be encouraged to give reasons and justify their solutions for every written statement. Reversed reasoning helps learners identify what they should really prove (see example in the Learner's Book).

Learners should complete this exercise on their own, but can discuss their methods and answers in pairs or small groups.

Suggested answers

1.1 Complementary angles are a pair of angles which add up to 90°.
Supplementary angles are a pair of angles which add up to 180°.
1.2 Adjacent angles share a vertex and a line and lie in the same plane.
Vertically opposite angles share only a vertex and lie non-adjacently between intersecting lines.
2 $\hat{A}_2 = 87°$ (vertically opposite angles)
In ΔABC:
$\hat{A}_2 + \hat{B} + \hat{C} = 180°$ (interior ∠s of Δ)

$\therefore 87° + 74° + \hat{C} = 180°$
$\therefore \hat{C} = 180° - 87° - 74° = 19°$ (subtract 87° and 74° on both sides)

3.1 In ΔDEF and ΔHGF:
(1) $\hat{E} = \hat{G} = 90°$ ($DE \perp EG$; $EG \perp GH$)
(2) $\hat{F}_1 = \hat{F}_2$ (vertically opposite $\angle$s)
(3) $DE = HG$ (given)
$\therefore \Delta DEF \equiv \Delta HGF$ (A;A;S)

3.2 $EF = GF$ (corresponding parts of congruent Δs)
$\therefore$ F bisects EG
$DF = HF$ (corresponding parts of congruent Δs)
$\therefore$ F bisects DH

3.3 In quadrilateral DEHG:
$FE = FG$ (proven)
$FD = FH$ (proven)
$\therefore$ DEHG is a parallelogram (its diagonals bisect each other)

4.1 $52° + \hat{J}_1 + 44° = 180°$ ($\angle$s on a straight line)
$\therefore \hat{J}_1 = 180° - 52° - 44° = 84°$ (subtract 52° and 44° on both sides)
In ΔJKL:
$\hat{J}_1 + \hat{K} + \hat{L} = 180°$ (interior $\angle$s of Δ)
$\therefore 84° + 48° + \hat{L} = 180°$
$\therefore \hat{L} = 180° - 84° - 48° = 48°$ (subtract 84° and 48° on both sides)

4.2 In Δ JKL:
$\hat{K} = 48°$ (given)
$\hat{L} = 48°$ (proven)
$\therefore \hat{K} = \hat{L}$
$\therefore JK = JL$ (base $\angle$s are equal)

5.1 In ΔMQN and ΔMNP:
(1) $\hat{M} = \hat{M}$ (common part of both Δs)
(2) $\hat{Q}_1 = M\hat{N}P = 90°$ (NQ is a perpendicular; given)
(3) $\hat{N}_1 = \hat{P}$ ($\angle$s of each $\Delta = 180°$)
$\therefore \Delta MQN \,|||\, \Delta MNP$ (A;A;A)

5.2 $\hat{N}_2 + \hat{N}_1 = 90°$ (given)
$\hat{M} + \hat{N}_1 = 90°$ (complementary $\angle$s in ΔMQN)
$\therefore \hat{N}_2 + \hat{N}_1 = \hat{M} + \hat{N}_1$
$\therefore \hat{N}_2 = \hat{M}$ (subtract $\hat{N}_1$ on both sides)

6 The three acute angles inside the triangle add up to 180° (interior $\angle$s of an acute-angled Δ)
Each acute angle inside the triangle is vertically opposite to an acute angle outside the triangle
$\therefore$ the sum of the acute angles outside the triangle = 180° (three pairs of vertically opposite $\angle$s).

7 $D\hat{A}B + B\hat{A}C + E\hat{A}C = 180°$ ($\angle$s on a straight line)
In ΔABC:
$\hat{B} + B\hat{A}C + \hat{C} = 180°$ (interior $\angle$s of Δ)
$\therefore D\hat{A}B + B\hat{A}C + E\hat{A}C = \hat{B} + B\hat{A}C + \hat{C}$
$\therefore D\hat{A}B + E\hat{A}C = \hat{B} + \hat{C}$ (subtract $B\hat{A}C$ on both sides)

Review Copy

Remedial
Start with simple problems where only one property of triangles is involved. Only allow learners to progress to the more challenging examples once they have mastered the simpler one triangle problems.

Extension
Provide faster learners with more examples like those in Exercise 5.

Unit 2 Parallel lines

Learner's Book page 191

Unit focus:
- describing the relationship between angles involving parallel lines
- solving problems using the relationships between pairs of angles formed by parallel, intersecting and perpendicular lines.

Background informamation on parallel lines
Learners have been exposed to parallel lines before in Grade 7 where they learnt how to construct parallel lines and classify quadrilaterals in terms of parallel sides. The focus in Grade 8 is on the angles created when a transversal cuts parallel lines. Learners should perform as many of the investigations as possible, as they are better able to take in and fully understand new concepts and information if they get practically involved in learning the subject material in some way.

Review Copy

Investigation Corresponding angles

Learner's Book page 191

Guidelines on how to implement this activity
Learners should each perform the investigation themselves. Ensure they work neatly and accurately. Learners must measure carefully using a protractor.
The learners should discover the following:
- When a pair of lines is crossed by a transversal, corresponding angles lie on the same side of the transversal and on corresponding sides of the pair of lines (either above or below both lines).
- When parallel lines are crossed by a transversal, any pair of corresponding angles is equal.
- When two lines are crossed by a transversal and a pair of corresponding angles are equal, the lines are parallel.

Remedial
Inaccurate measurement and construction can result in disappointment and negatively affect this investigation. Learners have to get equal measurements for the corresponding angles. Ensure learners use sharp pencils and work neatly in order to get the desired result.

Exercise 1

Learner's Book page 193

Guidelines on how to implement this activity

Revise the concept of parallel lines (Chapter 7 Unit 1). In order to manage this exercise learners should know the following angle relationships:

- If parallel lines are cut by a transversal, then corresponding angles are equal.
- If lines are cut by a transversal and corresponding angles are equal, the lines are parallel.
- Equal corresponding angles are obtained by translating (sliding) an angle in the direction of one of its sides (see drawing in the Learner's Book).

Do examples as a class of identifying corresponding angles.

Suggested answers

1 $\hat{1} = \hat{2}$ (parallel lines make equal corresponding ∠s)
$\hat{2} = \hat{3}$ (parallel lines make equal corresponding ∠s)
$\hat{3} = \hat{4}$ (intersecting lines make vertically opposite ∠s)
$\hat{4} = \hat{5}$ (parallel lines make equal corresponding ∠s)
$\therefore \hat{1} = \hat{5}$

2 $\hat{1} = \hat{2}$ (parallel lines make equal corresponding ∠s)
$\hat{2} = \hat{3}$ (intersecting lines make vertically opposite ∠s)
$\hat{3} = \hat{4}$ (parallel lines make equal corresponding ∠s)
$\hat{4} = \hat{5}$ (parallel lines make equal corresponding ∠s)
$\therefore \hat{1} = \hat{5}$

3 $\hat{1} = \hat{2}$ (parallel lines make equal corresponding ∠s)
$\hat{2} = \hat{3}$ (parallel lines make equal corresponding ∠s)
$\hat{3} = \hat{4}$ (intersecting lines make vertically opposite ∠s)
$\hat{4} = \hat{5}$ (parallel lines make equal corresponding ∠s)
$\therefore \hat{1} = \hat{5}$

4 $\hat{1} = \hat{2}$ (parallel lines make equal corresponding ∠s)
$\hat{2} = \hat{3}$ (parallel lines make equal corresponding ∠s)
$\hat{3} = \hat{4}$ (parallel lines make equal corresponding ∠s)
$\hat{4} = \hat{5}$ (parallel lines make equal corresponding ∠s)
$\therefore \hat{1} = \hat{5}$

5 $\hat{1} = \hat{2}$ (parallel lines make equal corresponding ∠s)
$\hat{2} = \hat{3}$ (parallel lines make equal corresponding ∠s)
$\hat{3} = \hat{4}$ (parallel lines make equal corresponding ∠s)
$\hat{4} = \hat{5}$ (opposite ∠s of ||m are equal)
$\therefore \hat{1} = \hat{5}$

Remedial

Encourage learners to use colours to help highlight the corresponding angles. This helps learners to see the necessary angles

Review Copy

Investigation Alternate angles

Learner's Book page 193

Guidelines on how to implement this activity

Learners should each perform the investigation themselves. Ensure learners work neatly and accurately. Learners must measure carefully using a protractor.
Learners should discover the following:

- When a pair of lines is crossed by a transversal, alternate angles lie on different sides of the transversal and between (inside) the pair of lines.
- When parallel lines are crossed by a transversal, any pair of alternate angles are equal.
- When two lines are crossed by a transversal and a pair of alternate angles are equal, the lines are parallel.

Remedial

Inaccurate measurement and construction can result in disappointment and negatively affect this investigation. Learners have to get equal measurements for the alternate angles. Make sure learners use sharp pencils and work neatly in order to get the desired result.

Exercise 2

Learner's Book page 194

Review Copy

Guidelines on how to implement this activity

Discuss what learners have discovered in the investigation. Learners should know the following angle relationship:

- If parallel lines are crossed by a transversal, then alternate angles are equal.
- If lines are cut by a transversal and alternate angles are equal, the lines are parallel.
- Equal alternate angles are obtained by rotating (turning) an angle through 180° around any point on one of its sides, but not the vertex (see drawing in the Learner's Book).

Do a worked example together as a class, where you use alternate angles in a geometric problem. Prove seemingly unrelated angles as equal through the use of alternate angles. Learners can complete this exercise in pairs.

Suggested answers

1 $\hat{1} = \hat{2}$ (parallel lines make equal alternate ∠s)
$\hat{2} = \hat{3}$ (parallel lines make equal alternate ∠s)
$\hat{3} = \hat{4}$ (intersecting lines make vertically opposite ∠s)
$\hat{4} = \hat{5}$ (parallel lines make equal alternate ∠s)
$\therefore \hat{1} = \hat{5}$

2 $\hat{1} = \hat{2}$ (parallel lines make equal alternate ∠s)
$\hat{2} = \hat{3}$ (parallel lines make equal alternate ∠s)
$\hat{3} = \hat{4}$ (intersecting lines make vertically opposite ∠s)
$\hat{4} = \hat{5}$ (parallel lines make equal alternate ∠s)
$\therefore \hat{1} = \hat{5}$

3 $\hat{1} = \hat{2}$ (parallel lines make equal alternate ∠s)
$\hat{2} = \hat{3}$ (parallel lines make equal corresponding ∠s)

$\hat{3} = \hat{4}$ (parallel lines make equal alternate ∠s)
$\hat{4} = \hat{5}$ (intersecting lines make vertically opposite ∠s)
$\therefore \hat{1} = \hat{5}$

4 $\hat{1} = \hat{2}$ (parallel lines make equal alternate ∠s)
$\hat{2} = \hat{3}$ (parallel lines make equal alternate ∠s)
$\hat{3} = \hat{4}$ (parallel lines make equal alternate ∠s)
$\hat{4} = \hat{5}$ (parallel lines make equal alternate ∠s)
$\therefore \hat{1} = \hat{5}$

5 $\hat{1} = \hat{2}$ (parallel lines make equal alternate ∠s)
$\hat{2} = \hat{3}$ (parallel lines make equal alternate ∠s)
$\hat{3} = \hat{4}$ (parallel lines make equal alternate ∠s)
$\hat{4} = \hat{5}$ (opposite ∠s of ||m are equal)
$\therefore \hat{1} = \hat{5}$

Remedial

Provide photocopies of the diagrams in the Learner's Book, or ask learners to replicate the diagrams in their exercise book. Encourage learners to use colour to highlight the alternate angles and a different colour to highlight the co-interior angles.

Investigation Co-interior angles

Learner's Book page 195

Review Copy

Guidelines on how to implement this activity

Learners should each perform the investigation themselves. Ensure learners work neatly and accurately. Learners must measure carefully using a protractor.
Learners should discover the following:

- When a pair of lines are crossed by a transversal, co-interior angles lie on the same side of the transversal and between the pair of lines.
- When parallel lines are crossed by a transversal, any pair of co-interior angles are supplementary.
- When two lines are crossed by a transversal and a pair of co-interior angles are supplementary, the lines are parallel.

Remedial

Inaccurate measurement and construction can cause disappointment and negatively affect this investigation. Learners have to get supplementary measurements for the co-interior angles. Ensure learners use sharp pencils and work neatly in order to get the desired result.

Exercise 3

Learner's Book page 196

Guidelines on how to implement this activity

Discuss what learners have discovered in the investigation.
Learners should know the following angle relationship:

- If parallel lines are cut by a transversal, then co-interior angles are supplementary.
- If lines are cut by a transversal and co-interior angles are supplementary, the lines are parallel.

Work through an example showing how to find the missing angles using co-interior angles. Integrate the knowledge of all three types of angles formed using parallel lines and a transversal to find missing angles. Encourage learners to work on their own, but allow them to work in pairs if necessary.

Suggested answers

1.1 AB || DC (given)
$\therefore \hat{A} + \hat{D} = 180°$ (co-interior ∠s are supplementary)
$\therefore \hat{A} = 180° - 90° = 90°$
$\hat{B} + \hat{C} = 180°$ (co-interior ∠s are supplementary)
$\therefore \hat{C} = 180° - 115° = 65°$ (subtract $\hat{B} = 115°$ on both sides)

1.2 KL || NM (given)
$\therefore \hat{K} + \hat{N} = 180°$ (co-interior ∠s are supplementary)
$\therefore \hat{N} = 180° - 78° = 102°$ (subtract $\hat{K} = 78°$ on both sides)
KN || LM (given)
$\therefore \hat{K} + \hat{L} = 180°$ (co-interior ∠s are supplementary)
$\therefore \hat{L} = 180° - 78° = 102°$ (subtract $\hat{L} = 78°$ on both sides)
KN || LM (given)
$\therefore \hat{N} + \hat{M} = 180°$ (co-interior ∠s are supplementary)
$\therefore \hat{M} = 180° - 102° = 78°$ (subtract $\hat{N} = 102°$ on both sides)
Note: This proves that the opposite angles of a parallelogram are equal.

1.3 PQ || SR (given)
$\therefore \hat{P} + \hat{S} = 180°$ (co-interior ∠s are supplementary)
$\therefore t + 50° + 3t + 10° = 180°$
$\therefore 4t + 60° = 180°$
$\therefore 4t = 180° - 60° = 120°$ (subtract 60° on both sides)
$\therefore t = 120° \div 4 = 30°$ (divide both sides by 4)
$\therefore \hat{P} = 30° + 50° = 80°$
and $\hat{S} = 3(30°) + 10° = 100°$
PS || QR (given)
$\therefore \hat{P} + \hat{Q} = 180°$ (co-interior ∠s are supplementary)
$\therefore \hat{Q} = 180° - 80° = 100°$ (subtract $\hat{P} = 80°$ on both sides)
PS || QR (given)
$\therefore \hat{S} + \hat{R} = 180°$ (co-interior ∠s are supplementary)
$\therefore \hat{R} = 180° - 100° = 80°$ (subtract $\hat{S} = 100°$ on both sides)
Note: This proves that the opposite angles of a parallelogram are equal.

2.1 an acute angle: $\hat{1}$; $\hat{2}$; $\hat{3}$
2.2 a right angle: none
2.3 an obtuse angle: $\hat{4}$; $\hat{5}$
2.4 a straight angle: $\hat{1} + \hat{4}$; $\hat{4} + \hat{2}$; $\hat{2} + \hat{5}$; $\hat{5} + \hat{1}$
2.5 a reflex angle: $\hat{1} + \hat{4} + \hat{2}$; $\hat{4} + \hat{2} + \hat{5}$; $\hat{2} + \hat{5} + \hat{1}$; $\hat{5} + \hat{1} + \hat{4}$
2.6 a revolution: $\hat{1} + \hat{4} + \hat{2} + \hat{5}$
2.7 adjacent angles: $\hat{1}$ and $\hat{4}$; $\hat{4}$ and $\hat{2}$; $\hat{2}$ and $\hat{5}$; $\hat{5}$ and $\hat{1}$
2.8 complementary angles: none
2.9 supplementary angles: $\hat{1} + \hat{4} = 180°$; $\hat{4} + \hat{2} = 180°$; $\hat{2} + \hat{5} = 180°$; $\hat{5} + \hat{1} = 180°$; $\hat{5} + \hat{3} = 180°$
2.10 vertically opposite angles: $\hat{1} = \hat{2}$; $\hat{4} = \hat{5}$

Review Copy

2.11 corresponding angles: $\hat{2} = \hat{3}$
2.12 alternate angles: $\hat{1} = \hat{3}$
2.13 co-interior angles: $\hat{3} + \hat{5} = 180°$
3.1 eight pairs of adjacent angles: $\hat{1}$ and $\hat{2}$; $\hat{2}$ and $\hat{3}$; $\hat{3}$ and $\hat{4}$; $\hat{4}$ and $\hat{1}$; $\hat{5}$ and; $\hat{6}$ and $\hat{7}$; $\hat{7}$ and $\hat{8}$; $\hat{8}$ and $\hat{5}$
3.2 four pairs of corresponding angles: $\hat{1} = \hat{7}$; $\hat{2} = \hat{6}$; $\hat{3} = \hat{5}$; $\hat{4} = \hat{8}$
3.3 two pairs of alternate angles: $\hat{3} = \hat{7}$; $\hat{4} = \hat{6}$
3.4 two pairs of co-interior angles: $\hat{3} + \hat{6} = 180°$; $\hat{4} + \hat{7} = 180°$
3.5 four pairs of vertically opposite angles: $\hat{1} = \hat{3}$; $\hat{2} = \hat{4}$; $\hat{5} = \hat{7}$; $\hat{6} = \hat{8}$
3.6 ten pairs of supplementary angles: $\hat{1} + \hat{2} = 180°$; $\hat{2} + \hat{3} = 180°$; $\hat{3} + \hat{4} = 180°$; $\hat{4} + \hat{1} = 180°$; $\hat{5} + \hat{6} = 180°$; $\hat{6} + \hat{7} = 180°$; $\hat{7} + \hat{8} = 180°$; $\hat{8} + \hat{5} = 180°$; $\hat{3} + \hat{6} = 180°$; $\hat{4} + \hat{7} = 180°$
4.1 $\hat{1} = \hat{2}$ (parallel lines make equal alternate ∠s)
$\hat{2} = \hat{3}$ (parallel lines make equal alternate ∠s)
$\hat{3} = \hat{4}$ (intersecting lines make vertically opposite ∠s)
$\therefore \hat{1} = \hat{4}$
but $\hat{4} + \hat{5} = 180°$ (parallel lines make supplementary co-interior ∠s)
$\therefore \hat{1} + \hat{5} = 180°$ (substitute $\hat{4}$ by $\hat{1}$)
4.2 $\hat{1} = \hat{2}$ (parallel lines make equal alternate ∠s)
$\hat{2} = \hat{3}$ (parallel lines make equal corresponding ∠s)
$\hat{3} = \hat{4}$ (parallel lines make equal corresponding ∠s)
$\therefore \hat{1} = \hat{4}$
but $\hat{4} + \hat{5} = 180°$ (adjacent supplementary ∠s)
$\therefore \hat{1} + \hat{5} = 180°$ (substitute $\hat{4}$ by $\hat{1}$)
4.3 $\hat{1} = \hat{2}$ (parallel lines make equal alternate ∠s)
$\hat{2} = \hat{3}$ (parallel lines make equal corresponding ∠s)
$\therefore \hat{1} = \hat{3}$
$\hat{3} + \hat{4} = 180°$ (parallel lines make supplementary co-interior ∠s)
$\therefore \hat{1} + \hat{4} = 180°$ (substitute $\hat{3}$ by $\hat{4}$)
but $\hat{4} = \hat{5}$ (intersecting lines make vertically opposite ∠s)
$\therefore \hat{1} + \hat{5} = 180°$ (substitute $\hat{4}$ by $\hat{5}$)

Review Copy

Remedial

Learners might find questions 2 – 4 confusing. Work through the first few with learners together as a class to ensure they know how to approach such questions.

Exercise 4

Learner's Book page 199

Guidelines on how to implement this activity

This section brings together drawings that contain triangles and/or quadrilaterals with intersecting and parallel lines.

Learners need to solve geometric problems to find unknown angles using the angle relationships between complementary, adjacent supplementary, vertically opposite, corresponding, alternate and co-interior angles, as well as other known properties of triangles and quadrilaterals. Learners should be encouraged to give reasons and justify their solutions for every written statement. Discuss the four steps of a problem-solving strategy in the Learner's Book. Remember that reversed reasoning helps learners

identify what they should really prove (see example in the Learner's Book). Point out that, from Exercise 4 onwards, lines are shown without arrowheads at their ends in order to make more complex drawings less cluttered and confusing.

Suggested answers

Note that steps and reasons may differ from those provided below.

1.1 $\hat{1} + \hat{2} = 80°$ (intersecting lines make vertically opposite ∠s)
but $\hat{1} = \hat{2}$ (given)
$\therefore \hat{1} = \hat{2} = 80° \div 2 = 40°$
$\hat{3} = \hat{1} = 40°$ (parallel lines make equal corresponding ∠s)
$\hat{4} = \hat{2} = 40°$ (parallel lines make equal alternate ∠s)

1.2 $2 \times \hat{1} + 70° = 180°$ (∠s on a straight line)
$\therefore 2 \times \hat{1} = 110°$ (subtract 70° on both sides)
$\therefore \hat{1} = 110° \div 2 = 55°$ (bisected ∠ given)
$\hat{2} = 55°$ (parallel lines make equal alternate ∠s)
$\hat{3} = 70°$ (parallel lines make equal corresponding ∠s)
$\hat{4} = \hat{3} = 70°$ (intersecting lines make vertically opposite ∠s)

1.3 $\hat{1} = 56°$ (intersecting lines make vertically opposite ∠s)
$\hat{1} + \hat{2} + \hat{3} = 180°$ (parallel lines make supplementary co-interior ∠s)
$\therefore \hat{2} + \hat{3} = 124°$ (subtract $\hat{1} = 56°$ on both sides)
but $\hat{2} = \hat{3}$ (given)
$\therefore \hat{2} = \hat{3} = 124° \div 2 = 62°$
$\hat{4} = \hat{2} = 62°$ (parallel lines make equal alternate ∠s)

2.1 $\hat{1} = 78°$ (parallel lines make equal corresponding ∠s)
$\hat{2} = \hat{1} + 35° = 113°$ (intersecting lines make vertically opposite ∠s)
$\hat{3} + \hat{2} = 180°$ (adjacent supplementary ∠s)
$\therefore \hat{3} = 67°$ (subtract $\hat{2} = 113°$ on both sides)
$\hat{4} = \hat{3} = 67°$ (intersecting lines make vertically opposite ∠s)

2.2 $\hat{1} = 29°$ (parallel lines make equal corresponding ∠s)
$\hat{2} + 90° + \hat{1} = 180°$ (parallel lines make supplementary co-interior ∠s)
$\therefore \hat{2} = 61°$ (subtract 90° and $\hat{1} = 29°$ on both sides)
$\hat{3} = \hat{2} = 61°$ (parallel lines make equal corresponding ∠s)

2.3 $\hat{1} = 38°$ (parallel lines make equal corresponding ∠s)
$\hat{2} + \hat{1} = 90°$ (complementary ∠s)
$\therefore \hat{2} = 52°$ (subtract $\hat{1} = 38°$ on both sides)
$\hat{3} + \hat{1} = 180°$ (parallel lines make supplementary co-interior ∠s)
$\therefore \hat{3} = 142°$ (subtract $\hat{1} = 38°$ on both sides)
$\hat{4} + \hat{2} = 180°$ (parallel lines make supplementary co-interior ∠s)
$\therefore \hat{4} = 128°$ (subtract $\hat{2} = 52°$ on both sides)
$\hat{5} + \hat{4} + \hat{3} = 360°$ (revolution)
$\therefore \hat{5} = 90°$ (subtract $\hat{4} = 128°$ and $\hat{3} = 142°$ on both sides)
Note: The position of $\hat{5}$ in the drawing corresponds with that of the right angle, but they are not corresponding angles because the translation to move from one position to the other was not performed in the direction of one of the sides of the right angle.

3.1 $\hat{1} + 90° + 53° = 180°$ (∠s of a Δ)
$\therefore \hat{1} = 37°$ (subtract 90° and 53° on both sides)
$\hat{2} = \hat{1} = 37°$ (parallel lines make equal alternate ∠s)

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$\hat{3} + 90° + \hat{1} = 180°$ (parallel lines make supplementary co-interior ∠s)
$\therefore \hat{3} = 53°$ (subtract 90° and 37° on both sides)
$\hat{4} = \hat{3} = 53°$ (parallel lines make equal alternate ∠s)

3.2 $\hat{1} = 65°$ (opposite ∠s of ||m are equal)
$\hat{2} = 65°$ (parallel lines make equal alternate ∠s)
$\hat{3} + 2 \times \hat{1} = 180°$ (∠s of an isosceles Δ)
$\therefore \hat{3} = 50°$ (subtract $2 \times 65° = 130°$ on both sides)
$\hat{4} = \hat{3} = 50°$ (parallel lines make equal alternate ∠s)

3.3 $\hat{1} + 126° = 180°$ (parallel lines make supplementary co-interior ∠s)
$\therefore \hat{1} = 54°$ (subtract 126° on both sides)
$2 \times \hat{2} + 126° = 180°$ (∠s of an isosceles Δ)
$\therefore 2 \times \hat{2} = 54°$ (subtract 126° on both sides)
$\therefore \hat{2} = 54° \div 2 = 27°$ (base ∠s of an isosceles Δ are equal)
$\hat{3} = \hat{2} = 27°$ (parallel lines make equal alternate ∠s)
$\hat{4} = 90°$ (parallel lines make equal alternate ∠s)

4.1 $\hat{R}_1 = \hat{S}$ (given)
but $\hat{R}_1 = \hat{T}_2$ (parallel lines make equal alternate ∠s)
and $\hat{S} = \hat{Q}$ (opposite ∠s of a ||m are equal)
$\therefore \hat{T}_2 = \hat{Q}$ (proven)
$\therefore RQ = RT$ (base ∠s of Δ are equal)
$\therefore$ ΔRTQ is an isosceles triangle

4.2 $\hat{Q} = 72°$ (opposite ∠s of a ||m are equal)
$\therefore \hat{T}_2 = 72°$ ($\hat{Q} = \hat{T}_2$; proven)
$\therefore \hat{R}_2 + \hat{T}_2 + \hat{Q} = 180°$ (∠s of ΔRTQ)
$\therefore \hat{R}_2 = 36°$ (subtract $2 \times 72°$ on both sides)

Extension

Challenge faster learners to prove any property of a parallelogram using only the fact that the opposite sides are parallel.

Review Copy

Consolidation

Learner's Book page 201

Before doing this consolidation exercise, encourage learners to review the work covered in this chapter. Advise learners to use the summary and to revise their work. This exercise can be used as an informal assessment task for you to track how learners are coping with the chapter and the concepts covered. The mark allocation provides guidelines on how to assess learners.

Suggested answers

1.1 Complementary angles add up to 90° while supplementary angles add up to 180°. (2)
1.2 Alternate angles lie on opposite sides of the transversal, but co-interior angles lie on the same side of the transversal. (2)
2.1 1 and 2: co-interior angles (1)
2.2 5 and 7: corresponding angles (1)
2.3 3 and 8: alternate angles (1)
2.4 3 and 6: vertically opposite angles (1)
2.5 4 and 8: adjacent supplementary angles (1)
2.6 2, 3, 7 and 6: revolution (1)
3.1 Corresponding angles are equal; therefore AB and CD should be parallel.

3.2 Alternate angles are not equal; therefore KL and MN cannot be parallel.

3.3 Co-interior angles add up to 187°; therefore XV and ZY should, if extended, meet somewhere above PQ, not below as the drawing shows. (3)

4.1 $a + 11° = 131°$ (intersecting lines make vertically opposite ∠s)
$\therefore a = 120°$ (subtract 11° on both sides)
$b + a + 11° = 180°$ (parallel lines make supplementary co-interior ∠s)
$\therefore b = 49°$ (subtract 131° on both sides)
$c = b = 49°$ (parallel lines make equal alternate ∠s)
$2d - 11° = c = 49°$ (intersecting lines make vertically opposite ∠s)
$\therefore 2d = 60°$ (add 11° on both sides)
$\therefore d = 30°$ (divide both sides by 2)

4.2 $e + 79° = 180°$ (adjacent supplementary ∠s)
$\therefore e = 101°$ (subtract 79° on both sides)
$f = 39°$ (parallel lines make equal corresponding ∠s)
$e + f + g = 180°$ (∠s on a straight line)
$\therefore 101° + 39° + g = 180°$ (proven)
$\therefore g = 40°$ (subtract 140° on both sides)
$h = g = 40°$ (parallel lines make equal alternate ∠s)
$k = e = 101°$ (intersecting lines make vertically opposite ∠s)

4.3 $m = 105°$ (parallel lines make equal corresponding ∠s)
$n + 92° = 180°$ (parallel lines make supplementary co-interior ∠s)
$\therefore n = 88°$ (subtract 92° on both sides)
$p = n = 88°$ (intersecting lines make vertically opposite ∠s) (3)

5.1 In ΔCAB:
$\hat{A}_1 + 36° = 112°$ (exterior ∠ = sum of two opposite interior ∠s)
$\therefore \hat{A}_1 = 76°$ (subtract 36° on both sides)
$\hat{E}_2 = \hat{A}_1 = 76°$ (parallel lines AB and ED make equal corresponding ∠s)
In ΔCDE:
$36° + \hat{D}_3 + 76° = 180°$ (∠s of Δ)
$\therefore \hat{D}_3 = 68°$ (subtract 36° and 76° on both sides)
$\hat{C}_4 = \hat{E}_2 = 76°$ (parallel lines ED and FC make equal alternate ∠s) (4)

5.2 In ΔCAB and Δ CED:
(1) $A\hat{C}B = E\hat{C}D$ (common ∠ of both Δs)
(2) $\hat{A}_1 = \hat{E}_2$ (proven above)
(3) $C\hat{B}A = C\hat{D}E$ (∠s of both Δs = 180°)
∴ ΔCAB ||| Δ CED (A;A;A) (3)

6.1 $\hat{R}_1 + 130° = 180°$ (parallel lines PT and SU make supplementary co-interior ∠s)
$\therefore \hat{R}_1 = 50°$ (subtract 130° on both sides)
$\hat{R}_2 + 65° + \hat{R}_1 = 180°$ (∠s on a straight line)
$\therefore \hat{R}_2 + 65° + 50° = 180°$ ($\hat{R}_1 = 50°$; proven)
$\therefore \hat{R}_2 = 65°$ (subtract 65° and 50° on both sides)
$\hat{P}_3 = \hat{R}_2 = 65°$ (parallel lines PQ and SR make equal alternate ∠s)
$\hat{P}_4 = 65°$ (parallel lines PS and QR make equal alternate ∠s)
$\hat{S}_5 = \hat{R}_1 = 50°$ (parallel lines PS and QR make equal corresponding ∠s) (4)

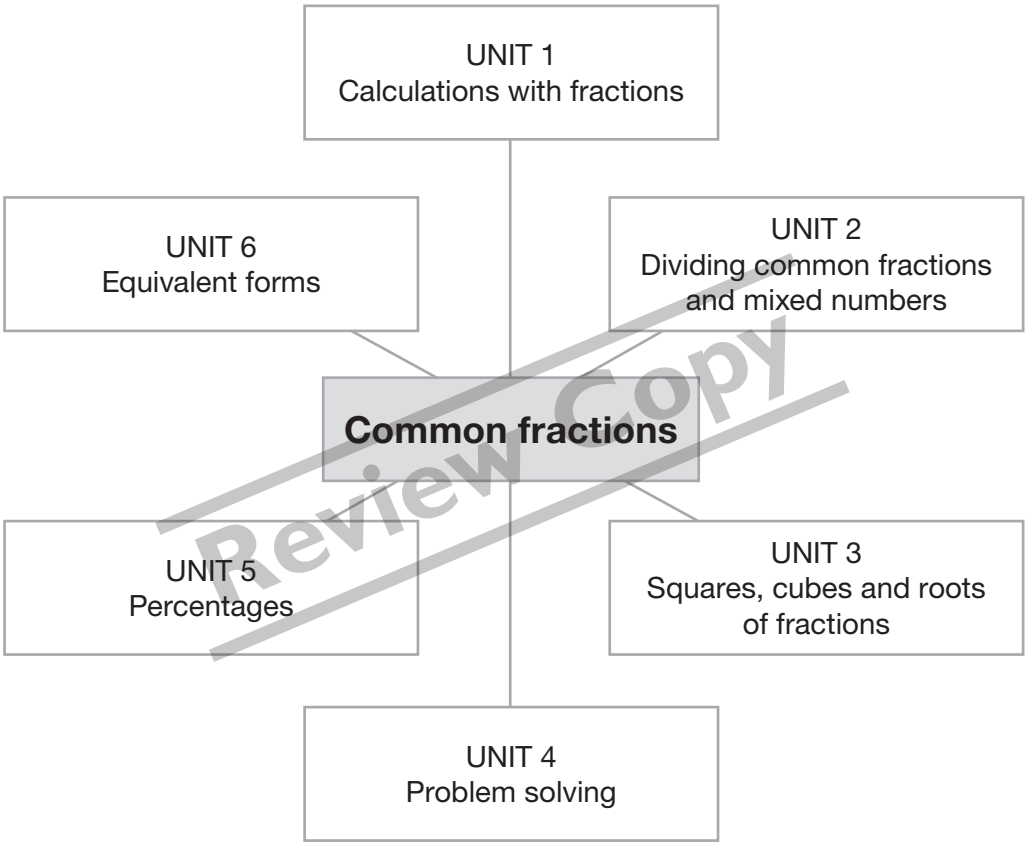
6.2 PQRS is a ||m (two pairs of opposite sides are parallel)
$Q\hat{P}R = Q\hat{R}P = 65°$ (proven above)
∴ QR = QP (base ∠s are equal)
∴ PQRS is a ||m with equal adjacent sides
∴ PQRS is a rhombus (3)

Total marks [30]

Review Copy

Chapter 10 Common fractions

Overview of concepts



Review Copy

Content		Time allocations	LB page
Unit 1	Calculations with fractions	1 hour	203
Unit 2	Dividing common fractions and mixed numbers	2 hours	208
Unit 3	Squares, cubes and roots of fractions	1 hour	211
Unit 4	Problem solving	1 hour	213
Unit 5	Percentages	1 hour	216
Unit 6	Equivalent forms	1 hour	219

Background information on common fractions

The following background information is crucial for learners to know and revise. Common fractions include proper fractions and improper fractions. A mixed number is an improper fraction written as a whole number and a proper fraction. Proper fractions have a numerator smaller than the denominator. Examples are:

Proper fractions: $\frac{2}{3}$ and $\frac{7}{8}$ and $\frac{69}{80}$

Improper fractions: $\frac{3}{2}$ and $\frac{9}{5}$ and $\frac{99}{58}$

Mixed numbers: $2\frac{1}{3}$ and $1\frac{7}{8}$ and $9\frac{1}{10}$

Proper fractions can be brought to their simplest form, for example, $\frac{6}{10} \div \frac{2}{2} = \frac{2}{5}$

$\frac{6}{10}$ and $\frac{3}{5}$ are equivalent fractions.

Table 1 Fraction wall showing fiths and tenths

$\frac{1}{5}$		$\frac{2}{5}$		$\frac{3}{5}$		$\frac{4}{5}$		$\frac{5}{5}$	
$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{4}{10}$	$\frac{5}{10}$	$\frac{6}{10}$	$\frac{7}{10}$	$\frac{8}{10}$	$\frac{9}{10}$	$\frac{10}{10}$

Equivalent fractions can be brought to their simplest form by dividing or by multiplying, as shown above.

Review Copy

Generic teaching guidelines for teaching common fractions

A fraction is a part of a whole. For example, we often talk about eating $\frac{1}{2}$ of an apple or taking a $\frac{1}{4}$ spoon of medicine. Show examples of fractions to the learners, for example, cut a sheet of paper in half. Ask them to think of times they use fractions in their lives.

Show learners that decimals are also fractions. For example, $0{,}1 = \frac{1}{10}$ and $0{,}4 = \frac{4}{10} = \frac{2}{5}$

We use decimals every day when talking about money (R1,25), distance (kilometres), mass (kilograms) etc.

Show learners that percentages are also fractions. A percentage is an amount out of 100. For example a learner might get $\frac{79}{100}$ for a test. That is 79%.

Learners must work with equivalent fractions as often as possible, so that they become familiar with the processes.

Resources

Fraction number cards; cardboard and colour pens to draw fractions and create fraction cards; fraction number lines and fraction walls. Each learner should also have a calculator and scrap paper to draw fractions to help them with their working out initially.

Unit 1 Calculations with fractions

Learner's Book page 203

Unit focus:

- revision of converting mixed numbers to improper fractions
- revision of equivalent fractions
- adding and subtracting common fractions, including mixed numbers
- finding fractions of whole numbers
- multiplying common fractions, including mixed numbers.

Background information on calculations with fractions

Adding or subtracting with fractions is simplified when equivalent fractions are found. For example, if you want to add $\frac{2}{3}$ to $\frac{1}{5}$ first find the equivalent fractions that have the same denominator. For example, $\frac{2}{3} \times \frac{5}{5} = \frac{10}{15}$ and $\frac{1}{5} \times \frac{3}{3} = \frac{3}{15}$. Now add the equivalent fractions. So: $\frac{10}{15} + \frac{3}{15} = \frac{13}{15}$. This means $\frac{2}{3} + \frac{1}{5} = \frac{13}{15}$.

The same is true for subtraction. For example, if you want to subtract $\frac{1}{3}$ from $\frac{7}{8}$ first find the equivalent fractions that have the same denominator. For example, $\frac{1}{3} \times \frac{8}{8} = \frac{8}{24}$ and $\frac{7}{8} \times \frac{3}{3} = \frac{21}{24}$. Now subtract the equivalent fractions. So: $\frac{21}{24} - \frac{8}{24} = \frac{13}{24}$. This means $\frac{7}{8} - \frac{1}{3} = \frac{13}{24}$.

Review Copy

Mixed numbers cannot be added, subtracted or multiplied. They must first be rewritten as improper fractions, for example:

$4\frac{1}{2} + 1\frac{1}{3} = \frac{9}{2} + \frac{4}{3} = \frac{27}{6} + \frac{8}{6} = \frac{35}{6} = 5\frac{5}{6}$

$4\frac{1}{2} - 1\frac{1}{3} = \frac{9}{2} - \frac{4}{3} = \frac{27}{6} - \frac{8}{6} = \frac{19}{6} = 3\frac{1}{6}$

$4\frac{1}{2} \times 1\frac{1}{3} = \frac{9}{2} \times \frac{4}{3} = \frac{36}{6} = \frac{6}{1} = 6$

To multiply fractions, cancel where possible. For example $\frac{3}{5} \times \frac{10}{21}$. Remember cancelling always happens from numerator to denominator. So $3 \div 3 = 1$ and $21 \div 3 = 7$. And $5 \div 5 = 1$ and $10 \div 5 = 2$.

$\frac{{}^{1}\cancel{3}}{\cancel{5}_{1}} \times \frac{{}^{2}\cancel{10}}{\cancel{21}_{7}} = \frac{2}{7}$

Once you have cancelled, multiply numerator by numerator and denominator by denominator.

CHAPTER 10

Exercise 1

Learner's Book page 205

Guidelines on how to implement this activity

Revise basic fractions. Ensure learners know that the top number in a fraction is called the numerator, and the bottom number is the denominator. Revise how to convert mixed numbers to improper fractions, and vice versa. Do a few examples together as a class on the board. Discuss equivalent fractions and how to find equivalent fractions. Do a few examples together. Next show learners how to use equivalent fractions to solve for unknown numbers. Learners should complete this exercise on their own.

Suggested answers

1.1 $4\frac{2}{5} = \frac{(4 \times 5) + 2}{5} = \frac{22}{5}$

1.2 $1\frac{4}{9} = \frac{(1 \times 9) + 4}{9} = \frac{13}{9}$

1.3 $6\frac{2}{3} = \frac{(6 \times 3) + 2}{3} = \frac{20}{3}$

1.4 $9\frac{1}{3} = \frac{(9 \times 3) + 1}{3} = \frac{28}{3}$

1.5 $17\frac{7}{8} = \frac{(17 \times 8) + 7}{8} = \frac{143}{8}$

1.6 $14\frac{12}{15} = \frac{(14 \times 15) + 12}{15} = \frac{222}{15}$

1.7 $21\frac{5}{8} = \frac{(21 \times 8) + 5}{8} = \frac{173}{8}$

1.8 $101\frac{4}{5} = \frac{(101 \times 5) + 4}{5} = \frac{509}{5}$

1.9 $712\frac{13}{18} = \frac{(712 \times 18) + 13}{18} = \frac{12\,829}{18}$

2.1 $\frac{32}{5} = 32 \div 5 = 6$ remainder $2 = 6\frac{2}{5}$

2.2 $\frac{121}{7} = 121 \div 7 = 17$ remainder $2 = 17\frac{2}{7}$

2.3 $\frac{90}{8} = 90 \div 8 = 11$ remainder $2 = 11\frac{2}{8}$

2.4 $\frac{43}{8} = 43 \div 8 = 5$ remainder $3 = 5\frac{3}{8}$

2.5 $\frac{777}{45} = 777 \div 45 = 17$ remainder $12 = 17\frac{12}{45}$

2.6 $\frac{212}{15} = 212 \div 15 = 14$ remainder $2 = 14\frac{2}{15}$

2.7 $\frac{540}{65} = 540 \div 65 = 8$ remainder $20 = 8\frac{20}{65}$

2.8 $\frac{76}{3} = 76 \div 3 = 25$ remainder $1 = 25\frac{1}{3}$

2.9 $\frac{321}{18} = 321 \div 18 = 17$ remainder $15 = 17\frac{15}{18}$

3.1 $\frac{5}{6} = \frac{10}{12} = \frac{15}{18} = \frac{20}{24} = \frac{25}{30} = \frac{30}{36}$

3.2 $\frac{4}{7} = \frac{8}{14} = \frac{12}{21} = \frac{16}{28} = \frac{20}{35} = \frac{24}{42}$

3.3 $\frac{9}{10} = \frac{18}{20} = \frac{27}{30} = \frac{36}{40} = \frac{45}{50} = \frac{54}{60}$

3.4 $\frac{6}{9} = \frac{12}{18} = \frac{18}{27} = \frac{24}{36} = \frac{30}{45} = \frac{36}{54}$

3.5 $\frac{4}{13} = \frac{8}{26} = \frac{12}{39} = \frac{16}{52} = \frac{20}{65} = \frac{24}{78}$

3.6 $\frac{11}{14} = \frac{22}{28} = \frac{33}{42} = \frac{44}{56} = \frac{55}{70} = \frac{66}{84}$

4.1 $\frac{4}{5} = \frac{4}{5} \times \frac{4}{4} = \frac{16}{20}$ $a = 20$

4.2 $\frac{9}{15} = \frac{9}{15} \times \frac{3}{3} = \frac{27}{45}$ $x = 27$

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4.3 $\frac{16}{19} = \frac{16}{19} \times \frac{4}{4} = \frac{64}{76}$ $y = 76$

4.4 $\frac{34}{40} = \frac{34}{40} \div \frac{2}{2} = \frac{17}{20}$ $u = 17$

4.5 $\frac{56}{64} = \frac{56}{64} \div \frac{8}{8} = \frac{7}{8}$ $b = 8$

4.6 $\frac{22}{66} = \frac{22}{66} \div \frac{11}{11} = \frac{2}{6}$ $k = 2$

Remedial

Provide Grade 6 and 7 work for learners to revise basic fractions, if they had any difficulty with this exercise.

Extension

Ask learners to make a wall chart demonstrating calculations with fractions. For example draw a picture of a cake and cut it into 2 halves. Then cut one of the halves into two equal parts (quarters). Stick the half next to one quarter to show $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$. Continue with other increasingly challenging ideas.

Exercise 2

Learner's Book page 207

Review Copy

Guidelines on how to implement this activity

Explain to learners that when adding or subtracting fractions with the same denominator, for example $\frac{1}{8} + \frac{5}{8}$ you keep the same denominator and add (or subtract) the numerators.

So: $\frac{1}{8} + \frac{5}{8} = \frac{6}{8}$. By dividing through by $\frac{2}{2}$ this can be simplified to $\frac{3}{4}$.

When adding or subtracting fractions with denominators that are multiples of each other you find the *lowest common denominator* like this:

$\frac{1}{4} + \frac{5}{8} = \frac{1}{4} \times \frac{2}{2} + \frac{5}{8} = \frac{2}{8} + \frac{5}{8} = \frac{7}{8}$ so $\frac{1}{4} + \frac{5}{8} = \frac{7}{8}$ The LCM is 8 in this case.

When adding or subtracting fractions that have denominators that are not the same and are not multiples of each other, such as $\frac{1}{3} + \frac{1}{5}$, list the multiples of 3 and of 5 to assist in finding the common denominator. So:

3 = 3; 6; 9; 12; 15; 18…. 5 = 5; 10; 15; 20…..

The lowest common multiple (LCM) is 15, so 15 must be the common denominator.

$\frac{1}{3} \times \frac{5}{5} = \frac{5}{15}$ and $\frac{1}{5} \times \frac{3}{3} = \frac{3}{15}$ $\frac{5}{15} + \frac{3}{15} = \frac{8}{15}$ therefore $\frac{1}{3} + \frac{1}{5} = \frac{8}{15}$

Work through the examples in the Learner's Book for adding fractions. Discuss the denominator and how to find the LCD. Remind learners that when we add fractions, we only add the numerators and not the denominators.

To revise multiplying fractions, start with examples of finding fractions of whole numbers. Work through some examples as a class.

- Show learners that we can write the denominator of a whole number as 1, and then multiply.
- Show learners, by means of worked examples how to multiply fractions and mixed numbers.
- Remind learners they have to convert mixed numbers before multiplication and

that we multiply both the numerator and denominator. Advise learners to supply their answers in simplest form.

Suggested answers

1.1 $1\frac{1}{8} + \frac{3}{4} = 1\left[\frac{1}{8} + \frac{3}{4} \times \frac{2}{2}\right] = 1\left[\frac{1}{8} + \frac{6}{8}\right] = 1\frac{7}{8}$

1.2 $\frac{1}{3} + 2\frac{4}{6} = 2\left[\frac{1}{3} + \frac{2}{3}\right] = 2\frac{3}{6} = 2\frac{1}{2}$

1.3 $3\frac{2}{5} + 1\frac{3}{10} = 4\left[\frac{4}{10} + \frac{3}{10}\right] = 4\frac{7}{10}$

1.4 $3\frac{3}{7} + 1\frac{5}{14} = 4\left[\frac{6}{14} + \frac{5}{14}\right] = 4\frac{11}{14}$

1.5 $\frac{3}{5} + \frac{9}{10} + \frac{3}{4} = \frac{3}{5} \times \frac{4}{4} + \frac{9}{10} \times \frac{2}{2} + \frac{3}{4} \times \frac{5}{5} = \frac{12}{20} + \frac{18}{20} + \frac{15}{20} = \frac{45}{20} = 2\frac{5}{20} = 2\frac{1}{4}$

1.6 $1\frac{1}{2} + 3\frac{5}{6} + 4\frac{1}{3} = 8\left[\frac{1}{2} \times \frac{3}{3} + \frac{5}{6} + \frac{1}{3} \times \frac{2}{2}\right] = 8\left[\frac{3}{6} + \frac{5}{6} + \frac{2}{6}\right] = 8\frac{10}{6} = 9\frac{4}{6} = 9\frac{2}{3}$

1.7 $7\frac{1}{6} + 2\frac{3}{4} + 3\frac{1}{8} = 12\left[\frac{1}{6} \times \frac{4}{4} + \frac{3}{4} \times \frac{6}{6} + \frac{1}{8} \times \frac{3}{3}\right] = 12\left[\frac{4}{24} + \frac{18}{24} + \frac{3}{24}\right] = 12\left[\frac{25}{24}\right] = 13\frac{1}{24}$

1.8 $4\frac{4}{7} + 7\frac{1}{14} + 5\frac{1}{21} = 16\left[\frac{4}{7} \times \frac{6}{6} + \frac{1}{14} \times \frac{3}{3} + \frac{1}{21} \times \frac{2}{2}\right] = 16\left[\frac{24}{42} + \frac{3}{42} + \frac{2}{42}\right] = 16\left[\frac{29}{42}\right]$

2.1 $\frac{3}{4} - \frac{1}{2} = \frac{3}{4} - \frac{1}{2} \times \frac{2}{2} = \frac{3}{4} - \frac{2}{4} = \frac{1}{4}$

2.2 $\frac{7}{8} - \frac{1}{4} = \frac{7}{8} - \frac{1}{4} \times \frac{2}{2} = \frac{7}{8} - \frac{2}{8} = \frac{5}{8}$

2.3 $\frac{7}{10} - \frac{1}{5} = \frac{7}{10} - \frac{1}{5} \times \frac{2}{2} = \frac{7}{10} - \frac{2}{10} = \frac{5}{10} = \frac{1}{2}$

2.4 $2\frac{2}{7} - 1\frac{1}{2} = \left[\frac{16}{7} \times \frac{2}{2} - \frac{3}{2} \times \frac{7}{7}\right] = \left[\frac{32}{14} - \frac{21}{14}\right] = \frac{11}{14}$

2.5 $3\frac{5}{6} - 2\frac{1}{3} = \frac{23}{6} - \frac{7}{3} = \frac{23}{6} - \frac{7}{3} \times \frac{2}{2} = \frac{23}{6} - \frac{14}{6} = \frac{9}{6} = 1\frac{3}{6} = 1\frac{1}{2}$

2.6 $\frac{24}{25} - \frac{7}{10} = \frac{24}{25} \times \frac{2}{2} - \frac{7}{10} \times \frac{5}{5} = \frac{48}{50} - \frac{35}{50} = \frac{13}{50}$

2.7 $5\frac{3}{4} - 2\frac{3}{5} = \frac{23}{4} - \frac{13}{5} = \frac{23}{5} \times \frac{5}{5} - \frac{13}{5} \times \frac{4}{4} = \frac{115}{20} - \frac{52}{20} = \frac{63}{20} = 3\frac{3}{20}$

2.8 $7\frac{1}{3} - 2\frac{5}{6} = \frac{22}{3} - \frac{17}{6} = \frac{22}{3} \times \frac{2}{2} - \frac{17}{6} = \frac{44}{6} - \frac{17}{6} = \frac{27}{6} = 4\frac{3}{6} = 4\frac{1}{2}$

3.1 $\frac{4}{7}$ of $65 = \frac{4 \times 65}{7} = \frac{260}{7} = 37\frac{1}{7}$

3.2 $\frac{7}{8}$ of $43 = \frac{7 \times 43}{8} = \frac{301}{8} = 37\frac{5}{8}$

3.3 $\frac{7}{8} \times \frac{2}{5} = \frac{14}{40} = \frac{7}{20}$

3.4 $\frac{3}{4} \times \frac{5}{6} = \frac{15}{24}$

3.5 $\frac{7}{8} \times \frac{24}{25} = \frac{168}{200} = \frac{21}{25}$

3.6 $2\frac{1}{2} \times 3\frac{3}{4} = \frac{5}{2} \times \frac{15}{4} = \frac{75}{8} = 9\frac{3}{8}$

3.7 $5\frac{2}{3} \times 1\frac{4}{7} = \frac{17}{3} \times \frac{11}{7} = \frac{187}{21} = 8\frac{19}{21}$

3.8 $3\frac{3}{11} \times 2\frac{1}{7} = \frac{36}{11} \times \frac{15}{7} = \frac{540}{77} = 7\frac{1}{77}$

Review Copy

Remedial

Encourage learners to add and subtract fractions with the same denominator; for example, $\frac{1}{3} + \frac{1}{3}$ and $\frac{1}{5} + \frac{2}{5}$ and $\frac{9}{10} - \frac{2}{10}$. Remind them that the idea is the same as any simple addition or subtraction. In words: One third plus one third equals two thirds, just as one cake plus one cake equals two cakes.

Once they are very confident with this concept give an example with different denominators, but where one is a multiple of the other, for example $\frac{3}{5} + \frac{1}{10}$. In this example you cannot simply add the numerators and keep the same denominator. It would be like adding beans and books. To get the same denominator, list the multiples of each denominator:
5; 10; 15; … and 10; 20; … This shows 10 is the LCM, so the lowest common denominator is 10.

$\frac{3}{5} \times \frac{2}{2} = \frac{6}{10}$ so $\frac{6}{10} + \frac{1}{10} = \frac{7}{10}$

Extension

Tell learners to experiment with increasingly challenging calculations. Start with something simple such as $\frac{5}{8} + \frac{3}{5}$. Calculate the sum. Then extend it by another term: $(\frac{5}{8} + \frac{3}{5}) - \frac{9}{10}$. Calculate at each step before adding another calculation, for example: $(\frac{5}{8} + \frac{3}{5} - \frac{9}{10}) \times \frac{3}{4} = (\frac{25}{40} + \frac{24}{40} - \frac{36}{40}) \times \frac{3}{4} = (\frac{13}{40})$ * $\frac{3}{4}$ and so on.

Review Copy

Unit 2 Dividing common fractions and mixed numbers

Learner's Book page 208

Unit focus:

- learning about dividing whole numbers by common fractions
- revising reciprocals
- learning how reciprocals are used to divide
- dividing common fractions by each other.

Background information on dividing common fractions and mixed numbers

- Dividing a whole number by a common fraction:
  You are finding out how many times the fraction fits into the number. For example 1 ÷ ¼. You know that there are 4 quarters in 1 whole. You also know that subtraction and division are related. How many times can you subtract ¼ from 1? $1 = \frac{4}{4}$ and $\frac{4}{4} - \frac{1}{4} = \frac{3}{4}$ and $\frac{3}{4} - \frac{1}{4} = \frac{2}{4}$ and $\frac{2}{4} - \frac{1}{4} = \frac{1}{4}$ and $\frac{1}{4} - \frac{1}{4} = 0$. You have subtracted $\frac{1}{4}$ four times from 1 so $1 \div \frac{1}{4} = 4$. This is a complicated way to divide. Instead we apply the multiplicative inverse. That means $2 \div \frac{1}{4} = \frac{2}{1} \div \frac{1}{4} = \frac{2}{1} \times \frac{4}{1} = 8$.
- Dividing a common fraction by a whole number:
  If you have a sheet of paper and you draw lines on it to divide it into 4 equal parts, each part will equal $\frac{1}{4}$ of the whole. What will you get if you divide one quarter by 2?

Draw lines on the paper to divide each $\frac{1}{4}$ into 2 equal parts. How many parts will you have altogether? Eight. $\frac{1}{4} \div 2 = \frac{1}{4} \div \frac{2}{1} = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$.

- Dividing a common fraction by a common fraction:
  In order to divide fractions by fractions we multiply by the reciprocal, or multiplicative inverse. For example:
  $\frac{2}{5} \div \frac{1}{3} = \frac{2}{5} \times \frac{3}{1} = \frac{6}{5} = 1\frac{1}{5}$. This means $\frac{1}{3}$ fits $1\frac{1}{5}$ times into $\frac{2}{5}$.

Revision

Learner's Book page 208

Guidelines on how to implement this activity

Use this exercise to assess learners' ability with multiplying and finding equivalent fractions.

Suggested answers

1.1 $\frac{1}{3} \times \frac{4}{5} = \frac{4}{15}$

1.2 $\frac{5}{6} \times \frac{1}{10} = \frac{5}{60} = \frac{1}{12}$

1.3 $\frac{2}{5} \times \frac{5}{8} = \frac{10}{40} = \frac{1}{4}$

1.4 $\frac{3}{8} \times \frac{4}{9} = \frac{12}{72} = \frac{1}{6}$

2.1 $\frac{2}{3} \times \frac{5}{5} = \frac{2}{3} \times \frac{100}{100} = \frac{2}{3}$

2.2 All the fractions multiplied by $\frac{2}{3}$ equal 1.

3.1 $\frac{7}{8} = \frac{x}{16} = \frac{21}{y} = \frac{2}{800} = \frac{z}{8} = \frac{14}{16} = \frac{21}{24} = \frac{700}{800}$

3.2 $\frac{99}{100} = \frac{a}{1\,000} = \frac{9{,}9}{b} = \frac{198}{c} = \frac{990}{1\,000} = \frac{9{,}9}{10} = \frac{198}{200}$

3.3 $\frac{1}{2} = \frac{x}{44} = \frac{17}{y} = \frac{9}{z} = \frac{1}{2} = \frac{22}{44} = \frac{17}{34} = \frac{9}{18}$

Review Copy

Remedial

Refer learners back to the material covered in Unit 1. Have learners redo the exercises in this unit. It is necessary that learners can do the work in this exercise before progressing.

Exercise 1

Learner's Book page 209

Guidelines on how to implement this activity

Show the learners a sheet of paper. Cut it into 2 halves. Ask volunteers to do a sketch on the board to show the rectangle divided in 2. Each piece of paper is a half. If a half is divided by a half, the answer is one: $\frac{1}{2} \div \frac{1}{2} = \frac{1}{2} \times \frac{2}{1} = 1$. A half goes into a half once.

Remind learners: $1 \div 1 = 1$ and $3 \div 3 = 1$ and $25 \div 25 = 1$.

Reciprocals. Show learners by repeated examples on the board, that dividing by a half is the same as multiplying by 2, and that dividing by $\frac{1}{3}$ is the same multiplying by 3.

Make sure that learners can see that dividing is the same as multiplying by the reciprocal. If necessary spend some time revising the reciprocal.

This exercise is limited to a whole number divided by a fraction. Learners should do this exercise on their own.

Suggested answers

1 $18 \div \frac{4}{5} = \frac{18}{1} \times \frac{5}{4} = \frac{90}{4} = 22\frac{1}{2}$

2 $5 \div \frac{3}{7} = \frac{5}{1} \times \frac{7}{3} = \frac{35}{3} = 11\frac{2}{3}$

3 $14 \div \frac{5}{6} = \frac{14}{1} \times \frac{6}{5} = \frac{84}{5} = 16\frac{4}{5}$

4 $9 \div \frac{2}{3} = \frac{9}{1} \times \frac{3}{2} = \frac{27}{2} = 13\frac{1}{2}$

5 $3 \div \frac{14}{15} = \frac{3}{1} \times \frac{15}{14} = \frac{45}{14} = 3\frac{3}{14}$

6 $8 \div \frac{9}{10} = \frac{8}{1} \times \frac{10}{9} = \frac{80}{9} = 8\frac{8}{9}$

7 $34 \div \frac{8}{17} = \frac{34}{1} \times \frac{17}{8} = \frac{578}{8} = 72\frac{1}{4}$

8 $108 \div \frac{9}{11} = \frac{108}{1} \times \frac{11}{9} = \frac{1188}{9} = 132$

9 $6 \div \frac{4}{9} = \frac{6}{1} \times \frac{9}{4} = \frac{54}{4} = 13\frac{1}{2}$

10 $10 \div \frac{9}{13} = \frac{10}{1} \times \frac{13}{9} = \frac{130}{9} = 14\frac{4}{9}$

11 $90 \div \frac{4}{25} = \frac{90}{1} \times \frac{25}{4} = \frac{2\,250}{4} = 562\frac{1}{2}$

12 $2 \div \frac{1}{14} = \frac{2}{1} \times \frac{14}{1} = \frac{28}{1} = 28$

Remedial

Spend additional time revising reciprocals and showing learners how dividing by a half, third and a quarter are the same as multiplying by two, three and four respectively. If necessary show learners that dividing by 2, 3, and 4 are the same as multiplying by $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$.

Review Copy

Extension

Remind learners of what they have learnt before. Tell learners to experiment with increasingly challenging calculations. Start with something such as $\frac{5}{8} + \frac{3}{5}$. Calculate the sum. Then divide the sum by a fraction, for example: $(\frac{5}{8} + \frac{3}{5}) \div \frac{2}{7}$. Calculate at each step before adding another calculation. In these calculations they must include division by fractions.

Exercise 2

Learner's Book page 210

Guidelines on how to implement this activity

Extend learners' knowledge and experience from the previous exercise to include the division of fractions by fractions. Show learners that when you divide one fraction by another fraction you change the operation from division by the fraction to multiplication by the reciprocal. Do a few examples on the board until learners are confident with dividing fractions.

Introduce mixed numbers. Explain to learners that just as when we multiply mixed numbers, when we divide mixed numbers, we have to convert all mixed numbers to improper fractions. Do a few examples of dividing with mixed numbers together as a class.

Revise BODMAS and what the acronym means. Do a few examples of mixed operations with fractions using BODMAS.
Encourage learners to complete this exercise on their own, but allow learners to discuss their answers in groups.

Suggested answers

1.1 $\frac{1}{5} \div 2 = \frac{1}{5} \times \frac{1}{2} = \frac{1}{10}$

1.2 $\frac{9}{10} \div 3 = \frac{9}{10} \times \frac{1}{3} = \frac{9}{30}$

1.3 $\frac{15}{16} \div 5 = \frac{15}{16} \times \frac{1}{5} = \frac{15}{80} = \frac{3}{16}$

1.4 $6 \div \frac{1}{2} = 6 \times \frac{2}{1} = \frac{12}{1} = 12$

1.5 $3 \div \frac{1}{3} = 3 \times \frac{3}{1} = \frac{9}{1} = 9$

1.6 $2 \div \frac{1}{5} = 2 \times \frac{5}{1} = \frac{10}{1} = 10$

1.7 $\frac{1}{4} \div \frac{1}{5} = \frac{1}{4} \times \frac{5}{1} = \frac{5}{4} = 1\frac{1}{4}$

1.8 $\frac{7}{8} \div \frac{1}{4} = \frac{7}{8} \times \frac{4}{1} = \frac{28}{8} = 3\frac{1}{2}$

1.9 $\frac{5}{6} \div \frac{1}{3} = \frac{5}{6} \times \frac{3}{1} = \frac{15}{6} = 2\frac{3}{6} = 2\frac{1}{2}$

2.1 $1\frac{1}{2} \div 3\frac{1}{3} = \frac{3}{2} \times \frac{3}{10} = \frac{7}{20}$

2.2 $2\frac{3}{5} \div 1\frac{3}{10} = \frac{13}{5} \times \frac{10}{13} = \frac{130}{65} = 2$

2.3 $4\frac{3}{8} \div 2\frac{1}{4} = \frac{35}{8} \times \frac{4}{9} = \frac{140}{72} = 1\frac{68}{72} = 1\frac{17}{18}$

2.4 $7\frac{4}{5} \div 3\frac{5}{6} = \frac{39}{5} \times \frac{6}{23} = \frac{234}{115} = 2\frac{4}{115}$

2.5 $3\frac{4}{7} \div 2\frac{1}{8} = \frac{25}{7} \times \frac{8}{17} = \frac{200}{119} = 1\frac{81}{119}$

2.6 $9\frac{1}{2} \div 2\frac{2}{11} = \frac{19}{2} \times \frac{11}{24} = \frac{209}{48} = 4\frac{17}{48}$

3.1 $3\frac{4}{7} \times (6\frac{1}{2} + 9\frac{3}{5}) = \frac{25}{7} \times [15(\frac{1}{2} \times \frac{5}{5} + \frac{3}{5} \times \frac{2}{2})] = \frac{25}{7} \times [15(\frac{5}{10} + \frac{6}{10})] = \frac{25}{7} \times 16\frac{1}{10}$
$= \frac{25}{7} \times \frac{161}{10} = \frac{4\,025}{70} = 57\frac{1}{2}$

3.2 $5\frac{2}{5} \times (1\frac{1}{9} + 1\frac{2}{3}) = \frac{27}{5} \times [2(\frac{1}{9} + \frac{2}{3} \times \frac{3}{3})] = \frac{27}{5} \times [2(\frac{1}{9} + \frac{6}{9})] = \frac{27}{5} \times 2(\frac{7}{9}) = \frac{27}{5} \times \frac{25}{9}$
$= \frac{675}{45} = 15$

3.3 $11\frac{1}{4} \times (2\frac{2}{9} + 1\frac{1}{5}) = \frac{45}{4} \times [3(\frac{2}{9} \times \frac{5}{5} + \frac{1}{5} \times \frac{9}{9})] = \frac{45}{4} \times [3(\frac{10}{45} + \frac{9}{45})] = \frac{45}{4} \times 3\frac{19}{45}$
$= \frac{45}{4} \times \frac{154}{45} = \frac{6\,930}{180} = 38\frac{1}{2}$

3.4 $(4\frac{1}{5} + 3\frac{3}{10}) \div 2\frac{1}{2} = [7(\frac{1}{5} \times \frac{2}{2} + \frac{3}{10})] \div 2\frac{1}{2} = [7(\frac{2}{10} + \frac{3}{10})] \div 2\frac{1}{2} = \frac{15}{2} \times \frac{2}{5} = \frac{30}{10} = 3$

3.5 $(3\frac{1}{2} + 3\frac{1}{2}) \div 1\frac{3}{4} = 7 \div 1\frac{3}{4} = \frac{7}{1} \times \frac{4}{7} = \frac{28}{7} = 4$

3.6 $(6\frac{3}{4} + 2\frac{2}{5}) \times 3\frac{1}{3} = [8(\frac{3}{4} \times \frac{5}{5} + \frac{2}{5} \times \frac{4}{4})] \times 3\frac{1}{3} = [8(\frac{15}{20} + \frac{8}{20})] \times 3\frac{1}{3} = 9\frac{3}{20} \times 3\frac{1}{3}$
$= \frac{183}{20} \times \frac{10}{3} = \frac{1\,830}{60} = 30\frac{1}{2}$

3.7 $2\frac{6}{7} \times (2\frac{4}{5} - 1\frac{2}{3}) = 2\frac{6}{7} \times [1(\frac{4}{5} \times \frac{3}{3} - \frac{2}{3} \times \frac{5}{5})] = 2\frac{6}{7} \times [1(\frac{12}{15} - \frac{10}{15})] = 2\frac{6}{7} \times 1\frac{2}{15}$
$= \frac{20}{7} \times \frac{17}{15} = \frac{340}{105} = 3\frac{5}{21}$

3.8 $3\frac{3}{7} \times (4\frac{2}{3} - \frac{11}{5}) = 3\frac{3}{7} \times [2(\frac{2}{3} \times \frac{5}{5} - \frac{1}{5} \times \frac{3}{3})] = 3\frac{3}{7} \times [2(\frac{10}{15} - \frac{3}{15})] = 3\frac{3}{7} \times 2\frac{7}{15}$
$= \frac{24}{7} \times \frac{37}{15} = \frac{888}{105} = 8\frac{48}{105}$

Review Copy

3.9 $(3\frac{3}{5} - 1\frac{1}{10}) \div 2\frac{2}{5} = [2(\frac{3}{5} \times \frac{2}{2} - \frac{1}{10})] \div 2\frac{2}{5} = [2(\frac{6}{10} - \frac{1}{10})] \div 2\frac{2}{5} = 2\frac{1}{2} \div 2\frac{2}{5} = \frac{5}{2} \times \frac{5}{12}$

$= \frac{25}{24} = 1\frac{1}{24}$

3.10 $(3\frac{3}{5} - 3\frac{3}{8}) \div 2\frac{1}{4} = (\frac{3}{5} \times \frac{8}{8} - \frac{3}{8} \times \frac{5}{5}) \div 2\frac{1}{4} = (\frac{24}{40} - \frac{15}{40}) \div 2\frac{1}{4} = \frac{9}{40} \times \frac{4}{9} = \frac{36}{360} = \frac{1}{10}$

Remedial

If learners experience problems, work with them in a separate small group, while the rest of the class continues with this exercise. Explain that when dividing a fraction by a fraction, multiply the first fraction by the reciprocal of the second. Give a simple example such as $\frac{1}{2} \div \frac{1}{4} = \frac{1}{2} \times \frac{4}{1} = 2$ (there are 2 quarters in a half) or $\frac{1}{3} \div \frac{1}{6} = \frac{1}{3} \times \frac{6}{1} = 2$ (there are 2 sixths in a third).

Use diagrams on the board to demonstrate these calculations. Do as many examples as necessary until learners are able to cope with the prescribed activity.

Extension

Learners who are confident about division of fractions can be asked to make wall charts demonstrating division. If they are dividing $\frac{1}{2}$ by $\frac{1}{4}$ tell them to cut out a piece of paper to represent a half and a piece to represent a quarter and to show how many times the quarter goes into the half.

Review Copy

Unit 3 Squares, cubes and roots of fractions

Learner's Book page 211

Unit focus:

- finding squares and cubes of fractions
- finding the square roots and cube roots of fractions.

Background information on squares, cubes and roots of fractions

Learners should know the following information:

- Any number multiplied by itself is called a squared number, for example 3×3 can be written 3^2 and called 3 to the power 2 or 3 squared.
- $3 \times 3 = 9$, which is called a perfect square because it is the square of an integer.
- Any number times itself times itself is called a cube: for example. $5 \times 5 \times 5$ can be written 5^3 and called 5 to the power 3 or 5 cubed. $5 \times 5 \times 5 = 125$, which is a perfect cube.

Revision

Learner's Book page 211

Guidelines on how to implement this activity

Ask volunteers to write the squares of numbers on the board, for example squares of 1; 2; 3; 4 which are counting numbers. Their squares are 1; 4; 9; 16 which are also perfect squares. Do the same exercise with cubed numbers.

Ask: How do you write 1 squared? (1^2). How do you write 3 cubed? (3^3). How do you write 4 to the power of five? (4^5). What does this mean? $4 \times 4 \times 4 \times 4 \times 4$. Use this exercise to assess learners' knowledge of squares and cubes.

Suggested answers

1.1	$5^2 = 5 \times 5 = 25$	1.2	$11 = 11 \times 11 = 121$	1.3	$9^2 = 9 \times 9 = 81$
2.1	$10^3 = 10 \times 10 \times 10 = 1\,000$	2.2	$12^3 = 12 \times 12 \times 12 = 1\,728$		
2.3	$100^3 = 100 \times 100 \times 100 = 1\,000\,000$				
3.1	$\sqrt{144} = 12$	3.2	$\sqrt{10\,000} = 100$	3.3	$\sqrt{64} = 8$
4.1	$\sqrt[3]{1} = 1$	4.2	$\sqrt[3]{1\,000} = 10$	4.3	$\sqrt[3]{27} = 3$
5.1	$\frac{5}{10} = \frac{1}{2}$	5.2	$\frac{9}{12} = \frac{3}{4}$	5.3	$\frac{49}{63} = \frac{7}{9}$
5.4	$\frac{100}{1000} = \frac{1}{10}$	5.5	$\frac{20}{60} = \frac{1}{3}$	5.6	$\frac{56}{72} = \frac{8}{9}$

Remedial

Have learners who are having problems with this concept draw a number of freehand squares on a piece of paper. Label the length and breadth of the first square 2 and 2, the second square 3 and 3 and the third square 4 and 4. Ask them to divide the squares into the appropriate number of small squares, for example, $3 \times 3 = 9$. Nine is the square of 3. 3 is the square root of 9.

Review Copy

Extension

Some learners might enjoy making an artwork demonstrating squares and roots of numbers. Encourage them to work out how they would do this.

Exercise 1

Learner's Book page 212

Guidelines on how to implement this activity

- Show learners that when you write a fraction as a power you do this: $\left(\frac{1}{2}\right)^2$ $\left(\frac{1}{3}\right)^3$ $\left(\frac{1}{2}\right)^4$. The brackets are necessary so that you can see that the whole fraction is a power, not just the numerator.
- Demonstrate that $\frac{1}{2} \times \frac{1}{2}$ can also be written as a half of a half. $\frac{1}{2}$ of $\frac{1}{2} = \frac{1}{4}$. $\left(\frac{1}{2}\right)^2 = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ and $\left(\frac{1}{3}\right)^3 = \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} = \frac{1}{27}$ and so on. The square and cube apply to both the numerator and the denominator.

- Show learners that if $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$, then the square root of a quarter is a half, or $\sqrt{\frac{1}{4}} = \frac{1}{2}$.
- If $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{8}{27}$ then the cube root of $\frac{8}{27}$ is $\frac{2}{3}$, or $\sqrt[3]{\frac{8}{27}} = \frac{2}{3}$
- Demonstrate that when we find the square root or cube root of a fraction, we find the root of the numerator and the denominator.

Do examples as a class and then get learners to complete this exercise on their own.

Suggested answers

1.1 $\sqrt{\frac{36}{4}} = \frac{6}{2} = 3$ 1.2 $\sqrt{\frac{81}{16}} = \frac{9}{4} = 2\frac{1}{4}$ 1.3 $\sqrt{\frac{36}{9}} = \frac{6}{3} = 2$

1.4 $\sqrt{\frac{16}{25}} = \frac{4}{5}$ 1.5 $\sqrt{\frac{9}{49}} = \frac{3}{7}$ 1.6 $\sqrt{\frac{81}{36}} = \frac{9}{6} = 1\frac{1}{2}$

1.7 $\sqrt{\frac{25}{100}} = \frac{5}{10} = \frac{1}{2}$ 1.8 $\sqrt{\frac{64}{4}} = \frac{8}{2} = 4$ 1.9 $\sqrt{\frac{121}{144}} = \frac{11}{12}$

2.1 $1 \times 1 = 1^2 = 1$ 2.2 $22 \times 22 = 22^2 = 484$

2.3 $29 \times 29 = 29^2 = 841$ 3.1 $2 \times 2 \times 2 = 2^3 = 8$

3.2 $4 \times 4 \times 4 = 4^3 = 64$ 3.3 $9 \times 9 \times 9 = 9^3 = 729$

4.1 $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ 4.2 $\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$ 4.3 $\frac{5}{7} \times \frac{5}{7} = \frac{25}{49}$

4.4 $\frac{4}{5} \times \frac{4}{5} = \frac{16}{25}$ 4.5 $\frac{8}{9} \times \frac{8}{9} = \frac{64}{81}$ 4.6 $\frac{12}{13} \times \frac{12}{13} = \frac{144}{169}$

5.1 $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$ 5.2 $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{8}{27}$ 5.3 $\frac{5}{7} \times \frac{5}{7} \times \frac{5}{7} = \frac{125}{343}$

5.4 $\frac{10 \times 10 \times 10}{100 \times 100 \times 100} = \frac{1\,000}{1\,000\,000}$ 5.5 $\frac{20 \times 20 \times 20}{30 \times 30 \times 30} = \frac{8\,000}{27\,000}$

6.1 $\sqrt[3]{\frac{8}{27}} = \frac{2}{3}$ 6.2 $\sqrt[3]{\frac{1}{125}} = \frac{1}{5}$ 6.3 $\sqrt[3]{\frac{64}{216}} = \frac{4}{6}$

Review Copy

Remedial

If learners are experiencing problems with finding square and cube roots, give them simple examples to square, for example, $\frac{1}{5}; \frac{2}{5}; \frac{2}{3}; \frac{1}{6}; (\frac{1}{25}; \frac{4}{25}; \frac{4}{9}; \frac{1}{36})$. Next ask them to find the square roots of: $\frac{1}{25}; \frac{4}{25}; \frac{4}{9}; \frac{1}{36}$ $(\frac{1}{5}; \frac{2}{5}; \frac{2}{3}; \frac{1}{6})$. Be sure to remind learners that 1 squared can be written 1^2 or 1 to the power of 2. $\frac{1}{2}$ squared can be written $(\frac{1}{2})^2$ or $\frac{1}{2}$ to the power of 2. The square root of 1 can be written as $\sqrt{1}$, and the square root of a half can be written as $\sqrt{\frac{1}{2}}$.

Extension

Encourage learners to make up and do calculations involving squared and cubed numbers and square and cube roots, for example, $(\frac{1}{2})^3 + \sqrt{\frac{1}{2}}$.

Unit 4 Problem solving

Learner's Book page 213

Unit focus:

- applying your knowledge of fractions to solve problems.

Background information on problem solving

Problem solving with fractions involves a number of basic rules:

- Read the question carefully.
- Take careful note of what information has been given.
- Take careful note of what has been asked.
- Make sure you know what unit the answer will be in.
- Think about what operation you will use to solve the problem.

Learners should be familiar with these, however they are worth revising.

Revision

Learner's Book page 213

Review Copy

Guidelines on how to implement this activity

Use this exercise to assess learners' work on fractions. The work covered in this exercise is required in order for learners to be able to continue with this unit.

Suggested answers

1.1 $\frac{1}{2} = \frac{30}{60}$ 1.2 $\frac{2}{3} = \frac{40}{60}$ 1.3 $\frac{7}{15} = \frac{28}{60}$

1.4 $\frac{15}{25} = \frac{3}{5} = \frac{36}{60}$ 2.1 $75\% = \frac{75}{100} = \frac{3}{4}$ 2.2 $40\% = \frac{40}{100} = \frac{2}{5}$

2.3 $12{,}5\% = \frac{12{,}5}{100} = \frac{1}{8}$ 2.4 $4\% = \frac{4}{100} = \frac{1}{25}$

3.1 Thandi gets $20\% = \frac{20}{100} = \frac{1}{5}$ 3.2 Mary gets $25\% = \frac{25}{100} = \frac{1}{4}$

3.3 $20\% + 25\% = 45\%$. Peter gets $100\% - 45\% = 55\% = \frac{55}{100} = \frac{11}{20}$

3.4 Thandi gets $\frac{1}{5}$ of $\frac{60}{1} = 12$ sweets Mary gets $\frac{1}{4}$ of $60 = 15$ sweets

Peter gets $\frac{11}{20}$ of $60 = 33$ sweets

4.1 $\frac{240}{800} = \frac{3}{10}$ play tennis

4.2 $\frac{560}{800} = \frac{7}{10}$ play soccer

Remedial

If learners struggle with this exercise, identify their problem areas and refer them to the relevant units in this chapter, or even to the Grade 7 Learner's Book to help them work though the areas where they have difficulty.

Exercise 1

Learner's Book page 214

Guidelines on how to implement this activity

Discuss the rules that should be applied to problem solving. Ask learners to give their ideas. Give an example of a simple problem and work through it with the learners.

Use an example, such as: In a class with 20 learners, each learner is given $3\frac{1}{2}$ slices of bread for breakfast and $4\frac{1}{2}$ slices for lunch. How many slices does each learner get per day? How many slices does the class get? If there are twenty-five slices in a loaf of bread, how many loaves are used per day? Work through this example and discuss going through of the steps in problem solving.

Facts given:

1 20 learners in a class.

2 Each gets $3\frac{1}{2} + 4\frac{1}{2}$ slices per day.

3 Class gets 20 $(3\frac{1}{2} + 4\frac{1}{2})$ slices per day.

4 There are 25 slices in a loaf.

5 $[20(3\frac{1}{2} + 4\frac{1}{2})] \div 25$ loaves are used per day.

Questions asked:

How many slices does each learner get per day? See point 2 above.

Each gets $3\frac{1}{2} + 4\frac{1}{2} = 8$ slices per day.

How many loaves are used per day? See point 5 above.

$[20(3\frac{1}{2} + 4\frac{1}{2})] \div 25$

$= 160 \div 25 = 6\frac{10}{25} = 6\frac{2}{5}$ loaves are used per day.

Do additional such examples until learners feel able to cope on their own. Allow learners to work in pairs, but each provide all their own working for this exercise.

Review Copy

Suggested answers

1.1 $\frac{95}{100} = \frac{19}{20}$ 1.2 $\frac{480}{800} = \frac{3}{5}$ 1.3 $\frac{2750}{4500} = \frac{11}{18}$

2.1 $\frac{3}{5} = \frac{99}{165} = \frac{21}{35} = \frac{33}{55} = \frac{120}{200}$

3.1 Mary earns R8 000,00 per month. If $\frac{3}{5}$ = R8 000 then $\frac{1}{5}$ = R2 667
Fuad earns 5 × R2 667 = R13 333

3.2 Jane earns $\frac{1}{2}$ of R8 000 = R4 000

3.3 Jane's salary is $\frac{\text{R4 000}}{\text{R13 333}} \approx \frac{3}{10}$ of Faud's.

4 There are 700 learners in a school. Each class has $\frac{1}{20}$ of the learners.

4.1 $\frac{1}{20}$ of 700 = $\frac{700}{20}$ = 35 per class

4.2 $\frac{49}{700} = \frac{7}{100}$ play chess.

4.3 $\frac{70}{700} = \frac{1}{10}$ play the piano.

4.4 $\frac{140}{700} = \frac{1}{5}$ do drama.

5.1 Ferdinand earns R7 540 ($\frac{1}{10}$ = R754 so $\frac{10}{10}$ = 10 × R754 = R7 540)

5.2 He gives his mother $\frac{\text{R754}}{\text{R7 540}} = \frac{1}{10}$

5.3 Ferdinand pays 25% of R7 540 for board $\frac{25}{100}$ × R7 540 = R1 885

5.4 Ferdinand pays 15% of R7 540 for transport $\frac{15}{100}$ × R7 540 = R1 131

5.5 Board and transport make up 25% + 15% = 40% of R7540 = $\frac{2}{5}$

6

Total	Fraction in simplest form	Equivalent fraction	Decimal fraction	Percentage
500 of 1 000	$\frac{1}{2}$	$\frac{2}{4}$	0,5	50%
44 of 100	$\frac{11}{25}$	$\frac{22}{50}$	0,44	44%
15 of 125	$\frac{3}{25}$	$\frac{45}{375}$	0,12	12%
32 of 40	$\frac{4}{5}$	$\frac{96}{120}$	0,8	80%

Review Copy

7.1 Team A plays $\frac{1}{3}$ × 30 = 10 games

7.2 Team B plays $\frac{2}{3}$ × 30 = 20 games

7.3 Team A wins $\frac{3}{5}$ × 10 = 6 games

7.4 Team B wins $\frac{1}{2}$ × 20 = 10 games

8.1 Lulundi = 30 cupcakes
Mother 30 × 3 = 90 cupcakes
Sister $\frac{90}{1} \times \frac{1}{3}$ = 30

Altogether they bake 30 + 90 + 30 = 150 cupcakes

8.2 Lulundi bakes $\frac{30}{150} = \frac{1}{5}$

9.1 $\frac{3}{5}$ × 625 = 375 tickets are bought by women.

9.2 Men buy 625 – 375 = 250 tickets

9.3 Men buy $\frac{250}{625} = \frac{2}{5}$

10

Decimal fraction	Fraction with denominator of 100	Fraction with denominator of 1 000	Fraction in simplest form	Percentage
0,25	$\frac{25}{100}$	$\frac{250}{1\,000}$	$\frac{1}{4}$	25%
0,35	$\frac{35}{100}$	$\frac{350}{1\,000}$	$\frac{7}{20}$	35%
0,75	$\frac{75}{100}$	$\frac{750}{1\,000}$	$\frac{3}{4}$	75%
0,6	$\frac{60}{100}$	$\frac{600}{1\,000}$	$\frac{3}{5}$	60%
0,82	$\frac{82}{100}$	$\frac{820}{1\,000}$	$\frac{41}{50}$	82%

Remedial

Give learners a list of the rules that should be applied to problem solving.
Give additional simple examples:

3 learners each have $2\frac{1}{2}$ apples. How many apples do they have altogether?

- What are you told? 3 learners each have $2\frac{1}{2}$ apples.
- What are you asked? How many apples do they have altogether?

Ask volunteers to do sketches on the board for the others to see; for example, 3 sets with $2\frac{1}{2}$ apples in each. Calculate $3 \times 2\frac{1}{2} = \frac{3}{1} \times \frac{5}{2} = \frac{15}{2} = 7\frac{1}{2}$ apples.

Review Copy

Extension

Encourage learners to make up and solve some word problems involving fractions.

Unit 5 Percentages

Learner's Book page 216

Unit focus:

- revision of percentages
- revision of percentage increase and decrease
- calculations with percentages.

Background information on percentages

It is important that learners have the following background information in order to manage this exercise. Common fractions and decimal fractions can be written as fractions with a denominator of 100.
A percentage is a value out of 100. For example:

- Equivalent common fractions, such as $\frac{1}{2}$; $\frac{3}{5}$; $\frac{7}{10}$; $\frac{19}{20}$ can be converted to:

  $\frac{1}{2} \times \frac{50}{50} = \frac{50}{100} = 50\%$ and $\frac{3}{5} \times \frac{20}{20} = \frac{60}{100} = 60\%$ and $\frac{7}{10} \times \frac{10}{10} = \frac{70}{100} = 70\%$ and $\frac{19}{20} \times \frac{5}{5} = \frac{95}{100} = 95\%$.

- Decimal fractions can be converted to common fractions and percentages. For example: $0{,}1 = \frac{1}{10} \times \frac{10}{10} = \frac{10}{100} = 10\%$ and $0{,}78 = \frac{78}{100} = 78\%$ and $0{,}125 = \frac{125}{1000} = \frac{12{,}5}{100} = 12{,}5\%$

Finding a percentage of a number is done this way. 20% of 50 is $\frac{20}{100} \times \frac{50}{1} = 10$.

Exercise 1

Learner's Book page 217

Guidelines on how to implement this activity

- Write the following on the board and discuss: $\frac{7}{10}$ is a common fraction.
  To find an equivalent fraction with a denominator of 100 do the following:
  $\frac{7}{10} \times \frac{10}{10} = \frac{70}{100}$. $\frac{70}{100}$ is 70%. It is also 0,70 and 0,7.
  The following are all equivalent: $\frac{7}{10}$; $\frac{70}{100}$; 70%; 0,70; 0,7.
- Ask learners to choose a decimal or common fraction or a percentage and find all its equivalent fractions.
- Allow volunteers to write their selection on the board and ask: If a learner gets 15 out of 20 for a test, find the percentage, and find what the equivalent fraction is with a denominator of 100.
  So: $\frac{15}{20} \times \frac{5}{5} = \frac{75}{100} = 75\%$
  Another way to do this is to multiply by 100. So $\frac{15}{20} \times \frac{100}{1} = 75\%$
  To find the percentage of an amount we do this: 50% of 40 is $\frac{50}{100} \times \frac{40}{1} = 20$.
  25% of 300 is $\frac{25}{100} \times \frac{300}{1} = 75$. Do as many examples as a class, and encourage learners to complete this exercise on their own.

Review Copy

Suggested answers

1.1 $\frac{2}{5} = 40\%$ 1.2 $\frac{7}{10} = 70\%$ 1.3 $\frac{19}{10} = 190\%$

2.1 $80\% = \frac{4}{5}$ 2.2 $24\% = \frac{24}{100} = \frac{6}{25}$ 2.3 $111\% = \frac{111}{100}$

3.1 $50\% \times 160 = \frac{50}{100} \times 160 = 80$

3.2 $25\% \times 44 = \frac{25}{100} \times 44 = 11$

3.3 $62\% \times 300 = \frac{62}{100} \times 300 = 186$

3.4 $85\% \times 500 = \frac{85}{100} \times 500 = 425$

4.1 $R18 \times \frac{50}{100} = R9$ so R18 less 50% = R18 − R9 = R9

4.2 $R120 \times \frac{30}{100} = R36$ so R120 less 30% = R120 – R36 = R84

4.3 R25 $\times \frac{20}{100}$ = R5, so R25 increased by 20% = R25 + R5 = R30

4.4 R45 $\times \frac{10}{100}$ = R4,50 so R45 increased by 10% = R45 + R4,50 = R49,50

5.1 5% of 120 = 120 $\times \frac{5}{100}$ = 6

5.2 55% of 300 = 300 $\times \frac{55}{100}$ = 165

5.3 150% of 180 = 180 $\times \frac{150}{100}$ = 270

6.1 240 $\times \frac{40}{100}$ = 96: 40% increase = 240 + 96 = 336

6.2 90 $\times \frac{40}{100}$ = 36: 40% increase = 90 + 36 = 126

6.3 15 $\times \frac{40}{100}$ = 6: 40% increase = 15 + 6 = 21

7.1 240 $\times \frac{40}{100}$ = 96: 40% decrease = 240 – 96 = 144

7.2 90 $\times \frac{40}{100}$ = 36: 40% decrease = 90 – 36 = 54

7.3 15 $\times \frac{40}{100}$ = 6: 40% decrease = 15 – 6 = 9

8 If Vuyo sees a computer for sale at R5 648,80, a TV for 75% of the price of the computer, and cellphone for $\frac{1}{5}$ of the price of the computer:

8.1 the TV costs $\frac{75}{100}$ × R5 648,80 = R4 236,60

8.2 the cellphone costs $\frac{1}{5}$ × R5 648,80 = R1 129,76

8.3 all three items altogether cost R5 648,80 + R4 236,60 + R1 129,76 = R11 015,16

9.1

Item for sale	Marked price	Amount of discount	Discounted price	Percentage markdown as a fraction
TV	R7 468,20	25%	R5 601,15	$\frac{1}{4}$
Computer	R15 875,98	50%	R7 937,99	$\frac{1}{2}$
Cellphone	R1 025,50	20%	R820,40	$\frac{1}{5}$

9.2 TV: 0,25 = 25%
Computer: 0,5 = 50%
Cellphone: 0,2 = 20%

Remedial

If learners are experiencing problems, revise the fact that a percentage is an amount out of 100.

Review Copy

If a learner gets $\frac{15}{20}$ for a test, to find the percentage, find what the equivalent fraction is with a denominator of 100. So: $\frac{15}{20} \times \frac{5}{5} = \frac{75}{100} = 75\%$. Give learners a worksheet with a number of simple examples of common fractions to convert to percentages. For example, $\frac{5}{10}$; $\frac{7}{20}$; $\frac{19}{25}$.

Extension

Ask learners to pretend they own a factory selling scarves. The price of the scarves is R80,00 but they sell to retailers less 20%. The retailers then sell at 50% more than the price for which they buy. They must design a poster advertising the scarves. If they would rather, they can make up their own business and prices. This is good practice for learners to work with percentage in a practical way.

Unit 6 Equivalent forms

Learner's Book page 219

Unit focus:

- revision of equivalent forms of fractions
- converting common fractions to decimals
- converting common fractions to percentages.

Background information on equivalent forms

Learners should know by now that fractions, decimals and percentages can all be written as equivalent forms of one another. Learners have worked with percentage and decimals since Grade 6, however they may need to revise the conversions between these forms.

Review Copy

Exercise 1

Learner's Book page 221

Guidelines on how to implement this activity

Start this section by giving learners fraction sequences to complete, showing equivalent fractions:

$\frac{3}{7} = \frac{a}{14} = \frac{9}{b} = \frac{12}{c} = \frac{d}{35}$ ($a = 6$; $b = 21$; $c = 28$; $d = 15$).

Next ask learners to find equivalent fractions with denominators of 10 or powers of 10: $\frac{4}{5} = \frac{a}{10} = \frac{b}{100} = \frac{c}{1\,000}$ and $\frac{23}{25} = \frac{x}{100} = \frac{y}{1\,000}$.

Ask learners to rewrite the fractions with denominators that are powers of 10 as decimal fractions. Demonstrate to learners how to divide the numerator by the denominator to convert the following fractions to decimal fractions: $\frac{2}{5}$; $\frac{1}{8}$; $\frac{3}{50}$.

As a class, discuss and complete a table, like the one below, by filling in the equivalent fractions, decimals and percentages.

Common fraction	Common fraction with denominator of 10	Common fraction with denominator of 100	Decimal fraction	Percentage
$\frac{1}{5}$				
	$\frac{6}{10}$			
		$\frac{70}{100}$		
			0,4	
				80%

Encourage learners to work on their own to complete this exercise.

Suggested answers

1.1 $\frac{1}{6} = \frac{2}{12} = \frac{6}{36} = \frac{30}{180} = \frac{7}{42}$

1.2 $\frac{9}{8} = \frac{72}{64} = \frac{45}{40} = \frac{180}{160} = \frac{495}{440}$

1.3 $\frac{3}{4} = \frac{15}{20} = \frac{75}{100} = \frac{144}{192}$

1.4 $\frac{2}{5} = \frac{12}{30} = \frac{68}{170} = \frac{208}{520}$

2 Copy and complete the following by filling in the common fraction:

2.1 $0{,}1 = \frac{1}{10}$ 2.2 $0{,}03 = \frac{3}{1000}$ 2.3 $1{,}7 = 1\frac{7}{10}$

3.1 $\frac{20}{100} = \frac{2}{10}$ 3.2 $\frac{84}{100} = \frac{21}{25}$ 3.3 $155\% = 1\frac{11}{20}$

Review Copy

4

Fraction	Equivalent fraction with denominator 10	Equivalent fraction with denominator 100	Equivalent fraction with denominator 20	Decimal fraction	Percentage
$\frac{1}{2}$	$\frac{5}{10}$	$\frac{50}{100}$	$\frac{10}{20}$	0,5	50%
$\frac{1}{4}$	$\frac{2{,}5}{10}$	$\frac{25}{100}$	$\frac{5}{20}$	0,25	25%
$\frac{3}{5}$	$\frac{6}{10}$	$\frac{60}{100}$	$\frac{12}{20}$	0,6	60%
$\frac{28}{40}$	$\frac{7}{10}$	$\frac{70}{100}$	$\frac{14}{20}$	0,7	70%

Fraction	Equivalent fraction with denominator given	Equivalent fraction with denominator 120	Decimal fraction	Percentage
$\frac{2}{3}$	$\frac{66}{99}$	$\frac{80}{120}$	0,667	66,7%
$\frac{1}{6}$	$\frac{30}{180}$	$\frac{20}{120}$	0,167	16,7%
$\frac{1}{8}$	$\frac{12}{96}$	$\frac{15}{120}$	0,125	12,5%
$\frac{4}{5}$	$\frac{44}{55}$	$\frac{96}{120}$	0,8	80%

Remedial

If learners require additional revision of decimals, remind learners of H T U. These letters stand for the place value of whole numbers. To show decimals a comma is placed right of the Units, and to the right of the comma we get tenths, hundredths and thousandths, which are all fractions.

Therefore:

	H	T	U	,	t	h	th
3 tenths ($\frac{3}{10}$) =			0	,	3		
4 hundredths ($\frac{4}{100}$) =			0	,	0	4	
7 thousandths ($\frac{7}{1\,000}$) =			0	,	0	0	7

Review Copy

Consolidation exercise

Learner's Book page 223

Before doing this consolidation exercise, encourage learners to review the work covered in this chapter. Advise learners to use the summary and to revise their work. This exercise can be used as an informal assessment task for you to track how learners are coping with the chapter and the concepts covered. The mark allocation provides guidelines on how to assess learners.

Suggested answers

1.1 $\frac{18}{20} + \frac{2}{3} = \frac{18}{20} \times \frac{3}{3} + \frac{2}{3} \times \frac{20}{20} = \frac{54}{60} + \frac{40}{60} = \frac{94}{60} = 1\frac{34}{60} = 1\frac{17}{30}$

1.2 $\frac{7}{15} + \frac{5}{21} = \frac{7}{15} \times \frac{21}{21} + \frac{5}{21} \times \frac{15}{15} = \frac{147}{315} + \frac{75}{315} = \frac{222}{315}$

1.3 $\frac{21}{25} - \frac{3}{5} = \frac{21}{25} - \frac{3}{5} \times \frac{5}{5} = \frac{21}{25} - \frac{15}{25} = \frac{6}{25}$

1.4 $\frac{16}{21} - \frac{1}{3} = \frac{16}{21} - \frac{1}{3} \times \frac{7}{7} = \frac{16}{21} - \frac{7}{21} = \frac{9}{21} = \frac{3}{7}$ (4)

2.1 $\frac{1}{3} \times \frac{4}{5} = \frac{4}{15}$

2.2 $\frac{1}{7} \times \frac{7}{10} = \frac{7}{70} = \frac{1}{10}$

2.3 $\frac{2}{5} \times \frac{15}{18} = \frac{30}{90} = \frac{1}{3}$ (3)

3.1 $\frac{1}{3} \div 2 = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$ 3.2 $5 \div \frac{1}{3} = \frac{5}{1} \times \frac{3}{1} = 15$

3.3 $\frac{3}{4} \div \frac{1}{8} = \frac{3}{4} \times \frac{8}{1} = \frac{24}{4} = 6$ (6)

4.1 $\sqrt{\frac{24}{6}} = \sqrt{4} = 2$ 4.2 $\sqrt{\frac{16}{25}} = \frac{4}{5}$ 4.3 $\sqrt{\frac{27}{3}} = \sqrt{9} = 3$ (3)

5.1 $\frac{18}{30} = \frac{9}{15} = \frac{3}{5} = 0{,}60 = 60\%$ 5.2 $\frac{39}{120} = \frac{13}{40} = 0{,}325 = 32{,}5\%$

5.3 $\frac{2}{3} = \frac{34}{51} = 0{,}667 = 66{,}67\%$ (3)

6.1 Trudy gets $25\% = \frac{25}{100} = \frac{1}{4}$ of the sweets. (1)

6.2 Mary gets $\frac{1}{5} = \frac{20}{100} = 20\%$ (1)

6.3 Bheki gets $100\% - 25\% - 20\% = 55\% = \frac{55}{100} = \frac{11}{20}$ (2)

6.4 $\frac{1}{2} = 40$ sweets so total $= 40 \times 2 = 80$ sweets

Trudy gets 25% of $80 = \frac{25}{100} \times 80 = 20$ sweets

Mary gets 20% of $80 = \frac{20}{100} \times 80 = 16$ sweets

Bheki gets 55% of $80 = \frac{55}{100} \times 80 = 44$ sweets (1)

7.1 25% of $360 = \frac{25}{100} \times 360 = 90$; so 25% increase $= 360 + 90 = 450$ (1)

7.2 25% of $800 = \frac{25}{100} \times 800 = 200$; so 25% increase $= 800 + 200 = 1\,000$ (1)

7.3 25% of $12 = \frac{25}{100} \times 12 = 3$; so 25% increase $= 12 + 3 = 15$ (1)

8.1 25 % of $320 = \frac{25}{100} \times 320 = 80$; so 25% decrease $= 320 - 80 = 240$ (1)

8.2 25 % of $900 = \frac{25}{100} \times 900 = 225$; so 25% decrease $= 900 - 225 = 675$ (1)

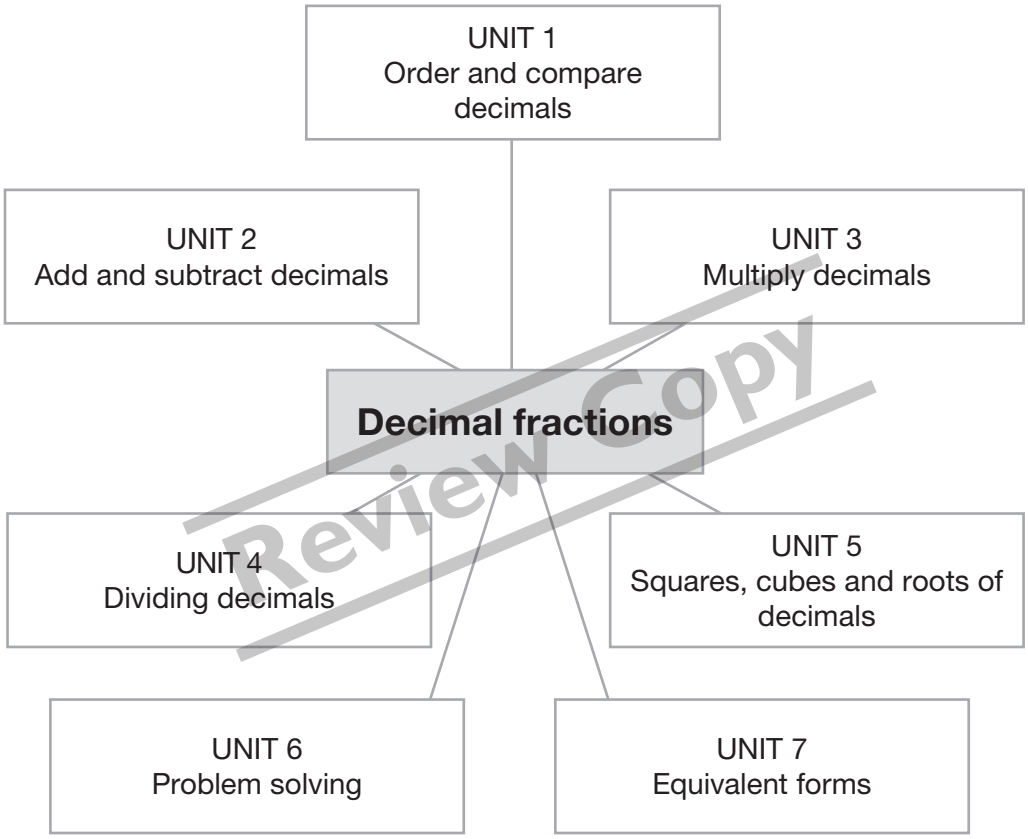
8.3 25 % of $8 = \frac{25}{100} \times 8 = 2$; so 25% decrease $= 8 - 2 = 6$ (1)

Total marks [30]

Review Copy

Chapter 11 Decimal fractions

Overview of concepts



Review Copy

Content		Time allocations	LB page
Unit 1	Order and compare decimals	$\frac{1}{2}$ hour	225
Unit 2	Add and subtract decimals	1 hour	228
Unit 3	Multiply decimals	1 hour	230
Unit 4	Divide decimals	1 hour	233
Unit 5	Squares, cubes and roots of decimals	$\frac{1}{2}$ hour	236
Unit 6	Problem solving	1 hour	239
Unit 7	Equivalent forms	1 hour	242

Background information on decimal fractions

The decimal number system has 10 as a base. It is the most commonly used number system in the modern world. The numbers in the decimal system are positioned in specific ways to give their values. For example 3 has a different value in 30 than its value in 3 000. These values are based on the powers of 10. So we work with, for example:

TTh Th H T U

The value of a number in each column is one tenth of the value of the number in the column to its left. For example:

10 000 ÷ 10 = 1 000 (TTh ÷ 10)
1 000 ÷ 10 = 100 (Th ÷ 10)
100 ÷ 10 = 10 (H ÷ 10)
10 ÷ 10 = 1 (T ÷ 10)

To continue with the division below 1, we can no longer deal with whole numbers. One Unit divided by 10 gives $\frac{1}{10}$. Written as a decimal number this has to go in the tenths column. The pattern simply continues, each number being one tenth the value of the number to its left. The comma divides whole numbers from fractions.

… TTh Th H T U , t h th …

Generic teaching guidelines for teaching decimal fractions

Ensure that learners grasp the concept of the value of the digits in the H, T and U columns. Encourage volunteers to draw columns on the board and then to fill in numbers like:

34 760 2 354 $\frac{1}{10}$ $\frac{3}{100}$

Review Copy

Revise the idea of adding and subtracting numbers in columns, starting at the right end, for example:

	H	T	U
	1	3	5
+	4	2	7
	5	6	2

When working with decimal fractions, exactly the same system is used.

Unit 1 Order and compare decimals

Learner's Book page 225

Unit focus:

- revision of the decimal system
- counting in decimals
- ordering and comparing decimals.

Background information on Ordering and comparing decimals

The number system we use is the decimal system. Digits are arranged in ordered columns, their value depending on the column they are in. For example, take the

number 11. Eleven has 1 Ten and 1 Unit. 11 = 10 + 1. We arrange numbers in columns to show their value. Copy this down: T U

Now write down the following numbers in the correct column. 5 Tens, 7 Units.

Where you have written 5 under the Tens column, you will need a place holder (0) in the Units column. Talk about these numbers:

H	T	U	,	t	h
	5	0	,	0	
		7	,	0	
	2	9	,	0	
3	0	6	,	0	
		3	,	4	
	1	0	,	0	7

Does there have to be a 0 after the decimal point? No, 50,0 is the same as 50.

Does there have to be a 0 in 306? Yes, otherwise it would be 36.

Revision

Learner's Book page 225

Guidelines on how to implement this activity

Use this exercise to assess learners' understanding of decimals.

Suggested answers

1

Given number	Thousands	Hundreds	Tens	Units	tenths	hundredths	thousandths
0,5					5		
0,001							1
1000	1						
345,567		3	4	5	5	6	7
84,89			8	4	8	9	
6,002				6			2

Review Copy

2 1 000; 345,567; 84,89; 6,002; 0,5; 0,001

3 0,001; 0,5; 6,002; 84,89; 345,567; 1 000

4.1 56,076 → 56,096 → 56,116 → 56,136 → 56,156 → 56,176 → 56,196

4.2 0,024 → 0,012 → 0 → –0,012 → –0,024 →–0,036 → –0,048

5.1 45,65; 45,05; 44,45; 43,85; 43,25; 42,65; 42,05

5.2 23,234; 24,335; 25,436; 26,537; 27,638; 28,739; 29,840

5.3 0,781; 0, 670; 0,559; 0,448; 0,337; 0,226; 0,115

5.4 0,876; 0,476; 0,076; –0,324; –0,724; –1,124; –1,524

6.1 0,79

6.2 99,65

6.3 890 768,89

6.4 569 990,88

6.5 450,00

6.6 875 990 009,89

Remedial

Learners should manage this activity, as it is all revision. If learners do have problems, provide Grade 6 and 7 activities for learners to revise.

Exercise 1

Learner's Book page 227

Guidelines on how to implement this activity

Revise with learners how to order and compare decimals using the place value. Work through the worked example and have learners complete this exercise on their own.

Suggested answers

1

Th	H	T	U	t	h	th
			0	,9		
			9	,0		
				,0	9	
				,0	0	1
				,1	9	

0,001; 0,09; 0,19; 0,9; 9,0

2

Th	H	T	U	t	h	th
		6	7	,0		
			6	,7		
			7	,6		
			0	,6	7	
			0	,7	7	7

67,0; 7,60; 6,70; 0,777; 0,67

3.1 5,6 < 6,6 3.2 0,99 > 0,9 3.3 8003,3 < 8033,9
3.4 9,00 = 9 3.5 1,007 > 1,006

Review Copy

Remedial

If learners have problems comparing decimals provide Grade 6 and 7 material for learners to practise with.

Unit 2 Add and subtract decimals

Learner's Book page 228

Unit focus:

- adding and subtracting decimals to at least three decimal places.

Background information on adding and subtracting decimals

When adding or subtracting decimals and decimal fractions, only digits with the same place value can be added or subtracted. To do this it is best to write the calculations in columns. For example:

	H	T	U	t	h
		2	3,	4	9
+		8	6.	2	4
	1	0	9,	7	3

Always start the calculation on the right hand side, so start at the hundredths column. Notice, the sum of the digits 9 + 4 = 13. So 3 goes in the hundredths column and the 1 is added to the tenths column.

When subtracting:

	H	T	U,	t	h
	1	2	3,	4	9
–		8	6,	2	4
		3	7,	2	5

9 – 4 = 5 and 4 – 2 = 2 are easy. In the Units column you can't take 6 from 3 so you have to use 1 of the Tens. Now you have 1 in the Tens column and (10 + 3) – 6 in the Units column.

Exercise 1

Learner's Book page 229

Guidelines on how to implement this activity

Revise how to add and subtract with decimals. Work through the worked examples in the Learner's Book and the example provided above. Discuss any problems learners may have. Learners should complete this exercise on their own.

Review Copy

Suggested answers

1 Calculate the following:

1.1 1,23 + 3,65

	T	U	t	h
		1,	2	3
+		3,	6	5
		4,	8	8

1.2 34,78 + 465,29

	H	T	U	t	h
		3	4,	7	8
+	4	6	5,	2	9
	5	0	0,	0	7

1.3 89,67 – 45,24

	T	U	t	h
	8	9,	6	7
–	4	5,	2	4
	4	4,	4	3

1.4 756,34 – 99,67

H	T	U	t	h
7	5	6,	3	4
–	9	9,	6	7
6	5	6,	6	7

1.5 7 823,02 – 678,99

T	H	T	U	t	
7	8	2	3,	0	2
–	6	7	8,	9	9
7	1	4	4,	0	3

2.1 17,009 + 7,8 (3 digits) = 24,809
2.2 403,802 + 45,096 (3 digits) = 448,898
2.3 56,019 + 23,82 (3 digits) = 79,839
2.4 90,009 – 89,99 (3 digits) = 0,019
2.5 88,977 – 76,65 (3 digits) = 12,327
3 Calculate in columns and then round off answers to 2 decimal places:

3.2
```
  289,007
+  23,987
  312,994 = 312,99 (rounded)
```

3.2
```
  459,281
+ 346,909
  809,190 = 809,19 (rounded)
```

3.3
```
  780,026
– 697,128
   82,898 = 82,90 (rounded)
```

3.4
```
  623,111
– 475,222
  197,889 = 197,89 (rounded)
```

4.1
```
  23,456
+ 41,0999
+  8,978
  73,5339
≈ 73,534 (rounded)
```

4.2
```
  567,349
+ 67,9123
+  2,8803
  638,1416
≈ 638,142 (rounded)
```

4.3
```
  800,0001
– 799,9999
    0,0002
≈ 0.000 (rounded)
```

4.4
```
  52,6789
– 31,7799
  20,8990
≈ 20,899 (rounded)
```

5.1
```
  R23,89
+ R46,92
+  R 3,67
  R74,48
```

5.2
```
  R99,15
– R27,79
  R71,36
```

5.3
```
  572,025 km
+  12,356 km
+   0,005 km
  584,386 km
```

5.4
```
  900,012 km
– 879,444 km
   20,568 km
```

5.5
```
  1 200,5751
+   678,6781
  1 879,2531
```

5.6
```
  1 575,575 g
–   157,5575 g
  1 418,0175 g
```

Remedial

If learners have problems with this exercise provide examples from Grade 6 and 7 for learners to revise and practise.

Review Copy

Unit 3 Multiply decimals

Learner's Book page 230

Unit focus:

- revision of multiplying decimals
- multiplying decimals up to four decimal places
- multiply negative decimals.

Background information on multiplying decimals

When you multiply by a power of 10 (100, 1 000, and so on) the digits move to the left. For example 5 × 10 = 50 and 0,3 × 10 = 3,0 or 3 and 0,004 × 10 = 0,04. This means the value of the number is greater. For example 50 > 5 and 3 > 0,3.

When you multiply two decimals, you first count the number of digits after the decimal points in the two numbers you are multiplying. For example 1,2 × 1,3. There are two digits after the decimal points, 2 and 3. This means there will be two digits after the decimal point in the answer. So 12 × 13 = 156 and with two decimal points after the comma the answer will be 1,56.

Exercise 1

Learner's Book page 231

Review Copy

Guidelines on how to implement this activity

Revise with learners multiplying by powers of ten. Discuss how multiplying by 10 changes the place value of numbers. Do examples of multiplying whole numbers by 10 and then multiplying decimals by powers of 10. Ensure learners understand that the comma is not moving, but that the place value is changing. Learners should complete this exercise on their own.

Suggested answers

1.1	234,567 × 10 = 2 345,67	1.2	234,567 × 100 = 23 456,7
1.3	43,012 × 20 = 860,24	1.4	43,012 × 200 = 8 602,4
1.5	23,2306 × 10 = 232,306	1.6	23,2306 × 100 = 2 323,06
1.7	23,2306 × 1 000 = 23 230,6		
2.1	214,33 × 10 = 2 143,3	2.2	364,413 × 100 = 36 441,3
2.3	1 365,6452 × 1 000 = 1 365 645,2	2.4	34,23 × 20 = 684,6
2.5	83,452 × 300 = 25 035,6	2.6	1,26422 × 2 000 = 2 528,44

Remedial

If learners experience problems, provide HTU grids for learners to use to help them see how the place value changes.

Exercise 2

Learner's Book page 232

Guidelines on how to implement this activity

Revise multiplying decimals by whole numbers. Next revise multiplying decimals by decimals. Show learners how to multiply out the decimals and then to divide back

in. Work through the examples in the Learner's Book. Remind learners that when working with negative numbers they have to remember their signs. Revise the effect of multiplication on the signs of the answer. Learners should complete this exercise on their own.

Suggested answers

1.1	42,13 × 2,4:	4213 × 24 = 101 112	∴ = 101,112
1.2	521,2 × 1,32:	5212 × 132 = 678 984	∴ = 678,984
1.3	1,42 × 1,143:	142 × 1143 = 162306	∴ = 1,62306
1.4	32,32 × 41,41:	3232 × 4141 = 13383712	∴ = 1 338,3712
2	101,11	678,98 1,62	1338,37
3.1	2,1 × 3 = 6,3		
3.2	4,23 × 4 = 16.92		
3.3	6,331 × 7 = 44,317		
3.4	1,322 × 20 = 26,440		
3.5	23,23 × 300 = 6 969,0		
3.6	12,4321 × 2 000 = 24 864,2		
4.1	34,76 × 5,002 = 173,86952		
4.2	609,718 × 0,273 = 166,453014		
4.3	7,98 × (–1,78) = –14,2044		
4.4	–65,121 × (–2,7) = 175,8267		
4.5	–8,987 × 0,765 = –6,875055		
4.6	–0,095 × (–9,008) = 0,855760		

Remedial

If learners struggle when multiplying negative numbers, revise multiplying simple integers. Provide practice examples before learners retry multiplying decimals.

Review Copy

Unit 4 Divide decimals

Learner's Book page 233

Unit focus:

- revision of dividing decimals by 10 and powers of 10
- learning how to divide decimals by decimals
- working with negative decimals and division.

Background information on dividing decimals

When dividing decimal numbers by 10 or a power of 10 we do the reverse to what we do when multiplying. When you divide by 10 the digits move to the right. In other words their value gets less, for example, 25 or 25,0 ÷ 10 = 2,5. The 5 moves right to the tenths position and the 2 moves right to the units position.

When dividing a decimal number by a decimal number remember that you cannot divide by a decimal fraction. You have to divide by a whole number. So:
1,44 ÷ 1,2 cannot be done as it is. The divisor has to be a whole number. To make the divisor a whole number you must multiply it by 10. What you do to the divisor you must do to the dividend. This means: (1,44 × 10) ÷ (1,2 × 10) = 14.4 ÷ 12 = 1,2

Exercise 1

Learner's Book page 234

Guidelines on how to implement this activity

Revise dividing by 10, powers of 10 and multiples of 10. Do examples as a class on the board, using an HTU grid to show learners how dividing by 10 decreases the place value by 1 column. Learners should complete this exercise on their own.

Suggested answers

1.1 300 ÷ 10 = 30
1.2 300 ÷ 100 = 3
1.3 300 ÷ 1 000 = 0,3
1.4 444 ÷ 10 = 44,4
1.5 444 ÷ 100 = 4,44
1.6 444 ÷ 1 000 = 0,444
1.7 4,48 ÷ 20 = 0,224
1.8 366,55 ÷ 500 = 0,7331
1.9 1 246,66 ÷ 3 000 = 0,41555
1.10 3 333,33 ÷ 3 000 = 1,11111
2.1 214,33 ÷ 10 = 21,433
2.2 364,413 ÷ 100 =3,64413
2.3 1 365,6452 ÷ 1 000 = 1,3656452
2.4 64,24 ÷ 20 = 3,212
2.5 96,453 ÷ 300 = 0,32151
2.6 14 664,22 ÷ 2 000 = 7,33211
The answers get smaller as you divide by 10, 100 and 1000

Remedial

If learners are experiencing problems, encourage them to use an HTU grid on tracing paper, to help them see the change in place value. Provide additional examples until learners are confident with dividing by 10 and powers of 10. Otherwise learners will not be able to cope with the rest of this activity.

Review Copy

Exercise 2

Learner's Book page 235

Guidelines on how to implement this activity

Discuss dividing decimals. Do an example of long division with whole numbers on the board. Ensure learners remember how to do division of whole numbers. Remind learners that we cannot divide decimals directly; we need to multiply out the divisor. Do examples together as a class on the board.

Have learners come up to the board to complete examples. Learners should complete this exercise on their own.

Suggested answers

1.1 213,4 ÷ 10 = 21,34
1.2 213,4 ÷ 100 = 2,134
1.3 213,4 ÷ 1 000 = 0,2134
1.4 888,68 ÷ 20 = 44,434
1.5 888,68 ÷ 200 = 4,4434
1.6 888,68 ÷ 2 000 = 0,44434
2.1 14,52 ÷ 1,2 = 145,2 ÷ 12 = 12,1
2.2 42,613 ÷ 1,3 = 426,13 ÷ 13 = 32,779
2.3 3,641 ÷ 0,11 = 364,1 ÷ 11 = 33,1
2.4 77,6525 ÷ 0,25 = 7 765,25 ÷ 25 = 310,61
2.5 9,145 ÷ 2,8 = 91,45 ÷ 28 = 3,266

2.6 $25{,}83 \div 6{,}3 = 258{,}3 \div 63 = 4{,}1$
2.7 $18{,}38 \div 5{,}9 = 183{,}8 \div 59 = 3{,}115$
2.8 $0{,}3096 \div 0{,}43 = 30{,}96 \div 43 = 0{,}72$
2.9 $7{,}89 \div (-0{,}32) = 789 \div (-32) = -24{,}656$
2.10 $-3{,}0625 \div (-0{,}14) = -306{,}25 \div (-14) = 21{,}875$
2.11 $-11{,}9534 \div 2{,}36 = -1195{,}34 \div 236 = -5{,}065$
2.12 $-0{,}42768 \div 0{,}12 = -42{,}768 \div 12 = -3{,}564$

Remedial

Provide additional examples for learners to practise. Remind learners of the rule of signs, and that the rules for multiplying decimals with negative signs applies to dividing decimals as well.

Unit 5 Squares, cubes and roots of decimals

Learner's Book page 236

Unit focus:

- squaring and cubing whole numbers, and finding their square and cube roots
- squaring and cubing decimal numbers, and finding their square and cube roots.

Background information on squares, cubes and roots of decimals

To find the square of any number you multiply the number by itself.
This means $1^2 = 1 \times 1 = 1$ and $2^2 = 2 \times 2 = 4$
This works in just the same way with decimal numbers.
$1{,}1^2 = 1{,}1 \times 1{,}1 = 1{,}32$ and $1{,}2^2 = 1{,}2 \times 1{,}2 = 1{,}44$
To find the cube of any number you multiply the number by itself and again by itself.
This means $1^3 = 1 \times 1 \times 1 = 1$ and $2^3 = 2 \times 2 \times 2 = 8$
This works in just the same way with decimal numbers.
$1{,}1^3 = 1{,}1 \times 1{,}1 \times 1{,}1 = 1{,}542$ and $1{,}2^3 = 1{,}2 \times 1{,}2 \times 1{,}2 = 1{,}728$

When multiplying decimals numbers it is important to remember that in the product there must be as many digits after the decimal point as there are in the multiplier and multiplicand together.

Remember the square root of 1 is written $\sqrt{1}$ and the cube root is written $\sqrt[3]{1}$. If you can write a number as the product of two identical factors, then those factors are the square root of the product. For example, $3^2 = 3 \times 3 = 9$ and $\sqrt{9} = 3$.

If you can write a number as the product of three identical factors, then those factors are the cube root of the product. For example, $3^3 = 3 \text{ x } 3 \text{ x } 3 = 27$ and $\sqrt[3]{27} = 3$.

Revision

Learner's Book page 236

Guidelines on how to implement this activity

Revise squared and cubed numbers, square roots and cube roots. Learners should know the all the perfect squares up to 144 and cubes up to 216. Have quizzes in the class to test learners' knowledge of these numbers.

Review Copy

Suggested answers

1 $22^2 = 484$
2 $22^3 = 10\ 648$
3.1 $4 \times 4 \times 4 = 4^3 = 64$
3.2 $10 \times 10 \times 10 \times 10 = 10^4 = 10\ 000$
3.3 $53 \times 53 = 53^2 = 2\ 809$
4.1 $\sqrt{361} = 19$
4.2 $\sqrt[3]{12\ 768} = 23{,}37 \ldots$

Remedial

Have learners create squared and cubed number charts for learners to memorise.

Exercise 1

Learner's Book page 237

Guidelines on how to implement this activity

Demonstrate to learners, by means of examples, how to find the square and cube of decimal numbers. Show learners how to apply the rules for multiplying decimals to finding squares and cubes of decimals. Do additional examples together as a class. Have learners come up to the board to complete examples. Ask learners to explain their working to the class. Learners should complete this exercise on their own.

Suggested answers

1

Exponential number	Rule	Answer
1^2	1×1	1
2^2	2×2	4
$(0, 1)^2$	$(0, 1) \times (0, 1)$	0,01
$(0, 2)^2$	$0{,}2 \times 0{,}2$	0,04
$(0, 3)^2$	$0{,}3 \times 0{,}3$	0,09
1^3	$1 \times 1 \times 1$	1
2^3	$2 \times 2 \times 2$	8
$(0, 1)^3$	$0{,}1 \times 0{,}1 \times 0{,}1$	0,001
$(0, 2)^3$	$0{,}2 \times 0{,}2 \times 0{,}2$	0,008
$(0, 3)^3$	$0{,}3 \times 0{,}3 \times 0{,}3$	0,027

2.1 $2{,}1^2 = 4{,}41$ 2.2 $3{,}4^2 = 11{,}56$
2.3 $2{,}1^3 = 9{,}261$ 2.4 $3{,}4^3 = 39{,}304$
2.5 $4{,}21^2 = 17{,}724$ 2.6 $23{,}34^2 = 544{,}7556$
2.7 $33{,}11^3 = 36\ 297{,}569321$ 2.8 $13{,}45^3 = 2\ 433{,}138625$
3 1; 4; 9; 16; 25; 36; 49; 64; 81; 100
3.1 Perfect squares
3.2 0,01; 0,04; 0,09; 0,16; 0,25; 0,36

Remedial

Provide square and cube examples of whole numbers for learners to practise with. Ensure learners are competent with whole numbers before retrying this exercise.

Review Copy

Exercise 2

Learner's Book page 238

Guidelines on how to implement this activity

Revise finding square roots and cube roots of whole numbers. Explain to learners how to find square roots and cube roots of decimals using examples. Explain the steps in the Learner's Book. Do additional examples together as a class. Learners should complete this exercise on their own.

Suggested answers

1.1	$\sqrt{4} = 2$	1.2	$\sqrt{25} = 5$	1.3	$\sqrt{49} = 7$
1.4	$\sqrt{100} = 10$	1.5	$\sqrt{81} = 9$	1.6	$\sqrt{144} = 12$
2.1	$\sqrt{0{,}04} = 0{,}2$	2.2	$\sqrt{0{,}16} = 0{,}4$	2.3	$\sqrt{0{,}36} = 0{,}6$
2.4	$\sqrt{0{,}0064} = 0{,}08$	2.5	$\sqrt{2{,}25} = 0{,}5$	2.6	$\sqrt{1{,}44} = 1{,}2$
2.7	$\sqrt{0{,}0169} = 0{,}13$	2.8	$\sqrt{0{,}0025} = 0{,}05$	2.9	$\sqrt[3]{0{,}027} = 0{,}3$
2.10	$\sqrt[3]{0{,}125} = 0{,}5$	2.11	$\sqrt[3]{0{,}216} = 0{,}6$	2.12	$\sqrt[3]{0{,}000216} = 0{,}06$

Remedial

Revise perfect squares, for example, 1; 4; 9; 16; 25; 36 …

A perfect square is the product of a number multiplied by itself, such as:. $1 \times 1 = 1$ and $2 \times 2 = 4$ and $3 \times 3 = 9$ and $4 \times 4 = 16$ and so on.

If $1 \times 1 = 1$ what will $0{,}1 \times 0{,}1$ equal? First count the number of digits after the decimal point in both numbers. 0,1 has 1 digit after the decimal point, and the other 0,1 also has 1 digit after the decimal point. This means there are 2 digits after the decimal point.

Leave away the decimals and multiply 1 by 1. $1 \times 1 = 1$.
You must have 2 digits after the decimal point so write 0,01.
Next try $1{,}2 \times 1{,}2 \times 12 \rightarrow 12 = 144$ and 2 digits after decimal point so we get 1,44.
If you are multiplying $2{,}12 \times 2{,}12$ there will be 4 digits after the decimal points.
$212 \times 212 \rightarrow 44\,944$. With 4 digits after the decimal point the answer is 4,4944.

Review Copy

Extension

Learners who are confident with this concept should be encouraged to find roots of large numbers such as 3 375, and 248 832 and 1 764.

All sorts of jobs require people to understand squares and square roots. Some learners might like to find out how electricians use square roots. But some might like to pretend they are going to build a house. Work with the fact that the land they are building on is roughly square and has an area of 4 000m^2. Plan the house to fit onto the land and work out the lengths of all walls and the area of each room.

Unit 6 Problem solving

Learner's Book page 239

Unit focus:

- solving problems involving decimal fractions.

Background information on problem solving

Learners should be familiar with the steps for problem solving. These involve:

1 Identifying what you have been asked to find.
2 Identifying what you have been given in the sum.
3 Creating a number sentence to describe the problem.
4 Substituting in the values and performing the calculation.
5 Checking the reasonableness of the solution.
6 Writing a final sentence, using correct SI units.

Exercise 1

Learner's Book page 240

Guidelines on how to implement this activity

Revise with learners the way to approach problem solving. Discuss the steps outlined above in the background information. Do the example in the Learner's Book and additional examples. If necessary, revise adding, subtracting, multiplying and dividing with decimals. Discuss the SI units and their application to decimals. Have learners discuss the problems in pairs; however each learner has to provide their own working and answer in their exercise books.

Suggested answers

1 Sam earns 0,5 × R560,60 → 5 × R5606 → R2803 = R280,30
2 Thabo runs 0,15km in 1 minute,
so in 15 minutes he runs 15 × 0,15 = 2,25 km.
3 Jasmin lives 3,45 + 0,67 km = 4,12 from school.
4 Mary gets R4,55 + R3,35 + R7,50 = R15,40 pocket money.
5 Thandi walks 1,75 + 0,29 = 2,04km.
6.1 Thandeka works 2,5 + 3,25 = 5,75 hours a week
6.2 She works 4 × 5,75 = 23 hours in a month with four weekends.
4.3 She earns 23 × R15,60 = R358,80 in a month.
7.1 Simon tiles 3,25 × 2,8 = 9,1 m2
7.2 His uncle pays him 9,1 m^2 × R6,00 per m^2 = R54,60
7.3 He earns 3 × R54,60 = R163,80
8.1 Novuyo lives 0,5 × 2,34 km = 1,17 km from the school.
8.2 Megan lives $\frac{1}{3}$ × 2,34 km = 0,78 km from the school.
8.3 2,34 km – 0,78 km = 1,56 km is the difference between the distance Peter and Megan live from the school.
8.4 3 km ÷ $\frac{1}{3}$ = 9 km 8.5 5 km ÷ $\frac{1}{2}$ = 10 km
9.1 Mr Mbeki earns R18 371,65 ÷ 0,875= R20 996,17
9.2 Mrs Sithole earns $\frac{\text{R18 371,65}}{1} \times \frac{2}{3} = \frac{\text{R36 743,30}}{3}$ = R12 247,77
10 4,92 m^2 ÷ 0,03 m^2 = 164 tiles
11.1 In 1 day he reads 1 415,25 pages 17 days = 83,25 pages in 1 day.
11.2 In 15 days he reads 83,25 × 15 = 1 248,75 pages.
11.3 In 7,5days he reads 7,5 × 83,25 = 624,38 pages.
12.1 He fills 167 370 ÷ 400 = 418,43 bottles.
12.2 He gets R12,45 × 418 = R5 204,10 for each full bottle.
12.3 R5,43 × 418 = R2 269,74 costs,
R5 204,10 – R 2 269,74 = R2 934,36 profit

Review Copy

Remedial

Provide simpler Grade 7 examples for learners to practise before doing this exercise. Assign learners to mixed ability groups, where a strong learner can help the other learners to identify the correct equation and help learners solve the problem.

Unit 7 Equivalent forms

Learner's Book page 242

Unit focus:

- revision of equivalent forms of decimals, common fractions and percentages.

Background information on equivalent forms

Learners should know that decimal fractions, such as 0,1 and 0,01 can be written as common fractions. For example, $0{,}1 = \frac{1}{10}$ and $0{,}01 = \frac{1}{100}$. These fractions have the same value and therefore they are equivalent. Learners should also be familiar with the fact that decimal fractions can also be written as percentages.

For example $0{,}03 = \frac{3}{100} = 3\%$. Remember that a percentage is always a value out of 100. So, $0{,}03 = \frac{3}{100} = 3\%$. These numbers are all equivalent.

Review Copy

Exercise 1

Learner's Book page 243

Guidelines on how to implement this activity

Revise H T U, t h and ensure that learners recognise that digits have positional value. For example a 3 in the tens column is worth 30, whereas a 3 in the tenths column is worth $\frac{3}{10}$. Revise with learners that all the decimal fractions can be written as common fractions; for example, $0{,}1 = \frac{1}{10}$ and $0{,}01 = \frac{1}{100}$. These are equivalent fractions because they have the same value. Do other examples as a class of equivalent decimal and common fractions.

Ask volunteers to work on the board to convert some decimal fractions to common fractions. Say: Write down 0,7 and then write it as a common fraction $\frac{7}{10}$. Next give them decimal fractions that, when converted, can be simplified.

For example $0{,}5 = \frac{5}{10} \div \frac{5}{5} = \frac{1}{2}$. Explain that these are all equivalent.

Common fractions can have many numbers as denominators: $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$, and these denominators need to be converted to either 10, 100 or 1 000.

When a common fraction has 100 as the denominator it is a percentage. For example $2\% = \frac{2}{100}$ and $78\% = \frac{78}{100}$ and $103\% = \frac{103}{100}$. So $0{,}09 = \frac{9}{100} = 9\%$.

Again have learners come up to the board and practise converting between fractions, decimals and percentage. Learners should complete this exercise on their own.

Suggested answers

1.1

Decimal fraction	Common fraction (simplest form)	Percentage
0,1	$\frac{1}{10}$	10%
0,5	$\frac{1}{2}$	50%
0,75	$\frac{3}{4}$	75%
0,68	$\frac{17}{25}$	68%
0,666	$\frac{2}{3}$	66,67%
1,37	$1\frac{37}{100}$	137%
2,555	$2\frac{555}{1\,000}$	255,5%
0,16667	$\frac{1}{6}$	16,67%
0,88	$\frac{22}{25}$	88%
1 234,655	$1\,234\frac{655}{100}$	1 234,655%

1.2

Decimal fractions	Common fraction in simplest form	Percentages
3,8	$3\frac{4}{5}$	380%
1,36	$1\frac{9}{25}$	136%
0,055	$1\frac{55}{1\,000}=\frac{11}{200}$	5,5%
9,99	$\frac{999}{1\,000}$	999%
0,54	$\frac{54}{100}=\frac{27}{50}$	54%
1,24	$1\frac{6}{25}$	124%
0,667	$\frac{2}{3}$	66,67%
0,9	$\frac{90}{100}$	90%
0,125	$\frac{125}{1\,000}$	12,5%

Review Copy

2.1

0	0,05	0,15	0,25	0,35	0,45
0	$\frac{1}{20}$	$\frac{3}{20}$	$\frac{1}{4}$	$\frac{7}{20}$	$\frac{9}{20}$
0%	5%	15%	25%	35%	45%

3.1 $75\% = \frac{3}{4} = 0{,}75$

3.2 $12\% = \frac{3}{25} = 0{,}12$

3.3 $250\% = 2\frac{1}{2} = 2{,}50$

3.4 $12{,}5\% = \frac{1}{8} = 0{,}125$

Remedial

Learners should not have problems converting, and if they do, provide plenty of Grade 7 examples for learners to revise this section. It is important that learners master this unit.

Give a worksheet with only decimal fractions, for example 0,3 and 0,9 and 0,01 and 0,7 etc. They must write these as common fractions $\left(\frac{3}{10}, \frac{9}{10}, \frac{1}{100} \text{ and } \frac{7}{10}\right)$.

Give a worksheet with only fractions that can simplify, for example 0,6 and 0,2 and 0,05 and 0,8 etc. They must write these as common fractions and simplify them. $\left(\frac{6}{10} = \frac{3}{5} \text{ and } \frac{2}{10} = \frac{1}{5} \text{ and } \frac{5}{100} = \frac{1}{20} \text{ and } \frac{6}{10} = \frac{4}{5}\right)$

Let them write the equivalent fractions in pairs: $0{,}3 = \frac{3}{10}$ etc.

Consolidation

Learner's Book page 246

Review Copy

Before doing this consolidation exercise, encourage learners to review the work covered in this chapter. Advise learners to use the summary and to revise their work. This exercise can be used as an informal assessment task for you to track how learners are coping with the chapter and the concepts covered. The mark allocation provides guidelines on how to assess learners.

Suggested answers

1 Ascending order: 9,1999; 9,99; 10,001; 10,09; 10,7 (1)

2 Descending order: 76,70; 67,887; 67,08; 7,67; 7,666 (1)

3.1 237,0909 + 16,18 = 253,2709 (2)

3.2 563,888 + 42,66 = 606,548 (2)

4.1
$$\begin{array}{r} 459{,}0236 \\ +\quad 3{,}5460 \\ \hline 462{,}5696 \end{array} = 462{,}570$$

4.2
$$\begin{array}{r} 679{,}2877 \\ +\quad 493{,}6767 \\ \hline 1172{,}9644 \end{array} = 1172{,}964$$

4.3
$$\begin{array}{r} 80{,}1029 \\ -\quad 67{,}1280 \\ \hline 12{,}9749 \end{array} = 12{,}975$$

4.4
$$\begin{array}{r} 444{,}9546 \\ -\quad 54{,}0870 \\ \hline 390{,}8676 \end{array} = 390{,}868$$

(8)

5.1
$$\begin{array}{r} 563{,}5630 \\ +\quad 87{,}0787 \\ +\quad 9{,}5550 \\ \hline 660{,}1967 \end{array} = 660{,}197$$

(2)

5.2
$$\begin{array}{r} 303{,}0202 \\ -\quad 299{,}8880 \\ \hline 3{,}1322 \end{array} = 3{,}132$$

(3)

6.1 $31{,}13 \times 4{,}5 \rightarrow 311{,}3 \times 45 = 1\ 4008{,}5 \rightarrow$ so $31{,}13 \times 4{,}5 = 140{,}085$ (3)

6.2 $761{,}22 \times 2{,}33 \rightarrow 7\ 6122 \times 233 = 1\ 7736426 \rightarrow$ so $761{,}22 \times 2{,}33 = 1\ 773{,}6426$ (3)

7.1 $2{,}52 \div 1{,}2 = 25{,}2 \div 12 = 2{,}1$ (3)

7.2 $0{,}2788 \div 0{,}34 = 27{,}88 \div 34 = 0{,}82$ (3)

8.1 $\sqrt{0{,}64} = 0{,}8$ (2)

8.2 $\sqrt{0{,}64} = 0{,}6$ (2)

8.3 $\sqrt{19{,}9} = 4{,}46$ the answers must be between $\sqrt{16}$ and $\sqrt{25}$ (2)

8.4 $\sqrt{12{,}8} = 3{,}58$ the answers must be between $\sqrt{9}$ and $\sqrt{16}$ (2)

9.1 Altogether they run 6,3 + 7,3 + 4,85 km =18,45 km (1)

9.2 Peter runs the furthest. (1)

9.3 The difference between the distance Peter runs and Ronald runs is 7,3 – 4,85 km = 2,45 km. (3)

10

Decimal fractions	Common fraction in simplest form	Percentages
1,6	$\frac{8}{5}$	160%
1,75	$\frac{7}{4}$	175%
0,025	$\frac{1}{40}$	2,5%
0,383	$\frac{383}{1\ 000}$	38,3%
0,58	$\frac{29}{50}$	58%

(8)

Total marks [50]

Review Copy

Chapter 12 The Theorem of Pythagoras

Overview of concepts

UNIT 1
The Theorem of Pythagoras

UNIT 2
Solve triangles using Pythagoras

The Theorem of Pythagoras

Content		Time allocations	LB page
Unit 1	The Theorem of Pythagoras	2 hours	248
Unit 2	Solve triangles using Pythagoras	1 hours	252

Review Copy

Background information on the Theorem of Pythagoras

Learners have already learnt about right-angled triangles and how to construct them in Chapters 7 and 8. This is learners' first introduction to a formal mathematical theorem. It is thus important that it is introduced correctly, in order to lay a foundation for future geometry and trigonometry work.

Pythagoras was a famous Ancient Greek mathematician and philosopher. He established the Pythagorean School of Mathematics at the Greek sea port of Cortona, in Southern Italy. Although this theorem is named after Pythagoras, it was known and used by the Babylonians at least 1 000 years before the Greek mathematician and philosopher adopted it. The theorem of Pythagoras showed the relationship of the hypotenuse to its sides, but it also showed that irrational numbers could exist. The notion of irrational numbers was considered so disturbing and revolutionary, that it became a secret that was kept hidden for many years due to its supposed power to destabilize the understanding of mathematics at the time.

Generic teaching guidelines for teaching the theorem of Pythagoras

Ensure all learners can draw a square, a rectangle and a right-angled triangle. Assess the learners' knowledge of the properties of each of these shapes. Learners should also know

that the area of a square is the square of its side length. Revise this if necessary. Make sure that the learners understand which side of a right-angled triangle is called the *hypotenuse*.

Provide context for learners in terms of the ancient builders and craftsmen who wanted to calculate the lengths of diagonals, of stones or doorways, for example. The difficulty in calculating the diagonal is what drove Pythagoras to determine his theorem.

Resources

Cardboard strip; colour pens, A4 paper or card; grid paper; scissors; measuring tape; ruler; and a metre rule. Each learner should have their own calculator.

Unit 1 The theorem of Pythagoras

Learner's Book page 248

Unit focus:

- learning about the relationship between the lengths of the sides of a right-angled triangle, known as the theorem of Pythagoras
- learning how to use Pythagoras' theorem to determine whether a given triangle is right-angled or not.

Exercise 1

Learner's Book page 249

Review Copy

Guidelines on how to implement this activity

Prepare cardboard strips as required in this exercise as well as sheets of A4-paper, preferably with a ΔABC already drawn in as required in step 3. Make enough copies so that pairs of learners can work with it on their desks.

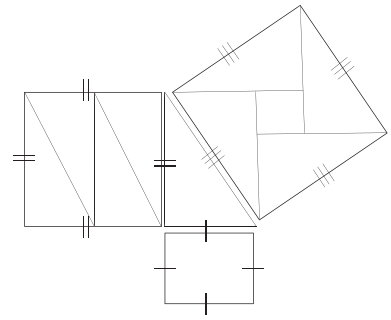
Write the wording of Pythagoras' theorem on the board and tell the learners that they are to demonstrate the meaning of this theorem with the material you are going to hand out.

Hand out the pairs of cardboard strips and scissors to the learners in pairs and tell them to cut out the 8 triangles and two 3 cm by 3 cm squares as accurately as possible, from each strip.

Now hand out the A4 sheets with ΔABC drawn or printed in its middle. Let the learners do steps 4 and 5 of Exercise 1. The results should look like that in the answer to Exercise 1 (the so-called 'windmill figure'). Make sure that the learners see the relationship between their 'windmill' figures and the wording of Pythagoras theorem.

Suggested answers

This is an indication of what the learners pieces should look like:



Remedial

For learners who lack background knowledge about squares and rectangles and how to calculate their areas, provide guidance and

Grade 7 material for learners to revise with. Worksheets of squares and rectangles drawn on 1 cm grids is a useful aid.

Some learners may lack drawing skills with pencil, ruler and protractor. Let them draw squares, rectangles and triangles on paper using these instruments.

Be available to assist as required, any learners who did not succeed in building the 'windmill figure' of Exercise 1.

Exercise 2

Learner's Book page 250

Guidelines on how to implement this activity

Work through Worked example 1 as a class. Discuss the Pythagoras relationship for a ΔPQR.

Let the learners do Exercise 2 on their own, and discuss the answers afterwards.

Suggested answers

1.1

Triangle	a	b	c	a^2	b^2	$a^2 + b^2$	c^2
1	8 cm	6 cm	10 cm	64 cm^2	36 cm^2	100 cm^2	100 cm^2
2	12cm	5 cm	13 cm	144 cm^2	25 cm^2	169 cm^2	169 cm^2
3	10 m	10 m	15 m	100 m^2	100 m^2	200 m^2	225 m^2
4	9 m	12 m	15 m	81 m^2	144 m^2	225 m^2	225 m^2
5	5 mm	12 mm	15 mm	25 mm^2	144 mm^2	169 mm^2	225 mm^2

1.2 1; 2; 4

2.1. $EF^2 = 7^2 = 49$ cm^2
$FG^2 + EG^2 = 6^2 + 5^2 = 36 + 25 = 61$ cm^2

2.2 acute angle

2.3 ΔEFG is not a right-angled triangle

3.1 $PQ^2 = 7^2 = 49$ cm^2
$QR^2 + PR^2 = 5^2 + 3^2 = 25 + 9 = 34$ cm^2

3.2 obtuse angle

3.3 ΔPQR is not a right-angled triangle

4 "The sum of the squares of the shorter sides of a triangle is only equal to the square of the longest side if the triangle is a *right-angled* triangle."

Review Copy

Remedial

Some learners may have difficulty with units of length and area. Revise these by showing centimetres and millimetres on a ruler and metres with a measuring tape, or metre ruler. Also drawings of square centimetres, a square metre and square millimetres.

For learners who lack background knowledge about triangles, revision must be done so that they can distinguish between right-angled triangles, acute-angled triangles, obtuse-angled triangles, isosceles triangles and equilateral triangles.

Extension

Encourage learners to find out about other ancient mathematicians and the theorems they may have contributed to 21st century mathematics.

Exercise 3

Learner's Book page 251

Guidelines on how to implement this activity

The worked examples and exercise in this unit is an extension of the worked examples and Exercise 2 of Unit 1. So refer to the statement of the answer to Question 4 of that exercise namely "The sum of the squares of the shorter sides of a triangle is only equal to the square of the longest side if the triangle is a *right-angled* triangle."

Draw a rough sketch of ΔPQR of Worked example 1, Question 1 on the board that does not show that $\hat{R}$ is a right angle.

Ask the learners: 'Which side is the longest side and opposite which angle does it lie?' Ask the learners: 'How can we find out whether ΔPQR is right-angled at vertex R?'. Answer: 'By checking it against Pythagoras theorem'; that is, by checking whether $PQ^2 = PR^2 + QR^2$.

Do the calculations on the board and ask the learners to draw a conclusion. Do the same with Worked example 1 Question 2

Let the learners do Exercise 3 on their own.

Suggested answers

1.1 In ΔABC: AC is the longest side
$AC^2 = (15)^2 = 225\ cm^2$
$AB^2 + BC^2 = 9^2 + 12^2 = 81+144 = 225\ cm^2$
So $AC^2 = AB^2 + BC^2$ therefore ΔABC is right-angled and $A\hat{B}C = 90°$

1.2 In ΔDEF: DF is the longest side
$DF^2 = (17)^2 = 289\ mm^2$
$DE^2 + EF^2 = 12^2 + 12^2 = 144 + 144 = 288\ mm^2$
So $DF^2 \neq DE^2 + EF^2$ therefore ΔDEF is not right-angled

2.1 In ΔPQR: PR is the longest side
$PR^2 = (11)^2 = 121\ m^2$
$PQ^2 + QR^2 = 5^2 + 7^2 = 25 + 49 = 74\ m^2$
So $PR^2 \neq PQ^2 + QR^2$ therefore ΔPQR is not right-angled

2.2 In ΔXYZ: YZ is the longest side
$YZ^2 = (39)^2 = 1\ 521\ cm^2$
$XY^2 + XZ^2 = 15^2 + 36^2 = 225 + 1\ 296 = 1\ 521\ cm^2$
So $YZ^2 = XY^2 + XZ^2$ therefore ΔXYZ is right-angled and $Y\hat{X}Z = 90°$

2.3 In ΔKLM: KM is the longest side
$KM^2 = (\sqrt{8})^2 = 8\ m^2$
$KL^2 + LM^2 = 2^2 + 2^2 = 4 + 4 = 8\ m^2$
So $KM^2 = KL^2 + LM^2$ therefore ΔKLM is right-angled and $K\hat{L}M = 90°$

2.4 In ΔSTU: SU is the longest side
$SU^2 = (18)^2 = 324\ mm^2$
$ST^2 + TU^2 = 12^2 + 16^2 = 144 + 256 = 400\ mm^2$
So $SU^2 \neq ST^2 + TU^2$ therefore ΔSTU is not right-angled

2.5 In ΔFGH,FH is the longest side.
$FH^2 = (10\sqrt{13})^2 = 1\ 300\ m^2$
$FG^2 + GH^2 = 20^2 + 30^2 = 400 + 900 = 1\ 300\ m^2$
So $FH^2 = FG^2 + GH^2$ therefore ΔFGH is right-angled and $F\hat{G}H = 90°$

Review Copy

Remedial

Some learners may struggle to write down the Pythagoras relationship that has to be checked for a given triangle. Ask such a learner 'Which side of this triangle is the longest of the three sides?' Then explain that we have to check whether the square of this longest side is equal to the sum of the squares of the two other sides.

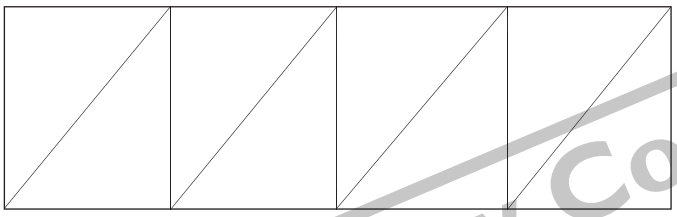
Demonstrate the format of this checking process as set out in Worked examples 1 and 2.

Some learners may be inclined to make mistakes in calculating the squares and /or sum of squares of side lengths. Encourage them to check their calculations.

Extension

On cm^2-dotted paper let learners draw a right-angled triangle with, for example, side lengths 3 cm, 4 cm and 5 cm, and then add a few more congruent triangles as shown below.

Challenge the learners to create other shapes by arranging the triangles in different patterns.



Review Copy

Unit 2 Solving triangles using Pythagoras

Learner's Book page 252

Unit focus:

- learning to apply the theorem of Pythagoras to calculate the unknown sides of right-angled triangles.

Background information on solving triangles using Pythagoras

The specific background for this unit is Unit 1 of this chapter. When tackling this unit learners should have first mastered Unit 1.

Exercise 1

Learner's Book page 253

Guidelines on how to implement this activity

Let the learners close their text books so that they have to think for themselves when answering your questions. Alternatively you can choose to use worked examples of you own, different from those of the textbook.

Draw a rough sketch of ΔABC of Worked example 1 on the board and ask the learners "Which side of ΔABC is the hypotenuse?". Then write 'AC^2 = 'on the board and ask them to tell you what to write on the R.H.S. according to Pythagoras.

Complete the Worked example 1 up to '$x^2 = 18\ cm^2$' on the board with participation of the learners.

Let the learners do Exercise 1 numbers 1.1, 1.2 and 1.3 that are similar to Worked example 1. Follow the same procedure with Worked examples 2 and 3.Let the learners do a selection of Exercise 1 from number 2 onwards. Some problems can be given as homework.

Suggested answers

1.1
$$AC^2 = AB^2 + BC^2$$
$$b^2 = 24^2 + 10^2$$
$$b^2 = 576 + 100$$
$$b^2 = 676$$
$$\sqrt{b^2} = \sqrt{676}$$
$$b = 26\ cm$$

1.2
$$DF^2 = EF^2 + DE^2$$
$$x^2 = 10^2 + 10^2$$
$$x^2 = 100 + 100$$
$$x^2 = 200$$
$$\sqrt{x^2} = \sqrt{200}$$
$$x = \sqrt{100 \times 2}$$
$$x = 10\sqrt{2}$$

1.3
$$ML^2 = KM^2 + KL^2$$
$$k^2 = 2{,}5^2 + 1{,}5^2$$
$$k^2 = \frac{25}{16} + \frac{9}{4}$$
$$k^2 = \frac{34}{4}$$
$$k = \frac{\sqrt{34}}{2}$$

2.1
$$AB^2 = AC^2 + BC^2$$
$$15^2 = 9^2 + a^2$$
$$a^2 = 15^2 - 9^2$$
$$= 225 - 81$$
$$= 144$$
$$\therefore a = \sqrt{144} = 12$$

2.2
$$HJ^2 = HI^2 + JI^2$$
$$8^2 = j^2 + 5^2$$
$$j^2 = 8^2 - 5^2$$
$$= 64 - 25$$
$$= 39$$
$$\therefore j = \sqrt{39}$$

2.3
$$TU^2 = TV^2 + UV^2$$
$$v^2 = 7^2 + 5^2$$
$$= 49 + 25$$
$$= 74$$
$$\therefore v = \sqrt{74}$$

3
$$DE^2 = DF^2 + EF^2$$
$$10^2 = y^2 + y^2$$
$$100 = 2y^2 \therefore y^2 = \sqrt{74}$$
$$y^2 = \frac{100}{2}$$
$$y^2 = 50$$
$$\sqrt{y^2} = \sqrt{25 \text{ x } 2}$$
$$y = 5\sqrt{2}\ m$$

Review Copy

4 $KN^2 = KM^2 + MN^2$

$x^2 = 2{,}5^2 + 6^2$

$x^2 = \frac{25}{4} + 36$

$x^2 = \frac{169}{4}$

$\sqrt{x^2} = \sqrt{\frac{169}{4}}$

$x = \frac{13}{4}$ m = 3,25 m

5.1 $AC^2 = AB^2 + BC^2$

$8^2 = AB^2 + 4^2$

$64 = AB^2 + 16$

$AB^2 = 64 - 16$

$AB^2 = 48$

$\sqrt{AB^2} = 4\sqrt{48}$

$AB = \sqrt{16 \times 3}$

$AB = 4\sqrt{3}$ cm

5.2 $PQ^2 = PR^2 + QR^2$

$PQ^2 = 6^2 + 12^2$

$PQ^2 = 36 + 144$

$PQ^2 = 180$

$\sqrt{PQ^2} = \sqrt{180}$

$PQ = \sqrt{36 \times 5}$

$PQ = 6\sqrt{5}$ m

5.3 $LM^2 = KL^2 + KM^2$

$3 = 1 + KM^2$

$KM^2 = 3 - 1$

$KM^2 = 2$

$KM = \sqrt{2}$

6 $FH^2 = HG^2 + FG^2$

$FH^2 = 3^2 + 3^2$

$FH^2 = 9 + 9$

$FH^2 = 18$

$\sqrt{FH^2} = \sqrt{18}$

$FH = \sqrt{9 \times 2}$

$FH = 3\sqrt{2}$ m

7.1 Area of the square on PR = 4 cm² + 16 cm² = 20 cm²

7.2 $PQ = \sqrt{4} = 2$ cm

$QR = \sqrt{16} = 4$ cm

$PR = \sqrt{20} = 4{,}47$ cm

Remedial

Some learners may have difficulty in writing an equation to calculate the length of a side other than the hypotenuse of a right-angled triangle. Revise Worked examples 1 Questions 2 and 3 or go through similar exercises step-by-step with them.

Some learners may have difficulty in interpreting and writing side lengths in surd form. Revise how composite numbers (divisible by at least one integer) can be factorized into prime factors of which some may come in pairs, for example:

$12 = 2^2 \times 3$; therefore $\sqrt{12} = \sqrt{2^2 \times 3} = 2\sqrt{3}$

Extension

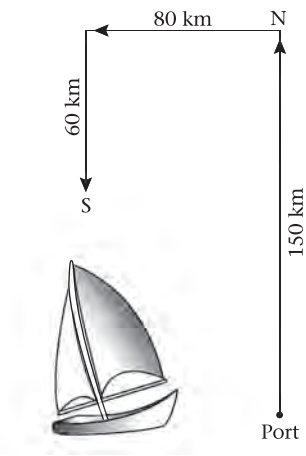
Let the learners work out practical applications of Pythagoras' theorem such as this navigation problem:

Review Copy

A ship sails 150 km due (directly) north from the port, then 80 km due west, then 60 km due south.

1 Calculate the longest distance that the ship was away from the port.

2 Calculate the direct distance from the port to the final position of the ship.



Exercise 2

Learner's Book page 255

Guidelines on how to implement this activity

Revise with learners how to prove similarity in triangles, by showing that corresponding sides of triangles are in proportion.

Revise the conditions for congruency. Refer learners back to the chapters where they covered these concepts.

Explain to learners that this exercise uses Pythagoras to solve for unknown sides, in order to help us prove similarity and congruency. Work through the worked example together as a class.

Discuss how Pythagoras be brought into the process of finding unknown sides of triangles. Allow learners to work in pairs, although each learner must supply their own working and answers in their exercise books.

Review Copy

Suggested answers

1.1 $PQ^2 = PT^2 + TQ^2$
$5^2 = PT^2 + 3^2$
$PT^2 = 5^2 - 3^2$
$= 25 - 9$
$= 16$
$\therefore PT = \sqrt{16} = 4$ m
Similarily, TR = 4 m

1.2 In ΔRTQ and ΔPTQ:
QT common
PQ = QR (given)
$P\hat{T}Q = R\hat{T}Q = 90^o$
$\therefore \Delta PQT \equiv \Delta RQT$ (RHS)

1.3 $PS^2 = SR^2 + PR^2$
$= 6^2 + (4 + 4)^2$
$= 6^2 + 8^2$
$= 36 + 64$
$= 100$
$\therefore PS = \sqrt{100} = 10$

1.4 In ΔPRS and ΔPTQ:
PR : PT = 8 : 4 = 2 : 1
SR : TQ = 6 : 3 = 2 : 1
PS : PQ = 10 : 5 = 2 : 1
All corresponding sides are in proportion, so $\Delta PRS \parallel\!| \Delta PTQ$ is.

2.1 $XY = KL = 10\sqrt{3}$ cm

2.2 Pythagoras only applies to right-angled triangles, so if the square of the longest side is equal to the sum of the squares of the other sides, then the triangle is right-angled.

$XZ^2 = ZY^2 + XY^2$

$20^2 = 10^2 + (10\sqrt{3})^2$

$20^2 = 100 + 300$

$400 = 100 + 300$

LHS = RHS so ΔXYZ is a right-angled triangle.

Remedial

Spend as much time as necessary revising similarity and congruency. Learners need to be able to use Pythagoras to help prove these concepts.

Consolidation

Learner's Book page 257

Before doing this consolidation exercise, encourage learners to review the work covered in this chapter. Advise learners to use the summary and to revise their work. This exercise can be used as an informal assessment task for you to track how learners are coping with the chapter and the concepts covered. The mark allocation provides guidelines on how to assess learners.

Review Copy

Suggested answers

1.1. In ΔPQR: PR is the longest side (2)

$PR^2 = 25^2 = 625$ cm^2

$PQ^2 + QR^2 = 15^2 + 20^2 = 225 + 400 = 625$ cm^2

So $PR^2 = PQ^2 + QR^2$ and therefore ΔPQR is right-angled and $P\hat{Q}R = 90°$

1.2 In ΔKLM: LM is the longest side (2)

$LM^2 = (6)^2 = 36$ m^2

$KL^2 + KM^2 = 3^2 + 5^2 = 9 + 25 = 34$ m^2

So $LM^2 \neq KL^2 + KM^2$ and therefore ΔKLM is not right-angled.

1.3 In ΔABC: BC is the longest side (2)

$BC^2 = (12\sqrt{2})^2 = 288$ cm^2

$AB^2 + AC^2 = 12^2 + 12^2 = 144 + 144 = 288$ cm^2

So $BC^2 = AB^2 + AC^2$ and therefore ΔABC right-angled and $B\hat{A}C = 90°$

1.4 In ΔDEF: EF is the longest side (2)

$EF^2 = (\sqrt{5})^2 = 5$ km^2

$DE^2 + DF^2 = 1^2 + 2^2 = 1 + 4 = 5$ km^2

So $EF^2 = DE^2 + DF^2$ and therefore ΔDEF is right-angled and $E\hat{D}F = 90°$

1.5 In ΔXYZ: XZ is the longest side (2)

$XZ^2 = (20)^2 = 400$ mm^2

$YZ^2 + XY^2 = 16^2 + 16^2 = 256 + 256 = 512$ mm^2

So $XZ^2 \neq YZ^2 + XY^2$ and therefore ΔXYZ is not right-angled.

2.1 $AC^2 = AB^2 + BC^2$
$50^2 = 30^2 + a^2$
$a^2 = 2\ 500 - 900$
$a^2 = 1600$
$\sqrt{a^2} = \sqrt{1600}$
$a = 40$ m

2.2 $ED^2 = FD^2 + EF^2$
$9^2 = 6^2 + x^2$
$x^2 = 81 - 36$
$x^2 = 45$
$\sqrt{x^2} = \sqrt{45}$
$x = \sqrt{9 \times 5} = 3\sqrt{5}$ cm (3)

2.3 $JL^2 = JK^2 + KL^2$
$24^2 = y^2 + 7^2$
$y^2 = 576 - 49$
$= 527$
$y = \sqrt{527}$

2.4 $PR^2 = PQ^2 + QR^2$
$17^2 = 15^2 + QR^2$
$QR^2 = 289 - 225$
$QR^2 = 64$
$\sqrt{QR^2} = \sqrt{64}$
QR = 8 cm (2)

3 In ΔPQR: $PR^2 = PQ^2 + QR^2$
$PR^2 = 40^2 + 9^2$
$PR^2 = 1\ 600 + 81 = 1\ 681$
$\sqrt{PR^2} = \sqrt{1\ 681}$
PR = 41 mm (3)

4.1 $5^2 = 4^2 + x^2$
$x^2 = 5^2 - 4^2$
$x^2 = 25 - 16$
$x^2 = 9$

Review Copy

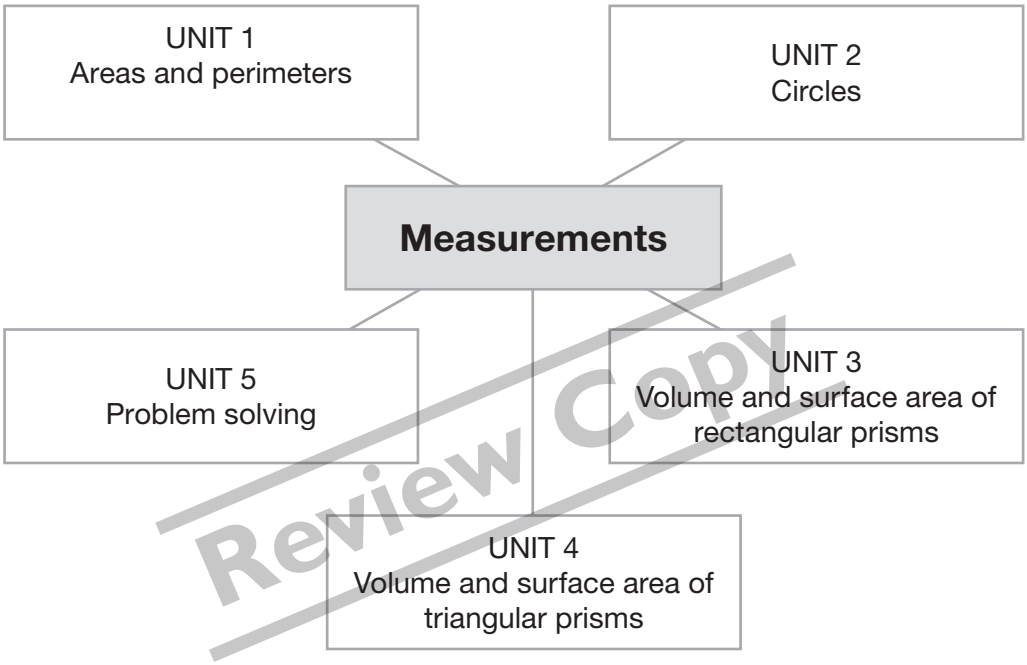
$\therefore x = 3$ m (3)
You must put your ladder 3 m from the base of the wall.

4.2 $x^2 = 8^2 + 6^2$
$x^2 = 64 + 36$
$x^2 = 100$
$\therefore x = 10$ m
The wire is 10 m long. (4)

Total marks [25]

Chapter 13 Measurement

Overview of concepts



Review Copy

Content		Time allocations	LB page
Unit 1	Areas and perimeters	4 hours	259
Unit 2	Circles	1 hour	266
Unit 3	Volume and surface area of rectangular prisms	2 hours	272
Unit 4	Volume and surface area of triangular prisms	2 hours	276
Unit 5	Problem solving	1 hour	283

Background information on measurement

- Learners have learnt about 2D geometric shapes such as squares, rectangles and triangles, as well as 3D objects such as cubes and rectangular prisms in Grade 7 and in the intermediate phase.
- Learners should also be familiar with the concepts perimeter, area and volume as well as the metric units for these.

Generic teaching guidelines for teaching measurement

- Measurement is one of the most practical topics in the curriculum and can, and should be used to show how mathematics is applied to practical situations with which the learners are familiar, such as measuring a door and calculating its perimeter, its area and its volume.
- Learners' knowledge of the metric units and skill in converting between different units is crucial to doing measurement calculations. Revise and demonstrate this as often as necessary.
- Interpreting and drawing geometric drawings of 3D objects on paper or on the board can be quite a challenge to both teachers and learners. Physical models of prisms and nets of these are useful to overcome this problem.

Resources

Cardboard; colour pens; scissors; polygon pictures; protractors; circular objects to measure; string; ruler; 3D objects; boxes; glue; conversion charts. Each learner should have their own calculator.

Unit 1 Areas and perimeters

Learner's Book page 259

Review Copy

Unit focus:

- calculating the perimeter of regular and irregular polygons
- calculating the perimeter and area of squares, rectangles and triangles using appropriate formulae and correct to at least one decimal place
- using and converting between appropriate SI units including
  - mm^2 to cm^2 and vice versa
  - cm^2 to m^2 and vice versa.

Background information on areas and perimeters

Learners should be familiar with the concepts of area and perimeter from the intermediate phase. Perimeter is known as the distance around the shape. In the intermediate phase learners understood area in terms of 1 cm by 1 cm squares inside a shape. In Grade 7 learners extended their knowledge of area to include formulae. They learnt the area formulae for squares, rectangles and triangles.

Exercise 1

Learner's Book page 262

Guidelines on how to implement this activity

Begin the lesson by evaluating the learners' knowledge of the formulas they learnt in grade 7 by drawing a square, a rectangle and a triangle on the board and asking them what the formulas for perimeter and area of these shapes are.

- Show learners that in the case of the triangle, there are three possible base lengths and heights that can be used. Worked example 1 is relevant for this.
- Be sure to assess the learners' knowledge of the units they learnt in Grade 7 by starting with the main metric unit for length / distance, namely the metre.
- Ask questions such as: "Give an example of an object for which you will use square millimetres to express its area?" The same for square centimetres, square metres and square kilometres.

- Either demonstrate the Worked Examples 2.1, 2.2 and 2.3 on the board or choose your own examples to work through with participation of the learners. Monitor the learners' progress when they do problems from Exercise 1.

Suggested answers

1 Perimeter and area calculation

1.1 $P = 2(78 + 35) = 2(113) = 226$ cm

$A = 78 \times 35 = 2\,730$ cm^2

1.2 $P = 8 + 8 + 8 = 24$ cm

$A = \frac{1}{2} \times 8 \times 6{,}9 = 27{,}6$ m^2

2.1 $PS^2 = PR^2 - RS^2 = (13)^2 - (5)^2 = 169 - 25 = 144$

$PS\ \sqrt{144} = 12$ mm

2.2 Area $= \frac{1}{2} \times 10 \times 12 = 60$ mm^2

3.1 $P = 7 + 7 + 12 + 12 = 38$ cm

3.2 $P = 1{,}45 + 1{,}45 + 3{,}71 + 3{,}71 = 10{,}32$ m

3.3 $P = 27 + 15 + 19 + 15 = 76$ mm

3.4 $P = 10 \times 0.75 = 7{,}5$ m

3.5 $P = 5{,}955 + 3{,}31 + 4{,}72 + 2{,}08 + 2{,}43 = 18{,}495$ km

3.6 $P = 4 \times 20 + 4 \times 14{,}1 = 136{,}4$ cm

4.1 $P = 3 \times 15 = 45$ m

4.2 $P = 4 \times 27{,}8 = 111{,}2$ m

4.3 $P = 4 \times 1{,}5 = 6$ km

4.4 $P = 5 \times 17 = 85$ cm

4.5 $P = 6 \times 51 = 306$ mm

4.6 $P = 8 \times 35{,}5 = 284$ mm

Review Copy

Remedial

Some learners may have difficulty in applying the formula for the area of a triangle. Revise the different types of triangles: acute-angled, right-angled and obtuse-angled and how a 'base' and a 'height' can be identified in each case.

For those learners who struggle with writing down the steps of a solution to a problem, first choose simple examples for them to do and then follow up with problems requiring more steps.

Some learners may lack sufficient knowledge of the SI units and how to convert between these. Revise the appropriate units by also showing them these units in practice, for example 1 cm^2 = (10 mm × 10 mm) = 100 mm^2.

Extension

Challenge learners to identify triangular and other geometric shapes in the environment and to measure and calculate the perimeters and areas of these.

Exercise 2

Learner's Book page 265

Guidelines on how to implement this activity

- Revise the terms *parallelogram*, *trapezium*, *pentagon*, *hexagon* and *octagon* by drawing rough sketches on the board.
- Discuss the strategies for breaking up these complex polygons into simple shapes – such as triangles, squares and rectangles. Revise using Pythagoras to find unknown sides of right-angled triangles.

Suggested answers

1.1 $A = 8{,}5 \times 6 = 51\ cm^2$

1.2 $A = \frac{1}{2}(25 + 45) + 31{,}5 = 1\ 102{,}5\ mm^2$

$A = 11{,}03\ cm^2$

1.3 $x^2 = (1{,}5)^2 - (1{,}2)^2 = 2{,}25 - 1{,}44 = 0{,}81$
$x = \sqrt{0{,}81} = 0{,}9\ m$
$AC = 2 \times 0{,}9 = 1{,}8\ m$
$BE^2 = (1)^2 - (0{,}9)^2 = 1 - 0{,}81 = 0{,}19$
$BE = \sqrt{0{,}19} = 0{,}44\ m$

2.1 $MR = MS = \frac{1}{2}\ QT = 60$ $QT = 120\ cm$
$h = (60)^2 - (30)^2 = 3600 - 900 = 2700$
$h\ \sqrt{2\ 700} = 51{,}96\ cm$
$\Delta MRS\ \frac{1}{2} = 60 \times 51{,}96 = 1\ 558{,}8\ cm^2$
Area $\Delta PQRSTU = 6 \times 1{,}558{,}8 = 9\ 352{,}8\ cm^2$

2.2.1 $h^2 = (8)^2 - (5)^2 = 64 - 25 = 39$
$h = \sqrt{39} = 6{,}24\ cm$

2.2.2 $\Delta OIH = \frac{1}{2} \times 10 \times 6{,}24 = 31{,}2\ m^2$

2.2.3 Area $\Delta FGHIJ = 5 \times 31{,}2 = 156\ m^2$

Review Copy

Remedial

If learners experience difficulty, demonstrate again how to break up polygons into familiar shapes in order to work out the area.

Extension

Provide a worksheet of geometric shapes like a regular octagon of which the perimeter and/or area has to be determined.

Unit 2 Circles

Learner's Book page 266

Unit focus:

- using and describing the relationship between the radius, diameter and circumference of a circle in calculations

- using and describing the relationship between the radius and area of a circle in calculations.

Background information on circles

In Grade 7, as part of the geometry of 2D shapes, learners learnt to describe and name parts of a circle namely centre, arc, diameter and radius. Learners should have also learnt how to construct circles using a compass in Grade 7.

Investigation

Learner's Book page 266

Guidelines on how to implement this activity

- Since the concepts of circumference and area of a circle are new to Grade 8 it is appropriate to introduce it by means of a practical investigation. This investigation requires the learners to measure the circumferences and diameters of a few round objects like tins or the lids of cooking pots.
- By calculating the ratio of circumference to diameter for at least three round objects, the learners will come to realize that this ratio is always slightly more than 3. This is the appropriate stage to tell them about Pi (written as π) and that its precise value can only be approximated by rational values such as $\frac{22}{7}$ or the value that a calculator displays when you press the π-key.

Exercise 1

Learner's Book page 268

Review Copy

Guidelines for implementing this activity

- Learners have learnt the relationship between π, diameter and circumference from the Investigation.
- Work through the Worked examples on the board as a class and show learners how they can calculate the circumference when given either the diameter or radius of a circle.

Allow learners to work in pairs for Exercise 1.

Suggested answers

1 Circumference calculation without calculator

Diameter	14 cm	70 cm	8,4 m	1,75 km
$C = \pi d$	$\frac{22}{7} \times \frac{14}{1}$ $= 22 \times 2$ $= 44$ cm	$\frac{22}{7} \times \frac{70}{1}$ $= 22 \times 10$ $= 220$ cm	$\frac{22}{7} \times \frac{84}{10}$ $\frac{22}{1} \times \frac{12}{10} = \frac{264}{10}$ $= 26{,}4$ m	$\frac{22}{7} \times \frac{175}{100}$ $\frac{11}{1} \times \frac{25}{50}$ $\frac{11}{1} \times \frac{1}{2} = \frac{11}{2}$ $= 5{,}5$ km

2 Circumference calculation with calculator

Radius	7 cm	35 cm	4,2 m	0,875 km
$C = 2\pi r$	43,98 cm	219,91 mm	26,39 m	5,50 km

3 The answers are almost the same.

Remedial

Some learners may have difficulty with calculations where $\pi = \frac{22}{7}$ is used. Revise the multiples of 7 with them so that they can simplify fractions where some numerators are multiples of 7.

Extension

Challenge learners to identify circular objects in their environment, measure diagonals and/or circumferences and calculate radii, circumferences and areas.

Exercise 2

Learner's Book page 269

Guidelines on how to implement this activity

- Discuss how to use the formulae for calculating the circumference of a circle. And how we can use the circumference to find the diameter and radius of a circle. It is important that the learners understand the formulae $d = 2 \times r$, $C = \pi d$ and $C = 2\pi r$ and how they can be applied.
- Work through the Worked examples in the Learner's Book. Monitor the learners' progress when doing problems from Exercises 1 and 2.

Review Copy

Suggested answers

1.1 $d = C \div \pi = 165 \div \frac{22}{7} = 165 \times \frac{7}{22} = \frac{1\,155}{22} = \frac{105}{2}$ cm

1.2 $r = C \div 2\pi = 143 \div \frac{2 \times 22}{7} = 143 \times \frac{7}{44} = \frac{1001}{44} = \frac{91}{4}$ mm

1.3 $d = C \div \pi = 198 \div \frac{22}{7} = 198 \times \frac{7}{22} = 9 \times 7 = 63$ m

2.1 $d = C \div \pi = \frac{561}{\pi} = 178{,}57$ mm

2.2 $r = \frac{C}{2\pi} = \frac{31{,}416}{2\pi} \approx 5$

2.3 $C = \pi d = \pi \times 70 = 219{,}91$ cm

$219{,}91 \times 100 = 21991{,}14$ cm

$\frac{2199114}{100} = 219{,}91$ m

3.1 $r = \frac{C}{2\pi} = \frac{1}{2\pi} = 0{,}16$ m

3.2 $P = 120 + 120 + 90 + 90 = 420$ m

Remedial

Some learners may also need help with using a calculator value of π and to express answers to the required number of decimals. Individual attention to such a learner is required.

Extension

Recommend books and/or web sites that learners can read or visit to learn more about the interesting history of π.

Exercise 3

Learner's Book page 271

Guidelines on how to implement this activity

- Discuss the concept of the area of a circle. The formula for the area of a circle is not so easy to find by means of a practical investigation as in the case of the circumference of a circle. For the learners to follow the reasoning in the Learner's Book, it is advised that the teacher draw replicas of Figures 1 and 2 on the board, discuss each step orally and write it on the board.
- Use as many worked examples as possible to demonstrate how to use the area formula correctly.

Review Copy

Suggested answers

1.1 $A = \pi r^2 = \frac{22}{7} \times (7)^2 = \frac{22}{7} \times \frac{49}{1} = 22 \times 7 = 154 \text{ cm}^2$

1.2 $A = \pi r^2 = \frac{22}{7} \times (28)^2 = \frac{22}{7} \times \frac{784}{1} = 22 \times 112 = 2\,464 \text{ mm}^2$

1.3 $A = \pi r^2 = \frac{22}{7} \times (14)^2 = \frac{22}{7} \times \frac{196}{1} = 22 \times 328 = 616 \text{ m}^2$

1.4 $A = \pi r^2 = \frac{22}{7} \times (1{,}4)^2 = \frac{22}{7} \times \frac{196}{100} = \frac{11}{1} \times \frac{28}{50} = \frac{308}{50} = \frac{154}{25} \text{ km}^2$

2.1 $C = 2\pi r = 2\pi(2{,}5) = 15{,}71 \text{ m}$
$A = \pi r^2 = \pi(2{,}5)^2 = \pi \times 6{,}25 = 19{,}63 \text{ m}^2$

2.2 $C = \pi d = \pi(16{,}5) = 51{,}84 \text{ cm}$
$A = \pi r^2 = \pi\left(\frac{16{,}5}{2}\right)^2 = \pi(8{,}25)^2 = 50{,}82 \text{ cm}^2$

3.1 $A = \pi r^2 = \pi(4)^2 = \pi(16) = 50{,}27 \text{ cm}^2$
$A = \pi r^2 = \pi(2)^2 = \pi(4) = 12{,}57 \text{ cm}^2$
Area in between $= 50{,}27 - 12{,}57 = 37{,}7 \text{ cm}^2$

3.2 $C = 2\pi r = 2\pi(4) = 25{,}13 \text{ cm}$
$C = 2\pi r = 2\pi(2) = 12{,}57 \text{ cm}$

4.1 Area = Area rectangle + 2 semi-circles
$100 \times 25 \times 2\left(\frac{1}{2} \times \pi \times 25\right)$
$= 2\,500 + 2(32{,}27) = 2\,500 + 78{,}54 = 2\,578{,}54 \text{ m}^2$

4.2 Big area = $100 \times 30 \times 2\left(\frac{1}{2} \times \pi \times 30\right)$
= 3 000 + 2(47,12) = 3 000 + 98,24 = 3 098,24 m^2
Track area = 3 098,24 cm^2 – 2 578,54 = 519,7 m^2
4.3 Cost = 519,7 × 87,50 = R45 473,75

Remedial

For those learners who struggle with writing down the steps of a solution to a problem, first choose simple examples for them to do, and then follow up with problems requiring more steps.

Unit 3 Volume and surface area of rectangular prisms

Learner's Book page 272

Unit focus:

- revise the concepts of surface area, volume and capacity of cubes and rectangular prisms, and the units we use for these quantities.

Background information on volume and surface area of rectangular prisms

Learners have worked with volume and surface area in Grade 7. This unit is mostly revision of that work. Learners will work with increasingly complex numbers, involving decimals up to three decimal places in Grade 8.

Review Copy

Exercise 1

Learner's Book page 273

Guidelines on how to implement this activity

Demonstrate 3D objects, such as cubes and rectangular prisms and their nets with practical models such as a matchbox and a cubic object. By using nets the concept of surface area becomes clearer to learners.
If the learners understand how the formula for the total surface area of a rectangular prism is worked out, they should be able to do calculations of total surface areas even without remembering and writing down the formula:

$$A = 2(lb + lh + bh)$$

- Do a few worked examples together as a class, and if necessary revise how the formulae for surface area of a rectangular prism are deduced. Learners should complete this exercise on their own.

Suggested answers

1 Surface are of rectangular prisms
1.1 A = 2(48 × 21 + 30 × 48 + 21 × 30) = 2(1 008 + 1 440 + 630) = 2(3 078) = 6 156 cm^2
1.2 A = 2(0,75 × 4,5 + 1,5 × 0,75 + 4,5 × 1,5)
= 2(3,375 + 1,125 + 6,75) = 2(11,25) = 22,5 m^2
1.3 A = 2(0,9 × 0,9 + 0,9 × 9 + 0,9 × 9) = 2(0,81 + 8,1 + 8,1)
= 2(17,01) = 34,02 cm^2
2 Surface area of cubes

2.1 $A = 6(35)^2 = 6(1225) = 7\ 350\ cm^2$

$\frac{7\ 350}{10\ 000} = 0{,}735\ m^2$

2.2 $A = 6(1{,}25)^2 = 6(1{,}5625) = 9{,}375\ m^2$

$9{,}375 \times 10\ 000 = 93\ 750\ cm^2$

Remedial

- Some learners may have difficulty with conceptualising or visualising the 3D objects printed on 2D paper. Relate real 3D cubes and rectangular prisms to drawings in perspective of these to these learners.
- Some learners may also need help with the conversion of units. Revise the relationships between these units and how to convert from one to the other.

Extension

Challenge learners to identify rectangular prism objects in the environment, estimate and measure dimensions and calculate surface areas and volumes.

Exercise 2

Learner's Book page 275

Review Copy

Guidelines on how to implement this activity

- The purpose of the diagram of the rectangular 'see-through container' and the accompanying notes in the Learner's Book is to show how we arrive at the formula:
  Volume of rectangular prism = length × breadth × height.
- If learners can visualize this container and the layers of cubic centimetre cubes that fill it, they will have a good concept of volume and the formula for volume.
- Ensure learners understand the difference between volume and capacity.
- Pay special attention to the units of length, area and volume and how to convert from one to the other. Also the units litre, millilitre and kilolitre for capacity needs special attention.
- Demonstrate by means of worked examples how to determine the volume and capacity of objects.

Suggested answers

1 Volume calculations

1.1 $V = l \times b \times h = 12 \times 17 \times 5 = 1\ 020\ mm^3$

1.2 $V = l \times b \times h = 2 \times 4 \times 7{,}5 = 60\ dm^3$

1.3 $V = l \times b \times h = 5{,}2 \times 2{,}3 \times 0{,}5 = 5{,}98\ m^3$

2 Volume calculations

2.1 $V = 1\ 020$ mm3 $= 1{,}02\ cm^3 = 1{,}02\ m^3$

2.2 $V = 60\ dm^3 = 60\ l$

2.3 $V = 5{,}98\ m^3 = 5{,}98\ kl$

3 Unit conversions

3.1 $7{,}4\ cm^3 = 7\ 400\ mm^3$

3.2 $0{,}76\ l = 760\ ml$

3.3 $3\ 540\ l = 3{,}54\ k\ l$

3.4 $3\ 450\ dm^3 = 3{,}45\ m^3$

3.5 $0{,}25\ m^2 = 2\ 500\ cm^2$
3.6 $12{,}5\ cm^2 = 1\ 250\ mm^2$
4.1

Prism	l	b	h	$A = 2(lb + lh + bh)$	Base area	Volume
1	10 cm	10 cm	10 cm	600 cm^2	100 cm^2	1 000 cm^3
2	12 cm	11 cm	20 cm	1 184 cm^2	132 cm^2	2 640 cm^3
3	40 m	36 m	5 m	3 640 m^2	1 440 m^2	7 200 m^3

4.2 base area $= \frac{v}{h}$

Remedial

For those learners who struggle with writing down the steps of a solution to a problem, first choose simple examples for them to do and then follow it up with problems requiring more steps.

Extension

Recommend books and/or web sites that learners can read or visit to learn about non-metric units of area, volume and capacity such as square inches, cubic feet, pints and gallons.

Review Copy

Unit 4 Volume and surface area of triangular prisms

Learner's Book page 276

Unit focus:

- calculating the volume and surface area of triangular prisms
- converting between units of area
- converting between units of volume and capacity.

Background information on volume and surface area of triangular prisms

- As part of the Geometry of 3D objects, learners learnt in Grade 7 to describe, sort and compare polyhedrons as well as using nets to create models of cubes and prisms.
- As part of measurement, learners learnt in Grade 7 to calculate the surface area, volume and capacity of cubes and rectangular prisms, using appropriate formulae. Also how to use and how to convert between appropriate SI units.

Exercise 1

Learner's Book page 278

Guidelines on how to implement this activity

- Since this topic (volume and surface area of triangular prisms) is new to Grade 8, it is appropriate to show the learners a physical model of a triangular prism and its net, so that they can better interpret the drawings in the text book. Some of the drawings show a triangular prism standing on a triangular face while others show this kind of prism standing on one of its rectangular faces. This should be demonstrated with the aid of a physical model.

- Demonstrate using worked examples to show how to determine the volume of a triangular prism.

Suggested answers

1 Calculation of volumes

1.1 $V = \text{area } \Delta ABC \times l = \frac{1}{2} \times 20 \times 17{,}3 \times 30 = 5\ 190 \text{ cm}^3$

1.2 $V = \text{area } \Delta PQR \times l = \frac{1}{2} \times 4 \times 2{,}24 \times 5 = 22{,}4 \text{ m}^3$

1.3 $V = \text{area } \Delta KLM \times l = \frac{1}{2} \times 30 \times 22{,}5 \times 50 = 16\ ,875 \text{ m}^3$

Remedial

Some learners may have difficulty with interpreting the 2D drawings of 3D objects. Help them by demonstrating the correspondence with a physical model of a triangular prism.

Extension

Challenge learners to identify triangular prism objects in the environment, estimate and measure dimensions, and calculate surface areas and volumes.

Review Copy

Exercise 2

Learner's Book page 279

Guidelines on how to implement this activity

- Show learners a net of a triangular prism and discuss how one could go about determining the surface area of a triangular prism.
- Work through the worked examples and help learners determine the formula.

Learners should be able to complete this exercise on their own.

Suggested answers

1 Calculation of surface areas

1.1 $SA = bh + \text{perimeter } \Delta ABC \times l$
$= 20 \times 17{,}3 + 20 + 20 + 2\ 030 = 346 + 1\ 800 = 2\ 146 \text{ cm}^2$

1.2 $SA = bh + \text{perimeter } \Delta PQR \times l$
$= 4 \times 22{,}4 + 5 + 5 + 45 = 8{,}96 + 70 = 78{,}96 \text{ m}^2$

1.3 $SA = bh + \text{perimeter } \Delta KLM \times l$
$30 \times 22{,}5 + 30 + 22{,}5 + 37{,}550 = 675 + 4\ 500 = 5\ 175 \text{ m}^2$

2.1 $h^2 = PQ^2 - QM^2 = (1{,}3)^2 - (0{,}5)^2 = 1{,}69 - 0{,}25 = 1{,}44$
$h = \sqrt{144} = 1{,}2 \text{ m}$

2.2 $MQ^2 = PQ^2 - h^2 = (1{,}3)^2 - (1{,}2)^2 = 1{,}69 - 1{,}44 = 0{,}25$
$MQ = \sqrt{0{,}25} = 0{,}5 \text{ m}$
$QR = MQ + MR = 0{,}5 + 1{,}1 = 1{,}6 \text{ m}$

2.3 $PR^2 = h^2 + MR^2 = (1{,}2)^2 + (1{,}1)^2 = 1{,}44 + 1{,}21 = 2{,}65$
$PR = \sqrt{2{,}65} = 1{,}63 \text{ m}$

2.4 $SA = bh + (\text{perimeter } \Delta PQR) \times l$
$= 1{,}6 \times 1{,}44 - (1{,}3 + 1{,}6 + 1{,}63)(2{,}4) = 2{,}304 + 10{,}872 = 13{,}81 \text{ m}^2$

3.1 V = area base × h = $\frac{1}{2} \times h$ = (35)(30,3)(208) = 110 292 mm^3

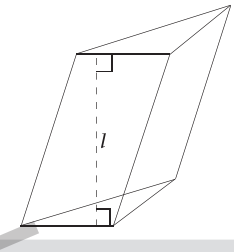
3.2 SA = perimeter of Δ × l = (35 × 35) + (35 + 35 + 35)(208)
= 1 225 + 21 840 = 23 065 mm^2

Remedial

Ensure learners know how to substitute the correct values into the formula. Take note of learners experiencing difficulty with determining the SA, if they are having trouble using their calculators, show them how to input the values correctly into their calculators.

Extension

Show some learners a skew rectangular prism and a skew triangular prism of which the perpendicular distance between parallel horizontal faces are given as well as the dimensions of all the faces. Challenge them to find formulas to calculate the volume and the surface area of these prisms.



Exercise 3

Learner's Book page 281

Guidelines on how to implement this activity

- The conversion of units of area and units of volume and capacity is very important and can be confusing for learners. Draw up a chart together as a class, showing the conversions of area.
- Work through as many worked examples as necessary to ensure earners become fluent in converting between the SI units. Learners should have ample opportunity to practise this skill.

Review Copy

Suggested answers

1 2 506 mm^2 ÷ 100 = 25,06 cm^2
2 5 070 cm^2 ÷ 10 000 = 0,507 m^2
3 0,0975 cm^2 × 100 = 9,75 mm^2
4 4 800 000 m^2 ÷ 1 000 000 = 4,8 km^2
5 1,039 m^2 × 10 000 = 10 390 cm^2
6 0,30405 km^2 × 1 000 000 = 304 050 m^2

Remedial

Some learners may also need help with the conversion of units where you have to decide whether to multiply or divide with some power of ten.

Revise the relationship between the various multiples and sub-multiples (tenths, hundredths, etc.) of units with these learners.

Exercise 4

Learner's Book page 282

Guidelines on how to implement this activity

This section extends learners' conversion skills to units of volume.

- Draw up a chart as a class showing the conversions.

- Do a few examples together as a class converting between the different units.

Learners should complete this exercise on their own.

Suggested answers

1 50 430 $mm^3 \div 1\,000 = 50{,}43\ cm^3$
2 50 430 $cm^3 \div 1\,000\,000 = 0{,}05043\ m^3$
3 $6{,}4\ cm^3 \times 1\,000 = 6\,400\ mm^3$
4 $0{,}49\ m^3 \times 1\,000\,000 = 490\,000\ cm^3$
5 $939\ mm^3 \div 1\,000 = 0{,}939\ cm^3$
6 $10^5\ mm^3 \div 1\,000 = 100\ cm^3$

Exercise 5

Learner's Book page 282

Guidelines on how to implement this activity

This section extends learners conversion to units of capacity.

- Draw up a chart as a class showing the conversions.
- Do a few examples together as a class converting between the different units.

Learners should complete this exercise on their own.

Suggested answers

1 9 430 m*l* ÷ 1 000 = 9,43 *l*
2 0,0759 *l* × 1000 = 75,9 m*l*
3 1,405 k*l* × 1000 = 1 405 *l*
4 10560 *l* ÷ 1000 = 10,56 k*l*
5 0,001 k*l* × 1000 = 1 *l* × 1 000 = 1 000 m*l*
6 $1{,}5 \times 10^5$ m*l* ÷ 1 000 = 150 *l*

Review Copy

Unit 5 Problem solving

Learner's Book page 283

Unit focus:

- problems involving perimeter and area of polygons
- problems involving surface area, volume and capacity of cubes and rectangular prisms

Background information on problem solving

- In previous grades learners have learnt to use and convert between appropriate SI units, including:
  - mm^2 to cm^2 and vice versa
  - cm^2 to m^2 and vice versa
  - mm^3 to cm^3 and vice versa
  - cm^3 to m^3 and vice versa
  - the equivalences: $1\ cm^3 = 1$ m*l* and $1\ m^3 = 1$ k*l*
- Most of the problems in this unit are designed to apply the skills of calculating perimeters, areas, volumes and capacities (dealt with in the previous units of this chapter) in practical situations. Similar situations in the environment of the school should also be used as examples.

- The last problems of both exercises are more challenging than the first problems. So it is advisable to discuss these to make sure that the learners understand the problems.

Exercise 1

Learner's Book page 284

Guidelines on how to implement this activity

- This exercise focuses on problem solving 2D problems involving area and perimeter. Revise by working through the worked examples, together as a class, how to break down complex shapes into simple squares, rectangles and triangles.
- Revise working with circles and using the calculator to work with complex decimals.
- Discuss the steps to problem solving learners have learnt in previous chapters. This involves:
  - identifying what they are being asked to find
  - identifying what they are given
  - writing up the correct equation or number sentence to solve the problem
- Revise these steps as you work through the examples.

Suggested answers

1.1 $P = 6 + 6 + 6 + 6 + 6 = 30$ m

1.2 $A = 6 \times 6 - 3 \times 2 = 36 - 6 = 30$ m^2

1.3 $h^2 = 6^2 - 3^2 = 36 - 9 = 27$

$h = \sqrt{27} = 5{,}196$ m

$A = \frac{1}{2} \times 6 \times 5{,}2 = 15{,}6$ m^2

2.1 $P = 7 \times 54 = 378$ mm

2.2 $C = \frac{2\pi R}{2} + 2\pi r = \pi R + \pi r = \pi(20) + 2\pi(10) = 125{,}66$ mm

3.1 $\frac{17}{5} = 3{,}4$ cm

3.2 $11 = 2\pi r$

$\frac{11}{2\pi} = r$

$r = 1{,}75$ m

4.1 1 dm = 10 cm

4.2 7,4 dm^2 = 740 cm^2

4.3 1 m^2 = 10 000 cm^2

4.4 15 700 cm^2 = 1,57 m^2

4.5 0,01 km^2 = 10 000 m^2

4.6 0,5 m^2 = 500 000 mm^2

5.1 $A = l \times b + \frac{\pi r^2}{2} = 2 \times 2 + \frac{\pi(1)^2}{2} = 4 + 1{,}57 = 5{,}57$ cm^2

5.2 $A = \pi r^2 - l \times b =$

$= 9\pi - (4{,}243)^2$

$= 10{,}27$ m^2

Review Copy

Remedial

Some learners have difficulty in knowing where to start solving a problem. Ask questions that will help them to decide on a strategy.

Extension

Ask learners who can make use of Internet to find out what the perimeter and the area of a country or a province is. Also, for example, the minimum and maximum volume of water flowing over a waterfall.

Exercise 2

Learner's Book page 286

Guidelines on how to implement this activity

- Working with 3D objects is often challenging for learners. If necessary ask learners to make rough models of the objects being discussed in the problem.
- Work through the examples together as a class.
- Be sure to explain your thinking and mark off on the diagram what you found as you progress.
- Remind learners that the sketch is very important when working with measurement and geometry, as is the order you are asked to find certain missing sides. Learners must follow the order and use the sketch to help them determine the next steps.

Learners can work in pairs or small groups to discuss the problems and their strategies.

Review Copy

Suggested answers

1.1 $\text{Vol} = l \times b \times h + \frac{1}{2} \times b \times h \times \text{H}$

$h = \sqrt{27} = 5{,}196$ m

$= 243\ 000 + 24\ 300 = 267\ 300\ \text{cm}^3 = 267\ 300\ \text{m}l$

1.2 $267\ 300\ \text{m}l = 267{,}3\ l$

1.3 $\text{SU}^2 = \text{UT}^2 + \text{TS}^2 = 36^2 + 15^2 = 1\ 296 + 225 = 1\ 521$

$\text{SU} = \sqrt{1\ 521} = 39$ cm

$\text{SA} = 90 \times 36 + 2 \times 36 \times 75 + 2 \times \frac{1}{2} \times 36 \times 15 + 39 \times 90$

$= 3\ 240 + 5\ 400 + 540 + 3\ 510 = 12\ 690\ \text{cm}^2$

$h^2 = (3{,}25)^2 - (3)^2 = 1{,}5625$

$h = \sqrt{1{,}5625} = 1{,}25$ m

2.1 $\text{Vol} = l \times b \times h + \frac{1}{2} \times b \times h \times \text{H}$

$= 8 \times 6 \times 3 + \frac{1}{2} \times 6 \times 1{,}25 \times 8 = 144 + 30 = 174\ \text{m}^3$

2.2 $174\ \text{m}^3 = 174\ 000\ l$

2.3 $\text{A} = 2 \times \frac{1}{2} \times 6 \times 1{,}25 + 2 \times 3{,}25 \times 8 = 7{,}5 + 52 = 59{,}5\ \text{m}^2$

2.4 $\text{A} = 8 \times 6 = 48\ \text{m}^2$

tile area $= 0{,}25 \times 0{,}2\ 5 = 0{,}0625\ \text{m}^2$

Number of tiles $= \frac{48}{0{,}0625} = 768$ tiles

2.5 $A = 2 \times 8 \times 3 + 2 \times 6 \times 3 - 12 = 48 + 36 - 12 = 72\ m^2$

2.6 Litres of paint $= \frac{72}{5} = 14{,}4\ l$

Remedial

Some learners may have difficulty with interpreting the 2D drawings of 3D objects. Help them by demonstrating the relationship with a physical model.

Extension

Challenge learners to identify objects in their environment to which they can apply their measurement skills to calculate perimeters, circumferences, areas and volumes; for example, the amount of paint required to paint the walls and ceiling of a classroom.

Consolidation

Learner's Book page 288

Before doing this consolidation exercise, encourage learners to review the work covered in this chapter. Advise learners to use the summary and to revise their work. This exercise can be used as an informal assessment task for you to track how learners are coping with the chapter and the concepts covered. The mark allocation provides guidelines on how to assess learners.

Review Copy

Suggested answers

1.1 $P = 2 \times 25 + 2 \times 21 + 8 \times 7 = 50 + 42 + 56 = 148\ mm = 14{,}8\ cm$ (2)

1.2 $A = 35 \times 39 - 4 \times 7 \times 7 = 1\ 365 - 196 = 1\ 169\ mm^2 = 11{,}69\ cm^2$ (2)

2.1 $C = 2\pi r = 2\pi(22) = 138{,}23\ mm$ (2)

2.2 $P = 6 \times 22 = 132\ mm$ (2)

2.3 $A = \pi r^2 = \pi(22)^2 = 15\ 20{,}53\ mm^2$ (2)

2.4 $a = 6 \times \frac{1}{2} \times 22 \times 19 = 1\ 254\ mm^2$ (2)

3.1 $P = 5 + 5 + 2\pi(2{,}5) = 10 + 15{,}71 = 25{,}71\ m$ (2)

3.2 $A = 30 \times 30 - \frac{\pi(30)^2}{4} = 900 - 706{,}86 = 193{,}14\ m^2$ (3)

4.1 $MC = \frac{1}{2} b = 3\ m$ (3)

$AC^2 = 3^2 + 3^2 = 9 + 9 = 18$

$AC = 4{,}24\ m$

4.2 $A = 2 \times \frac{1}{2} \times 6 \times 3 + 2 \times 4{,}2 \times 9 = 18 + 76{,}32 = 94{,}32\ m^2$ (3)

4.3 $A = 6 \times 9 = 54\ m^2$ (3)

4.4 $Vol = l \times b \times h + \frac{1}{2} \times b \times h \times H = 6 \times 3 \times 9 + \frac{1}{2} \times 6 \times 3 \times 9 = 162 + 81 = 243\ m^3$ (3)

5.1 $0{,}75\ cm^2 = 75\ mm^2$

5.2 $2\ 465\ cm^2 = 0{,}2465\ m^2$

5.3 $9\ dm^3 = 0{,}009\ m^3$

5.4 $0{,}065\ m^3 = 650\ cm^3$

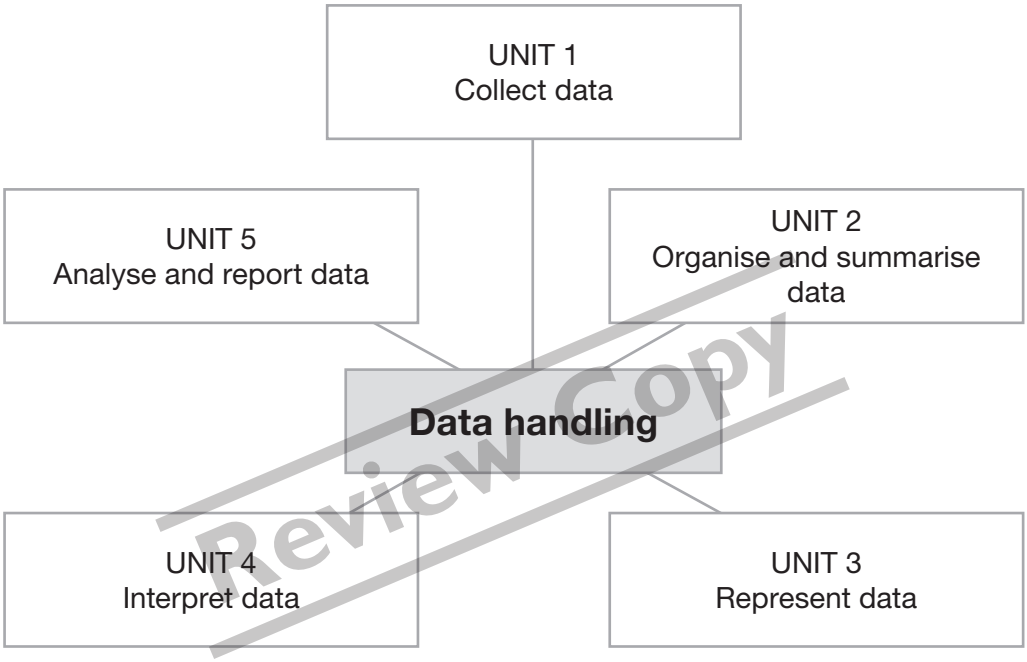
5.5 $34\ 557\ l = 3\ 4{,}557\ kl$

5.6 $1{,}085\ km^2 = 1\ 085\ 000\ m^2$ (6)

Total marks [35]

Chapter 14 Data handling

Overview of concepts



Review Copy

Content		Time allocations	LB page
Unit 1	Collect data	2 hours	290
Unit 2	Organise and summarise data	2 hours	294
Unit 3	Represent data	3 hours	304
Unit 4	Interpret data	2 hours	315
Unit 5	Analyse and report data	1,5 hours	318

Background information on data handling

Data handling involves asking questions and finding answers in order to describe events and the social, technological and economic environment. In this chapter learners will develop the skills to collect, organise, represent, analyse, interpret and report data. In completing data processing and presentation cycle, learners will have the opportunity to apply all these skills.

The following is new to learners in Grade 8:
- extremes
- broken line graphs
- dispersion of data
- error and bias in data

Generic teaching guidelines for teaching data handling

- By using everyday examples relating to the learners lives as your sources, this will be a fun chapter to teach.
- Learners will automatically engage with the content if it relates to their interest.
- This chapter also lends itself to healthy discussions as different topics are covered.
- Group work is a good way to consolidate the material taught. Each group can explore a different topic and complete a data cycle.
- Learners will learn about issues at school, in their community and our country as they work with content covering various socio-economic and health and environmental issues.

Resources

Examples of data and graphs from newspapers and magazines; examples of questionnaires; posters of relevant community issues that learners could investigate; cardboard; colour pens; blank tally charts; blank graph; grid paper; vocabulary cards; and poster board. Each learner should have their own calculator.

Review Copy

Unit 1 Collect data

Learner's Book page 290

Unit focus:
- posing questions
- selecting appropriate data sources
- distinguishing between population and sample
- creating a simple questionnaire.

Background information on collecting data
- Collecting data has to do with the compilation of information on a specific topic or issue. This can be done by asking questions about the specific topic or issue from a group that would best answer these questions.
- Data collection can also be done by consulting texts, published statistics, or provided results. The topic or issue should be well understood, as it would form the basis for the questions to be drawn up, people approached, and the materials sourced.
- Decision making (questionnaires, questions, sources), and planning go hand in hand with data collecting. It ensures that you collect data that is useful and meaningful.

In Grade 8 collecting data will include: posing questions, selecting sources and designing questionnaires.

Exercise 1

Learner's Book page 291

Guidelines on how to implement this activity

- Revise concepts such as questionnaire, sample, population and pictogram. A questionnaire is a list of questions used to collect data, and effective questionnaires use clear questions to collect useful information. Questions can be closed-ended, open-ended and combinations of the two. A closed-ended question is a question where the answer is limited to prescribed answers. In other words, a choice of different options is given and one is selected. An open-ended question on the other hand leaves room for an independent response.
- This topic lends itself to some fun practical examples. To introduce the topic, collect data using the learners as your sample group. Example, "Which university or college would you like to attend to complete your studies?"
- Give them some options, and collect data on a flipchart for further use.
- Have a short Question and Answer game to revise and explain concepts such as data, data source, survey, frequency, data collection sheet.
- Remind learners what a tally/frequency table looks like.
- Mention career opportunities such as social work and working in the entertainment industry as a radio DJ or broadcaster. Let learners investigate career and study opportunities related to their own interests and abilities.

Review Copy

Suggested answers

1 By asking if you have any brothers or sisters, their answer would be a "yes" or "no". By asking how many girls there are in your family, they would not know that only sisters are meant and could add mothers, aunts and girl cousins. When asking how big your family is they would not know that you only want to know how many brothers and sisters you have, and would add parents and maybe cousins and other family members.

2.1 Good questions might include:
Do you recycle waste materials at home?
If yes, do you recycle paper? glass? plastic?
Any other questions that result in specific answers.

2.2 Possible bad questions to ask when collecting the data might include:
Do you think recycling is important?
Why do you recycle wastes?
Any questions that don't give a specific answer are bad questions.

Extension

Learners can extend the activities in this unit in the learner book. In one activity data is to be collected about recycling in their community. They can collect data about schools recycling in their area, (advantages, disadvantages, obstacles etc.) so to start recycling at school.

Exercise 2

Learner's Book page 292

Guidelines on how to implement this activity

- Expose the learners to a variety of contexts to work with to compile data; including social, environmental, economic, school-based and health issues.
- Ensure that data sets are represented in a variety of ways, including big number ranges, decimals, fractions and percentages.
- When collecting data learners will have to pose questions (open-ended/closed-ended) and choose appropriate sources for their collection of data (family, peers, community newspapers, books, magazines or the Internet).

Suggested answers

	Information		Source
1	Tax information	B	South African Revenue Service website
2	Hourly traffic update	D	Radio
3	Location of Baragwanath hospital in Johannesburg	E	Map of the City of Johannesburg
4	Job listings	A	Local newspaper
5	Classmates' favourite colour	C	Learners

Extension

Let learners pair up to read some articles or pamphlets to identify questions and answers as well as data sources. Encourage learners to collect information from sources such as the Internet and organisations or institutions listed above. Highlight some social issues you think most appropriate in terms of their own environments, for example domestic violence, substance abuse, HIV/Aids.

Review Copy

Exercise 3

Learner's Book page 293

Guidelines on how to implement this activity

- Discuss questionnaires with learners and what types of questions can be included in questionnaires.
- Discuss the questionnaire provided in the worked example. Ask learners if they think it is a good or bad questionnaire. Discuss how you would improve the questionnaire.

Divide learners into pairs or groups to complete this exercise.

Suggested answers

1 Learners work in groups and choose a topic from the list below:
   - recycling in their community
   - bullying at school
   - how children spend their pocket money.

2 Learners select a data source such as their classmates or other pupils and how many children they will include in their questionnaire.

3 An example questionnaire the learners design

Pocket money questionnaire	
Do you get pocket money? (circle the answer)	Yes/No
Do you have to earn your pocket money?	Yes/No
If you said yes to question 1, tick (√) what you spend your pocket money on. You can tick more than one.	
Games	
Entertainment like movies	
Clothing	
Thank you for participating in this questionnaire.	

Extension

Further work on the pocket money topic can involve collecting data from different banks to determine which institution would give best service options for students.

Learners in this group can also explore the bullying issue by collecting data on groups that can be contacted to come and assist the school in eradicating this issue.

Exercise 4

Learner's Book page 293

Review Copy

Guidelines on how to implement this activity

This exercise extends the work completed in Exercise 3. Before allowing learners to carry out their questionnaire surveys, discuss appropriate collection methods, good manners and safety. Learners work in the same groups as before.

Suggested answers

Learners use their questionnaire from Exercise 3 to collect data.

Remedial

- Possible problems could include reading the assignment, writing up the questions and the lack of resources. Working in groups will assist with learning barriers as learners with the learning barriers can fulfil whatever role they can master. They can ask the questions in the collecting process. The stronger learners can write up the questions and the responses. Inevitably, peer learning will also take place as groups include different ability groups.
- Let learners who need help collect data by using a simple questionnaire with three different types of questions: closed-ended, open-ended and a combination question. They can for example, conduct a survey about favourite SA celebrities, by giving some choices (such as the first African in space – Mark Shuttleworth) and asking an open-ended question such as: "Why is that person your favourite?"

Unit 2 Organise and summarise data

Learner's Book page 294

Unit focus:

- organising and recording data
- grouping data into intervals
- summarising data using measures of central tendency
- summarising data using measures of dispersion.

Background information on organising and summarising data

- The organising and summarising of data takes place once all the data has been collected. We organise data so to read the collected data in an easier way.
- Once the data is organised we can use it to make summaries, and determine conclusions from the collected data. In Grade 8 learners will organise data using tallies, tables, stem-and-leaf displays.
- Data will be grouped into intervals. They will use measures of central tendency and measures of dispersion to summarise data.

Exercise 1

Learner's Book page 296

Review Copy

Guidelines on how to implement this activity

The aim of this exercise is to organise data by means of tallies and tables.

- Review the concepts of discrete and continuous data and remind learners that numerical data can be classified as discrete or continuous.
  - Discrete data consists of separate units that can be counted.
  - Continuous data results from a measuring process for example temperature, time, cost, length and mass.
- Draw up a tally table for the data collected in the class at the beginning of this chapter (Universities/Colleges) as an example to remind learners how to work with tally tables. Learners should complete this exercise on their own.

Suggested answers

1.1 The male listeners think drinking and driving is least "cool".

1.2 The female listeners think drug abuse is least "cool".

1.3

Uncool Activities		
Activity	Tally	f
Smoking	I	1
Drug abuse	IIII	4
Drinking and driving	III	3
Unsafe sex	II	2
Total		10

1.4 Smoking has the lowest overall frequency.

1.5 Drug abuse has the highest overall frequency.

Extension

Learners in this group continue with the topics chosen in extension of Unit 1. They can work in pairs or independently. They should investigate and compile information to distribute among the learner population about bullying, reasons to recycle and the power of saving. Distribute pamphlets and use posters to get information out.

Exercise 2

Learner's Book page 297

Guidelines on how to implement this activity

This exercise helps to consolidate the work learners did in Exercise 1.

Suggested answers

Learners use the data collected in Exercise 3 of Unit 1 and organise responses in a tally and frequency table. Learners show their own work.

Title		
Activity	Tally	*f*
Total		

Review Copy

Exercise 3

Learner's Book page 298

Guidelines on how to implement this activity

- Introduce learners to *stem-and-leaf displays*. This type of display is a statistical method for organising and displaying a set of numerical data to make it easier to order the numbers. It also makes it easy to draw conclusions from data that has been organised in a stem-and-leaf display.
- Work through the worked example in the learners book. Have learners complete this exercise on their own.

Suggested answers

Over a 12-week period a public hospital collected the given data. The data reflects the number of Foetal Alcohol Spectrum Disorder (FASD) cases tracked.

3, 12, 4, 3, 4, 5, 8, 10, 12, 14, 8, 9, 21

1 The data in a stem-and-leaf display.

Stem	Leaf
0	3; 3; 4; 4; 5; 8; 9
1	0; 2; 2; 4
2	1

2 The data in a frequency table.

Stems	Frequency (f)
0	7
1	4
2	1

3 Two conclusions from the display:
- Most weeks there are less than 10 FASD cases.
- In the worst week there were 21 cases tracked.

Remedial

Some learners might struggle with the following: organising data elements into stem- and leaf displays. Give them extra data sets to organise.

Exercise 4

Learner's Book page 300

Guidelines on how to implement this activity

- Explain the concept of grouping data and how intervals are useful groupings of data. A data element can only be assigned to one interval. When more than ten discrete values occur in a data set, it is useful to group the data into intervals.
- Work through examples together as a class and only allow learners to continue with the exercise when they understand and can group data.

Learners should do this exercise on their own.

Review Copy

Suggested answers

1

Interval	Elements
0–10	8; 9;
10–20	13; 15; 17; 19; 19
20–30	20; 25; 25; 25; 28; 29;
30–40	32; 38; 38;
40–50	42; 45; 47; 49;

2

Interval	Tally	Frequency
0–10	II	2
10–20	~~IIII~~	5
20–30	~~IIII~~ I	6
30–40	III	3
40–50	IIII	4
Total		20

3 Most learners got between 10 and 20 for the test, with a frequency of 6. Four learners got between 40 and 50.

Remedial

Some learners may find it challenging to group data. Let them go over the worked example on their own again. Then let them explain it to a learner who understands it, so they can determine the consolidation of the knowledge. In general, assist learners who require more help to consolidate a topic.

Exercise 5

Learner's Book page 301

Guidelines on how to implement this activity

There are different ways to describe or summarise a data set and it is important to make meaningful summaries.

- Learners should already be familiar with calculating the mean, mode and median values of a data set.
  These values are called *measures of central tendency* and are used to describe what the middle part of a data set "looks like".
  - The mean value = $\frac{\textit{sum of all elements}}{\textit{number of elements}}$
  - The mode is the value with the highest frequency (appears most often).
  - The median is the middle element in a sorted data set.
- Revise each of these concepts and demonstrate by means of the worked example.

Learners complete this exercise on their own.

Suggested answers

1 mean, mode, median:

The mean is

17 + 18 + 19 + 16 + 20 + 20 + 21 + 22 + 24 + 25 + 25 + 28 + 29 + 29 + 32 + 35 + 35 + 36 + 36 + 36 + 38 + 39

$= \frac{600}{22} = 27{,}27$

The mode =36.

The median $= \frac{25 + 28}{2} = 26{,}5$

2 The data set in descending order is 39; 38; 36; 36; 36; 35; 35; 32; 29; 29; 28; 25; 25; 24; 22; 21; 20; 20; 19; 18; 17; 16.

Stem (tens)	Leaf (units)	Frequency (f)
1	7 8 9 6	4
2	0 0 1 2 4 5 5 8 9 9	10
3	2 5 5 6 6 6 8 9	8
		22

Review Copy

Remedial

Learners have done these concepts before. Provide Grade 7 examples of finding the mode, median and mean to help learners revise.

Exercise 6

Learner's Book page 301

Guidelines on how to implement this activity

This exercise serves to consolidate the work done in Exercise 5. Give this exercise to learners to complete at home.

Suggested answers

1 True, mean value = 1,25

$1 + 0 + 2 + 0 + 1 + 1 + 1 + 0 + 1 + 2 + 1 + 5 = 15$ and $\frac{15}{12} = 1{,}25$

2 True, the mode = 1, as it occurs most frequently: 6 times.

3 False, the median ≠ 2; it is equal to 1, where

$\frac{1+1}{2} = 1$ [0; 0; 0; 1; **1**; **1**; 1; 1; 1; 2; 2; 5]

Exercise 7

Learner's Book page 303

Guidelines on how to implement this activity

- Introduce learners to the measures of dispersion. Learners will work with grouped and ungrouped numerical data.
- Remind learners that numerical data can be discrete or continuous.
- Introduce the concept of an *extreme value* and *range*, and that these are called *measures of dispersion*.
- It will be useful for learners to see how the mean value is sensitive to (easily affected by) extreme values. A data set containing extreme values is therefore not accurately described by the mean value.
- Work through the worked example and show learners how to use the range and extremes to summarise the data.

Learners complete this exercise on their own.

Review Copy

Suggested answers

Annual rainfall (in mm) was recorded for a few regions: 2 600; 1 080; 1 100; 890; 800; 780; 860; 0; 900; 870.

1 0 and 2600 are extreme values in the data set.

2 The extreme values would indicate drought and flooding, as 0 mm annual rainfall would indicate a drought and 2 600mm would indicate a flood.

3 The range is 2 600 – 0 = 2 600.

Exercise 8

Learner's Book page 303

Guidelines on how to implement this activity

This exercise continues the work done in Exercise 7. Prescribe this exercise for homework.

Suggested answers

The measures of dispersion are the range and extreme values of a data set.

In this case the range is 18 to 69, with the mean = $18 + 19 + 19 + 20 + 20 + 21 + 22 + 24 + 25 + 25 + 28 + 29 + 29 + 32 + 35 + 35 + 36 + 36 + 36 + 69 = \frac{578}{20} = 28{,}9 \approx 29$

69 is an extreme value relative to he mean.

Unit 3 Represent data

Learner's Book page 304

Unit focus:

- drawing bar graphs, double-bar graphs, histograms, pie charts and broken line graphs
- interpreting data.

Background information on representing data

- We represent data that has been collected and organised. Once data is represented in a graph or display the data is much easier to read.
- Conclusions and deductions can be made.
- Some graphs represent data in a better way than others. For example:
  - a broken-line graph is best to display continuous data
  - a double-bar graph can be used to represent two sets of data so they can be compared easily.

In Grade 8 learners will represent data by drawing bar graphs, double-bar graphs, histograms, pie charts and broken-line graphs.

Exercise 1

Learner's Book page 310

Review Copy

Guidelines on how to implement this activity

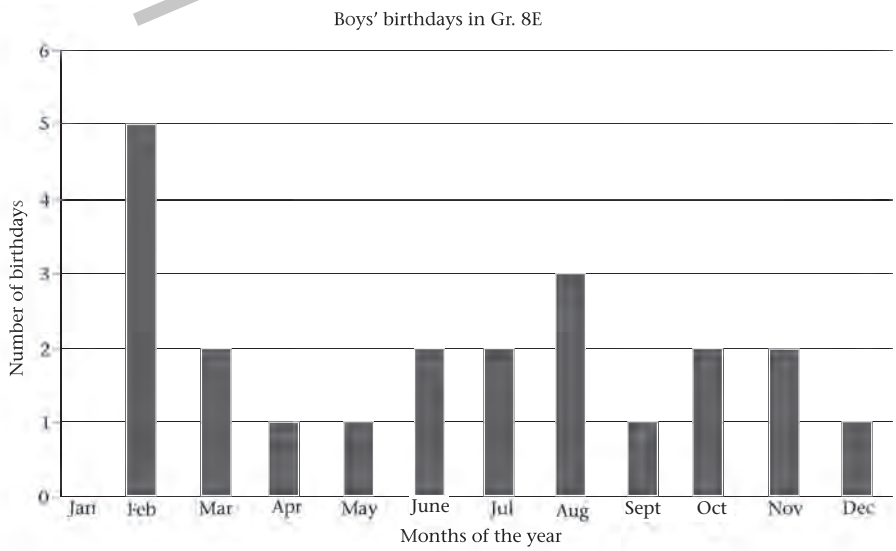
Data was collected at the start of this chapter concerning universities and colleges.

- Use the tally and frequency table to draw a bar graph.
- Explain the difference between a bar graph, a double-bar graph and a histogram.
- Work through each of the worked examples in the learner book together as a class.
- Ensure learners understand the difference between bar graphs and histograms.

Learner's should complete this exercise on their own.

Suggested answers

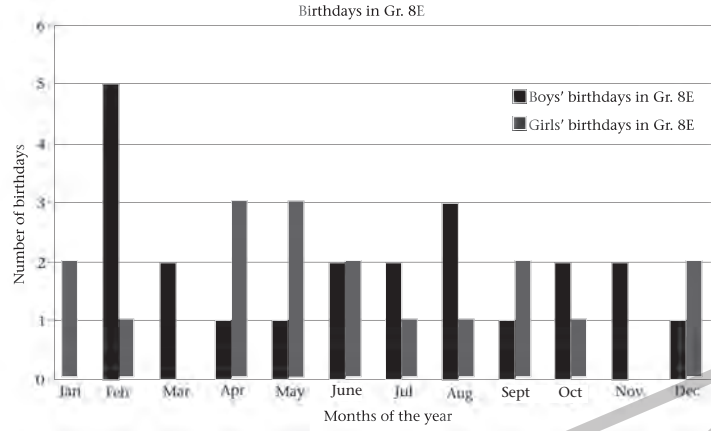
1



2

Discrete data	Continuous data
Volunteers educated 15 people	Siya travelled 100 km
Mbelu makes two new friends	The movie lasted an hour
	Kevin spent R38,00 on a gift
	The oven temperature is 180°C

3.1



3.2 The scale is 0 – 6.

3.3 The mean is 1,67.

$0 + 5 + 2 + 1 + 1 + 2 + 2 + 3 + 1 + 2 + 2 + 1 + 2 + 1 + 0 + 3 + 3 + 2 + 1 + 1 + 2 + 1 + 0 + 2 = 40$

$\frac{40}{24} = 1{,}67$

The median is 2.

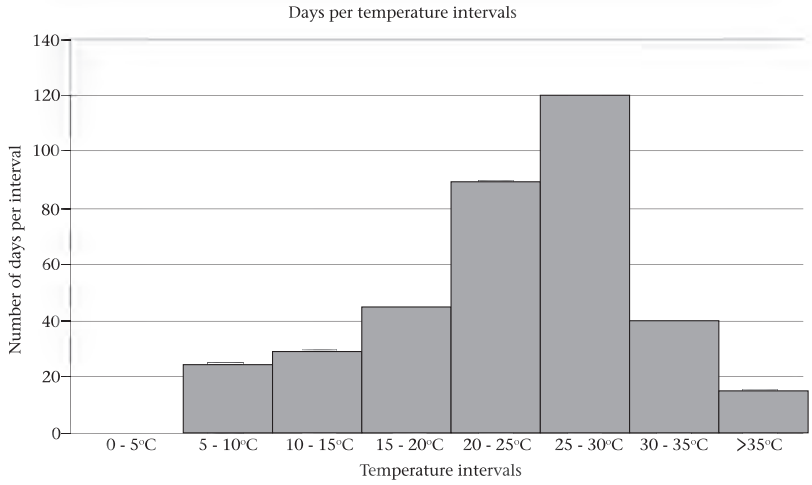
[0; 0; 0; 1; 1; 1; 1; 1; 1; 1; 1; **2; 2**; 2; 2; 2; 2; 2; 2; 2; 2; 3; 3; 3; 5]

The mode is 2.

[0; 0; 0; 1; 1; 1; 1; 1; 1; 1; 1; **2; 2; 2; 2; 2; 2; 2; 2; 2**; 3; 3; 3; 5]

3.4 The data range is 5.

4



Review Copy

4.1 The intervals are 5 ^{0}C.
4.2 The data shows a heat wave, where there were 16 days above 35 ^{0}C.
4.3 The interval 25–30 ^{0}C has the highest f.
4.4 366 days are recorded in this data set. This means the data was recorded in a leap year.

5.1 & 5.2

Intervals	Elements per interval	Tally	f
0–500	0; 10; 60	III	3
500-1 000	680; 700; 750; 850; 960	~~IIII~~	5
1 000-1 500	1 100	I	1
1 500-2 000			0
2 000-2 500			0
2 500-3 000	2 900; 3 000	II	2
3 000-3 500	3 300	I	1
Total		12	12

5.3 The extreme values in the data set are: 0; 10; 60; 2900; 3 000 and 3 300.
5.4 The low numbers 0; 10 and 60 are linked to droughts and the high numbers 2 900; 3 000; and 3 300 are linked to floods.

Extension

Learners can use A3 paper to draw their graphs. The graphs can be put up in the class.

Review Copy

Exercise 2

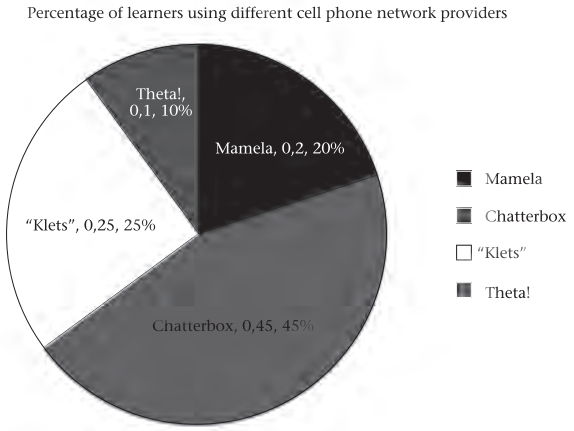
Learner's Book page 313

Guidelines on how to implement this activity

- Show learners how we can represent data in a pie chart.
- Discuss how we create segments using percentage and the degrees of a circle. Work through the worked example together as a class.
- Have learners work in pairs to complete this exercise.

Suggested answers

1



2 The majority of the learners make use of Chatterbox.
The least learners use Theta!
Each of the 40 learners makes use of some provider.

Remedial

Some learners might struggle with drawing the pie chart (protractor). Pair them up with a learner that exceeds expectation on this practical aspect. With extra practice they will master this, in their own time frame.

Extension

Represent the data for your school's Grade 8 pass rate of last year (girls/boys) in either a double-bar graph or a pie chart. Also, represent the last year's result compared with the results of five years ago.

Exercise 3

Learner's Book page 314

Guidelines on how to implement this activity

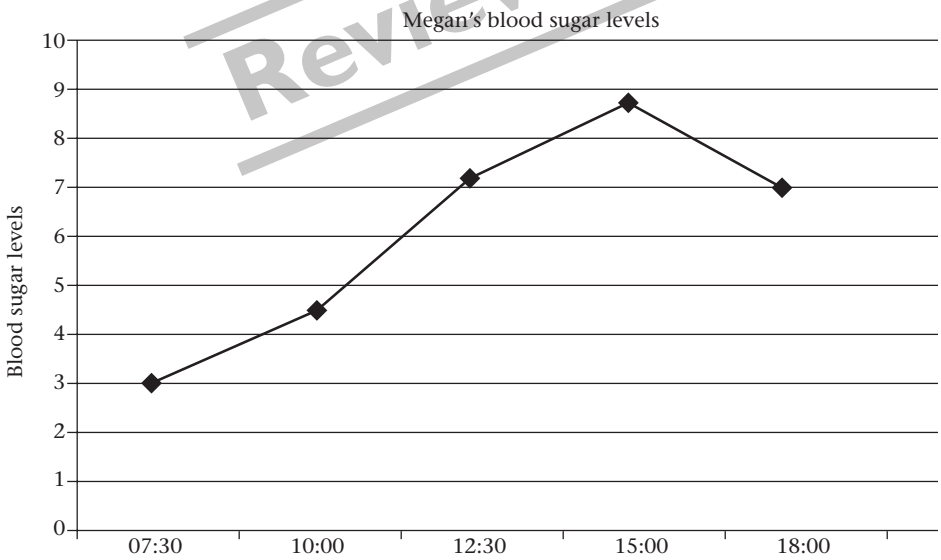
Introduce learners to broken-line graphs. Discuss instances of using broken-line graphs. Work through the worked examples and ensure learners know how to plot these graphs.

Have learners complete this exercise on their own.

Review Copy

Suggested answers

1.1



1.2 Megan's blood sugar levels will probably decrease overnight.

2.1 The *x*-axis shows months of the year and the *y*-axis shows kilolitres of water used.

2.2 (Jan; 30); (Feb; 18); (Mar; 41); (Apr; 29); (May; 43); (June; 40); (July; 38); (Aug; 50); (Sep; 38); (Oct; 35); (Nov; 23); (Dec; 20).

2.3 The *x*-points do not have numerical values; they are months of the year.

Extension

In 2012 the world gathered for the 30th Olympic Games in London. South Africa was represented well. Research our performance (medals) in these Games since re-admittance in 1992 and display in a graph of your choice.

Use the data you can collect on host nations since 1992 and display their performances in the best graph.

Unit 4 Interpret data

Learner's Book page 315

Unit focus:

- revision of different types of graphs and practising interpreting them.

Background information on interpreting data

- Data that has been represented tell us certain things, puts information across. In this unit we will study the represented data and make interpretations from it.
- In Grade 8 learners will read and interpret data; in words, bar graphs, double bar graph, pie charts, histograms and broken-line graphs.
- Present learners with the opportunity to interpret the same data represented in different displays. In this way you can determine what information is shown and what is hidden.
- Compare graphs on the same topic with different samples used. Discuss the impact of error and bias on the data results.

Review Copy

Exercise 1

Learner's Book page 316

Guidelines on how to implement this activity

- Engage the learners as far as possible. Get them involved in the interpretation of the data. It's important that learners understand the context of the graphs. Explain where the data comes from and how it was collected, as this will help with the interpretation. You could also ask them to come up with possible answers.
- Do not assume that learners understand the questions addressed, so make time to listen to their understanding of the question. Give learners time to think, process, write and ask questions if needed.
- Allow for time to clear up possible misunderstandings and explain terminology that may come up (axis, axis labels, scales, content displayed in graph). Some learners will have a greater experience to draw from than others. Use good visuals of graphs discussed so learners can follow without confusion.
- Explain a term or idea when you are physically referring to it, to reduce confusion. Work through worked examples in the Learner's Book with the class. They can then work through it in pairs so they have a chance to consolidate. Then move on to the activities in the Learner's Book.

Suggested answers

1 Comparing representations of data.

1.1 Graph B shows the best representation for the data displayed.

1.2 It is easier to see the differences between the data in Graph B.
1.3 Percentage for export for platinum is 28%; percentage for export for gold is 30%; percentage for export for chrome ore is 29% and percentage for export for diamonds is 13%.
1.4 The time period when these exports occurred is not shown and what the actual amounts exported are not shown (only percentages are given).
2 Information released by a real estate company.
2.1 The pie chart is misleading whereas the line graph and bar graph represent the numbers more accurately. In the pie chart it appears that more flats are sold than houses, which is not true.
2.2 The pie chart segments were not labelled. The title of the pie chart features flats first which makes it appear as if the large segment on the left represents flats.
2.3 Data is sometimes presented in a misleading way to make it appear to support a point of view. For instance, in this example the agent may want it to appear as if flats are more popular.
2.4 The pie chart does not show comparisons.
2.5 Double bar charts are better for showing comparisons.
2.6 Learner's own responses

Remedial

Some learners could struggle with the challenge of the abstraction of the graph. Make it as real or relatable for them. Other may have difficulty reading the questions, pairing strong with less strong learners will work here. Some will have difficulty answering the question; they can do it verbally for you or make use of a scribe (a faster learner) to get their answer across.

Review Copy

Extension

Consult newspapers (Business Times, MG etc.), Internet, libraries for graph on current issue. Allow learners in this group to make interpretation from the exemplars found. They can present it to the class and display their graph with its interpretation in class.

Unit 5 Analyse and report data

Learner's Book page 318

Unit focus:

- analysing data presented in different ways
- reporting on your findings.

Background information on analyse and report data

In Grade 8 learners will analyse data by answering questions related to:

- data categories
- data sources
- central tendencies
- scales used on graphs, samples and population
- dispersion of data and error
- bias in data

Learners should compare different ways of summarising the same data. Graphs showing same data, using different scales must also be studied so to analyse.
In Grade 8 learners will report data by:

- summarising data to draw conclusions
- making predictions based on data
- identifying sources of error and bias in data
- choosing appropriate summary statistics
- understanding the role of extremes in data

Encourage learners to write up short reports on data analysed.

Exercise 1

Learner's Book page 320

Guidelines on how to implement this activity

- Discussions are very important in the conveying of this topic to learners. Involve learning so they feel that they helped in making the analysis or drawing the conclusions.
- Use graphs that will bring out the topics you want to teach. Ensure that learner can fully understand with your example graph. In this way they can actively participate in the lesson. Topics to discuss include: summary statistics provided (does it best represent the data), the form in which the data is presented (manipulated), scales used on the graphs, samples and population chosen, dispersion of data and error and bias in data.
- Once learners feel confident with voicing their contributions and making their own deductions, they can be encouraged to write it up in report form. Model a report by: after class discussion wraps up put all the analysis points in writing on the board or a flip chart. The success of reporting on the data depends on the conclusions made.

Learners can complete this exercise in pairs.

Review Copy

Suggested answers

1 Vehicle energy graph

1.1 Electrical and ethanol cars show growth over the 10 year time period.

1.2 Electrical vehicles increased from 200 000 to 350 000 and ethanol vehicles from 70 000 to 120 000.

1.3 70 000 is an extreme value being the only value under 100 000.

1.4 The smallest range is for electrical cars, with a range of 50 000

1.5 Petroleum vehicles decrease in the future.
Electrical cars increase in the future.
Ethanol cars increase in the future.

2 Real estate data

2.1 Two of the following

flats	Houses	
3	5.25	mean
4	4	median
3	4	mode

2.2 The range for each

Flats	houses
2	2

2.3 There are no extreme values in this data set. Not every data set has extreme values.

3 Workforce analysis

3.1 The gender categories are male and female.

3.2 Female % of total workforce increased from 2001 to 2011.

Male % of total workforce decreased in this time period.
Male transport workers increased in the time period.
Female transport workers also increased.
Male % construction workers stayed the same over the 10 year period.
Female % construction workers increased over this period.
Male % miners stayed the same over the 10 year period
Female % mine workers increased from 0 to 0,5% in the same period.

3.3 The pie diagram displays its data in a biased way as it does not show the increase over the 10 year period for women workers.

3.4 If the media were to use the pie chart it would not show the improvements of % of women in the workforce over the last 10 years.

Remedial

Some learners could struggle with the challenge of the abstract nature of the graph. Make it as real or relatable for them as possible. Others may have difficulty reading the questions, pairing learners will work here. Some will have difficulty answering the question, they can do it verbally for you or make use of a scribe (a faster learner) to get their answer across.

Extension

Consult newspapers (Business Times, MG etc.), Internet, libraries for graph on current issues. Allow learners in this group to make interpretations from the exemplars found. They can present it to the class and display their graph with its interpretation in class. Then learners can analyse and write up a report on the graphs they interpreted.

Review Copy

Consolidation

Learner's Book page 323

Before doing this consolidation exercise, encourage learners to review the work covered in this chapter. Advise learners to use the summary and to revise their work. This exercise can be used as an informal assessment task for you to track how learners are coping with the chapter and the concepts covered. The mark allocation provides guidelines on how to assess learners.

Suggested answers

1 Analysis of calls

1.1 How many calls did the social worker handle in one day?
Or, what is the total of the frequency column of the table? (1)

1.2 Which was the least discussed problem?
Or, which row had the lowest frequency? (1)

1.3 Which of the problems called in had the highest number in the frequency column? Or, which was the most common problem? (1)

2.1 Determine the mean, mode and median values for the following data set:
Mean 1 + 5 + 0 + 6 + 3 + 8 + 4 + 5 = 32 divided by 8:
$\frac{32}{8} = 4$ (2)
Median $= \frac{4 + 5}{2} = 4{,}5$ (2)
Mode = 5 (2)

2.2 0; 1; 3; 4; 5; 5; 6; 8 (2)

3 Mountain-top snowfall recorded (in mm) over 30- year period

3.1

Intervals	Elements per interval	Tally	*f*
1 500–1 800	1 550	I	1
1 800–2 100	1 900; 2 000; 2 050; 2 050; 2 100	~~IIII~~	5
2 100–2 400	2 200	I	1
2 400–2 700	2 500; 2 600; 2 700	III	3
2 700–3 000	2 800; 2 900	II	2
Total		**12**	**12**

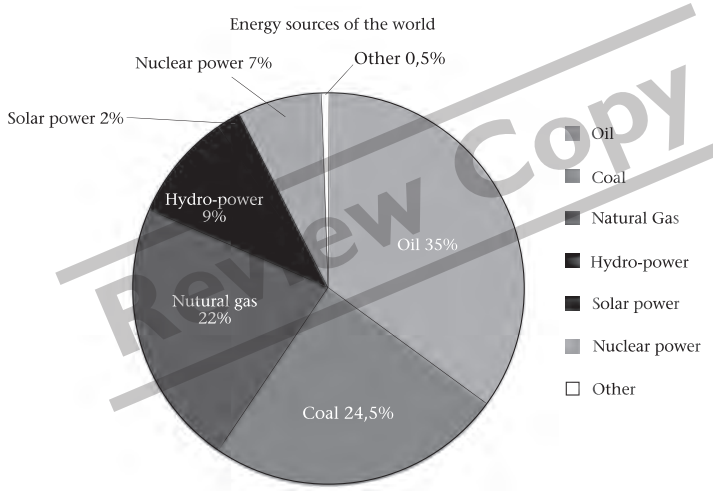
(10)

3.2 The extreme values are 1 550; 2 800 and 2 900. (2)

3.3 Droughts and floods can be linked to the extreme values. (2)

3.4 The majority of years the snowfall was between 1 900 and 2 100 mm, with an *f* value of 5. High values over 2 400 were recorded 5 of the 12 times. Any other conclusions the learners provide that are shown by the data can also be accepted. (3)

4.1



Review Copy

(10)

4.2 35% + 24.5% + 22% + 9% + 2% + 7% + 0.5% = 100%

5.1 By June the market share ratio is 50% for the 2 suppliers. (2)

5.2 Competitor activity caused Cindy and Karen's market share to drop from 100% to 50% within 6 months. (3)

Total marks [45]

Chapter 15 Functions and relationships

Overview of concepts

Functions and relationships

UNIT 1
Input and output values

UNIT 2
Equivalent forms

Content		Time allocations	LB page
Unit 1	Input and output values	3 hours	326
Unit 2	Equivalent forms	3 hours	335

Review Copy

Background information on functions and relationships

Both in the intermediate phase and in Grade 7 learners learnt about numeric and geometric patterns by doing investigations and by extending given patterns. The concepts "input and output values", "flow diagrams", "tables", "formulae" and "number sentences" should also not be new to the learners.

Generic teaching guidelines for teaching functions and relationships

It is advisable to first establish what the learners remember of flow diagrams, input and output values, tables showing the same relationships, etc.

The key concepts should be consolidated with a number of worked examples including examples other than those of this chapter in the Learner's Book. Although the descriptions of equivalent forms in Grade 7 and in Grade 8 of the CAPS document (2.2) seem to be almost identical, one cannot assume that the learners have mastered this topic. So it is important to do enough worked examples and exercises to consolidate this topic.

Resources

Blank flow diagrams, examples of input and output processes, cardboard, and colour pens.

Unit 1 Input and output values

Learner's Book page 326

Unit focus:

- determining input and output values when given the formula
- using flow diagrams, variables, formulae and equations to solve input and output problems.

Background information on input and output values

- This unit is mostly revision of the same topic done in Chapter 4 of this book, and in Grade 7.
- The most important objective of this unit is that learners come to understand that a mathematical relationship between an input variable and an output variable can be expressed in various ways. So the teacher should do several worked examples on the board with participation of the learners.

Exercise 1

Learner's Book page 326

Review Copy

Guidelines on how to implement this activity

- It is advisable to first establish what the learners remember of flow diagrams, input and output values, tables showing relationships and formulae. Discuss this as a class.
- Do examples of finding output values of flow diagrams when given input values. Then do examples finding input values when provided with output values. Show learners how to work backwards with the flow diagram.
- The exercises should be started under supervision of the teacher who monitors the learner's progress. The remainder of the exercises can be done as homework.

Suggested answers

1.1 $\boxed{6} \rightarrow \boxed{+5} \rightarrow \boxed{\times 4} \rightarrow \boxed{44}$

1.2 $\boxed{18} \rightarrow \boxed{\div 6} \rightarrow \boxed{+12} \rightarrow \boxed{15}$

1.3 $\boxed{31} \rightarrow \boxed{-15} \rightarrow \boxed{\times 3} \rightarrow \boxed{48}$

1.4 $\boxed{-2} \rightarrow \boxed{-3} \rightarrow \boxed{\times 5} \rightarrow \boxed{-25}$

1.5 $\boxed{6} \rightarrow \boxed{\times 3} \rightarrow \boxed{+1} \rightarrow \boxed{19}$

2.1 $\boxed{9} \rightarrow \boxed{+11} \rightarrow \boxed{\times 5} \rightarrow \boxed{100}$

2.2 $\boxed{9} \rightarrow \boxed{+11} \rightarrow \boxed{\times 5} \rightarrow \boxed{100}$

2.3 $\boxed{9} \Rightarrow \boxed{+11} \Rightarrow \boxed{\times 5} \Rightarrow \boxed{100}$

2.4 $\boxed{6} \Rightarrow \boxed{\div 2} \Rightarrow \boxed{-14} \Rightarrow \boxed{-11}$

2.5 $\boxed{-2} \rightarrow \boxed{+7} \rightarrow \boxed{\times 7} \rightarrow \boxed{35}$

Remedial

Some learners may have difficulty with interpreting and/or writing the rule of a flow diagram. Assist them to work through a few examples.

Exercise 2

Learner's Book page 328

Guidelines on how to implement this activity

- Introduce learners to a formula. Show learners formulae they have worked with before, such as the Area formula.
- Discuss how we use formulae – we substitute in known values to find the output. Revise substituting in values to find the output of the formula.
- Do a few examples as a class, and have learners complete the exercise on their own.

Review Copy

Suggested answers

1.1 $2(3) = 6$

1.2 $2(3) + 3 = 6 + 3 = 9$

1.3 $2(5) + 10 = 10 + 10 = 20$

1.4 $2(2a + 5) = 2(2(3) + 5) = 2(6 + 5) = 2(11) = 22$

1.5 $(a + b)(a - b) = (3 + 5)(3 - 5) = (8)(-2) = -16$

1.6 $2a - b + 1 = 2(3) - 5 + 1 = 6 - 5 + 1 = 2$

1.7 $3b \div a + 2 = 3(5) \div 3 + 2 = 15 \div 3 + 2 = 5 + 2 = 7$

1.8 $2b - (a + 1) = 2(5) - (3 + 1) = 10 - 4 = 6$

2.1 $3(2m + 3n) + 2(3m - 2n) = 3(2(3) + 3(2)) + 2(3(3) - (2)2)) = 3(6 + 6) + 2(9 - 4) = 3(12) + 2(5) = 36 + 10 = 46$

2.2 $4(3m - n) + 2(4m - 3n) + 6 = 4(3(3) - 2) + 2(4(3) - 3(2)) + 6 = 4(9 - 2) + 2(12 - 6) + 6 = 4(7) + 2(6) + 6 = 28 + 12 + 6 = 46$

2.3 $5m - 3n \times 2 - n = 5(3) - 3(2) \times 2 - 2 = 15 - 6 \times 2 - 2 = 15 - 12 - 2 = 1$

2.4 $(5m - 3) \times n = (5(3) - 3) \times 2 = (15 - 3) \times 2 = 12 \times 2 = 24$

3.1 $\frac{w}{x} = \frac{10}{20} = \frac{1}{2}$ 3.2 $\frac{x}{z} = \frac{20}{4} = 5$ 3.3 $\frac{x}{y} = \frac{20}{5} = 4$

3.4 $\frac{yx}{x} = \frac{5(20)}{20} = 5$ 3.5 $\frac{wx}{yz} = \frac{(10)(20)}{(5)(4)} = \frac{200}{20} = 10$

4.1 $T = 2(2) + 5(1) = 4 + 5 = 9$ 4.2 $T = 2(8) + 5(0) = 16 + 0 = 16$

4.3 $T = 2(3) + 5(3) = 6 + 15 = 21$ 4.4 $T = 2(5) + 5(4) = 10 + 20 = 30$

4.5 $T = 2(1) + 5(10) = 2 + 50 = 52$

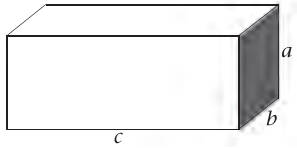
5.1 $P = 3 + 7 + 4 = 14$ 5.2 $P = 1 + 4 + 12 = 17$

5.3 $P = 10 + 15 + 20 = 45$

6.1 $v = 5 + 3(4) = 5 + 12 = 17$ 6.2 $v = 0 + 5(4) = 20$

6.3 $v = 3 + 2(10) = 3 + 20 = 23$ 6.4 $v = 11 + 3(6) = 11 + 18 = 29$

7.1 $V = 3(4)(10) = 120$ 7.2 $V = 5(6)(3) = 90$ 7.3 $V = 4(4)(9) = 144$



8.1 $Q = 3(4) + 7(2) = 12 + 14 = 26$ 8.2 $Q = 3^2 + 5 = 9 + 5 = 14$

8.3 $Q = (2)(7) + 5 = 14 + 5 = 19$ 8.4 $Q = 5(20) - 2(30) = 100 - 60 = 40$

9 Pay = (40 × R27,50) + R65 = R1 100 + R65 = R1 165

10 Perimeter = 2(4,5 + 5,2) = 2(9,7) = 19,4

Remedial

Some learners may need additional revision of substituting in values for variables. Revise this in more depth with these learners.

Extension

Challenge learners to create their own examples of input – output relationships using formulas from other parts of the curriculum such as distance = speed × time, where distance is a constant such as 20 km or speed is a constant such as 60 km/h, or time is a constant such as 1 hour.

Review Copy

Exercise 3

Learner's Book page 331

Guidelines on how to implement this activity

- Discuss how we used input values to find output values for formulae in the previous exercise.
- Now ask learners how we could use formulae to find input values, if we are given an output value.
- Show learners how to change the subject of the formula, using the properties of additive and multiplicative inverses.
- Demonstrate how changing the subject of the formula helps to find the input values.

Do a few examples of finding the input value together as a class, and encourage learners to attempt the exercise on their own.

Suggested answers

1.1 $y = x - 2$
$y + 2 = x - 2 + 2$
$x = y + 2$

1.2 $y = x + 7$
$y - 7 = x + 7 - 7$
$x = y - 7$

1.3 $y = 4x = \frac{y}{4} = \frac{4x}{4}$
$x = \frac{y}{4}$

1.4 $y = \frac{x}{3}$
$3y = \frac{x3}{3}$
$x = 3y$

1.5 $y = 2x + 1$
$y - 1 = 2x + 1 - 1$
$\frac{y-1}{2} = \frac{2x}{2}$
$x = \frac{y-1}{2}$

1.6 $y = 4x - 3$
$y + 3 = 4x - 3 + 3$
$\frac{y+3}{4} = \frac{4x}{4}$
$x = \frac{y+3}{4}$

1.7 $y = 2(x + 3)$
$\frac{y}{2} = \frac{2(x+3)}{2}$
$\frac{y}{2} - 3 = x + 3 - 3$
$x = \frac{y}{2} - 3$

1.8 $y = 3(x - 4)$
$\frac{y}{3} = \frac{3(x-4)}{3}$
$\frac{y}{3} + 4 = x - 4 + 4$
$x = \frac{y}{3} + 4$

1.9 $y = mx$
$\frac{y}{m} = \frac{mx}{m}$
$x = \frac{y}{m}$

1.10 $y = x + a$
$y - a = x + a - a$
$x = y - a$

1.11 $y = kx - c$
$\frac{y+c}{k} = \frac{kx - c + c}{k}$
$x = \frac{y+c}{k}$

1.12 $y = ax + b$
$\frac{y-b}{a} = x$
$x = \frac{y-b}{a}$

1.13 $y = \frac{x+2}{4}$
$4y = \frac{(x+2)4}{4}$
$4y - 2 = x + 2 - 2$
$x = 4y - 2$

1.14 $y = \frac{x-2}{5}$
$5y + 2 = x - 2 + 2$
$5y + 2 = x$
$x = 5y + 2$

1.15 $y = \frac{x-b}{a}$
$ay = x - b$
$ay + b = x$
$x = ay + b$

1.16 $y = a + b + x$
$y - a - b = x$
$x = y - a - b$

1.17 $y = \frac{ax-c}{b}$
$by = ax - c$
$\frac{by+c}{a} = \frac{ax}{a}$
$x = \frac{by+c}{a}$

1.18 $y = \frac{x-a+b}{c}$
$cy = x - a + b$
$cy + a - b = x$
$x = cy + a - b$

2.1 $y = ax + b$
$(y - b) = ax$
$\frac{y-b}{x} = a$
$a = \frac{y-b}{x}$

2.2 $y = ax + b$
$y - ax = b$
$b = y - ax$

Review Copy

3 $V = IR$

$\frac{V}{R} = \frac{IR}{R}$

$I = \frac{V}{R}$

$\frac{V}{I} = \frac{IR}{I}$

$R = \frac{V}{I}$

4 $F = ma$

$\frac{F}{a} = \frac{ma}{a}$

$m = \frac{F}{a}$

$\frac{F}{m} = \frac{ma}{m}$

$a = \frac{F}{m}$

5 The formula $C = 2\pi r$

$\frac{c}{2\pi} = r$

$r = \frac{c}{2}\pi$

6 $P = 2l + 2b + x$

6.1 $P - 2l - 2b = 2l + 2b - 2l - 2b + x$

$x = P - 2l - 2b$

6.2 $\frac{P - 2b - x}{2} = \frac{2l}{2}$

$l = \frac{P - 2b - x}{2}$

6.3 $P - 2l - x = 2b$

$\frac{P - 2l - x}{2} = b$

7.1 $A = \frac{h}{2}(a + b)$

$\frac{A}{a+b} = \frac{h}{2}$

$h = 2\left(\frac{A}{a + b}\right)$

7.2 $A = \frac{h}{2}(a + b)$

$A\left(\frac{2}{h}\right) - b = a$

8.1 $V = \pi r^2 h$

$\frac{V}{\pi h} = r^2$

$r = \sqrt{\frac{V}{\pi h}}$

8.2 $r = \sqrt{\frac{250}{3{,}14 \times 10}} = \sqrt{7{,}96} = 2{,}82$ cm

9.1 $v^2 = u^2 + 2as$

$v^2 - u^2 = 2as$

$\frac{(v^2 - u^2)}{2s} = a$

$a = \frac{v^2 - u^2}{2s}$

9.2 $s = ut + \frac{1}{2}at^2$

$s - ut = \frac{1}{2}at^2$

$2(s - ut) = at^2$

$a = \frac{2(s - ut)}{t^2}$

10.1 $h = \frac{V}{x^2}$

$\frac{A - 2x^2}{4x} = h$

$h = \frac{A - 2x^2}{4x}$

10.2 $h = \frac{24 - 2(2)^2}{4(2)}$

$= \frac{24 - 8}{8} = \frac{16}{8}$

$= 2$ cm

10.3 $h = \frac{V}{x^2} = \frac{250}{10(10)} = 2{,}5$ cm

Remedial

For those learners who struggle with finding an input value for a given output value, let them first attempt easy cases and then progress to more challenging cases, which include fractions. Learners who struggle with algebra with find this section problematic. Be available to revise the additive and multiplicative inverses and how we use them to solve equations.

Review Copy

Exercise 4

Learner's Book page 333

Guidelines on how to implement this activity

- Revise with learners how to construct a formula from a given verbal problem. Learners have done this before in algebra, but this now becomes more complex.
- Revise the key verbal descriptions and what they translate to algebraically.
- Do a few examples as a class, be sure to include a higher order problem as well.

Learners can work in pairs of small groups, but must do their own working.

Suggested answers

1 $x + 5$ 2 $x - 3$ 3 $x \times y$

4 $\frac{a}{b}$ 5 $2x + 8 = 12$ 6 $\frac{1}{4}x + \frac{1}{2}x = 3$

7 $(x) \times (x + 2)$ 8 $x + (x + 1) = 25$ 9 $x \times$ R30

10 R135 + (x minutes × R0,75)

11.1 R150 + (x hours × R75)

11.2 R150 + (3 × R75) = R150 + R225 = R375

11.3 R150 + (x hours × R75) = R525

(x hours) = $\frac{\text{R525} - \text{R150}}{\text{R75}} = \frac{\text{R375}}{\text{R75}}$ = 5 hours

12.1 $P = 3x$ $P = 3 \times 12 = 36$

12.2 $P = 4y$ $P = 4 \times 7{,}5 = 30$

12.3 $P = 2x + 4y$ $P = 2(12) + 4(7{,}5) = 24 + 30 - 54$

12.4 $P = 2y + x + z$ $P = 2(7{,}5) + 12 + 5{,}5 = 32{,}5$

13.1 Total ticket sales = R25 × x + R15 × y

13.2 R25(60) + R15(130) = R1 500 + R1 950 = R3 450

13.3 R5 500 = R25(100) + R15 × y

R5 500 – R2 500 = 15 × y

$\frac{\text{R3 000}}{15} = y$

y = 200 children bought tickets

14.1 $P = 2(x) + 2(x + 5)$ 14.2 $A = x \times (x + 5)$

15.1 C = R259 + x(15) 15.2 C = R259 + 80(R15) = R259 + R1 200 = R1 459

16.1 C = R10 + n(R2,50) 16.2 C = R10 + 20(R2,50) = R10 + R50 = R60

16.3 R40 = R10 + n(R2,50) = R40 – R10 = n(R2,50) = $\frac{\text{R30}}{\text{R2,50}}$ = 12

17.1 white slabs = $2x$ 17.2 white = $2x + 6$

18.1 $P = 2(l + b)$
$= 2[(10 - 2x) + (30 - 2x)]$
$= 2(40 - 4x)$
$= 80 - 8x$

18.2 $A = l \times b$
$= (10 - 2x) \times (30 - 2x)$
$= 300 - 20x - 60x + 4x^2$
$= 4x^2 - 80x + 300$

19.1 $C = \frac{5}{9}(7 - 32) = \frac{5}{9}(-25) = -\frac{125}{9} = -13{,}9$ °C

19.2 $C = \frac{5}{9}(98 - 32) = \frac{5}{9}(66) = \frac{330}{9} = 36{,}7$ °C

19.3 If $C = \frac{5}{9}(F - 32)$
Then $C \times \frac{9}{5} + 32 = F$
$40 \times \frac{9}{5} + 32 = 72 + 32 = 104$ °F

20 $4x + x + 50 = 180 \rightarrow 5x = 180 - 50$
$5x = 130$ or
$x = \frac{130}{5} = 26$

Review Copy

Extension

Learners can also be asked to find examples of input – output relationships that occur in other school subjects such as science and technology.

Unit 2 Equivalent forms

Learner's Book page 335

Unit focus:

- revision of representing functions in different forms.

Background information on equivalent forms

This unit is mostly revision of the same topic done in Chapter 4 of this book, and in Grade 7 except that 'number sentences' is replaced by 'equations'.

The most important objective of this unit is that learners come to understand that a mathematical relationship between an input variable and an output variable can be expressed in various *equivalent* ways. So the teacher should do several worked examples on the board with participation of the learners.

The exercises should be started under supervision of the teacher who monitors the learners' progress. The remainder of the exercises can be done as homework.

Exercise 1

Learner's Book page 336

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Guidelines on how to implement this activity

This exercise is a revision exercise. Revise number sentences and remind learners that we now use the word equations in place of number sentences. Show learners equations with variables like x and y. Revise using formulae, substitution and equations. Work through the worked examples together as a class. Encourage learners to complete this exercise on their own.

Suggested answers

1.1 $2^2 + (-4)^2 - (-5) = 4 + 16 + 5 = 25$

1.2 $2(-4) + (-5) = -8 - 5 = -13$

1.3 $2(2) - 4(-5) = 4 - (-20) = 24$

1.4 $\frac{(4)(-5)}{2} \times (-4) = \frac{20}{2}(-4) = -40$

1.5 $2(-4) - (-4)(-5) = -8 - (20) = -28$

1.6 $3(-5) + 4(2) = -15 + 8 = -7$

2.1 $\sqrt{15 + 2(-3)} = \sqrt{15 - 6} = \sqrt{9} = 3$

2.2 $\sqrt{-7 + 2(35{,}5)} = \sqrt{-7 + 71} = \sqrt{64} = 8$

3.1 $F = 32 + \frac{9(-100)}{5} = 32 + -\frac{900}{5} = 32 - 180 = -148$

3.2 $F = 32 + \frac{9(6000)}{5} = 32 + \frac{54\,000}{5} = 32 + 10\,800 = 10\,832$

3.3 $F = 32 + \frac{9(-20)}{5} = 32 + -\frac{180}{5} = 32 - 36 = -4$

3.4 $F = 32 + \frac{9(-10)}{5} = 32 + -\frac{90}{5} = 32 - 18 = 14$

4.1 $Q = \frac{ab + bc}{abc} = \frac{2(-6) + (-6)(-4)}{(2)(-6)(-4)} = \frac{-12 + 24}{48} = \frac{12}{48} = \frac{1}{4}$ true

4.2 $Q = \frac{bc - ab}{abc} = \frac{(-6)(-4) - (2)(-6)}{(2)(-6)(-4)} = \frac{24 - (12)}{48} = \frac{36}{48} = \frac{3}{4}$ not the equation

4.3 $Q = \frac{1}{a} - \frac{1}{c} = \frac{1}{2} - \frac{1}{-4} = \frac{2}{4} - -\frac{1}{4} = \frac{3}{4}$ not the equation

4.4 $Q = \frac{1}{c} + \frac{1}{a} = \frac{1}{-4} + \frac{1}{2} = -\frac{1}{4} + \frac{2}{4} = \frac{1}{4}$ true

4.5 $Q = \frac{1}{a} + \frac{1}{b} = \frac{1}{2} + \frac{1}{-6} = \frac{3}{6} - \frac{1}{6} - \frac{2}{6} = \frac{1}{3}$ not the equation

Remedial

Learners should manage this exercise as it is mainly revision. If learners experience problems it may be necessary to provide remediation activities for learners to work on at home to ensure they practise the necessary skills required.

Exercise 2

Learner's Book page 339

Guidelines on how to implement this activity

Learners need to be able to work smoothly between the different representations of functions and relationships. In earlier grades the emphasis was on tables and flow diagrams. In Grade 8 the focus is on equations.

Revise the different equivalent forms and do examples of representing the relationship in the different forms.

Learners should complete this exercise on their own.

Review Copy

Suggested answers

1.1 verbal (in words) 1.2 algebraic form 1.3 formula

1.4 equation 1.5 flow diagram 1.6 table

2.1 $x = (3)(4) - 7 = 12 - 7 = 5$

2.2 $x = (10 - 8) \times 7$
$= 2 \times 7 = 14$

2.3 $x = (8 - 5)(5)$
$= 3(5) = 15$

2.4 $x = (7 + 2) \div 7$
$= 9 \div 7 = 1{,}29$

3.1 $(x + 1) \times 4 = 12$
$x + 1 = \frac{12}{4}$
$x + 1 - 1 = 3 - 1$
$x = 2$

3.2 $\frac{x+1}{4} = 4$
$x + 1 = 4 \times 4$
$x = 16 - 1$
$x = 15$

3.3 $x \times 4 + 9 = 37$
$4x = 37 - 9$
$x = \frac{28}{4}$
$x = 7$

3.4 $(x)9 - 20 = 34$
$9x = 34 + 20$
$x = \frac{54}{9} = 6$

3.5 $\frac{x-6}{7} = 9$
$x - 6 = 9(7)$
$x = 63 + 6 = 69$

3.6 $x(1) + 7 = -3$
$x = -3 - 7$
$x = -10$

4.1 $\frac{8(7) - 5}{2} = \frac{51}{2} = 25{,}5$

4.2 $\frac{x(7) - 5}{2} = 22$
$7x - 5 = 22(2)$
$7x = 44 + 5$
$x = \frac{49}{7} = 7$

4.3 $\frac{x(7) - 5}{2} = -13$
$7x - 5 = -13(2)$
$7x = -26 + 5$
$7x = -21$
$x = -\frac{21}{7} = -3$

5.1

x	1	2	3	4	5	6	7
$5x + 3$	$5(1) + 3 = 8$	$5(2) + 3 = 13$	$5(3) + 3 = 18$	$5(4) + 3 = 23$	$5(5) + 3 = 28$	$5(6) + 3 = 33$	$5(7) + 3 = 38$

5.2

x	1	2	3	4	5	6	7
$3x + 10$	$3(1) + 10 = 13$	$3(2) + 10 = 16$	$3(3) + 10 = 19$	$3(4) + 10 = 22$	$3(5) + 10 = 25$	$3(6) + 10 = 28$	$3(7) + 10 = 31$

5.3

x	1	2	3	4	5	6	7
$4x - 2$	$4(1) - 2 = 2$	$4(2) - 2 = 6$	$4(3) - 2 = 10$	$4(4) - 2 = 14$	$4(5) - 2 = 18$	$4(6) - 2 = 22$	$4(7) - 2 = 26$

5.4

x	1	2	3	4	5	6	7
$6(x + 1)$	$6(1 + 1) = 12$	$6(2 + 1) = 18$	$6(3 + 1) = 24$	$6(4 + 1) = 30$	$6(5 + 1) = 36$	$6(6 + 1) = 42$	$6(7 + 1) = 48$

5.5

x	1	2	3	4	5	6	7
$\frac{2x + 6}{x}$	$\frac{2(1) + 6}{1} = 8$	$\frac{2(2) + 6}{2} = 5$	$\frac{2(3) + 6}{3} = 4$	$\frac{2(4) + 6}{4} = 3{,}5$	$\frac{2(5) + 6}{5} = \frac{16}{5} = 3{,}2$	$\frac{2(6) + 6}{6} = \frac{18}{6} = 3$	$\frac{2(7) + 6}{7} = \frac{20}{7} = 2{,}86$

6.1 $x^3 + 2$ 6.2 $x^2 + 5$ 6.3 $x^5 - 2x^2$

6.4 $x^2 \times x^3 = 32$ 6.5 $\frac{2x^3}{x^2} = 6$

Review Copy

Remedial

Some learners may have difficulty with understanding the concept 'equivalent'. Demonstrate this concept with a simple example such as "double the first five natural numbers" shown by a table, a flow diagram and an equation. Also demonstrate how a description like $y = 2x$ with domain {2; 4; 6; 8; 10} is *not* equivalent to $y = 2x$ with domain {1; 2; 3; 4; 5}.

Some learners may need help with the verbal descriptions of relationships. Let them first attempt easy examples and gradually tackle more complicated problems.

Extension

Challenge learners to create *equivalent* forms of input – output relationships using formulae from other parts of the curriculum or relationships that occur in other school subjects, such as science and technology.

Consolidation

Learner's Book page 342

Before doing this consolidation exercise, encourage learners to review the work covered in this chapter. Advise learners to use the summary and to revise their work. This exercise can be used as an informal assessment task for you to track how learners are coping with the chapter and the concepts covered. The mark allocation provides guidelines on how to assess learners.

Suggested answers

1 $\boxed{x} \xrightarrow{\times 2} \boxed{+8} \xrightarrow{\times 3} \boxed{-12} \xrightarrow{\div 6} \boxed{-x}$

The answer will always be 2. (5)

2.1 $\boxed{3} \longrightarrow \boxed{+12} \longrightarrow \boxed{15}$

2.2 $\boxed{35} \longrightarrow \boxed{-15} \longrightarrow \boxed{20}$

2.3 $\boxed{100} \longrightarrow \boxed{-15} \longrightarrow \boxed{85}$

2.4 $\boxed{15} \longrightarrow \boxed{-15} \longrightarrow \boxed{0}$

2.5 $\boxed{5} \longrightarrow \boxed{\times 5} \longrightarrow \boxed{25}$ (5)

3.1 $\boxed{-33} \longrightarrow \boxed{+53} \longrightarrow \boxed{20}$

3.2 $\boxed{34} \longrightarrow \boxed{-22} \longrightarrow \boxed{12}$

3.3 $\boxed{-3} \longrightarrow \boxed{\times(-3)} \longrightarrow \boxed{15}$

3.4 $\boxed{-80} \longrightarrow \boxed{\div 4} \longrightarrow \boxed{-20}$

3.5 $\boxed{65} \longrightarrow \boxed{-22} \longrightarrow \boxed{43}$ (5)

Review Copy

4.1 $x = \frac{(15-12)15}{9} = \frac{(3)(15)}{9} = \frac{45}{9} = 5$ 4.2 $x = \frac{\frac{75+12}{29}}{9} = \frac{\frac{87}{29}}{9} = \frac{3}{9} = \frac{1}{3}$

4.3 $\frac{125}{25} \times 15 \div 5 = \frac{5 \times 15}{5} = \frac{75}{5} = 15$

4.4 $x = (400 \times 2) + 20 - 650 = 800 + 20 - 650 = 170$

4.5 $\frac{(\sqrt{169} - 5) \times 8}{16} = \frac{(13 - 5) \times 8}{16} = \frac{8 \times 8}{16} = \frac{64}{16} = 4$ (5)

5.1 $-6 + 5 + 2 = 1$

5.2 $-6(-3 \div 3) - (-6) = -6(-1 + 6) = -6(5) = -30$

5.3 $6 \div 6 + 1 + 1 - 5 = 1 + 1 + 1 - 5 = -2$

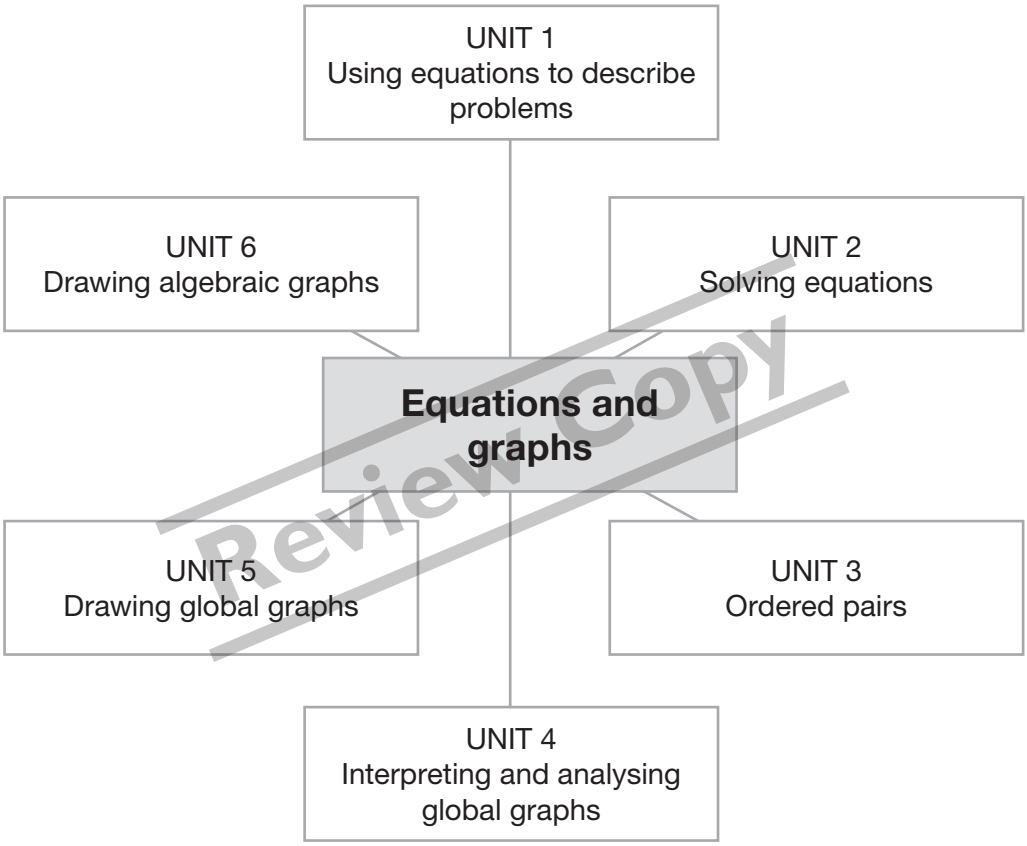
5.4 $6(2 + 3) + 4 - 5 = -6(5) + 4 - 5 = 30 + 4 - 5 = 29$

5.5 $11 + 5 - (5 - 1) + 15 = 16 - (4) + 15 = 27$ (10)

Total marks [30]

Chapter 16 Equations and graphs

Overview of concepts



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Content		Time allocations	LB page
Unit 1	Using equations to describe problems	$\frac{1}{2}$ hour	344
Unit 2	Solving equations	1,5 hours	346
Unit 3	Ordered pairs	1 hour	354
Unit 4	Interpreting and analysing global graphs	3 hours	357
Unit 5	Drawing global graphs	3 hours	368
Unit 6	Drawing algebraic graphs	3 hours	375

Background information on equations and graphs

In the intermediate phase and in Grade 7 learners learnt:

- to write, analyse and interpret number sentences to describe problem situations
- how to solve number sentences by trial and improvement
- to complete tables of ordered pairs using substitution
- to analyse and interpret global graphs as well as drawing these.

Equations were also dealt with in Chapters 6, 11 and 12 of the Grade 8 Learner's Book. What is new to the learners is the Cartesian plane and how to draw algebraic graphs.

Graphs often appear in the media, especially to illustrate economic and statistical data. Examples of such graphs on a notice board in the class room can help to make the learners aware of the importance of this topic.

Generic teaching guidelines for teaching equations and graphs

The key concepts in every unit should be consolidated with a number of worked examples including examples other than those of this chapter in the Learner's Book.

It is important that the learners participate when the teacher does examples on the board. This can be achieved by asking questions that lead to answers that suggest what the teacher should write on the board as 'a next step'.

Ideally the learners should do exercises in their books or work sheets under supervision of the teacher so that he/she can monitor their progress and provide help and corrective intervention where required. Unfinished exercises can be given as homework but must be followed up by the teacher at a next lesson.

Review Copy

Resources

Cardboard and colour pens; vocabulary cards; grid paper; blank Cartesian planes. Each learner should have their own calculator.

Unit 1 Using equations to describe problems

Learner's Book page 344

Unit focus

- transforming given information into algebraic language.

Background information on using equations to describe problems

In Grade 7 the learners learnt to:

- write number sentences to describe problem situations
- analyse and interpret number sentences that describe a given situation.

In Chapter 14 of this book learners also had to work with equations and verbal descriptions of relationships relevant to descriptions of problems.

Exercise 1

Learner's Book page 345

Guidelines on how to implement this activity

This unit is mostly revision of the same topic done in Grade 7. First establish what the learners remember of number sentences. Revise with learners how to write

an equation to describe a problem situation and how to analyse and interpret an equation that describes a given situation.

Do several worked examples on the board with participation of the learners. The exercises should be started under supervision of the teacher who monitors the learners' progress. The remainder of the exercises can be done as homework.

Suggested answers

1.1 $x + 8$

1.2 $4x$

1.3 $4z$

1.4 $x + (x + 2)$

1.5 R67y

2.1 $g + 7 = 62$

2.2 $\frac{x}{y} = 13$

2.3 $5 + 6x = 35$

2.4 R35x = R840

2.5 0,25y = R15

$y = \frac{\text{R15}}{0{,}25}$

Remedial

Learners may need some revision of formulae and concepts done previously such as the measurement formulae (volume, area, etc.), Pythagoras theorem and simple interest.

Some learners may have difficulty with interpreting a problem situation and to translate it into an equation. Let them first attempt easy cases and gradually progress to more difficult cases.

If some learners seem to struggle with solving an equation, let them first attempt easy cases and then progress to more challenging cases.

Review Copy

Extension

Challenge learners to find examples of problem situations that occur in other school subjects such as Economic and Management Science and Technology, and to translate these into equations.

Learners can also be asked to interpret problem situations described in the financial and economic pages of newspapers and, where appropriate, to translate these into equations.

Unit 2 Solving equations

Learner's Book page 346

Unit focus:

- Solving equations by:
  - inspection
  - substitution
  - application of rules of inverses
  - elimination of brackets.
- Solving equations with:
  - fractions
  - exponents.

Background information on solving equations

In Grade 7 the learners learnt to:

- solve and complete number sentences by inspection and by trial and improvement
- identify variables and constants in given formulae or equations.

When studying Chapters 6, 11 and 12 of the Learner's Book, learners also dealt with equations.

Exercise 1

Learner's Book page 347

Guidelines on how to implement this activity

Use this exercise to assess whether the learners can solve simple equations by inspection. Do the worked examples together as a class, and then encourage learners to continue with the exercise on their own.

Suggested answers

1 $x + 12 = 20$
$8 + 12 = 20$
$\therefore x = 8$

2 $x - 17 = 12$
$29 - 17 = 12$
$\therefore x = 29$

3 $2x = 10$
$2(5) = 10$
$\therefore x = 5$

4 $\frac{x}{10} = 2$
$\frac{20}{10} = 2$
$\therefore x = 20$

5 $x + 5 = 3$
$-8 + 5 = 3$
$\therefore x = -8$

6 $x - 3 = -8$
$-8 - 3 = -8$
$\therefore x = -5$

7 $6x = 42$
$6(7) = 42$
$\therefore x = 7$

8 $2x + 6 = 16$
$2(5) + 6 = 16$
$\therefore x = 5$

9 $\frac{2x}{3} = 4$
$\frac{2(6)}{3} = 4$
$\therefore x = 6$

10 $3x - 4 = 2$
$3(2) - 4 = 2$
$\therefore x = 2$

11 $4x - 2 = -10$
$4(-2) - 2 = -10$
$\therefore x = -2$

12 $\frac{20}{2x} = 5$
$\frac{20}{2(2)} = 5$
$\therefore x = 2$

Review Copy

Remedial

Identify learners that have struggled and give them Grade 7 material and allow them to practise the skill of solving equations with simple examples. Once learners are proficient at a Grade 7 level, have them rework through Exercise 1 again.

Exercise 2

Learner's Book page 348

Guidelines on how to implement this activity

Demonstrate to learners how to use substitution to solve equations. Be sure to show learners how important substitution is in determining valid solutions.

Work through Worked examples 4 and 5, and additional examples of your own design.

Have learners come up to the board and work through some of the worked examples with you, in order to achieve learner participation.

Encourage learners to complete the exercise on their own.

Suggested answers

1 $2 + 5 = 9 - 2$
$7 = 7$
$\therefore$ true

2 $14 + 3 = 18 - 3$
$17 \neq 15$
$\therefore$ not true

3 $25 - 5 = 4(5)$
$20 = 20$
$\therefore$ true

4 $\frac{80}{5} = 4(5) - 4$
$16 = 20 - 4$
$16 = 16$
$\therefore$ true

5 $2(4) + 12 = \frac{40}{4}$
$8 + 12 = 10$
$20 \neq 10$
$\therefore$ not true

6 $\frac{20(6)}{3} = 7(6)$
$40 \neq 42$
$\therefore$ not true

7 $7^2 + 7 = 8(7)$
$49 + 7 = 56$
$56 = 56$
$\therefore$ true

8 $\frac{27}{9} + 4 = 9 - 3$
$3 + 4 = 9 - 3$
$7 \neq 6$
$\therefore$ not true

9 $\frac{10^2}{10 + 10} = \frac{1}{2} \times 10$
$\frac{100}{20} = 5$
$5 = 5$
$\therefore$ true

10 $5(8) \times 2 = \frac{3(8)}{8 \times 25}$
$40 \times 2 = \frac{24}{200} \times 25$
$80 \neq \frac{24}{200}$
$\therefore$ not true

11 $8\left(\frac{1}{2}\right) + 4\left(\frac{1}{2}\right) = \frac{60}{10}$
$4 + 2 = 6$
$\therefore$ true

12 $\sqrt{16} + 4 = \frac{16}{2}$
$4 + 4 = 8$
$8 = 8$
$\therefore$ true

Review Copy

Remedial

Learners should be able to manage at this level. If learners experience problems refer them back to Grade 7 material and earlier chapters in this book.

Exercise 3

Learner's Book page 351

Guidelines on how to implement this activity

Revise the concepts *additive inverse* and *multiplicative inverse* of integers and fractions numbers with the learners. Ensure learners understand each of these concepts.

Demonstrate by means of worked examples how to use these concepts to solve equations. This is the method that learners will use to solve all future equations in mathematics. It is necessary to spend time on it and do as many examples as learners require.

Make it clear to the learners that we want to find a value of the variable (the unknown number) that will *satisfy* the equation. Ensure learners know the golden rule "What you do to the LHS you must also do to the RHS" for solving an equation. Demonstrate this when working through the worked examples. Show learners how to check whether an answer satisfies the given equation, by substitution. Learners should complete this exercise on their own.

Suggested answers

1.1 $x + 4 = 16$
$x = 16 - 4 = 12$

1.2 $x - 5 = 3$
$x = 3 + 5 = 8$

1.3 $2x = 22$
$x = \frac{22}{2} = 11$

1.4 $3x = 21$
$x = \frac{21}{3} = 7$

1.5 $\frac{x}{5} = 5$
$x = 5 \times 5 = 25$

1.6 $\frac{x}{6} = 2$
$x = 2 \times 6 = 12$

2.1 $4x + 4 = 16$
$4x = 16 - 4$
$x = \frac{12}{4} = 3$

2.2 $3x + 8 = 20$
$3x = 20 - 8$
$x = \frac{12}{3} = 4$

2.3 $11x - 4 = 29$
$11x = 29 + 4$
$x = \frac{33}{11} = 3$

2.4 $5x - 3 = 37$
$5x = 37 + 3$
$x = \frac{40}{5} = 8$

2.5 $6x + 2 = -10$
$6x = -10 - 2$
$x = -\frac{12}{6} = -2$

2.6 $3x - 3 = -12$
$3x = -12 + 3$
$x = -\frac{9}{3} = -3$

3.1 $x + 2 = 2x - 1$
$2 + 1 = 2x - x$
$x = 3$

3.2 $5x - 3 = 2x + 6$
$5x - 2x = 6 + 3$
$3x = 3x$
$\frac{3x}{3} = \frac{9}{3}x$
$x = 3$

3.3 $3x + 15 = 8x - 5$
$15 + 5 = 8x - 3x$
$20 = 5x$
$\frac{5x}{5} = \frac{20}{5}$
$x = 4$

3.4 $4x + 6 = 10x - 6$
$6 + 6 = 10x - 4x$
$12 = 6x$
$x = \frac{12}{6} = 2$

3.5 $5x + 7 = 6x - 14$
$7 + 14 = 6x - 5x$
$21 = x$
$x = 21$

3.6 $3x + 2 + 5x = x + 44$
$3x + 5x - x = 44 - 2$
$7x = 42$
$x = \frac{42}{7} = 6$

Review Copy

Remedial

It is crucial that learners are able to solve equations in this way. Provide additional material, at a similar level as this exercise, for learners to practise at home. Continue with exercises until learners are proficient.

Exercise 4

Learner's Book page 352

Guidelines on how to implement this activity

This section extends learners to solving equations with brackets and fractions. Learners need to integrate their knowledge and skill of working with *brackets* (applying the distributive rule), *algebra* and *fractions* with their knowledge of solving equations. These equations can be complex, and learners often need to keep BODMAS in mind, so that they tackle the equation in a systematic way.

Work through as many worked examples as necessary, ensuring you include brackets, fractions and the need for simplifying like terms. Learners may require assistance in the form of reminders as they work through this exercise.

Suggested answers

1 $3(x + 2) = 12$
$3x = 12x - 6$
$x = \frac{6}{3} = 2$

2 $5(2x - 3) = 15$
$10x - 15 = 15$
$10x = 15 + 15$
$10x = 30$
$x = \frac{30}{10} = 3$

3 $35 = 5(7 + 2x)$
$35 = 35 + 10x$
$35 - 35 = 10x$
$\frac{0}{10} = x$

4 $56 = 7(2 - 3x)$
$56 = 14 - 21x$
$56 - 14 = -21x$
$\frac{42}{-21} = x$
$x = -2$

5 $3(x + 2) = 5(x - 2)$
$3x + 6 = 5x - 10$
$6 + 10 = 5x - 3x$
$16 = 2x$
$x = \frac{16}{2} = 8$

6 $4(x - 2) = 3(x + 2)$
$4x - 8 = 3x + 6$
$4x - 3x = 6 + 8$
$x = 14$

7 $\frac{x}{2} + 5 = 13$
$\frac{x}{2} = 8$
$x = 8(2) = 16$

8 $14 = \frac{x}{3} - 10$
$14 + 10 = \frac{x}{3}$
$24(3) = x$
$x = 72$

9 $\frac{x + 5}{2} = -9$
$x + 5 = (-9) \times 2$
$x = -18 - 5$
$x = -23$

10 $3 - \frac{x}{4} = -5$
$3 + 5 = \frac{x}{4}$
$4(8) = x$
$x = 32$

11 $\frac{x - 9}{5} = -2$
$x - 9 = -2(5)$
$x = -10 + 9 = -1$

12 $\frac{2x}{3} + 1 = 13$
$\frac{2x}{3} = 13 - 1$
$2x = 12(3)$
$x = \frac{36}{2} = 18$

Review Copy

Remedial

If learners experience problems, help them to break up the equation into manageable steps. Show learners how to tackle the equation in a step by step way. This simplifies the process and builds learner confidence.

Extension

Challenge some learners to try and solve non-linear equations such as $\pi r^2 = 66$, $x^3 = 100$, and so on.

Exercise 5

Learner's Book page 353

Guidelines on how to implement this activity

Learners must extend their knowledge of solving equations even further by bringing in the concepts of squares, cubes, square roots and cube roots. Revise perfect squares and cubes with learners. Learners should know their squares up to 144 and their cubes up to 216. Revise these with learners. Also revise 1 000 as 10^3. Work through the worked examples that integrate solving equations with exponents.

Learners should complete this exercise on their own.

Suggested answers

1 $x^2 = 49$
$\sqrt{x^2} = \sqrt{49}$
$x = 7$ or $x = -7$

2 $x^2 = 81$
$\sqrt{x^2} = \sqrt{81}$
$x = 9$ or $x = -9$

3 $x^3 = 64$
$\sqrt[3]{x^3} = \sqrt[3]{4^3}$
$x = 4$

4 $x^3 = 1\,000$
$\sqrt[3]{x^3} = \sqrt[3]{10^3}$
$x = 10$

5 $x^2 - 25 = 75$
$x^2 = 75 + 25$
$\sqrt{x^2} = \sqrt{100}$
$x = +25$ or $x = -10$

6 $x^2 - 4 = 77$
$x^2 = 77 + 4$
$x^2 = 81$
$\sqrt[2]{x^2} = \sqrt[2]{9^2}$
$x = 9$ or $x = -9$

7 $\frac{x^2}{3} = 12$
$x^2 = 12(3)$
$\sqrt{x^2} = 36$
$x = \sqrt{6^2}$

8 $\frac{x^3}{4} = 16$
$x^3 = 16(4)$
$x^3 = 4^3$
$x = 4$

9 $2x^2 = 50$
$x^2 = \frac{50}{2}$
$\sqrt{x^2} = \sqrt{5^2}$
$x = +5$ or $x = -5$

10 $4x^2 = 4\,000$
$x^3 = \frac{4\,000}{4} = 1\,000$
$\sqrt[3]{x^3} = \sqrt[3]{10^3}$
$x = 10$

Remedial

If learners cannot remember their squares and cubes, ask them to redraw up their squares and cubes chart. Have learners memorise these and test them daily until all learners know these numbers.

Extension

Provide talented learners with examples of Maths Olympiad questions requiring the solution of equations.

Encourage talented learners to take part in appropriate Maths Olympiads.

Review Copy

Unit 3 Ordered pairs

Learner's Book page 354

Unit focus:

- learning how to put together ordered pairs
- using ordered pairs to write coordinates.

Background information on ordered pairs

In Grade 7 the learners learnt (as part of 'Functions and relationships') to complete tables of ordered pairs using substitution in a given rule or equation.

This was also done in Chapter 14 of the Grade 8 Learner's Book.

Exercise 1

Learner's Book page 355

Guidelines on how to implement this activity

Revise with learners how they have worked with equations that have two variables.

Remind learners that the value of the one variable, usually y, is dependent on the input variable, usually x. Show learners some of the examples they worked through in Chapter 14. Revise by doing the worked examples on the board, especially where a x-value (input value) has to be calculated when a y-value (output value) is given. Be sure to show learners that the values are only valid alongside the other variable's value. Learners should complete this exercise on their own.

Suggested answers

1.1 $x + y = 3$ if $x = 1$ and $y = 2$; $x = 0$ and $y = 3$; $x = -1$ and $y = 4$; and others

1.2 $x + y = 7$ if $x = 1$ and $y = 6$; $x = 2$ and $y = 5$; $x = 3$ and $y = 4$ and others

1.3 $x + y = 8$ if $x = 1$ and $y = 7$; $x = 2$ and $y = 6$; $x = 3$ and $y = 5$; $x = 4$ and $y = 4$; and others

1.4 $2x + 2y = 6$ or $2(x + y) = 6$ if $x = 1$ and $y = 2$; $x = 0$ and $y = 3$; $x = 2$ and $y = 1$ and others

1.5 $3x - y = 11$ if $x = 2$ and $y = -5$; if $x = 3$ and $y = -2$; if $x = 4$ and $y = 1$; if $x = 5$ and $y = 4$ and others

2.1 $2x + 3y = 13$
$x = 2$; $y = 3$ and $x = 5$; $y = 1$ and others

2.2 $5x - y = 3$
$x = 1$; $y = 2$ and $x = 2$; $y = 7$ and $x = 3$; $y = 12$ and $x = 4$; $y = 17$ and others

2.3 $3x + 5y = 11$
$x = 2$; $y = 1$ and others

2.4 $10x - 4y = 2$
$x = 1$; $y = 2$ and $x = 3$; $y = 7$ and others

2.5 $x + 7y = 9$
$x = 2$; $y = 1$ and others

2.6 $3x - 4y = 1$
$x = 5$; $y = 1$ and $x = 9$; $y = 2$ and $x = 13$; $y = 3$ and others

2.7 $8x - 2y = 8$
$x = 1$; $y = 0$ and $x = 2$; $y = 4$ and $x = 3$; $y = 8$ and $x = 4$; $y = 12$ and others

Review Copy

Remedial

Some learners may be inclined to make errors in substituting x-values into an equation in order to calculate y-values. Let them use brackets, especially where negative numbers are involved.

Extension

Challenge some learners to create tables of number pairs for non-linear equations such as $y = x^3 - 10$ and $y = \sqrt{x + 10}$.

Exercise 2

Learner's Book page 356

Guidelines on how to implement this activity

Explain to learners the ordered pairs – that the value of y is only true for the given value of x. Show learners how we use a table to group our pairs.

Ask learners where they have used this table before – when working with functions and relationships. Do a few worked examples as a class. Show learners that just

because we have integers for x, does not mean that y-values will always be integers. Include worked examples with fractions and decimals. The exercise should be started under supervision of the teacher who monitors the learners' progress. The remainder of the exercises can be done as homework.

Suggested answers

1 $y = x + 8$
$y = -3 + 8 = 5$
$y = -2 + 8 = 6$
$y = -1 + 8 = 7$
$y = 0 + 8 = 8$
$y = 1 + 8 = 9$
$y = 2 + 8 = 10$
$y = 3 + 8 = 11$

x	–3	–2	–1	0	1	2	3
y	5	6	7	8	9	10	11

2 $y = 2x - 1$
$y = 2(-3) - 1 = -6 - 1 = -7$
$y = 2(-2) - 1 = -4 - 1 = -5$
$y = 2(-1) - 1 = -2 - 1 = -3$
$y = 2(0) - 1 = 0 - 1 = -1$
$y = 2(1) - 1 = 2 - 1 = 1$
$y = 2(2) - 1 = 4 - 1 = 3$
$y = 2(3) - 1 = 6 - 1 = 5$

x	–3	–2	–1	0	1	2	3
y	–7	–5	–3	–1	1	3	5

3 $y = 2x + 1$
$y = 2(-3) + 1 = -6 + 1 = -5$
$y = 2(-2) + 1 = -4 + 1 = -3$
$y = 2(-1) + 1 = -2 + 1 = -1$
$y = 2(0) + 1 = 0 + 1 = 1$
$y = 2(1) + 1 = 2 + 1 = 3$
$y = 2(2) + 1 = 4 + 1 = 5$
$y = 2(3) + 1 = 6 + 1 = 7$

x	–3	–2	–1	0	1	2	3
y	–5	–3	–1	1	3	5	7

4 $x + y = 10$ or $y = 10 - x$
$y = 10 - (-3) = 13$
$y = 10 - (-2) = 12$
$y = 10 - (-1) = 11$
$y = 10 - (0) = 10$
$y = 10 - (1) = 9$
$y = 10 - (2) = 8$
$y = 10 - 3 = 7$

Review Copy

x	–3	–2	–1	0	1	2	3
y	13	12	11	10	9	8	7

5 $x + y = 4$ or $y = 4 - x$

$y = 4 - (-3) = 7$

$y = 4 - (-2) = 6$

$y = 4 - (-1) = 5$

$y = 4 - (0) = 4$

$y = 4 - (1) = 3$

$y = 4 - (2) = 2$

$y = 4 - (3) = 1$

x	–3	–2	–1	0	1	2	3
y	7	6	5	4	3	2	1

6 $y = 2 - \frac{x}{10}$

$y = 2 - \frac{(-3)}{10} = 2{,}3$

$y = 2 - \frac{(-2)}{10} = 2{,}2$

$y = 2 - \frac{(-1)}{10} = 2{,}1$

$y = 2 - \frac{(0)}{10} = 2{,}0$

$y = 2 - \frac{(1)}{10} = 1{,}9$

$y = 2 - \frac{(2)}{10} = 1{,}8$

$y = 2 - \frac{(3)}{10} = 1{,}7$

x	–3	–2	–1	0	1	2	3
y	2,3	2,2	2,1	2,0	1,9	1,8	1,7

7 $y = \frac{x}{2} + 5$

$y = \frac{-3}{2} + 5 = 3{,}5$

$y = \frac{-2}{2} + 5 = 4$

$y = \frac{-1}{2} + 5 = 4{,}5$

$y = \frac{0}{2} + 5 = 5$

$y = \frac{1}{2} + 5 = 5{,}5$

$y = \frac{2}{2} + 5 = 6$

$y = \frac{3}{2} + 5 = 6{,}5$

x	–3	–2	–1	0	1	2	3
y	3,5	4	4,5	5	5,5	6,0	6,5

Review Copy

8 $y = 2x + \frac{3}{2}$

$y = 2(-3) + \frac{3}{2} = -6 + \frac{3}{2} = -4{,}5$

$y = 2(-2) + \frac{3}{2} = -4 + \frac{3}{2} = -2{,}5$

$y = 2(-1) + \frac{3}{2} = -2 + \frac{3}{2} = -0{,}5$

$y = 2(0) + \frac{3}{2} = 0 + \frac{3}{2} = 1{,}5$

$y = 2(1) + \frac{3}{2} = 2 + \frac{3}{2} = 3{,}5$

$y = 2(2) + \frac{3}{2} = 4 + \frac{3}{2} = 5{,}5$

$y = 2(3) + \frac{3}{2} = 6 + \frac{3}{2} = 7{,}5$

x	–3	–2	–1	0	1	2	3
y	–4,5	–2,5	–0,5	1,5	3,5	5,5	7,5

9 $3x + 2y = 19$ or $y = \frac{19 - 3x}{2}$

$y = \frac{19 - 3(-3)}{2} = \frac{19 + 9}{2} = \frac{28}{2} = 14$

$y = \frac{19 - 3(-2)}{2} = \frac{19 + 6}{2} = \frac{25}{2} = 12{,}5$

$y = \frac{19 - 3(-1)}{2} = \frac{19 + 3}{2} = \frac{22}{2} = 11$

$y = \frac{19 - 3(0)}{2} = \frac{19}{2} = 9{,}5$

$y = \frac{19 - 3(1)}{2} = \frac{19 - 3}{2} = \frac{16}{2} = 8$

$y = \frac{19 - 3(2)}{2} = \frac{19 - 6}{2} = \frac{13}{2} = 6{,}5$

$y = \frac{19 - 3(3)}{2} = \frac{19 - 9}{2} = \frac{10}{2} = 5$

x	–3	–2	–1	0	1	2	3
y	14	12,5	11	9,5	8	6,5	5

10 $y = -3x + 5$

$y = -3(-3) + 5 = 9 + 5 = 14$

$y = -3(-2) + 5 = 6 + 5 = 11$

$y = -3(-1) + 5 = 3 + 5 = 8$

$y = -3(0) + 5 = 0 + 5 = 5$

$y = -3(1) + 5 = -3 + 5 = 2$

$y = -3(2) + 5 = -6 + 5 = -1$

$y = -3(3) + 5 = -9 + 5 = -4$

x	–3	–2	–1	0	1	2	3
y	14	11	8	5	2	–1	–4

Review Copy

Remedial

Learners may need assistance when working with decimals and fractions. Encourage learners to refer back to the relevant chapters and to work through the necessary concepts in order to cope with this exercise.

If learners have problems the concept of ordered pairs, keep the examples simple and confine learners to doing examples similar to numbers 1 – 5 of this exercise, until learners are adept. Gradually introduce more complex examples until learners can manage with numbers 6 – 10.

Extension

Provide talented learners with examples of Maths Olympiad questions involving the creation of sets of number pairs.

Unit 4 Interpreting and analysing global graphs

Learner's Book page 357

Unit focus:

- understanding and reading global graphs
- understanding different aspects of graphs:
  - maximum and minimum values
  - discrete and continuous data.

Review Copy

Background information on interpreting and analysing global graphs

In Grade 7 the learners learnt to analyse and interpret global graphs of problem situations, with special focus on the following trends and features:

- linear or non-linear
- constant, increasing or decreasing.

Exercise 1

Learner's Book page 357

Guidelines on how to implement this activity

Determine what the learners remember about global graphs that they learnt in grade 7 by asking questions related to graphs you draw on the board, for example:

- "Is this graph linear or non-linear?"
- "Is this graph increasing, decreasing or constant?"
- Sketch some graphs of your own choice.
- Let the learners do Exercise 1 and then discuss it by asking what answers they gave to each question.

Suggested answers

1 State whether linear or non-linear
1.1 a linear graph
1.2 a non-linear graph
2.1 Cost is shown on the vertical axis.
2.2 Number of units is shown on the horizontal axis.
2.3 As the number of units increase, the cost increases.

2.4 The cost of 5 units = R10.
2.5 The cost of 15 units will be R2 x 15 = R30.
3.1 The vertical axis displays cell growth.
3.2 The horizontal axis displays time in hours.
3.3 As time increases the number of cells increases.
3.4 There are 8 cells after 3 hours.
3.5 After each hour the cells are doubled, so after 5 hours there will be 32 cells ($1 \times 2 \times 2 \times 2 \times 2 \times 2 = 32$).
4.1 Graph A shows shoe size versus age, graph B shows test scores versus hours studies, Graph C shows the distance driven versus petrol in the tank and graph D shows bus fare versus distance travelled.
4.2 In Graph A the relationship is increasing; in Graph B it is increasing; in Graph C it is decreasing and in Graph D it is constant.
4.3 In Graph A, the points show how shoe size increases with age.
In Graph B the points show how test score increase with increasing hours of study.
Graph C shows how there is less petrol in the tank with greater distances driven.
Graph D shows that the bus fare stays the same for the two distances travelled.

Remedial

Some learners may be inclined to have difficulty in interpreting non-linear graphs, especially the hyperbola. Explain that each point represents a number pair which is the length and the breadth of some rectangle with an area of 10 cm^2. The graph shows that a rectangle with a long breadth has a short length and, as the breadth decreases, the length increases so that the area is a constant 10 cm^2.

Review Copy

Extension

Challenge some learners to create tables and draw graphs of relationships such as $C = \pi d$, $A = \pi r^2$ and SI = R100 $\times$ n $\times$ (0.05).

Exercise 2

Learner's Book page 359

Guidelines on how to implement this activity

Show learners examples of increasing, decreasing and constant graphs. Do a few examples together as a class, interpreting graphs in this way. Include some linear and non-linear graphs as well. Revise how we can use graphs to predict outcomes by following the trend of the graph.

Exercise 2 should be started under supervision of the teacher who monitors the learner's progress. The remainder of the exercises can be done as homework.

Suggested answers

1.1 The graph is linear.
1.2 The graph is increasing.
1.3 The graph increases 8 units on the vertical axis for each 1 unit on the horizontal axis .
2.1 The graph is non-linear.
2.2 The graph is increasing.

2.3 The rate of increase is not constant. The increase starts slow and then speeds up, before slowing down again. The first two units on the horizontal axis results in a five unit increase on the vertical axis. The next two units on the horizontal axis result in a ten unit increase on the vertical axis and the last section slows in the increase.
3.1 The graph is non-linear.
3.2 This is an increasing graph.
3.3 The area of the square increases each time the length increases.
3.4 With a square with sides of 5,5 cm, the area of a square will be 5,5 cm × 5,5 cm = 30,25 cm^2.
3.5 The length of the sides of a square with an area of 64 cm^2 will be $\sqrt{64\ cm^2}$ = 8 cm
4.1 The graph is linear.
4.2 The graph is decreasing.
4.3 The amount of water in the tank decreases over time.
4.4 After 7 hours the tank will be empty.

Extension

Ask learners to find examples of global graphs in newspapers, periodicals and on web sites and bring copies to school to discuss.

Exercise 3

Learner's Book page 365

Review Copy

Guidelines on how to implement this activity

Review the graphs learners have covered in this unit. Discuss maximum and minimum values. Ask learners to identify these values from the graphs they have looked at so far. Discuss discrete and continuous data.

Ensure learners understand these terms, and the difference between them. If learners need more context, provide examples of categories for discrete data, and measurement or scores for continuous data. Use a few examples to help learners understand the distinction.

Integrate all learners' knowledge of analysing graphs, and show them the checklist when analysing graphs. Work through the examples that bring everything together. Allow learners to discuss their answers in pairs, but to write all their own workings and answers in their own exercise books.

Suggested answers

1 Graph 1.1 shows the rainfall decreases over time, stops for a period, and then increases again.
Graph 1.2 shows that the cost in rand increases for the number of items bought at a constant rate.
Graph 1.3 shows that height (in m) decreases to zero as time increases, increases and decreases again to zero, increases again, but each time at half the height of the previous time (this is similar to a bouncing ball).
Graph 1.4 shows that for the population size to increase one unit from 0-1 takes much longer than the next increase of one unit. The population size increases at a faster rate over time.

2 Glass a = Graph C
Glass b = Graph B
Glass c = Graph A
Glass d = Graph A
Glass e = Graph D
Glass f = Graph B

3.1 Lumba won the race.
3.2 Lumba ran with the best strategy starting slow and increasing the pace over time.
3.3 Fouza ended last in the race as she started the race very fast, stopped for a period, ran fast again, stopped again and carried on to the finish at a slower pace.
3.4 Fouza and Lumba's runs show a non linear with Taryn's run approximating a linear trend.
4.1 The maximum temperature was experienced in February.
4.2 The maximum temperature was 31 °C.
4.3 The minimum temperature recorded was 16 °C.
4.4 The minimum temperature was recorded in August.
4.5 The winter season occurred between June and September.
4.6 Temperatures showed a decrease from February to August.
4.7 The temperature increased the most from August to December.
4.8 The temperature remained constant between June and July.
5.1 The volume of water was decreasing in the tank in the first five days.
5.2 Between Monday and Thursday the tank volume was decreasing by 50 litres per day.
5.3 The maximum volume recorded was 650 litres.
5.4 The minimum volume of water that the tank contained over the 2 week period is 300 litres.
5.5 The tank was never empty over the two week period.
5.6 The volume of water remained constant between the first Friday and Saturday.
5.7 The lowest volume the tank contained was 300 litres.

Review Copy

Remedial

Encourage learners to write the checklist on a piece of paper that they keep next to them as they complete this exercise. Learners should write their analysis according to the checklist.

Extension

Learners can analyse examples of graphs from newspapers and magazines to practise their skills in a real world context.

Unit 5 Drawing global graphs

Learner's Book page 368

Unit focus:

- learning to draw global graphs
- understanding the relationship between two variables shown on global graphs
- studying specific global graphs.

Background information on drawing global graphs

In Grade 7 the learners learnt to draw global graphs from given descriptions of a problem situation, including:

- linear or non-linear graphs
- constant, increasing or decreasing graphs.

Exercise 1

Learner's Book page 369

Guidelines on how to implement this activity

Determine what the learners remember about plotting points on graph paper from number pairs shown in a table. Explain the scale and the labeling of the axes of Worked example 1 and how the number pairs of the table correspond to the points plotted on the grid. Ask the learners whether points between those shown on the grid can also provide information about the temperature at times not exactly on the hour, for example at 09:30. This means we can join the points with a smooth curve from which temperatures can be read for any time from 05:00 to 17:00.

Let the learners do Exercise 1 under supervision of the teacher who monitors the learners' progress. Then discuss it by comparing it with Worked example 1.

Review Copy

Suggested answers

1 Study the two graphs shown on the same set of axes and answer the following questions:

1.1 Graph A is increasing and Graph B is decreasing.

1.2 The rate of increase is greater than the rate of decrease as the distance covered in Graph B is 700 km and in Graph B it is 350km over the same time period.

1.3 Both travelled 8 – 1 = 7 hours.

1.4 Vehicle B travelled the shorter distance(350 km).

1.5 Vehicle A Speed = $\frac{700}{7}$ = 100 km/h

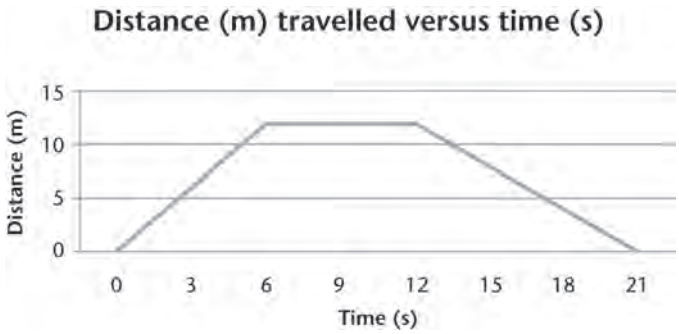
Vehicle B Speed = $\frac{350}{7}$ = 50 km/h

2 Distance travelled (km) = speed (km/h) × time (h)

Time (h)	Distance travelled by Train (km)	Distance travelled by car (km)
0	0	0
1	85 × 1 = 85	100 × 1 = 100
2	85 × 2 =170	100 × 2 = 200
3	85 × 3= 255	100 × 3 = 300
4	85 × 4 = 340	100 × 4 = 400
5	85 × 5 = 425	100 × 5 = 500

3 The graph shows the distance of 3 blocks on the vertical axis against time travelled. It is likely that Tevin waited to cross a busy street (32) between points B and C as the time scale fits this assumption.

4



4.1 Between points 0 and 6 s, the vehicle travelled 12 m.
4.2 12 s
4.3 Speed = $\frac{\text{distance}}{\text{time}} = \frac{12\text{ m}}{6\text{s}} = 2$ m/s.
4.4 Between 6 and 12 seconds the distance remained constant.
4.5 Speed = 0 m/s
4.6 Between points 12 and 21 for time, the vehicle travelled 12 m.
4.7 This took 9 s.
4.8 Speed = $\frac{\text{distance}}{\text{time}} = \frac{12}{9} = 1\frac{1}{3}$ m/s

4.9

	Time s	Distance m	Speed
1	4	12	$\frac{12}{4} = 3$
2	9	18	2
3	7	21	3
4	8	40	5
5	2	20	10
6	6	18	3
7	15,2	60,8	4
8	5	125	25
9	45	18	0,4

Review Copy

5

Graph A	(5.2) Thando travels at a constant speed away from her home for 15 minutes.
Graph B	(5.6) Thando waits 5 minutes before beginning her journey and then travels away from her home at a constant speed for 15 minutes.
Graph C	(5.3) Thando begins her journey 90 m away from her home and moves at a constant speed towards her home for 20 minutes.
Graph D	(5.4) Thando moves away from her home at a constant speed for 10 minutes, and then moves at a faster constant speed for 15 minutes.
Graph E	(5.5) Thando accelerates away from her home for 25 minutes.

Remedial

Some learners may need help to draw and label axes on a grid, to plot points and to join the points with a smooth curve. Provide hands on help with this.

Extension

Challenge some learners to create tables and draw graphs of relationships like speed-time graphs for an athletics track of 800 m.

Exercise 2

Learner's Book page 374

Guidelines on how to implement this activity

Explain the scale and the labeling of the axes of Worked example 2 (or a similar time-distance graph of the teacher's choice). Then discuss each section of the graph with participation of the learners.

Exercise 2 should be started under supervision of the teacher who monitors the learners' progress. Learners can complete this exercise as homework.

Suggested answers

1.1 The most accidents occur in December.

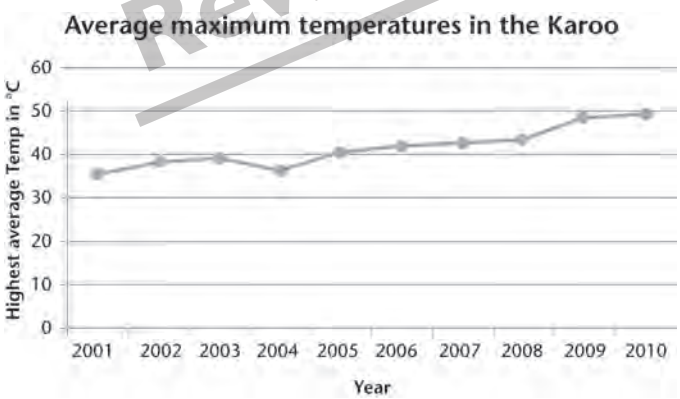
1.2 There were a total of 867 + 402 + 352 + 589 + 235 + 301 + 801 + 324 + 205 + 341 + 651 + 1 256 = 6 324 accidents in 2012. (Rounded off numbers should also be accepted.)

1.3 The number of accidents starts off high in January, decreases over the next two months, increases again in April, decreases in May and June, increases in July, decreases in August and September, after which there is a steady increase until the highest number of accidents is located in December.

1.4 The accidents appear to be highest when it is holiday period with January and December having the highest number of accidents, followed by July and April.

2.1 2010 had the highest temperature.

2.2



2.3 48,6 °C

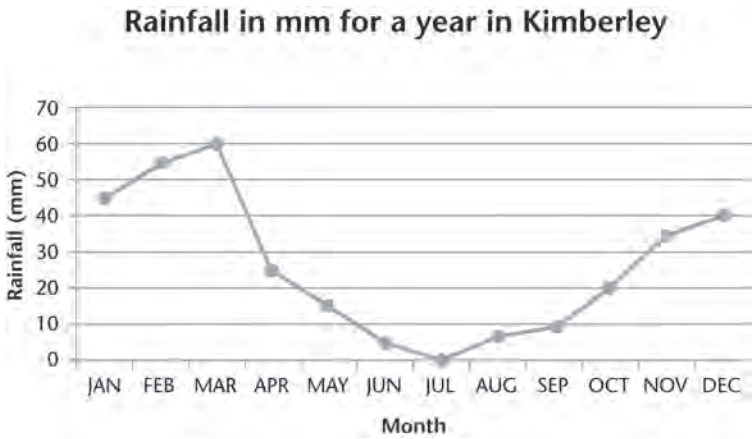
2.4 49,5 °C – 35,9 °C = 13,6 °C

2.5 The general trend in temperature is an increasing one from 2001 to 2010.

2.6 This increase in average maximum temperatures may be as a result of global warming.

Review Copy

3



The following are examples of questions that can be asked related to the graph. Additional questions may also be relevant.

3.1 In which month is the highest rainfall found? (March)
3.2 In which month does the least rainfall occur? (July)
3.3 Are there any months where no rainfall took place in Kimberley? (yes, July)
3.4 What is the total rainfall for the year? (316 mm)
3.5 In how many months of the year had less that 30 mm of rain? (7 months)

Remedial

Some learners may struggle to interpret and draw the time-distance graphs. Provide individual attention to these learners and help them set up the graph.

Extension

Ask learners to find data of rainfall or temperature variations from own measurements or from a weather station and to represent this graphically.

Review Copy

Unit 6 Drawing algebraic graphs

Learner's Book page 375

Unit focus:

- understanding how to read and work with the Cartesian plane
- plotting ordered pairs (coordinates) on a Cartesian plane
- drawing line graphs.

Background information on drawing algebraic graphs

Learners have worked with grids and coordinates, but have been limited to the first quadrant. They have only worked with grids providing for non-negative coordinates. The exposure to the full Cartesian plane is important and integral to much future mathematical work.

Exercise 1

Learner's Book page 377

Guidelines on how to implement this activity

Introduce the learners to the Cartesian plane that provides for coordinates of any numbers: positive, zero or negative. Exercise 1 is designed to help learners to gain the skill of reading the coordinates of given points and plot the points of which the coordinates are given.

Discuss Worked example 1 by asking questions such as: "Is this graph increasing or decreasing?", "Is the equation $y = 2x$ also satisfied by other points lying between those plotted?". "What do the 'arrows' on either end of the straight line indicate?"

Let the learners do Exercise 1 under supervision of the teacher who monitors the learners' progress. Then discuss it by comparing it with Worked example 1.

Suggested answers

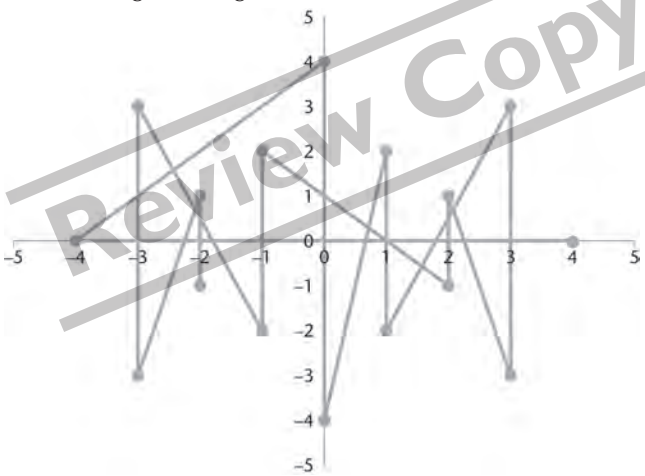
1.1 (–7; 5) 1.2 (13; 8) 1.3 (2; –3) 1.4 (–6; –3)

2.1.1 (2; 3) 2.1.2 (4; –4) 2.1.3 (–5; –3) 2.1.4 (–5; 4) 2.1.5 (–1; –3)

2.2 Sibulele started at the tennis court and finished at the camp site.

2.3 Lukhanyo cannot walk in a straight line from the bird watch to the campsite as this route would go through the water.

3.1 and 3.2



Review Copy

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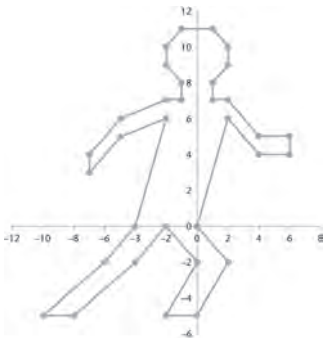

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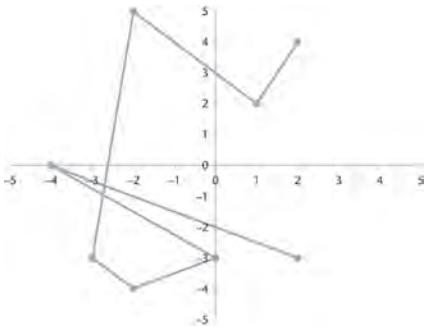

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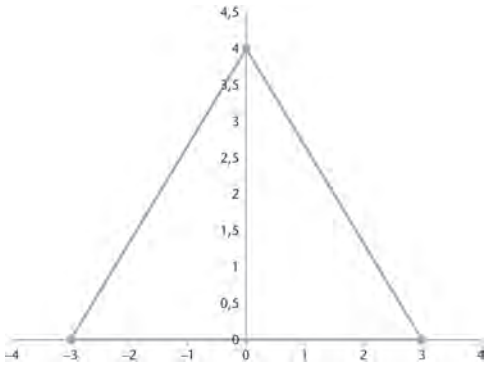
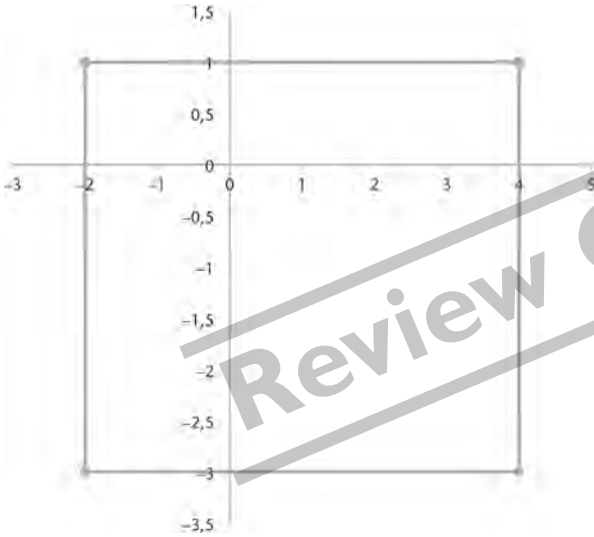


The joined coordinates form a triangle.

7.1 (–3; –2) 7.2 (2; 5) 7.3 (6; 2) 7.4 (1; –4) 7.5 (– 2; –5)

8 The coordinates of the 4th corner are (4; -3)



Review Copy

Remedial

Some learners may need help to draw and label axes on a Cartesian grid, to plot points and to join the points with a straight line or curve.

Provide learners with square grid or graph paper to help them with this. Revise how to plot points by following first the x coordinate and then matching to the corresponding y coordinate. Show learners how to use two fingers on the plane to do this.

When drawing a straight line learners must use a ruler on the graph.

Extension

Challenge some learners to create tables and draw graphs of relationships such as $y = \frac{12}{x}$ and $y = 2x$ for both negative and positive values of x.

Exercise 2

Learner's Book page 381

Guidelines on how to implement this activity

Show learners how to draw a graph using a table. Learners should recognize this table of ordered pairs, and how it is used to produce coordinate values, from Unit 3.

Discuss as a class coordinates and the relation to the equation, function and graph. Ensure that learners recognise that an algebraic graph is simply another representation of an algebraic function.

Learners should also recognise the use of the table as an intermediate step in showing this relationship. Work through the worked examples for using the table to plot the points on the Cartesian plane. This is known as the table method.

Learners should start this exercise in class, and do at least the first two examples under supervision. Learners can complete the rest of the exercise for homework.

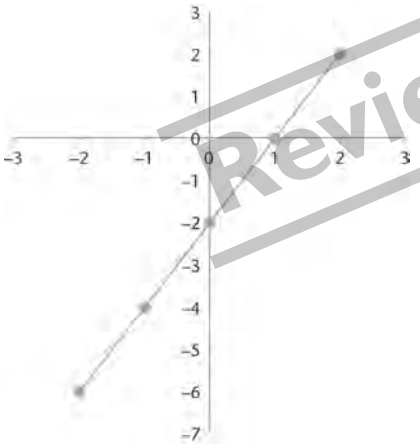
Suggested answers

1 For $y = 2x - 2$

1.1

x	–2	–1	0	1	2
y	–6	–4	–2	0	2
$(x; y)$	–2; –6	–1; –4	0; –2	1; 0	2; 2

1.2



Review Copy

2 Ordered pairs tables

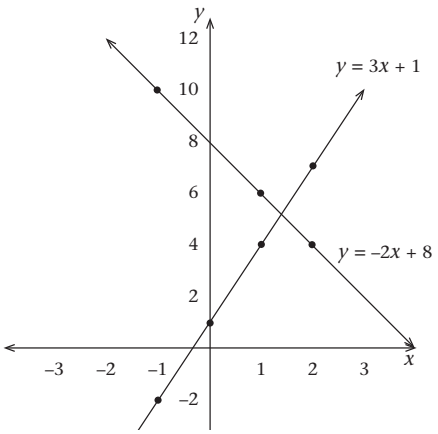
2.1

x	–2	–1	0	1	2
y	–5	–2	1	4	7
$(x; y)$	–2; –5	–1; –2	0; 1	1; 4	2; 7

2.2

x	–2	–1	0	1	2
y	12	10	8	6	4
$(x; y)$	–2; 12	–1; 10	0; 8	1; 6	2; 4

2.3



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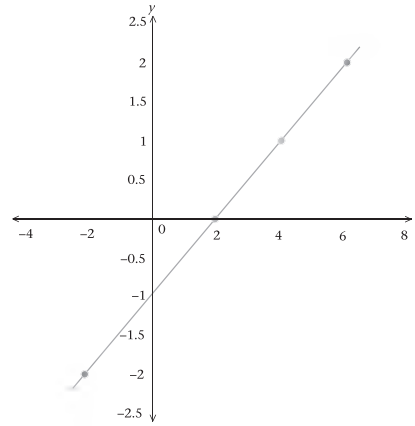
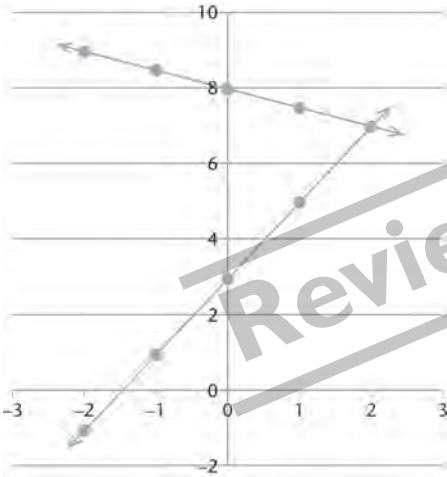


4.1 $y = 2x + 3$ and $y = -\frac{1}{2}x + 8$



Review Copy

4.2 The two graphs cross each other at (2; 7).

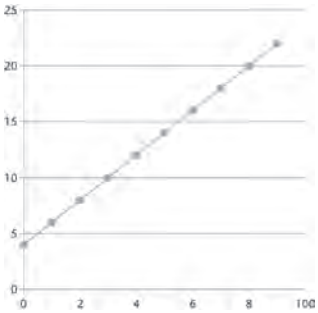
5.1

WEEK	PLANT HEIGHT (cm)
0	4
1	6
2	8
3	10
4	12
5	14
6	16
7	18
8	20
9	22

5.2 $y = 2x + 4$

5.3 (0; 4); (1; 6); (2; 8); (3; 10); (4; 12); (5; 14); (6; 16); (7; 18); (8; 20); (9; 22)

5.4



5.5 The points are continuous as the plant grows continually.

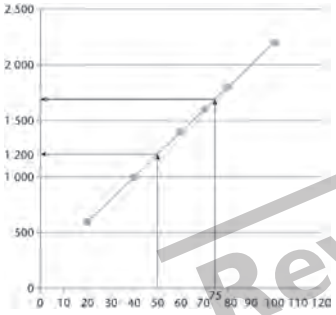
5.6 The graph is a linear graph.

6.1 C = 20 × k + R200

6.2

k	20	40	60	70	80	100
C	600	1 000	1 400	1 600	1 800	2 200

6.3



Review Copy

6.4 From the graph read off on the vertical axis R1 200

6.5 From the graph read off R1 700 on the vertical axis to show 75 km on the horizontal axis.

Remedial

Provide graph paper to help learners to construct the Cartesian plane. Revise substituting values into equations if this is where learners are experiencing difficulty.

Consolidation

Learner's Book page 385

Before doing this consolidation exercise, encourage learners to review the work covered in this chapter. Advise learners to use the summary and to revise their work. This exercise can be used as an informal assessment task for you to track how learners are coping with the chapter and the concepts covered. The mark allocation provides guidelines on how to assess learners.

Suggested answers

1.1	$x - 25 = 42$	1.2	$x + 10 = 14$	1.3	$x - 21 = 37$
1.4	$x^3 = 125$	1.5	$x - 8 = 16$	1.6	$x - 5 = 47$
1.7	$x + 5 = 26$	1.8	$x + 11 = 14$	1.9	$x + 12 = 18$

1.10 $x + 8 = 15$ (10)

2.1.1 $2(5) + 13 = 10 + 13 = 23$ (1)

2.1.2 $5(5) - 5 = 25 - 5 = 20$ (1)

2.1.3 $3 + 6(5) = 3 + 30 = 33$ (1)

2.2 LHS: $3(5) + 7 = 15 + 7 = 22$ RHS: $2(5) + 12 = 10 + 12 = 22 = \text{LHS}$ (2)

3.1 $3y + 13 = 2(y + 9)$
$3y + 13 = 2y + 18$
$y = 5$
(2)

3.2 $2(2y + 3) = y - 13$
$4y + 6 = y - 13$
$4y - y = -13 - 6$
$3y = -18$
$y = \frac{18}{3} = -6$ (2)

3.3 $\frac{10 + 2y}{3} = 4$
$10 + 2y = 4(3)$
$2y = 10 - 12$
$2y = 2\ y = \frac{2}{2} = 1$ (2)

3.4 $\frac{9y - 15}{4} = 3$
$9y - 15 = 3(4)$
$9y = 12 + 15$
$9y = 27$
$y = \frac{27}{9} = 3$ (2)

3.5 $3(2y - 1) - 2(y - 4) = 19$
$6y - 3 - 2y + 8 = 19$
$6y - 2y = 19 + 3 - 8$
$4y = 14$
$y = \frac{14}{4} = 3{,}5$ (2)

4.1 $x + y = 12$ and $x - y = 4$
$x = 12 - y$ $x = 4 + y$
$12 - y = 4 + y$
$12 - 4 = 2y$
$y = \frac{8}{2} = 4$
$\therefore x = 12 - 4 = 8$ (2)

4.2 $x - y = 3$ $x + y = 13$
$x = 3 + y$ $x = 13 - y$
$\therefore (3 + y) = 13 - y$
$2y = 13 - 3 = 10$
$y = \frac{10}{2} = 5$
$\therefore x = 3 + 5 = 8$ (2)

4.3 The difference in speeds = 158 – 112 = 46 km/h
Wind speed = $\frac{46}{2}$ = 23 km/h
Plane speed = 158 – 23 = 135 km/h
or 112 + 23 = 135 km/h (3)

4.4 Student = x Adult = y
$3x + y$= R38 and $3x + 2y$ = R52
Difference: $0x + 1y$ = R52 – R38 = R14
$\therefore y$ = R14 (Adult ticket)
$\therefore 3x$ + R14 = R38
$3x$ = R38 – R14 = R24
$x = \frac{\text{R24}}{3}$ = R8 (Student ticket)
Check: $3 \times 8 = 2 \times 14$ = R28 (3)

4.5 The number is 34
3 + 4 = 7
The reverse of 34 = 43 The increase from 34 to 43 = 9 (4)

4.6 210 km ÷ 2 = 105 km one way
Downstream = $\frac{105\text{ km}}{10\text{ h}}$ = 10,5 km/h
Upstream = $\frac{105\text{ km}}{70\text{ h}}$ = 1,5 km/h
Still water boat speed = $\frac{10{,}5 + 1{,}5}{2}$ = 6 km/h
Speed of current = 6 km/h – 1,5 km/h = 4,5 km/h (10)

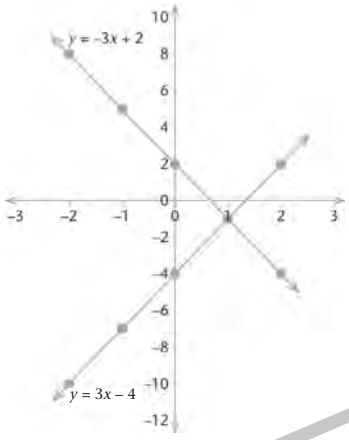
Review Copy

5.1 & 5.2

(i)

x	–2	–1	0	1	2
y	–10	–7	–4	–1	2
(x; y)	–2; –10	–1; –7	0; –4	1; –1	2; 2

x	–2	–1	0	1	2
y	8	5	2	–1	–4
(x; y)	–2; 8	–1; 5	0; 2	1; –1	2; –4

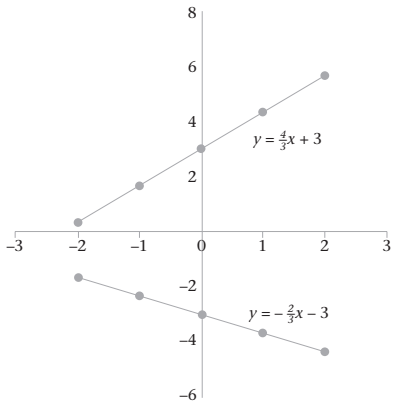


(ii)

x	–2	–1	0	1	2
y	0,33	1,67	3	4,33	5,67
(x; y)	–2; 0,33	–1; 1,67	0; 3	1; 4.33	2; 5,67

	–2	–1	0	1	2
	–1,67	–2,33	–3	–3,67	–4,33
(x; y)	–2; –1,67	–1; –2,33	0; –3	1; –3,67	2; –4,33

(iv)

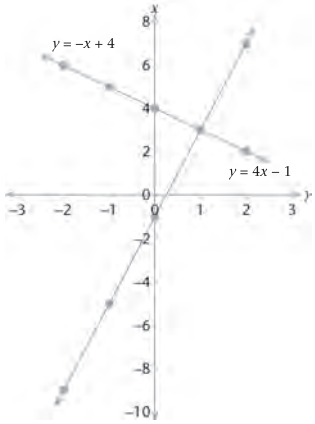


Review Copy

(iii)

x	–2	–1	0	1	2
y	–9	–5	–1	3	7
$(x; y)$	–2; –9	–1; –5	0; –1	1; 3	2; 7

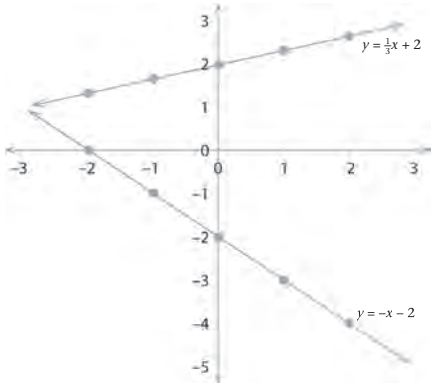
	–2	–1	0	1	2
	6	5	4	3	2
$(x; y)$	–2; 6	–1; 5	0; 4	1; 3	2; 2



(iv)

x	–2	–1	0	1	2
y	1,33	1,67	2	2,33	2,67
$(x; y)$	–2; 1,33	–1; 1,67	0; 2	1; 2,33	2; 2,67

x	–2	–1	0	1	2
y	0	–1	–2	–3	–4
$(x; y)$	–2; 0	–1; –1	0,–2	1; –3	2; –4



(10)

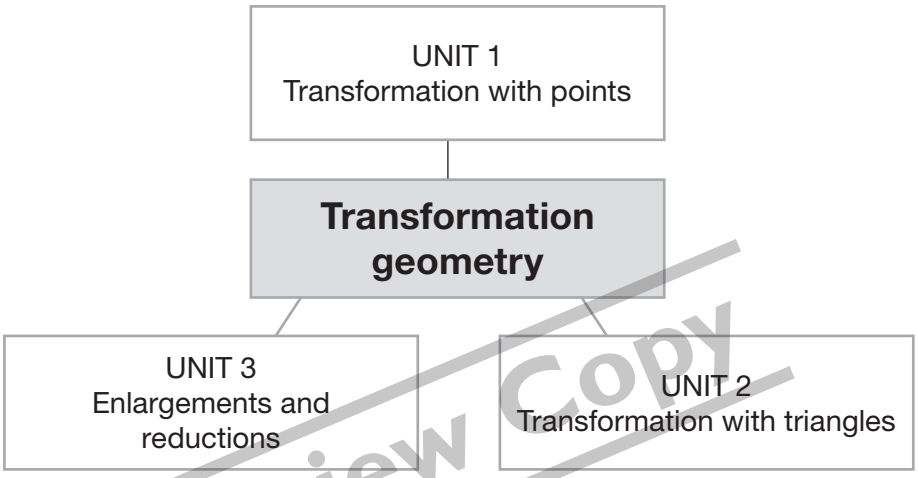
6.1 A child runs at a constant speed for 40 seconds before stopping – Graph A
6.2 A panda moves slowly at a constant speed for 40 seconds before stopping – Graph D
6.3 A cheetah stalks his prey, moving slowly, stopping occasionally – Graph B
6.4 A rabbit runs away from a farmer, not stopping for anything – Graph C (4)

Total marks [65]

Review Copy

Chapter 17 Transformation geometry

Overview of concepts



Review Copy

Content		Time allocations	LB page
Unit 1	Transformation with points	2 hours	388
Unit 2	Transformation with triangles	2 hours	393
Unit 3	Enlargements and reductions	2 hours	398

Background information on transformation geometry

Transformation geometry has been extensively used throughout history by cultures all over the world to create works of art and decorate their architectural buildings. This use of transformations helps historians and anthropologists to research different cultures and their traditions.

- The stepping stone for modern coordinate geometry was laid by René Descartes (1596–1650), a French mathematician and philosopher, who invented the Cartesian coordinate system. By allocating two numbers that represent the position of every point is a plane, Descartes showed that there is a link between geometry and algebra, and therefore algebraic techniques could be used to solve geometric questions. Read more about this topic on the Internet.

Generic teaching guidelines for teaching transformation geometry

- Wherever possible have learners perform the transformation on graph paper.
- For reflections and rotations provide learners with transparencies or tracing paper to help them.
- Exposure to physically transforming the figures helps learners to visualise the process before having to work abstractly. It is important that learners use the concrete transformations for as long as they need; this helps them to work correctly when abstracting as they have a thorough and sufficient understanding of the transformation process.

Resources

Learner's Book, workbook, poster that illustrates reflections and translations of points and triangles in the *x*- and *y*-axes, and rotations of triangles around the origin, poster that illustrates enlargement and reduction and the factor of enlargement and reduction

Unit 1 Transformation with points

Learner's Book page 388

Review Copy

Unit focus:

- work with ordered pairs on the coordinate plane
- reflect points in the *y*-axis and the *x*-axis
- translate points within quadrants and across quadrants.

Background information on transformations with points

Transforming points lays the groundwork for all future transformations. Learners need to be able to work accurately with points before they can transform line segments and figures.

Exercise 1

Learner's Book page 390

Guidelines on how to implement this activity

Revise the Cartesian plane, and plotting and reading points from the plane. Before continuing with the other exercises in this chapter, it is crucial that learners should be able to:

- plot points on the coordinate plane
- read coordinates of points off the axes.

Use this exercise to ensure learners have mastered these skills.

Suggested answers

1 Learners find the point with coordinates:
1.1 (2; 3): C; (3; 2): B
1.2 (–3; –1,3): K; (–1,3; –3): M
1.3 (2,5; –4): R; (4; –2,5): S
1.4 (0; 2): U; (2; 0): T

2 A(4; 1); B(3; 2); C(2; 3); D(1; 4); E(–1; 1); F(–2; 2); G($-3\frac{1}{2}$; $3\frac{1}{2}$); H(–4; –1); K(–3; $-1\frac{3}{5}$); L(–2; –2); M($-1\frac{1}{3}$; –3); N(–1; –4); P($2\frac{1}{2}$; –1); Q(1; $-2\frac{1}{2}$); R($2\frac{1}{2}$; –4); S(4; $-2\frac{1}{2}$); T(2; 0); U(0; 2); V(–3; 0); W(0; –4)

3.1 The point is in the second quadrant: E; F; G.
3.2 The points have the same x-coordinate: A and S; C and T; D and Q; E and N; F and L; P and R; U and W; V and K.
3.3 The points have the same y-coordinate: A and E; B, F and U; H and P; N,R and W; Q and S; T and V.
3.4 The x-coordinate is 0: U; W.
3.5 The x- and y-coordinates are equal: L.
3.6 The x-coordinate is 2 more than the y-coordinate: T.
3.7 The y-coordinate is four times the x-coordinate: D; N.
3.8 The x-coordinate is the additive inverse of the y-coordinate: E; F.
3.9 The product of the x- and y-coordinates is equal to –10: R; S.
3.10 The horizontal distance between the points is five units: A and E/N; B and F/L; T and V.
3.11 The vertical distance between the points is 4 units: B and L; C and H/P; D and T/V; E and M; F and L; T/V and N/R/W.
3.12 The point lies closest to the origin: E.
3.13 The point lies furthest from the origin: G (use the theorem of Pythagoras).

Remedial

Provide graph paper for learners to use, instead of measuring up their own grids in their exercise books, which is time consuming.

Review Copy

Exercise 2

Learner's Book page 391

Guidelines on how to implement this activity

- Learners do not have to learn general rules for transformations at this stage, but should explore the way the coordinates of points change when performing different transformations. Rules provided in square brackets below are for the benefit of the teacher only.
- Ensure learners know the correct notations, so that if point P is the object under transformation, its image is denoted by P'.
  Learners must know that:
  - a transformation is a change carried out under specific rules. Examples of transformations are reflections, translations and rotations (enlargements and reductions are discussed in Unit 17.3).
  - A reflection of point P(a; b) is a transformation such that P and its image P' are both the same distance from a fixed straight line called the mirror line, but on opposite sides of it.
- Encourage learners to physically transform points using tracing paper or transparencies. Learners should record the change in co-ordinates each time, and discuss the general rule as a class.

- Learners should complete the following statements:
  - If point P(a; b) is reflected in the y-axis, coordinate a changes its sign and coordinate b stays the same. [P(a; b) → P′(–a; b)]
  - If point P(a; b) is reflected in the x-axis, coordinate a stays the same and coordinate b changes its sign: [P(a; b) → P′(a; –b)]

Suggested answers

1 Reflected in y-axis

1.1 E(4; 3); E′(–4; 3)

1.2 F(–6; 2); F′(6; 2)

1.3 G(–9; –12); G′(9; –12)

1.4 H(7; –1); H′(–7; –1)

1.5 J(–8; $5\frac{1}{2}$); J′(8; $5\frac{1}{2}$)

1.6 K($8\frac{1}{4}$; 0); K′(–$8\frac{1}{4}$; 0)

1.7 L(0; –6); L′(0; –6)

1.8 M(–4,4; 0); M′(4,4; 0)

1.9 N(0; 5,2); N′(0; 5,2)

2 Reflected in x-axis

2.1 P(3; 4); P′(3; –4)

2.2 Q(–2; 6); Q′(–2; –6)

2.3 R(–12; –9); R′(–12; 9)

2.4 S(1; –7); S′(1; 7)

2.5 T($5\frac{1}{2}$; –8); T′($5\frac{1}{2}$; 8)

2.6 U(0; –$8\frac{1}{4}$); U′(0; $8\frac{1}{4}$)

2.7 V(6; 0); V′(6; 0)

2.8 W(0; 4,4); W′(0; –4,4)

2.9 Z(–5,2; 0); Z′(–5,2; 0)

3.1 E(1; 2) → E′(1; –2): reflected in the x-axis

3.2 F(3; –4)→ F′(–3; –4): reflected in the y-axis

3.3 G(–5; 6) → G′(–5; –6): reflected in the x-axis

3.4 H(–7; –8) → H′(7; –8): reflected in the y-axis

3.5 K(0; –3) → K′(0; 3): reflected in the x-axis

3.6 L(–4; 0) → L′(4; 0): reflected in the x-axis

4.1 Both coordinates a and b change signs.

4.2 Both coordinates a and b change signs.

Review Copy

Remedial

Provide additional opportunities for learners to perform actual translations.

Exercise 3

Learner's Book page 392

Guidelines on how to implement this activity

- Explain a translation to learners. A *translation* of point P(a; b) is a transformation such that P slides over a certain distance in a specific direction to the position of its image P′. The translation is described in terms of the horizontal and vertical changes in position.
- Work through the worked examples, and have learners complete the following statements for $h > 0$ and $k > 0$: If point P(a; b) is translated:
  - h units to the right then a increases by h units and b stays the same [P(a; b) → P′($a + h$; b)]
  - h units to the left then a decreases by h units and b stays the same [P(a; b) → P′($a - h$; b)]
  - k units upwards then a stays the same and b increases by k units [P(a; b) → P′(a; $b + k$)]
  - k units downwards then a stays the same and b decreases by k units [P(a; b) → P′(a; $b - k$)]

- h units to the right and k units upwards then a increases by h units and b increases by k units [P(a; b) → P′($a + h$; $b + k$)]
- h units to the left and k units downwards then a decreases by h units and b decreases by k units [P(a; b) → P′($a - h$; $b - k$)].

Encourage learners to complete the exercise on their own.

Suggested answers

1.1 E(1; 2) → E′(3; 2); F(–3; 4) → F′(–1; 4); G(–5; –6) → G′(–3; –6); H(7; –8) → H′(9; 8)
1.2 E(1; 2) → E′–2; 2); F(–3; 4) → F′(–6; 4); G(–5; –6) → G′(–8; –6); H(7; –8) → H′(4; –8)
1.3 E(1; 2) → E′(1; 6); F(–3; 4) → F′(–3; 8); G(–5; –6) → G′(–5; –2); H(7; –8) → H′(7; 4)
1.4 E(1; 2) → E′(1; –3); F(–3; 4) → F′(–3; –1); G(–5; –6) → G′(–5; –11); H(7; –8) → H′(7; –13)
1.5 E(1; 2) → E′(2; 0); F(–3; 4) → F′(–2; 2); G(–5; –6) → G′(–4; –8); H(7; –8) → H′(8; –10)
1.6 E(1; 2) → E′(–2; 6); F(–3; 4) → F′(–6; 8); G(–5; –6) → G′(–8; –2); H(7; –8) → H′(4; –4)
1.7 E(1; 2) → E′(6; 8); F(–3; 4) → F′(2; 10); G(–5; –6) → G′(0; 0); H(7; –8) → H′(12; –2)
1.8 E(1; 2) → E′(–6; –6); F(–3; 4) → F′(–10; –4); G(–5; –6) → G′(–12; –14); H(7; –8) → H′(0; –16)
1.9 E(1; 2) → E′(3; –2); F(–3; 4) → F′(–1; 0); G(–5; –6) → G′(–3; –10); H(7; –8) → H′(9; –12)
1.10 E(1; 2) → E′(–2; 3); F(–3; 4) → F′(–6; 5); G(–5; –6) → G′(–8; –5); H(7; –8) → H′(4; –7)
2.1 K(1; 2) → K′(1; –2): translated 4 units downwards
2.2 L(3; –4) → L′–3; –4): translated 6 units to the left
2.3 M(–5; 6) → M′(–5; –6): translated 12 units downwards
2.4 N(–7; –8) → N′(7; –8): translated 14 units to the right
2.5 P(0; –3) → P′(0; 3): translated 6 units upwards
2.6 Q(–4; 0) → Q′(4; 0): translated 8 units to the right
3.1 R(1; 2) → R′(3; 4): translated 2 units to the right and 2 units upwards
3.2 S(3; –4) → S′(–5; –6): translated 8 units to the left and 2 units downwards
3.3 T(–5; 6) → T′(–7; 8): translated 2 units to the left and 2 units upwards
3.4 U(–7; –8) → U′(9; –10): translated 16 units to the right and 2 units downwards
3.5 V(0; –3) →V′–3; 0): translated 3 units to the left and 3 units upwards
3.6 W(–4; 0) → W′(0; –4): translated 4 units to the right and 4 units downwards

Sample Copy

Remedial

If learners struggle to form a concept of where the new coordinates are, have learners physically perform the translation on grid paper. Have learners do this as many times as necessary until they are able to conceptualise the change in coordinates.

Extension

Challenge learners to translate a point three times so that a parallelogram is formed.

Unit 2 Transformation with triangles

Learner's Book page 393

Unit focus:

- recognising and describing the transformations of triangles
- reflecting triangles in the y-axis and the x-axis
- translating triangles within quadrants and across quadrants
- rotating triangles around the origin.

Background information on transformations with triangles

This section extends learners knowledge of transformation from transforming points to transforming figures, in this case triangles.

Exercise 1

Learner's Book page 393

Guidelines on how to implement this activity

- Revise the following concepts:
  - reflection: flip something over a mirror line to a new position
  - translation; slide something without any turning to a new position
  - rotation: turn something around a point of rotation (fixed point) to a new position.
- Revise the transformations learners did in Unit 1.
- Discuss the effect of the transformations on the coordinates.
- Work through the worked example in the Learner's Book.

Learners should complete this exercise on their own.

Suggested answers

1.1 P → B: translation from the first to the second quadrant
1.2 P → D: translation across quadrants
1.3 P → E: rotation through 180° around the origin
1.4 P → F: reflection in the x-axis
1.5 P → H: rotation through 90° around the origin
1.6 P → K: translation within the first quadrant
2.1 reflection in the y-axis: A and P; E and F
reflection in the x-axis: A and E; P and F
2.2 translation within quadrants: P and K
2.3 translation across quadrants: P/K and B/G/D; B and D/G/K/P; D and B/G/K/P; G and B/D/K/P
2.4 rotation around the origin: P and C/E/H; C and E/H/P; E and C/H/P
3 Reflection, translation and rotation change the position of a figure, but not its shape and size, therefore the figures are congruent.

Review Copy

Remedial

Provide graph paper and encourage learners to perform each transformation to help them 'see' how the coordinates are affected.

Exercise 2

Learner's Book page 394

Guidelines on how to implement this activity

- Discuss that a reflection is a transformation such that any two corresponding points in the object and the image are both the same distance from the mirror line but on opposite sides.
- Show learners that refelcting triangles means reflecting each vertex.

- Work through the worked example, and ensure that learners can complete the following statements:
  - If a triangle is reflected in the x-axis the x-coordinates of its vertices stay the same and the y-coordinates of its vertices change sign.
  - If a triangle is reflected in the y-axis the x-coordinates of its vertices change sign and the y-coordinates of its vertices stay the same.

Learners should complete this exercise on their own.

Suggested answers

1.1 reflected in the x-axis: D(4; 5) → D′(4; –5); E(1; 3) → E′(1; –3); F(6; 2) → F′(6; –2)

1.2 reflected in the y-axis: D(4; 5) → D′(–4; 5); E(1; 3) → E′(–1; 3); F(6; 2) → F′(–6; 2)

2.1 reflected in the x-axis: G(–1; 6) → G′(–1; –6); H(–1; 2) → H′(–1; –2); K(–5; 2) → K′(–5; –2)

2.2 reflected in the y-axis: G(–1; 6) → G′(1; 6); H(–1; 2) → H′(1; 2); K(–5; 2) → K′(5; 2)

3.1 reflected in the x-axis: L(–4; –2) → L′(–4; 2); M(–6; –6) → M′(–6; 6); N(–1; –1) → N′(–1; 1)

3.2 reflected in the y-axis: L(–4; –2) → L′(4; –2); M(–6; –6) → M′(6; –6); N(–1; –1) → N′(1; –1)

4.1 reflected in the x-axis: P(6; –2) → P′(6; 2); Q(1; –6) → Q′(1; 6); R(6; –6) → R′(6; 6)

4.2 reflected in the y-axis: P(6; –2) → P′(–6; –2); Q(1; –6) → Q′(–1; –6); R(6; –6) → R′(–6; –6)

5 If a triangle is reflected in the x-axis, the x-coordinates of its vertices stay the same and the y-coordinates of its vertices change sign. If it is reflected in the y-axis, the x-coordinates of its vertices change sign and the y-coordinates of its vertices stay the same.

Review Copy

Remedial

If learners struggle to determine the coordinates have them perform the transformation using tracing paper or transparencies.

Exercise 3

Learner's Book page 395

Guidelines on how to implement this activity

Discuss that a translation is a transformation such that all points in the object move the same distance and in the same direction to reach their corresponding points in the image.

- Work through the worked example and do additional examples if necessary.
- Ensure that learners are able to translate a triangle within, and across quadrants and describe the changes in the coordinates of its vertices.
- Have learners complete the following statement: If a triangle is translated h units horizontally and k units vertically, the x-coordinates of its vertices change to $x + h$ and the y-coordinates of its vertices change to y + k.

Learners can complete this exercise on their own.

Suggested answers

1 A(1; 4) → (1 – 4; 4 – 6) = (–3; –2), which are the coordinates of A″
B(1; 3) → (1 – 4; 3 – 6) = (–3; –3), which are the coordinates of B″
C(3; 2) → (3 – 4; 2 – 6) = (–1; –4), which are the coordinates of C″
∴ ΔA″B″C″ will be the image of ΔABC

2.1 B′ lies at (0; 0): –1 horizontally and –3 vertically
2.2 A′ lies at (–5; –5): –6 horizontally and –9 vertically
3.1 ΔABC → ΔDEF: translate ΔABC –5 horizontally and $-\frac{1}{2}$ vertically
3.2 ΔABC → ΔGHK: translate ΔABC $+\frac{1}{2}$ horizontally and $-5\frac{1}{2}$ vertically
3.3 ΔDEF → ΔGHK: translate ΔDEF $+5\frac{1}{2}$ horizontally and –5 vertically
3.4 ΔGHK → ΔDEF: translate ΔGHK $-5\frac{1}{2}$ horizontally and +5 vertically
4.1 $(x; y) \rightarrow (x - 4; y - 3)$: D(4; 5) → D′(0; 2); E(1; 3) → E′(–3; 0); F(6; 2) → F′(2; –1)
4.2 $(x; y) \rightarrow (x + 1; y + 5)$: G(–1; 6) → G′(0; 11); H(–1; 2) → H′(0; 7); K(–5; 2) → K′(–4; 7)
4.3 $(x; y) \rightarrow (x; y + 6)$: L(–4; –2) → L′(–4; 4); M(–6; –6) → M′(–6; 0); N(–1; –1) → N′(–1; 5)
4.4 $(x; y) \rightarrow (x - 6; y)$: P(6; –2) → P′(0; –2); Q(1; –6) → Q′(–5; –6); R(6; –6) → R′(0; –6)
5 If a triangle is translated h units horizontally and k units vertically, the x-coordinates of its vertices change to $x + h$ and the y-coordinates of its vertices change to $y + k$.

Remedial

If learners struggle to determine the coordinates have them perform the transformation using tracing paper or transparencies.

Exercise 4

Learner's Book page 397

Review Copy

Guidelines on how to implement this activity

By the end of this section learners should be able to rotate a triangle through multiples of 90° around the origin and describe the changes in the coordinates of its vertices.

- Discuss as a class that a rotation is a transformation such that all points in the object turns through the same angle around a fixed point so as to reach their corresponding points in the image.
- Before learners attempt the exercise, ensure learners can complete the following statement:
  - Every time a triangle is rotated through 90° around the origin, the coordinates of its vertices swap around before they take the signs of x and y of the new quadrant.

Learners should complete this exercise on their own.

Suggested answers

1 For ΔGHK: G(1; 4) → G′(4; –1); H(1; 1) → H′(1; –1); K(4; 1) → K′(1; –4)
For ΔLMN: L(–4; 4) → L′(4; 4); M(–4; 1) → M′(1; 4); N(–1; 4) → N′(4; 1)
For ΔPQR: P(–1; –1) → P′(–1; 1); Q(–4; –3) → Q′(–3; 4); R(–3; –4) → R′(–4; 3)
For ΔSTU: S(2; –2) → S′(–2; –2); T(0; –3) → T′(–3; 0); U(4; –4) → U′(–4; –4)

2 For ΔGHK: G(1; 4) → G′(–4; 1); H(1; 1) → H′(–1; 1); K(4; 1) → K′(–1; 4)
For ΔLMN: L(–4; 4) → L′(–4; –4); M(–4; 1) → M′(–1; –4); N(–1; 4) → N′(–4; –1)
For ΔPQR: P(–1; –1) → P′(1; –1); Q(–4; –3) → Q′(3; –4); R(–3; –4) → R′(4; –3)
For ΔSTU: S(2; –2) → S′(2; 2); T(0; –3) → T′(3; 0); U(4; –4) → U′(4; 4)

3 For ΔGHK: G(1; 4) → G′(–1; –4); H(1; 1) → H′(–1; –1); K(4; 1) → K′(–4; –1)
For ΔLMN: L(–4; 4) → L′(4; –4); M(–4; 1) → M′(4; –1); N(–1; 4) → N′(1; –4)
For ΔPQR: P(–1; –1) → P′(1; 1); Q(–4; –3) → Q′(4; 3); R(–3; –4) → R′(3; 4)
For ΔSTU: S(2; –2) → S′(–2; 2); T(0; –3) → T′(0; 3); U(4; –4) → U′(–4; 4)

4 Every time a triangle is rotated through 90° around the origin, the coordinates of its vertices swap around before they take on the signs of x and y of the new quadrant.

Remedial

Learners use "flag shapes" like in the example in the Learner's Book until they can handle rotation around the origin with confidence.

Extension

Learners rotate triangles, of which the three vertices lie in different quadrants, around the origin.

Unit 3 Enlargements and reductions

Learner's Book page 398

Unit focus:

- calculating the area and perimeter of geometric figures on a coordinate plane
- investigating the effect of enlargement and reduction on the area and perimeter of geometric figures
- working out the factor of enlargement or reduction of a geometric figure.

Background information on enlargements and reductions

Learners need to know the formulae for calculating area and perimeter of triangles and quadrilaterals in order to manage this unit. Learners should understand that shapes that have been enlarged or reduced are similar to the original.

Review copy

Exercise 1

Learner's Book page 399

Guidelines on how to implement this activity

- Revise the formulae for area and perimeter of squares, rectangles and triangles. Show learners how to find area and perimeter of shapes on the Cartesian plane.
- Learners need to be shown how to use the coordinates to calculate the sides of the shapes.
- Work through examples together as a class on the board. Discuss the steps and formulae used. At the end of this section learners should be able to calculate the area and perimeter of geometric figures on a coordinate plane.

Learners must complete this exercise on their own, but can discuss their thinking and working in pairs or small groups.

Suggested answers

1 Area of $\Delta ABC = (2 \times 2) - \frac{1}{2}(2 \times 1) - \frac{1}{2}(1 \times 2) = 4 - 1 - 1 = 2$ units^2

$AB = \sqrt{20}$ units; $BC = 4$ units; $CA =$ units

Perimeter of $\Delta ABC = \sqrt{20} + 4 + \sqrt{20} = 12{,}94$ units

2 Area of $\Delta DEF = (9 \times 9) - \frac{1}{2}((9 \times 4) - \frac{1}{2}(5 \times 5) - \frac{1}{2}(4 \times 9) = 81 - 18 - 12{,}5 - 18 = 32{,}5$ units^2

$DE = \sqrt{97}$ units; $EF = \sqrt{50}$ units; $CA = \sqrt{97}$ units

Perimeter of $\Delta ABC = \sqrt{97} + \sqrt{50} + \sqrt{97} = 26{,}77$ units

3 Area of KLMN = $(8 \times 8) - \frac{1}{2}(4 \times 4) - \frac{1}{2}(4 \times 4) - \frac{1}{2}(4 \times 4) - \frac{1}{2}(4 \times 4) = 64 - 8 - 8 - 8 - 8 = 32$ units^2
KL = $\sqrt{32}$ units; LM = $\sqrt{32}$ units; MN = $\sqrt{32}$ units; NK = $\sqrt{32}$ units
Perimeter of ΔABC = $4 \times \sqrt{32} = 22{,}63$ units

4 Area of PQRS = $(8 \times 8) - \frac{1}{2}(8 \times 4) - \frac{1}{2}(8 \times 4) = 64 - 16 - 16 = 32$ units^2
PQ = $\sqrt{80}$ units; QR = 4 units; RS = $\sqrt{80}$ units; SP = 4 units
Perimeter of ΔABC = $2 \times \sqrt{80} + 2 \times 4 = 25{,}89$ units

5 Area of TUVW = $(6 \times 8) - \frac{1}{2}(3 \times 4) - \frac{1}{2}(4 \times 3) - \frac{1}{2}(3 \times 4) - \frac{1}{2}(4 \times 3) = 48 - 6 - 6 - 6 - 6 = 24$ units^2
TU = 5 units; UV = 5 units; VW = 5 units; WT = 5 units
Perimeter of ΔABC = $4 \times 5 = 20$ units

Remedial

Learners may need to be reminded how to use Pythagoras with triangles to find the perpendicular height, in order to calculate area. Learners may also require help working with the resultant surds from working with Pythagoras. Learners do not need to simplify the surds, but must be shown how to input them correctly into their calculators.

PoA | Investigation

Learner's Book page 399

This investigation is compulsory. The teacher can decide to include this investigation as part of the formal POA or simply as an in class task activity.

Review Copy

Guidelines on how to implement this activity

- At the end of this investigation learners should be able to:
  - describe how a geometric shape is affected when the coordinates of each of its vertices are multiplied or divided by a factor of 2
  - describe the effect of enlargement or reduction on the angles, shape, sides, perimeter and area of geometric figures
- Read through the investigation with learners and ensure learners know what is expected of them. Learners can discuss the process with their groups, but must record their own workings and findings.
- Learners should summarise their findings by writing statements such as the following:
  - If the coordinates of the vertices of a geometric figure are multiplied by a factor of 2, its angles stay the same, its shape stays the same, the lengths of its sides are doubled, its perimeter is doubled and its area is four times larger than before.
  - If the coordinates of the vertices of a geometric figure are multiplied by a factor of $\frac{1}{2}$ (divided by 2) its angles stay the same, its shape stays the same, the lengths of its sides are halved, its perimeter is halved and its area is four times smaller than before.
- Learners must check the validity of the following statements:
  - If the dimensions of a geometric figure are doubled, its perimeter is doubled and its area is four times larger.
  - If the dimensions of a geometric figure are halved, its perimeter is halved and its area is four times smaller.

If you are using this investigation as part of the formal programme of assessment, ensure correct procedure is followed. For marking purposes please refer to the rubric in the POA at the back of this Teacher's Guide.

Exercise 2

Learner's Book page 401

Guidelines on how to implement this activity

- Learners should know that an enlargement or reduction is a transformation in which the distances between every pair of points in the object are multiplied by the same amount to produce the image.
- Discuss with learners, that the factor of enlargement or reduction is the number by which the dimensions are multiplied to produce the enlargement or reduction. To find this factor we divide any dimension of the image by the corresponding dimension of the object.
- Explain that a factor of enlargement is always bigger than 1, and a factor of reduction lies between 0 and 1. Show learners how this information can be used to determine the area and perimeter of an enlarged or reduced shape without having to perform the enlargement. We can also determine the scale factor for given shapes.
- Work through the worked example and then have learners complete this exercise on their own.

Suggested answers

1.1 $PR^2 = PQ2 + QR^2$ (Pythagoras; $P\hat{Q}R = 90°$)
$= 602 + 802 = 10\ 000$
$\therefore PR = \sqrt{10\ 000} = 100$ mm

Factor of reduction $= \frac{SU}{PR} = \frac{50\text{ mm}}{100\text{ mm}} = \frac{1}{2}$

1.2 $\frac{SU}{PQ} = \frac{1}{2}$; $= \frac{1}{2}$; $ST = \frac{1}{2} \times 60\text{ mm} = 30\text{ mm}$
$\frac{TU}{QR} = \frac{1}{2}$; $= \frac{1}{2}$; $TU = \frac{1}{2} \times 80\text{ mm} = 40\text{ mm}$

Perimeter of $\Delta PQR = 6\text{ cm} + 8\text{ cm} + 10\text{ cm} = 24\text{ cm}$
Perimeter of $\Delta STU = 3\text{ cm} + 4\text{ cm} + 5\text{ cm} = 12\text{ cm}$

1.3 Area of $\Delta PQR = \frac{1}{2} \times 8\text{ cm} \times 6\text{ cm} = 24\text{ cm}^2$

Area of $\Delta STU = \frac{1}{2} \times 4\text{ cm} \times 3\text{ cm} = 6\text{ cm}^2$

2 The perimeter of a square ABCD = 48 cm.

2.1 Perimeter of new square $= 2 \times 48\text{ cm} = 96\text{ cm}$ (dimensions have doubled)

2.2 The area of the new square will be four times larger than the area of the original square.
Perimeter of original square $= 4 \times$ side length $= 48$ cm
$\therefore$ side length of original square $= 48\text{ cm} \div 4 = 12\text{ cm}$
$\therefore$ side length of new square $= 2 \times 12\text{ cm} = 24\text{ cm}$ (side length is doubled)
Area of original square = side × side $= 12\text{ cm} \times 12\text{ cm} = 144\text{ cm}^2$
Area of new square = side × side $= 24\text{ cm} \times 24\text{ cm} = 576\text{ cm}^2$, which is four times larger.

3 If the dimensions of a rectangle are divided by 3, the perimeter of the new rectangle will be one third of the original perimeter and the area of the new rectangle will be nine times smaller than the original area.

Review Copy

Remedial

If learners struggle with this work, start with simple similar drawings such as squares and rectangles, and work up to more complex shapes.

Extension

Provide more complicated problems where the theorem of Pythagoras has to be applied before the factor of enlargement or reduction can be calculated.

Consolidation

Learner's Book page 403

- Before doing this consolidation exercise, encourage learners to review the work covered in this chapter.
- Advise learners to use the summary and to revise their work.

This exercise can be used as an informal assessment task for you to track how learners are coping with the chapter and the concepts covered. The mark allocation provides guidelines on how to assess learners.

Suggested answers

1.1 P(–3; 2) reflected in the y-axis: P′(3; 2)

1.2 P(–3; 2) reflected in the x-axis: P′(–3; –2)

1.3 P(–3; 2) reflected in both axes: P′(3; –2)

1.4 P(–3; 2) translated +3 units horizontally: P′(0; 2)

1.5 P(–3; 2) translated +5 units vertically: P′(–3; 7)

1.6 P(–3; 2) translated –6 units horizontally as well as vertically: P′(–9; –4) (6)

2.1 Translate Q($1\frac{1}{2}$; $-2\frac{1}{2}$) $2\frac{1}{2}$ units upwards ($+2\frac{1}{2}$ units vertically)

2.2 Translate Q($1\frac{1}{2}$; $-2\frac{1}{2}$) $1\frac{1}{2}$ units to the left ($-1\frac{1}{2}$ units horizontally)

2.3 Translate Q($1\frac{1}{2}$; $-2\frac{1}{2}$) $-1\frac{1}{2}$ units horizontally and $+2\frac{1}{2}$ units vertically

2.4 Translate Q($1\frac{1}{2}$; $-2\frac{1}{2}$) $-4\frac{1}{2}$ units horizontally and $+4\frac{1}{2}$ units vertically

2.5 Translate P(–3; 2) $+4\frac{1}{2}$ units horizontally and $-4\frac{1}{2}$ units vertically (5)

3.1 reflected in the y-axis: A(–4; 4) → A′(4; 4); B(–2; –2) → B′2; –2); C(2; 2) → C′(2; 2)

3.2 reflected in the x-axis: A(–4; 4) → A′(–4; –4); B(–2; –2) → B′(–2; 2); C(2; 2) → C′(2; –2)

3.3 translated +2 units horizontally: A(–4; 4) → A′(–2; 4); B(–2; –2) → B′(0; –2); C(2; 2) → C′(4; 2)

3.4 translated –4 units vertically: A(–4; 4) → A′(–4; 0); B(–2; –2) → B′(–2; –6); C(2; 2) → C′(2; –2)

3.5 translated so that A′ lies on the origin: A(–4; 4) → A′(0; 0); B(–2; –2) → B′(2; –6); C(2; 2) → C′(6; –2) (5)

4 For the rectangles:

4.1 Factor of enlargement $= \frac{GH}{CD} = \frac{90\text{ mm}}{30\text{ mm}} = 3$

Review Copy

4.2 Factor of enlargement = 3; $\frac{FG}{BG} = \frac{60\text{ mm}}{BG} = 3$; BC = 60 mm ÷ 3 = 20 mm

Perimeter of ABCD = 3 cm + 2 cm + 3 cm + 2 cm = 10 cm

Perimeter of EFGH = 9 cm + 6 cm + 9 cm + 6 cm = 30 cm

4.3 Area of ABCD = 3 cm × 2 cm = 6 cm^2

Area of EFGH = 9 cm × 6 cm = 54 cm^2

For the triangles:

4.1 PR = 15 cm (Pythagoras; $P\hat{Q}R = 90°$)

Factor of reduction = $\frac{SU}{PR} = \frac{75\text{ mm}}{150\text{ mm}} = \frac{1}{2}$

4.2 Factor of reduction = $\frac{1}{2}$; $\frac{ST}{9\text{ cm}} = \frac{1}{2}$; ST = $\frac{1}{2} \times 9$ cm = 4,5 cm

Factor of reduction = $\frac{1}{2}$; $\frac{TU}{12\text{ cm}} = \frac{1}{2}$; TU = $\frac{1}{2} \times 12$ cm = 6 cm

Perimeter of ΔPQR = 9 cm + 12 cm + 15 cm = 36 cm

Perimeter of ΔSTU = 4,5 cm + 6 cm + 7,5 cm = 18 cm

4.3 Area of ΔPQR = $\frac{1}{2} \times 12\text{ cm} \times 9\text{ cm} = 54\text{ cm}^2$

Area of ΔSTU = $\frac{1}{2} \times 6\text{ cm} \times 4{,}5\text{ cm} = 13{,}5\text{ cm}^2$ (8)

5.1 Dimensions multiplied by 3:

Perimeter = 3 × 28 = 84 cm

Area = 9 × 48 cm^2 = 432 cm^2 (6)

5.2 Dimensions divided by 4:

Perimeter = $\frac{1}{4} \times 28 = 7$ cm

Area = × 48 cm^2 = 3 cm^2 (4)

6 Reflections, translations and rotations deliver congruent triangles. (1)

7 Enlargements and reductions deliver similar triangles. (1)

Total marks [30]

Review Copy

Chapter 18 Geometry of 3D objects

Overview of concepts

Geometry of 3D objects

UNIT 1
Classifying 3D objects

UNIT 2
Building 3D models

Content		Time allocations	LB page
Unit 1	Classifying 3D objects	3 hours	405
Unit 2	Building 3D models	4 hours	410

Review Copy

Background information on geometry of 3D objects

The Platonic solids are named after Plato, who wrote about them in the dialogue Timaeus in 360 BC. He associated each of the four classical elements with one of the Platonic solids.

Earth was associated with the cube, because its solidity was believed to be due to the fact that the cube is the only regular solid that tessellates. Air was associated with the octahedron, because its minuscule components are so smooth that one can barely feel it. Water was associated with the icosahedron, because it flows out of one's hand when picked up, as if it is made of tiny little balls. Fire was associated with the tetrahedron, because the heat of fire feels sharp and stabbing like little tetrahedrons. Plato had the idea that the dodecahedron was used to arrange the constellations on the whole heaven. Read more about this topic on the Internet.

Generic teaching guidelines for teaching geometry of 3D objects

When teaching 3D objects, try to have as many objects in the classroom as possible. Learners should be encouraged to investigate these objects, observe them, touch them and identify the properties of them.

Having concrete examples is crucial to helping learners visualise 3D forms. Learners need to work abstractly with these objects, and having concrete objects they can see and handle helps them to form accurate concepts of the solid shapes.

Resources

- Learner's Book; workbook; poster with drawings of spheres, cylinders and cones as well as various types of right prisms and pyramids; poster with drawings of the five Platonic solids and their nets; poster that shows some 3D objects and their nets.

Unit 1 Classifying 3D objects

Learner's Book page 405

Unit focus:

- describing, naming and comparing 3D objects specified in earlier grades
- describing, naming and comparing the five Platonic solids.

Background information on classifying 3D objects

Learners should know the basic forms of prisms, pyramids, spheres and cylinders. They should also understand the basic properties of these objects in terms of straight or curved sides, and number of faces, edges and vertices.

Learners should also be able to identify the shapes that make up the prisms and pyramids.

Learners should know that regular shapes are those with equal sides and equal angles, such as squares and equilateral triangles.

Review Copy

Exercise 1

Learner's Book page 406

Guidelines on how to implement this activity

Revise the properties of the different objects that are shown in the Learner's Book. Focus on the following:

- shape and number of faces (flat surfaces on the objects)
- number of edges (straight lines formed where two faces meet)
- number of vertices (angular points where three or more edges meet).

Introduce the concepts of polygon and polyhedron.
At the end of this section learners should be able to describe, name and compare spheres, cylinders, cones, prisms and pyramids.
Learners can complete this exercise in pairs.

Suggested answers

1.1 Objects in group 1 are neither prisms nor pyramids.

1.2 Objects in group 2 are all prisms (objects with identical faces at the top and bottom and with rectangles or squares on the side).

1.3 Objects in group 3 are all pyramids (objects with a polygon as a base and triangles on the sides which all meet at a common vertex called the apex).

2.1 All objects in group 1 have surfaces that are curved.

2.2 All objects in group 2 have identical faces at the top and bottom with either rectangles or squares on their sides.

2.3 All objects in group 3 have triangles on their sides which meet at a common vertex.

3 The objects in groups 2 and 3 only have polygons as faces (they are polyhedrons).

4 A: equilateral triangle; B: rectangle; C: heptagon; D: right-angled triangle; E: hexagon; F: square; G: octagon; H: pentagon

5 Groups 2 and 3 contain only polyhedrons.

6.1 triangular prism: J
6.2 rectangular pyramid: V
6.3 heptagonal prism: Q
6.4 hexagonal pyramid: X
6.5 square prism: L
6.6 octagonal pyramid: Z
6.7 pentagonal prism: N
6.8 triangular pyramid: S
6.9 rectangular prism: M
6.10 heptagonal pyramid: Y
6.11 hexagonal prism: P
6.12 square pyramid: U
6.13 octagonal prism: R
6.14 pentagonal pyramid: W
6.15 cube: K
6.16 tetrahedron: T

7.1

3D Object	Information on base		Information on object		
	Polygon	sides	Vertices	Faces	Edges
triangular prism	triangle	3	6	5	9
cube	square	4	8	6	12
square prism	square	4	8	6	12
rectangular prism	rectangle	4	8	6	12
pentagonal prism	pentagon	5	10	7	15
hexagonal prism	hexagon	6	12	8	18
heptagonal prism	heptagon	7	14	9	21
octagonal prism	octagon	8	16	10	24

Review Copy

7.2 Number of vertices = 2 × number of sides of the base
Number of faces = 2 + number of sides of the base
Number of edges = 3 × number of sides of the base

8.1

3D Object	Information on base		Information on object		
	Polygon	sides	Vertices	Faces	Edges
triangular pyramid	triangle	3	4	4	6
tetradron	triangle	3	4	4	6
square pyramid	square	4	5	5	8
rectangular pyramid	rectangle	4	5	5	8
pentagonal pyramid	pentagon	5	6	6	10
hexagonal pyramid	hexagon	6	7	7	12
heptagonal pyramid	heptagon	7	8	8	14
octagonal pyramid	octagon	8	9	9	16

8.2 Number of vertices = 1 + number of sides of the base
Number of faces = 1 + number of sides of the base
Number of edges = 2 × number of sides of the base

9.1 A decagonal prism has 20 vertices, 12 faces and 30 edges.

9.2 A decagonal pyramid has 11 vertices, 11 faces and 20 edges.

PoA | Investigation

Learner's Book page 407

The relationship between the number of vertices, faces and edges of the five Platonic solids

Guidelines on how to implement this activity

This investigation is compulsory.

The teacher should decide whether this will be part of the Programme of Assessment. If not, learners should still hand in their work as a proof that they have completed the investigation.

Suggested answers

Step 1: Learners build models of the five Platonic solids.

Step 2: Learners complete the table below.

Platonic solid	Number of vertices	Number of faces	Number of edges
tetrahedron	4	4	6
hexahedron	8	6	12
octahedron	6	8	12
dodecahedron	20	12	30
icosahedron	12	20	30

Review Copy

Step 3: Vertices + Faces − Edges = 2

Step 4: Learners complete the table below and confirm that Euler's formula is valid for prisms and pyramids.

Polyhedron	Number of vertices (V)	Number of faces (F)	Number of edges (E)
triangular prism	6	5	9
triangular pyramid	4	4	6
rectangular prism	8	6	12
rectangular pyramid	5	5	8
pentagonal prism	10	7	15
pentagonal pyramid	6	6	10
hexagonal prism	12	8	18
hexagonal pyramid	7	7	12
heptagonal prism	14	9	21
heptagonal pyramid	8	8	14
octagonal prism	16	10	24
octagonal pyramid	9	9	16

Step 5: The difference between the number of edges and the sum of the number of vertices and faces is 2: $V + F - E = 2$
If you are using this investigation as part of the formal program of assessment ensure correct procedure is followed. For marking purposes please refer to the rubric in the POA at the back of the Teacher's Guide.

Exercise 2:

Learner's Book page 409

Guidelines on how to implement this activity

Revise the properties of a tetrahedron and a hexahedron (cube). Learners have seen these forms before. Introduce the octahedron, dodecahedron and icosahedron. Explain to learners that these are the platonic solids. Ask learners to identify why they are special. Learners should be able to see that these are the only five solids that can be made with identical regular shapes as faces. Ask learners to complete the table for the Platonic solids and their faces, edges and vertices. Learners should look for a relationship between these parts of the objects.
Ensure that by the end of the section learners are able to:

- describe, name and compare the five Platonic solids in terms of the shape and number of faces, the number of vertices and the number of edges
- make sense of Euler's formula: $V + F - E = 2$

Learners should complete this exercise on their own.

Suggested answers

1 $V + F - E = 2$; $E = V + F - 2 = 18 + 11 - 2 = 27$; 11 faces – 2 faces at the top and bottom = 9 faces on the side; the prism is nine-sided (nonagonal).

2 $V + F - E = 2$; $E = V + F - 2 = 13 + 13 - 2 = 24$; 13 faces – 1 face at the bottom = 12 faces on the side; the prism is twelve-sided.

Review Copy

Extension

Challenge faster learners to obtain more information on the five Platonic solids from the Internet. Learners can then create informative posters to put up around the classroom.

Unit 2 Building 3D models

Learner's Book page 410

Unit focus:

- describing nets of prisms and pyramids
- matching nets with different prisms and pyramids
- making models of geometric solids, including cubes, prisms and pyramids.

Background information on building 3D models

Learners must always have the necessary materials at hand in order to construct accurate 3D models. Ensure learners have sharp pencils, rulers and sharp scissors.

Learners must work neatly and accurately in order for their models to be accurate representations.

Learners know what a net is, and know how it is a 2D representation of a 3D form. They have built models before and should be familiar with the techniques; however it is always helpful to remind them.

Exercise 1

Learner's Book page 411

Guidelines on how to implement this activity

- Revise the concept of a net. Do examples of matching nets to their 3D forms.
- Ensure that learners are able to:
  - describe the unique properties of different prisms and pyramids
  - describe nets of different prisms and pyramids.

Suggested answers

For drawings of the nets, refer to the next section in the Learner's Book.

1 Object B is a cube because all its faces are congruent squares; drawing XVI.
2 Object L is a tetrahedron because all its faces are congruent equilateral triangles; drawing XV.
3 Object D is a pentagonal prism because its faces at the top and bottom are congruent pentagons and all faces on its side are congruent rectangles; drawing IX.
4 Object Q is a hexagonal pyramid because its face at the bottom is a hexagon and all faces on its side are congruent triangles; drawing XII.
5 Object F is a heptagonal prism because its faces at the top and bottom are congruent heptagons and all faces on its side are congruent rectangles; drawing III.
6 Object S is an octagonal pyramid because its face at the bottom is a octagon and all faces on its side are congruent triangles; drawing XIII.

Review Copy

Remedial

Have examples of each of the 3D objects available in the class for learners to cut up and investigate the nets.

Exercise 2

Learner's Book page 412

Guidelines on how to implement this activity

This exercise extends learners understanding of nets and 3D forms. Work through the steps and example in the Learner's Book.

Suggested answers

Learners match each net with a prism or pyramid in the previous section.

I – triangular prism (A)
II – square pyramid (M);
III – heptagonal prism (F)
IV – pentagonal pyramid (P);
V – heptagonal pyramid (R)
VI – square prism (H);
VII— triangular pyramid (K)
VIII – hexagonal prism (E);
IX – pentagonal prism (D)
X – rectangular pyramid (N);
XI – octagonal prism (G)
XII – hexagonal pyramid (Q);
XIII – octagonal pyramid (S)
XIV – rectangular prism (C);
XV – tetrahedron (L)
XVI – cube (B)

Exercise 3

Learner's Book page 415

Guidelines on how to implement this activity

Learners should be able to:

- construct an accurate net of a given prism or pyramid
- build a neat model of any prism or pyramid.

Discuss the construction of models. Remind learners to work neatly, to have all the necessary equipment and to keep tabs available on surface edges to stick the models together.

Have learners work in groups.

Suggested answers

1 Nets A to M can be used to build a model of a cube.
Nets N to R cannot be used to build a model of a cube.

2 Use a pair of compasses to construct an equilateral triangle with sides 4 cm long. Attach a 3 cm by 6 cm rectangle to one of the sides of the triangle. Attach another triangle on the opposite side of the rectangle. Attach two more rectangles to the existing one.

3 Construct a 4 cm by 4 cm square. Attach a 4 cm by 8 cm rectangle to one of the sides of the square. Attach another square to the opposite side of the rectangle. Attach three more rectangles to the existing one.

4 Use a front view of the pyramid and the theorem of Pythagoras to prove that each triangle on the side of the pyramid must be $\sqrt{3^2 + 4^2} = 5$ cm high. Construct a 6 cm by 6 cm square. Construct perpendicular bisectors of 5 cm on each side of the square. Complete the triangles.

5 Use the method which is explained in the third worked example in the Learner's Book. Construct a triangle with a base of 3 cm and base angles of $(180° - 72°) \div 2 = 54°$ each. Draw a circle around the triangle with its centre at the top of the triangle. Construct the pentagon with sides 3 cm long by marking off the edges along the circle. Attach a 3 cm by 5 cm rectangle to one of the sides of the pentagon. Attach another pentagon to the opposite side of the rectangle. Attach four more rectangles to the existing one.

6 Use a front view of the pyramid and the theorem of Pythagoras to prove that each triangle on the side of the pyramid must be $\sqrt{3^2 + 4^2} = 5$ cm high. Construct a circle with a radius of 3 cm. Construct a hexagon with sides of 3 cm by marking off the edges along the circle. Construct perpendicular bisectors of 5 cm on each side of the hexagon. Complete the triangles.

7 Each triangular prism should have an isosceles right-angled triangle as base. Construct a right-angled triangle with its shortest sides 4 cm long. On each short side, construct a 4 cm by 4 cm square. On the hypotenuse, construct a rectangle which is 4 cm high. Construct another right-angled triangle on the opposite side of the rectangle. To build the cube, join the rectangular faces of the two prisms.

8 Each pyramid should have a square base. Construct a 5 cm by 5 cm square. On each side, construct an equilateral triangle with sides 5 cm long. To build the octahedron, join the bases of the two pyramids.

Review Copy

Extension

Use examples where the theorem of Pythagoras produces irrational numbers.

Consolidation

Learner's Book page 417

Before doing this consolidation exercise, encourage learners to review the work covered in this chapter. Advise learners to use the summary and to revise their work. This exercise can be used as an informal assessment task for you to track how learners are coping with the chapter and the concepts covered. The mark allocation provides guidelines on how to assess learners.

Suggested answers

1.1

Type of prism	Vertices (V)	Faces (F)	Edges (E)	Type of pyramid	Vertices (V)	Faces (F)	Edges (E)
triangular	6	5	9	triangular	4	4	6
rectangular	8	6	12	rectangular	5	5	8
pentagonal	10	7	15	pentagonal	6	6	10
hexagonal	12	8	18	hexagonal	7	7	12
heptagonal	14	9	21	heptagonal	8	8	14
octagonal	16	10	24	octagonal	9	9	16

(12)

1.2 $V = 2n$; $F = n + 2$; $F = 3n$ (1)

1.3 $V = n + 1$; $F = n + 1$; $E = 2n$ (2)

2.1 A tetrahedron has 4 faces which are congruent equilateral triangles, 4 vertices and 6 edges.

2.2 A hexahedron (cube) has 6 faces which are congruent squares, 8 vertices and 12 edges.

2.3 An octahedron has 8 faces which are congruent equilateral triangles, 6 vertices and 12 edges.

2.4 A dodecahedron has 12 faces which are congruent pentagons, 20 vertices and 30 edges.

2.5 An icosahedron has 20 faces which are congruent equilateral triangles, 12 vertices and 30 edges. (5)

3.1 four equilateral triangles: tetrahedron

3.2 six squares: cube

3.3 eight equilateral triangles: octahedron

3.4 three pairs of congruent rectangles: rectangular prism

3.5 two congruent pentagons and five squares: pentagonal prism

3.6 one hexagon and six isosceles triangles: hexagonal pyramid

3.7 two heptagons and seven rectangles: heptagonal prism (7)

4.1 A cube could have been built with a net like this, but there is no face in a position to close the cube at the top.

4.2 A square pyramid could have been built with a net like this, but the triangles are not high enough to meet at a point above the base.

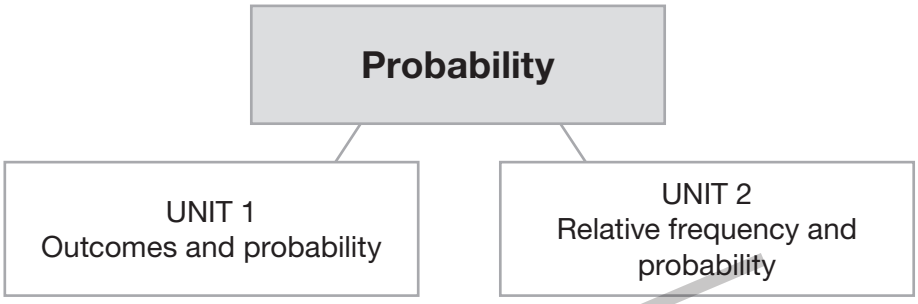
4.3 A rectangular pyramid could have been build with a net like this, but the triangles on the left and right are not high enough to meet the other two triangles in the same point. (3)

Total marks [30]

Review Copy

Chapter 19 Probability

Overview of concepts



Content		Time allocations	LB page
Unit 1	Outcomes and probability	2 hours	419
Unit 2	Relative frequency and probability	2,5 hours	424

Review Copy

Background information on probability

In Grade 7 the learners only listed possible outcomes and determined the relative frequency of actual outcomes for a series of trials, by means of experiments.

In Grade 8 they have to predict with reason the relative frequency of the possible outcomes, as well as compare relative frequency with probability.

Actual experiments will feature less in Grade 8, as learners must consider probability for hypothetical events.

They will use this formula to determine relative frequency of an outcome:

$$\text{relative frequency} = \frac{\text{number of times event occurred}}{\text{number of trials}}$$

The study of probability enables the learner to develop skills and techniques for making informed predictions, and describing randomness and uncertainty. Probability is a number that tells us the chance or likelihood that a random event will occur.

Ensure that learners are clear on the terminology that is used in this chapter such:

- Throwing of a die is called an *event*.
- A trial is an action that can have more than one *outcome*.
- An *experiment* is more than one trial (such as throwing a die six times).
- The result of the event is called the *outcome*.
- The *frequency of an event* indicates how many times that event occurs.
- A *frequency of an outcome* is the number of times that outcome occurs when the trial is performed.

Probability is used in many real-life scenarios, including insurance, weather forecasting, playing the Lotto, industrial quality control, study of genetics and quantum mechanics.

Generic teaching guidelines for teaching probability

Probability comes up in many everyday life and concrete situations. There are simple probability exercises that can easily be done in the classroom to teach learners the basic concepts. The topic of probability lends itself to everyday examples.

To introduce the topic talk about any of the following; weather forecast, playing the Lotto, entering competitions and insuring our homes, cars and cellphones.

Let them discuss in small groups how this relates to probability

Allow learner to mention other ways that probability occurs. List them on the board.

Point out that probability involves outcomes, predictions and relative frequency.

Make sure that learners take note of the word *predict* when associated with the relative frequency of an outcome. If asked to predict, they should know to use the probability formula.

Resources

Dice, coins, spinners and a deck of cards to use for probability experiments. Cardboard, colour pens and scissors to create posters. Each learner should have their own calculator.

Review Copy

Unit 1 Outcomes and probability

Learner's Book page 419

Unit focus:

- how to list all possible outcomes of a random event
- how to determine the probability of each possible outcome using the definition of probability.

Background information on outcomes and probability

Outcomes are the results of an experiment or trial. We can list all possible outcomes for a situation. The chance of an outcome is a description of the likelihood of it happening. Outcomes can be impossible, unlikely, very likely, equally likely, likely or certain. Tree diagrams are used to show all the outcomes visually.

Exercise 1

Learner's Book page 421

Guidelines on how to implement this activity

As an introduction throw a die to explain the different terminology listed in the Learner's Book (for example, throwing this die is an event, our results are outcomes). Use a learner to record the outcomes in a table on the board or flip chart.

	Outcomes for rolling of die experiment					
	Trial no. 1	Trial no. 2	Trial no. 3	Trial no. 3	Trial no. 4	Trial no. 5
Throw die/dice	3	5	6	1	2	3

Now use the table to question learners on the terminology. Draw a tree diagram to show all the outcomes. The following terminology words can be written on flashcards and displayed in the classroom.

Event	The name of the game, in this case "flip a coin".
Experiment	A series of trials, for example, tossing a coin three times.
Outcome	The actual result of an event such as tossing a coin.
Trial	The act of letting an event take place.

Learners can do this exercise in groups, but should do their own recordings in their exercise book.

Suggested answers

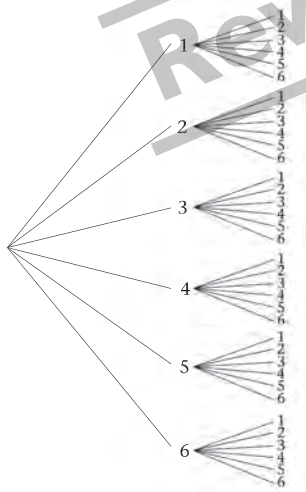
	Outcomes for each trial		
	Trial no. 1	Trial no. 2	Trial no. 3
Flip a coin	T	H	T
Pick a card	10 of hearts	5 of spades	2 of diamonds
Throw dice	6; 2	1; 5	3; 2

The possible outcomes for:

1 Flipping a coin → Heads, Tails

2 Picking one of four cards → Card 1, Card 2, Card 3, Card 4

3 Throwing two dice



Review Copy

Extension

Ask learners to perform their own experiments. Provide learners with objects like dice, coins, packs of cards, and paper bags and coloured balls. Explain to each group how to conduct their experiments. Each group should set up their respective experiments;

and each group should perform 5 trials. Each member of the group needs to perform a task: either performing the trial, recording, drawing up tree diagrams, or reporting back to the class.

Exercise 2

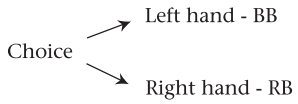
Learner's Book page 422

Guidelines on how to implement this activity

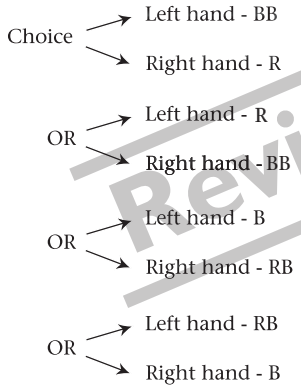
Show learners, by means of the worked example, how to calculate the probability of an event. Ask learners to devise possible experiments, and then determine the probability of the outcomes. Learners complete the exercise on their own.

Suggested answers

1



1.1 Lolo decides to add only one blue pen to one of her hands.



Review Copy

1.2 R; RB; BB; B 1.3 BB; R; B; RB 1.4 They are the same

2.1 The probability of the outcome being two blue pens is $\frac{1}{4} = 25\%$;

2.2 The probability of getting a blue and red pen in the left hand $\frac{1}{4} = 25\%$

3.1 Probability of a boy being chosen at random out of a class of 37 learners, with 21 boys and 16 girls is:

probability $= \frac{21}{37}$

3.2 Probability of a red ball being drawn out of a box 66 balls, with the ratio of balls being red: blue: green being 1: 2: 3 is:

probability = 1 out of $(1 + 2 + 3 = 6) = \frac{1}{6} = 16{,}67\%$

3.3 Probability of two left socks chosen out a drawer of 27 pairs of socks is:

probability $= \frac{2}{54} = \frac{1}{27} = 3{,}7\%$

4 Probability of rolling three 4s:

probability $= \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{216}$

5.1 Possible outcomes B; B2; B3 and B4

5.2 After Chumani adds another ball the possible outcomes are: B1; B2; B3; B4 or B5. The probability of Raeez choosing the ball Chumani added is:
No. of favourable outcomes = 1 (B5)
No. of possible outcomes = 5
Probability $= \frac{1}{5} = 20\%$

Remedial

Learners can construct spinners, by drawing a circle on a piece of paper and dividing it into quarters. Learners can spin a pen/pencil and see where it points to. Ask learners to collect this data for a few outcomes and represent their findings in a table. Learners can compare their outcomes with the probability.

Extension

Research why gambling is controversial. Keep the following in mind: Some people think gambling exploits the poor; some countries ban gambling altogether and casinos, lotteries and internet gambling are different forms of gambling.

Learners discuss what ethics are and what good ethics are. (Ethics refer to people's moral standards or principles. Certain behaviour can be called "ethical" or "unethical".)

Review Copy

Unit 2 Relative frequency and probability

Learner's Book page 424

Unit focus:

- revision of calculating probability
- learning
  - about relative frequency
  - how to calculate relative frequency
  - how to compare relative frequency with probability
  - how to explain possible differences between them.

Background information on relative frequency and probability

A prediction by definition is about a future event – relating to an outcome that has not yet happened. When making a prediction about the relative frequency of an outcome, you have to make use of probability calculations because the relative frequency of an outcome can only be determined when the experiment has been performed and the outcome has happened.

Make sure the learners understand this key concept:

$$\text{Probability} = \frac{\text{number of favourable outcomes}}{\text{the total number of possible outcomes}}$$

Relative frequency is the observed number of successful outcomes for a series of trials.

When we determine the relative frequency of something happening, we work out the approximate probability by doing actual practical experiments.
The formula used to calculate relative frequency is:

$$\text{Relative frequency} = \frac{\text{number of times a particular outcome takes place}}{\text{the number of trials}}$$

The more trials used to find the relative frequency, the better the estimate for the probability.

Exercise 1

Learner's Book page 425

Guidelines on how to implement this activity

To make predictions about the relative frequency of a possible outcome, you must rely on probability. The formula to calculate probability can be used when making predictions:

$$\text{Probability} = \frac{\text{the number of favourable outcomes}}{\text{the total number of possible outcomes}}$$

As an introduction solve a prediction scenario with the whole class. For example: predict the possible outcomes when a coin is tossed three times. Engage learners by asking, "what is a possible outcome for each toss?" It will either be heads or tails. For three tosses it will be heads, heads, heads, tails, tails, tails. Six outcomes are listed, but with three tosses only three outcomes can be reached.

Make clear that with each throw heads or tails have an equal chance of occurring. To calculate the relative frequency we have to perform the experiment.

Review Copy

Per toss there can only be one outcome. If a coin is tossed 3 times, there can only be 3 outcomes. Each outcome (H or T) has an equal chance of occurring.
Probability for T as outcome (use probability formula)

Probability (T) $= \frac{3}{6} = \frac{1}{2} = 50\%$

Probability for H as outcome is the same:

Probability (H) $= \frac{3}{6} = \frac{1}{2} = 50\%$

Ensure learners know this difference between probability and relative frequency. Compare relative frequency and probability by working through the worked example. Learners should perform this exercise on their own, but can discuss their answers in pairs or small groups.

Suggested answers

1.1 $\frac{2}{7}$ 1.2 $\frac{1}{7}$

1.3

Colour of tie	Tally	Relative frequency
Blue	|||	$\frac{3}{10}$
Grey	||	$\frac{3}{10} = \frac{1}{5}$
Green	|||	$\frac{3}{10}$
Yellow	||	$\frac{2}{10} = \frac{1}{5}$

1.4 In a small number of trials, the difference between the probability and relative frequency may be large.

2

	White ball	Green ball
Learner 1	X	
Learner 2		X
Learner 3		X
Learner 4	X	
Learner 5	X	

Discovering the difference between probability and relative frequency:

2.1 The probability of the white ball being chosen for each trial = $\frac{1}{2}$ = 50%

2.2 The probability of the green ball being chosen for each trial= $\frac{1}{2}$ =50%

2.3 The relative frequency of the white ball = $\frac{3}{5}$ = 60%

2.4 The relative frequency of the green ball = $\frac{2}{5}$ =40%

2.5 The relative frequency of white balls chosen was 60% and was higher than the probability of 50%. The relative frequency of the green balls chosen was 40% and was lower than the probability of 50%.

2.6 Relative frequency is the actual outcome whereas probability is the possible outcome.

Review Copy

Remedial

Make sure learners take note of the word "predict" when associated with the relative frequency of an outcome. When asked to predict the relative frequency of an outcome, they should know to use the probability formula. Give them more exercises to do where they can apply this skill and knowledge.

Learners who need more practice should have plenty of opportunity to do experiments with for example pens or dice.

Encourage them to always draw tree diagrams to help them find all possible outcomes.

Extension

Learners who grasp the work can come up with compound events and predict the relative frequencies of a few outcomes.

Learners find a partner in this ability group to complete this exercise.

Each person gets a turn to be blindfolded.

They put two objects in front of the blindfolded person that he/she can select, by saying for example "I choose the object on my left". The other person records the outcomes.

They record the outcomes in a data collection sheet for a series of six trials per person, for example as in the table:

Tally table		
	Object A	Object B
Person 1	||||	||
Person 2	|||	|||

Investigation

Learner's Book page 426

Guidelines on how to implement this activity

Give learners an additional chance to complete an experiment, this time with many trials. Divide the class into teams of four. Each person rolls the die 15 times and records the outcomes.

Team member	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Trial 6	Trial 7	Trial 8	Trial 9	Trial 10	Trial 11	Trial 12	Trial 13	Trial 14	Trial 15
1															
2															
3															
4															

- Learners should find:
  For each member: The probability of each number 1 – 6 appearing = $\frac{1}{6}$ = 16,67%.
- Learners calculate the relative frequency of each number appearing on the dice.
- Put all the scores together, and calculate the relative frequency of the outcomes.
- Learners should notice that the more trials one does the closer the relative frequency comes to the probability.

Review Copy

Challenge

Learner's Book page 427

Guidelines on how to implement this activity

Learners should be encouraged to put together the results of all the class members' trials with the dice. Ask learners to compare the prediction with the relative frequency for one person, for the team and for the class. Ask learners what they notice.

Suggested answers

Learners should have further proof that there is a relationship between the number of trials and how closely the relative frequency agrees with the calculated probability? Learners should note that the more trials they do they more accurate their relative frequency will be.

Consolidation

Learner's Book page 429

Before doing this consolidation exercise, encourage learners to review the work covered in this chapter. Advise learners to use the summary and to revise their work. This exercise can be used as an informal assessment task for you to track how learners

are coping with the chapter and the concepts covered. The mark allocation provides guidelines on how to assess learners.

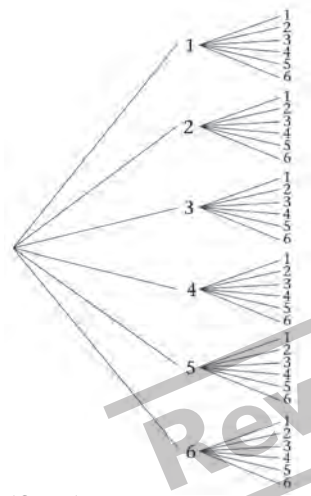
Suggested answers

1

Experiment		The name of the game, in this case "flip a coin".
Outcome		A series of trials, for example, tossing a coin 3 times.
Trial		The actual result of an event such as tossing a coin.
Event		The act of letting an event take place.

(4)

2 Throwing two dice:



Review Copy

(4)

3.1 $\frac{13}{52} = \frac{1}{4}$ is its simplest form. (1)

3.2.1 There are 51 cards left in the pack after the first card is drawn. (1)

3.2.2 There are 12 heart cards left after the first one is drawn. (1)

3.2.3 The probability of drawing a second heart card is therefore $\frac{12}{51}$. (1)

4.1 True (1)

4.2 True (1)

5.1 Possible combinations that will add up to R300:

- Pair of running shoes (R130) + Sun hat (R20) + Matching top and shorts (R80) + T-shirt (R35) + Pair of shorts (R35)
- Shorts (R35) + tennis racquet (R235) + packet of balls (R30)
- Tshirt (R35) + tennis racquet (R235) + packet of balls (R30) (2)

5.2 The probability of Liesl's choice = $\frac{1}{3}$ = 33,3% (2)

Total marks [30]

Chapter 20 Programme of Assessment

This chapter provides all the resources you need to ensure your learners meet the requirements for promotion.

- Two options for each of the required formal programme of assessment tasks.
- Exemplar exams for learners to use to practise for their exams. These exemplars are in the Learner's book with the memoranda supplied in this section of the Teacher's Guide.
- Control tests for you to use as part of the POA, along with the memoranda.
- A June and a December exam paper, with memoranda.

The following table lays out the programme for you. The shaded cells are only in the Teacher's Guide to ensure the tests and exams are unseen by the learners.

Term	Task		Learner's Book page	Teacher's Guide page
1	Assignment	Option 1 Ratio, rate and integers	431	334
		Option 2 Exponents and algebra	432	336
	Control Test 1			338
2	Investigation	Option 1 Constructions	147, 150, 152	139, 142, 143, 341
		Option 2 Geometry	156	145, 342
	Control Test 2			343
	Exemplar June exam	Exemplar paper for revision purposes	434	356
	June Exam			347
3	Assignment	Option 1 Fractions	437	360
		Option 2 Solving triangles	438	361
	Project	Option 1 Measurement	439	362
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4	Assignment	Option 1 Algebra	443	369
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	Exemplar December exam	Exemplar paper for revision purposes	445	374
	December exam			378

Review Copy

We suggest that in order to prepare learners adequately for formal assessment you use the allocated Revision time prescribed in the CAPS for revising work.
This Series advises that you use the Consolidation exercises and Summaries at the end of each chapter to revise. The Consolidation exercises have mark allocations to enable informal assessment of how learners are managing the specific content area.

Assignment: Option 1 Ratio, rate and integers

1.1	F	(1)
1.2	T	(1)
1.3	T	(1)
1.4	T	(1)
1.5	F	(1)
1.6	T	(1)
1.7	T	(1)
1.8	T	(1)
		[8]
2.1	$7 : 9 = \frac{7}{9} = \frac{a}{81}$	$(\frac{1}{2})$
	$\frac{7}{9} \times \frac{9}{9} = \frac{63}{81}$	$(\frac{1}{2})$
	The smaller number is 63	(1)
		[2]
2.2	$4 : 5 = \frac{4}{5} = \frac{44}{b}$	$(\frac{1}{2})$
	$\frac{4}{5} \times \frac{11}{11} = \frac{44}{55}$	$(\frac{1}{2})$
	There are 55 toffees	(1)
		[2]
3	Tom : Thandi = 40 : 30 = $\frac{40}{30}$	(1)
	$\frac{40}{30} \div \frac{10}{10} = \frac{4}{3}$	(1)
	4 + 3 = 7	(1)
	$\frac{4}{7}$ of R7 700 = $\frac{4}{7} \times \frac{7\,700}{1}$ = R4 400	(1)
	$\frac{3}{7}$ of R7 700 = $\frac{3}{7} \times \frac{7\,700}{1}$ = R3 300	(1)
	R4 400,00 + R3 300,00 = R7 700,00	(1)
		[6]
4.1	R21,00 ÷ 3 = c	(1)
	The rate is R7,00 per kg	(1)
		[2]
4.2	910 km ÷ 7 = d	(1)
	The rate is 130 km/h	(1)
		[2]
5	R12,00 ÷ 1 000 g = 1 200 cents ÷ 1 000 g	$(\frac{1}{2})$
	1,2 c/g	(1)
	R4,00 ÷ 250 g = 400 cents ÷ 250 g	$(\frac{1}{2})$
	1,6 c/g	(1)
	R12,00/kg is cheaper	(1)
		[4]
6	90 km ÷ 1 h = 90 000 m ÷ 60 min = 1 500 m/min	
	1 500m/min is faster than 1 360m/min	[3]

Review Copy

7.1	$36 - 7 = 29$	$36 - (-7) = 36 + 7 = 43$	
	$36 + (-7) = 36 - 7 = 29$	$-36 + (-7) = -36 - 7 = -43$	(4)
7.2	$0 + 45 = 45$	$0 + (-45) = 0 - 45 = -45$	
	$0 - 45 = -45$	$0 - (-45) = 0 + 45 = 45$	(4)
7.3	$(-52) + (-9) = -61$	$(-34) + (+8) = -26$	
	$(72) + (-63) + (10) + (-19) = 0$	$28 - (-30) = 58$	
	$28 - (+30) = -2$	$(72) - (-63) - (10) - (-19) = 144$	(6)
			[14]
8.1	$7 \times (-9) = -63$	$(-17) \times 5 = -85$	
	$(-12) \times (-13) = 156$	$21 \times 22 = 462$	(4)
8.2	$34 + (-34) = 0$		
	We call +34 and –34 additive inverses		
	$x = 1$		(3)
			[7]
			Total Marks [50]

Review Copy

Assignment: Option 2 Exponents and Algebra

1.1 exponent (1)
1.2 base (1)
2.1 $49 = 7^2$; 7×7 ($\frac{1}{2}$)
2.2 $2ab \times 2ab \times 2ab = 2^3a^3b^3$; $8a^3b^3$ ($\frac{1}{2}$)
2.3 $1\ 000\ 000 = 10^6$; $10 \times 10 \times 10 \times 10 \times 10 \times 10$ ($\frac{1}{2}$)
2.4 5^3; $5 \times 5 \times 5$ ($\frac{1}{2}$)
2.5 $3 \times 3 \times 3 \times c \times c \times c \times c = 3^3$; $c4$; $27c^4$ ($\frac{1}{2}$)
2.6 $169 = 13^2$; 13×13 ($\frac{1}{2}$)
2.7 $144 = 12^2$; 12×12 ($\frac{1}{2}$)
2.8 $5 \times 5 \times 5 \times 5 \times 5 \times 5 = 5^6 = 15\ 625$ ($\frac{1}{2}$)
3.1 $x^7 \times x^5 = x^{12}$ (1)
3.2 $3a^4b^5 \times 4a^3b^4 = 12a^7b^9$ (2)
3.3 $\frac{y^{10}}{y^2} = y^8$ (1)
3.4 $\frac{6y^6}{y^8} = \frac{6}{y^2}$ (1)
3.5 $7c^2d^7 \times 2c^4e^6 = 14c^6d^7e^6$ (2)
3.6 $5x^4y^4z^8 \times x^4y^3 = 20x^8y^7z^8$ (2)
3.7 $\frac{10m^{12}n^6}{5m^2n^3} = 2m^{10}n^3$ (2)
3.8 $\frac{18r^9s^5}{6rs^3} = 3r^8s^2$ (2)

4.1

Standard form	Exponential form	Expanded form	Scientific form
64	8^2	8×8	$6{,}4 \times 10^1$
1 000	10^3	$10 \times 10 \times 10$	$1{,}0 \times 10^3$
625	$\frac{5^4}{25^2}$	$5 \times 5 \times 5 \times \frac{5}{25} \times 25$	$62{,}5 \times 10^1$
100 300 000	$2^4 \times 5^4$	$2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 5$	$1{,}003 \times 10^8$
10 000	10^4	$10 \times 10 \times 10 \times 10$	$1{,}0 \times 10^4$

(10)

5.1 $(10 - 8)^3 = 10^3 - 8^3$ FALSE $(10 - 8)^3 = 2^3 = 8$ (2)
5.2 $\sqrt{25 + 11} = \sqrt{36}$ TRUE (1)
5.3 $x^1 = x$ TRUE (1)
5.4 $y^0 = 0$ FALSE $y^0 = 1$ (2)
6.1 $(0{,}006)^2$
$0{,}000036$ (2)
6.2 $\sqrt{16a^{16}b^4}$
$= 4a^8b^2$ (2)

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6.3 $9{,}4 \times 10^3 - 5{,}2 \times 10^{-2} \times 2{,}2 \times 10^2$

$= 9\ 400 - 5{,}2 \times 2{,}2 \times 10^{-2+2}$

$= 9\ 400 - 11{,}44 \times 1$

$= 9\ 400 - 11{,}44$

$= 9\ 388{,}56$ (4)

6.4 $(-3y^3)^2 = (-3y^3)(-3y^3)$

$= 9y^6$ (2)

6.5 $\sqrt[3]{3^6 \times 5^3}$ $\qquad$ $\sqrt[3]{3^6 \times 5^3}$

$= \sqrt[3]{729 \times 125}$ $\qquad$ $= 3\frac{6}{3} \times 5\frac{3}{3}$

$= 45$ $\qquad$ $= 45$ (2)

6.6 $2{,}6 \times 10^4 \div (1{,}2 \times 10^2)$

$= \frac{26\ 000}{1\ 200}$

$= \frac{1\ 300}{6}$ (3)

Total marks [50]

Review Copy

Test 1

Time: 1 hour Marks: 50

Question 1

1.1 True or false. If false provide the correct statement.

1.1.1 $(65 \times 54) \times 20 = (65 \times 20) \times 54$ (1)

1.1.2 $45(5 + 9) = (45 + 5) \times (45 + 9)$ (1)

1.2 Determine $\sqrt[3]{1\,728}$ by prime factors. You MAY NOT use a calculator. (3)

1.3. Mr Jobias wants to take a loan of R50 000. He visits two banks and receives the following options: He can either borrow the money over 3 years and pay 7,5% interest or over 2 years and pay 9% interest. Determine which option Mr Jobias should choose in order to repay the lowest amount to the bank. (3)

Question 2

Calculate:

2.1 $(-6) - (-7) + 2$ (1)

2.2 $4 \times (-2) \times (-3)$ (2)

2.3 $(-10) \div (-2) + 5$ (2)

2.4 $(\sqrt[3]{216} + (-\sqrt{4})$ (2)

2.5 $(-1\,200) \div (-4) + [(-2) \times 4]$ (3)

Question 3

3.1 Write in expanded notation and calculate the answer of the following:

3.1.1 $(-3)^3 \times (4)^2 \times (-2)^5$ (2)

3.1.2 $(2)^7 \times (-4)^4 \times (-7)^1$ (2)

3.2 Use the laws of exponents to find the value of the following:

3.2.1 $(-4^2)^3$ (1)

3.2.2 $3^2 \times 3^3$ (1)

3.2.3 $5^{13} \div 5^{10}$ (1)

3.2.4 $(3 \times 5)^2$ (1)

3.3 Write the following in scientific notation.

3.3.1 56 000 000 000 000 (1)

3.3.2 768 890 453 (1)

3.3.3 1 001 (1)

Question 4

4.1 William is offered the following options by his parents. He can either receive R50 a week for 10 weeks as pocket money, or he can start with R5 in the first week, and have this amount double every week.

4.1.1 Draw a table to show how much William will receive each week for both options. (2)

4.1.2 Determine a number sentence to calculate how much money William will have in total at the end of the 10 weeks with each option. (3)

4.1.3 Which option should William choose and why? (1)

4.2 Determine the missing output values in the following flow diagrams.

4.2.1 $-5 \rightarrow \times 3 \rightarrow \times (-2) \rightarrow a$ (1)

4.2.2 $75 \rightarrow \div 5 \rightarrow + (-2) \rightarrow b$ (1)

Review Copy

4.3 Determine the missing input values in the following flow diagrams.

4.3.1 $c \rightarrow \times (-4) \rightarrow + 7 \rightarrow 23$ (1)

4.3.2 $d \rightarrow \div 3 \rightarrow \times (-2) \rightarrow -20$ (1)

Question 5

Simplify:

5.1 $8(n + 4m) + 6(2n + 5m)$ (4)

5.2 $-2(x + 4 - 4y) + 4y - x$ (3)

Question 6

Write down an algebraic equation the following:

6.1 Eight times a increased by b equals 54. (2)

6.2 The sum of the cubes of x and y is equal to 3 more than x. (2)

Total marks [50]

Review Copy

Test 1 Memo

Question 1

1.1.1 True (1)

1.1.2 False $45(5 + 9) = (45 \times 5) + (45 \times 9)$ (1)

1.2 $\sqrt[3]{1\,728} = \sqrt[3]{2^6 \times 3^3} = 2^2 \times 3 = 12$ (3)

1.3 Option 1: $A = 50\,000(1 + 0{,}075 \times 3) = R61\,250$

Option 2: $A = 50\,000(1 + 0{,}09 \times 2) = R59\,000$

Option 2 is the better option. (3)

Question 2

2.1. $(-6) - (-7) + 2 = 3$ (1)

2.2 $4 \times (-2) \times (-3) = 24$ (2)

2.3 $(-10) \div (-2) + 5 = 10$ (2)

2.4 $(\sqrt[3]{216}) + (-\sqrt{4}) = 4$ (2)

2.5 $(-1\,200) \div (-4) + [(-2) \times 4] = 292$ (3)

Question 3

3.1.1 $(-3)^3 \times (4)^2 \times (-2)^5 = -3 \times -3 \times -3 \times 4 \times 4 \times -2 \times -2 \times -2 \times -2 \times -2 = 13\,824$ (2)

3.1.2 $(2)^7 \times (-4)^4 \times (-7)^1 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 4 \times 4 \times 4 \times 4 \times -7 = -229\,376$ (2)

3.2.1 $(-4^2)^3 = (-4)^6 = 4\,096$ (1)

3.2.2 $3^2 \times 3^3 = 3^5 = 243$ (1)

3.2.3 $5^{13} \div 5^{10} = 5^3 = 125$ (1)

3.2.4 $(3 \times 5)^2 = 225$ (1)

3.3.1 $56\,000\,000\,000\,000 = 5{,}6 \times 10^{13}$ (1)

3.3.2 $768\,890\,453 = 7{,}68890453 \times 10^8$ (1)

3.3.3 $1\,001 = 1{,}001 \times 10^3$ (1)

Review Copy

Question 4

4.1.1

Option 1:

Week	1	2	3	4	5	6	7	8	9	10
Amount	R50	R50	R50	R50	R50	R50	R50	R50	R50	R50

Option 2:

Week	1	2	3	4	5	6	7	8	9	10
Amount	R5	R10	R20	R40	R80	R160	R320	R640	R1280	R2560

(2)

4.1.2 Determine a number sentence to calculate how much money William will have in total at the end of the 10 weeks with each option.

Option 1: Total = R50 × 10 = R500

Option 2: Total = R5 + R10 + R20 + R40 + R80 + R160 + R320 + R640 + R1 280 + R2 560 = R5 115 (3)

4.1.3 Option 2 as he will receive more pocket money. (1)

4.2.1 $a = 30$ 4.2.2 $b = 13$ 4.3.1 $c = -4$ 4.3.2 $d = 30$ (4)

Question 5

5.1 $8(n + 4m) + 6(2n + 5m) = 8n + 32m + 12n + 30m = 20n + 62m$ (4)

5.2 $-2(x + 4 - 4y) + 4y - x = -2x - 8 + 8y + 4y - x = -3x + 12y - 8$ (3)

Question 6

6.1 $8a + b = 54$ 6.2 $x^3 + y^3 = x + 3$ (4)

Total marks [50]

Investigation: Option 1 Constructions

The following investigations can be used as formal assessment tasks for the formal Programme of Assessment. **All** these investigations form part of the curriculum for the year and **all three** should be covered in class; however only one of these three investigations needs to be recorded and assessed as part of the formal Programme of Assessment. You as the teacher can choose which to use for this purpose.

1. Investigation: Properties of triangles by construction

Please find this investigation in the Learner's Book page 147 and Teacher's Guide page 139.
If using this investigation for formal assessment please use the following rubric to assess learners.

Criteria	5	4	3	2	1
Did the learner construct appropriate diagrams and see that the interior angles of a triangle always equal 180°?					
Did the learner construct the appropriate diagram and accurately measure the size of an angle of an equilateral triangle as 60°?					
Did the learner construct the appropriate diagram and see that base angles and sides of an isosceles triangle are equal?					
Did the learner construct the appropriate diagram and see that the angles of a right-angled triangle also add up to 180°?					
Did the learner construct the appropriate diagram and bisect correctly the interior angles of a triangle?					
Did the learner construct the appropriate diagram and see that the exterior angle of a triangle is equal to the sum of the two interior opposite angles?					

Total marks [30]

Review Copy

2. Investigation: Construction of congruent and similar triangles

Please find this investigation in the Learner's Book pages 150 and 152 and Teacher's Guide pages 142 and 143.
If using this investigation for formal assessment please use the following rubric to assess learners.

Criteria	5	4	3	2	1
Did the learner construct appropriate diagrams and see that the SSS is condition for congruency?					
Did the learner construct the appropriate diagram and see that SAS is a condition for congruency?					
Did the learner construct the appropriate diagram and see that AA and corresponding S, is a condition for congruency?					
Did the learner construct the appropriate diagram and see that RHS is a condition for congruency?					
Does the learner understand that these are the only conditions that can be used to prove triangles congruent?					
Did the learner construct the appropriate diagram and see that AAA is a condition for similarity in triangles?					
Did the learner construct the appropriate diagram and see that corresponding sides in proportion is a condition for congruency?					
Does the learner understand that these are the minimum conditions for similarity?					

Total marks [40]

3. Investigation: Properties of quadrilaterals

Please find this investigation in the Learner's Book page 156 and Teacher's Guide page 145. If using this investigation for formal assessment please use the following rubric to assess learners.

Criteria	5	4	3	2	1
Did the learner construct appropriate diagrams and discover that pairs of opposite sides of a parallelogram, square, rhombus and rectangle are parallel?					
Did the learner construct the appropriate diagrams and discover that pairs of opposite sides of a parallelogram, square, rhombus and rectangle are equal in length?					
Did the learner construct the appropriate diagrams and discover that pairs of adjacent sides of a square, rhombus and kite are equal in length?					
Did the learner construct the appropriate diagrams and see that all sides of a rhombus and a square are equal?					
Did the learner construct appropriate diagrams and discover that pairs of opposite angles of a parallelogram, square, rhombus and rectangle are equal?					
Did the learner construct the appropriate diagrams and discover that all angles of a square and a rectangle are equal all equal to 90°?					
Did the learner construct appropriate diagrams and discover that the diagonals of a parallelogram, square, rhombus and rectangle are equal?					
Did the learner construct appropriate diagrams and discover that the diagonals of a parallelogram, square, rhombus, and rectangle bisect each other?					
Did the learner construct appropriate diagrams and discover that the diagonals of a square, rhombus and kite are perpendicular to each other?					

Total marks [45]

Review Copy

Test 2

Time: 1 hour **Marks: 50**

Question 1

1.1 Simplify the following:

1.1.1 $4ab(9b^2 - 5ab - 7a^3)$ (2)

1.1.2 $-8m(mn^2 + 5m - 3n^3)m^2n$ (2)

1.1.3 $\frac{-14f(f^2 + gh^4)}{7fg^2}$ (2)

1.1.4 $\frac{-6x^2y + 9xy^6}{3xy^2}$ (2)

1.1.5 $(15a^2b - ab)3a - b(8a^2 + 3a^3)$ (3)

1.2 If $a = 1$, $b = 2$ and $c = 0$, determine the values of the following expressions.

1.2.1 $\frac{c}{a^4b^6}$ (1)

1.2.2 $b^3 - a^c + c$ (2)

1.2.3 $\sqrt{16} + \sqrt{a}$ (1)

1.3 Solve the following equations:

1.3.1 $3x - 7 = 8$ (1)

1.3.2 $6a - 2a + 5 = 10$ (2)

1.3.3 $\frac{18x^2}{3x} \times 2 = 24$ (2)

1.3.4 $a^2 \times a^3 - 12 = 20$ (2)

[22]

Review Copy

Question 2

2.1 Construct line AB parallel to line CD. (3)

2.2 Construct equilateral triangle ABC, with each side equal to 4 cm. (3)

2.3 Construct an angle of 45° without using a protractor. (3)

2.4.1 Construct trapezium DEFG with DE = 6 cm, FG = 4,5 cm and with $D\hat{E}F = 67°$. (3)

2.4.2 Write down the lengths of GD and EF. (2)

[14]

Question 3

3.1 State whether the following are true or false. If false provide the correct statement.

3.1.1 An equilateral triangle has a 90° angle.

3.1.2 A rhombus is a special parallelogram.

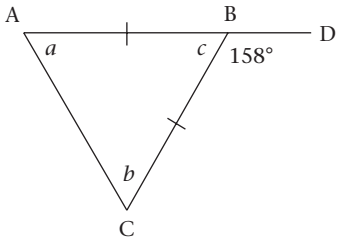
3.1.3 The interior angles of quadrilaterals add up to 180°.

3.1.4 The exterior angle of a triangle is equal to the sum of the two interior opposite angles.

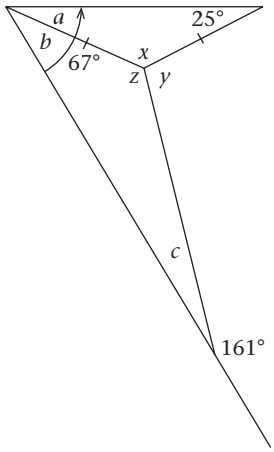
3.1.5 The diagonals of a kite bisect each other at 90°. (5)

3.2 For each of the following find the values of the missing variables.

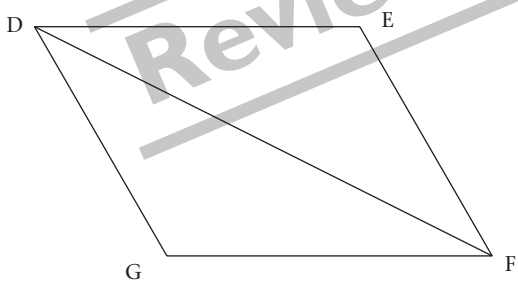
3.2.1 (2)



3.2.2 (4)



3.3 If DEFG is a parallelogram, prove that ΔDEF ≡ ΔFGD. (3)



Review Copy

[14]

Total marks [50]

Test 2 Memo

Time: 1 hour **Marks: 50**

Question 1

1.1.1 $4ab(9b^2 - 5ab - 7a^3) = 36ab^3 - 20a^2b^2 - 28a^4b$ (2)

1.1.2 $-8m(mn^2 + 5m - 3n^3)m^2n = -8m^3n(m^2n = + 5m - 3n^3)$

$= -8m^4n^3 - 40m^4n + 24m^3n^4$ (2)

1.1.3 $\frac{-14f(f^2 + gh^4)}{7fg^2} = \frac{-14f^3 - fgh^4}{7fg^2} = \frac{-2f^2}{g^2} - \frac{2h^4}{g}$ (2)

1.1.4 $\frac{-6x^2y + 9xy^6}{3xy^2} = \frac{-2x}{y} + 3y^4$ (2)

1.1.5 $(15a^2b - ab)3a - b(8a^2 + 3a^3) = 45a^3b - 3a^2b - 8a^2b - 3a^3b$

$= 42a^3b - 11a^2b$ (3)

1.2.1 $\frac{c}{a^4b^6} = 0$ (1)

1.2.2 $b^3 - a^c + c = (2)^3 - (1)^0 + 0 = 7$ (2)

1.2.3 $\sqrt{16} + \sqrt{a} = 4 + 1 = 5$ (1)

1.3.1 $3x - 7 = 8$

$x = 5$ (1)

1.3.2 $6a - 2a + 5 = 10$

$4a = 5$

$a = \frac{5}{4}$ (2)

1.3.3 $\frac{18x^2}{3x} \times 2 = 24$

$12x = 24$

$x = 2$ (2)

1.3.4 $a^2 \times a^3 - 12 = 20$

$a^5 = 32$

$a = 2$ (2)

Review Copy

Question 2

2.1 Accurate construction of line AB parallel to line CD. (3)

2.2 Construction of equilateral triangle ABC, with each side equal to 4 cm. (3)

2.3 Construction of an angle of 45° without using a protractor. (3)

2.4.1 Construction of trapezium DEFG with DE = 6 cm and FG = 4,5 cm, with $D\hat{E}F = 67°$ (3)

2.4.2 Accurate lengths of GD and EF. (2)

Question 3

3.1.1 False, an equilateral triangle has three 60° angles.

3.1.2 True.

3.1.3 False, the interior angles of quadrilaterals add up to 360°.

3.1.4 True.

3.1.5 False, the diagonals of a kite intersect at 90°. (5)

3.2.1 $c = 22°$ (straight line angles)
$a = b = 79°$ (exterior angle isosceles triangle OR sum angles isosceles triangle) (2)

3.2.2 $a = 25°$ (isosceles triangle)
$b = 42°$
$x = 130°$ (sum angles isosceles triangle)
$c = 19°$ (straight line angles)
$z = 119°$ (sum angles triangle)
$y = 111°$ (angles around a point equal to 360°) (4)

3.3 DE = FG (opposite sides parallelogram)
EF = GD (opposite sides parallelogram)
$\hat{E} = \hat{G}$ (opposite angles parallelogram)
$\therefore \Delta DEF \equiv \Delta FGD$ (S; A; S) (3)

Total marks [50]

Review Copy

June exam

Time: 2 hours **Marks: 100**

Question 1

1 List the following:

1.1 The first ten whole numbers. (1)

1.2 List all the factors of 210.
List all the factors of 165.
What is the HCF of 210 and 165? (2)

1.3 The first ten multiples of 20.
The first ten multiples of 25.
What is the LCM of 20 and 25? (2)

1.4 What is the LCM of 15 and 25. (1)

[6]

Question 2

2 Calculate and check by applying the distributive property:

2.1 $7\ (23 + 14)$ (1)

2.2 $\dfrac{(81 + 54)}{5}$ (1)

[2]

Review Copy

Question 3

Divide R360,00 between Thandi and Mary in the ratio 4 : 5. [2]

Question 4

Calculate:

4.1 $(+17) \times (+5)$ 4.2 $(-17) \times (-5)$

4.3 $(-17) \times (+5)$ 4.4 $(+17) \times (-5)$ [4]

Question 5

Calculate:

5.1 $(+36) \div (+3)$ 5.2 $(-445) \div (-5)$

5.3 $(+82) \div (-2)$ 5.4 $(-168) \div (+12)$ [4]

Question 6

6.1 Arrange the following in ascending order:

–357 –10 38 –156 5 (1)

6.2 Arrange the following in descending order:

–99 –160 0 –93 1 (1)

[2]

Question 7

Calculate:

7.1 $10^3 - (-7)^2$

7.2 $12^2 + (-5)^3$ [2]

Question 8
Find the square roots of:
8.1 64
8.2 121 [2]

Question 9
Find the cube roots of:
9.1 125
9.2 216 [2]

Question 10
Calculate the value of the following expressions without a calculator:
10.1 $\sqrt{625}$
10.2 $\sqrt[3]{729}$ [2]

Question 11
Calculate:
11.1 $(3^2)^3$
11.2 $(7 \times 3)^2$ [2]

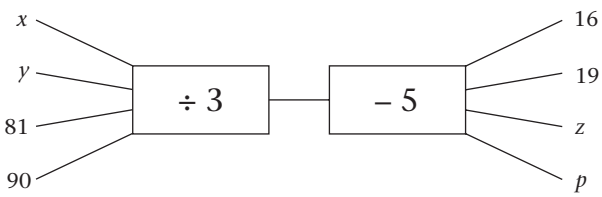
Question 12
Calculate:
12.1 $3^5 \div 3^2$
12.2 $4^5 \div 4^2$ [2]

Question 13
Calculate:
13.1 $\sqrt{139 + 5} + \sqrt{25}$
13.2 $\sqrt{110 - 10} + \sqrt{25}$ [2]

Question 14
Find the square root of 21. [1]

Question 15
Write 54 267 852 m^2 in scientific notation. [1]

Question 16
Complete the flow diagam by filling in the variables:



Review Copy

Question 17

If the area of a room is 36 m^2 and the length is 9 m:

17.1 What is the breadth?

17.2 What formula is used to find area of a rectangle?

17.3 What formula is used to find perimeter of a rectangle?

17.4 What is the perimeter of the room? [4]

Question 18

18.1 Complete the following table by filling in the missing numbers: (2)

Fraction	Denominator of 10	Percentage
$\frac{3}{5}$		
$\frac{72}{180}$		
$\frac{18}{30}$		

18.2 Which of the common fractions in the first column are equivalent? (1)

[3]

Review Copy

Question 19

Consider:

$-2a^2 + 6 - 5a - 8a^3 + 3a^6$

19.1 How many terms in the algebraic expression? (1)

19.2 Give the coefficient of a? (1)

19.3 Name this type of expression. (1)

19.4 Rewrite the expression in descending powers of a. (1)

19.5 Write the constant in the expression. (1)

[5]

Question 20

Simplify:

20.1 $3ab + 2bc + ab + 4dc - 5bc - 3bc$ (2)

20.2 $(3x^2 + 7xy - 8) + (-5x^2 - 12xy + 4)$ (3)

[5]

Question 21

Simplify:

21.1 $\sqrt{36a^{36}}$ (2)

21.2 $\dfrac{2pq^2 \times 3p^3q^6}{6p^5q^4}$ (2)

[4]

Question 22

If a = −2 and b = 4, determine the following value:

22.1 $a(b + a)a(b + a)$ (2)

22.2 $\frac{6a \times 5b}{ab} \frac{6a \times 5b}{ab}$ (2)

[4]

Question 23

Solve the following equations for x:

23.1 $8x = 248x = 24$ (1)

23.2 $\frac{8}{-x} = 9\frac{8}{-x} = 9$ (2)

[3]

Question 24

Write an algebraic equation to solve the following problem:

Lizo scored 18 goals for the season. Jeremy scored an unknown number of goals for the team. Together their goals total 26. How many goals did Jeremy score? (3)

[3]

Question 25

Do the following construction on one drawing:

25.1 Construct ΔABC with BC = 8 cm, AC = 6 cm and $\hat{C}$ = 90°. Measure AB. (3)

25.2 Construct the bisector of $\hat{A}$. Let it cut BC in D. Measure $D\hat{A}C$. (3)

25.3 Construct the perpendicular bisector of BC. Let it cut AB in E. (2)

25.4 Construct a circle with centre E and radius equal to DC. (2)

[10]

Review Copy

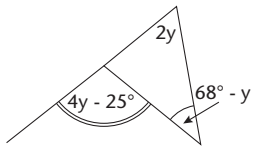
Question 26

26.1 Complete the following statements:

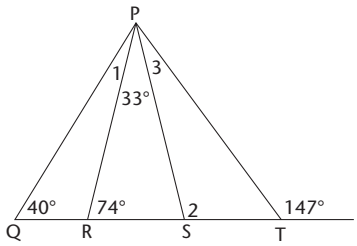
26.1.1 The sum of the interior angles of a triangle is equal to … (1)

26.1.2 The exterior angle of a triangle is equal to … (1)

26.2 Determine the size of the exterior angle in the drawing below. (3)



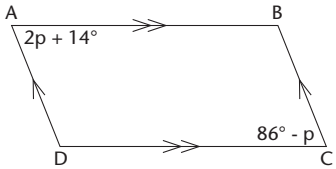
26.3 Look at the drawing below:



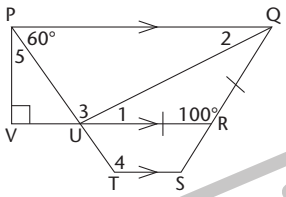
26.3.1 Calculate the size of the three numbered angles. (3)
26.3.2 Is ΔPQR similar to ΔPTS? Explain your answer. (2)
[10]

Question 27

27.1 Complete the following statements:
27.1.1 The sum of the interior angles of a quadrilateral is equal to … (1)
27.1.2 The opposite angles of a parallelogram are … (1)
27.2 In the drawing below AB || DC and DA || CB. Calculate the size of $\hat{D}$. (4)



27.3 In the drawing below PQ || VR || TS and RQ = RV. Calculate the sizes of the five numbered angles. (5)



[11]

Review Copy

June Exam Memo

Question 1

1.1 0; 1; 2; 3; 4; 5; 6; 7; 8; 9 (1)

1.2 Factors of 210 are 1; 2; 3; 5; 6; 7; 10; 14; 15; 21; 30; 35; 70; 42; 105; 210
Factors of 165 are 1; 3; 5; 11; 15; 33; 55; 165
The HCF of 210 and 165 is 15. (2)

1.3 The first ten multiples of 20 are 20; 40; 60; 80; 100; 120; 140; 160; 180; 200
The first ten multiples of 25 are 25; 50; 75; 100; 125; 150; 175; 200; 225; 250
The LCM of 20 and 25 is 100. (2)

1.4 The LCM of 15 and 25 is 75. (1)

Question 2

2.1 $7(23 + 14) = 259$
$7(23 + 14) = 7 \times 23 + 7 \times 14 = 2\ 597(23 + 14) = 7 \times 23 + 7 \times 14 = 259$

2.2 $\frac{(81 + 54)}{5} = 27$
$\frac{(81 + 54)}{5} = 81 \div 5 + 54 \div 5 = 27$
(1)

Question 3

Thandi's share $= \frac{4}{9} \times 360 = R160$

Mary's share $= \frac{5}{9} \times 360 = R200$ (2)

Question 4

4.1 $(+17) \times (+5) = 85$

4.2 $(-17) \times (-5) = 85$

4.3 $(-17) \times (+5) = -85$

4.4 $(+17) \times (-5) = -85$ (4)

Question 5

5.1 $(+36) \div (13) = 12$

5.2 $(-445) \div (-5) = 89$

5.3 $(+82) \div (-2) = -41$

5.4 $(-168) \div (112) = -14$ (4)

Question 6

6.1 –357; –156; –10; 5; 38 (1)

6.2 1; 0; –99; –160; –93 (1)

Question 7

7.1 $10^3 - (-7)^2 = 951$

7.2 $12^2 + (-5)^3 = 19$ (2)

Question 8

8.1 8

8.2 11 (2)

Review Copy

Question 9
9.1 5
9.2 6 (2)

Question 10
10.1 $\sqrt{625} = 25$
10.2 $\sqrt[3]{729} = 9$ (2)

Question 11
11.1 $(3^2)^3 = 36 = 729$
11.2 $(7 \times 3)^2 = 441$ (2)

Question 12
12.1 $3^5 \div 3^2 = 3^3 = 27$
12.2 $4^5 \div 4^2 = 4^3 = 64$ (2)

Question 13
13.1 $\sqrt{139} + 5 + \sqrt{25} = 21{,}79$
13.2 $\sqrt{110} - 10 + \sqrt{25} = 5{,}49$ (2)

Question 14
$\sqrt{21} = 4{,}582575695$ (1)

Question 15
54 267 852m^2 = 5,4267852 × 10^7 m2 (1)

Question 16
$x = 63$
$y = 72$
$z = 22$
$p = 25$ (2)

Question 17
17.1 Breadth = 4 m
17.2 Area = length × breadth
17.3 Perimeter = 2(length + breadth)
17.4 Perimeter = 26 m (4)

Question 18
18.1 (2)

Fraction	Denominator of 10	Percentage
$\frac{3}{5}$	$\frac{6}{10}$	60%
$\frac{72}{180}$	$\frac{4}{10}$	40%
$\frac{18}{30}$	$\frac{6}{10}$	60%

18.2 $\frac{3}{5}$ and $\frac{18}{30}$ (1)

Review Copy

Question 19

19.1 5 terms (1)

19.2 –5 (1)

19.3 A polynomial (1)

19.4 $3a^6 - 8a^3 - 2a^2 - 5a + 6$ (1)

19.5 6 (1)

Question 20

20.1 $3ab + 2bc + ab + 4dc - 5bc - 3bc = 4ab - 6bc + 4dc$ (2)

20.2 $(3x^2 + 7xy - 8) + (-5x^2 - 12xy + 4) = -2x^2 - 5xy - 4$ (3)

Question 21

21.1 $\sqrt{36a^{36}} = 6a^{18}$ (2)

21.2 $\dfrac{2pq^2 \times 3p^8q^6}{6p^5q^4} = \dfrac{q^4}{p}$ (2)

Question 22

If $a = -2$ and $b = 4$, determine the following value:

22.1 $a(b + a)^b = -2(4 - 2)^4 = -32$ (2)

22.2 $\dfrac{6a \times 5b}{ab} = \dfrac{6(-2) \times 5(4)}{(-2)(4)} = 30$ (2)

Question 23

23.1 $8x = 24$

$x = 3$ (1)

23.2 $\dfrac{8}{2x} = 9$

$x = -\dfrac{8}{9}$ (2)

Question 24

$x + 18 = 26$

$x =$ goals (3)

Review Copy

Question 25

25.1 Construction ΔABC with BC = 8 cm, AC = 6 cm and $\hat{C} = 90°$, AB = 10 cm. (3)

25.2 Construction of the bisector of $\hat{A}$ so that it cuts BC in D, $D\hat{A}C = 63°$. (3)

25.3 Construction of the perpendicular bisector of BC so that it cuts AB in E. (2)

25.4 Construction of a circle with centre E and radius equal to DC. (2)

Question 26

26.1.1 The sum of the interior angles of a triangle is equal to 180°. (1)

26.1.2 The exterior angle of a triangle is equal to the sum of the opposite interior angles. (1)

26.2 $4y - 25° = 2y + 68° - y$

$3y = 93°$

$y = 31°$

The exterior angle is 99°. (3)

26.3.1 1 = 34° (ext angle triangle PQR)
2 = 107° (ext angle triangle PRS)
3 = 40° (ext angle triangle PST) (3)

26.3.2 No because their angles are not equal. (2)

Question 27

27.1.1 The sum of the interior angles of a quadrilateral is equal to 360°. (1)

27.1.2 The opposite angles of a parallelogram are equal. (1)

27.2 $2p + 14° = 86° - p$

$3p = 72°$

$p = 24°$

$\hat{D} = 180° - (2 \times 24° + 14°) = 118°$ $\hat{D} = 180° - (2 \times 24° + 14°) = 118°$ (4)

27.3 1 = 40° (sum angles isosceles triangle)
2 = 40° (alternate angles, PQ || VR)
3 = 80° (sum angles triangle)
4 = 120° (corresponding angles, TS || VR)
5 = 30° (sum angles triangle) (5)

Review Copy

June Exemplar exam Memo

Question 1

1.1 Peter's amount = $\frac{5}{20} \times 188\ 000 = R47\ 000$

Mpho's amount = $\frac{6}{20} \times 188\ 000 = R56\ 400$

Davis' amount = $\frac{9}{20} \times 188\ 000 = R84\ 600$ (3)

1.2 Total including VAT = R432,30

VAT = $\frac{14}{100} \times 432{,}30 = R60{,}522$

Total excluding VAT = R432,30 – R60,52 = R371,78 (3)

1.3 Original selling price = $\frac{1\ 399{,}50}{0{,}68} = R2\ 058{,}09$ (2)

Question 2

2.1.1 –9 786; –6 986; –6 865; –7; 3 (2)

2.1.2 –10 000; –1 000; –1; 100; 100 000 (2)

2.2.1 $(-1\ 200) + (670) - (-450) = -80)$ (2)

2.2.2 $(-9)[(-3) + 4] + (-6)[(-2) - (-1)] = -3$ (2)

2.2.3 $(-30) \div (-15) + (-4) = -2$ (2)

2.2.4 $(-3)^2 - (-2)^3 + 5^2 = 42$ (2)

Question 3

3.1 Write the following in expanded form.

3.1.1 $7^4 = 7 \times 7 \times 7 \times 7$ (1)

3.1.2 $22^1 = 22$ (1)

3.1.3 $4^6 = 4 \times 4 \times 4 \times 4 \times 4 \times 4$ (1)

3.1.4 $a^7 = a \times a \times a \times a \times a \times a \times a$ (1)

3.2.1 7^3; 5^3; 10^2; 3^4; 4^3 (2)

3.2.2 12^2; 9^2; 19^0; $(-1)^7$ (2)

Review Copy

3.3.1 $56\ 000\ 000\ 000\ 000 = 5{,}6 \times 10^{13}$ (1)

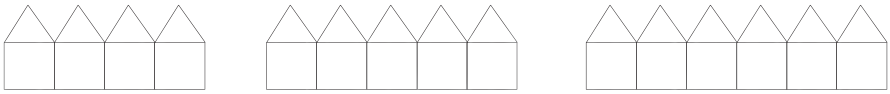
3.3.2 $2\ 300 = 2{,}3 \times 10^{3}$ (1)

3.4.1 $4{,}54 \times 10^{12} = 4\ 540\ 000\ 000\ 000$ (1)

3.4.2 $8{,}9 \times 10^{7} = 89\ 000\ 000$ (1)

Question 4

4.1.1



Term 4 Term 5 Term 6 (1)

4.1.2 (2)

Term number	1	2	3	4	5	10	15
Number of matches	6	11	16	21	26	51	76

4.1.3 By adding five matches to the previous term. (1)

4.1.4 $T_n = 5n + 1$ (1)

4.1.5 501 matches (1)

4.1.6 The 45th term will have 226 matches. (1)

4.2.1 $V = u + at$

$at = V - u$

$t = \frac{V - u}{a}$ (2)

4.2.2 $t = \frac{15 - 3}{4} = 3$ (1)

Question 5

5.1.1 $\frac{3x + 7}{2}$ (1)

5.1.2 $\frac{x + x + 1}{3} = 6$ or $2\frac{x + 1}{3} = 7$ (1)

5.1.3 $\frac{12x}{2} + 15$ (1)

5.2.1 $6a(a^2 + 3a - 4) - a^2(4 - a) = 6a^3 + 18a^2 - 24a - 4a^2 + a^3$

$= 7a^3 + 14a^2 - 24a$ (2)

Review Copy

5.2.2 $(-3n^2 - 4nm + 5m^2)(2n^2m^3) = -6n^4m^3 - 8n^3m^4 + 10n^2m^5$ (2)

5.2.3 $\frac{5x^2 - 10x + 25}{5x} = x - 2 + \frac{5}{x}$ (2)

5.2.4 $\frac{2a^8b^4 + 6ab^7}{2ab^6} + \frac{9a^4b^8}{3(a^2b^4)^2} = \frac{a^2}{b^2} + 3b + 3$ (4)

5.3.1 $y = 4x^3 - 3x + 1 = 4(-2)^3 - 2(-2) + 1$

$= -25$ (3)

5.3.2 $y = \sqrt[3]{27} + 3a + 2a^3 = 3 - 3(4) + 2(4)^3$

$= 119$ (3)

5.4.1 $3x - 3 + x = x - 9$

$3x = -6$

$x = -2$ (2)

5.4.2 $6x + 5 = 3x - 13$

$3x = -18$

$x = -6$ (2)

5.4.3 $5a^2 - 3a^2 - 6 = -4$

$2a^2 = 2$

$a^2 = 1$

$a = \pm 1$ (3)

Review Copy

Question 6

6.1 Accurate construction of line AB as the perpendicular bisector of CD. (3)

6.2 Accurate construction of rhombus DEFG, with sides equal to 5 cm and DEFG equal to $D\hat{E}F = 135°$ (4)

6.3 Accurate construction without a protractor of an angle of 45°. (2)

Question 7

7.1 $x = 52°$ (alternate angles, PR||ST

$5 = 52°$ (sum angles triangle SQT)

$2 = 76°$ (alternate angles, PR||ST (3)

7.2 $x = 105°$ (corresponding angles, GH||IJ)

$y = 88°$ (straight line angles and alternate angles,

PR||ST or co-interior angles)

$z = 88°$ (vertical opposite angles) (3)

7.3 $w = 24°$ (alternate angles, BC||EF)

$x = 156°$ (straight line angles and corresponding angles, AD||BC or co-interior angles)

$y = 24°$ ($y = w$, given)

$z = 156°$ (straight line angles and corresponding angles, HD||IE or co-interior angles (4)

7.4 $z = 45°$ (sum angles triangle TRU)

$y = 70°$ (alternate angles, QT||RU)

$x - 5° = 45°$ (straight line angles)

$x = 50°$ (4)

7.5 $w + w + 14° = 180°$ (co–interior angles, JK||CD $w = 83°$

$x = 83°$ (corresponding angles, CD||EF))

$y = 83°$ (corresponding angles, AH||BI)

$z = 97°$ (straight line angles) (3)

7.6 $w = 62°$ (alternate angles, UV||WX)

$x = 28°$ (sum angles triangle)

$y = 28°$ (sum angles triangle)

$z = 34°$ (alternate angles, WX||YZ) (5)

Review Copy

Assignment: Option 1 Common and decimal fractions

1.1 $\frac{2}{3} = \frac{6}{9} = \frac{14}{21} = \frac{150}{225} = \frac{100}{150}$ (4)

1.2 $\frac{4}{5} = \frac{8}{10} = \frac{80}{100} = \frac{800}{1\,000}$ (3)

[7]

2 0,8 0,80 0,800 (3)

[3]

3.1 0,125 0,375 0,96 (3)

3.2 0,333 recurring 0,833 recurring 0,333 recurring (3) [6]

4.1 $\frac{45}{100} = \frac{9}{20}$ $\frac{86}{100} = \frac{43}{50}$ $\frac{2}{10} = \frac{1}{5}$ $\frac{625}{1\,000} = \frac{5}{8}$ (4) [4]

5.1 $\frac{2}{5} + \frac{3}{10} = \frac{7}{10}$ $\frac{3}{7} + \frac{3}{5} = \frac{36}{35} = \frac{11}{35}$ $1\frac{3}{4} + 3\frac{3}{4} = 4\frac{3}{4} + \frac{3}{4} = 5\frac{1}{2}$ (3)

5.2 $\frac{4}{9} - \frac{1}{3} = \frac{1}{9}$ $\frac{19}{21} - \frac{1}{6} = \frac{31}{42}$ $5\frac{3}{4} - 2\frac{1}{2} = 3\frac{3}{4} - \frac{1}{2} = 3\frac{1}{4}$ (3)

5.3 $\frac{2}{3} \times \frac{9}{10} = \frac{3}{5}$ $\frac{27}{35} \times \frac{7}{18} = \frac{3}{10}$ $3\frac{1}{4} \times \frac{12}{13} = \frac{3}{1} = 3$ (3)

5.4 $\frac{1}{4} \div \frac{2}{5} = \frac{5}{8}$ $\frac{1}{3} \div \frac{5}{6} = \frac{2}{5}$ $\frac{5}{12} \div \frac{15}{16} = \frac{4}{9}$ (3)

[12]

6.1 0,23 + 5,7 = 5,93 19,01 – 3,22 = 15,79 (2)

6.2 1,3 × 3,01 = 3,913 27,99 × 0,36 = 10,0764 (2)

6.3 1,536 ÷ 0,48 = 3,2 1,775 ÷ 0,25 = 7,1 (2)

[6]

7

Common fractions	Decimal fractions	Percentages
$\frac{23}{50}$	0,46	46%
$\frac{49}{100}$	0,49	49%
$\frac{123}{100}$	1,23	123%

(6)

[6]

8

Input	Rule	Output	Decimal
$\frac{4}{5}$	$\times \frac{2}{2}$	$\frac{8}{10}$	0,8
$\frac{72}{80}$	$\div \frac{8}{8}$	$\frac{9}{10}$	0,9
$\frac{74}{200}$	$\div \frac{2}{2}$	$\frac{37}{100}$	0,37

(6)

[6]

Total marks [50]

Review Copy

Assignment: Option 2 Solving triangles

1.1 $AC = \sqrt{1^2 + 1^2} = \sqrt{2}$

$AD = \sqrt{(\sqrt{2})^2 + 1^2} = \sqrt{3}$

$AE = R(\sqrt{3})^2 + 1^2 = \sqrt{4} = 2$

$AF = \sqrt{2^2 + 1^2} = \sqrt{5}$ (4)

1.2 $AG = \sqrt{(\sqrt{5})^2 + 1^2} = \sqrt{6}$

$AH = \sqrt{(\sqrt{6})^2 + 1^2} = \sqrt{7}$ (2)

[6]

2.1 $PR = \sqrt{20^2 + 20^2} = 28{,}28$ m (2)

2.2 $RT = \sqrt{(28{,}8)^2 + 10^2} = 30{,}49$ m (3)

[5]

3.1 $MN = \sqrt{2^2 - \sqrt{3^2}} = 1$ cm (2)

3.2 In ΔKLM and ΔOMN:
$KL = OM$ (given)
$LM = NM$ (given)
$\hat{L} = O\hat{M}N = 90°$ (ΔKLM and ΔOMN lie on the same line)
$\therefore \Delta KLM \equiv \Delta OMN$ (SAS) (3)

[5]

4.1 AB is parallel to DE (corresponding angles are equal)
$\hat{A} = E\hat{D}C$ (corresponding angles, AB || DE)
$\hat{C}$ is common
$\therefore \Delta ABC \,|||\, \Delta DEC$ (AAA) (3)

4.2 $DE = \sqrt{20^2 - 16^2} = 12$ cm (2)

4.3 $BE = 8$ cm (2)

4.4 $AB = 36$ cm (2)

[9]

5.1 $\hat{R}_1 = \hat{R}_2$ (vertical opposite angles)
$\hat{P} = \hat{S}$ (alternate angles, PQ || ST)
$\hat{Q} = \hat{T}$ (alternate angles, PQ || ST)
$\therefore \Delta PQR \,|||\, \Delta STR$ (AAA) (3)

5.2 $\frac{x}{9} = \frac{8}{12}$
$x = 6$ m (2)

[5]

Total marks [30]

Review Copy

Project: Option 1 Pythagoras and circles

Part 1:

Use this rubric to assess Part 1.

Criteria	5	4	3	2	1
The life of Pythagoras					
Pythagoras' work with triangles					
The importance of Pythagoras in mathematics					
Overall neatness and correctness of report					

Part 2:

Use the memorandum below to assess this investigation.

1.1 $A = \frac{1}{2} \times \pi \times 10^2 = 50\pi$ (2)

1.2 $A = \frac{1}{2} \times \pi \times 8^2 = 32\pi$ (2)

1.3 $A = \frac{1}{2} \times \pi \times 6^2 = 18\pi$ (2)

1.4 $50\pi : 32\pi : 18\pi = 25 : 16 : 9$ (3)

1.5 $32\pi + 18\pi = 50\pi$. This is the same as the answer to 1.1. (2)

2.1 $A = \frac{1}{2} \times \pi \times 13^2 = \frac{169\pi}{2}$ (2)

2.2 $A = \frac{1}{2} \times \pi \times 12^2 = 72\pi$ (2)

2.3 $A = \frac{1}{2} \times \pi \times 5^2 = \frac{25\pi}{2}$ (2)

2.4 $\frac{169\pi}{2} : 72\pi : \frac{25\pi}{2} = 169 : 144 : 25$ (3)

2.5 $72\pi + \frac{25\pi}{2} = \frac{169\pi}{2}$. This is the same as the answer to number 2.1. (2)

3 The area of a semicircle on the hypotenuse of any right-angled triangle is equal to the sum of the areas of the semicircles on the other two sides of the triangle. (2)

4.1 Learners choose their own positive integers p and q with $p > q$, for example: $p = 6$ and $q = 5$ (1)

4.2 Learners calculate the values of a, b and c, for example:

$a = p^2 - q^2 = 36 - 25 = 11$; $b = 2pq = 60$;

$c = p^2 + q^2 = 36 + 25 = 61$ (3)

Review Copy

4.3 Learners must test their Pythagorean triplet, for example:

$a^2 + b^2 = 11^2 + 60^2 = 3\ 721$

$c^2 = 61^2 = 3\ 721$

$\therefore a^2 + b^2 = c^2$ (2)

Part 1 marks: 20

Part 2 marks: 30

Total marks [30]

Review Copy

Project: Option 2 Completing a data cycle

Step 1: Questionnaire (example)
(20 questionnaires had to be completed)

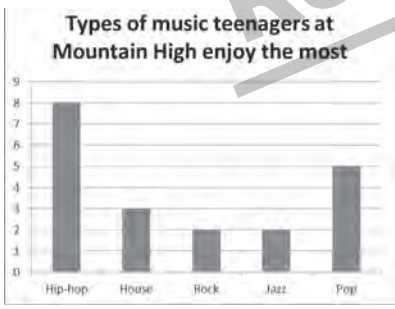
Which type of music do you enjoy most?	
Mark a cross (X) next to the type of music that you enjoy most.	
Hip-hop	
House	
Rock	
Jazz	X
Pop	
Thank you for participating in this questionnaire.	

Step 4: Frequency and tally table (example) (reflect the questionnaire results)

Music choices of teenagers at Mountain High		
Music type	Tally	Frequency (f)
Hip-hop	~~IIII~~ III	8
House	III	3
Rock	II	2
Jazz	II	2
Pop	~~IIII~~	5
Total		20

Review Copy

Step 5: Bar graph (example)



Step 6: Report (example)
Most of the learners enjoy listening to hip-hop music.
Pop music is the next popular choice.
Only three learners chose house music.
The same amounts of learners listen to rock and jazz music. These selections are also the least in the survey.

Use the rubric in the Learner's Book page 442 to assess this project.

Total marks [50]

Test 3

Time: 1 hour **Marks: 50**

Question 1

1.1 Calculate:

1.1.1 $1\frac{3}{4} \times 2\frac{3}{7}$ (2)

1.1.2 $4\frac{3}{5} \div 2\frac{6}{7}$ (3)

1.2 Determine without a calculator:

1.2.1 $\frac{\sqrt{25}}{\sqrt[3]{64}}$ (2)

1.2.2 $\sqrt{\frac{81}{100}} + \sqrt[3]{\frac{27}{125}}$ (2)

[9]

Question 2

2.1 Copy and complete the table.

Common fraction	Decimal fraction	Percentage
$\frac{3}{4}$		
	0,125	
		$66\frac{2}{3}\%$
	0,4	

(4)

2.2 Calculate without a calculator.

2.2.1 9,145 × 0,654 (2)

2.2.2 8,541 ÷ 9,65 (2)

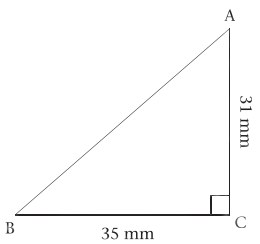
[8]

Review Copy

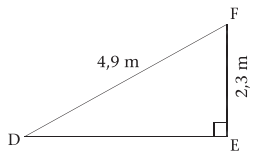
Question 3

3.1 Solve for the missing sides in the following triangles.

3.1.1



3.1.2



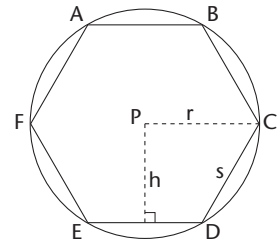
3.2 A lighthouse, that is 5 m tall, stands on the shore and shines its light on a boat that is 250 m away from the shore. Calculate the length of the beam of light. Use a sketch to help you.

[7]

CHAPTER 20

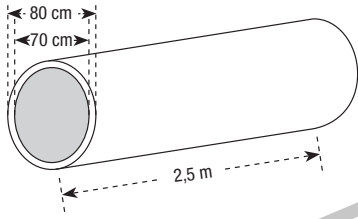
Question 4

4.1 Look at the diagram on the right.
In the diagram r = 13, h = 13 and s = 11



4.1.1 Calculate the circumference of the circle. (1)
4.1.2 Calculate the perimeter of the hexagon. (1)
4.1.3 Calculate the area of the circle. (2)
4.1.4 Calculate the area of the hexagon. (3)

4.2 A factory makes large concrete pipes and plans to make 1 000 storm water pipes like the one below. The inside diameter of the pipe is 70 cm, the outside diameter 80 cm and the length 2,5 m. The production manager wants to know how much concrete makes one pipe so that they can work out how much concrete makes one pipe, so that they can determine how much concrete is needed to make any number of pipes.



4.2.1 Calculate the volume of the solid part of one pipe. Express your answer in dm^3. (3)
4.2.2 Calculate how much concrete they will need to make 1 000 pipes. Write your answer in m^3 and round off to the nearest 100. (3)

[13]

Review Copy

Question 5

Paliswa wanted to find out how tall people in her high school were. She recorded the data of 30 pupils. The table below shows her information. She recorded the heights in metres.

1,66	1,32	1,89	1,33	1,65	1,54
1,72	1,12	1,64	1,19	1,78	1,65
1,43	1,62	1,34	1,29	1,73	1,80
1,32	1,71	1,73	1,65	1,54	1,43
1,76	1,87	1,65	1,63	1,59	1,71

5.1 Organise Paliswa's data in a stem and leaf plot. (3)
5.2 Determine the modal interval. (1)
5.3 Determine the range of the data. (1)
5.4 Determine the median and mean of the data. (3)
5.5 Draw a histogram of the data. (3)
5.6 Do you think Paliswa's findings are representative of all learners in her school? Give a reason for your answer. (2)

[13]

Test 3 memo

Question 1

1.1.1 $1\frac{3}{4} \times 2\frac{3}{7} = \frac{7}{4} \times \frac{17}{7} = \frac{17}{4} = 4\frac{1}{4}$ (2)

1.1.2 $4\frac{3}{5} \div 2\frac{6}{7} = \frac{23}{5} \times \frac{7}{20} = \frac{161}{100} = \frac{161}{100}$ (3)

1.2.1 $\frac{\sqrt{25}}{\sqrt[3]{64}} = \frac{5}{4}$ (2)

1.2.2 $\sqrt{\frac{81}{100}} + \sqrt[3]{\frac{27}{125}} = \frac{9}{10} + \frac{3}{5} = \frac{15}{10} = 1\frac{1}{2}$ (2)

Question 2

2.1

Common fraction	Decimal fraction	Percentage
$\frac{3}{4}$	0,75	75%
$\frac{1}{8}$	0,125	12,5%
$\frac{2}{3}$	$0,\dot{6}$	$66\frac{2}{3}$%
$\frac{2}{5}$	0,4	40%

(4)

2.2.1 $9,145 \times 0,654 = 5,98083$ (2)

2.2.2 $8,541 \div 9,65 = 0,8850777202$ (2)

Review Copy

Question 3

3.1.1 $AB = \sqrt{35^2 + 31^2} = 46,75$ mm (2)

3.1.2 $DE = \sqrt{4,9^2 - 2,3^2} = 4,33$ m (2)

3.2 Length $= \sqrt{5^2 + 250^2} = 250,05$ m (3)

Question 4

4.1.1 Circumference = $2\pi r = 2 \times r \times 13 = 81{,}68$ mm (1)

4.1.2 Perimeter = $6 \times 13 = 78$ mm (1)

4.1.3 Area = $\pi r^2 = \pi \times 13^2 = 530{,}93$ mm^2 (2)

4.1.4 Area = $6 \times \frac{1}{2} \times 13 \times 11 = 429$ mm2 (3)

4.2.1 Volume = $\pi \times 0{,}42 \times 2{,}5\ 2\ \pi \times 0{,}352 \times 2{,}5 = 0{,}29452$ m^3 = 294,52 dm^3 (3)

4.2.1 $0{,}29452 \times 1\ 000 = 294{,}52$ m^3 = 300 m^3 (3)

Question 5

5.1 (3)

Stem	Leaf
1,0	
1,1	2 9
1,2	9
1,3	2 2 3 4
1,4	3 3
1,5	4 4 9
1,6	2 3 4 5 5 5 5 6
1,7	1 1 2 3 3 6 8
1,8	0 7 9

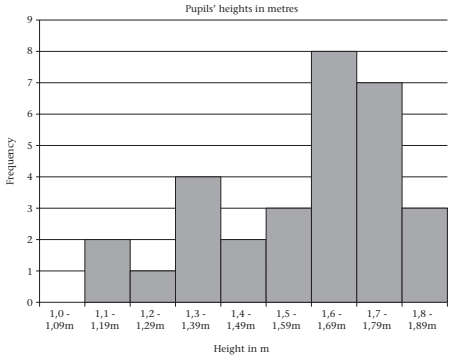
5.2 Modal height = 1,65 m (1)

5.3 Range = 1,89 – 1,12 = 0,77 m (1)

5.4 Median = 1,645 m

Mean = $\frac{47{,}29}{30} = 1{,}58\frac{47{,}29}{30} = 1{,}58$ m (3)

5.5 (3)



5.6 Not necessarily – it depends on how Paliswa selected her sample. If the sample was chosen randomly across all grades then the data would represent her school, but if only from a few grades then the data would not represent her school.
OR Yes because the heights cover a wide range. (2)

Review Copy

Assignment: Option 1 Algebra

1.1 $\frac{x}{4} + \frac{x}{3}$

$= \frac{3x + 4x}{12}$

$= \frac{7x}{12}$ (2)

1.2 $\frac{4}{x} + \frac{2}{x} = \frac{6}{x}$ (1)

1.3 $\frac{7}{2y} - \frac{2}{y}$

$= \frac{7 - 4}{2y}$

$= \frac{3}{2y}$ (2)

1.4 $\frac{4x}{5} - \frac{x}{10}$

$= \frac{8x - x}{10}$

$= \frac{7x}{10}$ (2)

1.5 $5\frac{1}{5}x + 2\frac{1}{2}x = \frac{26x}{5} + \frac{5x}{2} = \frac{52x}{10} + \frac{52x}{10} = \frac{77x}{10} = 7\frac{7}{10}x$ (4)

[11]

2.1 $\frac{24x^{10}}{6x}$

$= 4x^9$ (1)

2.2 $\frac{12a^7b^6}{3a^2b^4}$

$= 4a^5b^2$ (1)

2.3 $\frac{3x^3y^5}{6x^2y^2}$

$= \frac{xy^3}{2}$ (1)

2.4 $\frac{6a^4b^5c^4}{12a^2bc} = \frac{a^2b^4c^3}{2}$ (1)

[4]

3.1 $5(2x + 3y - z) = 10x + 15y - 5z$ (1)

3.2 $5ac(3a + 2c) = 15a^2c + 10a^2c$ (1)

3.3 $2x^2(x^2 - 3x + 5) = 2x^4 - 6x^3 + 10x^2$ (1)

[3]

Review Copy

4.1 $3x + 2(x - 4) = 11$
$3x + 2x - 8 = 11$
$5x = 11 + 8$
$x = 19 \div 5$
$x = 3\frac{4}{5}$

4.2 $7 - 3x = 5 - 2x$
$-3x + 2x = 5 - 7$
$-x = -2$
$x = 2$
$y = -\frac{1}{3}$

4.3 $2(3y - 1) = 3(y - 1)$
$6y - 2 = 3y - 3$
$6y - 3y = -3 + 2$
$3y = -1$

(2)(2)(3)

4.4 $\frac{(2x - 1)}{3} = \frac{x}{2}$
$2(2x - 1) = 3x$
$4x - 2 = 3x$
$4x - 3x = 2$
$x = 2$

4.5 $\frac{(a + 3)}{2} = \frac{(a - 4)}{5}$
$5(a + 3) = 2(a - 4)$
$5a + 15 = 2a - 8$
$5a - 2a = -15 - 8$
$3a = -23$
$a = -7\frac{2}{3}$

4.6 $\frac{4}{z} + 5 = 3$
$\frac{4}{z} = 3 - 5$
$= -2$
$\frac{z}{4} = -\frac{1}{2}$
$z = 4(-\frac{1}{2})$
$z = -2$

(2)(2)(3)

[14]

5.1 Let the number be x
$15 - 2x = 3x + 4$
$-2x - 3x = 4 - 15$
$-5x = -11$
$x = 2\frac{1}{5}$

5.2 $2(x + 8 + x) = 48$ cm
$2x + 16 + 2x = 48$
$4x = 48 - 16$
$x = 32 \div 4 = 8$
breadth is 8c
length is $8 + 8 = 16$ cm

5.3.1 $3x + 4 = 5x - 2$
$3x - 5x = -2 - 4$
$-2x = -6$
$x = 3$

5.3.2 $BC = 4(3) + 1$
$= 13$

(4)(4)(2)(2)

[12]

6.1 $6\frac{1}{x} = 18$
$\frac{1}{x} = 18 \div 6 = 3$
$\frac{x}{1} = \frac{1}{3}$
$x = \frac{1}{3}$

6.2 $10(\frac{1}{x}) + 7 = 12$
$\frac{10}{x} = 12 - 7 = 5$
$10 = 5x$
$x = 2$

(3)(3)

[6]

Total marks [50]

Review Copy

Assignment: Option 2 Graphs

1.1 Section BC: Musa stopped for $\frac{1}{4}$ hour / 15 minutes (3)

Section CD: Musa cycled 20 km in 1 hour (3)

Section DE: Musa cycled 10 km in $\frac{1}{4}$ hour / 15 minutes (3)

Section EF: Musa stopped for $\frac{3}{4}$ hour / 45 minutes (3)

Section FG: Musa cycled back 30 km in $\frac{1}{2}$ hour / 30 minutes (3)

Section GH: Musa cycled 20 km back in $1\frac{1}{4}$ hour / 75 minutes (3)

1.2 Total distance home and back = 50 km + 50 km = 100 km (2)

[20]

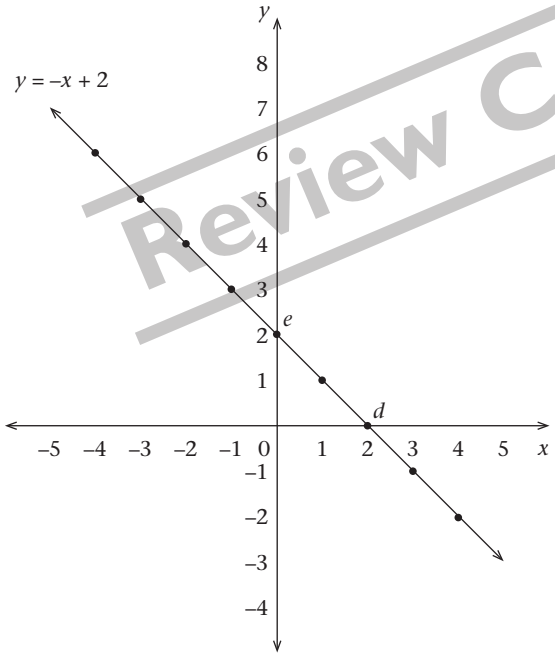
2

x	–4	–3	–2	–1	0	1	2	3	4
y = –x + 2	6	5	4	3	2	1	0	–1	–2

(2)

2.1 Cartesian plane (4)

2.2 Plot points and draw and label graph (10)



Review Copy

2.3 Co-ordinates are (2; 0) (2)

2.4 Co-ordinates are (0; 2) (2)

[20]

Total marks [40]

Investigation 2

The following investigations can be used as formal assessment tasks for the formal Programme of Assessment. Both these investigations form part of the curriculum for the year and both should be covered in class; however only one of these two investigations needs to be recorded and assessed as part of the formal Programme of Assessment. You as the teacher can choose which to use for this purpose.

1. Investigation: The effect of enlargement and reduction on the area and perimeter of geometric shapes

Please find this investigation in the Learner's Book page 399 and Teacher's Guide page 312. If using this investigation for formal assessment please use the following rubric to assess learners.

	Criteria	5	4	3	2	1
Enlargement	Did the learner correctly multiply the coordinates of the vertices by 2?					
	Did the learner correctly calculate the new lengths of the sides?					
	Did the learner accurately calculate the perimeter of the new shape?					
	Did the learner accurately calculate the area of the new shape?					
	Did the learner provide a correct and insightful statement as to how doubling the coordinates impacts on the area and perimeter of a shape?					
Reduction	Did the learner correctly divide the coordinates of the vertices by 2?					
	Did the learner correctly calculate the new lengths of the sides?					
	Did the learner accurately calculate the perimeter of the new shape?					
	Did the learner accurately calculate the area of the new shape?					
	Did the learner provide a correct and insightful statement as to how halving the coordinates impacts on the area and perimeter of a shape?					

Review Copy

Total marks [50]

2. Investigation: The relationship between the number of vertices, faces and edges of the five Platonic solids

Please find this investigation in the Learner's Book page 407 and Teacher's Guide page 319. If using this investigation for formal assessment please use the following rubric to assess learners.

Criteria	5	4	3	2	1
Did the learner accurately identify the net, number of faces, vertices and edges of a cube?					
Did the learner accurately identify the net, number of faces, vertices and edges of an octohedron?					
Did the learner accurately identify the net, number of faces, vertices and edges of a tetrahedron?					
Did the learner accurately identify the net, number of faces, vertices and edges of a dodecahedron?					
Did the learner accurately identify the net, number of faces, vertices and edges of a icosahedron?					
Did the learner accurately identify the relationship between the number of faces, vertices and edges (Euler's law)?					
Did the learner discover for which other 3D forms Euler's law can be applied?					
Did the learner provide a clear and concise statement about Euler's law?					

Total marks [40]

Review Copy

December Exemplar exam Memo

Question 1

1.1.1 $R229 - \frac{37}{100} \times R229 = R144{,}27$

1.1.2 $R5\ 999 - \frac{37}{100} \times R5\ 999 = R3\ 779{,}37$

1.1.3 $R899 - \frac{37}{100} \times R899 = R566{,}37$

1.1.4 $R8\ 695 - \frac{37}{100} \times R8\ 695 = R5\ 477{,}85$ (4)

1.2.1 $(-4) + (-8) \times (-3) = 20$ (2)

1.2.2 $\frac{((-4)^2 + \sqrt{81})}{((-5))} = -5$ (2)

1.2.3 $\frac{(6^2 + (-8) - (-8))}{\sqrt{169}} = \frac{30}{13}$ (2)

1.3.1 $12x^4y^2z^3 \times 3x^2y^5z^0 = 36x^6y^7z^3$ (2)

1.3.2 $(3a^3b^5)^2 = 9a^6b^{10}$ (2)

1.3.3 $\frac{(5a^4b^6c^5)}{(10a^2c^8)} = \frac{(a^2b^6c^2)}{2}$ (2)

Question 2

2.1.1. (2)

No. of hours worked	0	1	2	3	4	5
Amount of money earned (R)	100	175	250	325	400	475

2.1.2 Amount = $100 + 75h$ (1)

2.1.3. $700 = 100 + 75h$

$75h = 600$

PQ = hours (2)

Review Copy

Question 3

3.1.1 $4a(3a^2 - a + 1) - 4(a^3 + 1) = 12a^3 - 4a^2 + 4a - 4a^3 - 4$

$= 8a^3 - 4a^2 + 4a - 4$ (2)

3.1.2 $\frac{(3x^2(x - y) + 6y(x + y))}{3xy} = \frac{(3x^3 - 3x^2y + 6xy + 6y^2)}{3xy} = \frac{x^2}{y - x} + \frac{2 + 2y}{x}$ (2)

3.1.3 $\sqrt{(64n^6)} + (27n^9) = 8n^3 + 3n^3 = 11n^3$ (2)

3.2.1 $4x + 4 - 3x = 65$

$x = 61$ (2)

3.2.2 $12(x - 7) = 26 - 2$

$12x - 84 = 24$

$12x = 108$

$x = 9$ (2)

3.2.3 $4a(a - 2) - 4a^2 = 56$

$4a^2 - 8a - 4a^2 = 56$

$-8a = 56$

$a = -7$ (2)

Question 4

4.1 Construction of JK parallel to GH. (2)

4.2 Construction of triangle DEF with $\hat{E} = 90°$, and DE = EF = 5 cm. (2)

Question 5

5.1 In ΔPQR and ΔTSR:

PQ = TS (given)

$\hat{Q} = \hat{S}$ (alt angles, PQ || TS)

$\hat{P} = \hat{Q}$ (alt angles, PQ || TS)

∴ ΔPQR = ΔTSR (SAA)

NOTE: learners can also use vertically opposite angles for the angle at R. (3)

5.2 If two parallel lines are cut by a transversal, the alternate and corresponding angles are equal. (1)

5.3 $2x - 8° = 50°$ (corresp angles, parallel lines)

$x = 29°$

$y = 10°$ (alt angles, parallel lines) (2)

5.4 $a = 60°$ (co-interior angles, parallel lines OR alt angles and straight line angles)

$b = 58°$ (alt angles, parallel lines)

$c = 62°$ (straight line angles)

$d = 62°$ (alt angles, parallel lines)

$e = 60°$ (corresp angles, parallel lines)

$f = 60°$ (alt angles, parallel lines) (3)

5.5 $k = \sqrt{10^2 - 6^2} = 8$ (Pythagoras)

$i = \sqrt{0^2 - 15^2} = 17$ (Pythagoras) (3)

Review Copy

Question 6

6.1 $1\frac{2}{3} \times 2\frac{3}{4} = \frac{5}{3} \times 1\frac{1}{4} = 5\frac{5}{12} = 4\frac{7}{12}$ (2)

6.2 $5\frac{6}{7} \div 2\frac{2}{14} = 4\frac{1}{7} \times \frac{14}{30} = \frac{41}{15} = 2\frac{11}{15}$ (2)

6.3 $98{,}765 \times 0{,}45 = 44{,}44425$ (2)

6.4 $0{,}654 \div 2{,}34 = 0{,}2(794871)$ (2)

Question 7

7.1 $AM = \sqrt{(8^2 - 6^2)} = 5{,}29$ m (2)

7.2 Surface area $= 2 \times 8 \times 15 + 2 \times \frac{1}{2} \times 12 \times 5{,}29 = 303{,}48$ m^2 (2)

7.3 Volume $= \frac{1}{2} \times 12 \times 5{,}29 \times 15 = 476{,}10$ m^3 (2)

Question 8

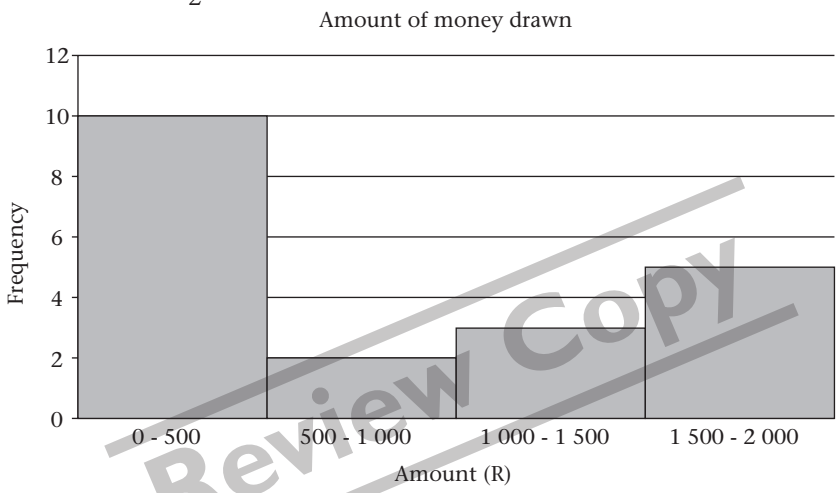
8.1 (3)

Rands	Frequency
R0 ≤ A < R500	10
R500 ≤ A < R1 000	2
R1 000 ≤ A < R1 500	3
R1 500 ≤ A ≤ R2 000	5

8.2 Mean $= \frac{(15\ 950)}{20} =$ R797,50

Median $= \frac{(350 + 700)}{2} =$ R525 (2)

8.3 (2)



Review Copy

8.4 They can make sure they have the correct notes available for the most popular amounts. (1)

Question 9

9.1 The input value multiplied by negative three and then added to two and then multiplied by negative one gives the output value. (1)

9.2 $y = -1(-3x + 2) = 3x - 2$ (2)

9.3 $43 = 3x - 2$

$3x = 45$

$x = 15$ (1)

Question 10

10.1.1 $3(x - 7) = 2(x - 22)$

$3x - 21 = 2x - 44$

$x = -23$ (2)

10.1.2 $5a(a - 1) = 4a^2 - 5a + 9$

$5a^2 - 5a = 4a^2 - 5a + 9$

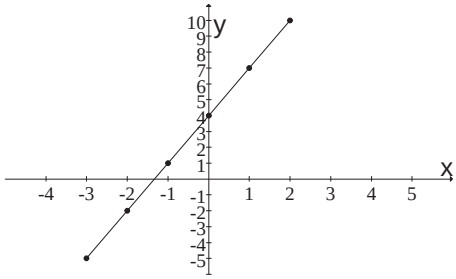
$a^2 = 9$

$a = \pm 3$ (2)

10.2.1 (2)

x	−3	−2	−1	0	1	2
y	−5	−2	1	4	7	10

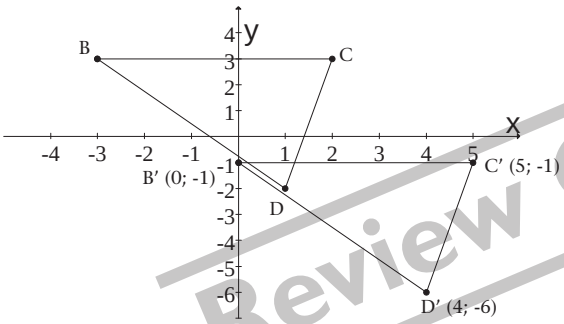
10.2.2 (2)



Question 11

11.1 and 11.2 (2)

(2)



Review Copy

Question 12

12.1 Cube, tetrahedron, octahedron, dodecahedron, icosahedron (2)

12.2 Each face is the same regular polygon and the same number of faces meet at each vertex. (1)

12.3 A prism is a 3D shape with two identical polygon bases, joined by rectangular faces.
A pyramid is also a 3D shape, but it has one polygon base with at least three triangular faces. (1)

Question 13

13.1 P (girl's name) = $\frac{1}{3}$ (1)

13.2 P (blue eyes) = $\frac{1}{4}$ (1)

13.3 P (girl with blue eyes) = $\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$ (2)

December exam

Time: 2 hours **Marks: 100**

Question 1

Calculate: (2)

1.1 $\frac{10}{21} + \frac{2}{7} = a$

1.2 $\frac{3}{5} \times \frac{5}{6} = c$

1.3 $\frac{2}{5}\left(\frac{9}{10} + \frac{3}{5}\right) = f$

1.4 $3\frac{2}{3} \div 2\frac{1}{5} = h$

Question 2

Calculate: (2)

2.1 $\sqrt[3]{\frac{27}{64}} = q$

2.2 $\sqrt[3]{\frac{8}{125}} = r$

Question 3

Mannie works in a bank. He earns $\frac{1}{2}$ of what Themba earns. Penny earns $\frac{3}{4}$ of what Mannie earns. If Themba earns R12 000,00 per month: (2)

3.1 What does Mannie earn?

3.2 What does Penny earn?

3.3 What fraction is Penny's salary of Themba's?

3.4 Themba gets a 10% increase. How much is this?

Review Copy

Question 4

Calculate: (4)

4.1 $434{,}13 \times 1{,}4 = e$ (to 2 decimal places)

4.2 $451{,}271 \times 2{,}22 = f$ (to 3 decimal places)

4.3 $19{,}98 \div 0{,}9 = g$

4.4 $0{,}5075 \div 0{,}25 = \text{h}$

Question 5

Do the following construction on one drawing:

5.1 Construct, without using a protractor, ΔABC with $\hat{B} = 30°$, AB = 60 mm and $AC \perp BC$. Measure AC. (4)

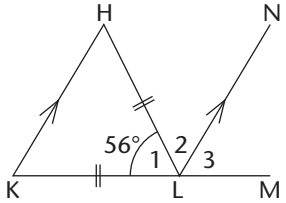
5.2 Construct the bisector of $\hat{A}$. Let it cut BC at D. Measure $D\hat{A}B$ and $D\hat{A}C$. (3)

5.3 Construct the perpendicular bisector of AB. Let it cut AB at E. Measure AE. (3)

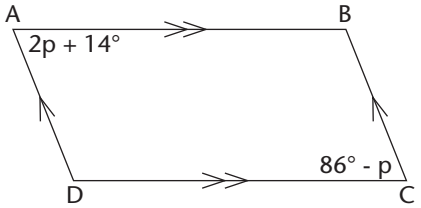
5.4 Explain why $\Delta ADE \equiv \Delta ADC$. (1)

Question 6

6.1 Given: ΔHKL with HL = KL. Side KL is extended to M. Line LN is drawn parallel to KH. If $\hat{L}_1 = 56°$, find the sizes of the four remaining angles in the drawing. (4)

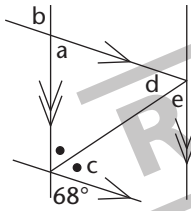


6.2 Quadrilateral ABCD is a parallelogram. Line CE is drawn so that CE = CB. Show that $\hat{C}_1 = \hat{D}$. (3)



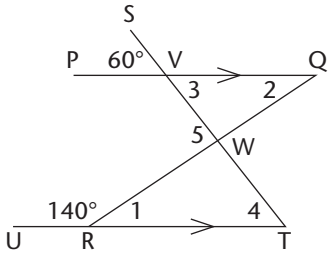
Question 7

7.1 In the drawing below the dots indicate that an angle has been bisected. Use the information that is given and find the values of a, b, c, d and e in the drawing. (5)



Review Copy

7.2 In the drawing below PQ || UT, $P\hat{V}S = 60°$ and $U\hat{R}W = 140°$. Calculate the sizes of the five numbered angles. Give reasons for your answers. (5)

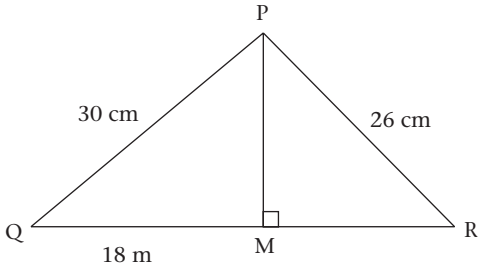


Question 8

8.1.1 In ΔABC, $\hat{C} = 90°$, AB = 12 cm and BC = 8cm. Calculate the length of AC without using a calculator. Leave the answer in the simplest surd form. (2)

8.1.2 Determine, by applying Pythagoras' theorem, whether ΔDEF is right-angled and if so, which angle is the right angle. DE = 8 cm, EF = 15 cm and DF = 17 cm. (2)

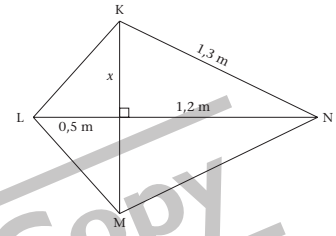
8.2 Calculate:



8.2.1 side PM of ΔPQM
8.2.2 side MR of ΔPMR
8.2.3 the perimeter of ΔPQR. (3)

Question 9

9.1 Calculate, correct to two decimal places:



9.1.1 the length of KE of kite KLMN (1)
9.1.2 the area of ΔKLM (2)
9.1.3 the area of kite KLMN. (3)

9.2 Using $\pi = \frac{22}{7}$ calculate correct to two decimal places:

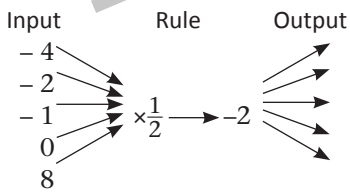
9.2.1 the diameter of a circle of which the circumference is 112 cm (1)

9.2.2 the area of a circle of which the diameter is 42 m. (1)

Review Copy

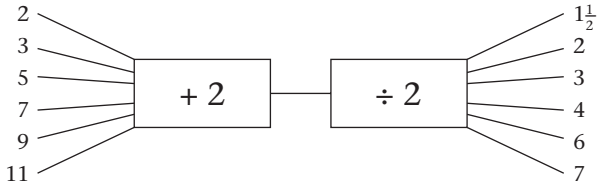
Question 10

10.1.1 Copy and complete this flow diagram.



10.1.2 Represent the same data in the form of a table.
10.1.3 Write an equation to show this relationship. (4)
10.2.1 Check whether the following descriptions of a relationship are equivalent. If not equivalent, do corrections so that it becomes equivalent.

(i)



(ii)

p	2	3	5	7	11
n	1,5	2	3	4	5,5

(iii) $n = (p + 1) \div 2$ with domain the first six prime numbers. (3)

10.3 Describe the relationship in words. (1)

Question 11

11.1 Solve the following equations:

11.1.1 $7x + 5 = 2x - 25$ 11.1.2 $\frac{y-1}{3} = 5 - y$ (4)

11.2.1 Copy and complete the table of ordered pairs for the equation $y = \frac{3x}{2} + 1$

x	–4	–2	–1	0	1	2	3
y							

(2)

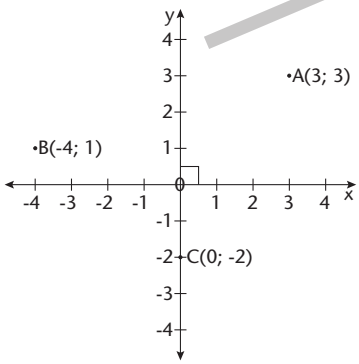
11.2.2 Create a Cartesian plane with x-axis ranging from –5 to 5 and a y-axis ranging from –6 to +6. (2)

11.2.3 Plot the points with co-ordinates as in the table and join them to form the graph of $y = \frac{3x}{2} + 1$. (3)

Review Copy

Question 12

A(3; 3), B(–4; 1) and C0; –2) are three points on a co-ordinate plane.



12.1 Write down the co-ordinates of A if A is reflected in the y-axis. (1)

12.2 Write down the co-ordinates of B if B is reflected in the x-axis. (1)

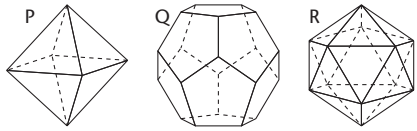
12.3 Write down the co-ordinates of C if C is translated two units vertically. (1)

12.3 Write down the co-ordinates of ΔABC if ΔABC is rotated around the origin through 90° against the clock. (3)

12.4 The co-ordinates of all the vertices of ΔABC are divided by 2. What will happen to the perimeter and area of ΔABC? (1)

Question 13

13.1 Look at the Platonic solids below:



13.1.1 Identify each solid. (3)
13.1.2 How many vertices does object P have? (1)
13.1.3 How many edges does object Q have? (1)
13.1.4 How many faces does object R have? (1)
13.2 A square pyramid is 4 cm high. The sides of it base are 6 cm long. Draw a sketch of the net that you would use to build this pyramid. Show all the important dimensions on the sketch. (1)

Question 14

The data below was collected when rainfall for the year was recorded.

Yearly rainfall recorded (in mm)

Month	Jan	Feb	March	Apr	May	Jun	July	Aug	Sept	Oct	Nov	Dec
Rainfall	7	3	5	25	79	83	98	85	67	23	12	5

14.1 Organise the data set into four intervals. (2)
14.2 Draw up a tally and frequency table. (2)
14.3 Use the data set to determine the following:
the mean (1)
the median (1)
the mode (1)
the range (1)
14.4 Write down two conclusions from your display. (2)

Review Copy

Question 15

The following table shows the selections made by consumers at a frozen yoghurt store, just after lunch time:

SELECTIONS	FREQUENCY
English toffee	12
Fruits of the forest	8
Chocolate	15
Blueberry	13
Caramel	8

15.1 What is the probability of choosing chocolate as a flavour? (2)
15.2 Calculate the relative frequency for the selection of a fruity flavour? (2)

December Exam paper Memo

Question 1 (2)

1.1 $\frac{10}{21} + \frac{2}{7} = \frac{10 + 6}{21} = \frac{16}{21}$

1.2 $\frac{3}{5} \times \frac{5}{6} = \frac{1}{2}$

1.3 $\frac{2}{5}\,\frac{9}{10} + \frac{3}{5} = \frac{2}{5} \times \frac{15}{10} = \frac{3}{5}$

1.4 $3\frac{2}{3} \div 2\frac{1}{5} = \frac{11}{3} \times \frac{5}{11} = \frac{5}{3}$

Question 2 (2)

2.1 $\sqrt[3]{\frac{27}{64}} = \frac{3}{4}$

2.2 $\sqrt[3]{\frac{8}{125}} = \frac{2}{5}$

Question 3 (2)

3.1 Mannie earns R6 000 per month.

3.2 Penny earns R4 500 per month.

3.3 Penny's salary is $\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$ of Themba's.

3.4 $\frac{10}{100} \times 12\ 000 =$ R1 200 (so his new salary is R13 200)

Review Copy

Question 4 (4)

4.1 $434{,}13 \times 1{,}4 = 607{,}78$

4.2 $451{,}271 \times 2{,}22 = 1\ 001{,}822$

4.3 $19{,}98 \div 0{,}9 = 22{,}2$

4.4 $0{,}5075 \div 0{,}25 = 2{,}03$

Question 5

5.1 Construction of ΔABC with $\hat{B} = 30°$, AB = 60 mm and $AC \perp BC$. AC = 36 mm. (4)

5.2 Construction of the bisector of $\hat{A}$ to cut BC at D. $D\hat{A}B = D\hat{A}C = 30°$. (3)

5.3 Construction of the perpendicular bisector of AB to cut AB at E. AE = 30 mm. (3)

5.4 $\Delta ADE \equiv \Delta ADC$ because they have two equal angles and one shared side (SAA). (1)

Question 6

6.1 $\hat{K} = \hat{H} = 62°$ (sum angles isosceles triangle KHL)

$\hat{L}_2 = 62°$ (alternate angles, KH || LN)

$\hat{L}_3 = 62°$ (angles on a straight line) (4)

6.2 $\hat{C}_1 = \hat{E}_2$ (alternate angles, AB || CD)

$\hat{E}_2 = \hat{B}$ (isosceles triangle BCE, CE = CB)

$\hat{B} = \hat{D}$ (opposite angles of parallelogram ABCD)

$\therefore \hat{D} = \hat{C}_1$ (3)

Review Copy

Question 7

7.1 $a = 68°$ (corresponding angles, parallel lines)

$b = 68°$ (vertical opposite angles)

$c = 56°$ (straight line angles)

$d = 56°$ (alternate angles, parallel lines)

$e = 56°$ (alternate angles, parallel lines) (5)

7.2 1 = 40° (straight line angles)

2 = 40° (alternate angles, PQ||UT)

3 = 60° (vertical opposite angles)

4 = 60° (alternate angles, PQ ||UT)

5 = 100° (external angle triangle TRW or triangle QVW) (5)

Question 8

8.1.1 $AC = \sqrt{12^2 - 8^2} = R80 = 4\sqrt{5}$ (2)

8.1.2 DE = 8 cm, EF = 15 cm and DF = 17 cm

$8^2 + 15^2 = 289 = 17^2$

Therefore it is a right-angled triangle and $\hat{E}$ is the right angle. (2)

8.2.1 $PM = \sqrt{30^2 - 18^2} = 24$ cm

8.2.2 $MR = \sqrt{26^2 - 24^2} = 10$ cm

8.2.3 Perimeter of $\Delta PQR = 84$ cm (3)

Question 9

9.1.1 $KE = \sqrt{1{,}3^2 - 1{,}2^2} = 0{,}5$ m (1)

9.1.2 Area $\Delta KLM = \frac{1}{2} \times 1 \times 0{,}5 = 0{,}25$ m^2 (2)

9.1.3 Area of kite KLMN $= 0{,}25 + \frac{1}{2} \times 1 \times 1{,}2 = 0{,}85$ m^2 (3)

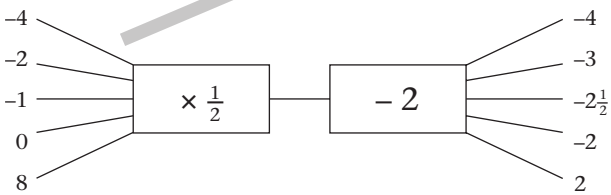
9.2.1 Diameter $= \frac{112}{\pi} = \frac{112}{\frac{22}{7}} = 35{,}63$ cm (1)

9.2.2 Area $= \pi \times 21^2 = 1\,386$ cm^2 (1)

Review Copy

Question 10

10.1.1 (2)



10.1.2

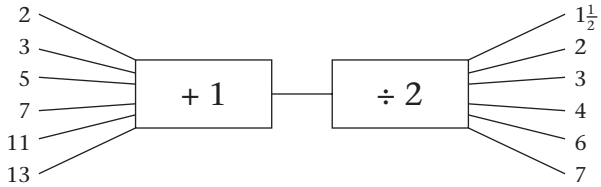
Input (x)	–4	–2	–1	0	8
Output (y)	–4	–3	–2,5	–2	2

(1)

10.1.3 $y = \frac{1}{2}x - 2y = \frac{1}{2}x - 2$ (1)

10.2 They are not equivalent – corrections done below:

10.2.1



10.2.2

p	2	3	5	7	11
n	1,5	2	3	4	6

10.2.3 $n = (p + 1) \div 2$ with domain the first five prime numbers. (3)

10.3 The output values are obtained by adding one to the input values and then dividing by two. (1)

Question 11

11.1.1 $7x + 5 = 2x - 25$

$5x = -30$

$x = -6$ (2)

11.1.2 $\frac{y-1}{3} = 5 - y$

$y - 1 = 15 - 3$

$4y = 16$

$y = 4$ (2)

Review Copy

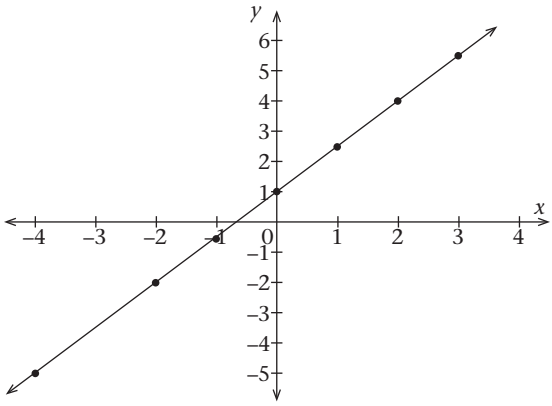
11.2.1

x	–4	–2	–0,5	0	1	2	3
y	–5	–2	–1	1	2,5	4	5,5

(2)

11.2.2 Cartesian plane drawn with x-axis from –5 to 5 and y-axis from –6 to +6. (2)

11.2.3 (3)



Question 12

12.1 A′ (–3; 3) (1)

12.2 B′ (–4; –1) (1)

12.3 C′ (0; 0) (1)

12.4 A″ (–3; 3), B″ (–1; –4), C″ (2; 0) (3)

12.5 The perimeter will halve and the area will be four times smaller. (1)

Question 13

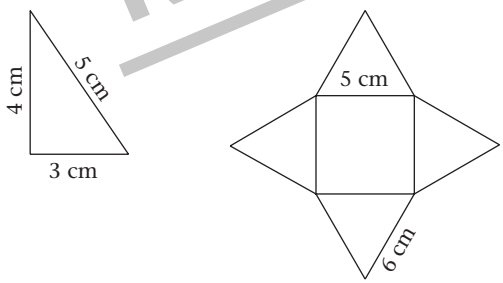
13.1.1 P is a tetrahedron.
Q is a dodecahedron.
R is an icosahedron. (3)

13.1.2 P has 6 vertices. (1)

13.1.3 Q has 30 edges. (1)

13.1.4 R has 20 faces. (1)

13.2 (1)



Question 14

14.1 Rainfall in mm: $0 \leq r < 25$; $25 \leq r < 50$; $50 \leq r < 7\ 550 \leq r < 75$;
$75 \leq r < 100$ (2)

14.2

Rainfall (mm)	Tally (no of months)	Frequency
$0 \leq r < 25$	~~||||~~ |	6
$25 \leq r < 50$	|	1
$50 \leq r < 75$	|	1
$75 \leq r < 100$	||||	4

(2)

Review Copy

14.3

Mean rainfall = $\frac{407}{12}$ = 33,9 mm (1)

Median = 24 mm (1)

Mode = 5 mm (1)

Range = 95 mm (1)

14.4 There is a wide range of monthly rainfall. The monthly rainfall is highest in the middle of the year and lower at the beginning and end of the year. (2)

Question 15

15.1 P(choosing chocolate) = $\frac{1}{5}$ (2)

15.2 Relative frequency of a fruity flavour = $\frac{21}{56}$ (2)

Review Copy