HEADSTART

Mathematics

TEACHER'S GUIDE

GRADE

8

Review Copy

K. Kriel • L. du Toit

It is illegal to photocopy any pages from this book
without the written permission of the Publisher.

OXFORD
UNIVERSITY PRESS
SOUTHERN AFRICA

OXFORD
UNIVERSITY PRESS
SOUTHERN AFRICA

Oxford University Press Southern Africa (Pty) Ltd

Vasco Boulevard, Goodwood, Cape Town, Republic of South Africa
P O Box 12119, N1 City, 7463, Cape Town, Republic of South Africa

Oxford University Press Southern Africa (Pty) Ltd is a subsidiary of
Oxford University Press, Great Clarendon Street, Oxford OX2 6DP.

The Press, a department of the University of Oxford, furthers the University's objective of excellence in research, scholarship, and education by publishing worldwide in

Oxford New York

Auckland Cape Town Dar es Salaam Hong Kong Karachi
Kuala Lumpur Madrid Melbourne Mexico City Nairobi
New Delhi Shanghai Taipei Toronto

With offices in

Argentina Austria Brazil Chile Czech Republic France Greece
Guatemala Hungary Italy Japan Poland Portugal Singapore South Korea
Switzerland Turkey Ukraine Vietnam

Oxford is a registered trade mark of Oxford University Press
in the UK and in certain other countries

Published in South Africa
by Oxford University Press Southern Africa (Pty) Ltd, Cape Town

Oxford Headstart Mathematics Grade 8 Teacher's Guide
ISBN PROM 199044139

© Oxford University Press Southern Africa (Pty) Ltd 2013

The moral rights of the author have been asserted
Database right Oxford University Press Southern Africa (Pty) Ltd (maker)

First published 2013

All rights reserved. No part of this publication may be reproduced,
stored in a retrieval system, or transmitted, in any form or by any means,
without the prior permission in writing of Oxford University Press Southern Africa (Pty) Ltd,
or as expressly permitted by law, or under terms agreed with the appropriate
designated reprographics rights organization. Enquiries concerning reproduction
outside the scope of the above should be sent to the Rights Department,
Oxford University Press Southern Africa (Pty) Ltd, at the address above.

You must not circulate this book in any other binding or cover
and you must impose this same condition on any acquirer.

Publisher / Commissioning editor: Megan Carver / Sharon Villette
Editor: Nadine Botha
Project Manager: Chantel Parry
Designer: Jade Benjamin
Illustrators: Rob Foote / Eva Nossek

Set in ITC Stone Serif 9.5 pt on 12 pt by Stronghold Publishing CC
Reproduction by
Cover reproduction by
Printed and bound by

The authors and publisher gratefully acknowledge permission to reproduce copyright material in this book. Every effort has been made to trace copyright holders, but if any copyright infringements have been made, the publisher would be grateful for information that would enable any omissions or errors to be corrected in subsequent impressions.

Review Copy

Contents

Introduction 7

How this Teacher's Guide should be used 7
How this course works 8

Curriculum and Assessment Policy Statement 8

Instructional time allocation 9
The CAPS for Mathematics 9

Planning and assessment 12

Types of planning tool 12
Assessment in the Senior Phase 16
Types of assessment 17
Formal assessment requirements of Mathematics 18
Formal Programme of Assessment plan (PoA) for Mathematics 21
Inclusive assessment 22
Recording and reporting assessment 22

Teaching and learning Mathematics 23

What is Mathematics? 23
Content area focus in the Senior Phase 24
Metacognitive strategies 25
Mental maths in the Senior Phase 26
Teaching Mathematics in the Senior Phase 27
Inclusive teaching 27

Chapter 1 Whole numbers 30

Unit 1 Ordering and comparing whole numbers 31
Unit 2 Properties of whole numbers 32
Unit 3 Calculations with whole numbers 35
Unit 4 Multiples and factors 44
Unit 5 Ratio and rate 49
Chapter 1 Revision 54

Chapter 2 Integers 57

Unit 1 Counting, ordering and comparing integers 58
Unit 2 Calculations with integers 61
Unit 3 Properties of integers 68
Unit 4 Financial Mathematics 70
Chapter 2 Revision 78

Review Copy

Chapter 3 Exponents 80

Unit 1 Square and cube numbers 81
Unit 2 Comparing and representing numbers in exponential form 84
Unit 3 Calculations with exponents 88
Unit 4 Scientific notation 94
Chapter 3 Revision 98

Chapter 4 Patterns, functions and algebra 99

Unit 1 Investigate and extend patterns 99

Chapter 5 Functions and relationships 105

Unit 1 Input values, output values and flow diagrams 105
Unit 2 Equivalent forms of relationships 108
Chapter 5 Revision 112

Chapter 6 Algebraic expressions 113

Unit 1 Algebraic language 114
Unit 2 Expand and simplify algebraic expressions 123

Chapter 7 Algebraic equations 126

Unit 1 Equations 126
Unit 2 Solving equations 129
Chapter 7 Revision 132

Chapter 8 More algebraic expressions 134

Unit 1 Interpreting algebraic expressions 135
Unit 2 Working with algebraic expressions 138
Unit 3 Simplifying algebraic expressions 141
Chapter 8 Revision 150

Chapter 9 More algebraic equations 152

Unit 1 Solving for the variable 155
Chapter 9 Revision 162

Review Copy

Chapter 10 Geometry 164

Unit 1 Geometry of straight lines 165
Unit 2 Constructing geometric figures 173
Unit 3 Properties of geometric figures 180
Unit 4 Geometry of 2D shapes: Triangles 187
Unit 5 Geometry of 2D shapes: Quadrilaterals 193
Unit 6 Geometry of 2D shapes: Congruent and similar figures 197
Chapter 10 Revision 204

Exam Exemplar 1 (June) Memorandum 206
Exam exemplar 1 (additional): June 209
Exam exemplar 1 (additional): June Memorandum 211
Exam exemplar 2 (additional): June 212
June Exam exemplar 2 (additional): Memorandum 214

Chapter 11 Common fractions 216

Unit 1 Revision of common fractions 217
Unit 2 Adding and subtracting common fractions 219
Unit 3 Multiplying and dividing common fractions 225
Unit 4 Percentages 229
Chapter 11 Revision 233

Review Copy

Chapter 12 Revision: Decimal fractions 234

Unit 1 Ordering and comparing decimal fractions 235
Unit 2 Adding and subtracting decimal fractions 240
Unit 3 Multiplying and dividing decimal fractions 242
Unit 4 Equivalent forms 245
Chapter 12 Revision 249

Chapter 13 Geometry of 2D and 3D 250

Unit 1 Theorem of Pythagoras 251
Unit 2 Area and perimeter 256
Unit 3 Circles 264
Unit 4 Surface area and volume 271
Chapter 13 Revision 278

Chapter 14 Data handling 281

Unit 1 Collecting data 282
Unit 2 Organising and summarising data 288
Unit 3 Representing data 297
Unit 4 Interpreting, analysing and reporting data 304
Chapter 14 Revision 311

Chapter 15 More algebraic expressions 317

Unit 1 Input and output values....317
Unit 2 Equivalent forms....320
Unit 3 Problem-solving with substitution....322
Chapter 15 Revision....324

Chapter 16 Graphs 326

Unit 1 Interpreting graphs....326
Unit 2 Understanding and drawing graphs....330
Chapter 16 Revision....337

Chapter 17 Transformation geometry 338

Unit 1 Transformations....338
Unit 2 Enlargements and reductions....345
Chapter 17 Revision....350

Chapter 18 Geometry of 3D objects 351

Unit 1 Classifying 3D objects....351
Unit 2 Platonic solids....355
Unit 3 Building 3D models....358
Chapter 18 Revision....361

Chapter 19 Probability 362

Unit 1 Theoretical probability....362
Unit 2 Relative frequency....366
Chapter 19 Revision....371

Exam exemplar 2 (November) Paper 1 Memorandum....372
Exam exemplar 2 (November) Paper 2 Memorandum....373
Exam exemplar 1 (additional): November Paper 1....377
Exam exemplar 1 (additional): November Paper 1 Memorandum....380
Exam exemplar 2 (additional): November Paper 2....384
Exam exemplar 2 (additional): November Paper 2 Memorandum....386
Exam exemplar 2 (additional): November Paper 3....388
Exam exemplar 2 (additional): November Paper 3 Memorandum....393

Resources 398

Review Copy

Introduction

How this Teacher's Guide should be used

Mathematics Grade 8 Teacher's Guide is a Mathematics course that provides rich resources to ensure complete curriculum coverage and the successful development of mathematical concepts and skills.

The Teacher's Guide supports you by:

- defining subject, the National Curriculum and Assessment Policy Statement (CAPS) and teaching terminology
- indicating pacing, content and resources in content *overviews*, and structuring the course into lessons with *clear teaching guidelines* according to the CAPS
- providing *background information* (prior knowledge and skills covered in previous grades and/or interesting subject/topic background
- providing suitable *remediation and extension* activities for each lesson
- providing *suggested answers* for every activity
- providing Formal Assessment Tasks as well as *marking guidelines* and *rubrics* as required by the CAPS
- providing the chapter and unit at the bottom of the page to enable easy referencing between components
- offering additional resources that support core content.

Review Copy

Five easy-to-navigate sections

Section A: How this Teacher's Guide should be used and how the course works

- Introduction and the CAPS
- An overview of the CAPS
- The CAPS for Mathematics

Section B: Planning and assessment in the Senior Phase

- Planning tools and teaching plans
- Types of assessment, including the Formal Programme of Assessment for Mathematics
- Recording and reporting assessment
- Teaching and learning Mathematics in the Senior Phase
- Inclusivity

Section C: Lesson plans for the teaching of Mathematics

Section D: Resources for the teaching of Mathematics

How this course works

The _____ series meets the requirements of the CAPS for the Senior Phase. In Grade 8, Mathematics consists of two core components: a Teacher's Guide and a Learner's Book.

The Learner's Book

The 2-colour Learner's Book provides content knowledge, core concepts and skills development. It includes activities for learners to develop, practise and consolidate their knowledge and skills. Teachers receive guidance on how to teach important concepts. Written texts are supported by illustrations that help to explain content. All examples, activities and illustrations are representative of all cultural groups.

Activities steadily become more challenging so that learners progressively develop their understanding of concepts.

The Teacher's Guide

The Teacher's Guide provides you, the teacher, with all the planning, teaching and assessment tools you need to successfully teach this subject.

Curriculum and Assessment Policy Statement

Review Copy

Overview

This series is based on the *National Curriculum Statement Grades R–12* (NCS, January 2012) which is the policy document for learning and teaching in South Africa. The NCS consists of three documents, namely:

- Curriculum and Assessment Policy Statements (CAPS) for all approved subjects for Grades R–12
- *National Policy pertaining to the Programme and Promotion Requirements of the National Curriculum Statement Grades R–12*
- *National Protocol for Assessment Grades R–12* (January 2012).

Each CAPS document has four sections:

- Section 1 – Introduction to the Curriculum and Assessment Policy Statements for the specific subject
- Section 2 – The specific subject's aims, time allocations and requirements to offer it as a subject
- Section 3 – Overview of topics, teaching plan and content clarification for the specific subject
- Section 4 – Assessment guidelines in the specific subject.

Sections 2, 3 and 4 of the CAPS document, together with the National Policy pertaining to the Programme and Promotion Requirements of the NCS, represent the norms and standards of the *National Curriculum Statement Grades R–12*. Together these documents are the basis for determining minimum outcomes, processes and procedures for the assessment of learner achievement in public and independent schools.

Instructional time allocation

The instructional time in the Senior Phase is as follows:

Subject	**Teaching hours per week**
Home Language	5
First Additional Language	4
Mathematics	4,5
Natural Sciences	3
Social Sciences	3
Technology	2
Economic and Management Sciences	2
Life Orientation	2
Creative Arts	2
Total	27,5

Review Copy

The CAPS for Mathematics

Each CAPS document provides:
- an overview of topics and content areas for its subject (see below)
- the weighting prescribed for each content area (see below)
- a teaching plan for the subject (see Section C – Planning and assessment).

The following content areas comprise the Senior Phase Mathematics curriculum:
- Numbers, Operations and Relationships
- Patterns, Functions and Algebra
- Space and Shape (Geometry)
- Measurement
- Data Handling.

Each content area has a prescribed weighting to ensure complete curriculum coverage.

Content Area	Grade 7	Grade 8	Grade 9
Numbers, Operations and Relationships	30%	25%	15%
Patterns, Functions and Algebra	25%	30%	35%
Space and Shape (Geometry)	25%	25%	30%
Measurement	10%	10%	10%
Data Handling (Statistics)	10%	10%	10%
Total	100%	100%	100%

Topic overview

	Grade 7	Grade 8	Grade 9
Term 1	Mental calculations Order and compare whole numbers (9 digits) Properties of whole numbers Calculations with whole numbers Addition and subtraction (6 digits) Multiplication and division (4-digit by 2-digit) Multiples and factors (of 2- and 3-digit whole numbers) Prime factors LCM and HCF (3-digit whole numbers) Solve problems (ratio and rate; percentages, decimal fractions; financial context) Exponents Measure angles Construct geometric figures Classify 2D shapes Similar and congruent 2D shapes Solve problems	Mental calculations Order and compare whole numbers (prime numbers to 100) Properties of whole numbers Calculations with whole numbers Multiples and factors Solve problems (ratio and rate; percentages, decimal fractions; financial context) Count, order and compare integers Calculations with integers Properties of integers Solve problems Represent numbers in exponential form Calculations in exponential form Laws of exponents Numeric and geometric patterns Input and output values or rules for patterns and relationships Equivalent forms Algebraic language Expand and simplify algebraic expressions Set up equations and solve by inspection	Mental calculations Properties of whole numbers Calculations with whole numbers Multiples and factors Solve problems (ratio and rate; direct and indirect proportion; percentages, decimal fractions; financial context) Calculations with integers Properties of integers Solve problems Common fractions Decimal fractions Exponents Calculations in exponential form Solve problems Numeric and geometric patterns Input and output values or rules for patterns and relationships Equivalent forms Algebraic language Expand and simplify algebraic expressions Equations (use factorisation; of the form where a product of factors = 0)

Review Copy

Term 2	Mental calculations Common fractions Percentages Decimal fractions Equivalent forms Solve problems Input and output values for patterns and relationships Equivalent forms (verbal, flow diagrams, tables, formulae, number sentences) Area and perimeter of 2D shapes Convert SI units Surface area and volume of 3D objects	Mental calculations Algebraic language Expand and simplify algebraic expressions Set up equations and solve by using additive and multiplicative inverses Construct and investigate geometric figures Classify 2D shapes Similar and congruent triangles Angle relationships Solve problems	Mental calculations Investigate properties of geometric figures by construction Classify 2D shapes Similar and congruent triangles Solve problems Angle relationships Use the Theorem of Pythagoras Area and perimeter of 2D shapes (polygons and circles)
Term 3	Mental calculations Numeric and geometric patterns Input and output values for patterns and relationships Equivalent forms Algebraic language Number sentences Interpret and draw graphs Transformations Classify 3D objects Build 3D models	Mental calculations Common fractions Percentages Decimal fractions The Theorem of Pythagoras Area and perimeter of 2D shapes Surface area and volume of 3D objects Solve problems Data handling	Mental calculations Input and output values or rules for patterns and relationships Algebraic language Expand and simplify algebraic expressions Factorise algebraic expressions Equations Draw and interpret graphs Draw linear graphs from given equations Surface area and volume of 3D objects (include cylinders)
Term 4	Mental calculations Integers Numeric and geometric patterns Input and output values for patterns and relationships Algebraic language Number sentences Data handling Probability	Mental calculations Input and output values or rules for patterns and relationships Equivalent forms Solve algebraic equations Interpret and draw graphs Transformations Enlargements and reductions Classify 3D objects Build 3D models Probability	Mental calculations Transformations Enlargements and reductions Classify 3D objects Build 3D models Data handling Probability

Review Copy

Planning and assessment

Types of planning tools

The following planning tools are provided:

- a teaching plan that follows the CAPS time allocations
- sample lesson plans.

Teaching plan for Mathematics Grade 8

This teaching plan shows:

- the pacing of the topics for the course by term
- where to find the relevant content and activities in the Learner's Book
- when Formal Assessment takes place, cross-referenced to suitable activities in the Learner's Book.

Chapter	Content/topics (as per the CAPS)	Learner's Book	Time allocation	LB page ref	TG page ref
TERM 1					
1	**Whole numbers**	**Unit 1** Ordering and comparing whole numbers **Unit 2** Properties of whole numbers **Unit 3** Calculations with whole numbers **Unit 4** Multiples and factors **Unit 5** Ratio and rate	6 hours	6	30
Chapter 1 Revision				34	54
2	**Integers**	**Unit 1** Counting, ordering and comparing integers **Unit 2** Calculations with integers **Unit 3** Properties of integers **Unit 4** Financial Mathematics	9 hours	37	57
Chapter 2 Revision				70	79
3	**Exponents**	**Unit 1** Square numbers and cube numbers **Unit 2** Comparing and representing numbers in exponential form **Unit 3** Calculations with exponents **Unit 4** Scientific notation	9 hours	72	80
Chapter 3 Revision				96	98
PoA Assignment 1: Whole numbers				36	56
4	**Numeric and geometric patterns**	**Unit 1** Investigate and extend patterns	4,5 hours	98	99

Review Copy

Chapter	Content/topics (as per the CAPS)	Learner's Book	Time allocation	LB page ref	TG page ref
Chapter 4 Revision				See Revision 5	
5	**Functions and relationships**	**Unit 1** Input values, output values and flow diagrams **Unit 2** Equivalent forms of relationships	3 hours	103	105
Chapter 5 Revision				114	112
6	**Algebra**	**Unit 1** Algebraic language **Unit 2** Expand and simplify algebraic expressions	4,5 hours	115	113
Chapter 6 Revision				See Revision 7	
7	**Algebraic equations**	**Unit 1** Equations **Unit 2** Solving equations	3 hours	130	126
Chapter 7 Revision				138	132
PoA Assignment 2: Patterns, Functions and Relationships				140	133
PoA Test 1					134
TERM 2					
8	**More algebraic expressions**	**Unit 1** Interpreting algebraic expressions **Unit 2** Working with algebraic expressions **Unit 3** Simplifying algebraic expressions	9 hours	142	136
Chapter 8 Revision				155	152
9	**Algebraic equations**	**Unit 1** Solving for the variable	3 hours	160	153
Chapter 9 Revision				169	162
10	**Geometry**	**Unit 1** Geometry of straight lines **Unit 2** Constructing geometric figures **Unit 3** Properties of geometric figures **Unit 4** Geometry of 2D shapes: Triangles **Unit 5** Geometry of 2D shapes: Quadrilaterals **Unit 6** Geometry of 2D shapes: Congruent and similar figures	25 hours	170	164
Chapter 10 Revision				224	204
PoA Investigation 1				199	184
PoA Exam exemplar 1 (June)				226	206
PoA Exam exemplar 1 (additional) June					209
PoA Exam exemplar 2 (additional) June					212

Review Copy

Chapter	Content/topics (as per the CAPS)	Learner's Book	Time allocation	LB page ref	TG page ref
TERM 3					
11	**Revision: Common fractions**	**Unit 1** Revision of common fractions **Unit 2** Adding and subtracting common fractions **Unit 3** Multiplying and dividing common fractions **Unit 4** Percentages	7 hours	229	216
Chapter 11 Revision				247	233
12	**Revision: Decimal fractions**	**Unit 1** Ordering and comparing decimal fractions **Unit 2** Adding and subtracting decimal fractions **Unit 3** Multiplying and dividing decimal fractions **Unit 4** Equivalent forms	6 hours	249	234
Chapter 12 Revision				266	249
13	**Geometry of 2D and 3D**	**Unit 1** Theorem of Pythagoras **Unit 2** Area and perimeter **Unit 3** Circles **Unit 4** Surface area and volume	15 hours	267	250
Chapter 13 Revision				294	278
PoA Investigation 2				280	265
PoA Investigation 3				283	268
14	**Data handling**	**Unit 1** Collecting data **Unit 2** Organising and summarising data **Unit 3** Representing data **Unit 4** Interpreting, analysing and reporting data	10,5 hours	296	281
POA Project				302	287–309
Chapter 14 Revision				327	311

Review Copy

Chapter	Content/topics (as per the CAPS)	Learner's Book	Time allocation	LB page ref	TG page ref
TERM 4					
15	**More algebraic expressions**	**Unit 1** Input and output values **Unit 2** Equivalent forms **Unit 3** Problem-solving with substitution	9 hours	328	317
Chapter 15 Revision				337	324
PoA Assignment 3: Algebra				338	325
16	**Graphs**	**Unit 1** Interpreting graphs **Unit 2** Understanding and drawing graphs	9 hours	339	326
Chapter 16 Revision				352	337
17	**Transformation geometry**	**Unit 1** Transformations **Unit 2** Enlargements and reductions	6 hours	353	338
PoA Investigation 4				364	347
Chapter 17 Revision				368	350
18	**Geometry of 3D objects**	**Unit 1** Classifying 3D objects **Unit 2** Platonic solids **Unit 3** Building 3D models	7 hours	369	351
Chapter 18 Revision				381	361
19	**Probability**	**Unit 1** Theoretical probability **Unit 2** Relative frequency	4,5 hours	382	362
Chapter 19 Revision				395	371
PoA Exam exemplar 2 (November) Paper 1				396	372
PoA Exam exemplar 2 (November) Paper 2				399	373
PoA Exam exemplar 2 (additional) November Paper 1					384
PoA Exam exemplar 2 (additional) November Paper 2					386
PoA Exam exemplar 2 (additional) November Paper 3					388

Review Copy

Sample lesson plan for Grade 8

Some may find daily lesson plans useful, although these are not a formal policy requirement. An example of how to complete a lesson plan is below.

Date:	**Grade:** 8	**Term:** 1
Unit: 2	**Unit title:** Number sentences	**Contact time:** 3 hours
Content/concept: (Teacher to complete)	**Activities:** 1–5	**Resources required:** Learner's Book; exercise book
Activity 1:		
Links with previous activity: n/a		
Links with next activity: (Teacher to complete)		
Teaching plan (Teacher to complete; tips provided below) Mental mathematics Check prior knowledge: class discussion Teaching procedures (methods you will use): Differentiation (how you will deal with extension/remedial tasks): Progress checks:		
Assessment: Teacher, peer		
Confidential information/Teacher reflection: (Teacher to complete)		

Review Copy

Assessment in the Senior Phase

Assessment is about collecting evidence of the learners' learning. It is an integral part of teaching and learning, and should be planned when planning the lesson content. Assessment helps to identify the needs of the learners. It also provides evidence of progress, enables teachers to reflect on what they are doing and provides for feedback and reporting to all stakeholders. Good assessment practice in Mathematics includes:

- assessing whether skills and aims are applied to content knowledge
- determining whether learners can apply this knowledge to procedures and problems
- providing feedback.

The four steps of assessment

1. Generating and collecting evidence of achievement
2. Evaluating the evidence
3. Recording the findings
4. Using the findings to guide future learning and teaching

Types of assessment

Type of assessment	Description
Baseline assessment	Establishes whether learners meet basic skills and knowledge level required Helps teacher plan for the year and for each learner Is administered at the beginning of the year and before a particular topic Results used as a guide for teaching and not for promotion purposes.
Diagnostic assessment	Informs the teacher about certain specific problem areas that may hinder performance May help determine whether a learner's problems are content or psycho-social based Appropriate interventions should follow on from diagnostic assessment Results should inform interventions and not be used for promotion purposes
Formative assessment	Used to aid the learning process and not for promotion purposes Usually informal, to provide the teacher and learner with a more frequent account of where the learner is at Teachers can use this form of assessment to modify and adapt their own teaching
Summative evaluation	Carried out after completion of a topic or cluster of topics Is an assessment of learning that has taken place Recorded and used for promotion Usually formal assessment, making up the Formal Programme of Assessment

Review Copy

Informal or daily assessment

Informal assessment is a daily monitoring of learners' progress. This is done through observations, discussions, practical demonstrations, learner-teacher conferences, informal classroom interactions, etc. The Learner's Book is packed with activities that can be used for informal assessment, once learners have mastered the basic concepts.

The CAPS tells us that informal assessment should be used to provide feedback to the learners and to inform planning for teaching, but need not be recorded or taken into account for promotion. It should not be seen as separate from learning activities taking place in the classroom. Learners or teachers can mark these assessment tasks.

Self-assessment and peer assessment actively involve learners in assessment. This is important as it allows learners to learn from and reflect on their own performance.

Learners often experience difficulty completing extended writing. Learners should read and write regularly, starting with sentences and paragraphs and building up to extended pieces of work. Much of this may be structured by working through activities.

Formal assessment

All assessment tasks that make up a Formal Programme of Assessment for the year are regarded as formal assessment. Formal assessment tasks are marked and formally recorded by the teacher for progression and certification purposes. All formal assessment tasks are subject to moderation for the purpose of quality assurance and to ensure that appropriate standards are maintained.

Formal assessment provides teachers with a systematic way of evaluating how well learners are progressing in a grade and in a particular subject. Examples of formal assessments include tests, examinations, practical tasks, projects, oral presentations, demonstrations, performances, etc. Formal assessment tasks form part of a year-long Formal Programme of Assessment in each grade and subject.

Formal assessment requirements of Mathematics

The forms of assessment used should be appropriate for the learners' ages and developmental levels.

Learners must complete formal assessments each term. Formal assessments include formally assessed tasks, along with projects and examinations.

The Formal Programme of Assessment as prescribed by the CAPS is shown below. This Programme of Assessment is generic across the three grades in the Senior Phase, and lists the types of formal assessment required each term.

Minimum requirements for formal assessment

	Forms of assessment	Minimum requirements per term				Number of tasks per year	Weighting
		Term 1	Term 2	Term 3	Term 4		
SBA (School-based Assessment)	Tests	1	1	1		3	40%
	Examination		1			1	
	Assignment	1		1	1	3	
	Investigation		1		1	2	
	Project			1		1	
	Total	2	3	3	2	10	
Final examination		End of year				1	60%

Review Copy

Types of formal assessment for Mathematics

Tests and examinations

These are individual assessment tasks. Tests and examinations for formal assessment should cover a substantial amount of content. Tests and examinations must be completed under strictly controlled conditions.

Each test and examination must cater for a range of cognitive levels in the correct allocation (see the table on the next page).

Cognitive level	Description of skill to be demonstrated
Knowledge ≈ 25%	Estimation and appropriate rounding of numbers Straight recall Identification and direct use of correct formulae Use of mathematical facts Appropriate use of mathematical vocabulary
Routine procedures ≈ 45%	Performing of well-known procedures Simple applications and calculations which might involve many steps Derivation from given information may be involved Identification and use (after changing the subject) of correct formulae generally similar to those encountered in class
Complex procedures ≈ 20%	Problems involving complex calculations and/or higher-order reasoning Investigate elementary axioms to generalise them into proofs for straight-line geometry, congruence and similarity No obvious route to the solution Problems not necessarily based on real-world contexts Making significant connections between different representations Require conceptual understanding
Problem-solving ≈ 10%	Unseen non-routine problems (which are not necessarily difficult) Higher-order understanding and processes are often involved Might require the ability to break the problem down into its constituent parts

Review Copy

Projects

Learners complete *one* project in Mathematics in each grade. Projects can be used to test a range of skills and competencies. It is prescribed that learners complete a project in Term 3 of each grade. Projects must provide learners with the ability to demonstrate their understanding of a mathematical concept and apply it to a real-life situation. Be wary of prescribing projects that are beyond the cognitive level of the learners, or that will simply involve duplicated facts and data from reference material.

Assignments

An assignment is also an individual task, similar to tests and examinations. However, the assignment should be an extended piece of work with a focus on more demanding work than that covered in class. *Three* assignments per year are required by the CAPS. The assignment can include past questions, but should also include more challenging aspects encouraging the learner to use additional material to help them. The assignment can be completed at home.

Investigations

An investigation should be used to discover rules or concepts. It is recommended that learners conduct investigations in class as much as possible, and that the final written task definitely be done in class. Rubrics are used to assess investigations. *Two* investigations per year are required by the CAPS.

The skills involved in investigations include:

- organising and recording ideas and discoveries in tables and diagrams
- explaining ideas in appropriate forms
- showing clear understanding of concepts and procedures through calculations
- generalising and drawing conclusions.

Tests and examinations

Assessment tasks should be designed to cover the content and concepts of the subject and include a variety of activities selected to assess the identified aims and skills.

Before handing out an assessment task to learners, teachers should ensure that they are able to answer all the questions themselves. When teachers set an assessment task, they should draw up a memorandum of answers and/or a rubric for the assessment. Refer to the seven-point rating code or scale of achievement in this Teacher's Guide when constructing a rubric.

Feedback should acknowledge strengths and identify areas of weakness for learners' developmental needs. Action plans on how learners will be supported should accompany this feedback. It is important that the feedback provided to learners encourages them to do better, and builds their self-confidence.

Review Copy

Formal Programme of Assessment (PoA) plan for Mathematics

We have provided a full Formal Programme of Assessment plan. These assessments cater for a range of cognitive levels and abilities as required by the CAPS, as described in this section.

Programme of Assessment for Grade 8 Mathematics

Forms of assessment	**Term 1**	**Term 2**	**Term 3**	**Term 4**
*Revisions	At the end of every chapter			
Tests	LB p. 140; TG p. 134		TG p. 313	
Exam exemplar 1 (June)		LB p. 225		
Exam exemplar 1: (**additional**) June		TG p. 206		
Exam exemplar 2: (**additional**) June		TG p.209		
Exam exemplar 2 Paper 1 (November)				LB p. 372
Exam exemplar 2 Paper 2 (November)				LB p. 373
Exam exemplar 1: (**additional**) November				TG p. 377
Exam exemplar 2: (**additional**) November				TG p. 384
Exam exemplar 3: (**additional**) November				TG p. 388
Assignment 1 Assignment 2 Assignment 3	LB page 36 **LB page 140			LB p. 335
Investigation 1 Investigation 2 Investigation 3 Investigation 4		LB p. 197	***LB p. 278 ***LB p. 281	LB p. 361–363
Project			LB pp. 299–307	
Note: *Revision exercises at the end of every chapter can be used for consolidation and/or as a form of assessment (assignment). **Additional assignment for Term 1 *** Two investigations in Term 3: Additional assignments tor Term 3.				

Review Copy

Inclusive assessment

Teachers need to develop adaptive and alternative methods to assess learners with barriers to learning, so that learners are given opportunities to demonstrate competence in ways that suit their needs. Here are some examples of how to assess these learners, while still maintaining the validity of the assessment.

- Some learners may need concrete apparatus for a longer time than their peers.
- Assessment tasks, especially written tasks, may have to be broken up into smaller sections for learners who cannot concentrate or work for a long time, or short breaks may be given during the tasks. Learners can also be given extra time to complete tasks.
- Some learners may need to do their assessment tasks in a separate venue to limit distractions.
- A variety of assessment instruments should be used, as a learner may find that a particular assessment instrument does not allow them to show what they can do.
- Learners who cannot read can have tasks read to them and they can orally dictate answers. Assessment can also include a practical component in which learners can demonstrate their competence without having to use language.
- A sign-language interpreter can be used.
- Assessment tasks can be available in Braille or enlarged with bolded text.
- Assessment can include the use of dictaphones or computers with voice synthesisers.
- The forms of assessment used should be appropriate for age and developmental levels. The design of these tasks should cover the content of the subject and include a variety of tasks designed to achieve the objectives of the subject.

Review Copy

Recording and reporting assessment

Recording

Recording documents the level of a learner's performance in a specific assessment task. It indicates learner progress towards the achievement of the knowledge as prescribed in the curriculum. Records of learner performance should be used to verify the progress made by teachers and learners in the teaching and learning process.

Reporting

Learners' performance can be reported in a number of ways. These include report cards, parents' meetings, school visitation days, parent-teacher conferences, phone calls, letters, class or school newsletters, etc. Teachers in all grades report in percentages against the subject. The various achievement levels and their corresponding percentage bands are as shown in the table below.

Rating code	Description of competence	Marks %
7	Outstanding achievement	80–100
6	Meritorious achievement	70–79
5	Substantial achievement	60–69
4	Adequate achievement	50–59
3	Moderate achievement	40–49
2	Elementary achievement	30–39
1	Not achieved	0–29

Teaching and learning Mathematics

What is Mathematics?

Mathematics is a language. It uses symbols and notations to describe relationships. Mathematics is a human activity that involves observing, representing and investigating patterns and relationships in both the physical and social dimensions. Mathematics helps develop key mental processes such as logical and critical thinking, accuracy and problem-solving. All of these processes contribute to a learner's decision-making ability.

Review Copy

The specific aims of Mathematics

The aims of Mathematics are to develop:

- a critical awareness of mathematical relationships
- confidence and competence in Mathematics without fear of the subject
- curiosity and a love of Mathematics
- appreciation for the beauty and elegance of Mathematics
- recognition of the subject as a creative art
- the deep conceptual understanding required to understand Mathematics
- the acquisition of specific skills in order to apply Mathematics
- the ability to study related subject matter and to study further in Mathematics.

Specific skills required in Mathematics

In order to develop the essential skills the learner should:

- develop correct mathematical language use
- develop number vocabulary, number concept and application skills
- learn to listen, communicate, think, reason logically and apply the knowledge gained
- learn to investigate, analyse, represent and interpret information
- learn to pose and solve problems
- be aware that Mathematics plays a key role in real-life situations.

Content area focus in the Senior Phase

Content area	General focus	Specific focus
Numbers, Operations and Relationships	Meaning of different kinds of numbers Relationships between different kinds of numbers Relative sizes of different numbers Representations of numbers in various ways Operations with numbers Estimation and checking solutions	Represent numbers in a variety of ways and move flexibly between representations Recognise and use properties of operations with different number systems Solve a variety of problems, use an increased range of numbers and are able to perform multiple operations correctly and fluently
Patterns, Functions and Algebra	Achieve efficient manipulation skills which will carry over into other domains of the subject Describe patterns and relationships through the use of symbolic expressions, graphs and tables Identify and analyse regularities and change in patterns Make predictions and solve problems	Investigate numerical and geometric patterns to establish the relationships between the variables Express rules governing patterns in algebraic language or symbols Develop algebraic manipulative skills to recognise the equivalence between different representations of the same relationship Analyse situations in a variety of contexts Use of different and equivalent representations – algebraic language, formulae, expressions, equations and graphs
Space and Shape (Geometry)	Properties of shapes and objects Relationships between these properties Orientations, positions and transformations of two-dimensional shapes and three-dimensional objects	Draw and construct a wide range of geometric figures and solids using appropriate instruments Appreciate the use of constructions to investigate the properties of geometric figures and solids Develop clear and precise descriptions and classification categories of geometric figures and solids Solve geometric problems drawing on known properties of geometric figures and solids
Measurement	Select and use appropriate units, instruments and formulae Make sensible estimates Be aware of reasonableness of measurement and results	Use formulae for measuring area, perimeter, surface area and volume of geometric figures and solids Select and convert between units of measurement Use the Theorem of Pythagoras to solve problems involving right-angled triangles

Review Copy

Data Handling	Ask questions and find answers in order to describe events and the social, technological and economic environment Collect, organise, represent, analyse, interpret and report data Enabled to make informed predictions by the study of probability Describe randomness and uncertainty	Pose questions for investigation Collecting, summarise and represent and critically analyse data Interpret, report and make predictions about situations Probability – include single and compound events and their relative frequency in simple experiments

Metacognitive strategies

What are metacognitive strategies and how can I use them?

Metacognition is the process of thinking about how you think. Adults often do this automatically. Before taking on something new, we may ask ourselves: What do I already know about this? What will help me understand it better? How is it structured? As we engage with a text or action, we may ask ourselves: Did I understand that? Why do I think that? How does this connect with what I already know? How could I apply this in my life? Then we evaluate what we have learnt or done by asking questions like: Did I understand that well? What strategies helped and what strategies didn't help? What should I do the next time I take on a task like this?

Learners, however, are often unaware of how they think and engage with learning material. You help learners to learn independently by explicitly guiding them to plan, monitor, and evaluate their reading and learning strategies. This is particularly effective for those learning in English as a second language and for learners who are struggling. It can dramatically improve their performance.

You teach metacognitive skills by asking learners to explain what they are thinking and what strategies they are using to understand material. This is best done is small groups. You can also use "think aloud" strategies when engaging with texts and images. "Think-alouds" are often effective when reading texts to learners, and during small-group and pair reading exercises. Here is an example of how to teach metacognitive strategies using a "think aloud":

1. Choose a short piece of text and note where you will stop during reading to model your thought processes.
2. Things to include in this planning stage could be:
   - reading the text title and the table of contents
   - looking at the images and predicting what the text may be about
   - skim-reading the text looking for headings, words in bold, and summaries. As you skim read, think about what you already know about the subject and what more you would like to know.
3. In class, explain what you will be doing to the learners. Start by explaining how you planned before reading the text.

Review Copy

4. To monitor understanding during reading, you can explain where you stopped to ask yourself whether you understood the content. If the text has a long or complex sentence, describe how you divided it up to understand it. Find places where you could ask questions such as:
   - Why would this ...?
   - Is this similar to ...
   - How can I figure out what this new word means?
   - What does the writer want me to know?
   - What do I think will happen next? Why do I think that?
   - Do I need to re-read this for detailed information?
5. Now show learners how to evaluate their metacognitive strategies by asking and answering questions such as:
   - Did I read and understand this well?
   - What helped me to understand? What didn't help?
   - What should I do next time I read about this topic?
   - What will help me remember what I read?

By engaging with how learners think, you can better prepare them for their lives and learning in the future.

Mental maths in the Senior Phase

Review Copy

Mental maths-type activities are provided at the start of every unit in Grades 7 and 8 in this series.

Spread these activities across the duration of the unit, and do a small chunk every day alongside any additional ideas for mental maths that are provided. These activities are adaptable, and are designed to accommodate any additional ideas you may have.

It is important that the questions provided are extended to include additional questions, possibly in quick 10-minute mental maths tests and quizzes.

There are two aspects to mental calculation. The first is the *rapid recall* of number facts and the other is encouraging learners to figure out a way to solve calculations (*strategies*). Learners have to memorise some number facts as this helps them to develop strategies.

It is important that learners are given the opportunity to discuss how they reached their answer. This is how they learn that there is a *range of possible strategies*, but some are more *effective* than others.

Often in the Senior Phase, mental maths activities provided are not strictly speaking mental maths but comprise practice activities related to current and forthcoming concepts. For example, writing down an algebraic expression using algebraic language is not a mental maths-type activity but aims to provide the learner with an opportunity to practise this skill, and the teacher with an opportunity to quickly assess learners' progress in this regard. A 10-minute mental maths "space" should be created for these types of activities at the start of every lesson. It is still important in the Senior Phase.

Mental maths sessions should be fast-paced, interactive and engaging. Ensure that a range of activity, as explained above, are included in every session as well as questions and activities that range from simple practice to the more challenging (extension) activities. However, while you are striving for a lively "warm-up" pace, learners should still be allowed sufficient time to work through strategies if necessary.

Teaching Mathematics in the Senior Phase

Grade 8 is usually the first year in senior school and in the middle of the Senior Phase. The Grade 8 learners are the "junior" learners in the school and as such they feel quite nervous at first.

The transition to Grade 8 is an important one, and a BIG step. For many learners , for many different reasons, going to "high school" is a combination of being exciting, yet challenging and even stressful.

Learners in the Senior Phase do Mathematics every day. They must be challenged to think abstractly and critically, and not merely to copy formulae and do substitutions.

The writing of formal tests and exams becomes even more important. The Mathematics teacher must spend time developing exam techniques which include unpacking terminology used in exams, such as *determine, identify, deduce, predict, present, summarise, expand, suggest, illustrate*, and so on.

The Learner's Book provides many built-in opportunities for learners to engage with these. Section E of the Teacher's Guide also provides a list of important terminology.

Presenting answers, time management, exam stress management, etc. are all important areas in which learners must receive constant coaching. Mathematics teachers should work very closely with Life Orientation teachers in order to support learners with these issues.

The volume of work increases in Grade 8, and the expectations are higher. Learners are also expected to mark their own work (from the board) and this is new to many Grade 8 learners.

Grade 9 is the most challenging grade in this phase. While being on the cusp of the final and another extremely important phase of their high school career, learners are also dealing with other issues important to them, such as sexuality. Girls develop faster than boys. Some schools separate the boys and girls in Grade 9 in order to address this issue.

Grade 9 is a crucial year in the teaching of Mathematics: learners are required to make a choice between Mathematics and Mathematical Literacy in Grade 10. This choice will be based on their experience and level of success achieved in Grade 9. For learners who have some idea of their future career, the choice between these two might be somewhat more straightforward.

It is of upmost importance that teachers lay a good foundation for basic algebra and geometry in Grades 8 and 9 in order to facilitate suitable and appropriate subject choice.

Review Copy

Inclusive teaching

What is inclusive teaching?

In the Senior Phase, it is crucial that learners find themselves in an environment where they can develop an interest in learning and the belief that they can learn. Inclusive education is defined as a learning environment that promotes the full personal, academic and professional development of all learners irrespective of race, class, gender, disability, religion, culture, sexual preference, learning styles and language.

Inclusion is about acknowledging and respecting that:

- all children have the right to learn
- all children are able to learn
- all learners need support
- all learners are unique and have different, but equally valued, learning needs
- all learners need the opportunity to build on their own unique strengths
- the learner is the centre of the teaching and learning process
- there are differences in learners, for example, age, gender, language, culture, learning styles, disabilities, HIV status and so on.

Inclusion is also about:

- enabling educational structures, systems and learning methodologies to meet the needs of all learners
- more than just formal schooling – it embraces learning that occurs in the home, community and so on
- changing attitudes, behaviour, methodologies and environments to meet the needs of all learners
- ensuring maximum participation of all learners in the culture and curriculum of all educational institutions
- identifying and minimising barriers to learning that can occur at any level of the system.

Some of the learners in your class may already suffer from exclusion or think negatively about education. There is no reason for their exclusion from class activities. It is the responsibility of the teacher to ensure the inclusion of these learners. This means adapting activities to suit their needs and capabilities. It is equally important that the class is not divided because of this. Rather, learners with these challenges should be accepted and helped where possible by their peers. Learners should at all times be discouraged from teasing, bullying or ignoring learners with special needs. When these attitudes are directed towards a learner, they create a barrier to learning in that learner.

Review Copy

Practical guidelines for inclusive teaching

- Have a true understanding of each learner's background, strengths, unique abilities, needs and barriers. Then use this information to inform your planning and give a clearer focus.
- Remember that the teacher is a facilitator of learning.
- Keep the content and material as relevant as possible.
- Break down learning into small, manageable and logical steps. Keep instructions clear and short (plan beforehand).
- Grade activities according to the different levels and abilities of learners. Try to ensure that learners remain challenged enough without undue stress.
- Develop a balance between individual teaching, peer tutoring, cooperative learning and whole-class teaching.Use learners to help one another in the form of group types, peer-assisted learning, buddy systems and so on. Ensure that learners feel included and supported in the classroom by both the teacher and their peers.
- Set up pairs and groups of learners where members can have different tasks according to strengths and abilities. Promote self-management skills and responsibility through group roles and the types of tasks you set.

- Motivate learners and affirm their efforts and individual progress. Build confidence. Encourage questioning, reasoning, experimentation with ideas and risking opinions.
- Determine the learner's Zone of Proximal Development (ZPD) and use it for effective teaching and learning. Vygotsky described the ZPD as the distance between what the learner already knows and understands and what they can understand with adult support. Learning is thus a social interaction as the teacher mediates and supports the learner as they understand a new concept.
- Spend time on consolidating new learning. Use different ways to do this until all learners understand the concept. Make time to go back to tasks so that learners can learn from their own and others' experiences and methods.
- Use and develop effective language skills (expressive and receptive, verbal and non-verbal).
- Experiment with a variety of teaching methods and strategies to keep learners interested and to cater for and develop different learning styles. Use games, cooperative group work, brainstorming, problem-solving, debates, presentations and so on.

Learners with barriers to learning

A barrier to learning is anything that prevents a learner from participating fully and learning effectively. This includes learners who were formerly disadvantaged and excluded from education because of the historical, political, cultural and health challenges facing South Africans. Some other examples of barriers to learning may be learners who are visually or hearing impaired, or learners who are intellectually challenged. Barriers to learning cover a wide range of possibilities and learners may often experience more than one barrier. Some barriers, therefore, require more than one adaptation in the classroom and varying types and levels of support.

Review Copy

These learners may require and should be granted more time for:
- completing tasks
- acquiring thinking skills (own strategies)
- assessment activities.

Teachers need to adapt the number of activities to be completed without interfering with the learners gaining the required language skills.

CHAPTER

1 Whole numbers

Chapter overview

Learner's Book pages 6 to 36
Recommended pacing: 6 hours

This chapter focuses on the following:

Unit 1: Ordering and comparing whole numbers	45 minutes
Comparing and ordering whole numbers	
Prime numbers	
Problem-solving	
Unit 2: Properties of whole numbers	45 minutes
Properties of 0	
Properties of 1	
Commutative, associative and distributive properties	
Unit 3: Calculations with whole numbers	2,5 hours
Estimating solutions	
Addition and subtraction of whole numbers	
Round off and compensate	
Multiplication and division of whole numbers	
Round off and compensate	
Revision: The order of operations	
Assignment 1: Whole numbers (POA)	
Unit 4: Multiples and factors	1 hour
Multiples	
Factors	
Factorisation: Numbers as products of prime factors	
Unit 5: Ratio and rate	1 hour
Ratio	
Ratio: Dividing quantities into parts	
Increase/decrease a number in a given ratio	
Rate	
Rate: Time, distance and speed	
Exchange rates	
Chapter 1 revision	

Review Copy

UNIT 1

Ordering and comparing whole numbers

Unit overview

Learner's Book page 7
Recommended pacing: 45 minutes

This unit focuses on the following:

- defining and finding prime numbers
- comparing whole numbers to decide which number is larger
- writing whole numbers in ascending and descending order
- solving problems involving whole numbers

Resources: Learner's Book; calculator

Mental Maths

(10 minutes per day)

Suggested answers — Learner's Book page 7

1	a	12	b	45	c	56	d	20	e	96
	f	60	g	44	h	42	i	144	j	40

2 a > b <

3 2; 3; 5; 7; 11; 23; 29; 41; 43; 47; 61; 67; 83; 89; 101; 113; 131

4 991; 971; 907; 769; 733; 701; 359; 347; 199; 179; 113; 107; 79; 17; 13

5 33; It is not a prime number.

Review Copy

Background information

In Grades 6 and 7, learners did the following:

- represented, compared and ordered up to 9-digit numbers
- recognised and represented prime numbers up to 100
- rounded off numbers to the nearest five, ten, hundred and thousand.

In Grade 8, number knowledge is revised and consolidated.

Teaching guidelines

Although learners should have encountered whole numbers and their properties in Grade 7, they often come from different primary schools, and so it is very important to make sure that this section is revised. In particular, it is important to make sure that all learners are comfortable with the terminology and symbols.

Ask learners to write down the terms with the definitions. This will be at the beginning of their exercise books, and so they can refer back to these easily. Alternatively, they can start their own glossary at the back of their exercise books. Words and symbols that learners must pay particular attention to include: greater than; smaller than; equal to; descending order; ascending order and prime numbers.

Comparing and ordering whole numbers; Prime numbers; Problem-solving

Activities 1–2 Compare and order whole numbers

Learner's Book page 8

Guidelines for implementing these activities

- Learners already encountered prime numbers, and comparing and ordering whole numbers in Grade 7. This therefore serves as revision.
- When comparing and ordering larger numbers, learners should first look at the first three digits of numbers in their hundred thousands, e.g. 123 675 is smaller than 124 908. The last three digits do not matter in this case because it is obvious that 123 000 < 124 000. Do the examples as set out in the Learner's Book.
- Learners can draw a number grid with numbers from 1 to 100, and cross out all the multiples of 2, all the multiples of 3, etc. until they have found all the prime numbers between 1 and 100.

Remedial and extension

Remedial: If learners struggle with large numbers, e.g. 3 100; 3 200; 3 300; …, let them first ignore the thousands and focus on 100; 200; 300; 400; etc. Then they can re-write these numbers as 3 100; 3 200; etc.
Extension: Activity 1 question **4** is a challenge.

Review Copy

Suggested answers

Activity 1

1 **a** > **b** < **c** < **d** <
2 **a** 124 230; 124 620; 124 780; 125 100; 125 114; 125 350; 125 930
b R128 730 776; R248 712 594; R348 702 413; R378 720 685; R508 780 211
3 694 376 158 ml; 674 546 101 ml; 654 196 134 ml; 653 860 123 ml; 633 736 197 ml
4 97; 89; 83; 79; 73; 71; 67; 61; 59; 53; 47; 43; 41; 37; 31; 29; 23; 19; 17; 13; 11; 7; 5; 3; 2

Activity 2

1 **a** It decreased.
b R5 876 000 > R5 775 830
2 3 376 600; 3 980 600; 4 278 200; 4 675 000; 4 727 000; 4 786 500

UNIT

2 Properties of whole numbers

Unit overview

Learner's Book page 9
Recommended pacing: 45 minutes

This unit focuses on the following:

- recognising and using 0 in terms of its additive property
- recognising and using 1 in terms of its multiplicative property
- recognising and using the commutative, associative and distributive properties with whole numbers
- working with the inverse of the distributive property
- expanding numbers and then using the distributive property to find the answer
- using number sentences to find the unknown or the answer

Resources: Learner's Book; exercise book; calculator

Mental Maths

(10 minutes per day)

Suggested answers

Learner's Book page 9

1 a False; $5 \div 0$ = undefined
b True
c False; $5 + 0 = 5$
d True
e False; $1 \times 13 = 13$

2 a distributive law
b commutative law
c associative law
d distributive law
e associative law

3 a $10 \times 18 + 10 \times 40 = 180 + 400 = 580$
b $50 \times 15 - 50 \times 10 = 750 - 500 = 250$

4 a $12(3 + 18) = 12 \times 21 = 252$
b $35(16 - 10) = 35 \times 6 = 210$

5 a $8(30 + 4) = 8 \times 30 + 8 \times 4 = 240 + 32 = 272$
b $15(50 + 2) = 15 \times 50 + 15 \times 2 = 750 + 30 = 780$

6 a 70
b 720
c 50
d 2 250
e 680
f $25(30 - 2) = 25 \times 28 = 700$
g 40
h 300

Review Copy

Background information

- In Grades 6 and 7, learners studied the following:
  - commutative, distributive and associative properties
  - the *names* of the properties (for the first time in Grade 7)
  - 0 in terms of its additive property
  - 0 in terms of its division property (a number divided by 0 is undefined)
  - 1 in terms of its multiplicative property

- addition and subtraction as inverse operations
- multiplication and division as inverse operations.

- In Grade 8, revision of the above properties prepares learners for the introduction of numbers such as integers, and how these properties will be applied in these new contexts.
- Learners will also have to apply these properties of numbers in Algebra, when they work with variables.
- It is therefore very important that learners know and are able to use these properties.

Teaching guidelines

Again learners have encountered the properties that are dealt with in this unit, but it is important to ensure that everyone is comfortable with the terminology. These terms and phrases can also be added to learner's glossary if they have made one. Make sure learners give some of their own examples. You can ask learners to check the solutions on their calculators. Make sure they understand what "Maths error" means on a calculator.

When dealing with the properties of zero, adding, subtracting and multiplying zero should be well established in learners' minds already. There is often a problem with the difference between dividing zero by any number which still give zero, and dividing *by zero* which is undefined. Many learners do not always see the difference and think that dividing by zero is zero. It is really important to establish the difference between the two.

The properties of 1 do not usually present a problem. There are numerous situations where the 1 is "implied" and it is a good idea to remind learners about this often, for example $3 = 3^1 = \frac{3}{1} = 1 \cdot 3$ and so on.

Review Copy

The commutative, associative and distributive properties will be used again in Algebra and learners should be able to apply these rules then. Make sure that these laws are well-established now with numbers so that they can then be quickly revised.

Properties of 0; properties of 1; commutative, associative and distributive properties

Activity 1 **Investigate the properties of whole numbers**

Learner's Book page 11

Guidelines for implementing this activity

- To show learners that the properties are always true, ask them to make up their own examples, and to test the properties.
- Number sentences are used in this activity to make it easier for learners to see how the rules are applied.

Remedial and extension

Remedial: The number sentences lead learners and help them to use the laws and answer the questions.
Extension: Do another exercise with similar (and perhaps more difficult) questions. More difficult numbers that can be expanded in more than one way can be used (as in question 6a).

Suggested answers

1 a 16 b 12 c 0 d 21 *e 38
2 a 33; 55 b 16; 70 c 47; 470 d 8; 720
3 a 3 + 22 = 25 b 29 + 21 = 50 c 2 × 160 = 320 d 15 × 150 = 2 250
4 a 12; 6; 60 + 30 = 90 b 40; 20 × 6; 800 + 120 = 920
5 a 4(12 + 6) = 4 × 18 = 72 b 25(30 – 2) = 25 × 28 = 700
6 a 5(50 – 4) = 5 × 50 – 5 × 4 = 250 – 20 = 230
or 5(40 + 6) = 5 × 40 + 5 × 6 = 200 + 30 = 230
b 10(80 – 1) = 10 × 80 – 10 × 1 = 800 – 10 = 790
c 30(90 + 1) = 30 × 90 + 30 × 1 = 2 700 + 30 = 2 730
7 a 22 b 62 – 25 = 37 or 62 – 37 = 25
c 15 d 192; 16

UNIT 3

Calculations with whole numbers

Review Copy

Unit overview

Learner's Book page 12
Recommended pacing: 2,5 hours

This unit focuses on the following:

- estimating solutions by rounding off and by using doubling and halving
- adding and subtracting whole numbers without a calculator by first expanding the numbers
- multiplying and dividing whole numbers without a calculator by first expanding the numbers
- adding, subtracting, multiplying and dividing large whole numbers by using a calculator
- checking answers by doing the inverse operation and using calculators
- rounding off and compensating in order to check whether an answer has been calculated correctly
- solving problems that involve adding, subtracting, multiplying and dividing with whole numbers
- revising and practising the correct order of operations

Resources: Learner's Book; exercise book; calculator

Mental Maths

(10 minutes per day)

Suggested answers — Learner's Book page 12

1 a $480 + 230 = 710$ b $950 - 810 = 140$
c $300 \times 200 = 60\,000$ d $550 \div 110 = 5$

2 a $\approx 120 \times 50 \approx 60 \times 100 \approx 6\,000$ b $\approx 500 \times 480 \approx 1\,000 \times 240 \approx 240\,000$
c $\approx 220 \div 5 \approx 110 \div 10 \approx 11$ d $\approx 1\,000 \div 500 \approx 2$

3 a Estimate: $516 + 282 \approx 520 + 280 \approx 800$
Actual answer: $516 + 282 = 798$; Check: $798 - 282 = 516$
b Estimate: $369 - 259 \approx 370 - 260 \approx 110$
Actual answer: $369 - 259 = (300 + 60 + 9) - (200 + 50 + 9) = 110$;
Check: $110 + 259 = 369$
c Estimate: $9 \times 8 \approx 10 \times 8 \approx 80$
Actual answer: $9 \times 8 = 72$; Check: $72 \div 8 = 9$
d Estimate: $123 \times 5 \approx 120 \times 5 \approx 60 \times 10 \approx 600$
Actual answer: $123 \times 5 = 615$; Check: $615 \div 5 = 123$
e Estimate: $3\,980 \div 100 \approx 4\,000 \div 100 \approx 40$
Actual answer: $3\,980 \div 100 = 39{,}80$; Check: $39{,}80 \times 100 = 3\,980$
f Estimate: $875 \div 25 \approx 900 \div 25 \approx 36$
Actual answer: $875 \div 25 = 35$; Check: $35 \times 25 = 875$
g Estimate: $30 + 8 \times 5 \approx 30 + 10 \times 5 \approx 30 + 50 \approx 80$
Actual answer: $30 + 8 \times 5 = 30 + 40 = 70$; Check with calculator: 70
h Estimate: $6(11 - 3) + 12 \div 6 \approx 5(10) + 2 \approx 50 + 2 \approx 52$
Actual answer: $6(11 - 3) + 12 \div 6 = 6(8) + 2 = 48 + 2 = 50$;
Check with calculator: 50
*i Estimate: $250 - 12 \times 9 + 4 \approx 250 + 12 \times 10 + 0 \approx 250 - 120 \approx 130$
Actual answer: $250 - 12 \times 9 + 4 = 250 - 108 + 4 = (200 + 50) - (100 + 8) + (4)$
$= 100 + 50 - 4 = 146$; Check with calculator: 146

4 a True b True

Review Copy

Background information

- Learners have worked quite extensively with all the calculation strategies in the previous grades. The content in this unit is revision and practice of the strategies that they have learnt, and are designed to build confidence and polish their skills.
- The use of the calculator for calculations with larger numbers is encouraged.
- Estimation (judging the reasonableness of their answers) and checking answers (using inverse operations and calculators) using learnt methods are very important.

Teaching guidelines

Make sure learners get into the habit of estimating before they do calculations. Estimation is not a random guess, but involves judging the size or value of something, using the minimum amount of calculation possible. Rounding is very important in estimation and this should be revised. Learners often have misconceptions about rounding up and rounding down and tend to forget the 'cut-off' point. Most learners intuitively understand the concept of bigger and smaller than a half, so if they can clearly establish what the halfway mark is, they can more easily decide whether to round up or round down.

Estimation techniques include doubling and halving. Give learners mental maths activities to practise this technique throughout the year.

Spend some time revising the different methods of adding and subtracting, including the column method. This will be used again in Algebra, so it is important that the concept is well-grounded, with the more concrete numbers. Multiplying and dividing using the column method, and rounding off and compensating are also important.

Just as it is important that learners estimate before calculating, they should check their answers using the inverse operation: addition for subtraction and vice versa; multiplication for division and vice versa. Ask learners to write down the steps on page 11 of the Learner's Book and use them as a checklist until the habit has been well established.

Remind learners about the order of operations. They may have various methods of remembering the order, but establish the set of rules for Grade 8. These rules apply throughout Mathematics.

Estimating solutions

Rounding off to the nearest ten, hundred, thousand

Activity 1 **Estimating solutions**

Learner's Book page 13

Review Copy

Guidelines for implementing this activity

- Learners should be fairly competent at estimating answers using rounding off and doubling and halving, since this was focused on quite extensively in the Intermediate Phase.
- Revise rounding to the nearest ten and hundred. Halving and doubling is a useful method to quickly estimate the answer without using a calculator.
- Revise the problem-solving process with learners, also ensuring that they understand all vocabulary used in the word problems.

Remedial and extension

Remedial: Learners who struggle should write out more steps, e.g. in Activity 1 question **1j** we wrote out the steps in full.
Extension: Activity 1 questions **1i** and **1j** as well as Activity 2 pose a challenge to learners.

Suggested answers

1 a $318 + 281 \approx 320 + 280 \approx 600$
b $863 - 561 \approx 860 - 560 \approx 300$
c $198 \times 205 \approx 200 \times 200 \approx 40\,000$
d $991 \div 253 \approx 1\,000 \div 250 \approx 4$
e $2\,718 + 271 \approx 2\,700 + 270 \approx 2\,970$
f $3\,845 - 528 \approx 3\,800 - 530 \approx 3\,270$
g $1\,763 \times 103 \approx 1\,760 \times 100 \approx 176\,000$
h $4\,866 \div 504 \approx 5\,000 \div 500 \approx 10$
*i $2\,372 - 1\,250 + 111 \approx 2\,370 - 1\,250 + 110 \approx 1\,230$
*j $4\,987 - 3\,283 + 2\,183 \approx 4\,990 - 3\,280 + 2\,180 \approx 3\,890$

2 a $320 \times 50 = 160 \times 100 = 16\,000$
b $500 \times 128 \approx 500 \times 130 \approx 1\,000 \times 65 \approx 65\,000$
c $119 \times 48 \approx 120 \times 50 \approx 60 \times 100 \approx 6\,000$
d $498 \times 32 \approx 500 \times 32 \approx 1\,000 \times 16 \approx 16\,000$

3 $348 + 671 + 945 + 134 + 612 \approx 350 + 670 + 950 + 130 + 610 \approx 2\,710$

4 $48 \times$ R11,99 $\approx 50 \times$ R12 $\approx$ R600

Addition and subtraction of whole numbers

Adding, subtracting and checking solutions

Activity 2 Add and subtract whole numbers

Learner's Book page 14

Review Copy

Guidelines for implementing this activity

- Learners worked with the various calculation strategies extensively in the Intermediate Phase, and should be competent with at least one method (breaking down numbers into place-value parts; expanded column method; column method; rounding off and compensating). They often use the methods they are comfortable with, and should at this stage be encouraged to use column methods as far as possible.
- Emphasise the importance, purpose and, usefulness of first estimating the answer to a calculation, as well as checking using the inverse operation and/or a calculator.
- Learners can add or subtract numbers by breaking them down into place-value parts, if they still need to do this (remedial).
- Larger numbers may be added or subtracted with a calculator.

Remedial and extension

Remedial: Learners should write out the steps below one another so that they are less confusing.
Extension: Learners do questions **1e** and **1f**. Learners who complete the activity quickly and understand it well can make up their own "mini tests" and then give them to one another to complete. (Work in pairs.)

Suggested answers

1 a Estimate: $462 + 31 \approx 460 + 30 \approx 400 + 60 + 30 \approx 490$
Actual answer: $462 + 31 = 493$; Check: $493 - 31 = 462$

b Estimate: 514 + 385 ≈ 510 + 390 ≈ 500 + 300 + 10 + 90 ≈ 900
Actual answer: 899; Check: 899 – 385 = 514

c Estimate: 356 + 19 ≈ 360 + 20 ≈ 380
Actual answer: 375; Check: 375 – 19 = 356

d Estimate: 850 + 267 ≈ 850 + 270 ≈ (800 + 200) + (50 + 70) = 1 000 + 120 ≈ 1 120
Actual answer: 850 + 267 = 1 117; Check: 1 117 – 267 = 850

*e Estimate: 312 + 637 + 385 ≈ 310 + 640 + 390 ≈ 1 340
Actual answer: 312 + 637 + 385 = 1 334; Check: 1 334 – 385 – 637 = 312

*f Estimate: 825 + 132 + 677 ≈ 820 + 130 + 680 ≈ 1 630
Actual answer: 825 + 132 + 677 = 1 634; Check: 1 634 – 677 – 132 = 825

g Estimate: 469 – 349 ≈ 470 – 350 ≈ 120
Actual answer: 469 – 349 = 120; Check: 120 + 349 = 469

h Estimate: 798 – 506 ≈ 800 – 500 ≈ 300
Actual answer: 798 – 506 = 292; Check: 292 + 506 = 798

i Estimate: 149 – 137 ≈ 150 – 140 ≈ 10
Actual answer: 149 – 137 = 12; Check: 12 + 137 = 149

j Estimate: 585 – 372 ≈ 590 – 370 ≈ 220
Actual answer: 585 – 372 = 213; Check: 213 + 372 = 585

*k Estimate: 786 – 445 – 321 ≈ 790 – 450 – 320 ≈ 20
Actual answer: 786 – 445 – 321 = 20; Check: 20 + 321 + 450 = 786

*l Estimate: 875 – 550 – 24 ≈ 880 – 550 – 20 ≈ 310
Actual answer: 875 – 550 – 24 = 301; Check: 301 + 24 + 550 = 875

2 a 530 616 b 1 586 216
c 28 171 188 d 99 698 069
e 44 220 f 155 265 – 97 789 = 57 476
g 49 856 – 48 562 = 1 294 h 8 864 515 – 5 651 245 = 321 270

3 e A and C

4 a True b False

Review Copy

Round off and compensate

Activity 3 Round off and compensate

Learner's Book page 15

Guidelines for implementing this activity

- Explain that this method can also be used to check answers.
- Work carefully through the steps in the Learner's Book. This method is not unfamiliar to learners, but careful explanation may still be required.
- Revise the problem-solving process with learners, and ensure that they understand the questions and/or difficult words before proceeding. Encourage learners to use dictionaries for words that they do not understand.

Remedial and extension

Remedial: Learners should re-do these questions until they get them right. Alternatively, provide additional exercises. Short rounding-off and compensating exercises can also be included in the mental maths session every day, or as short 10-minute mental maths tests, e.g. round off one of the numbers to the nearest ten or hundred in the following calculations. Give the: i) estimated answer to 546 + 235; ii) Give the actual answer. Keep the calculations simple, and increase complexity with the last two or three questions of the 10 questions asked.
Extension: Learners should try doing question **3** without help.

Suggested answers

1 a 575 – 432 ≈ 575 – 430 ≈ (500 + 70 + 5) – (400 + 30) ≈ 100 + 40 + 5 ≈ 145; 145 – 2 = 143
b 679 – 257 ≈ 679 – 260 ≈ (600 + 70 + 9) – (200 + 60) ≈ 400 + 10 + 9 ≈ 419; 419 + 3 = 422
c 220 + 788 ≈ 220 + 790 = (200 + 700) + (20 + 90) = 900 + 110 = 1 010; 1 010 – 2 ≈ 1 008
d 766 + 103 ≈ 766 + 100 ≈ 866; 866 + 3 ≈ 869

2 a R4 102 + R3 895 + R5 012 + R4 568 – R11 987 ≈ R4 000 + R4 000 + R5 000 + R4 500 – R12 000 ≈ R5 500
b R5 590

*3 8 494 + 3 950 + 6 012 = 18 456

Review Copy

Multiplication and division of whole numbers

Activity 4 Multiply and divide whole numbers

Learner's Book page 17

Guidelines for implementing this activity

- Learners worked with the various multiplication and division calculation strategies quite extensively in the Intermediate Phase, and should be competent with at least one method.
- Smaller numbers must be multiplied and divided without using a calculator.
- They often use the methods they are most comfortable with, and at this stage should be encouraged to use column methods and long division as far as possible.
- Emphasise the importance, purpose and usefulness of estimating the answer to a calculation first, as well as checking using the inverse operation and/or a calculator.
- The "halving and doubling" method can't be used to estimate when dividing. Instead, learners must double halve both amounts. This method is important for Activity 8 question **3**.
- Larger numbers may be multiplied or divided using calculators where it is indicated.

Remedial and extension

Remedial: The best way to master calculation strategies is to practise them over and over. Learners should re-do the questions they got wrong until they can do the correct method and get the correct answers on their own. Also provide them with additional exercises to do for homework.
Extension: Learners that complete the activity quickly and understand it well can make up their own "mini tests" and then give them to one another to complete. (Work in pairs.)

Suggested answers

1 a 96 b 42 c 54 d 4 630
e 62 100 f 239 000

2 a Estimate: $324 \times 5 \approx 320 \times 5 \approx 160 \times 10 \approx 1\,600$
Actual answer: $324 \times 5 = 1\,620$; Check: $1\,620 \div 5 = 324$
b Estimate: $657 \times 5 \approx 660 \times 5 \approx 330 \times 10 \approx 3\,300$
Actual answer: 3 285; Check: $3\,285 \div 5 = 657$
c Estimate: $303 \times 50 \approx 300 \times 50 \approx 150 \times 100 \approx 15\,000$
Actual answer: 15 150; Check: $15\,150 \div 50 = 303$
d Estimate: $657 \times 500 \approx 650 \times 50 \approx 325 \times 100 \approx 32\,500$
Actual answer: 328 500; Check: $328\,500 \div 500 = 657$
e Estimate: $480 \times 25 \approx 500 \times 25 \approx 250 \times 50 \approx 125 \times 100 \approx 12\,500$
Actual answer: 12 000; Check: $12\,000 \div 25 = 480$
f Estimate: $816 \times 20 \approx 800 \times 20 \approx 1\,600 \times 10 \approx 16\,000$
Actual answer: 16 320; Check: $16\,320 \div 20 = 816$
g Estimate: $473 \times 200 \approx 500 \times 200 \approx 100\,000$
Actual answer: 94 600; Check: $94\,600 \div 200 = 473$
h Estimate: $1\,050 \times 250 \approx 1\,000 \times 250 \approx 250\,000$
Actual answer: 262 500; Check: $262\,500 \div 250 = 1\,050$
i Estimate: $921 \times 250 \approx 900 \times 250 \approx 450 \times 500 \approx 225 \times 1\,000 \approx 225\,000$
Actual answer: 230 250; Check: $230\,250 \div 250 = 921$

3 a 67; $67 \times 10 = 670$ b 61,2; $61{,}2 \times 10 = 612$
c 72,8; $72{,}8 \times 100 = 7\,280$ d 5,73; $5{,}73 \times 1\,000 = 5\,730$

4 a Estimate: $2\,855 \div 5 \approx 2\,900 \div 5 \approx 5\,800 \div 10 \approx 580$
Actual answer: 571; Check: $571 \times 5 = 2\,855$
b Estimate: $9\,765 \div 5 \approx 10\,000 \div 5 \approx 20\,000 \div 10 \approx 2\,000$
Actual answer: 1 953; Check: $1\,953 \times 5 = 9\,765$
c Estimate: $7\,650 \div 50 \approx 7\,700 \div 50 \approx 15\,400 \div 100 \approx 154$
Actual answer: 153; Check: $153 \times 50 = 7\,650$
d Estimate: $18\,500 \div 500 \approx 19\,000 \div 500 \approx 38\,000 \div 1\,000 \approx 38$
Actual answer: Check: $37 \times 500 = 18\,500$
e Estimate: $10\,775 \div 25 \approx 11\,000 \div 25 \approx 44\,000 \div 100 \approx 440$
Actual answer: 431; Check: $431 \times 25 = 10\,775$

5 a 2 563 b 489 c 569 d 657

Review Copy

Round off and compensate

Activity 5 Round off and compensate

Learner's Book page 18

Guidelines for implementing this activity

- Explain to learners that this method can also be used to check answers.
- Work carefully through the steps in the Learner's Book. This method is not unfamiliar to learners, but careful explanation may still be required. Mention that we are using the distributive law when we multiply 553 by each number in brackets.
- Learners should also be encouraged, and this should be monitored, to break down multiples of 10 into smaller numbers when multiplying, as in $553 \times 20 = (553 \times 2 \times 10)$.
- Encourage learners to use dictionaries for words that they do not understand when doing problem-solving.

Remedial and extension

Remedial: Learners who struggle with this concept should re-do the exercise until they get the method and answers correct for all calculations. They can use calculators to help them during all steps.
Extension: The checking of answers by rounding off and compensating is already a challenge for learners. Activity 8 is a problem-solving activity and therefore more challenging. Learners should write number sentences and then solve the problems.

Review Copy

Suggested answers

1 a $30 + 2$ b $20 + 1$ c $20 - 1$ d $200 - 4$
2 $6 \times 4 \times \square = 48$; $48 \div 6 \div 4 = 8 \div 4 = 2$. Two boxes can fit into the height of the crate.
3 $30 \times 1 \times 20 = \square$; $30 \times 20 = 600$; They will pick up 600 pieces of litter in a period of 20 days.
*4 a R3 357,50 ÷ 85 ≈ R4 000 ÷ 100 ≈ R40
b R39,50

Revision: The order of operations

Activity 6 Practise the correct order of operations

Learner's Book page 19

Guidelines for implementing this activity

- Revise the order of operations: brackets first, then multiply/divide, then add/subtract. Do examples with learners so they can see how to do calculations step-by-step.

- It is important to do examples where a number is next to a bracket, e.g. $2(15 + 10)$ or $(30 \div 15)20$, and to explain that a number next to a bracket means multiplication.

Remedial and extension

Remedial: The best way for leraners to master these methods is to practise them over and over. Learners should re-do the questions they got wrong until they can use the correct method and get the correct answer on their own. Provide additional exercises for homework.
Extension: Learners who complete the activity quickly and understand it well can make up their own "mini tests" and then give them to one another to complete. (Work in pairs.) They cazn also be given additional, more challenging calculations to do (without the use of a calculator).

Suggested answers

1 $250 \div 50 - 5 = 5 - 5 = 0$
2 $25 + 6 \times 2 = 25 + 12 = 37$
3 $24 + 80 \div 4 = 24 + 20 = 44$
4 $3(70 \div 7 \times 5) = 3(10 \times 5) = 3(50) = 150$
5 $44 - 6 \times 3 = 44 - 18 = 26$
6 $(9 \times 3 + 3) \times 40 = (27 + 3)40 = 30(40) = 1\,200$
7 $300 - 15 \times 4 + 1 = 300 - 60 + 1 = 240 + 1 = 241$
8 $12(8 - 3) + 4 \div 2 = 12(5) + 2 = 60 + 2 = 62$
9 $3 \times 3 - 2 \times 2 + 5 = 9 - 4 + 5 = 5 + 5 = 10$
10 $300 \div 6(4 + 3) = 600 \div 6 \times 7 = 100 \times 7 = 700$
11 $25 + 16 \div 4 + 8 = 25 + 4 + 8 = 29 + 8 = 37$
12 $12 + 1 \times 8 - 6 = 12 + 8 - 6 = 20 - 6 = 14$
13 $100 - (7 + 3 \times 20) = 100 - (7 + 60) = 100 - 67 = 33$
14 $20 - 16 \div 16 \times 17 = 20 - 1 \times 17 = 20 - 17 = 3$
*15 $16 + (14 - 7 \times 2)3 - 15 = 16 + (14 - 14)3 - 15 = 16 + 0(3) - 15 = 16 - 15 = 1$
*16 $230 - 15 \div 3 \times (16 \div 2 + 2) = 230 - 5(8 + 2) = 230 - 5(10) = 230 - 50 = 180$

Review Copy

UNIT 4

Multiples and factors

Unit overview

Learner's Book page 20
Recommended pacing: 1 hour

This unit focuses on the following:

- determining multiples and the Lowest Common Multiple
- determining factors and the Highest Common Factor
- defining prime numbers and prime factors
- writing numbers as products of their prime factors
- finding the HCF by first finding the number's prime factors
- solving problems involving multiples and factors

Resources: Learner's Book; exercise book; calculator

Mental Maths

(10 minutes per day)

Suggested answers

Learner's Book page 20

Review Copy

1 a M_{65} = {65; 130; 195; 260; 325; 390}
b M_{78} = {78; 156; 234; 312; 390; 468}
c M_{110} = {110; 220; 330; 440; 550; 660}
d M_{385} = {385; 770; 1 155; 1 540; 1 925; 2 310}
e M_{450} = {450; 900; 1 350; 1 800; 2 250; 2 700}
f M_{900} = {900; 1 800; 2 700; 3 600; 4 500; 5 400}

2 a LCM = 390 b LCM = 770 *c LCM = 11 700

3 a F_{48} = {1; 2; 3; 4; 6; 8; 12; 16; 24; 48}
b F_{142} = {1; 2; 71; 142}
c F_{230} = {1; 2; 5; 23; 46; 115; 230}
d F_{276} = {1; 2; 3; 4; 6; 12; 23; 46; 69; 92; 138; 276}
e F_{516} = {1; 2; 3; 4; 6; 12; 43; 86; 129; 172; 258; 516}

4 a PF_{48} = {2; 3} b PF_{142} = {2; 71} c PF_{230} = {2; 5; 23}
d PF_{276} = {2; 3; 23} e PF_{516} = {2; 3; 43}

5 a $48 = \mathbf{2} \times 2 \times 2 \times 2 \times 3$
$142 = \mathbf{2} \times 71$
HCF = 2
b $142 = \mathbf{2} \times 71$
$230 = \mathbf{2} \times 5 \times 23$
HCF = 2
*c $48 = \mathbf{2} \times \mathbf{2} \times 2 \times 2 \times \mathbf{3}$
$276 = \mathbf{2} \times \mathbf{2} \times \mathbf{3} \times 23$
$516 = \mathbf{2} \times \mathbf{2} \times \mathbf{3} \times 43$
HCF = $2 \times 2 \times 3 = 12$

6 (Use the "ladder" method)
a PF_{75} = {3; 5} b PF_{124} = {2; 31} c PF_{162} = {2; 3}
d PF_{210} = {2; 3; 5; 7} e PF_{320} = {2; 5} f PF_{500} = {2; 5}

7 a $75 = 3 \times 5 \times 5$
b $124 = 2 \times 2 \times 31$
c $162 = 2 \times 3 \times 3 \times 3 \times 3$
d $210 = 2 \times 3 \times 5 \times 7$
e $320 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5$
f $500 = 2 \times 2 \times 5 \times 5 \times 5$

8 a $75 = \mathbf{3} \times \mathbf{5} \times 5$
$210 = 2 \times \mathbf{3} \times \mathbf{5} \times 7$
HCF $= 3 \times 5 = 15$

b $124 = \mathbf{2} \times 2 \times 31$
$162 = \mathbf{2} \times 3 \times 3 \times 3 \times 3$
HCF $= 2$

c $320 = 2 \times 2 \times 2 \times 2 \times \mathbf{2} \times \mathbf{2} \times \mathbf{5}$
$500 = \mathbf{2} \times \mathbf{2} \times \mathbf{5} \times 5 \times 5$
HCF $= 2 \times 2 \times 5 = 20$

Background information

- This unit serves as revision of all work done in previous grades. The content covered in this unit will become important when learners calculate with fractions (finding the LCM of denominators that are different from each other).
- Factorising whole numbers also lays the foundation for factorising algebraic expressions.

Teaching guidelines

Learners have been doing multiples and factors for a number of years. This unit revises the work done so far. This work will be used for fractions – numerical and algebraic.

Ensure that learners understand that the first multiple of a number is the number itself. Have learners write down a few multiples for several different numbers and then start looking for common multiples. Once this is established, finding the Lowest Common Multiple (LCM) is not that difficult. Let learners practise a few of these and use the Think-Do examples.

Remind learners that the number itself and 1 are both factors of a number. Steps for finding factors are given on page 22 of the Learner's Book and learners can refer back to these if they forget the method. Once learners have found the common factors for various numbers, ask them to circle the highest common factors of two or three numbers.

The ladder method is a very useful way to factorise. It requires a number of skills and learners should practise these until they are able to do them easily.

Review Copy

Multiples

Lowest Common Multiple (LCM)

Activity 1 Write and work with multiples

Learner's Book page 21

Guidelines for implementing this activity

- Remind learners of their times tables. Say a few numbers' multiples aloud with them, e.g. 3; 6; 9… 36. Then show them how to get the multiples of larger numbers just by adding the same number again and again.
- Work through the examples in the Learner's Book. To find the LCM they should write out the multiples of the numbers until they see a common number in both sets.

Remedial and extension

Remedial: Learners can add numbers or use their calculators to find multiples, e.g. 6 + 6 = = = = = = = on a non-scientific calculator. Learners should ideally practise and master their multiplication tables.
Extension: Learners do question **7c**.

Review Copy

Suggested answers

1 a M_2 = {2; 4; 6; **8**; 10; 12; 14; **16**; 18; 20; 22; **24**}
  M_4 = {4; **8**; 12; **16**; 20; **24**; 28; 32; 36; 40; 44; 48}
  M_8 = {**8**; **16**; **24**; 32; 40; 48; 56; 64; 72; 80; 88; 96}
  b (common multiples in bold above)
  c 8
2 a M_3 = {3; 6; 9; 12; 15; **18**; 21; 24; 27; 30; 33; **36**; 39; 42; 45}
  M_6 = {6; 12; **18**; 24; 30; **36**; 42; 48; 54; 60; 66; 72; 78; 84; 90}
  M_9 = {9; **18**; 27; **36**; 45; 54; 63; 72; 81; 90; 99; 108; 117; 126; 135}
  b (common multiples in bold above)
  c 18
3 The second digit of the number is double the first digit of the number.
4 M_{11} = {11; 22; 33; 44; 55; 66; 77; 88; 99}
5 690 ÷ 30 = 23
6 a M_{50} = {50; 100; 150; 200; 250; 300}
  b M_{120} = {120; 240; 360; 480; 600; 720}
  c M_{315} = {315; 630; 945; 1 260; 1 575; 1 890}
  d M_{402} = {402; 804; 1 206; 1 608; 2 010; 2 412}
  e M_{875} = {875; 1 750; 2 625; 3 500; 4 375; 5 250}
7 a LCM = 560
  b LCM = 420
  *c LCM = 1 860

Factors

Steps for finding all the factors of a number; Highest Common Factor; Prime numbers and prime factors

Activity 2 Find and work with factors

Learner's Book page 22

Guidelines for implementing this activity

- The "thinking process" is shown in detail in the examples and the steps in the Learner's Book. It is important to work methodically.
- Work step-by-step through each example with the learners.
- Let a learner do an additional example of each concept on the board while the other learners tell him or her what the next step is if he/she struggles (finding factors; finding the HCF; prime numbers and prime factors).
- Learners worked with the divisibility rules in Grade 6, and these should also be revised here.

Remedial and extension

Remedial: Learners can use their calculators, e.g. 100 ÷ 1, 100 ÷ 2, 100 ÷ 3, 100 ÷ 4, etc. until they have found all the factors. This process is time-consuming but learners who struggle often find this method easier.
Extension: Question **11a** is an excellent challenge. Learners will have to work methodically to find *all* the factors.

Review Copy

Suggested answers

1 a F_{60} = {1; 2; 3; 4; 5; 6; 10; 12; 15; 20; 30; 60}
  b PF_{60} = {2; 3; 5}
  c CF_{60} = {4; 6; 10; 12; 15; 20; 30; 60}
2 No; F_{31} = {1; 31}; 1 is not a prime number
3 F_{93} = {1; 3; 31; 93}; CF_{93} = {93}
4 M_4 = {**4**; **8**; **12**; 16; **20**; **24**; 28; 32; 36; **40**; 44; 48; 52; 56; 60}
  F_{120} = {1; 2; 3; **4**; 5; 6; **8**; 10; **12**; 15; **20**; 24; 30; **40**; **60**; 120}
5 F_{70} = {1; 2; 5; 7; 10; 14; 35; 70} PF_{70} = {2; 5; 7}
  No; 70 has eight factors and only three prime factors.
6 a F_{50} = {1; 2; 5; 10; 25; 50}
  b F_{106} = {1; 2; 53; 106}
  c F_{215} = {1; 5; 43; 215}
  d F_{380} = {1; 2; 4; 5; 10; 19; 20; 38; 76; 95; 190; 380}
  e F_{450} = {1; 2; 3; 5; 6; 10; 15; 18; 25; 30; 45; 75; 90; 150; 225; 450}
7 a PF_{50} = {2; 5} b PF_{106} = {2; 53} c PF_{215} = {5; 43}
  d PF_{380} = {2; 5; 19} e PF_{450} = {2; 3; 5}
8 a $50 = \mathbf{2} \times 5 \times 5$
    $106 = \mathbf{2} \times 53$
    HCF = 2
  b $215 = \mathbf{5} \times 43$
    $380 = 2 \times 2 \times \mathbf{5} \times 19$
    HCF = 5

*c $50 = \mathbf{2} \times \mathbf{5} \times 5$
$380 = \mathbf{2} \times 2 \times \mathbf{5} \times 19$
$450 = \mathbf{2} \times 3 \times \mathbf{3} \times \mathbf{5} \times 5$
$HCF = 2 \times 5 = 10$

9 a 3; 6; 9; 12; 15; 18; 21; 24; 27; 30; 33; 36; 39; 42; 45; 48; 51; 54; 57; 60
b No. 28 is not a multiple of 3.

10 *a $F_{1\,440}$ = {1; 2; 3; 4; 5; **6**; 8; 9; 10; 12; 15; 16; 18; 20; 24; 30; 32; 36; 40; 45; 48; 60; 72; 80; 90; 96; 120; 144; 160; 180; **240**; 288; 360; 480; 720; 1 440}
b She must take her medicine six times per day.

Factorisation: Numbers as products of prime factors

Activity 3 Products of prime factors and HCFs

Learner's Book page 24

Guidelines for implementing this activity

Use the ladder method.

- **Step 1:** Write down the number.
- **Step 2:** Divide by the smallest prime number that can go into that number, e.g. if it is an even number, then divide by 2. Keep on dividing by 2 until it becomes an odd number.
- **Step 3:** Is 3 a factor? Divide by 3 until it becomes a number that is not a multiple of 3.
- **Step 4:** Continue dividing by all the prime numbers that are factors of the number, until 1 is left.

The following can be seen from the ladder:

- the prime factors of the number (use this for Activity 3, question **1**)
- all the prime numbers that must be multiplied to get the number (use this for Activity 3, question **2**).

Review Copy

Remedial and extension

Remedial: Let learners first practise the ladder method with easier "round" numbers like 200, 250, 1 000, etc.
Extension: Learners can write their own numbers as the product of their prime numbers. Every time they get it right, they can start choosing larger and larger numbers to challenge themselves.

Suggested answers

1 (ladder method)
a PF_{80} = {2; 5}
b PF_{100} = {2; 5}
c PF_{138} = {2; 3; 23}
d PF_{204} = {2; 3; 17}
e PF_{315} = {3; 5; 7}
f PF_{600} = {2; 3; 5}

2 a $80 = 2 \times 2 \times 2 \times 2 \times 5$
b $100 = 2 \times 2 \times 5 \times 5$
c $138 = 2 \times 3 \times 23$
d $204 = 2 \times 2 \times 3 \times 17$
e $315 = 3 \times 3 \times 5 \times 7$
f $600 = 2 \times 2 \times 2 \times 3 \times 5 \times 5$

3 a $80 = 2 \times 2 \times \mathbf{2} \times \mathbf{2} \times \mathbf{5}$
b $138 = \mathbf{2} \times 3 \times 23$
$100 = \mathbf{2} \times \mathbf{2} \times \mathbf{5} \times 5$
HCF $= 2 \times 2 \times 5 = 20$
$204 = \mathbf{2} \times 2 \times 51$
HCF $= 2$
c $315 = 3 \times \mathbf{3} \times \mathbf{5} \times 7$
$600 = 2 \times 2 \times 2 \times \mathbf{3} \times \mathbf{5} \times 5$
HCF $= 3 \times 5 = 15$

UNIT

5 Ratio and rate

Unit overview

Learner's Book page 25
Recommended pacing: 1 hour

This unit focuses on the following:

- writing and simplifying ratios
- dividing quantities into parts (ratios)
- increasing/decreasing a number in a given ratio
- writing and working with rate, including speed, distance and time
- working with exchange rates

Resources: Learner's Book; exercise book; calculator

Review Copy

Mental Maths

(10 minutes per day)

Suggested answers

Learner's Book page 25

1 a 1 : 2 b 3 : 1 c 1 : 6 : 1 d 7 : 10 : 16
2 a $5 + 1 = 6$ parts; one part $= 90 \div 6 = 15$; $15 \times 5 = 75$; 75 : 15
b $4 + 2 + 3 = 9$ parts; one part $= 513 \div 9 = 57$; $57 \times 4 = 228$; $57 \times 2 = 114$; $57 \times 3 = 171$; 228 : 114 : 171
3 a $24 \div 3 = 8$; $8 \times 5 = 40$
b $240 \div 6 = 40$; $40 \times 13 = 520$
c R3 000 ÷ 25 = 120; 120 × 26 = R3 120
4 a $52 \div 13 = 4$; $4 \times 9 = 36$
b $660 \div 30 = 22$; $22 \times 29 = 638$
c R5 120 ÷ 2 = R2 560; R2 560 × 1 = R2 560
5 a False b True c True

Background information

Learners continue to solve problems involving ratio and rate.

Teaching guidelines

The difficulty that learners experience with ratio and rate is the difference between the terms. Establish that ratio compares two (or more) similar quantities. Learners should make sure that both values are written with the same unit; however, the answer does not have units as the units cancel out. Ratio can be used to divide a number in a given ratio as well as to increase or decrease the number in a given ratio. Draw on learners' prior knowledge of mixing cold drinks or paint to make the concept more concrete. Show learners the different notations of writing ratios, for example 2 : 3 is equivalent to $\frac{2}{3}$.

Rate is when one quantity is measured in relation to another, for example distance in a particular amount of time (speed), exchange rates, and so on. It is important to give learners extra practice if necessary.

Ratio

Activity 1 Simplify ratios

Learner's Book page 26

Guidelines for implementing this activity

- Learners can simplify ratios by dividing all the numbers by their HCF.
- Let learners do a few examples on the board and follow the steps as shown in the examples in the Learner's Book.

Review Copy

Remedial and extension

Remedial: Learners who struggle with this concept should re-do the exercise until they can simplify all the ratios correctly, and/or be provided with additional exercises in order to practise.
Extension: Learners do questions **2** to **9**.

Suggested answers

1 **a** 1 : 2 **b** 3 : 1 **c** 3 : 13 **d** 32 : 71
e 41 : 57 **f** 15 : 19 ***g** 9 : 12 : 2 ***h** 7 : 9 : 3
2 3 : 5
3 3 440 : 215 = 16 : 1
4 6 : 4 : 8 = 3 : 2 : 4
5 R26 : R24 = 13 : 12
6 312 : 534 = 52 : 89
7 2 GB : 4 GB = 1 : 2
8 R175 : R245 = 5 : 7
9 R3 417 : R2 295 = 67 : 45

Ratio: Dividing quantities into parts

Activity 2 Divide quantities into parts

Learner's Book page 27

Guidelines for implementing this activity

- Explain the concept by drawing a long rectangle on the board and then dividing it into eight parts (to demonstrate the example in the Learner's Book). Shade two parts one colour, three parts another colour and three yet another colour to show that Joseph gets two parts, Ashwin gets three parts and Christina gets three parts but altogether there are eight parts. Then continue with steps as set out in the Learner's Book.

Remedial and extension

Remedial: Learners who struggle with this concept should re-do the exercise until they can remember how to follow all the steps and until they find the correct answers for all the questions. Practice is of the utmost importance.
Extension: Learners can work in pairs, make up their own exercises with more "dividing quantities into parts" problems and then give them to one another to do.

Review Copy

Suggested answers

1 a 2 + 3 = 5 parts; 1 part = 50 ÷ 5 = 10; 10 × 2 = 20; 10 × 3 = 30; 20 : 30
b 8 parts; 1 part is 11; 55 : 33
c 5 parts; 1 part is 15; 15 : 60
d 7 parts; 1 part is 30; 60 : 150
e 8 parts; 1 part is 50; 250 : 150
f 3 parts; 1 part is 12; 24 : 12
g 4 parts; 1 part is 16; 16 : 48
h 12 parts; 1 part is 12; 60 : 84
i 10 parts; 1 part is 30; 60 : 90 : 150
j 7 parts; 1 part is 8; 24 : 8 : 24

2 5 parts; 1 part is R15; Salome made a profit of R30 and Busisiwe made a profit of R45.
3 7 parts; 1 part is 6 fruit; there are 6 apples and 36 strawberries in the fruit basket.
4 9 parts; 1 part is 10 people; there are 20 doctors and 70 nurses at the hospital.
5 5 parts;1 part is 25 learners; 75 learners jogged and 50 learners cycled
6 3 parts; 1 part is 4 music concerts; Lila attended 4 music concerts and Mandisa attended 8 music concerts.
7 8 parts; 1 part is 5 CDs; Kaleb has 15 CDs and Naeem has 25 CDs.
8 10 parts; 1 part is 20 dancers; There are 100 hip-hop dancers, 40 contemporary dancers and 60 jazz dancers at the dance school.
9 9 parts; 1 part is 111 ml; Precious uses 555 ml of fresh fruit, 222 ml of yoghurt and 222 ml of orange juice.
10 7 parts; 1 part is 2 goals; Michael scored 2 goals, Lindiwe scored 8 goals and Zaid scored 4 goals.

Increase/decrease a number in a given ratio

Activity 3 Increase/decrease a number in a given ratio

Learner's Book page 29

Guidelines for implementing this activity

This method is basically the direct proportion method. Learners should first write down the ratio. The second number will always represent the old amount. Therefore write the old amount on the right-hand side underneath the ratio's amount. The first number represents the new amount, so write the question mark below the first number of the ratio. Then follow the steps as set out in the Learner's Book.

Remedial and extension

Remedial: Learners who struggle with this concept should re-do the exercise until they can remember how to follow all the steps and find the correct answers for all the questions.
Extension: Learners do Questions **3** to **5**.

Suggested answers

1 a $30 \div 6 = 5$; $5 \times 7 = 35$
  b R400 ÷ 8 = 50; 50 × 11 = R550
  c 740 ÷ 20 = 37; 37 × 23 = 851 learners
2 a $45 \div 9 = 5$; $5 \times 7 = 35$
  b R750 ÷ 5 = R150; R150 × 3 = R450
  c 320 ÷ 4 = 80; 80 × 1 = 80 racing cars
3 R150 000 ÷ 3 = R50 000; R50 000 × 4 = R200 000
4 400 ml ÷ 10 = 40 ml; 40 ml × 7 = 280 ml
5 6 trees ÷ 3 = 2 trees; 2 trees × 5 = 10 trees

Review Copy

Rate; Time, distance and speed

Activity 4 Write down rates, including speed, distance and time

Learner's Book page 31

Guidelines for implementing this activity

- Use everyday examples to explain this concept, e.g. a parent driving a certain speed in km/h; a parent earning a certain amount of money per hour, etc.
- Show learners how to substitute the correct numbers in given formulae, how to calculate the answers and then how to write the correct units.

Remedial and extension

Remedial: Learners who struggle with this concept should re-do the exercise until they find the correct answers for all the questions. Practice is important.
Extension: Learners do questions **12**, **13** and **14**.

Suggested answers

1 8 km/1 h = 8 km/h
2 R40/20 bags = R2/bag
3 R1 000/4 h = R250/h
4 28 songs/2 h = 14 songs/hour
5 R200/8 h = R25/h
6 R5 580/20 days = R279/day
7 20 burgers/15 min = 80 burgers/1 h = 80 burgers/h
(multiplied the number of burgers and the time by 4)
8 R54 000/12 months = R4 500/month
9 620 km/50 ℓ = 12,4 km/ℓ
10 400 km/6 h ≈ 66,67 km/h
11 30 km/h × 3,25 h = 97,5 km
***12** 10 km/(15 ÷ 60)h = 10 km/0,25 h = 40 km/h
***13** 100 km/38 km/h = 2,63… h; 2,63… h × 60 ≈ 158 minutes
***14** 8 km/65 km/h = 0,12… h; 20,12… h × 60 ≈ 8 minutes (round up as he won't be able to reach his mother's house in 7 minutes.)

Exchange rates

Activity 5 Work with exchange rates

Learner's Book page 33

Review Copy

Guidelines for implementing this activity

- Explain different currencies, e.g. R1 is not the same as $1. The rand is weaker than the dollar, so we have to pay many rand to buy a few dollars.
- To convert currencies: If the exchange rate is given as: 1 unit of of another currency = R…, we can:
  - convert from rand to any other currency by dividing
  - convert from any currency to rand by multiplying.
- Encourage learners to keep up to date with common currencies such as the US dollar, the Japanese yen, and so on, and to feed back to class about what the exchange rates are.
- Use the latest exchange rates to do quick conversions in class. Encourage learners to provide estimates as well.
- Use real life scenarios where possible, e.g. I want to buy a pair of shoes in London for £40. At the current exchange rate, how many rand will I be spending?

Remedial and extension

Remedial: Learners should follow the instructions to decide whether they have to multiply or divide. They can use their calculators.

Suggested answers

1 a R5 000 ÷ R8,4661/US$ ≈ US$590,59
b R3 500 ÷ R8,33502/Aus$ ≈ Aus$419,92
c R7 500 ÷ R1,0812/BP ≈ BP6 936,74
d R1 400 ÷ R0,106737/¥ ≈ ¥13 116,35
e R20 000 ÷ R12,9772/£ ≈ £1 541,16
f R800 ÷ R8,78813/SF ≈ SF91,03

2 a $250 × R8,4661/$ ≈ R2 116,53
b £400 × R12,9772/£ = R5 190,88
c €3 200 × R10,5563/€ = R33 780,16
d 6 000 Botswana pula × R1,0812/BP = R6 487,20
e 60 000 Malawian kwacha × R0,03111/MK = R1 866,60
f ¥50 000 × R0,106737/¥ = R5 336,85

Chapter 1 Revision

Learner's Book page 34

Review Copy

Encourage learners to review the content covered before attempting the revision activities. The revision activities should be used to assess learners' progress thus far, and to assess where remediation may be required.

Suggested answers

1 a < b >
2 R125 876 599; R189 657 841; R358 674 258; R456 987 325; R565 715 548; R589 632 574
3 a 313 b 0 c 6
4 a 4; 20 b 30; 720
5 a 20; 12 + 38 = 50 b 3; 18 × 12 = 216
6 50; 3; 550 + 33 = 583
7 14(20 – 5) = 14(15) = 210
8 a 5(30 + 4) = 5 × 30 + 5 × 4 = 150 + 20 = 170
b 10(30 – 2) = 10 × 30 – 10 × 2 = 300 – 20 = 280
c 20(100 – 3) = 20 × 100 – 20 × 3 = 2 000 – 60 = 1 940
9 a 44; 62 or 62; 44 b 1 050; 150
10 a ≈ 580 + 100 ≈ 680 b ≈ 50 × 300 ≈ 100 × 150 ≈ 15 000
c ≈ 14 000 ÷ 1 000 ≈ 14
11 a ≈ 70 × 50 ≈ 35 × 100 ≈ 3 500 b ≈ 500 × 32 ≈ 1 000 × 16 ≈ 16 000
c ≈ 900 ÷ 5 ≈ 1 800 ÷ 10 ≈ 180 d ≈ 600 ÷ 50 ≈ 1 200 ÷ 100 ≈ 12

12 **a** Estimate: 860 + 140 ≈ 1 000; Actual answer: 999; Check: 999 – 138 = 861
b Estimate: 780 – 530 ≈ 250; Actual answer: 248; Check: 248 + 527 = 775
c Estimate: 860 – 310 + 540 ≈ 1 090
Actual answer: 1 088; Check: 1 088 – 540 + 310 = 860
d Estimate: 10 × 4 ≈ 40
Actual answer: 36; Check: 36 ÷ 4 = 9
e Estimate: 700 × 5 ≈ 350 × 10 ≈ 3 500
Actual answer: 3 565; Check: 3 565 ÷ 5 = 713
f Estimate: 1 700 ÷ 25 ≈ 6 800 ÷ 100 ≈ 68
Actual answer: 66; Check: 66 × 25 = 1 650

13 **a** correct **b** incorrect, 258

14 **a** 30 + 3 **b** 20 – 3

15 **a** 112(30 + 3) = 112 × 30 + 112 × 3 = 3 360 + 336 = 3 696
b 728(20 – 3) = 728 × 20 – 728 × 3 = 14 560 – 2 184 = 12 376

16 **a** 130 – 48 + 8 = 82 + 8 = 90 **b** 12(6) + 4 = 72 + 4 = 76
c 34 + 11 × 8 = 34 + 88 = 122 **d** (88 – 80)3 – 8 = 8(3) – 8 = 24 – 8 = 16

17 **a** M_{65} = {65; 130; 195; 260; 325} **b** M_{156} = {156; 312; 468; 624; 780}
c M_{114} = {114; 228; 342; 456; 570} **d** M_{136} = {136; 272; 408; 544; 680}

18 **a** 780 **b** 1 080 **c** 408

19 **a** F_{100} = {1; 2; 4; 5; 10; 20; 25; 50; 100} **b** F_{175} = {1; 5; 7; 25; 35; 175}
c F_{230} = {1; 2; 5; 10; 23; 46; 115; 230} **d** F_{452} = {1; 2; 4; 113; 226; 452}

20 **a** PF_{100} = {2; 5} **b** PF_{175} = {5; 7} **c** PF_{230} = {2; 5; 23} **d** PF_{452} = {2; 113}

21 **a** 100 = 2 × 2 × 5 × 5 **b** 175 = 5 × 5 × 7
c 230 = 2 × 5 × 23 **d** 452 = 2 × 2 × 113

22 **a** 100 = 2 × 2 × **5** × **5**
175 = **5** × **5** × 7
HCF = 5 × 5 = 25
b 175 = 5 × **5** × 7
230 = 2 × **5** × 23
HCF = 5
c 100 = **2** × 2 × 5 × 5
230 = **2** × 5 × 23
452 = **2** × 2 × 113
HCF = 2

23 **a** M_{16} = {16; 32; 48; 64; 80; 96; 112; 128; 144; 160; 176; 192; 208; **224**; 240}
b 14 boxes must be bought. (224 is the 14th multiple of 16.)

24 **a** 1 : 5 **b** 157 : 7 **c** 4 : 1 **d** 6 : 9 : 5

25 55 min : 40 min = 11 : 8

26 **a** 16 parts; 1 part is 10; 50 : 110
b 6 parts; 1 part is R5 000; R5 000 : R15 000 : R10 000

27 **a** 55 ÷ 5 = 11; 11 × 7 = 77 **b** R650 ÷ 25 = R26; R26 × 28 = R728

28 **a** 80 ÷ 8 = 10; 10 × 5 = 50 **b** R2 000 ÷ 4 = R500; R500 × 3 = R1 500

29 315 ÷ 9 = 35; 35 × 10 = 350 learners

30 R86 : 2 kg = R43/kg

31 S = D ÷ T = 69 km ÷ 1,1 h ≈ 62,73 km/h

32 D = S × T = 8 km/h × 1,25 h = 10 km

33 T = D ÷ S = 50 km ÷ 9,5 km/h = 5,263… h
5,263… h × 60 ≈ 315,79 min

34 **a** £7 300 × R12,9772/£ = R94 733,56 **b** SF10 000 × R8,78813/SF = R8 7881,30

35 **a** R4 000 ÷ R0,106737/¥ ≈ ¥37 475,29 **b** R280 500 ÷ R8,4661/$ ≈ $33 132,14

Review Copy

PoA | Activity Assignment 1: Whole numbers

Learner's Book page 36

Time: 50 minutes **Total: 50 marks**

1 124 248 758; 125 220 904; 125 865 248; 125 875 258; 126 084 347; 126 548 335✓✓ (–1 per mistake; two or more mistakes = no mark) (2)

2 R7 878 000 > R7 779 999✓ (1)

3 a 38✓ b 47✓
c 0✓ d 46✓ (4)

4 498 × R5,99 ≈ 500 × R6 ≈ 1 000 × R3✓≈ R3 000✓ (2)

5 a 1 731✓✓ (2)
b 711✓; 230✓✓ (3)
c 19 822✓✓ (2)
d 683✓✓ (2)
e 175 – 21 ÷ 7 + (3 + 2 × 2)
= 175 – 3✓ + (3 + 4✓)
= 175 – 3 + 7✓ = 179✓ (4)

6 a M_{12} = {12; 24; 36; 48; 60}✓
M_{15} = {15; 30; 45; 60; 75}✓ (2)
b LCM = 60✓ (1)

7 a F_{18} = {1; 2; 3; 6; 9; 18}✓
F_{20} = {1; 2; 4; 5; 10; 20}✓ (2)
b HCF = 2✓ (1)
c

2	180
2	90
3	45
3	15
5	5
	1

✓✓{2 marks for doing ladder correctly}
180 = 2 × 2 × 3 × 3 × 5✓ (3)

8 R275 : R350✓ = 11 : 14✓ (2)

9 4 + 3 = 7 parts✓; 1 part = 42 ÷ 7 = 6✓; 24 : 18✓✓ (4)

10 3 + 2 + 1 = 6 parts✓(A; a part = 60 ÷ 6 = 10✓); 10 × 3 = 30✓ (3)

11 600 ÷ 12✓ × 7✓(M) = 350 ml black paint✓ (3)

12 R90/2 h✓(M) = R45/h✓ (2)

13 Speed = 50 km ÷ (50 ÷ 60) ✓ ✓= 60 km/h ✓ (3)

14 50 000 ÷ 8,23423✓ = $6 072,21✓ (2)

Review Copy

CHAPTER

2 Integers

Chapter overview

Learner's Book pages 37 to 71
Recommended pacing: 9 hours

This chapter focuses on the following:

Unit 1: Counting, ordering and comparing integers — 1 hour
- What are integers?
- Comparing integers
- Ordering integers

Unit 2: Calculations with integers — 4 hours
- Adding integers
- Subtracting integers
- Multiplying integers
- Dividing integers
- Mixed operation
- Square numbers and roots; cube numbers and roots

Unit 3: Properties of integers — 1 hour
- Properties involving operations
- The multiplicative inverse

Unit 4: Financial mathematics — 3 hours
- A percentage of a quantity
- Discount
- Income, expenses, profit and loss
- Budgets
- Accounts
- Loans
- Simple interest
- Hire-purchase

Chapter 2 revision

Review Copy

UNIT 1

Counting, ordering and comparing integers

Unit overview

Learner's Book page 38
Recommended pacing: 1 hour

This unit focuses on the following:

- identifying integers
- counting forwards and backwards in integers
- comparing integers to decide which number is larger
- writing integers in ascending and descending order

Resources: Learner's Book ; exercise book

Mental Maths

(10 minutes per day)

Suggested answers — Learner's Book page 38

1 **a** false **b** true

2 $\mathbb{Z} = \left\{25; -16; \frac{14}{7}; 38; -44\right\}$

3 **a** –10; –12; –14; –16 **b** 5; 0; –5; –10 **c** –18; –10; –2; 6

4 –26; –22; –18

5 **a** > **b** < **c** > **d** < **e** > **f** >

6 –95; –68; –59; –41; –23; –14; 32; 86

7 36; 30; 24; –12; –48; –51; –70; –94

Review Copy

Background information

The concepts and skills covered in this unit are revision of work done in Grade 7. Learners consolidate number knowledge and calculation techniques for integers, developed in Grade 7.

Teaching guidelines

Learners are introduced to integers in Grade 7, and it is important to revise this section thoroughly and consolidate prior knowledge. There may be misconceptions that need to be ironed out.

Learners usually have a good understanding of temperature. Try to have a few different types of thermometer available for learners to inspect. Ask learners to draw a large thermometer across the top of the page of their exercise books. Point out the similarities with a number line.

Give learners opportunities to identify integers. Once they are able to identify these, they should complete patterns using integers. This will assist when they are comparing and ordering integers. Some learners struggle with the concept of –7 being smaller than –2, but they should be referred to the thermometer or an equivalent number line.

What are integers?

Counting integers

Activity 1 Identify and count integers

Learner's Book page 39

Guidelines for implementing this activity

- Learners have already encountered integers in Grade 7.
- Learners often find counting negative numbers confusing, since it appears that numbers are getting smaller as opposed to getting bigger, or vice versa.
- It is important to emphasise that the values of the negative numbers are actually decreasing when we count "forward" (–1; –2; –3; etc.), and increasing when we count "backward" (–3; –2; –1; etc.), and to show this on a number line if necessary. The closer we move to zero from the left, the bigger the values of the numbers.
- Again, explain in terms of temperature, e.g. –20 °C is colder than –16 °C; –16 °C is colder than –12 °C, etc. so the numbers run –20; –16; –12; –8; …
- Work through the examples in the Learner's Book.
- Emphasise that fractions always have to be checked by simplifying them in order to see whether they are integers. The answer will tell us whether it is an integer or not.

Remedial and extension

Remedial: If learners struggle, let them draw and count on a number line. Provide as much practice as possible working with integers, especially negative integers.
Extension: Provide an activity where learners add and subtract larger negative numbers.

Review Copy

Suggested answers

1 **a** true **b** false **c** true **d** false

2 $\mathbb{Z} = \{-11; 22; 28\}$

3 **a** –28; –35; –42; –49 **b** 32; 26; 20; 14 **c** –25; –30; –35; –40
d –66; –55; –44; –33 **e** 2; 0; –2; –4 **f** 8; 4; 0; –4
g –15; –5; 5; 15 **h** –13; –26; –39; –52 **i** 0; –17; –34; –51

4 –3

5 **a** –14; –9; –4 **b** –14; –2; 4

6 –5 → –1 → –6 → –2 → –8 → –3 → –11 → –4 → –15

Comparing integers

Activity 2 Compare integers

Learner's Book page 40

Guidelines for implementing this activity

- Work through the example in the Learner's Book.
- Learners can use a number line to see which number is smaller, or they can reason as follows, "If these were temperatures, which one would be colder?"
- Once again it's also important that they realise a "more negative digit" such as –40 is actually smaller than a "less negative digit" such as –30.

Remedial and extension

Remedial: Use temperatures to determine which number amount is colder (smaller).

Suggested answers

1 a > b < c <
d > e > f <
g > h < i <

2 a > b < c > d >
e > f < g > h >
i < j > k > l <
m > n > o <

Review Copy

Ordering integers

Activity 3 Order integers

Learner's Book page 41

Guidelines for implementing this activity

Based on what they've learnt so far in this unit about comparing integers, learners should be able to write numbers in ascending order (smallest to largest) and descending order (largest to smallest).

Remedial and extension

Remedial: If learners struggle, a useful method is to spread out the numbers in one line. They circle, for example, all the negative numbers and put them first in ascending order, and then order the rest of the numbers. It helps to see in which order the numbers have to be when they are written out. Learners can also cross out numbers, e.g. if –80 is first, cross out –80 and then "rate" the numbers that are left over. This method will also prevent them from leaving out some of the numbers.

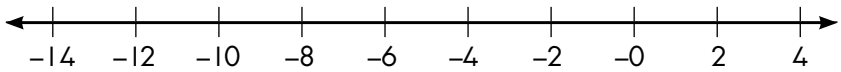
Suggested answers

1 a –8; –7; –5; –2; 3; 6; 7; 9
b –29; –28; –21; –20; 21; 22; 25; 30
c –100; –71; –65; –52; –33; –24; –13; –8

2 a 19; 16; 12; 9; 1; –4; –8; –13
b 71; 70; 69; 63; –61; –65; –70; –78
c –8; –10; –43; –53; –63; –71; –89; –94

3 –18; –12; –6; 0; 6; 12; 18

4



5

–10	–5	0	5	10
–20	–15	–10	–5	0
–30	–25	–20	–15	–10
–40	–35	–30	–25	–20
–50	–45	–40	–35	–30

Review Copy

UNIT 2

Calculations with integers

Unit overview

Learner's Book page 42
Recommended pacing: 4 hours

This unit focuses on the following:

- adding, subtracting, multiplying and dividing integers without using a calculator
- defining and working with the additive inverse
- solving problems that involve adding and subtracting integers
- revising and practising the correct order of operations
- working with integers and square numbers, square roots, cube numbers and cube roots

Resources: Learner's Book; exercise book

Mental Maths

(10 minutes per day)

Suggested answers — Learner's Book page 42

1	a	0	b	0	c	–1	d	12
2	a	false	b	true	c	false		
3	a	–5	b	–13	c	1	d	–6
	e	1	f	–2	g	9	h	–5
	i	2	j	–30	k	108	l	100
	m	2	n	–1	o	–4	p	12
	q	21	r	132	s	90	t	16
	u	–121	v	–125	w	–1	x	impossible
	y	–5	*z	–8				

Background information

The concepts and skills covered in this unit are revision of work done in Grade 7. Learners consolidate number knowledge and calculation techniques for integers, developed in Grade 7.

Note: Multiplying and dividing with integers, doing all four operations with integers and doing all four operations with squares, cubes as well as square roots and cube roots are new in Grade 8.

Teaching guidelines

Initially let learners draw a large number line from –10 to 10 on top of the page in their exercise books. They can refer to this as they do the exercises. This can help them establish how the addition of integers works. They should do a number of examples using the number line. Ask them to start moving towards finding rules that will assist them to do the operations mentally. Ask learners to down write their methods, so that you can check that they are not making erroneous rules and establishing bad habits.

This work will be used when learners do Algebra, so it would be useful to revise this section, using only numbers, before starting Chapter 6 (Algebra).

Review Copy

Adding integers

Activity 1 Add integers

Learner's Book page 43

Guidelines for implementing this activity

- The number ranges are small so that learners can easily add without using calculators.
- Go carefully through the worked examples, since learners need to be able to progress from using number lines.

- Learners can use a number line, and start at the first number in the question and then use their fingers to count to the:
  - right if they are adding a positive number
  - left if they are adding a negative number.

Remedial and extension

Remedial: Learners who struggle should use a number line and count with their fingers on the number line. However, it is important that they progress beyond using number lines and their fingers once they have built confidence. Many opportunities to practise these skills and the application of these rules should help with this progression, and their reliance on these concrete tools should be monitored and gradually discouraged.
Extension: Questions **10** to **15** have larger numbers or contain more than one step, and are more challenging.

Suggested answers

1 5	**2** –8	**3** 2	**4** –6	**5** 3
6 5	**7** –7	**8** 3	**9** 20	***1** –40
***11** –12	***12** –5	***13** –8	***14** 0	***15** –39

Subtracting integers

The meaning of the minus (–) sign; The additive inverse

Activities 2–3 **Subtract integers; Add and subtract integers**

Learner's Book page 45

Review Copy

Guidelines for implementing these activities

- The number ranges are small so that learners can easily subtract without using calculators. Learners need to use the rules for subtracting integers.
- The steps that we followed in each example are:
  - Write the problem, e.g. –1 – –4.
  - Change subtracting a number to adding its additive inverse, e.g. = –1 + 4
  - Subtract, e.g. (4 – 1 = 3). It is written in brackets because we normally do this step mentally. However, if it helps learners, they may write it down, as long as it is in brackets.
  - Determine the sign, e.g. 4 > –1 so the positive number is larger and the answer is positive. The final answer = 3.
- Reinforce these steps in each worked example, and whenever learners are required to subtract integers.

Remedial and extension

Remedial: Learners who struggle should expand the numbers by writing hundreds, tens and units together, and write steps neatly below one another.
Extension: Activity 2 questions **4i** to **4l** and Activity 3 questions **1** to **10** have larger numbers or contain more than one step and are more challenging. In Activity 3 question **2** learners must decide whether each number is a positive or a negative quantity and then add it.

Suggested answers

Activity 2

1	**a**	0	**b**	0	**c**	–6	**d**	17			
2	**a**	–3	**b**	9	**c**	–74	**d**	0	**e**	25	**f**	–300
3	**a**	2; 3; 4; 5; 6; 7; 8			**b**	2; 8	**c**	different			
4	**a**	–8	**b**	2	**c**	4	**d**	–6	**e**	19	**f**	–15
	g	11	**h**	–3	***i**	–1	***j**	–4	***k**	25	***l**	–70

Activity 3

1	**a**	11	**b**	–5	**c**	10	**d**	2
	e	4	**f**	23	**g**	1	**h**	–31
	i	17	**j**	–2	**k**	–27	***l**	50
	***m**	–125	***n**	–2	***o**	–106		

2 **a** R20 – R5 = +R15 **b** –R75 + R32 = –R43
c R120 + R42 – R80 = +R82 **d** R1 100 + R550 – R1 649 = +R1
e –R100 + R56 = +R44 **f** –R40 + R40 = R0
g R35 + R35 – R42 – R5 = +R23 **h** –R15 + –R24 + –R16 + –R10 = –R65

Review Copy

Multiplying integers

The rules for multiplying integers

Activity 4 Multiply integers

Learner's Book page 47

Guidelines for implementing this activity

- Learners need to use the rules when multiplying integers. It is therefore important that you go through them carefully and systematically with them.
- The pattern in Activity 5 question **1** helps them to see why a negative number multiplied by a negative number gives a positive answer.
- Let learners write general rules after the activity. The rules are:
  - When I multiply two numbers with the same sign, the answer is positive.
  - When I multiply two numbers with different signs, the answer is negative.

Remedial and extension

Remedial: Learners should re-do the questions they got wrong until they can get the correct answers on their own. They should also be supplied with additional exercises to give them as much practice as possible.
Extension: Learners do questions **4p** to **4t** in which there are more than two numbers that are multiplied.

Suggested answers

1 **a** –1; 0; 1; 2; 3
b Every answer increased by 1. They are consecutive numbers.
c 3; a positive answer

2 No; $5 \times -2 = -10$ and $-2 \times 5 = -10$

3 **a** R2 + R2 + R2 + R2 + R2 = R2 × 5 = R10
b –R5 + –R5 + –R5 = –R5 × 3 = –R15
c –R10 + –R10 + … 37 times = –R10 × 37 = –R370

4

a –32	**b** 0	**c** –18	**d** 20	**e** 70
f –60	**g** –1	**h** 60	**i** –3 000	**j** –36
k 60	**l** –180	**m** 240	**n** –16	**o** 1
***p** 100	***q** 48	***r** 120	***s** –300	***t** 370

Dividing integers

The rules for dividing integers

Activity 5 Divide integers

Learner's Book page 48

Review Copy

Guidelines for implementing this activity

- Learners need to use the rules when dividing integers. It is therefore important that you go through it together carefully and systematically.
- We used the inverse operation (multiplication) in each example to show what sign the answer will have.
- Let learners write general rules after the activity.
  - When I divide two numbers with the same sign the answer is positive.
  - When I divide two numbers with different signs the answer is negative.

Remedial and extension

Remedial: Learners should re-do the questions they got wrong until they can get the correct answers on their own. They should also be provided with additional exercises to give them as much practice as possible.
Extension: Learners do questions **16** to **21** in which more than two numbers are divided or multiplied.

Suggested answers

1	1	2	–7	3	–4
4	3	5	10	6	–12
7	–4	8	7	9	0
10	–2	11	10	12	undefined
13	–88	14	1	15	0
*16	–20	*17	2	*18	100
*19	48	*20	–74	*21	1

Mixed operations

Activity 6 Use the correct order of operations

Learner's Book page 49

Guidelines for implementing this activity

Revise the order of operations: brackets first, then multiply/divide, then add/subtract. Do examples with learners so they can see how to do calculations step-by-step.

Remedial and extension

Remedial: Learners should re-do the questions they got wrong until they can get the correct answers on their own, and be provided with lots of opportunities to practise.
Extension: Learners who complete the activity quickly and understand it well can make up their own "mini tests" and then give them to another to complete. (Work in pairs.)

Review Copy

Suggested answers

1 $-10 \div 2 + 5 = -5 + 5 = 0$
2 $9 \times -8 - 12 = -72 - 12 = -84$
3 $15 + -6 \times -4 = 15 + 24 = 39$
4 $-13 - -4 \div 4 = -13 - -1 = -13 + 1 = -12$
5 $-5 \times 12 \div -6 = -60 \div -6 = 10$
6 $-4 - (-1) + -5 = -4 + 1 + -5 = -3 + -5 = -8$
7 $-8 + -8 \times -8 = -8 + 64 = 56$
8 $6 + 2 \times -3 - 7 = 6 + -6 - 7 = 0 - 7 = -7$
9 $9 + (-4 + 2 \times -2) = 9 + (-4 + -4) = 9 + -8 = 1$
10 $(-10 + -10) \div -10 = -20 \div -10 = 2$
11 $6 \div -3 + -6 \times 2 = -2 + -12 = -14$
12 $(-5 \times -5)4 + -102 = (25)4 + -102 = 100 + -102 = -2$
13 $16 \div (-20 \div -4 + -5) = 16 \div (5 + -5) = 16 \div 0 =$ undefined
*14 $100 - (-22 \div -11 + 3) = 100 - (2 + 3) = 100 - 5 = 95$
*15 $3 - 5(-1 \times 1 - 1) = 3 - 5(-1 - 1) = 3 - 5(-2) = 3 + 10 = 13$
*16 $-8 - 2(-10 \div 10 + 9) = -8 - 2(-1 + 9) = -8 - 2(8) = -8 - 16 = -24$

Square numbers and roots; cube numbers and roots

The rules for integers in square numbers, square roots, cube roots and cube roots

Activity 7 Work with integers, squares and cubes

Learner's Book page 50

Guidelines for implementing this activity

- It is important that learners understand the difference between $(-5)^2$ and -5^2. The number $(-5)^2$ means -5×-5 and gives a positive answer, 25. The number -5^2 means $-1 \times 5 \times 5$ and therefore has a negative answer, –25. Take time to explain this, ensuring that learners understand the difference.
- Provide quick examples to practise the above on the board before moving on.
- Work carefully and systematically through the steps and the examples in the Learner's Book.
- Square roots of negative numbers do not exist because no number multiplied by itself gives a negative answer.

Remedial and extension

Remedial: Let learners practise by re-doing the questions in the activity. In mental maths at the beginning of every lesson, include exercises that practise the difference between examples such as -5^2 and $(-5)^2$ as illustrated above, and that practise the application of the rules covered in this section. This should benefit learners who require additional support, as well as those who need extension activities.
Extension: Questions **2m** to **2o** each have more than one square or cube number.

Review Copy

Suggested answers

1	**a**	False	**b**	True	**c**	False	**d**	True
2	**a**	25	**b**	49	**c**	–81	**d**	–121
	e	–64	**f**	–64	**g**	–1	**h**	impossible
	i	–2	**j**	–10	**k**	impossible	**l**	–100
	***m**	$144 \div 4 = 36$	***n**	$-27 \times -1 = 27$	***o**	$121 + -100 = 21$		

UNIT

3 Properties of integers

Unit overview

Learner's Book page 52
Recommended pacing: 1 hour

This unit focuses on the following:

- recognising and using the commutative, associative and distributive properties with integers
- working with the inverse of the distributive property
- finding the multiplicative inverse

Resources: Learner's Book; exercise book

Mental Maths

(10 minutes per day)

Suggested answers — Learner's Book page 52

1	a -17	b -30	c $-11 + -3 = -14$			
	d $-4 \times 24 = -96$	e -28	f -20			
2	a -4; -12 or -12; -4		b -7; -13			
	c -9; -11 or -11; -9		d -50; 150			
3	a $\frac{1}{5}$	b $\frac{6}{1}$	c -9	d $-\frac{5}{7}$	e 1	f $-\frac{1}{2}$
4	a $\frac{1}{12}$	b $\frac{5}{2}$	c 1	*d $-\frac{25}{16}$	*e $-\frac{35}{3}$	*f 1

Review Copy

Background information

In Term 4 of Grade 7, learners were introduced to the commutative and associative properties of integers.

Note: Working with the distributive property with integers, as well as the additive and multiplicative inverses, are new in Grade 8. However, the properties themselves are not new, and were applied extensively throughout the previous grades with whole numbers.

Teaching guidelines

Learners have worked with the commutative, associative and distributive properties with whole numbers in Chapter 1 as well as in previous grades. In this unit, learners investigate these properties with respect to integers.

Remind learners what the properties mean and then let them investigate how the laws are applied to integers. The commutative and associative properties only apply to addition and multiplication of integers. The distributive law is quite important in Algebra. One of the common mistakes made with the distributive law is that learners do not distribute the sign and number to all the numbers inside the brackets. Introduce the "dolphin leap" where learners draw curves from the number outside the bracket

to all the numbers inside the brackets (page 124 of the Learner's Book). Learners can also draw a different coloured curve from the sign to the signs (or implied signs) inside the bracket. The multiplicative inverse is the reciprocal of a number and results in a rational number if the initial number is an integer. When a number is multiplied by its multiplicative inverse the answer is 1. This is very important once learners start solving equations that require a few steps and cannot necessarily be solved by inspection alone.

Properties involving operations

The commutative property; the associative property; the distributive property

Activity 1 **Investigate the properties of integers**

Learner's Book page 53

Guidelines for implementing this activity

Learners have already worked with all these properties when they studied whole numbers. Working through the properties in the context of integers should show them that the rules still apply even though they are working with integers.

Remedial and extension

Remedial: The number sentences lead learners and help them to use the laws and answer the questions.

Review Copy

Suggested answers

1 a –5; –20 b 50 + –20 = 30
c 4; –40 d –6 × 8 = –48
2 a –4 + –2 = –6 b 5; –20 + 4 = –16
c –2 × 120 = –240 d 20; –3 × –80 = 240
3 a –6; 6; 54 + –54 = 0 b 8; –14; –8 – 14 = –22
4 a (–3 + 10); –7 × 7 = –49 b –1(–18 – 10) = –1(–28) = 28
5 a –24 b –8 – 9 = –17
c –4 d –16 + 9 = –7
e –15 f –144 ÷ –12 = 12
g –20; 100 h 2 × –40 = –80

The multiplicative inverse

Activity 2 **Find the multiplicative inverse**

Learner's Book page 54

Guidelines for implementing this activity

- Work through the introduction in the Learner's Book.
- Explain the word *reciprocal*: the reciprocal of a number is 1 divided by that number, e.g. $\frac{1}{3}$ is the reciprocal of 3 because $1 \div 3 = \frac{1}{3}$. It is important that learners understand this.

- Do a few additional examples: Find the multiplicative inverse of a number by dividing 1 by that number, e.g. the multiplicative inverse of 3 is $\frac{1}{3}$. The multiplicative inverse of $\frac{5}{4}$ is $1 \div \frac{5}{4} = \frac{4}{5}$.
- The multiplicative inverse of a negative number is also negative because when a number and its multiplicative inverse are multiplied, the answer is positive 1 (a negative number multiplied by a negative number gives a positive answer).

Remedial and extension

Remedial: Learners can check their answers on their calculators by doing the inverse: 1 ÷ (multiplicative inverse) = (number started with).
Extension: Learners do questions **2g** to **2i**.

Suggested answers

1	**a**	$\frac{1}{3}$	**b**	2	**c**	$\frac{1}{42}$	**d**	$-\frac{1}{8}$	**e**	$\frac{1}{71}$	**f**	$\frac{5}{27}$
	g	-4	**h**	$-\frac{1}{7}$	**i**	$-\frac{8}{3}$	**j**	$-\frac{1}{16}$	**k**	$\frac{11}{12}$	**l**	$-\frac{10}{23}$
2	**a**	$\frac{1}{3}$	**b**	$\frac{1}{14}$	**c**	$\frac{3}{2}$	**d**	1	**e**	$\frac{6}{5}$	**f**	$\frac{11}{17}$
	***g**	$-\frac{23}{3}$	***h**	18	***i**	1						
3	**a**	B	**b**	A and D								

UNIT 4

Financial Mathematics

Review Copy

Unit overview

Learner's Book page 55
Recommended pacing: 3 hours

This unit focuses on the following:

- calculating a percentage of a quantity
- working with discount and calculating discounted prices
- defining and working with income, expenses, profit and loss
- working with a family budget
- defining and identifying needs and wants
- working with accounts
- working with loans and loan tables
- calculating simple interest on an amount and the final amount of an investment
- working with hire–purchase
- solving problems that involve financial contexts and integers

Resources: Learner's Book; exercise book; calculator; Internet (optional); family budget (optional); family's cellphone bill or telephone bill (optional)

Mental Maths

(10 minutes per day)

Suggested answers

Learner's Book page 55

1 a 240 people b 225 tables
2 a R89,20 b 768,75 kg

Background information

The concepts and skills covered in this unit are revision of work done in Grade 7. Learners consolidate the skills required when working with problems related to Financial Mathematics.

Teaching guidelines

Learners have already been introduced to the concepts in Grade 7, but again spend some time consolidating them. These skills will be used in everyday life and learners should find them easier to relate to. Also, the skills that are covered in this unit are ones used in other subjects like Geography, EMS, Natural Sciences, Life Orientation and so on.

Try to make the section as authentic as possible by asking learners to bring in pamphlets, newspapers and magazines that have adverts in them. Have learners make budgets for themselves as well as the examples given in the Learner's Book. Cell phone accounts can confuse people and it is well worth working through another account, apart from the one given in Activity 5 on page 61 of the Learner's Book.

Encourage learners to think critically about loans and hire purchase. It is important to encourage responsible decisions. Make sure learners establish the difference in the cash price and the price that would be paid on hire purchase or if a loan is made. Challenge learners to work out how long it will take to save the amount at a particular interest rate. This again can link to budgeting, done earlier.

Review Copy

A percentage of a quantity; Discount

Activities 1–2 **Find a percentage of a quantity; Work with discount**

Learner's Book pages 56–57

Guidelines for implementing these activities

- Explain that a percentage is a fraction out of a hundred, e.g. 20% means $\frac{20}{100}$, and that the word "of", in this context, means that we have to multiply.
- Learners can also write 20% as a decimal fraction, i.e. 0,20.
- Discount is a concept that many learners experience in everyday life, therefore they themselves can be given the chance to explain this concept to the class.
- Work through the examples in the Learner's Book. Show them both methods of determining the amount payable if a certain discount is given. Learners can then choose which method they like/understand best.

Remedial and extension

Remedial: Learners who struggle with this concept should re-do the exercise until they find the correct answers for all the questions. They should also practise with real-life examples as often as possible.
Extension: Learners do Activity 2 question **6**.

Suggested answers

Activity 1

1 **a** $\frac{10}{100} \times 210 = 21$ people **b** 94 tables
c 418 babies **d** 482 children

2 **a** R65,99 **b** R412,50 **c** €477,40 **d** $371,51
e 400 kg **f** 250 ml **g** 675 kg **h** ¥139,86
i €797,25 **j** £371,73 **k** R567,73

Activity 2

1 **a** 40% 0f R5 999 = $\frac{40}{100} \times$ R5 999 = R2 399,60
b R5 999 – R2 399,60 = R3 599,40

2 100% – 45% = 55%; $\frac{55}{100} \times$ R560 = R308

3 100% – 60% = 40%; $\frac{40}{100} \times$ R199 = R79,60

4 $\frac{15}{100} \times$ R750 = R112,50

5 100% – 1% = 99%; $\frac{99}{100} \times$ R79,50 ≈ R78,71

***6** 100% – 60% = 40%; $\frac{40}{100} \times$ R2,50 = R1; R1 × 3 000 = R3 000

Review Copy

Income, expenses, profit and loss

Income and expenses; Types of expenses; Formula for profit and loss

Activity 3 Investigate cost, income, profit and loss

Learner's Book page 58

Guidelines for implementing these activities

- Work through each concept (income; expenses; profit; loss; break even) in the introduction. Learners themselves can offer explanations to the class, since they have been introduced to these concepts in earlier grades, but will also have encountered these concepts in their daily lives.
- Explain that profit and loss can be determined by using the formula: Profit or Loss = Income – Expenses. Show learners that if the expenses are more than the income, we suffer a loss. If the income is more than the expenses, we make a profit. If the income and expenses are exactly the same, we break even.

Remedial and extension

Remedial: The number sentences and tables are designed to help learners work systematically in order to find answers.
Extension: In question **3** learners need to write their own number sentences and then answer the questions.

Suggested answers

1 a Expenses = R50; Income = R80 × 10 = R800
b Profit = Income – Expenses = R800 – R50 = R750

2 a

Number of biscuits (dozens)	Expenses	Income	Difference	Profit or loss?
0	R60	R0	–R60	Loss
1	R76	R32	–R44	Loss
2	R92	R64	–R28	Loss
3	R108	R96	–R12	Loss
4	R124	R128	R4	Profit
5	R140	R160	R20	Profit
6	R156	R192	R36	Profit

b She needs to sell 4 dozen biscuits to start making a profit.
c Income = R32 × 24 = R768; Expenses = R60 + R16 × 24 = R444
Profit = Income – Expenses = R768 – R444 = R324

*3 a Expenses = R20 + R75 × □; Income = R135 × □
b

Number of blankets	Expenses	Income	Difference	Profit or loss?
0	R20	R0	–R20	Loss
1	R95	R135	R40	Profit
2	R170	R270	R100	Profit
3	R245	R405	R160	Profit
4	R320	R540	R220	Profit
5	R395	R675	R280	Profit

Review Copy

c She needs to sell 1 blanket to start making a profit.
d R280

Budgets

What is a budget and why should we budget?

Activity 4 Work with a family budget

Learner's Book page 60

Guidelines for implementing this activity

- Talk to learners about what their families use their money for: food; clothes; electricity; school fees; transport; etc. Explain that all these are called expenses. Alternatively, introduce the word "expenses" and ask learners to provide examples.
- Emphasise the importance of a budget: that we should budget in order to have enough money for all the necessities we need to buy during the week or month. Without a budget, the risk of overspending increases, which leads to financial difficulties. Discuss this with learners, exploring their understanding

of overspending and financial difficulties. Also explore, through discussion, the importance and benefits of saving a portion of your weekly/monthly income.

Remedial and extension

Remedial: Explain the concepts using basic examples that learners can identify with. Use basic income examples and familiar scenarios, such as earning pocket money (income) and how this can be budgeted.
Extension: Learners can ask their parents what their family budget looks like. (This is personal information, so learners don't have to share this with the teacher/class.)

Suggested answers

1 a R11 160 b R10 148,72
c R11 160 – R10 148,72 = R1 011,28; Yes
d

Needs	Wants
Rent	Cell phone bills
Municipal bill	Entertainment
Prepaid electricity	
Groceries	
Medical Aid contribution	
Pension fund contribution	
Medical expenses	
Insurance	
Child care	
School fees	
Unforeseen expenses	
Repayment on credit card	
Credit card interest	

Review Copy

e Answers will vary. Some possibilities: the family could use public transport instead of having their own car; use a cheaper cellphone package; try to cut down on groceries by buying specials or cheaper products; etc.
f Own answer. Depends on **1e**.

2 a
i R1 000 ÷ R37 300 × 100 ≈ 3%
ii R6 800 ÷ R37 300 × 100 ≈ 18%
iii R8 200 ÷ R37 300 × 100 ≈ 22%
vi R750 ÷ R37 300 × 100 ≈ 2%
*v (R2 950 + R1 200) ÷ R37 300 × 100 ≈ 11%
*vi Total expenses without savings = R34 330;
Income – expenses = R37 300 – R34 330 = R2 970;
R2 970 ÷ R37 300 × 100 ≈ 8%

b Total expenses without savings = R34 330 – R6 800 + R3 550 = R31 080;
Income – expenses = R37 300 – R31 308 = R6 220

c Answers will vary. A possibility is to spend less money on entertainment and clothes.

Accounts

Activity 5 Investigate a cellphone bill and a municipal account

Learner's Book page 62

Guidelines for implementing this activity

- Talk to learners about cellphone contracts. Explain the monthly fees, the free-minutes received and that the cost of calls will vary depending on the length of the call as well as to which service provider the call is being made.
- Explain how to calculate the cost of calls and how to convert cents to rand.

Remedial and extension

Remedial: Let learners work in small groups through the activities, and help each other with the concepts.
Extension: Learners can ask their parents what their cellphone bills are like. (This is personal information, so learners don't have to share this with the teacher/class.)

Review Copy

Suggested answers

1 a His name and surname should be on the account and be correct; his address should be on the account and be correct; his cellphone number should be correct.
b itemised billing; caller identity and emergency services
c R172,60
d (i) R6 ÷ 10 = R0,60; (ii) R8,40 ÷ 24 = R0,35; (iii) R15 ÷ 5 = R3
e R50 ÷ 20 = R2,50; he pays per minute
f R0; he has more than 104 free minutes
g yes
h No, it should be R29,40.
i The total amount owed is R377,29 but the top of the bill shows R377,28

2 a 18 765 kWh
b 20 054 kWh
c A = 20 054 – 18 765 = 1 289kWh
d 1 289 × R0,11 = R141,79
e R141,79 + R115,86 = R257,65
f 105 ÷ 100 × R257,65 = R270,53

Loans

Loan tables

Activity 6 Understand loans

Learner's Book page 65

Guidelines for implementing this activity

- Explain to learners what loans are, that interest is charged when borrowing money and that interest can vary depending on where you borrow the money from, how much money you borrow and over how long you have agreed to pay it off.
- Explain how a loan table works. What do they notice? Is paying off less over a longer period of time the best option? Discuss.
- Work through the examples in the Learner's Book.
- Another way to work through the examples is to organise learners into groups, and give each group a repayment period. Let them work out the total that they have to repay, and compare their answers with those of other groups. Do additional examples as practice, using real-life loan tables taken from the Internet, or junk mail that often comes in the post.

Remedial and extension

Remedial: Make sure that learners read the correct values from the table and follow the correct steps. Give individual help where needed.
Extension: Learners can find loan tables on the Internet and do further calculations using these tables.

Review Copy

Suggested answers

1 a (i) R70 × 24 = R1 680; (ii) R1 680 – R1 000 = R680
  b (i) R215 × 36 = R7 740; (ii) R7 740 – R5 000 = R2 740
  c (i) R120 × 48 = R5 760; (ii) R5 760 – R2 500 = R3 260
2 a R215 × 36 = R7 740
  b R7 740 – R5 000 = R2 740
3 a R2 050 + R200 + R500 + R300 = R3 050
  b R5 000
  c 2 years = 24 months; R230 × 24 = R5 520
4 36 months: 215 × 36 = R7 740; Interest = R7 740 – R5 000 = R2 740
  48 months: 200 × 48 = R9 600; Interest = R9 600 – R5 000 = R4 600
  Although she would pay a smaller amount per month if she repays the R5 000 over 48 months, she would pay substantially more interest and therefore I would recommend that she repay the money over 36 months.

Simple interest

Activity 7 Calculate simple interest

Learner's Book page 67

Guidelines for implementing this activity

- Explain that when we invest our money we receive interest, which is a percentage of the amount we invested.
- Explain the concept of invest, how money is invested (investment options, e.g. buying property; insurance; etc.) and discuss the benefits of investing money.
- Explain the concepts of simple interest and principle amount, as in the example.

Remedial and extension

Remedial: Make sure that learners who struggle follow the steps as given in the example. Learners can work in pairs where needed, and in this way support each other.
Extension: Learners do questions **4** and **5**.

Suggested answers

1 **a** 15% of R10 000 = 0,15 × R10 000 = R1 500
b R10 000 + R1 500 = R11 500
c R11 500 + R1 500 = R13 000

2 16,5% of R12 500 = 0,165 × R12 500 = R2 062,50
End of first year: R12 500 + R2 062,50 = R14 562,50
End of second year: R14 562,50 + R2 062,50 = R16 625
End of third year R16 625 + R2 062,50 = R18 687,50

3 **a** 14,75% of R3 525 = 0,1475 × R3 525 = R519,94
End of first year: R3 525 + R519,94 = R4 044,94
End of second year: R4 044,94 + R519,94 = R4 654,88
End of third year: R4 654,88 + R519,94 = R5 084,82
b R5 084,82 – R3 525 = R1 559,821

***4** **a** Interest for one year: 10% of R55 000 = R5 500;
R55 000 + 20 × R5 500 = R165 000
b You can say 20 times the interest instead of adding it 20 times.

***5** Mr Tshabalala: 9,5% of R10 000 = R950; R10 000 + 5 × R950 = R14 750
Mr Tshabalala's brother: 10% of R9 500 = R950; R9 500 + 5 × R950 = R14 250; No

Review Copy

Hire-purchase

Advantages of hire-purchase; Disadvantages of hire-purchase

Activity 8 **Work with hire-purchase**

Learner's Book page 68

Guidelines for implementing this activity

- Work carefully through the concept of hire-purchase in the Learner's Book.
- Since learners are also applying their knowledge of calculating percentages, give groups different scenarios similar to the one in the Learner's Book once you have worked through this example with them.
- Try to use actual examples of hire-purchase agreements that shops such as furniture stores offer/advertise. Let learners share their findings with the class.
- Discuss the advantages and disadvantages of hire-purchase.

Remedial and extension

Learners can make up their own exercises and swap them with classmates.

Review Copy

Suggested answers

1 a 17,5 ÷ 100 × R5 999 = R1 049,83
  b R1 049,83 + (R409,99 × 24) = R10 889,59
  c R4 890,59
2 a 10 ÷ 100 × R349,99 = R35
  b R35 + (R21 × 48) = R1 043
  c R1 043 – R349,99 = R693,01
3 a (no deposit) + (R80 × 12) = R960; R960 – R750 = R210
  b 210 ÷ 750 × 100 = 28%
*4 a i (30 ÷ 100 × R2 600) + (R505 × 6) = R780 + R3 030 = R3 810
  ii (25 ÷ 100 × R2 600) + (R305 ×12) = R650 + R3 660 = R4 310
  iii cash = R2 600
  b Cash, because then you don't pay any interest.
5 Ben's loss: R1 000 – R275 = R775. Ben's loss is R775.
6 a R1 320,46 – R800 = R520,46; Interest at 2,5% = R13,02; R520,46 + R13,02 = R533,47
7 a Income = R3 200 + R800 = R4 000; Expenses = R5 650
  Loss = Income – Expenses = R4 000 – R5 650 = –R1 650
  b If your expenses exceed your income, you end up with a negative total. Money that you have left, or profit that you make, yields a positive total.
  Learners' own answers

Chapter 2 Revision

Learner's Book page 70

Encourage learners to review the content covered before attempting the revision activities. These activities should be used to assess learners' progress thus far, and to assess where remediation may be required.

Suggested answers

1 a True b False c False d True

*2 $(-2)^2 = -2 \times -2 = 4$ and $-2^2 = -1 \times -2 \times -2 = -4$; Therefore $(-2)^2 \neq -2^2$

3 $\mathbb{Z} = \left\{16; -4; 0; \frac{18}{2}\right\}$

4 a –28; –32; –36; –40; –44; –48; –52
b –8; –17; –26; –35; –44; –53; –62
c –5; –20; –35; –50; –65; –80; –95

5 49; 8; –9; –14; –26; –52; –85; –90

6 a –11 b –10
c –3 d –88
e undefined f 3
g –2 h 3
i 0 j 144
k –81 l –8
m impossible n –5
o –10(5 + 200) = –10(205) = –2 050 p –6 × –12 = 72
q 3 – 5 + 28 = –2 + 28 = 26 r 77 ÷ –11 × 4 = –7 × 4 = –28

7 a o b –16 c $\frac{9}{8} = 1\frac{1}{8}$ d 1

8 a –2 b 5 c –85 d 0 e 13

9 a $\frac{1}{13}$ b $-\frac{1}{6}$ c 4 d $-\frac{7}{3} = -2\frac{1}{3}$ e $-\frac{11}{14}$

10 a 4; –5 b –1 × 34 = –34
c –18; 8; 90 + –40 = 50 d (–3 + 7); 4; –28
e 8; –9 f –20

11 a 0,25 × 800 = 200 people b 0,54 × 250 = 135 houses
c 0,165 × R345,45 ≈ R57 d 0,7175 × 150 = 107,625 kg

12 a 100% – 62% = 38%; 0,38 × R798 = R303,24

13 a

No. of meals	expenses	income	Difference	Profit/loss
0	250	0	–250	Loss
2	300	90	–210	Loss
4	350	180	–170	Loss
6	400	270	–130	Loss
8	450	360	–90	Loss
10	500	450	–50	Loss
12	550	540	–10	Loss
14	600	630	10	Profit
16	650	720	30	Profit
18	700	810	70	Profit

b She needs to sell 13 meals in order to start making a profit.
c Income = R45 × 25 = R1 125; Expenses = R250 + R25 × 25= R875;
Profit = R1 125 – R875 = R250

14 R120 × 48 = R5 760; R5 760 – R2 500 = R3 260

15 a 7,5% of R25 000 = R1 875
b R25 000 + 3 × R1 875 = R30 625

16 a 20% of R5 750 + 36 × R400 = R1 150 + R14 400 = R15 550
b R15 550 – R5 750 = R9 800

Review Copy

CHAPTER

3 Exponents

Chapter overview

Learner's Book pages 72 to 97
Recommended pacing: 9 hours

This chapter focuses on the following:

Unit 1: Square numbers and cube numbers	2 hours
Square numbers and square roots	
Cube numbers and cube roots	
Unit 2: Comparing and representing numbers in exponential form	2 hours
Representing whole numbers in exponential form	
Expanded and exponential notation	
Comparing integers written in exponential form	
Unit 3: Calculations with exponents	4 hours
Laws of exponents	
Using exponent laws to work with square numbers, square roots, cube numbers and cube roots	
Unit 4: Scientific notation	1 hour
Writing large numbers in scientific notation	
Chapter 3 Revision	
Term 1 Test (PoA)	

Review Copy

UNIT 1

Square numbers and cube numbers

Unit overview

Learner's Book page 73
Recommended pacing: 2 hours

This unit focuses on the following:

- revising and doing calculations with square numbers and square roots involving whole numbers and integers
- revising and doing calculations with cube numbers and cube roots involving whole numbers and integers

Resources: Learner's Book; exercise book

Mental Maths

(10 minutes per day)

Suggested answers — Learner's Book page 73

1	a	153	b	117	c	100	d	125
	e	impossible	f	–4	g	10	h	–40
	i	–2	j	68	k	–250		
	l	$\sqrt{\sqrt[3]{23^3} + \sqrt{27}^2 \times 2}$						
2	a	$6 < \sqrt{40} < 7$			b	$11 < \sqrt{130} < 12$		
	c	$4 < \sqrt[3]{70} < 5$			d	$2 < \sqrt[3]{9} < 3$		

Review Copy

Background information

In Grade 7, learners did the following:

- compared and represented numbers in exponential form
- determined squares (to at least 12^2) and square roots and cubes (to at least 6^3) and cube roots
- learnt the laws of exponents in order to perform calculations with exponents
- performed calculations with exponents involving all four operations, limited to exponents up to 5
- solved problems involving numbers in exponential form, and square and cube roots.

Note: In Grade 8, learners revise all of the above, and are introduced to integers and rational numbers in exponential form, as well as scientific notation of numbers.

Teaching guidelines

This unit is revision of the work done in Grade 7. It is useful to consolidate this important work and to make sure that learners understand the concepts, terminology and notation. Page 72 of the Learner's Book starts with a concrete example of what a perfect squares is. Learners should learn at least the first 12 perfect squares.

The concepts of squares and areas of squares can be used to introduce square roots. Give learners the areas of the squares and ask them to find the length of a side. Give learners a lot of mental maths practice with squares and square roots, as with cubes and cube roots.

Cubes are also introduced with a concrete 3D shape. It is useful to have some cubes available, because some learners struggle to "see" the layers that are not visible when 3D objects are drawn in 2D.

Square numbers and square roots

Square numbers; Square roots; Non-perfect squares

Activity 1 **Work with square numbers and square roots**

Learner's Book page 75

Guidelines for implementing this activity

- This should be a fast-paced session that revises all basic concepts learnt thus far.
- Learners worked with square numbers and square roots in Chapter 2. Revise the definitions of square and cube numbers, and square and cube roots by working through the introduction in the Learner's Book.
- Test learners with questions such as: What does 5^2 mean? (5×5); What is 5^2 called? (Power); What is the 5 called? (Base); and so on.
- Include questions such as: Does 7^2 3 14? Is this true? (No. 7^2 means 7 multiplied by itself.)
- Ensure that learners know these facts well.
- Work carefully through the examples showing calculations with square numbers and square roots, as well as non-perfect squares. Start by testing learners' basic knowledge, such as expanding, e.g. How could I make the answer $(-5)^2$?
- Revise the order of operations.
- Assist learners with writing out the steps for calculations with more than one term.

Review Copy

Remedial and extension

Remedial: Learners need to refer to all the examples and steps. Make sure they follow the order of operations correctly.
Extension: Activity 1 questions **1k** to **1o** and question **2** pose more of a challenge. Provide an additional activity that contains questions with more terms and more difficult terms where learners need to apply the order of operations.

Suggested answers

1 a $1 + 9 = 10$ b $100 - 25 = 75$ c $9 \times 100 = 900$
d $64 \div 4 = 16$ e $9 + 25 - 4 = 30$ f $36 \div 4 \times 9 = 9 \times 9 = 81$
g $2 + 5 = 7$ h $7 - 6 = 1$ i $4 \div 2 = 2$
j $8 \times 9 = 72$
*k $16 + 49 \times 0 = 16$
*l $144 - (9 + 5) = 144 - 14 = 130$
*m $4 \div 2 + 9 \times 5 = 2 + 45 = 47$
*n $2^2 \times 2^2 + 19 = 4 \times 4 + 19 = 16 + 19 = 35$
*o $\sqrt{11 + 25} = \sqrt{36} = 6$

2 a $2 < \sqrt{7} < 3$ b $6 < \sqrt{42} < 7$ c $11 < \sqrt{126} < 12$
d $3 < \sqrt{12} < 4$ e $8 < \sqrt{80} < 9$

Cube numbers and cube roots

Calculations with cube numbers and cube roots; Non-perfect cube numbers

Activities 2–3 **Work with cube numbers and cube roots; Work with squares and roots, cube numbers and roots**

Learner's Book pages 76–77

Guidelines for implementing this activity

Follow the same format for this lesson as for the previous lesson, since this is also revision of familiar skills and knowledge.

Review Copy

Remedial and extension

Remedial: Learners do Activity 3 in order to get more practice.
Extension: Activity 2 questions **1k** to **1o** and question **2** are more challenging.

Suggested answers

Activity 2

1 a $64 + -1 = 63$ b $-1\,000 - 27 = -1\,027$
c $-8 \times 1\,000 = -8\,000$ d $1\,000 \div 125 = 8$
e $64 - -1 + 1\,000 = 64 + 1 + 1\,000 = 1\,065$
f $1\,000 \div 1 \times -27 = 1\,000 \times -27 = -27\,000$
g $-5 + 2 = -3$ h $4 - 5 = -1$
i $10 \div -2 = -5$ j $-5 \times 1 = -5$
k $23 + -27 \times 0 = 23$ l $-8 - (6 + 5) = -18 - 11 = -19$
*m $-4 \div -8 + 27 = \frac{1}{2} + 27 = 27\frac{1}{2}$
*n $2^3 \times 2^3 + 36 = 8 \times 8 + 36 = 64 + 36 = 100$
*o $\sqrt[3]{10 + 2 \times 27} = \sqrt[3]{10 + 54} = \sqrt[3]{64} = 4$

2 a $4 < \sqrt[3]{89} < 5$ b $2 < \sqrt[3]{12} < 3$ c $3 < \sqrt[3]{38} < 4$

Activity 3

1 $8 + 25 = 33$
2 $64 \times 100 = 6\,400$
3 $144 - -125 = 144 + 125 = 269$
4 $0 \div 64 + 1 = 0 + 1 = 1$
5 $16 + 4 = 20$
6 $121 + 81 - 64 = 138$
7 $144 - 64 + 4 = 84$
8 $121 - -5 = 121 + 5 = 126$
9 $16 \times (0 + 1) - 4 = 16 \times 1 - 4 = 16 - 4 = 12$
10 $10 \div 2 \times 25 = 5 \times 25 = 125$
11 $-4 \div 4 + -64 = -1 + -64 = -65$
*12 $5 + 36 - 10 = 31$
*13 $(125 + 49) \div 2 - 8(10) = 174 \div 2 - 80 = 87 - 80 = 7$
*14 $49(18 - 17) = 49(1) = 49$
*15 $\sqrt{-10 + 64 + 27} = \sqrt{81} = 9$

UNIT 2

Comparing and representing numbers in exponential form

Unit overview

Learner's Book page 78
Recommended pacing: 2 hours

This unit focuses on the following:

- representing whole numbers in exponential form
- powers and co-efficients
- representing integers in exponential form
- revising, identifying and working with powers, variables and constants
- defining and working with coefficients
- writing in expanded notation and exponential notation
- comparing integers, including whole numbers, in exponential form

Resources: Learner's Book, exercise book; calculator

Review Copy

Mental Maths

(10 minutes per day)

Suggested answers — Learner's Book page 78

1 a constant b variable c constant
d variable e variable f constant

2 a $h \times h \times h \times h \times h \times h \times h$
b $-16 \times a \times b$
c $9 \times x \times y \times y \times y \times y$
d $-5 \times -5 \times -5 \times -5 \times -5 \times -5 \times m \times m \times m \times m \times m \times m$
e $-p \times -p \times y \times y \times z \times z$
f $3 \times 3 \times 3 \times 3 \times x \times x \times y \times y$

3	a	8^7	b	$8p^7q^9$	c $45p^4$
	d	m^4n^4	e	$-20hkz$	*f $-k$

4 a $3^4 = 81$; $5^3 = 125$; $3^4 < 5^3$
b $(-18)^4 = 18^4$; 18^2; $(-18)^4 > 18^2$
c $-4^6 = -4\ 096$; $(-4)^5 = -1\ 024$; $-4^6 < (-4)^5$

Background information

In Grade 7, learners did the following:

- compared and represented numbers in exponential form
- determined squares and square roots (to at least 12^2 and $\sqrt{144}$), and cubes and cube roots (to at least 6^3 and $\sqrt[3]{216}$)
- encountered most of these definitions: constants, variables, powers, coefficients, expanded notation and exponential notation.

Note: In Grade 8, learners revise all of the above, and are introduced to integers and rational numbers in exponential form, as well as scientific notation of numbers.

Teaching guidelines

Although most learners have encountered the definitions of constants, variables, powers, coefficients, expanded notation and exponential notation in earlier grades, it is really important to consolidate this knowledge and make sure that all the learners have in fact done this work.

Rules are given for writing numbers in expanded notation in the form of a Think-Do example on page 78 of the Learner's Book. The table in Activity 1 can be extended with various additional examples if learners need extra practice.

Let learners use the expanded form initially so that they can compare and order integers in exponential form and continue using this form until they can establish consistent rules from practice.

Review Copy

Representing whole numbers in exponential form

Revision: Powers; Variables and constants; Powers and co-efficients; Expanded and exponential notation; Rules for writing exponents in expanded notation; Rules for writing in exponential notation

Activity 1 **Represent whole numbers in exponential form**

Learner's Book page 80

Guidelines for implementing this activity

- Learners have already encountered most of the definitions and worked with constants, variables, powers, coefficients, expanded notation and exponential notation in Grade 7. Revise the terminology and all these concepts, and work carefully through all the examples in the Learner's Book.
- It is advisable to do one part of the activity at a time, after covering the relevant section of content.

Remedial and extension

Remedial: Question **2** is presented in a table to "break up" the question into parts and thereby make it easier.
Extension: Questions **2d** to **2g**, and **4h** are more challenging.

Suggested answers

1 **a** constant **b** variable **c** variable
d constant **e** variable **f** constant

2

	Coefficient	Base	Exponent
a	2	a	5
b	16	t	14
c	8	abc	3
***d**	4	k	1
***e**	3	g	1
***f**	1	x	5
***g**	2	ab	3

3 **a** $p \times p \times p$ **b** $3 \times k \times k$
c $8 \times p \times p \times p \times p \times q \times q$ **d** $5 \times x \times y \times y$
e $3 \times 3 \times x \times x$ **f** $2 \times 2 \times a \times a \times b$
g $a \times a \times a \times b \times b \times c \times c$ **h** $2 \times 2 \times 2 \times 2 \times x \times x \times y \times y$

4 **a** 40^5 **b** $3ab^2c$ **c** $4p^2q^2$ **d** $45k^3p$
e $56x^2y$ **f** $48a^2b^5$ **g** $120pq^3$ ***h** $3a^2b^4c^7d$

Review Copy

Representing integers in exponential form

Activity 2 Represent integers in exponential form

Learner's Book page 81

Guidelines for implementing this activity

Approach the teaching of this section similarly to the previous section since the only difference is that learners are now working with integers instead of only whole numbers.

Remedial and extension

Remedial: Question **2** is presented in a table to "break up" the question into parts and thereby make it easier.
Extension: Questions **2e** to **2g** and **3f** to **3h** are more challenging.

Suggested answers

1 a constant b variable c variable
d constant e variable

2

	Coefficient	Base	Exponent
a	–4	m	7
b	–7	k	–12
c	–2	abc	3
*d	12	mn	1
*e	16	pq	1
*f	–1	y	10
*g	–25	de	7

3 a $-30 \times p \times q$ b $-4 \times x \times x \times y \times y$
c $-1 \times x \times x \times y$ d 0
e $-1 \times x \times y \times y \times z \times z$ f $-2 \times -2 \times a \times a \times b$
g $-1 \times a \times a \times a \times a \times a \times b \times b \times c \times c$ h $3 \times 3 \times 3 \times 3 \times x \times x \times y \times y$

4 a $-9w^2$ b $3pqr$ c 0 d $-4hkj$
e $60p^2$ f $-x$ g $-8mpq^2$ h b^3c^3

Comparing integers written in exponential form

Activity 3 Compare integers in exponential form

Learner's Book page 82

Review Copy

Guidelines for implementing this activity

- Explain the following to learners: Sometimes it is difficult to see whether one power is larger than another, e.g. If there are two powers, one with a base of 4 and one with a base of 5, does it mean that $4^{\square} < 5^{\square}$? Not necessarily. It also depends on the exponent. That is why we first expand and then calculate the powers to see which one is larger.
- Work through the example in the Learner's Book. Do one or two additional examples on the board if necessary.
- Learners may use their calculators to work out the answers to larger powers, e.g. in questions **2h** and **2i**.
- In some cases we can say which one is larger without doing a calculation, e.g. we know that $(-8)^3$ will give a negative answer and $(-8)^2$ will give a positive answer, therefore $(-8)^3 < (-8)^2$.

Remedial and extension

Remedial: If learners can't see which power is larger, they can write it in expanded notation, e.g. for question **2a**: $t^7 = t \times t \times t \times t \times t \times t \times t$ and $t^4 = t \times t \times t \times t$. We can see that t^4 has fewer *t*s than t^7, therefore $t^7 > t^4$.
Extension: The fact that learners can see, e.g. in question **2c**, that the one power will give a negative answer, that the other power will give a positive answer and that they therefore won't need to do any calculations, is higher-order thinking.

Suggested answers

1 Ask learners to make estimates based on knowledge learnt so far. They should not make random guesses.

2 a $t^7 > t^4$
b $(-18)^1 < 18^1$
c $7^5 > -7^5$
d $-13^4 < 13^2$
e $-4^3 = (-4)^3$
f $(-10)^3 < (-3)^{10}$
g $2^3 = 8$; $3^2 = 9$; $2^3 < 3^2$
h $4^5 = 1\ 024$; $3^7 = 2\ 187$; $4^5 < 3^7$
i $-6^4 = -1\ 296$; $-4^6 = -4\ 096$; $-6^4 > -4^6$

UNIT 3

Calculations with exponents

Review Copy

Unit overview

Learner's Book page 83
Recommended pacing: 4 hours

This unit focuses on the following:

- learning Exponent Laws 1 to 4 (multiplication, division, brackets and zero as the exponent)
- using numbers to investigate the Exponent Laws
- using the Exponent Laws to work with square numbers, square roots, cube numbers and cube roots
- solving problems involving numbers in exponential form

Resources: Learner's Book; exercise book; calculator

Mental Maths

(10 minutes per day)

Suggested answers — Learner's Book page 83

1 a $12^2 = 144$ b $\sqrt{64} = 8$

2 a LHS $= x^{m+n} = 5^{2+3} = 5^5 = 3\ 125$
RHS $= x^m + x^n = 5^2 + 5^3 = 25 + 125 = 150$
LHS $\neq$ RHS; $\therefore x^{m+n} \neq x^m + x^n$; false

b LHS $= (x^m y)^n = (5^2 \times 1)^3 = 25^3 = 15\ 625$
RHS $= x^{mn} + y^n = 5^{2\times 3} + 1^3 = 5^6 + 1 = 15\ 625 + 1 = 15\ 626$
LHS $\neq$ RHS; $\therefore (x^m y)^n \neq x^{mn} + y^n$; false

3 a 7^{14} b 13^{11} c 18^{12} d $\frac{4}{3}$

e $-36m^{10}n^5$ f $-\frac{1}{2m^6p^3}$ g x^{150} h 1

i $(-4)^3 = -64$ j $-6p^{13}q^9$ k $-\frac{10x^4}{3y^3}$ l $\frac{1}{15^6}$

m $\sqrt{100} = 10$ n $a^{20}b^{24}$ o 2 p $9^2 = 81$

q $\frac{49}{144}$ r $-\frac{15m^{11}}{17n^{15}}$

Background information

In Grade 7, learners did the following:

- performed calculations with exponents involving all four operations, limited to exponents up to 5
- learnt the rules for doing calculations with exponents
- solved problems involving numbers in exponential form and square a cube roots.

Note: In Grade 8, learners revise all of the above, and are introduced to Laws of Exponents, integers and rational numbers in exponential form, as well as scientific notation of numbers.

Review Copy

Teaching guidelines

Make sure that learners can perform calculations with all four operations involving exponents. The mental maths section on page 82 of the Learner's Book can serve as a good prior knowledge exercise either as a test or as an individual activity. Identify any shortcomings especially concentrating on misconceptions that may have arisen from different teaching methodologies.

Encourage learners to use the expanded form if they need to. They should try to use the exponent laws. Make sure that each law is well consolidated before moving onto the next law. Remind learners that Algebra is used to 'summarise' our practices with numbers in a general law or rule. These rules make it easier to calculate other numbers, because we don't have to do the calculation with expanded form each time, but can move directly to the answer, using the rule.

Give learners ample practice with each of the laws before moving onto the next one. One of the problems that learners have is that they get used to using a variable as the base of the power and follow the rules very well, but when there is an actual number in the base, they multiply (or divide) the bases as well, for example the will say that $2^3 \times 2^5 = 4^8$. Similarly, with division. Spend some time ironing out any consistent errors like this and make sure that there is a mixture of examples with variables and numbers as bases.

The summary of the exponent laws is a useful tool for learners to use until they are used to all the rules. It could be photocopied or learners can preferably write it out onto a separate piece of paper that they can quickly refer to when doing the exercises. (Writing is a very important learning method).

Laws of exponents

Law 1 (Multiplication)

Activity 1 **Work with multiplying exponents**

Learner's Book page 84

Guidelines for implementing this activity

- Explain the exponent rule by first expanding the terms and then counting how many bases there are. Then write out what has been done, e.g. $6^3 \times 6^4 = 6^7$. Let the learners see 3 + 4 = 7 for themselves and make up their own rule before formally writing it.
- Work through the examples in the Learner's Book, providing additional examples on the board if necessary.

Review Copy

Remedial and extension

Remedial: Learners should redo question **1** until they can answer each question correctly, and do additional exercises as practice.
Extension: Question **2** has larger and more terms, making it more challenging.

Suggested answers

1	a	4^8	b	35^{18}	c	17^{17}
	d	25^{40}	e	b^{15}	f	c^7
	g	$99m^9$	h	$48p^4$	i	$-130k^7$
	j	$-48a^9b^{12}$	k	$-36p^3q^4$	l	$108x^{16}y^6$
*2	a	$50a^6$	b	$-800x^3y^4$	c	$-60a^9b^{14}$
	d	$450p^9q^7$	e	$216a^{16}b^{14}$	f	$-24p^{13}q^{11}$

Laws of exponents: Law 2 (Division)

Activity 2 Work with dividing exponents

Learner's Book page 85

Guidelines for implementing this activity

- It's very important to explain the process of "cancelling out" properly. First do simple examples like $\frac{2}{2} = 1$, $\frac{8}{8} = 1$, $\frac{-6}{-6} = 1$, $\frac{x}{x} = 1$ and $\frac{2x^2}{2x^2} = 1$. Show that a number or term divided by itself equals one. Therefore if we cancel out and there is "nothing left", it is actually 1 that is left.
- After learners understand that, explain the exponent rule by first expanding the terms, and then cancelling out and seeing what is "left over". Write out what has been done, e.g. $5^7 \div 5^4 = 5^3$. Let the learners see for themselves that $7 - 4 = 3$ and make up their own rule before writing it.
- Work through the examples in the Learner's Book.
- Learners must ask themselves these questions when doing division:
  - **Question 1:** Will I get a positive or negative answer when I divide the coefficients?
  - **Question 2:** What is the HCF of the coefficients? They should divide both numbers by the HCF.
  - **Question 3:** Are there more variables on top or at the bottom? If, e.g. more *b*s are on top then they should write b^2 at the top when writing the answer.

Remedial and extension

Remedial: Learners must ask themselves the above questions with every problem they tackle. When helping a learner, don't just give the answer or method but rather ask the same questions over and over. This will help learners to think about what they are doing, and to find the method and answers themselves.

Review Copy

Suggested answers

1 a 5^2 b 12^{14} c 82^5
d 33^{20} e p^{10} f $2x^3$
g $-4m^2$ h $-6x^2y^6$ i $6a^{10}b^{11}$

2 a $\frac{p^6q^7}{2}$ b $-\frac{mn^{13}}{5}$ c $\frac{-3c^5b^8}{5}$
d $\frac{-16m^6}{5}$ e $\frac{3x^2y^5}{2}$ *f $\frac{-9m^{28}n^{-4}}{11}$

Laws of exponents: Law 3 (Brackets)

Activity 3 Work with raising a bracket to a power

Learner's Book page 86

Guidelines for implementing this activity

- Again, explain these rules by first expanding and showing learners where the rule comes from. Then write the shorter rules formally.
- Work through the examples in the Learner's Book.

Remedial and extension

Remedial: Learners practise by redoing questions they got wrong, and doing additional exercises.
Extension: Provide a more challenging exercise that contains calculations involving all of the Exponent Laws.

Suggested answers

1 10^6	**2** 12^{40}	**3** 15^{72}	**4** 36^{55}
5 a^{24}	**6** p^{28}	**7** a^8b^{30}	**8** $c^{72}d^{48}$
9 $g^{40}h^{80}$	**10** $25k^2$	**11** $16m^{20}$	**12** n^{40}
13 $16p^{48}q^{60}$	**14** $400r^{70}t^{100}$	**15** $-v^{150}w^3$	

Laws of exponents: Law 4 (Zero as exponent)

Activities 4–5 Work with $a^0 = 1$; Work with all the Exponent Laws

Learner's Book pages 87–88

Review Copy

Guidelines for implementing this activity

- Use the following concept to explain this law:
  If $a = b$ and $a = c$, then $b = c$.
  This might not be an easy concept for learners to grasp, so do more than one example on the board.
- Ensure that learners understand that, in an example such as $(-8a^2b)^0$, the exponent applies to the whole bracket and the answer will be 1. However, if there is no bracket, then e.g., $-8a^2b^0$ means $-8a^2 \times b^0 = -8a^2 \times 1 = -8a^2$.

Remedial and extension

Remedial: Learners who struggle should have the mind map of exponent rules on page 86 of the Learner's Book handy when doing Activity 5.
Extension: Learners can work in pairs and make up their own mini tests for each other.

Suggested answers

Activity 4

1 1	**2** 1	**3** 1	**4** 1	**5** 1	**6** 1
7 1	**8** 1	**9** 1	**10** 1	**11** 1	**12** 5
13 12	**14** -4	**15** -20			

Activity 5

1 23^{13}	**2** 16^6	**3** $\frac{1}{13^8}$	**4** 36^{42}	**5** 1	**6** m^{25}
7 $-2m^7$	**8** g^{33}	**9** $-36p^{15}$	**10** $9a^{30}$	**11** 34	**12** $-n\frac{7}{9}$
13 $15^3r^{18}t^{21}$	**14** $-20xy^6$	**15** $160p^8q^4$	**16** $\frac{5x^4y}{4}$	**17** $-\frac{5m^3}{2n}$	**18** $-27a^7b^{27}$

Using numbers to test the Exponent Laws

Activity 6 Use numbers to investigate the Exponent Laws

Learner's Book page 88

Guidelines for implementing this activity

Learners often have some misconceptions about the Exponent Laws. Here we are trying to make them see which theories are true or false by substituting values into each side of the equations and then drawing conclusions.

Remedial and extension

Remedial: Let learners work in pairs. However, each of the learners should write the work down in his or her own book.
Extension: This is already a challenging exercise!

Suggested answers

1 LHS $= y^{m+n} = 3^{2+4} = 3^6$
RHS $= y^m \times y^n = 3^2 \times 3^4 = 3^6 \therefore$ LHS =RHS $\therefore$ True

2 LHS $= (x + y)^m = (2 + 3)^2 = 5^2 = 25$
RHS $= x^m + y^m = 2^2 + 3^2\ 4 + 9 = 13 \therefore$ LHS $\neq$ RHS $\therefore$ False

3 LHS $= x^m + x^n = 2^2 + 2^4 = 4 + 16 = 20$
RHS $= x^{m+n} = 2^2 + 4 = 2^6 = 64 \therefore$ LHS $\neq$ RHS $\therefore$ False

4 LHS $= \frac{x^m}{y^m} = \frac{22}{32} = \frac{4}{9}$
RHS $= \frac{x^m}{y} = \left(\frac{2}{3}\right)^2 = \frac{4}{9} \therefore$ LHS =RHS $\therefore$ True

5 LHS $= (x + y)^m = (2 + 3)^2 = 5^2 = 25$
RHS $= xy^m = 2 \times 3^2 = 2 \times 9 = 18 \therefore$ LHS $\neq$ RHS $\therefore$ False

6 LHS $= (x^m y^n)^p = (2^2 \times 3^4)^1 = 4 \times 81 = 324$
RHS $= (y^n)^p \times (x^m)^p = (3^4)^1 \times (2^2)^1 = 81 \times 4 = 324 \therefore$ LHS =RHS $\therefore$ True

Review Copy

Using the Exponent Laws to work with square numbers, square roots, cube numbers and cube roots

General rules for adding and subtracting, multiplying and dividing

Activity 7 Use the Exponent Laws to work with square and cube numbers, and roots

Learner's Book page 89

Guidelines for implementing this activity

- Work through the general rules for adding, subtracting, multiplying and dividing.
- It is important that learners first add and subtract before they calculate the square, cube or square root. Observe this carefully and correct where necessary.

Remedial and extension

Remedial: Learners redo questions they get wrong.
Extension: Learners can work in pairs and make up their own mini tests for each other.

Suggested answers

1 a $12^2 = 144$ b $\sqrt{64} = 8$ c $\left(\frac{5}{6}\right)^2 = \frac{25}{36}$
d $\sqrt[3]{\frac{27}{216}} = \frac{3}{6} = \frac{1}{2}$ e $(20 - 9)^2 = 11^2 = 121$

2 a $(11 + 9)^2 = 20^2 = 400$ b $(1 + 4 + 5)^3 = 10^3 = 1\,000$ c $42 = 16$
d $5^3 = 125$ e $\sqrt{100} = 10$ f $\sqrt[3]{1\,000} = 10$
g $\sqrt{16} = 4$ h $\sqrt[3]{125} = 5$ i $\frac{121}{81}$
j $\frac{1}{8}$ k $\frac{6}{7}$ l $\frac{5}{3}$

3 a After 40 minutes: No. of bacteria = $10 \times 10 = 100$
After 60 minutes: No. of bacteria = $100 \times 10 = 1\,000$
After 80 minutes: No. of bacteria = $1\,000 \times 10 = 10\,000$
After 100 minutes: No. of bacteria = $10\,000 \times 10 = 100\,000$

4 a After 3 minutes ($t = 3$): No. of viruses = $2(1 + 0{,}05)^3 \approx 2$
After 10 minutes ($t = 10$): No. of viruses = $2(1 + 0{,}05)^{10} \approx 3$
After 60 minutes ($t = 60$): No. of viruses = $2(1 + 0{,}05)^{60} \approx 37$
*b After 120 minutes ($t = 120$): No. of viruses = $2(1 + 0{,}05)^{120} \approx 698$

5 At the end of year 3 ($n = 3$): A = R50 000$(1 + 0{,}06)^3$ = R59 550,80
At the end of year 4 ($n = 4$): A = R50 000$(1 + 0{,}06)^4 \approx$ R63 123,85
At the end of year 7 ($n = 7$): A = R50 000$(1 + 0{,}06)^7 \approx$ R75 181,51
At the end of year 8 ($n = 8$): A = R50 000$(1 + 0{,}06)^8 \approx$ R79 692,40

Review Copy

UNIT 4

Scientific notation

Unit overview

Learner's Book page 92
Recommended pacing: 1 hour

This unit focuses on the following:
- writing large numbers in scientific notation
- converting scientific notation to normal notation
- comparing numbers written in scientific notation

Resources: Learner's Book; exercise book

Mental Maths

(10 minutes per day)

Suggested answers — Learner's Book page 92

1 a $5{,}25 \times 10^{21}$ b $4{,}86 \times 10^{17}$ c $8{,}42 \times 10^{19}$
2 a 500 000 000 b 1 380 000 000 000 c 685,7
3 a $5 \times 10^7 = 50\ 000\ 000$; $1{,}5 \times 10^6 = 1\ 500\ 000$; >
b $3{,}78 \times 10^4 = 37\ 800$; $2 \times 10^4 = 20\ 000$; >
c $1 \times 10^3 = 1\ 000$; $9{,}2 \times 10^4 = 92\ 000$; <

Background information

This content is new in Grade 8. Learners will learn to write very large numbers in scientific notation.

When writing numbers in scientific notation, learners have to understand and apply the relationship between the number of decimal places and the index of 10.
Scientific notation is limited to positive exponents.

Teaching guidelines

Scientific notation is introduced in Grade 8. Learners are fascinated by very large numbers and there are a few mentioned in the article on page 91 of the Learner's Book to start with. Others that fascinate learners are, for example, the distance around the earth, the moon, the sun, the distance between the planets and the sun, the speed of light and the number of people in the world. Ask learners to contribute two facts involving very large numbers. While they are reading these out, write the numbers on the board or ask learners to write the numbers on the board.

Once learners have seen how many zeros these numbers can have, they will see that it makes sense to find an abbreviated version for them. Also, when we try to do any calculations with these numbers, it becomes very difficult.

Review Copy

The skills that scientific notation will require are:

- rounding off (to one or two decimal places)
- an understanding of the relationship between the number of decimal places and the index of 10.

Also show learners how to convert numbers that are in scientific notation to normal notation and how to compare very large numbers written in scientific notation.

Writing large numbers in scientific notation

Steps for writing large numbers in scientific notation

Activity 1 Write large numbers in scientific notation

Learner's Book page 93

Guidelines for implementing this activity

- Write two very large numbers, e.g. with 23 or 30 digits. Ask learners first to add them and then to multiply them (virtually impossible).
- Explain that (as the learners saw) it is very difficult to work with such large numbers and that is why we use scientific notation.
- Explain scientific notation and work through the worked examples in the Learner's Book.

Remedial and extension

Remedial: Work in pairs.

Suggested answers

1 a 1 b 2 c 3 d 4
e 5 f 6 g 7 h 10

2 a $7{,}39 \times 10^{22}$ b 6×10^{13}
c 3×10^{8} d 7×10^{8}
e $1{,}33 \times 10^{9}$ f $3{,}61 \times 10^{8}$
g 1×10^{7} h $1{,}2 \times 10^{14}$
i $2{,}54 \times 10^{4}$ j $5{,}8 \times 10^{10}$

Review Copy

Converting scientific notation to normal notation

Activity 2 Write in normal notation

Learner's Book page 94

Guidelines for implementing this activity

- Work through the example in the Learner's Book.
- Let learners practise with more numbers by writing a lot of zeros at the back of the number, and then shift the comma the same amount of places as the exponent.

Remedial and extension

Remedial: Use the "lots of zeros" method as stated above.

Suggested answers

1 78 700 km^3
2 24 000 puzzle pieces
3 172,7 cm
4 5 130 minutes
5 28 288 m^2
6 12 272 kg
7 247,2 km
8 1 433 characters
9 4 499,7 kg
10 110,3 m

Comparing numbers written in scientific notation

Activity 3 Compare numbers written in scientific notation

Learner's Book page 95

Guidelines for implementing this activity

- Sometimes it is difficult to see which number will be larger. Therefore learners must write each number in normal notation first and then compare it.
- Work through the example in the Learner's Book.

Remedial and extension

Remedial: Use the "lots of zeros" method to write numbers in normal notation. Learners compare numbers as they did with whole numbers.
Extension: Let learners find examples on the Internet of interesting facts where scientific notation is used.

Review Copy

Suggested answers

1 a $3{,}18 \times 10^5 = 318\ 000$; $2{,}98 \times 10^6 = 2\ 980\ 000$; <
b $6{,}10 \times 10^4 = 61\ 000$; $7{,}4 \times 10^4 = 74\ 000$; <
c $8 \times 10^3 = 8\ 000$; $9{,}98 \times 10^2 = 998$; >
d $1{,}6 \times 10^8 = 160\ 000\ 000$; $1{,}2 \times 10^7 = 12\ 000\ 000$; >
e $9{,}9 \times 10^2 = 990$; $8 \times 10^2 = 800$; >
2 a $1{,}2272 \times 10^4$ kg = 12 272 kg; $4{,}4997 \times 10^3$ kg = 4 499,7 kg; >
b $2{,}4 \times 10^4 = 24\ 000$; $1{,}433 \times 10^3 = 1\ 433$; >
c $2{,}472 \times 10^2$ km = 247,2 km; $1{,}103 \times 10^2$ m = 110,3 m; 110,3 m ÷ 1 000 = 0,1103 km; >

Chapter 3 Revision

Learner's Book page 96

Encourage learners to review the content covered before attempting the revision activities. The activities should be used to assess learners' progress thus far, and to assess where remediation may be required.

1 **a** constant **b** variable **c** constant
d variable **e** variable

2

	Coefficient	Base	Exponent
a	4	q	20
b	8	xy	5
c	–2	k	–15
d	–1	y	6
e*	–14	bc	5

3 **a** $k \times k \times k \times k \times k \times k$
b $5 \times 5 \times a \times a$
c $-1 \times x \times x \times x \times x \times x \times y \times y \times y \times z \times z \times z$
d $-10 \times p \times p \times p \times p \times p \times q$

4 **a** 6^8 **b** $-16m^3n^3p^5q^3$ **c** $250p^{13}q^{19}$
d $6a^3b^6c$ **e** x^6y^6

5 **a** < **b** > **c** = **d** =
e $2{,}5 \times 10^5 = 250\ 000$; $2{,}15 \times 10^4 = 21\ 500$; >
f $6 \times 10^3 = 6\ 000$; $6{,}1 \times 10^5 = 610\ 000$; <

6 **a** LHS $= x^m \times x^n = 4^2 \times 4^1 = 16 \times 4 = 64$
RHS $= x^{m \times n} = 4^{2 \times 1} = 4^2 = 16$; LHS $\neq$ RHS; False
b LHS $= (x + y)^m = (4 + 3)^2 = 7^2 = 49$
RHS $= x^m + y^m = 4^2 + 3^2 = 16 + 9 = 25$; LHS $\neq$ RHS; False

7 **a** $(1 + 9)^2 = 10^2 = 100$ **b** $(20 - 11)^2 = 9^2 = 81$ **c** $\sqrt{100 - 64} = \sqrt{36} = 6$

8 **a** $5{,}82 \times 10^{12}$ **b** $4{,}1 \times 10^{10}$ **c** $9{,}88 \times 10^{17}$

9 **a** 68 500 000 000 **b** 700 000 000 **c** 897,6

10 **a** $16 + 36 = 52$ **b** $1 \times 25 = 25$
c 72^{20} **d** $-88a^{26}b^{20}$
e 12^9 **f** $-5m^2n^3$
g 15^{30} **h** $x^{36}y^{12}$
i $9a^{10}$ **j** 4
k $9 + 11 = 20$ **l** $-4 \div 1 = -4$
m $\frac{1}{12}$ ***n** $-24a^{20}b^{14}$
***o** $81 + 4 \times 4 = 81 + 16 = 97$ ***p** $\sqrt[3]{12 + 8 + 7} = \sqrt[3]{27} = 3$

11 $9 < \sqrt{90} < 10$

Review Copy

CHAPTER

4 Numeric and geometric patterns

Chapter overview

Learner's Book pages 98 to 102
Recommended pacing: 4 hours 30 minutes

This chapter focuses on the following:

Unit 1: Investigate and extend patterns	**4,5 hours**
Extending patterns by finding the rule	
Chapter 4 Revision	**45 minutes**

UNIT

1 Investigate and extend patterns

Review Copy

Unit overview

Learner's Book page 99
Recommended pacing: 4,5 hours

This unit focuses on the following:

- extending numeric and geometric patterns with constant differences or ratios
- extending numeric and geometric patterns without constant differences or ratios
- writing the rule for a numeric pattern in words
- creating own numeric patterns, with or without constant differences or ratios
- writing out the first few terms
- recognising a pattern from the written terms, finding the rule informally and writing a formula (equation)
- calculating larger terms by using formulae

Resources: Learner's Book; exercise book; calculator

Mental Maths

(10 minutes per day)

Suggested answers

Learner's Book page 99

1 a i Add 4 to each number to get the next number in the sequence

ii 32; 36; 40; 44

b i Multiply each number by –2 to get the next number in the sequence

ii 80; –160; 320; –640

c i The numerator stays 1. Subtract four from each denominator to get the next denominator in the sequence

ii $\frac{1}{-16}; \frac{1}{-20}; \frac{1}{-24}; \frac{1}{-28}$

d i Multiply each number by 3 to get the next number in the sequence

ii 27; 81; 243; 729

2 a i

n	1	2	3	4
T_n	6	9	12	15

ii $T_1 = 3(1) + 3 = 6$; $T_2 = 3(2) + 3 = 9$; $T_3 = 3(3) + 3 = 12$; $T_4 = 3(4) + 3 = 15$

iii $T_n = 3(n) + 3$

iv In the number sequence I need to add 3 each time to get the next number. Therefore I multiplied the term number by 3. For the first term $3 \times 1 = 3$, but the answer is 6. Therefore I need to add 3 to get to the correct answer.

v $T_{200} = 3(200) + 3 = 603$

b i

n	1	2	3	4
T_n	–10	–6	–2	2

ii $T_1 = 4(1) - 14 = -10$; $T_2 = 4(2) - 14 = -6$; $T_3 = 4(3) - 14 = -2$; $T_4 = 4(4) - 14 = 2$

iii $T_n = 4(n) - 14$

iv In the number sequence I need to add 4 each time to get the next number. Therefore I multiplied the term number by 4. For the first term $4 \times 1 = 4$, but the answer is –10. Therefore I need to subtract 14 to get to the correct answer.

v $T_{200} = 4(200) - 14 = 786$

Review Copy

Background information

Learners have worked extensively with patterns in previous grades. In Grade 8, the range of number patterns is extended to include patterns with multiplication and division with integers, and numbers in exponential form.

Teaching guidelines

Learners' descriptions of patterns are also expected to become more precise, and include the use of algebraic language to describe the general rule, descriptions of relationships between terms and their position in a sequence, and the justification of solutions. Learners are also introduced to the language of terms in a sequence in order to distinguish the term from the position of a term in a sequence.

In Chapter 3, learners used Algebra to generalise the patterns seen in numbers in exponential form. In this chapter, learners again use Algebra to devise a general rule. Once this is established, they then use it to predict other values or positions in the pattern without actually writing the pattern out to the nth term. Emphasise how useful Algebra can be in finding these shortcuts.

Extending patterns by finding the rule

Activities 1–2 **Extend numeric and geometric patterns; Create your own number sequence**

Learner's Book page 100

Guidelines for implementing these activities

Let the learners systematically find the rule.

Step 1: Let learners subtract consecutive numbers in the sequence to see if the same number was added each time. If not, go to Step 2.

Step 2: Let learners add consecutive numbers in the sequence to see if the same number was subtracted each time. If not, go to Step 3.

Step 3: Let learners divide consecutive numbers in the sequence to see if each number was multiplied by the same number. If not, go to Step 4.

Step 4: Let learners multiply consecutive numbers in the sequence to see if each number was divided by the same number. If not, go to Step 5.

Step 5: The pattern is not a constant difference or ratio. Learners need to find the pattern by inspection.

Review Copy

Remedial and extension

Remedial: Let learners work in pairs.
Extension: Questions **1g** to **1j** of Activity 1 are more challenging. Learners can also make more difficult rules in Activity 2.

Suggested answers

Activity 1

1 a i Add 6 to each number to get the next number in the sequence.
ii –2; 4; 10; 16
b i Subtract 7 from each number to get the next number in the sequence.
ii –22; –29; –36; –43
c i Add 15 to each number to get the next number in the sequence.
ii –35; –20; –5; 10
d i Multiply each number by 10 to get the next number in the sequence.
ii 4 000; 40 000; 400 000; 4 000 000
e i Multiply each number by –3 to get the next number in the sequence.
ii 81; –243; 729; –2 187
f i Divide each number by 4 to get the next number in the sequence.
ii 18 750; 4 687,5; 1 171,875; 292,96875

*g i The numerator stays 1. Add 3 to each denominator to get the next denominator in the sequence.

ii $\frac{1}{12}; \frac{1}{15}; \frac{1}{18}; \frac{1}{21}$

*h i The numerator stays 1. Divide by the cube number (negative) of each term number to get the denominator.

ii $-\frac{1}{64}; -\frac{1}{125}; -\frac{1}{216}; -\frac{1}{343}$

*i i Subtract consecutive numbers, i.e. subtract 1, then 2, then 3, etc.

ii 0; –5; –11; –18

*j i Add consecutive even numbers, i.e. add 2, then 4, then 6, etc.

ii 0; 10; 22; 36

2 a i Add 7 to each number to get the next number in the sequence

ii 28; 35; 42; 49

b i Subtract R50 from each number to get the next number in the sequence

ii R100; R50; R0; –R50

c i Multiply with consecutive numbers, i.e. multiply by 2, then by 3, then by 4, etc. to get the next number in the sequence

ii 72; 360; 2 160; 15 120

Activity 2

1–5 learners' own creations

Writing a general rule

Using tables to find terms in numeric sequences

Activity 3 Find terms in numeric patterns

Learner's Book page 102

Review Copy

Guidelines for implementing this activity

- We are not formally teaching learners to find the formula of a pattern. This is more of an informal way of finding the formula.
- Learners need to use the constant difference as a guide for how to work out the output value (T_n). They need to write out how to find the first few terms below one another. This will help them see the pattern easily.
- It will help the learners if you do more than two examples with them on the board. Learners understand better when they repeatedly see how they should be done.

Remedial and extension

Remedial: Learners should refer back to the example in the Learner's Book to help them. Alternatively, let learners work in pairs.
Extension: Questions **4** to **6** are more challenging.

Suggested answers

1 **a**

n	1	2	3	4
T_n	5	7	9	11

b $T_1 = 2(1) + 3 = 5$; $T_2 = 2(2) + 3 = 7$; $T_3 = 2(3) + 3 = 9$; $T_4 = 2(4) + 3 = 11$

c $T_n = 2(n) + 3$

d In the number sequence I need to add 2 each time to get the next number. Therefore I multiplied the term number by 2. For the first term, $2 \times 1 = 2$, but the answer is 5. Therefore I need to add 3 to get the correct answer.

e $T_{200} = 2(200) + 3 = 403$

2 **a**

n	1	2	3	4
T_n	10	15	20	25

b $T_1 = 5(1) + 5 = 10$; $T_2 = 5(2) + 5 = 15$; $T_3 = 5(3) + 5 = 20$; $T_4 = 5(4) + 5 = 25$

c $T_n = 5(n) + 5$

d In the number sequence I need to add 5 each time to get the next number. Therefore I multiplied the term number by 5. For the first term, $5 \times 1 = 5$, but the answer is 10. Therefore I need to add 5 to get the correct answer.

e $T_{200} = 5(200) + 5 = 1\,005$

3 **a**

n	1	2	3	4
T_n	2	5	8	11

b $T_1 = 3(1) - 1 = 2$; $T_2 = 3(2) - 1 = 5$; $T_3 = 3(3) - 1 = 8$; $T_4 = 3(4) - 1 = 11$

c $T_n = 3(n) - 1$

d In the number sequence I need to add 3 each time to get the next number. Therefore I multiplied the term number by 3. For the first term, $3 \times 1 = 3$, but the answer is 2. Therefore I need to subtract 1 to get the correct answer.

e $T_{200} = 3(200) - 1 = 599$

***4** **a**

n	1	2	3	4
T_n	–6	–4	–2	0

b $T_1 = 2(1) - 8 = -6$; $T_2 = 2(2) - 8 = -4$; $T_3 = 2(3) - 8 = -2$; $T_4 = 2(4) - 8 = 0$

c $T_n = 2(n) - 8$

d In the number sequence I need to add 2 each time to get the next number. Therefore I multiplied the term by 2. For the first term, $2 \times 1 = 2$, but the answer is –6. Therefore I need to subtract 8 to get the correct answer.

e $T_{200} = 2(200) - 8 = 392$

***5** **a**

n	1	2	3	4
T_n	–3	–9	–15	–21

b $T_1 = -6(1) + 3 = -3$; $T_2 = -6(2) + 3 = -9$; $T_3 = -6(3) + 3 = -15$; $T_4 = -6(4) + 3 = -21$

c $T_n = -6(n) + 3$

d In the number sequence I need to subtract 6 each time to get the next number. Therefore I multiplied the term number by –6. For the first term, $-6 \times 1 = -6$, but the answer is –3. Therefore I need to add 3 to get the correct answer.

e $T_{200} = -6(200) + 3 = -1\,197$

Review Copy

*6 a

n	1	2	3	4
T_n	–10	–6	–2	2

b $T_1 = -1(1 \times 1) = -1$; $T_2 = -1(2 \times 2) = -4$; $T_3 = -1(3 \times 3) = -9$; $T_4 = -1(4 \times 4) = -16$

c $T_n = -1(n^2)$ or $T_n = -1(n \times n)$

d I need to multiply the term number by itself and by –1 to get the answer.

e $T_{200} = -1(200 \times 200) = -40\ 000$

Chapter 4 Revision

Note

Learners will revise Chapter 4 content at the end of Chapter 5. Alternatively, allow learners to do questions **1** to **3** of Chapter 5 Revision at this point.

Review Copy

CHAPTER

5 Functions and relationships

Chapter overview

Learner's Book pages 103 to 114
Recommended pacing: 3 hours

This chapter focuses on the following:

Unit 1: Input values, output values and flow diagrams 1,5 hours
Recognise, describe and justify the rule

Unit 2: Equivalent forms of relationships 1,5 hours
Use equivalent forms to find output values
Use equivalent forms to find the rule

Chapter 5 Revision

UNIT

1 Input values, output values and flow diagrams

Review Copy

Unit overview

Learner's Book page 104
Recommended pacing: 1,5 hours

This unit focuses on the following:

- defining input and output values
- defining terms and term numbers
- working forward to find the output value by using the rule given in a flow diagram
- working backwards and doing the inverse operations to find the input value by using the rule given in a flow diagram
- solving problems that involve tables, flow diagrams and formulae

Resources: Learner's Book; exercise book; calculator

Mental Maths

(10 minutes per day)

Suggested answers

Learner's Book page 104

1 a $y = -4(-1) - 15 = -11$

b $y = 18 \div -2 + 27 = 18$

2 a $x = -5(-3) + 15 = 30$

b $x = 20(4) + 20 = 100$

3 a i $y = 3(n) - 2$

ii $10 \rightarrow \boxed{\times 3} \rightarrow \boxed{-2} \rightarrow 28$

iii $19 \leftarrow \boxed{\div 3} \xleftarrow{57} \boxed{+2} \leftarrow 55$

b i $y = -5(n) + 11$

ii $12 \rightarrow \boxed{\times -5} \rightarrow \boxed{+11} \rightarrow -49$

iii $+15 \leftarrow \boxed{\div -5} \xleftarrow{-75} \boxed{-11} \leftarrow -64$

Background information

Learners have worked extensively with input and output values, flow diagrams and tables in previous grades. In Grade 8, learners determine input and output values or rules for patterns and relationships using variables, formulae and equations. The rules and number patterns for which learners have to find input and output vales are extended to include patterns with multiplication and division of integers and numbers in exponential form.

They also begin to build awareness of the domain of input values to include natural numbers, integers and rational numbers.

Review Copy

Teaching guidelines

Flow diagrams are very useful to establish a rule. Learners have been using them for a number of years in different guises like the linear one, the "spider" or "butterfly" diagram and so on. It is useful to draw examples of the types that they may have encountered before, on the board.

Make sure that learners understand that the numbers that are on the left of the flow diagram are the input values when moving from left to right. The number that results at the end of the operations in the flow diagram are the output values.

Learners are given steps to writing the rule or general formula. This is a very important skill and is very useful in Algebra. Make sure that learners can use either the flow diagrams with the input and output values, or tables to generate the general formula.

Input values, output values and flow diagrams

Activity 1 Find input values and output values in flow diagrams

Learner's Book page 105

Guidelines for implementing this activity

- To find the output value learners need to work forward. They should first find the answer to each step in the flow diagram before going to the next step and then finally find the output value.
- To find the input value learners need to work in reverse, doing the inverse operation in each step and also finding the answer to each step before moving on to the next.
- Work through the examples in the Learner's Book, and revise the meaning of the symbols n and T_n.
- Provide additional exercises and flow diagrams and let learners practise writing the information in tables, and finding the general term. Confidence and skill will improve with practice.

Remedial and extension

Remedial: Learners need to work in a step-by-step manner to get each answer before moving on to the next.
Extension: Learners who understand the work easily and who complete the activity quickly can help fellow learners who are struggling. They can also work in pairs and make up flow diagrams for each other to solve.

Review Copy

Suggested answers

1 a $y = 15(4) + 3 = 63$
b $y = (13 + 7) \div 4 = 5$
c $y = 24 \div -2 - 8 = -20$
d $y = (72 + 8) \div -2 = -40$
e $y = -5(-3) + 7 = 22$

2 a $x = (99 \div 3) - 3 = 30$
b $x = 10(8) + 20 = 100$
c $x = (23 - 5) \div -3 = -6$
d $x = (-20 + 10) \div -2 = 5$
e $x = (30 \div -3) + 6 = -4$

Recognise, describe and justify the rule

Activity 2 Find the formula, input values and output values

Learner's Book page 107

Guidelines for implementing this activity

- Here we teach learners formally how to find the formula. Work carefully and systematically through the steps and examples in the Learner's Book.
- These steps may seem very long to learners. It is best to do at least three additional examples on the board with learners so that they can see how they are done step-by-step.

- When you do the third example, ask learners to tell you what the next step is and what you should write down on the board.
- Learners also need to use the formula they just wrote to determine input and output values.

Remedial and extension

Remedial: Learners may need lots of help. Refer back to steps and examples in the Learner's Book and let them work in pairs, if necessary.
Extension: Questions **1g** and **1h** and question **2h** are more challenging. Pair capable learners with learners who are struggling so that they can assist them.

Suggested answers

1 **a** $y = 2(n) + 1$; $a = 2(15) + 1 = 31$; $b = (41 - 1) \div 2 = 20$
b $y = 5(n) - 2$; $c = 5(20) - 2 = 98$; $d = (123 + 2) \div 5 = 25$
c $y = 3(n) - 4$; $e = 3(12) - 4 = 32$; $f = (23 + 4) \div 3 = 9$
d $y = -2(n) + 22$; $e = -2(10) + 22 = 2$; $f = (-14 - 22) \div -2 = 18$
e $y = -1(n) + 11$; $j = -1(15) + 11 = -4$; $k = (-8 - 11) \div -1 = 19$
f $y = -4(n) + 6$; $m = -4(8) + 6 = -26$; $p = (-42 - 6) \div -4 = 12$
***g** $y = 10(n^2)$; $q = 10(12^2) = 1\ 440$; $r = \sqrt{(810 \div 10)} = 9$
***h** $y = n^3 + 1$; $v = 7^3 + 1 = 344$; $w = \sqrt[3]{(730 - 1)} = 9$

2 **a** $C = 100(n) + 250$
b $12(100) + 50 = R1\ 250$
c $(2\ 050 - 250) \div 100 = 18$ bicycles
d R180
e $I = 180(n)$
f $12(180) = R2\ 160$
g $2\ 880 \div 180 = 16$ bicycles
***h** Income = 180(7) = R1 260
Expenses = 100(7) + 250 = R950
Profit = R1 260 – R950 = R310

Review Copy

UNIT 2

Equivalent forms of relationships

Unit overview

Learner's Book page 109
Recommended pacing: 1,5 hours

This unit focuses on the following:
- finding output values of numerical and geometrical patterns by drawing flow diagrams, using formulas and working with tables
- finding and describing the rule of numerical and geometrical patterns by describing it in words, using flow diagrams, using formulas and working with tables

Resources: Learner's Book; exercise book; calculator

Mental Maths

(10 minutes per day)

Suggested answers

Learner's Book page 109

1 a $t = 10p - 20$

b + c

p	1	2	3	4	5
t	–10	0	10	20	30

2 a

Term number	n	1	2	3	4
Number of squares	q	5	7	9	11

b $q = 2(n) + 3$

c I have to multiply the input value by 2 and then add 3 to get the output value.

3 a $t = 10(p) + 5$

b I have to multiply the input value by 10 and then add 5 to get the output value.

Background information

The focus in this term is to practise operations with integers, or include integers in the rules for finding output values.

It is important to note that when learners find input or output values for given rules, they are finding the numerical value of algebraic expressions using substitution.

Teaching guidelines

In Chapter 4, Patterns, functions and algebra, and Chapter 5, Functions and relationships, learners consolidated the equivalent forms that functions and relationships can be expressed in. In both chapters, learners generalise the patterns or relationships into rules which they can then use to predict other values or positions.

Learners have worked with some of these equivalent forms in previous grades. The flow diagram in various forms including the linear ones used here and the spider diagrams or butterfly diagrams used in earlier grades are very useful for establishing rules.

The rules can then be used to complete a table of input and output values. When the input and output values are given specific variables, the general rule or formula can be found. (Use the steps on page 112 of the Learner's Book again if necessary).

Learners should become quite comfortable moving between the different equivalent forms, including writing the general rule or formula.

Review Copy

Use equivalent forms to find output values

Activity 1 Use equivalent forms to find output values

Learner's Book page 111

Guidelines for implementing this activity

In this section learners will practise all the skills that they have already acquired.

Remedial and extension

Remedial: Learners should redo problems until they get everything right.
Extension: Question **4** is more challenging.

Suggested answers

1 **a** $t = 4(p) + 10$
b $t = 4(1) + 10 = 14; 30; 2; -10; -30$
c

p	1	2	3	4	5
t	14	18	22	26	30

2 **a** $t = (p \div 2) - 7$
b $t = (8 \div 2) - 7 = -3; -5; -8; -12; -16$
c

p	1	2	3	4	5
t	$-6\frac{1}{2}$	-7	$-7\frac{1}{2}$	-8	$-8\frac{1}{2}$

3 **a** $t = (p + 7) \times -3$
b $t = (3 + 7) \times -3 = -30; -24; -15; -12; 3$
c

p	1	2	3	4	5
t	-24	-27	-30	-33	-36

***4** **a** $t = (p - 4) \div 3$
b $t = (25 - 4) \div 3 = 21 \div 3 = 7; 1; -1; -4; -10$
c

p	1	2	3	4	5
t	-1	$-\frac{2}{3}$	$-\frac{1}{3}$	0	$1\frac{1}{3}$

Review Copy

Use equivalent forms to find the rule

Activity 2 Use equivalent forms to find and describe the rule

Learner's Book page 112

Guidelines for implementing this activity

Learners need to find the rule on their own. A clue they can look at is the output value when the input value is 0. This will be the number they can add/subtract. They can use that and another pair of input and output values to find the rule by working backwards.

Remedial and extension

Remedial: Learners work in pairs.
Extension: Learners work in pairs and make up problems with more difficult rules that peers must solve.

Suggested answers

1 a i

Term number	n	1	2	3	4
Number of leaves	l	2	4	6	8

ii $l = 2(n)$

iii I need to multiply the input value by 2 to get the output value.

b i

Term number	n	1	2	3	4
Number of clay pots	c	2	3	4	5

ii $c = n + 1$

iii I need to add 1 to the input value to get the output value.

c i

Term number	n	1	2	3	4
Number of recycled plastic bottles	b	4	9	14	19

ii $b = 5(n) - 1$

iii I need to multiply the input value by 5 and then subtract 1 to get the output value.

d i

Term number	n	1	2	3	4
Number of stones	s	1	4	9	16

ii $s = n^2$

iii I need to multiply the input value by itself to get the output value. (or: I need to square the input value to get the output value.)

e i

Term number	n	1	2	3	4
Number of owls	w	1	2	3	4

ii $w = n$

iii The input value is equal to the output value.

2 a i $t = 5(p) - 2$

ii I need to multiply the input value by 5 and then subtract 2 to get the output value.

b i $t = (p \div 2) + 2$

ii I need to divide the input value by 2 and then add 2 to get the output value.

c i $t = 6(p) + 24$

ii I need to multiply the input value by 6 and then add 24 to get the output value.

3 Learners' own work.

Review Copy

Chapter 5 Revision

Learner's Book page 114

The questions covered in the revision revise the content of Chapters 4 and 5.

Encourage learners to review the content before attempting the revision activities. The revision activities should be used to assess learners' progress thus far, and to assess where remediation may be required.

Suggested answers

1 a I need to subtract consecutive even numbers, i.e. subtract 2, then 4, then 6, etc. to get the next number.

b –8; –18; –30; –44

2 a

n	1	2	3	4
T_n	3	–2	–7	–12

b $T_n = -5(n) + 8$

c I need to subtract 5 from each number to get the next number in the sequence, so I multiplied the term number by –5. For term 1 $-5 \times 1 = -5$, but the answer is 3, so I had to add 8.

d $T_{100} = -5(100) + 8 = -492$

3 a $y = 2(n) - 11$

b $m = 2(12) - 11 = 24 - 11 = 13$

c $p = (27 + 11) \div 2 = 38 \div 2 = 19$

4 a $t = -1(p) + 4$

b $t = -1(5) + 4 = -5 + 4 = -1$; 1; 6; 8; 15

c

p	1	2	3	4	5
t	3	2	1	0	–1

5 a

Term number	n	1	2	3	4
Number of diamonds	d	5	8	11	14

b $d = 3(n) + 2$

c I need to multiply the input value by 3 and then add 2 to get the output value.

Review Copy

CHAPTER

6 Algebraic expressions

Chapter overview

Learner's Book pages 115 to 129
Recommended pacing: 4 hours 30 minutes

This chapter focuses on the following:

Unit 1:	Algebraic language	2 hours
	Writing in algebraic language	
	Variables and constants	
	Algebraic conventions	
	Like and unlike terms	
	Naming and ordering expressions	
Unit 2:	Expand and simplify algebraic expressions	2,5 hours
	Multiplying out brackets	
	Write in expanded notation	
	Adding and subtracting like terms	
	Use the property laws to add and subtract like terms	
Chapter 6 Revision		

Review Copy

UNIT 1

Algebraic language

Unit overview

Learner's Book page 116
Recommended pacing: 2 hours

This unit focuses on the following:

- writing in algebraic language by substituting words and phrases with symbols
- revising and identifying variables and constants in algebraic expressions
- using variables and constants to write algebraic expressions
- defining and identifying expressions, terms and coefficients
- learning some important algebraic conventions
- correcting expressions according the algebraic conventions
- counting how many terms there are in an expression
- defining and identifying like and unlike terms
- naming expressions
- ordering expressions in ascending, descending and alphabetical order

Resources: Learner's Book; exercise book; photocopies of coins and notes

Review Copy

Mental Maths

(10 minutes per day)

Suggested answers — Learner's Book page 116

1 **a** $16 + x$ (or $x + 16$) **b** $3x - 10$
c $\frac{2x}{10} + 5$ **d** $\frac{2}{3}n$ learners

2 **a** $2m$ **b** abz **c** $-6kl$ **d** $3tu + ty$

3 **a** 2 **b** 3 **c** 1 **d** 3

4

a	1	3	–5
b	3	1	1
c	0	–2	12
d	1	0	–1
e	0	–1	0

5

a	8	abc	3
b	–16	rs	1
c	1	x	5
d	–1	b	2

6 **a** $4yyx$ **b** $-18x$ **c** $\frac{2}{p}$

7 **a** unlike terms; there are two terms with a^3b^3 and one term with a^3b
b like terms

8 **a** trinomial **b** monomial
c trinomial **d** polynomial

9 $18 - 3x - 2x^2 + 9x^3 - 12x^4 + x^6$

10 $-3p^2 + 6pq + 5q^2 + 1$

Background information

In Grade 7, learners were introduced to formal algebraic language as symbolic language to develop the understanding of variables.

They recognised, wrote and interpreted rules and relationships represented in symbolic form through the use of symbols. They also identified variables and constants in formulae and equations.

In Grade 8, they revise the above and are introduced to the conventions of algebraic language. They also start manipulating algebraic expressions.

Teaching guidelines

Once again, thoroughly revise the work that was done in Grade 7 to make sure that all learners have the same understanding of the algebraic language and conventions as this may not have been consistently covered at different schools in Grade 7. You could use the Mental Maths section as a prior knowledge exercise to establish precisely where there are gaps in the knowledge or understanding of this very important section. At this early stage, time is best spent making sure that the terminology is well-consolidated before moving on. Learners will reap the benefit of a good grounding in this Algebra in FET Mathematics.

This is another opportunity for learners to write the new terminology and their definitions into their own glossary if they have established one. It is important that all the words like coefficient, constant, variable, base, power, like and unlike terms are used often in class so that learners become used to hearing them and using them. Constantly ask learners to identify the coefficient of terms, or how many variables there are, and so on.

Make sure that the conventions are discussed and ask learners to give extra examples of their own for each of the conventions. Make sure that all learners know the rules on page 116 of the Learner's Book.

Like and unlike terms can be quite confusing for learners especially when there are the same variables with different exponents as well as when there are many different variables. If learners follow the conventions of sign, coefficient, variables in alphabetical order and descending powers, it is easier to compare and identify if the terms are the same. This is another reason why this unit with its conventions is so important. If learners ignore the conventions, the manipulations of expressions will become very difficult.

Review Copy

Writing in algebraic language

Rules for replacing words/phrases with symbols

Activity 1 **Write in algebraic language**

Learner's Book page 117

Guidelines for implementing this activity

- Start the lesson informally by asking learners what they think words and phrases like sum of, product, etc. mean. They should say +, ×, etc. Learners should be familiar with these words and phrases, and their meanings.

- Explain how we can write algebraically by replacing words and phrases with symbols. Work through the examples and rules in the Learner's Book.
- Emphasise that a symbol is a sign that we use in place of words, and that when we write algebraically we write mathematically using combinations of symbols.
- Explain Rule 6 very carefully, ensuring learners understand this rule.

Remedial and extension

Remedial: Learners can work in pairs. Sometimes it helps for a partner to say the question out loud at first so that the other learner can hear it.
Extension: Questions **15** and **16** are more challenging.

Suggested answers

1 $x + 2$

2 $x - 16$

3 $2x$

4 $\frac{26}{x}$

5 $x + 40$ or $40 + x$

6 $x + 5 - 10$ (or $x - 5$)

7 $2x - 4$

8 $x + \frac{1}{2}x$

9 $x - 6 = 20 - 9$ (or $x - 6 = 11$)

10 $9 + 2 \times x = 13$

11 $\frac{50}{x} = 10$

12 $13 \times 5 \times x = 1\,170$ (or $65x = 1\,170$)

13 $x = 18 - 5$

14 $56 = 12 + x$

***15** $1 + x = 2 \times x$

***16** $\frac{1}{2} \times x \times 2 + 13$

Review Copy

Variables and constants

Activity 2 **Identify variables and constants and write algebraic expressions**

Learner's Book page 118

Guidelines for implementing this activity

- Learners have studied variables and constants in Chapter 3. Revise the definitions.
- Learners also need to learn how to use variables and constants in order to write algebraic expressions.
- Emphasise that an algebraic expression contains numbers and symbols, but no = sign.
- Call out a few examples such as $x + 5$; $32x + 3 = 35$, etc. and ask learners to say whether they are algebraic expressions or not.
- Work through the example in the Learner's Book.

Remedial and extension

Remedial: Learners can work in pairs. However, each learner should write down the answers in his or her own book. Provide additional exercises as practice.
Extension: Question **3** is more challenging.

Suggested answers

1 **a** constant **b** variable **c** variable
d constant **e** variable **f** constant

2 **a** $R55 - t$ **b** $7 \times x$ **c** $\frac{1}{3}g$ **d** $c + 11$
e $\frac{1}{2}t$ **f** $\frac{4}{5}p$ **g** $\frac{x}{5}$ **h** $(2 + x) \times 3$
i $\frac{9}{10}d$ **j** $\frac{13}{15}t$

***3** **a** $2x + 1$ **b** $2x + 1 + 5$ (or $2x + 6$) or $2x - 1 + 5$ or $2x + 4$
c $\frac{1}{2}(2x + 1)$ or $\frac{1}{2}(2x - 1)$ **d** x; $x + 1$
e x; $x + 1$; $x + 2$ **f** $x - 1$; x; $x + 1$

Algebraic conventions

Expressions, terms and coefficients; Some important algebraic conventions

Activity 3 Work with mathematical conventions, terms and coefficients

Learner's Book page 120

Review Copy

Guidelines for implementing this activity

- Revise terminology such as expression, term, constant, variable, base, exponent, power and coefficient. Ask learners to provide examples of each.
- Write a few examples on the board. Ask learners to identify the terms, coefficients, constants and variables. They should also say whether these are algebraic expressions or not.
- Ensure that learners understand when there is only one term and when there is more than one term.
- When working through the algebraic conventions, learners should always make sure that they write multiplication without brackets or any other multiplication symbols, and that the expression is written in descending and alphabetical order, unless stated otherwise.
- Work through the examples in the Learner's Book.

Remedial and extension

Remedial: Questions **3** and **4** are in tables to "break up" the work for learners and thereby make it easier to do.

Suggested answers

1 **a** $8m$ **b** ab **c** kmp **d** $8r$ **e** $5mn$ **f** $2abc$
g $4bc$ **h** $3gh$ **i** z **j** abc **k** $4ax + 6y$ **l** $\frac{4 + p}{2}$

2 **a** 2 **b** 3 **c** 2 **d** 1 **e** 2 **f** 1
g 2 **h** 4 **i** 2 **j** 1 **k** 2 **l** 1

3

	Expression	Coefficient of x^2	Coefficient of x	Constant
a	$x^2 + 6x + 5$	1	6	5
b	$2x - x^3 + 9x^2 - 1$	9	2	–1
c	$x^2 + 10x + 5$	1	10	5
d	$x^2 - 16x$	1	–16	0
e	$2x - 3y + z$	0	2	0
f	$10x^2 - y$	10	0	0
g	$x^5 - 6x^2 + x$	–6	1	0
h	$x^2 - 1$	1	0	–1

4

	Question	Coefficient	Base	Power
a	$5p^3$	5	p	3
b	$-2a^5$	–2	a	5
c	$8(abc)^3$	8	abc	3
d	$-(pq)^{-7}$	–1	pq	–7
e	$-16(rs)$	–16	rs	1
f	$3g$	3	g	1
g	x^5	1	x	5
h	$-b^2$	–1	b	2

Review Copy

Like and unlike terms

Activity 4 Identify like and unlike terms

Learner's Book page 121

Guidelines for implementing this activity

- Present a few bags of real or play coins (5c, 10c, 20c, 50c, R1, R2 and R5 coins). Photocopy real money and cut out the circles if you don't have real coins.
- Let learners work in groups. Let each group put all their money on the table or floor. The first team who can say how much money there is wins.
- Ask the winning team afterwards how they were able to get to the answer first. They will then probably say that they put all the R2 coins together, all the R1 coins together, etc., counted them separately and then added the totals.
- Explain to the learners that what they were actually doing was to sort all the "like coins" into groups and that with numbers we also add the "like terms" together. Do an example to show them what this means, e.g. $2x + 4x + 10x + 7x$ (they are all x, so we can add them together).
- Work through the introduction in the Learner's Book.

Remedial and extension

Remedial: Learners can highlight like terms with highlighters. It helps learners to see like terms more easily.
Extension: Questions **4** to **6** test the "gut feeling" of learners. This sets the stage for adding like terms, which they will do later in this chapter.

Review Copy

Suggested answers

1 a $2cb$ b $3x^3$ c $4p$
d a e $\frac{2}{x+z}$ f $2a^2b$
g $4cb$ h $3k$ i p
j $\frac{2}{y}$ k $3ap$ l $4a$

2 a Unlike. The first and second terms both contain p^2q^2, but the last term contains p^2q (the q doesn't have a square).
b Like
c Like
d Like
e Like
f Unlike. Although all the terms contain r^2, the exponent of t is different in every term.
g Like
h Like

3

Expression where all the terms are like terms	Expressions that contain unlike terms
a; e; f	b; c; d; g; h

4 Put the R2 coins together, count how many there are, multiply it by 2 and then find the total. Do the same for the R5 coins but multiply by 5. Then add the two totals.
5 We won't be able to count the money properly, so it's better to add just the like coins.
6 a I *may* add like terms.
  b I *may not* add unlike terms.

Naming and ordering expressions

Monomials; Binomials; Trinomials; Polynomials
The order of expressions

Activity 5 Name and order expressions

Learner's Book page 123

Guidelines for implementing this activity

- Work through the terminology monomial, binomial, trinomial and polynomial, ensuring that learners understand these.
- Write a few examples on the board, and ask learners to identify whether they are monomials, binomials, trinomials or polynomials, and to motivate their answers.
- Let learners first arrange 4, 32, 8, 2 and 16 in descending order. Then ask them what they notice about the exponents of the numbers they just ordered: 2^5; 2^4; 2^3; 2^2 and 2^1 (the exponents are in descending order.)
- Emphasise the fact that the correct order to write an expression is descending and alphabetical. Do the examples in the Learner's Book with learners on the board.

Review Copy

Remedial and extension

Remedial: Learners should re-do questions they got wrong until they can do them correctly. Practice is important. Provide as many opportunities for practice as possible.
Extension: Questions **3g** to **3j** are more challenging.

Suggested answers

1 a trinomial
  b monomial
  c binomial
  d trinomial
  e polynomial
  f trinomial
2 a $12a^5 + 13a^2 + a$
  b $9b^3 - b^2 - 14b + 8$
  c $3x^6 - x^4 + x^3 - x^2 - 8x + 1$
  d $6y^5 + 3y^4 - y^3 + 5y^2 + 5$
3 a $m^4 + 12m^3 + m^2 - 3m + 2$
  b $a^3 + 6a^2 + 4a + 7$
  c $-2a^5 - 4a^4 + a^2 + a$
  d $-5k^3 - 2k^2 + 7k + 9$
  e $p^5 - 16p^4 - p^3 + 8p^2$
  f $-5q^3 + 2q^2 + 3q + 6$
  *g $a^2 + 2ab + b^2$
  *h $x^4y^2 + x^2y^3 - xy^4$
  *i $-a^2b + ab - b^2 + 1$
  *j $-m^4n^2 - m^2n^4 + mn^6$

UNIT 2

Expand and simplify algebraic expressions

Unit overview

Learner's Book page 124
Recommended pacing: 2,5 hours

This unit focuses on the following:

- multiplying out brackets
- writing terms and expressions in expanded notation
- adding and subtracting like terms
- using the property laws to add and subtract like terms

Resources: Learner's Book; exercise book

Mental Maths

(10 minutes per day)

Suggested answers — Learner's Book page 124

Review Copy

1 **a** $6x + 6y$ **b** $120a + 90b$
c $-48a - 32b$ **d** $-11x + 110y$
e $12a - 144b + 12c - 132z$ **f** $-8w + 32x + 72y + 16z$
***g** $36v - 45w + 7x + 7y$ ***h** $-10a + 45b + 3c + 3d$

2 **a** $7 \cdot a \cdot b \cdot c$ **b** $2 \cdot p \cdot p \cdot p \cdot p \cdot p$ **c** $-10 \cdot a \cdot a \cdot a \cdot b \cdot b$
d $-1 \cdot a \cdot a \cdot a \cdot a \cdot b$ **e** $7 \cdot x \cdot y \cdot y$ **f** $2 \cdot a \cdot a \cdot b \cdot b \cdot c \cdot c$

3 **a** $2p^2 + 2p - 11$ **b** $23m^2n + 6mn^2 + 3$
c $3h^2 - 15h - 3$ **d** $10b$
e $(3a + 11b - 6c) - (-18a + 22b + 42c) = 3a + 11b - 6c + 18a - 22b - 42c$
$= 21a - 11b - 48c$

4 **a** $-5a; -20a$ **b** $30x + -20x = 10x$

5 **a** $-4x^2 + -2x^2 = -6x^2$ **b** $-4y^5 + (-y^5 + 5y^5) = -4y^5 + 4y^5 = 0$

6 **a** $-4a; 6a; 36a - 54a = -18a$ **b** $-8x; 2x; 8x - (-2x) = 10x$

7 **a** $-7(-3x + 5x) = -7(2x) = -14x$ **b** $-1(-15y - 10y) = -1(-25y) = 25y$

Background information

In Grade 7, learners were introduced to formal algebraic language as symbolic language to develop the understanding of variables.

They recognised, wrote and interpreted rules and relationships represented in symbolic form (rules and relationships represented through the use of symbols).

They also identified variables and constants in formulae and equations.

In Grade 8, they revise the above and are introduced to the conventions of algebraic language. They also start manipulating algebraic expressions.

Teaching guidelines

Learners start to manipulate algebraic expressions in this unit. This requires a variety of mathematical skills that need to be used simultaneously. Each chapter has been building on the previous ones to ensure that learners have the skills required. For example, relatively easy simplification like $7(a + c)$ requires knowledge of the Distributive Law, multiplication and algebraic conventions.

Although learners used the Distributive Law for whole numbers and integers in Chapters 1 and 2, it is a good idea to revise the numerical rule briefly before starting this unit. Make sure that the algebraic conventions in the previous unit are very well consolidated. Expanded notation is a useful tool to explain how manipulations work.

Make sure that learners understand from the previous unit what like and unlike terms are. They can underline, circle or highlight the terms that can be added together. This makes it initially easier to identify all the terms. When learners have to add a larger number of terms, there is a danger that some are "left behind" so it is useful to get them into the habit of ticking off the term as they add it to the total.

Multiplying out brackets

Activity 1 **Multiply out brackets**

Learner's Book page 125

Review Copy

Guidelines for implementing this activity

- Do $2(3 + 5)$ on the left-hand side and $(2 \times 3) + (2 \times 5)$ on the right-hand side. Show that both sides give the same answer.
- Write $2(x + y)$ and $(2 \times x) + (2 \times y)$. Ask the learners if they think that these two expressions will also give the same answer (yes).
- Explain that what we actually do with $2(x + y)$ is to "multiply it out" and "to get rid of" the bracket.
- Show learners the "dolphin" method. Draw the arrows to show that we first multiply 2 by x and then we multiply 2 by y. Make sure that the learners see that there are two terms in the bracket and therefore there should also be two terms in the answer.

Remedial and extension

Remedial: Learners should re-do the questions until they get them right. Provide as many opportunities for practice as possible.
Extension: Questions **10** to **14** are more challenging.

Suggested answers

1 $3x + 3y$
2 $20a + 40b$
3 $30b + 45c$
4 $-5k - 5m$
5 $-18a - 12c$
6 $40p - 90q$
7 $25a - 200b$
8 $-21x + 36y$
9 $36a - 27b + 54c$
*10 $-26x + 28y + 20z$
*11 $-2k - 3m - n$
*12 $-4w + 7x + 2y - 2z$
*13 $-6a + 2b + 12c - 3d$
*14 $2x + 10y$

Write in expanded notation

Activity 2 Write expressions in expanded notation

Learner's Book page 126

Guidelines for implementing this activity

- Learners have already encountered expanded notation. Revise expanded notation and ensure that learners know that numbers should be written first and then variables in alphabetical order.
- Explain the position of the multiplication dot (slightly raised from the bottom line).

Suggested answers

1 $3 \times p \times q \times r$
2 $3 \times k \times k \times k \times k$
3 $-2 \times p \times p \times q \times q \times q \times q$
4 $-1 \times x \times x \times y$
5 $14 \times a \times b \times c \times c$
6 $14 \times a \times a \times b \times b \times c \times c$
7 $4 \times x \div y$
8 $m \times m \times m \times m \times m$
9 $-30 \times p \times q$
10 $2 \times p \times p \times p \times q \times q \times q$
11 $2 \times a \times b \times c$
12 $-4 \times x \times x \times y \times y$
13 0
14 $-1 \times x \times y \times y \times z \times z$
15 $m \times m \times m \times m \times n \times n \times n \times n$
16 $\frac{a \times b}{x \times y}$

Review copy

Adding and subtracting like terms

Important rules for adding and subtracting terms; Adding like terms horizontally

Activity 3 Add like terms horizontally

Learner's Book page 126

Guidelines for implementing this activity

- Work through the rules in the Learner's Book.
- Do an easy addition calculation where there are only two letters and no exponents. Show learners that they can circle like terms in this expression using the same colour. This makes it easier to add.
- Do two or three simpler examples first before tackling two more challenging examples.

Suggested answers

1 $9a$
2 $24m$
3 $40k$
4 $4p - 5$
5 $-9g + 13$
6 $-3h + 5$
7 $7a - b$
8 $6x - 4y$
9 $29m - 12n$
10 $-2r + 7t - 21$
11 $29p + q - 7$
12 $-18v + 9w + 16$
13 $10ab - 3cd + 3$
14 $5pq - 20$
15 $4p^2 - 2pq - 2$
*16 $12x^3y - 11xy^2$
*17 $-8a^2b - 2$
*18 $10p^2 - 15pq^3 + 30$

Adding like terms vertically

Activity 4 Add like terms vertically

Learner's Book page 127

Guidelines for implementing this activity

Explain the vertical method by showing learners step-by-step on the board. (Show how to write like terms below one another and do it in descending order.) They might find this method easier since they used this method to add numbers in primary school.

Suggested answers

1 $4a^2 - 11a + 11$
2 $16b^2 - 17b + 9$
3 $3p^2 - 14p - 25$
4 $2q^2 - 7q + 2$
5 $-10x^2 - 21x + 1$
6 $11k^3 + 23k^2 - 2$
7 $17x^2 + 4xy - 9y^2$
8 $9a^2 + 4a + 6$
9 $4p^2q + 1$
10 $2a + 4b - c$
*11 $5x^2 - 2x - 19$
*12 $8y^4 - 7y^2 + 5$
*13 $-6a^2 - a + 25$
*14 $-7k^6 + 10k^3 - k + 21$

Review Copy

Subtracting like terms horizontally

Activity 5 Subtract like terms horizontally

Learner's Book page 128

Guidelines for implementing this activity

- Work through the example in the Learner's Book.
- Emphasise that we first need to multiply out the brackets, and then change subtracting to adding additive inverses. This simplifies the process since learners have already practised how to add like terms.

Remedial and extension

Remedial: Learners should highlight like terms with highlighters. They must remember that ( ) – ( ) means ( ) – 1( ), so they need to multiply –1 into the second bracket.
Extension: Questions **13** to **15** are more challenging.

Suggested answers

1 $q^2 + 4q + 7$
2 $8x^2 + 10x + 12$
3 $9m^2 + 7$
4 $2a^2 + 7a - 11$
5 $2pq + q^2$
6 $14k^4 + 7k^2 + 8$
7 $-7b^3 + b + 18$
8 $-5c^2 + 14c + 17$
9 $-11e^5 + 6e + 15$
10 $2f^5 + 3f^2 - 20$
11 $13x - 7y - 3z$
12 $-7a + 2ab - 18b$
*13 $-4m^2 + 14n^2 - 10$
*14 $2kp - 9p^2 + 8$
*15 $-4x^2 + 66y^2 - z^2$

Use the property laws to add and subtract like terms

The commutative property; The associative property; The distributive property

Activity 6 **Investigate the properties of like terms**

Learner's Book page 129

Guidelines for implementing this activity

- Learners have already applied the Commutative, Associative and Distributive laws to whole numbers and integers. They will now be applying these properties to like terms.
- Work through the examples in the Learner's Book. Provide additional examples of each property on the board.

Remedial and extension

Remedial: Learners work in pairs. However, each learner should write the answers in his or her own exercise books.

Review Copy

Suggested answers

1 a $8x$; $10x$
b $9a + -7a = 2a$
c $-3m + 8m = 5m$
d $-9k + -10k = -19k$
2 a $5n + 10n = 15n$
b $5p$; $-3p + 9p = 6p$
c $-4c + 10c = 6c$
d $3h$; $-20h + 2h = -18h$
3 a $-10a$; $6a$; $-32a + 18a = -12a$
b $8c$; $4c$; $40c - 20c = 20c$
c $-2e$; $6e$; $8e + -24e = -16e$
d $8g$; $2g$; $-8g + 2g = -6g$
4 a $5(-3k + 10k) = 5(7k) = 35k$
b $10(7n - n) = 10(6n) = 60n$
*c $-4p(-3q + 15q) = -4p(12q) = -48pq$
*d $-10r(-20t - 8t) = -10r(-28t) = 280rt$

Chapter 6 Revision

Note

Learners will revise Chapter 6's content at the end of Chapter 7. Learners can be asked to complete questions **1–3** of Chapter 7 Revision at this point

CHAPTER

7 Algebraic equations

Chapter Overview

Learner's Book pages 130 to 141
Recommended pacing: 3 hours

This chapter focuses on the following:

Unit 1:	Equations	1,5 hours
	Equations used to describe problems	
	Interpreting equations	
Unit 2:	Solving equations	1,5 hours
	Solving by means of inspection	
	Solve expressions by substitution	
PoA Assignment 2: Patterns, relationships and Algebra		30 minutes
Chapter 7 Revision		

Review Copy

UNIT

1 Equations

Unit overview

Learner's Book page 131
Recommended pacing: 1,5 hours

This unit focuses on the following:

- learning problem-solving skills in order to solve word problems
- writing and solving equations
- interpreting equations

Resources: Learner's Book and exercise book

Mental Maths

(10 minutes per day)

Suggested answers — Learner's Book page 131

1 a correct
  b incorrect; $5x - 1 + 4 = 30$
2 a $8x - 8 = 56$; $x = 8$
  b $x + x + 10 = 40$; $x = 10$; Zihle is 25 years old and her brother is 15 years old.
  c $x + x + 1 = 29$; $x = 14$; The two numbers are 14 and 15.

Background information

In Grade 7, learners studied the following:

- writing number sentences to describe problem situations
- analysing and interpreting numbers sentences that describe a given situation
- solving and completing number sentences by inspection and trial-and-improvement.
- number sentences including integers, square numbers and cube numbers
- determining the numerical value of an expression by substitution.

In Grade 8, learners revise the above and solve equations using additive and multiplicative inverses and laws of exponents.

Teaching guidelines

Learners have been doing "word sums" for many years and have probably not been using Algebra to solve them. They would have employed formulae (like Perimeter or Area) if available, but may not have written their own equations. In this unit, learners are taken through the steps of using problem-solving skills to write and solve word problems.

Mathematics has been developed to solve problems and, although the word problems found in Mathematics are necessarily contrived, they are good examples of how it can be used. Try to introduce a sense of excitement in this section. Explain to learners that real Mathematicians work on word problems'and that they are now getting to the exciting part of Mathematics. Often word problems are seen as problems and learners do not like them, so it is important to try to change that attitude. When word problems are practised often, they become word exercises and should no longer be problems!

Review Copy

Equations used to describe problems

Activity 1 **Solve word problems**

Learner's Book page 132

Guidelines for implementing this activity

- Revise what an equation is, and the difference between an algebraic expression and an algebraic equation (the latter has an "is equal to" sign; the left-hand side is equal to the right hand side).
- Work carefully through the steps and the example in the Learner's Book. Pause and check learners' understanding of words such as substitute and trial-and-improvement.
- Make up a few of your own word problems (similar to word problems in this activity) or search on the Internet for some word problems. Write them on the board. Use the problem-solving steps to show learners the how to write and solve the problem step-by-step.

Remedial and extension

Remedial: Learners should reread the problem several times if necessary to ensure that they understand the problem. They work in pairs. However, learners should write the answers in their own exercise books.
Extension: Questions **6** to **12** are more challenging.

Suggested answers

1 Let a certain number be x; $6x + 7 = 19$; $x = 2$; the number is 2.
2 Let a number be x; $10x + 7 = 57$; $x = 5$; the number is 5.
3 Let a number be x; $2x - 5 = 15$; $x = 10$; the number is 10.
4 Let a number be x; $(x + 11) \times 2 = 26$; $x = 2$; the number is 2.
5 Let the mystery number be x; $\frac{1}{2}x + 2 - 3 = 5$; $\frac{1}{2}x - 1 = 5$; $x = 12$; the mystery number is 12.
6 Let Sibongile be x years old; Sibongile: x; Thabiso: $x + 4$;
$(x) + (x + 4) = 28$; $x = 12$; therefore Sibongile is now 12 and Thabiso 16 years old.
7 Let Sipho score x goals; Sipho: x; Khaya: $x + 3$;
$(x) + (x + 3) = 17$; $x = 7$; therefore Sipho scored 7 goals.
8 Let Palisa cycle x kilometres; Palisa: x; Jakes: $x + 10$;
$(x) + (x + 10) = 70$; $x = 30$; therefore Palisa cycled 30 km and Jakes cycled 40 km.
9 Let the breadth of the rectangle be x centimetres long
Breadth: x; Length: $x + 2$; $(x) \times (x + 2) = 35$; $x = 5$;
therefore the breadth of the rectangle is 5 cm and the length is 7 cm.
10 Let the breadth of the rectangle be x centimetres.
Breadth: x; Length: $x + 1$; $(x) \times (x + 1) = 90$; $x = 9$;
therefore the breadth of the rectangle is 9 cm and the length is 10 cm.
11 Let the breadth of the rectangle be x centimetres.
Breadth: x; Length: $3 \times x$; $(x) \times (3 \times x) = 12$; $x = 2$;
therefore the breadth of the rectangle is 2 cm and the length is 6 cm.
12 Let the smallest number be x; $(x) + (x + 1) = 41$; $x = 20$;
therefore the two consecutive numbers are 20 and 21.

Review Copy

Interpreting equations

Activity 2 Interpret equations

Learner's Book page 134

Guidelines for implementing this activity

- Here we teach learners to judge whether an expression or equation that they write,is correct. This skill will help them in tests and exams when they write and solve word problems.
- Work through the example in the Learner's Book, and through additional examples on the board of necessary.

Remedial and extension

Remedial: Learners work in pairs. However, learners should write their answers in their own exercise books.
Extension: Question **8** is actually correct. We said that consecutive numbers are x and $x + 1$. However, $x - 1$ and x are also consecutive numbers.

Suggested answers

1 correct
2 incorrect; $6x + 2 = 20$
3 incorrect; $2x - 5 + 7 = 22$
4 correct
5 incorrect; $(x - 15) \times 2 = 10$
6 correct
7 correct
***8** correct

UNIT 2

Solving equations

Review Copy

Unit overview

Learner's Book page 135
Recommended pacing: 1,5 hours

This unit focuses on the following:
- solving equations by means of inspection
- solving equations by means of substitution

Resources: Learner's Book; exercise book

Mental Maths

(10 minutes per day)

Suggested answers — Learner's Book page 135

1 **a** $p = 6$ **b** $q = 14$ **c** $r = 6$
d $r = 5$ **e** $k = -2$ ***f** $a = 10$
***g** $b = 9$ ***h** $c = -5$ ***i** $m = 0$

2 **a** $5 + -2 = 3$ **b** $5 - 2(-2) = 5 + 4 = 9$
c $5 \times -2 = -10$ **d** $\frac{5}{-2}$
e $5^2 = 25$ **f** $4(-2)^2 = 4 \times 4 = 16$
g $5^2 \times (-2)^3 = 25 \times 8 = 200$
***h** $3(5)^2 + 5(-2) = 3(25) + -10 = 75 + -10 = 65$

Background information

In Grade 7, learners studied the following:

- writing number sentences to describe problem situations
- analysing and interpreting numbers sentences that describe a given situation
- solving and completing number sentences by inspection and trial-and-improvement; number sentences included integers, square numbers and cube numbers
- determining the numerical value of an expression by substitution

In Grade 8, learners revise the above and solve equations using additive and multiplicative inverses and the laws of exponents.

Teaching guidelines

This unit revises the work that learners have done in Grade 7, but again it is very important to establish that all learners, irrespective of their previous Maths backgrounds, master solving equations.

Remind learners that they have been solving equations since Grade 1 in the form of 5 + □ = 8 and that they can do straightforward equations using inspection. In Grade 8, learners use variables instead of the little squares used in the Foundation Phase, but the mental maths methods that they used in Grades 1, 2 and 3 should not be forgotten.

Solving expressions using substitution is also not new to learners. Ensure that all learners use brackets when they substitute. It is important to establish these good habits early on. As learners progress to Grade 12 Mathematics, they will be given more complex equations to substitute into, and the importance of using brackets increases.

Review Copy

Solving by means of inspection

Activity 1 **Solve equations by means of inspection**

Learner's Book page 136

Guidelines for implementing this activity

- Work through the examples in the Learner's Book.
- Write a few equations on the board with □ instead of variables. Ask learners to solve the equations by just looking at them. (Learners have worked with number sentences in previous grades.) Explain to learners what equivalent means, and how to solve equations through inspection.

Remedial and extension

Remedial: Use the inverse operation to find the variable.
Extension: Questions **2f** and **3** are more challenging

Suggested answers

1	a	$x = 2$	b	$a = 37$	c	$b = 5$
	d	$c = 24$	e	$k = 2$	f	$p = 6$
	g	$q = 40$	h	$r = 5$	i	$r = 8$
	j	$x = 83$	k	$x = 0$	l	$p = 144$
	m	$q = 60$	n	$m = 5$		
2	a	$x = 4$	b	$m = 3$	c	$x = 7$
	d	$k = 2$	e	$x = 2$	*f	$x = 4$
*3	a	$y = 0$	b	$k = 2$	c	$k = 10$
	d	$m = 10$	e	$k = 25$	f	$x = 2$

Solve expressions by means of substitution

Activity 2 Solve expressions by means of substitution

Learner's Book page 137

Guidelines for implementing this activity

Explain substitution by writing something easy such as $3x$ on the board. Then write:
If $x = 1$: $3 \times 1 = 3$
If $x = 2$: $3 \times 2 = 6$
If $x = 3$: $3 \times 3 = 9$
If $x = 4$: $3 \times 4 = 12$.
Explain the examples on the board. Learners may find questions containing cubes more difficult.

Review Copy

Remedial and extension

Remedial: Learners should write out the steps properly with the equals signs below one another.
Extension: Questions **1j** to **1n** are more challenging.

Suggested answers

1 a $-3 + 4 = 1$
b $-3 - 4 = -7$
c $4 - -3 = 7$
d $(-3)(4)(0) = 0$
e $(-3)^2 = 9$
f $\frac{0}{-3} = 0$
g $(-3)^3 + 7 = -27 + 7 = -20$
h $\frac{4}{0}$ = undefined
i $4^3 = 64$
*j $(-3)^2(4) - (4)(-3) = 9(4) - -12 = 36 + 12 = 48$
*k $4^2(0) - 12(4) - 4(-3) = 0 - 48 + 12 = -36$
*l $2(-3)^2(4) = 2(9)(4) = 2(36) = 72$
*m $-(-3 \times 4)^2 = -(-12)^2 = -1 \times 144 = -144$
*n $[(-3)^2(4) - 31]^3 = (9 \times 4 - 31)^3 = (36 - 31)^5 = 5^3 = 125$

2	a	-40	b	-161	c	2	d	68
	e	0	f	-24	g	48	h	undefined
	i	-60	j	-216	k	$-\frac{7}{32}$	l	72

Chapter 7 Revision

Learner's Book page 138

Please note that revision for Chapters 6 and 7 is combined into one revision exercise, on page 137 of the Learner's Book. Encourage learners to work through all the content in the chapters before doing the revision exercise. Because it is a combined exercise, consider doing questions **1–19** as one exercise and questions **20–23** as a second exercise.
The revision activities should be used to assess learners' progress thus far, and to assess where remediation may be required.

Suggested answers

1 **a** $40 + x$ (or $x + 40$) **b** $10x - 16$
c $\frac{50}{x} = 2$ ***d** $2x + 1 = 35$

2 **a** R250 + n **b** $p + 30$

3 **a** xyz **b** $25t$
c $10pq + 6q$ **d** $5(1 - p)$ or $(1 - p)5$

4 **a** 3 **b** 2 **c** 1 **d** 2

5

a	5	1	3
b	7	9	–6
c	–1	0	11

6

a	3	k	7
b	–1	xy	4
c	15	k	1
d	1	y	12

7 **a** $4x^{15}$ **b** $8p$ **c** $\frac{5}{2x}$ **d** $18p^2q$

8 **a** polynomial **b** monomial/term **c** binomial

9 **a** $3m^4 + 7m^3 + m^2 - 4m + 8$ ***b** $-18a^2b + 6ab - 4b^2$

10 **a** $36b - 18c$ **b** $-66p + 99q$
c $40k + 140m - 120n$ **d** $-48x + 80y + 28z$
e $6a + 42b - 36c + 6d - 6e$ ***f** $-3r + 13t - 9t + 18r = 15r + 4t$

11 **a** $-5 \cdot p \cdot p \cdot p \cdot p \cdot p \cdot q$ **b** $2 \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y$
c $\frac{4 \cdot x \cdot y \cdot y \cdot z}{a \cdot b}$

12 **a** $-3pq + 3$ **b** $6x^2 + 7x + 9$
c $7p^2 + 20pq - 10q^2$

13 $12m + -n - 11p$

14 $(14r^2 - 3r + 5) - (r^2 - 9r + 10) = 13r^2 + 6r - 5$

15 $(4x + 8y) + (10y + -12x) = -8x + 18y$; $(x + 5y) - (-8x + 18y) = 9x - 13y$

16 $10a + -3a = 7a$

17 $-10b$; $2b + -4b = -2b$

18 $-p$; $12p$; $-3p + 36p = 33p$

19 $5a(-3b + 10b) = 5a(7b) = 35ab$

20 **a** $x - 8 = 14$; $x = 22$
b $x + x + 5 = 21$; $x = 8$; Henry scored 13 goals and Thabo scored 8 goals.
***c** $x + x + 2 = 54$; $x = 26$; $x + 2 = 28$

21 **a** correct **b** incorrect; $x + x + 20 = 120$

22 **a** $z = 5$ **b** $y = 9$ **c** $x = -15$ **d** $p = 6$
***e** $n = -1$ ***f** $m = 12$ ***g** $k = 2$

Review Copy

23 a $-5 + 3 = -2$ b $5(-5) - 7 = -25 - 7 = -32$

c $-2(3) + 13 = -6 + 13 = 7$ d $\frac{-10}{3}$

e $20(-5) + 20(3) + 20 = -100 + 60 + 20 = -20$

•f $(-5 + 3)^2 + 6 = (-2)^2 + 6 = 4 + 6 = 10$

PoA | Assignment 2 Patterns, relationships and Algebra

Learner's Book page 140–141

Time: 50 minutes **Marks: 50**

Suggested answers

1 a I need to add consecutive numbers, i.e. first add 1, then add 2, then add 3, etc. to get the next number in the sequence✓ (1)

b –2; 3; 9; 16✓✓(1 mark per two correct values) (2)

2 a ✓(A, top row) ✓(A, bottom row) (2)

n	1	2	3	4
T_n	1	–3	–7	–11

b $T_1 = -4(1) + 5 = 1$; $T_1 = -4(1) + 5 = 1$; $T_1 = -4(1) + 5 = 1$; $T_4 = -4(4) + 5 = -11$✓✓ (1 mark per two correct terms) (2)

c $T_n = -4(n) + 5$✓(n term) ✓(+ 5) (2)

d $T_{100} = -4(100) + 5 = -395$✓✓ (2)

3 $t = (-4 + 5)3 = (1)(3) = 3$✓; 12✓; 0✓; 30✓ (1 mark per two correct values) (2)

4 $t = 3(n) - 1$✓(t =) ✓($3n - 1$) (2)

5 a $l = 2(n)$✓ (2)

b $2(7)$ ✓ $= 14$✓ (2)

c $24 \div 2$✓ $= 12$ weeks✓ (2)

6 $2(2x + 1)$ or $2(2x - 1)$✓($\times$ 2)✓(odd number) (2)

7 a $3mnp$✓ (1)

b $2xy + 3y$✓ (1)

8 one term✓ (1)

9 1✓ (1)

10 a –1✓ b xy✓ c 16✓ (3)

11 pac^2 ✓ (1)

12 trinomial✓ (1)

13 $m^4 + 5m^3 + 4m^2 - 5m + 18$✓ (1)

14 a $-15a$✓ $- 30b$✓ (2)

b $11n$✓ (1)

c $(x^2 + 3x^2) + (5x - 7x) + (-6 + 6) = 4x^2$✓ $- 2x$✓ (2)

d $(3q^2 + 4q - 5) - (6q^2 - 7q + 6)$
$= 3q^2 + 4q - 5 - 6q^2 + 7q - 6$✓
$= -3q^2$✓ $+ 11q$✓ $- 11$✓ (4)

15 a $x = 8$✓ (1)

b $x = 0$✓ (1)

c $x = 40$✓ (1)

16 $b^2c + -4a = (32)(1) + -4(-2)$ ✓ $= 9$✓ $+ 8$✓ $= 17$✓ (4)

17 $x + x + 5 = 29$✓
$x = 12$; Ella is 12 years old. ✓ (2)

Review Copy

PoA Test 1

Time: 50 minutes **Total: 50 marks**

1 Identify the integers in this set. Write Z ={$\frac{13}{9}$; –14; 28; $1\frac{1}{2}$; 25,555; 0,65; 15; –6; $\frac{10}{5}$} (2)

2 Write the next three numbers in the number sequence: –24; –17; –10; … (1)

3 Rewrite the following numbers in descending order: –13; 14; –8; –25; –37; 63; –43; –10 (2)

4 True or false:

a $\frac{8}{5}$ is the additive inverse of $\frac{5}{8}$

b $-\frac{2}{3}$ is the multiplicative inverse of $-\frac{3}{2}$ (2)

5 Calculate.

a $-5 - 13 + 1$ (2)

b $-7 + -9 - 2$ (2)

c $-60 \div -5 \times -9$ (2)

d $16 \div -4 + -15 \times 2$ (3)

e -12^2 (1)

f $\sqrt{-64}$ (1)

g $5^3 - 6^2$ (3)

h $(25 - 16)2$ (2)

i $32^8 \times 32^{20}$ (1)

j $\frac{9m^4n^2}{-18n^2m^{10}}$ (2)

k $(3x^7y^9)^2$ (2)

l $75(-13)^0x^0$ (2)

m $32 + 2\sqrt{64} \times \sqrt[3]{27}$ (5)

Review Copy

6 Vernon wants to buy his cousins old car. His cousin wants R75 250. However, Vernon convinces his cousin to give him a 35% discount since the car has not recently had a service, the tyres are worn down, the brake lights are not working and there is a crack in the wind shield. His cousin agrees. What is the new purchase price of the car? (2)

7 Estimate the answer of $\sqrt{50}$ (1)

8 Is $-ab$ a constant or a variable? (1)

9 Redraw and complete the table:

	Question	Coefficient	Base	Exponent
a	$-(xy)^2$			
b	m			

(2)

10 Expand $(-5x)^2y$ (1)

11 Write $-3p \times 4p \times -5p^7$ in exponential notation. (2)

12 Which one is larger?

a 5^6 or -6^5 (1)

b $2{,}58 \times 10^5$ or $1{,}9 \times 10^6$ (2)

13 Write 568 138 100 000 in scientific notation. (2)

14 Write $8{,}21 \times 10^5$ in normal notation. (1)

PoA Test 1 Memorandum

1 $Z = \{-14; 28; 15; -6; \frac{10}{5}\}$✓✓ (2)

2 –3; 4; 11✓ (1)

3 63; 14; –8; –10; –13; –25; –37; –43✓✓ (2)

4 a false✓

b true✓ (2)

5 a –18✓ + 1 = –17✓ (2)

b –16✓ – 2 = –18✓ (2)

c 12✓ × –9 = –108✓ (2)

d –4✓ + –30✓ = –34✓ (3)

e –144✓ (1)

f impossible✓ (1)

g 125✓ – 36✓ = 89✓ (3)

h 92✓ = 81✓ (2)

i 32^{28}✓ (1)

j $-\frac{1}{2m^6}$ ✓(✓ (2)

k $9x^{14}y^{18}$ ✓ ✓ (2)

l 75 × 1✓ = 75✓ (2)

m 9✓ + 2(8)✓ × 3✓ = 9 + 48✓ = 57✓ (5)

6 100% – 35% = 65 %; $\frac{65}{100}$ × R75 250✓ = R48 912,50✓ (2)

OR $\frac{35}{100}$ × R75 250 = R26 337,50✓; R75 250 – R26 337,50 = R48 912,50✓

7 $7 < \sqrt{50} < 8$✓ (1)

8 variable✓ (1)

9 a –1; *xy*; 2✓

b 1; *m*; 1✓ (2)

10 –5.–5.*x*.*x*.*y*✓ (1)

11 $60p^9$ ✓✓ (2)

12 a $5^6 > -6^5$ ✓ (1)

b $2{,}58 \times 10^5 = 258\ 000$; 1,9 × 106 = 1 900 000✓; $2{,}58 \times 10^5 < 1{,}9 \times 106$ ✓ (2)

13 $5{,}58138 \times 10^{11}$ ✓ ✓ (2)

14 821 000✓ (1)

Review Copy

CHAPTER

8 More algebraic expressions

Chapter overview

Learner's Book pages 142 to 156
Recommended pacing: 9 hours

This chapter focuses on the following:

Unit 1	**Interpreting algebraic expressions**	2 hours
	Recognising and using algebraic language	
	Interpreting algebraic expressions	
Unit 2	**Working with algebraic expressions**	3 hours
	Adding and subtracting algebraic expressions	
	Revision: Types of algebraic expression	
Unit 3	**Simplifying algebraic expressions**	4 hours
	Multiply monomials	
	Multiply binomials	
	Multiply trinomials	
	Divide monomials	
	Divide binomials	
	Divide trinomials	
	Simplifying complex expressions	
	Simplifying expressions with exponents	
	Simplifying using substitution	
Chapter 8 Revision		

Review Copy

UNIT

1 Interpreting algebraic expressions

Unit overview

Learner's Book page 143
Recommended pacing: 2 hours

This unit focuses on the following:

- recognising and using algebraic language by substituting words/phrases with symbols
- interpreting algebraic expressions by working with the general rules of
- number sequences
- interpreting algebraic expressions by solving word problems using substitution

Resources: Learner's Book; exercise book

Mental Maths

(10 minutes per day)

Suggested answers — Learner's Book page 143

1 a false b true

2 a $(x + 6) \times 12$ b $2x + 1$ c $\frac{x}{4} = 2$ *d $\frac{6 + 2x}{3}$

3 a $5n$ b $4n - 3$ *c $\frac{1}{3n}$

4 a 40 years old *b 42 pairs of socks

Review Copy

Background information

Learners were introduced to algebraic expressions and equations in Grade 7 and in Grade 8 Term 1, where they did the following:

- identified variables and constants in formulae and equations
- recognised and interpreted algebraic language
- wrote in algebraic language
- used number sentences to solve problems
- solved number sentences by trial-and-improvement and by inspection
- used substitution
- extended numeric and geometric patterns by finding the rule
- created their own number sequences
- wrote the general rule
- found the nth term in a sequence.

The content in this unit is therefore revision.

Teaching guidelines

This unit focuses on revising the use of algebraic language, and describing number sequences using algebraic expressions. The mental maths exercises at the beginning of

the unit provide the essential practice that learners require in order to master these basic skills, and can also be used to assess learners' prior knowledge and current understanding of this content. Doing these types of mental maths activities each day also serves the purpose of revising and consolidating these basic skills. Revise the following important mathematical conventions for writing variables and constants before commencing with the teaching of this unit.

- A constant is a number in an expression whose value does not change, for example –9.
- A variable is a letter of the alphabet, for example a
- A variable may represent one value, for example $a = 5$ in $a + 7 = 12$.
- A variable may represent many values, for example for $y = x + 8$, x and y could have many different values.
- $-n$ means $-1 \times n$.

Recognising and using algebraic language

Activity 1 Recognise and write algebraic expressions

Learner's Book page 144

Guidelines for implementing this activity

- Revise the words and phrases and their symbols by working through the Remember section in the Learner's Book with the learners.
- Discuss the example in the Learner's Book, and provide additional examples for learners to do orally, or to write down quickly in their exercise books and volunteer their answers, for example "90 decreased by a certain number multiplied by 2" ($90 - x \times 2$).
- Emphasise that an algebraic expression does not contain a = sign.

Review Copy

Remedial and extension

Remedial: Provide additional, simpler questions at first, such as the ones in questions **2a** and **2b** in order to build leaners' confidence. Learners can also work in pairs, though it is important to check their individual progress to ensure that they grasp the basic concepts. Simpler questions described above can also be worked through in pairs, before learners tackle questions **2c** to **2h**. Also include simple problems, and then gradually more challenging examples such as the ones in the mentals maths activity as well as the ones in questions **2a** and **2b**, in the 10-minute mental maths sessions every day.

Extension: Question **2h** is more challenging. In pairs, let learners create their own examples and swap these so that their partners can write the expressions.

Suggested answers

1 a true b false c true
d false e false f true

2 a $15 + x$ (or $x + 15$) b $(x - 5)7$ c $2x - 12$
d $x - 3x$ e $x = 16 - 4$ f $\frac{x}{10} = 2$
g $2x \times 6 = 24$ *h $\frac{2x+1}{2} + 5 \left(\text{or } \frac{2x-1}{2} + 5\right)$

Interpreting algebraic expressions

Activity 2 Interpret algebraic expressions

Learner's Book page 145

Guidelines for implementing this activity

- Revise finding the general term for the number pattern of a number sequence by working through the example in the Learner's Book.
- Do additional, simple examples such as the ones in questions **1** and **2** before learners attempt the activity.
- Invite learners to volunteer their own number sequences such as the ones in question **1**, and go through these together as a class.
- Additional examples such as the one in question **2a** can be done orally, or read them aloud for learners to do them in their exercise books and then volunteer their answers.
- Go through the summary in the Learner's Book as a form of revision and consolidation of the concepts covered, after learners have completed the activities.

Review Copy

Remedial and extension

Remedial: If necessary, work through questions **1a** and **2a** with the learners before they continue on their own. Provide additional, simpler questions at first, such as the ones in question **2a** in order to build leaners' confidence. Learners can also work in pairs, though it is once again important to check their individual progress to ensure that they grasp the basic concepts. Practice builds confidence and skill.
Extension: Activity 2 questions **1e**, **1f**, **2e** and **2f** are more challenging.

Suggested answers

1 a $2n$ b $2n - 1$ c $2n + 1$
d $3n + 1$ *e $\frac{1}{2^n}$ *f 3^n

2 a $7 - 5 = 2$ years old
b $675 + 50 = 725$ songs
c $5 + 1 = 6$ brothers
d $25 - 14 = 11$ dogs
*e 2 weeks = 14 days; $x = 14$ days; $2 \times 14 = 28$ days
*f $3x = 90$; $x = 30$; $90 + 30 = 120$ plants

UNIT 2

Working with algebraic expressions

Unit overview

Learner's Book page 146
Recommended pacing: 3 hours

This unit focuses on the following:

- adding and subtracting like terms in algebraic expressions
- identifying, simplifying and naming algebraic expressions

Resources: Learner's Book; exercise book

Mental Maths

(10 minutes per day)

Suggested answers — Learner's Book page 146

1 a binomial b monomial c trinomial
2 a $7x + 10$; binomial b $11m^2 - 3m + 12$; trinomial c $18n^2 - 8n$; binomial

Review Copy

Background information

Learners have been introduced to like and unlike terms as well as monomials, binomials and trinomials in the first term. They have also ordered expressions, added and subtracted like terms vertically and horizontally, and used the commutative, associative and distributive properties when working with like and unlike terms.

Teaching guidelines

The mental maths exercises at the beginning of the unit provide the essential practice that learners will require and can also be used to assess learners' prior knowledge and current understanding of this content. Doing these types of mental maths activityb each day also serves the purpose of revising and consolidating these basic skills.

Revise like and unlike terms thoroughly at the beginning of this unit since these concepts form the basis of the content to be covered. Work through examples such as the following with the learners, and include examples such as these in subsequent mental maths sessions.

- Which term does not fit in with the rest in each set?
  a abc; $3bac$; $2cb$ b $3x^2$; $3x^3$; $2x^2$ c $4p$; $3q$; $-4q$
  d 7;16; a e $2a^2b^2$; $3a^2b^2$; $4a^2b$

- Is each of the following a like or unlike term? If they are unlike terms, explain why this is the case.
  - **a** $9p^2q^2$; p^2q^2; $2p^2q$
  - **b** $2a^2b^3$; a^2b^3; $4a^2b^3$
  - **c** bac; $9bca$; $5acb$
  - **d** $7r^2t^5$; r^2t^7; $4r^2t^6$

Revise these important mathematical conventions.

- Coefficients first: Always write the number before the letter of the alphabet, for example $6n$.
- Order: If there is more than one variable in a term, it should be in alphabetical order. Also write the different terms in the expression in alphabetical order, for example $3a + 6b - c$
- No brackets or other signs when multiplying: $2x$ is the correct way to write $2 \times x$; $x \times 2$; $2(x)$; $(2)x$; $2 \cdot x$.
- Don't write 1: When there is only one of a variable, for example $1p$, just write p. (It is okay to write $1p$ at first while you're still working out an answer.)

Do additional examples as follows:

- Correct each expression according to mathematical conventions: $x \times -3$; $-1(t)$; $x \times -11 \times z$; tax; $xy(-3)$; $(-1)(x)(y)(z)$; $-2 \times x \times z$; and so on.
- How many terms are there in $3x + 2 + \frac{r}{2}$?
- What is the coefficient of y in $2y^3 + 7y^2 + 10y$?

Review Copy

Adding and subtracting algebraic expressions

Activity 1 Add and subtract algebraic expressions

Learner's Book page 147

Guidelines for implementing this activity

- Once like and unlike terms have been revised with the learners, remind learners that only like terms can be added. It is very useful for learners to mark like terms with either a highlighter or with circles, blocks or by underlining them.
- Work through the example in the Learner's Book, and remind learners that the final answer has to be written in descending and alphabetical order. Do additional examples with three or more variables to show this, since this can be difficult for learners to grasp at first.
- Emphasise the importance of focusing on the first alphabetical letter (for example, x if there is x and y) and then writing these variables in descending order before moving on to the next variable.
- Draw learners' attention to the notes in the Learner's Book, ensuring that they understand these basic rules and techniques for manipulating algebraic expressions.

Remedial and extension

Remedial: Ensure that learners highlight like terms at all times. Sometimes it also makes it easier to first group all the like terms together, for example:
$(c^2 + 25c + 24) - (5c^2 + 6c + 1)$
$= c^2 + 25c + 24 - 5c^2 - 6c - 1$
$= (c^2 - 5c^2) + (25c - 6c) + (24 - 1)$
$= -4c^2 + 19c + 23$
Extension: Activity 1 questions **10** to **12** are more challenging.

Suggested answers

1 $2a + 3$
2 $5b^2 - 24b + 5$
3 $-4c^2 + 19c + 23$
4 $m^2 - m + 12$
5 $10x - 9y + 6z$
6 $22e^2 + 22e$
7 $6p^2 + 8pq^3 + 8$
8 $17x^2 + 12xy - 6y^2$
9 $-3m^2 - 2mn + 5n^2 - 6$
***10** $22k^3 + 15k^2 - 4k + 17$
***11** $45r^2 + 3r - 5$
***12** $(15x + 18y) - (x + 14y + 8y - x) = 15x + 18y - 22y = 15x - 4y$

Revision: Types of algebraic expression

Review Copy

Activity 2 Identify algebraic expressions

Learner's Book page 147

Guidelines for implementing this activity

- Learners have been introduced to monomials, binomials and trinomials in the first term. Remind learners that "mono" means 1, "bi" means 2 and "tri" means 3.
- Emphasise that a term can contain multiplication, division and brackets. Terms are separated by + or – signs. For example, $2x \times -1$ is one term, and $2x + -1$ has two terms, namely $2x$ and -1.
- Revise these types of algebraic expression by working carefully through the notes in the Learner's Book. Provide additional examples of expressions on the board, similar to the ones in the Learner's Book, and let the learners count the terms and name the type of expression.
- Once learners are confident, they can attempt the activity on their own.

Remedial and extension

Remedial: Question **1** is simpler as learners can count the number of terms by inspection. Work through questions **2a** and **2b** with learners if necessary to build their confidence, before they continue on their own. Provide additional questions as homework for learners to practise.
Extension: In Question **2** learners first have to simplify the algebraic expressions before they can name them.

Suggested answers

1 a binomial
b trinomial
c binomial
d monomial
e trinomial
f monomial
g binomial
h binomial

2 a $14a + 10$; binomial
b $17n^2 - 22n - 12$; trinomial
c $2c^2 + 4c - 8$; trinomial
d $25a + 5b$; binomial

UNIT 3

Simplifying algebraic expressions

Unit overview

Learner's Book page 148
Recommended pacing: 4 hours

This unit focuses on the following:

- multiplying numbers and monomials with monomials
- multiplying numbers and monomials with binomials
- multiplying numbers and monomials with trinomials
- dividing monomials by numbers and monomials
- dividing binomials by numbers and monomials
- dividing trinomials by numbers and monomials
- simplifying complex algebraic expressions
- simplifying algebraic expressions with square and cube numbers and roots
- simplifying algebraic expressions by means of substitution

Resources: Learner's Book; exercise book

Review Copy

Mental Maths

(10 minutes per day)

Suggested answers

Learner's Book page 148

1 a $-60x^3y^5$
b $5x - 40y$
c $-21x + 36y$
d $15x^2 + 4x$
e $8x^4 - 12x^3y^3 - 24x^2y^4$
f $\frac{x^6}{2y^{12}}$
g $\frac{x^3y^3}{15} - 1$
h $\frac{5x^7}{y^4} + 4x^3 - \frac{3x}{y^5}$
i $-24x^2y^4 + 40x^2y^4 = 16x^2y^4$
j $-19x^3 + 14x^2 + 42x$
k $-6x^{21}y^7$
l $9x^3y^4$
m $-\frac{x}{2y^3}$

2 a $(-1)^2 + 5(4) - 4 = 17$
b $2(4)^2 - 12 = 20$
c $\{3(-1) + 2(4)\}^2 = 25$
d $\frac{6(-1) + 4}{3(4) - 5} = -\frac{2}{7}$
e $\sqrt{(-1)^2 \times (4)^3} = 8$

Background information

Learners have been introduced to exponents and the exponent laws in the first term and in previous grades, and have ordered, added and subtracted algebraic expressions. Multiplying and dividing monomials, binomials and trinomials are new.

Teaching guidelines

The mental maths exercises at the beginning of the unit provide the essential practice that learners will require in order to master these basic skills, and can also be used to assess learners' prior knowledge and current understanding of this content. Doing these types of mental maths activity each day also serves the purpose of revising and consolidating these basic skills.

Introduce the content of this unit by revising the exponent laws, as this knowledge forms the basis of the content of this unit.
Exponent Law 1: $8^5 \times 8^7 = 8^{5+7} = 8^{12}$. (When we multiply powers with the same bases we add the exponents.)
Exponent Law 2 (division with larger numerator): $18^{10} \div 18^8 = 18^{10-8} = 18^2$. $(10 > 8)$
Exponent Law 3 (division with larger denominator): $12^6 \div 12^{17} = \frac{1}{12^{17-6}} = \frac{1}{12^{11}}$.
$(17 > 6)$

Do quick, simple examples such as the ones above, that require learners to apply the above laws.

Include simple exercises as described above in mental maths sessions at the beginning of every lesson.

Review Copy

Multiply monomials

Activity 1 Multiply with monomials

Learner's Book page 149

Guidelines for implementing this activity

- Revise Exponent Law 1: When we multiply powers with the same bases we add the exponents.
- Revise multiplication of unlike terms as shown in the Learner's Book.
- Emphasise the importance of following the steps.
  - Multiply the signs.
  - Multiply the coefficients.
  - Multiply the variables and add the exponents.
- Turn learners' attention to the notes; the negative variable must be multiplied with each term inside the bracket. This is very important.

Remedial and extension

Remedial: Emphasise the importance of working step-by-step. Monitor learners while they are doing the activity to ensure that they adhere to the steps at all times, and in order to guide them through specific steps that they may find difficult.
Extension: Learners can work in pairs and make up mini-tests of three questions at a time for their partners to work through.

Suggested answers

1 $2b$	**2** $15yz$	**3** $12rt$	**4** $28ab$
5 $-48fg$	**6** $12kmn$	**7** $-56k^3$	**8** $-x^4$
9 0	**10** $33c^6f$	**11** $-34m^2n^6$	**12** $15r^6$
13 $190x^2y^5z^3$	**14** $-21a^4b^2c^3$		

Multiply binomials

Activity 2 Multiply with binomials

Learner's Book page 149

Review Copy

Guidelines for implementing this activity

- Revise multiplying out brackets. We multiply the coefficient with every term inside the bracket.
- Work through the notes in the Remember box with the learners, checking their understanding every step of the way.
- Remind learners to write answers in descending and alphabetical order.
- Do a few additional examples on the board. Let learners do two or three of these examples in their exercise books and volunteer their solutions by writing them out on the board.

Remedial and extension

Remedial: The most common challenge is remembering to multiply the coefficient with every term. Learners often make the mistake of multiplying the coefficient only with the first term inside the bracket. Walk around the classroom and cast an eye over learners' exercise books to see that they are multiplying the coefficient with all the terms, and that they are following the steps.
Extension: Provide an additonal exercise with more variables, larger exponents and larger coefficients for learners.

Suggested answers

1 $10x + 20y$	**2** $-70a + 20b$	**3** $-30c - 40d$
4 $-9g + 36h$	**5** $12p + 4q$	**6** $5x^2 - x$
7 $-9k^2 + 8k$	**8** $12x^2 + 10x$	**9** $-15x^2 + 33x$
10 $40p^2 + 25p$	**11** $-6k - 14p$	**12** $-m + n$
13 $12x^2y + 2xy^2$	**14** $-15x^2y + 6xy^2$	

Multiply trinomials

Activity 3 Multiply trinomials

Learner's Book page 150

Guidelines for implementing this activity

- Remind learners how to multiply out brackets. They need to multiply the coefficient with every term inside the bracket and follow the same steps as for multiplying binomials.
- Remind them to write the answers in descending and alphabetical order. Let learners do two or three of these examples in their exercise books and volunteer their solutions by writing them out on the board.

Remedial and extension

Remedial: Walk around the classroom and cast an eye over learners' exercise books to see that they are multiplying the coefficient with all the terms, and that they are following the steps.
Extension: Questions **7** to **10** contain variables in front of the bracket and are therefore more challenging.

Review Copy

Suggested answers

1 $60a + 35b - 40c$
2 $24d - 144e + 12f$
3 $-12g^2 - 24g + 80$
4 $-9k^2 + 54k - 27$
5 $60x^3y - 70x^2y - 120x$
6 $-12m^2 - 18m + 13$
***7** $-6p^3 + 21p^2q - 18pq^3$
***8** $63x^4 - 49x^3 - 7x^2$
***9** $24v^2w - 42vw^2 + 12vw^3$
***10** $-44x^4 + 11x^3y^3 + 11x^2y^4$

Divide monomials

Activity 4 Divide monomials

Learner's Book page 151

Guidelines for implementing this activity

- Revise Exponent Laws 2 and 3 by working through the notes in the *Remember* box with the learners.
- Work through the steps and the worked example with the learners.
  - Divide the signs.
  - Divide the coefficients by the HCF.
  - Divide the variables and subtract the coefficients.
- Remember that the answers lie "where the most are", for example, in $\frac{x^2}{x^5}$ there are more *x*s at the bottom, therefore the answer's power will be written at the bottom $\left(\frac{1}{x^3}\right)$.

Remedial and extension

Remedial: Emphasise the importance of following the steps. Monitor learners' progress and provide specific guidance and support with this important process while they are busy with the activity. Sit with small groups of learners and monitor their progress, providing support if necessary.
Extension: In Activity 4 questions **9** to **12** the bases are in different orders. Learners need to look carefully when simplifying.

Suggested answers

1 $2p^2q^6$	**2** $-b^4c^7$	**3** $-\frac{1}{b^4c^7}$	**4** $-3m^2n^5$
5 $\frac{x^2y^3}{4}$	**6** $-\frac{m^7n^8}{4}$	**7** $-\frac{p^5q^7}{4}$	**8** $-\frac{m^3n^4}{7}$
***9** $\frac{5k^4h^6}{2}$	***10** $-\frac{12a^6b^5}{5}$	***11** $-\frac{8q^4}{9p}$	***12** $\frac{3}{8}$

Divide binomials

Activity 5 Divide binomials

Learner's Book page 151

Review Copy

Guidelines for implementing this activity

- It is important for learners to "break down" the binomial into two monomials. Use this as the basis for teaching the division of binomials (and trinomials).
- Work through the example in the Learner's Book with the learners. Emphasise the following.
  - The same denominator should be written at the bottom of each fraction.
  - Each monomial should be written separately and only then should the answer be written down. (We have already put the terms in the correct order because with division it is difficult for learners to order algebraic expressions. For example, they don't yet know that $\frac{1}{x^2}$ means x^{-2} and this should therefore come after $3x$.)

Remedial and extension

Remedial: When learners "break down" binomials into monomials they must remember to divide the one term. Remind them to follow these steps for each monomial, and ensure that they do.
Extension: In questions **10** to **12** learners first need to multiply one of the terms and then do division.

Suggested answers

1 $8x + 4$

2 $7x^2 - 2x$

3 $2x - \frac{1}{5}$

4 $x^6 - \frac{x^4}{2}$

5 $7x^4 + 1$

6 $\frac{x^3}{3} + 2x^2$

7 $2x^5y + \frac{x^4y^5}{9}$

8 $\frac{x^6y}{8} - 2y$

9 $6x^5y + \frac{x^3}{4y}$

*10 $\frac{22x^2y^7}{6x^2y^7} - \frac{16x^2y}{6x^2y^7} = \frac{11}{3} - \frac{8}{3y^6}$

*11 $\frac{18x^2y}{15x^4y^8} + \frac{12x^5y^5}{15x^4y^8} = \frac{6}{5xy^7} + \frac{4x}{5y^3}$

*12 $\frac{12x^{12}y^9}{8x^3y^{10}} - \frac{6x^{10}y^2}{8x^3y^{10}} = \frac{3x^9}{2y} - \frac{3x^7}{4y^8}$

Divide trinomials

Activity 6 Divide with trinomials

Learner's Book page 151

Guidelines for implementing this activity

- Work through the example in the Learner's Book. Emphasise once again that we "break down" the trinomial into three monomials. The same denominator is written at the bottom of each fraction. Each monomial is worked with separately before the answer is written down.

Suggested answers

1 $3x^{11}y + 2x^5y^3 + x^2y$

2 $3x^2y^2 + 7x^4y^4 + 13x^6y^6$

3 $-\frac{8x}{y^5} + \frac{2}{y^5}$

4 $2x^4 + x - 5y^2$

5 $\frac{y^3}{3x^2} - \frac{y^3}{4z} - xy^4$

6 $\frac{x^3}{9y} - \frac{2x^2}{y^3} + \frac{1}{2xy}$

7 $\frac{x}{y} + \frac{2}{y^2} - \frac{1}{13y^2}$

8 $\frac{4x^{14}y^3}{5} + \frac{7x^9}{10} + \frac{3x^4y^6}{5}$

9 $\frac{y^2}{x^2} - \frac{x^2}{2y^3} - \frac{1}{4x^2y4}$

10 $\frac{x^4}{y^2} + \frac{2}{x^3y^3} + \frac{y}{4}$

11 $\frac{x^{19}}{3y^{16}} - \frac{x^3}{y^{17}} + \frac{1}{9y^9}$

12 $\frac{1}{4x^3y^2} + \frac{x^8}{3y^5} - \frac{xy^7}{6}$

Review Copy

Simplifying complex expressions

Activity 7* Simplify complex expressions

Learner's Book page 151

Guidelines for implementing this activity

- Revise the order of operations with the learners. When working with multiple operations, we first multiply out the brackets and then add like terms.
- Work though the example in the Learner's Book, and provide additional chalkboard examples if necessary, before learners proceed to do the activity.

Remedial and extension

Remedial: Let learners work in pairs. Encourage them to work carefully and systematically through all the steps. Working neatly is also very important.
Extension: Activity **7** is an extension activity.

Suggested answers

1 $-8x^2y^2 + 24x^2y^2 = 16x^2y^2$

2 $10x^7 + 20x^7 = 30x^7$

3 $\frac{6x^5}{5y^{12}} - \frac{3}{5x^2y^{12}} + \frac{4}{5x^7y^9}$

4 $\frac{x^{11}}{4y^5} + \frac{1}{3x^3y^5} - \frac{1}{6x^4y^2}$

5 $31x^3y - 42xy^2$

6 $-57x^2 + 17x$

7 $6x^5 + 15x^3 - 14x^2$

8 $60x^3 - 10x^2y - 18xy$

9 $6x^3 - 54x^2 + 56x + 24$

10 $-13x^4y + 45x^3y^2 + 40xy^3$

Simplifying expressions with exponents

Activity 8 Simplify expressions with exponents

Learner's Book page 153

Guidelines for implementing this activity

- Revise square and cube numbers. Then introduce variables that are squared or cubed, for example $(x)^2$ means $x \times x$; $(x^5)^2$ means $x^5 \times x^5 = x^{10}$; etc.
- Revise Exponent Law 3 by working through the notes in the Remember box with the learners. Emphasise the importance of multiplying the exponents, as opposed to the common mistake of adding them.
- Work through the example in the Learner's Book. Do additional, more challenging examples such as the ones in questions **9** and **10**, on the board before learners attempt the activity.
- Turn learners' attention to the notes in the Learner's Book, and provide similar, additional examples to test learners' grasp, for example $(-3x^3)^2 = 9x^6$ and not $9x^5$.

Review Copy

Remedial and extension

Remedial: Monitor learners' progress, ensuring that they follow the steps carefully and apply Exponent Law 3 correctly. Provide additional examples similar to the ones at the beginning of the activity for them in order to build confidence. Practice builds confidence and skill.

Extension: Activity 8 questions **9** to **12** contain more variables, larger exponents and larger coefficients.

Suggested answers

1 a^6b^6

2 $c^{15}d^{24}$

3 $121g^8h^{20}$

4 $125k^{30}m^3$

5 $49m^{20}n^{30}$

6 $-x^{27}y^{24}$

7 $150p^{14}q^{12}$

8 $32r^{60}t^{15}$

*9 $-144x^4y^{25}$

*10 $-5x^{16}y^6$

*11 $-16x^{19}y^{23}$

*12 $-10xy^2$

Simplify expressions containing roots

Activity 9 Simplify roots

Learner's Book page 154

Guidelines for implementing this activity

- Revise square and cube roots with the learners.
- Explain that just as the cube root has a "little 3" in the root sign, the square root actually has a "little 2" in the root sign. Why? For a cube we need to multiply a number by itself three times and for a square we need to multiply a number by itself two times.
- Learners need to know about the 2 and the 3 for the "inside divided by outside" rule. We could write everything out every time but that is time-consuming, which is why we divide the exponents.
- Explain the "inside divided by outside" rule: In $\sqrt{x^6}$ for example, we say the answer is $x^{6 \div 2} = x^3$. We took the exponent "inside the root sign" and divided it by the "exponent outside" the root sign. With this rule it doesn't matter how large the exponent is, we can still simplify, for example $\sqrt[3]{x^{30}} = x^{30 \div 3} = x^{10}$.
- Work though the examples in the Learner's Book on the board. Give learners an additional example or two to do before they attempt the activity.

Remedial and extension

Remedial: The importance of practice cannot be over-emphasised. Include examples such as the ones in Activity 9 in the 10-minute mental maths sessions every day.
Extension: Activity 9 questions **9** to **12** contain more variables, larger exponents and larger coefficients.

Review Copy

Suggested answers

1 $4x^2$
2 $2xy^2$
3 $7x^4y^5$
4 $4x^2y^3$
5 $2x^2y$
6 $-xy$
7 $-2x^4y^5$
8 $-3x^2y$
***9** $\sqrt{144x^{10}y^8} = 12x^5y^4$
***10** $\sqrt{81x^6y^4} = 9x^3y^2$
***11** $\sqrt[3]{27x^3y^6} = 3xy^2$
***12** $\sqrt[3]{\frac{-125}{x^3y^6}} = -\frac{5}{xy^2}$

Simplifying using substitution

Activity 10 Simplify by using substitution

Learner's Book page 155

Guidelines for implementing this activity

- Remind learners how to substitute variables with constants.
- They should also be reminded to put brackets around negative numbers because there is a difference between $(-3)^2$ and -3^2.
- Work through the example in the Learner's Book with them, and provide a few additional examples for them to do quickly in their exercise books. Monitor their understanding before they attempt the activity.
- Turn learners' attention to the notes in the Learner's Book and provide similar, additional examples to test their grasp of substitution.
- Work through the summary in the Learner's Book with the learners as a form of revision and consolidation.

Remedial and extension

Remedial: Provide additional exercises for learners to practise.
Extension: Provide more challenging exercises for the learners, for example, with larger numbers.

Review Copy

Suggested answers

1 $-3(1)^5 + 8(1)^3 - (1) = 4$

2 $4(-3)^2 - 16 = 20$

3 $8(1)^2 - 3 = 5$

4 $[2(-3) - (1)]^2 = 49$

5 $[(-3) + 7(1)]^3 = 64$

6 $5(-3)^2 + 4(-3) + 3 = 36$

7 $\frac{3(-3) - 10}{13(1)} = -\frac{19}{13}$

8 $\frac{2(-3) + 1}{3(1) - 1} = -\frac{5}{2}$

9 $\sqrt{(-3)^2 + 16(1)} = 5$

10 $\sqrt[3]{2(-3) + 5(1)} = -1$

Chapter 8 Revision

Learner's Book page 155

Encourage learners to review the content covered before attempting the revision activities. The activities should be used to assess learners' progress thus far, and to assess where remediation may be required.

Suggested answers

1 a false b true

2 a $\frac{10 + x}{3}$ b $\frac{50}{x} = 25$ c $2(6 + 2x)$

3 a $5n$ b $4n + 1$ c $\frac{1}{n^3}$

4 a $x = 10$; $x + 4x = 5x$; $5(10) = 50$ trees

5 a $8b^2 - 8b + 7$
b $-2c^2 - 21c + 16$
c $-7x^2 + 9x + 4$

6 a $12x - 9$; binomial b $6m - n$; binomial

7 a $-24kmn$ b $-60x^5y^{16}z^{12}$
c $27x - 18y$ d $\frac{2h^2}{3k^6}$
e $-8x + 4$ f $144m^8n^{18}$
g $-16p^3q - 32p^2q^2 + 8pq^3$ h $-6x^{13}y^{14} \times -x^{18}y^6 = 6x^{31}y^{20}$
i $-10x^5y$ j $\sqrt{x^4y^6} = 10x^2y^3$
k $\frac{16x^5y^5}{4x^4y^4} - \frac{14x^2y}{14x^4y^4} = 4xy - \frac{1}{x^2y^3}$ l $-8x^5y^3 + 30x^5y^3 = 22x^5y^3$
m $-38x^2y^2 + 18xy^3 + 5y^3$ n $-3x^3y^5 + 2xy^3 + \frac{y}{x}$

8 a $10(-2)^2 - 5(-2) + 2 = 52$
b $\sqrt[3]{3(-2) + 7(-3)} = -3$
c $\sqrt{(-2)^2 + (-3)^2 - (-3)} = 4$

Review Copy

CHAPTER

9 More algebraic equations

Chapter overview

Learner's Book pages 157 to 169
Recommended pacing: 3 hours

This chapter focuses on the following:

Unit 1 Solving for the variable 3 hours

Revision
Equations with variables on one side only
Equivalent or not equivalent?
Types of solutions
Equations with variables on both sides
Solve equations with brackets
Solving word problems

Chapter 9 Revision

Review Copy

UNIT

1 Solving for the variable

Unit overview

Learner's Book page 158
Recommended pacing: 3 hours

This unit focuses on the following:

- revising how to solve equations by means of inspection
- solving equations with variables on one side only
- determining whether the left-hand side and the right-hand side of an equation are equivalent by means of substitution
- finding different types of solution for equations
- solving equations with variables on both sides
- solving equations containing brackets on one side only
- solving equations containing brackets on both sides
- solving word problems, including general word problems, problems involving age, money, and perimeter and area

Resources: Learner's Book; exercise book; colour highlighters

Mental Maths

(10 minutes per day)

Suggested answers

Learner's Book page 158

1 LHS = 3(5) + 21 = 36; RHS = 31 + 5 = 36; yes

2 a $x = 3$ b $x = -10$
c $x = 80$ d $x = 0$
e no solution f $x = 0$
g $x = -13$ h $x = 9$
i $x = \frac{5}{3}$ j $x = -\frac{12}{5}$

3 Now: Nasipi = x; Kuhle = $5x$
In nine years' time: Nasiphi = $x + 9$; Kuhle = $5x + 9$
$5x + 9 = 3(x + 9)$; $x = 9$
Nasipi will be 18 years old in nine years' time.

4 B $\left(\text{side} = \frac{12}{4} = 3 \text{ cm}; 3^2 = 9 \text{ cm}^2\right)$

Background information

Learners were introduced to algebraic equations in Grade 7 and again in Chapter 7, where they did the following:

- used equations to describe problems
- interpreted and analysed equations
- solved equations by inspection
- solved equations by substitution (determining the numerical value of an expression).

In this chapter, learners will revise the above, and with this prior knowledge as a basis solve equations using additive and multiplicative inverses.

Review Copy

Teaching guidelines

The mental maths exercises at the beginning of the unit provide the essential practice that learners require in order to master these basic skills, and can also be used to assess learners' prior knowledge and current understanding of this content. Doing these types of mental maths activity each day also serves the purpose of revising and consolidating these basic skills. Revise the meaning of the following at the beginning of this unit.

- Solving an equation by inspection
- Solving an equation by substitution

Careful and systematic handling of the content in this unit, as well as the correct use of terminology, is very important.

Revision

Activity 1 Solve equations by means of inspection

Learner's Book page 159

Guidelines for implementing this activity

- Let learners work through the notes in the Remember box in pairs. Allow one pair to summarise their understanding orally while the rest of the class listens carefully and identifies mistakes/asks questions/fills gaps. Encourage interaction by asking key questions such as: How is an algebraic equation different from an algebraic expression? Is it an equation if the left-hand side is not equal to the right-hand side? What does it mean to solve an equation by inspection?
- Invite learners to provide additional examples for the rest of the class to solve by inspection, for example if $x = 13$ in the equation $x + 10 = 23$; then LHS $= x + 10 = 13 + 10 = 23$ and RHS $= 23$. If learners are unable to provide their own examples initially, write one or two examples on the board for them to solve, and in order to prompt them to think of their own examples.
- Do as many of these kinds of examples before working through the example in the book, in order to ensure that learners' understanding is sound.
- Emphasise the usefulness of doing the inverse operation to find the answer.

Remedial and extension

Remedial: The importance of practice cannot be over emphasised. Provide as many additional examples as described in the guidelines above for learners to practise.
Extension: In pairs, let learners create their own examples as described in the guidelines above, for the partner to solve.

Review Copy

Suggested answers

1 $x = 57$	**2** $c = 30$	**3** $k = -3$
4 $q = 5$	**5** $x = 10$	**6** $k = 9$
7 $x = 2$	**8** $y = 0$	**9** $k = -5$
10 $k = \frac{1}{2}$	**11** $k = 4$	***12** $x = 4$

Equations with variables on one side only

Activity 2 Solve equations with variables on one side only

Learner's Book page 160

Guidelines for implementing this activity

- Introduce this lesson by working through the example showing the scales in the Learner's Book, and emphasising the following:
  - We can write an equation to represent what is shown on the scale. We have $3 \times x + 1$ on the one side, and 19 on the other side. In other words, imagine we have three identical unknown masses, and a 1 kg mass on the left-hand side.

- On the right-hand side we have 19 masses of 1 kg each. The scale is balanced, so the total mass on the left-hand side is equal to the total mass on the right-hand side. We can write an equation to show this: $3x + 1 = 19$. In order to solve for x, we do the following.
  - x is the unknown value that we have to determine, therefore we have to isolate x (end up with only x on the left-hand side). Firstly, how would we go about removing the 1 on the left-hand side so that we have only the x values?
  - In order for an equation to remain "balanced" (the left-hand side = the right-hand side), what we do to the left-hand side we also have to do to the right-hand side at all times. This is fundamental to the understanding of equations.
  - When we remove the 1 kg mass on each side, we do it as follows: $3x + 1 - \mathbf{1} = 19 - \mathbf{1}$. The answer is: $3x = 18$. Remember, our scale is still balanced because we took away 1 kg from each side.
  - Remember, we have to solve for x, so our aim is to have only x on the left-hand side of the scale. What do we have to do next to achieve this? We can multiply $3x$ by $\frac{1}{3}$ or divide $3x$ by 3 but once again, what we do to the left-hand side we also have to do to the right-hand side: $\frac{3x}{3} = \frac{18}{3}$. Therefore, $x = 6$.
- Is our answer correct? We can check by substituting for x: $3(6) + 1 = 18 + 1 = 19$. Our answer is therefore correct.

- Work carefully through the worked example in the Learner's Book. Do additional examples on the board to consolidate the steps and learners' understanding of the process.
- Emphasise the importance of working neatly and lining up the = signs.

Review Copy

Remedial and extension

Remedial: Let learners work through as many examples as they need to by drawing scales as explained in the teaching guidelines above, and working though the process in this way. You can also do the first few questions in this way with them if they do not feel confident doing it on their own yet, or they can work in pairs with learners who are confident enough to do the "teaching" as described above.
Extension: Let learners work in pairs with learners who require help, by using the scales method described in the teaching guidelines above. Teaching their peers in this way will also help to consolidate and extend their own understanding.

Suggested answers

1 $2x = 16$; $x = 8$
2 $5x = 50$; $x = 10$
3 $4x = -44$; $x = -11$
4 $4x = 32$; $x = 8$
5 $6x = 66$; $x = 11$
6 $2x + 5 = 5$; $2x = 0$; $x = 0$
7 $46 = 3x + 1$; $45 = 3x$; $15 = x$
8 $-x - 6 = -28$; $-x = -22$; $x = 22$
9 $-2x = -20$; $x = 10$
10 $10 = \frac{2x}{6}$; $60 = 2x$; $30 = x$
11 $\frac{x}{4} = 5$; $x = 20$
12 $17 = \frac{17x}{-2}$; $-34 = 17x$; $x = -2$
13 $6 = \frac{3x}{5}$; $30 = 3x$; $x = 10$
14 $5x + 3 = -52$; $5x = -55$; $x = -11$
15 $32 = 13x + 6$; $26 = 13x$; $x = 2$

Equivalent or not equivalent?

Activity 3 Prove equivalence

Learner's Book page 160

Guidelines for implementing this activity

- It is important for learners to understand **why** they need to check equivalence. In Activity 2, we checked our answer by substituting 6 for x: $3(6) + 1 = 18 + 1 = 19$. Our answer was correct.
- In an example such as $3x + 1 = 6x - 11$ (in the Learner's Book), we see that $x = 4$, and substitute 4 for x in order to check that the left-hand side is equal to the right-hand side.
- If we get the same answer for both sides, we know that we have worked correctly and that $x = 4$. If we get different answers, we know that we have made a mistake and need to re-do the problem.
- Checking answers is very important in tests and exams, and learners should be reminded to check their answers in this way.

Tip

Emphasise the importance of writing out the LHS and RHS separately. Provide additional practice exercises for learners to do and monitor their progress very carefully. Repetition and practice are very important.

Review Copy

Suggested answers

1 LHS = $40(1) + 2 = 42$; RHS = $1 + 42 = 43$; LHS ≠ RHS; no
2 LHS = $3(0) + 10 = 10$; RHS = $2(0) + 10 = 10$; LHS = RHS; yes
3 LHS = $2(2) + 17 = 21$; RHS = $19 + 2 = 21$; LHS = RHS; yes
4 LHS = $\frac{1}{2}(8) \times 6 = 24$; RHS = $3 \times 8 = 24$; LHS = RHS; yes
5 LHS = $80 - 20 = 60$; RHS = $4 \times 20 = 80$; LHS ≠ RHS; no
6 LHS = $\frac{15}{3} = 5$; RHS = $15 - 5 = 10$; LHS ≠ RHS; no
7 LHS = $13 + 5 + 13 = 31$; RHS = $33 - 2 = 31$; LHS = RHS; yes
8 LHS = $\frac{4(1)}{4} = 1$; RHS = $\frac{10(1)}{10(1)} = 1$; LHS = RHS; yes
9 LHS = $\frac{20(6)}{3} = 40$; RHS = $7 \times 6 = 42$; LHS ≠ RHS; no
10 LHS = $0 + 5 = 5$; RHS = $0 - 5 = -5$; LHS ≠ RHS; no
11 LHS = $\frac{27}{9} + 4 = 7$; RHS = $9 - 3 = 6$; LHS ≠ RHS; no
12 LHS = $62x$; RHS = $62x$; LHS = RHS; yes
13 LHS = $\frac{45}{9} + 8(2) = 21$; RHS = $18 + 2 = 20$; LHS ≠ RHS; no
14 LHS = $\frac{2\,100}{100} = 21$; RHS = $100 - 80 + 1 = 21$; LHS = RHS; yes
15 LHS = $4(4)^2 = 64$; RHS = 64; LHS = RHS; yes

Types of solution

Activity 4 Find different solutions

Learner's Book page 161

Guidelines for implementing this activity

- Show learners that an answer isn't always "a nice number" or "a straightforward number". We deal with situations where x can be only zero, where x can have any value ($x \in \mathbb{R}$) or where x has no solution, for example, $x + 5 = x + 1$.
- This is important because learners may encounter solutions like these in future activities.

Tip

Let learners work in pairs. However, learners should write the answers in their own exercise boosk.

Suggested answers

1 $x = \frac{1}{5}$	**2** $x \in \mathbb{R}$	**3** $x \in \mathbb{R}$	**4** $x = 2$
5 $x \in \mathbb{R}$	**6** $x = 0$	**7** no solution	**8** $x \in \mathbb{R}$
9 $x = 16$	**10** $x = 0$	**11** $x = 5$	**12** $x = 0$
13 $x = 0$	**14** no solution	**15** no solution	**16** $x = 0$

Review Copy

Equations with variables on both sides

Activities 5–6 Solve equations with variables on both sides; Solve more equations

Learner's Book page 162

Guidelines for implementing these activities

- Go carefully through the worked example. Emphasise the following.
  - The aim is to isolate x.
  - What we do to the left-hand side we also do to the right-hand side.
- Important: Teachers often say "Take it over the equal sign and then the sign of the term changes". This is incorrect. Explain that if we need to cancel out $2x$ on one side, we do the inverse operation, which is $-2x$, on both sides because the equation should be balanced at all times.
- Do not encourage shortcuts since these occur at the risk of compromising accuracy and soundness, and impact negatively on learners' understanding of more complex concepts and procedures later on.
- Let learners test their answers by using substitution and re-do problems that were incorrect in order to see where they went wrong.

Remedial and extension

Remedial: Learners need to work neatly and write out the steps with = signs below one another. Learners should re-do the problems until they get all the answers correct. Activity 6 provides additonal practice of the same types of problems.
Extension: Activity 5 questions **11** and **12** are more challenging.

Suggested answers

Activity 5

1 $x = -13$

2 $2x = 20$; $x = 10$

3 $-2x = -12$; $x = 6$

4 $-4x = -10$; $x = \frac{5}{2}$

5 $14x = 28$; $x = 2$

6 $x = 0$

7 $5x = 45$; $x = 9$

8 $3x = 24$; $x = 8$

9 $24x = 36$; $x = \frac{3}{2}$

10 $-4x - 11 = -4x - 11$; $x \in \mathbb{R}$

***11** $2x - 3 = 6x - 1$; $-4x = 2$; $x = -\frac{1}{2}$

***12** $-2x + 7 = -4x + 49$; $2x = 42$; $x = 21$

Activity 6

1 $x = 15$

2 $x = 6$

3 $x = 15$

4 $-7x = -21$; $x = 3$

5 $x = 5$

6 $7x = -7$; $x = -1$

7 $x = 26$

8 $6x + 9 = 6x - 8$; no solution

9 $-3x = -8x$; $x = \frac{8}{3}$

10 $11x = 3x$; $x = 0$

11 $5x = 15$; $x = 3$

12 $-x = -4$; $x = 4$

13 $12x = 6$; $x = \frac{1}{2}$

14 $11x - 6 = 7x + 20$; $4x = 26$; $x = \frac{13}{2}$

Review Copy

Solve equations with brackets

Activities 7–8 **Solve equations with one bracket; Solve equations with two brackets**

Learner's Book page 163

Guidelines for implementing these activities

- Revise multiplying out brackets. Then learners should follow the usual steps for solving an equation.

Remedial and extension

Remedial: When learners multiply out the brackets (which is additional practice of this particular skill), they are practising the skills learnt in previous lessons.
Extension: Activity 8 contains questions with many terms, so learners should enjoy these!

Suggested answers

Activity 7

1 $3x + 6 = 27$; $3x = 21$; $x = 7$
2 $30 - 5x = -35$; $5x = 65$; $x = 13$
3 $-2x - 9 = -10$; $-2x = -1$; $x = \frac{1}{2}$
4 $-10x - 6 = -8x - 6$; $-2x = 0$; $x = 0$
5 $24 = 5x - 5 - 1$; $30 = 5x$; $x = 6$
6 $x - 67 = -22x - 44$; $23x = 23$; $x = 1$
7 $24x - 84 + 51 = 90x$; $-33 = 66x$; $x = -\frac{1}{2}$
8 $-28 - 6x + 32 = 12x$; $4 = 18x$; $x = \frac{2}{9}$
9 $-9 + 4x = 12x - 15$; $6 = 8x$; $x = \frac{3}{4}$
10 $-64x = -18 + 8x$; $18 = 72x$; $x = \frac{1}{4}$
11 $10 - 30x - 3 = 12x$; $7 = 42x$; $x = \frac{1}{6}$
12 $12 - x - 3 = -x + 9$; $9 = 9$; $x \in \mathbb{R}$

Activity 8

1 $3x + 12 = 2x + 14$; $x = 2$
2 $60 - 6x = 48 - 2x$; $12 = 4x$; $x = 3$
3 $-3x + 12 = 12 - 4x$; $x = 0$
4 $14 - 7x - 16 = 12x - 12$; $-7x - 2 = 12x - 12$; $10 = 19x$; $x = \frac{10}{19}$
5 $10x - 10 + 2 = 10x + 6$; $10x - 8 = 10x + 6$; no solution
6 $30 + 12 + 15x = -2x + 24$; $15x + 42 = -2x + 24$; $13x = -18$; $x = -\frac{18}{13}$
7 $17 - 4x - 9 = -9x - 7$; $-4x + 8 = -9x - 7$; $5x = -15$; $x = -3$
8 $24 + 3 - 7x = 12 - 12x$; $-7x + 27 = -12x + 12$; $5x = -15$; $x = -3$
9 $45x - 60 = 24 - 3x + 36$; $45x - 60 = -3x + 60$; $48x = 120$; $x = \frac{5}{2}$
10 $16 - 13x - 1 = -7x - 21$; $-13x + 15 = -7x - 21$; $-6x = -36$; $x = 6$
11 $35 - 15x = 19 - 7x - 56$; $-15x + 35 = -7x - 37$; $-8x = -72$; $x = 9$
12 $12 - 5 + 7x = -3x - 21 - 12$; $7x + 7 = -3x - 33$; $10x = -40$; $x = -4$

Review Copy

Solving word problems

Solving problems involving age; Solving word problems involving money; Solving word problems involving perimeter and area

Activities 9–12 **Solve word problems; Solve word problems (age); Solve word problems (money); Solve word problems (perimeter and area)**

Learner's Book pages 163–168

Guidelines for implementing these activities

- There are general steps for solving word problems but it has been found that when teaching, it is better to handle each type of word problem differently.
- Take the time to do each problem type, which means one period per type (four periods). Learners often struggle to grasp more than one type of word problem per day.
- Work through the appropriate example/s in the Learner's Book each day. If necessary, repeat and re-explain the example until the learners understand it.
- It is important that learners re-read a problem at least three times and try to write something down before getting help. It is quite important that they grapple with the problem on their own first.

Remedial and extension

Remedial: Read, read and read carefully! Learners can work in pairs.
Extension: Activities **9** to **12** are all challenging.

Suggested answers

Activity 9

1. $(4t - 6) - 2t = 2t - 6$
2. $(24m + 15) - (10m - 6) - (7m + 1) = 24m + 15 - 10m + 6 - 7m - 7 = 7m + 14$
3. $45 - (4x + 25) = 45 - 4x - 25 = -4x + 20$
4. $3t + r = 3\ 000$; $3t + 2r = 5\ 900$; $3\ 000 - r = 5\ 900 - 2r$; $r =$ R2 900
5. $(15p - 5) \div 5 = 3p - 1$
6. $\frac{1}{3}x + \frac{1}{2}x = 120$; $\frac{5}{6}x = 120$; $x = 120 \times \frac{6}{5} = 144$
7. $(5x) + (5x + x + 5) = 115$; $11x + 5 = 115$; $11x = 110$; $x = 10$; Jolene weighs 50 kg and Farhana weighs 65 kg.
8. $4 \times 2x + 4{,}5 \times x = 75$; $12{,}5x = 75$; $x = 6$; She sold six chocolate fudge blocks and 12 toffee fudge blocks.

Activity 10

1. Raymond $= x$; Mpho $= x + 4$; $x + x + 4 = 34$; $2x = 30$; $x = 15$; Mpho is 19 years old.
2. Taariq $= x$; Mohammed $= 3x$; $x + 3x = 80$; $4x = 80$; $x = 20$; Taariq is 20 years old and Mohammed is 60 years old.
3. Daphne $= x$; Brother $= 2x$; $x + 2x = 36$; $3x = 36$; $x = 12$; Daphne's brother is 24 years old.
4. Kim-Lee $= x$; Belle $= x + 1$; Bruce $= x + 2$; $x + x + 1 + x + 2 = 48$; $3x + 3 = 48$; $3x = 45$; $x = 15$; Belle is 16 years old.
5. Now: Raine $= x$; Sam $= 3x$. In 12 years' time: Raine $= x + 12$; Sam $= 3x + 12$
   $3x + 12 = 2(x + 12)$; $3x + 12 = 2x + 24$; $x = 12$; Raine will be 24 years old in 12 years' time.
6. Now: Dan $= x$; Nico $= 2x$. 10 years ago: Dan $= x - 10$; Nico $= 2x - 10$;
   $2x - 10 = 3(x - 10)$; $2x - 10 = 3x - 30$; $x = 20$; Dan is now 20 years old and Nico is now 40 years old.
7. Now: Tini $= x$; Pam $= 4x$. In 2 years' time: Tini $= x + 2$; Pam $= 4x + 2$;
   $4x + 2 = 2(x + 2)$; $4x + 2 = 2x + 4$; $2x = 2$; $x = 1$; Tini is now 1 year old.
8. Now: Nomcebo $= x$; Thomas $= x + 8$. Four years ago: Nomcebo $= x - 4$;
   Thomas $= x + 8 - 4 = x + 4$; $x + 4 = 5(x - 4)$; $x + 4 = 5x - 20$; $24 = 4x$; $x = 6$;
   Thomas was 10 years old four years ago.

Activity 11

1. Number of children $= x$; Number of adults $= 5x$;
   $40(x) + 100(5x) = 5\ 724\ 000$; $540x = 5\ 724\ 000$; $x = 10\ 600$; There were 53 000 adults.
2. Number of sweets $= x$; Number of chocolates $= 3x$;
   $0{,}5(x) + 4{,}5(3x) = 56$; $14x = 56$; $x = 4$; Xander bought 12 chocolates.
3. Number of sandwiches $= x$; Number of hamburgers $= x + 10$;
   Number of hotdogs $= 2(x + 10) = 2x + 20$; $8(x) + 12(x + 10) + 5(2x + 20) = 1\ 460$;
   $10x + 100 + 12x + 120 + 8x = 1\ 460$; $20x + 220 = 1\ 460$; $20x = 1\ 240$; $x = 62$;
   Patience sold 144 hotdogs.
4. Number of cats $= x$; Number of dogs $= x + 5$; Number of fish $= x + 4$
   $500(x + 5) + 280(x) + 50(x + 4) = 10\ 170$; $500x + 2\ 500 + 280x + 50x + 200 = 10\ 170$;
   $830x + 2\ 700 = 10\ 170$; $830x = 7\ 470$; $x = 9$; She sold 9 cats, 14 dogs, 13 fish and 36 pets altogether.

Review Copy

Activity 12

1 $x = \frac{60}{4} = 15$ mm

2 Breadth = x; Length = $4x - 15$
$2(x) + 2(4x - 15) = 900$; $2x + 8x - 30 = 900$; $10x = 930$; $x = 93$
$b = 93$ mm; $l = 4(93) - 15 = 357$ mm

3 Breadth = x; Length = $3x - 1$
$2(x) + 2(3x - 1) = 38$; $2x + 6x - 2 = 38$; $8x = 40$; $x = 5$
$b = 5$ m; $l = 3(5) - 2 = 13$ m

4 Breadth = x; Length = $3x - 5$
$2(x) + 2(3x - 5) = 54$; $2x + 6x - 10 = 54$; $8x = 64$; $x = 8$
$b = 8$ cm; $l = 3(8) - 5 = 19$ cm

5* Breadth = $x - 2$; Length = $x + 8$
$2(x - 2) + 2(x + 8) = 40$; $2x - 4 + 2x + 16 = 40$; $4x = 28$; $x = 7$
$b = 7 - 2 = 5$ cm; $l = 7 + 8 = 15$ cm

Chapter 9 Revision

Learner's Book page 169

Encourage learners to review the content covered before attempting the revision activities. The revision activities should be used to assess learners' progress thus far, and to assess where remediation may be required.

Review Copy

Suggested answers

1 a LHS = $20(1) + 5 = 25$; RHS = $1 + 24 = 25$; LHS = RHS; yes
b LHS = $3(3)2 = 27$; RHS = -27; LHS $\neq$ RHS; no

2 a $x = 38$
b $x = 0$
c $x = 2$
d $3x = 12$; $x = 4$
e $x = 0$
f $-x - 11 = 1$; $-x = 12$; $x = -12$
g $4 = \frac{2x}{5}$; $2x = 20$; $x = 10$
h $x \in \mathbb{R}$
i $3x = 9$; $x = 3$
j $x = 0$
k $-6x = 5$; $x = -\frac{5}{6}$
l $140 - 30x = -10$; $-30x = -150$; $x = 5$
m $-4x - 6 = 13x - 9$; $3 = 17x$; $x = \frac{3}{17}$
n $9 + 14 + 12x = -20x + 15$; $12x + 23 = -20x + 15$; $32x = -8$; $x = -\frac{1}{4}$
o $25 - 2 + x = -8x - 40 - 9$; $x + 23 = -8x - 49$; $9x = -72$; $x = -8$

3 a Let a dog cost x rand; dog = x; cat = y;
2 dogs + 1 cat = R2 000; $2x + y =$ R2 000; $y =$ R2 000 $- 2x$
1 dog + 1 cat = R1 250; $x + y =$ R1 250; $y =$ R1 250 $- x$
2 000 $- 2x =$ 1 250 $- x$; $750 = x$; One dog will cost R750
b Now: Melvin = x; Jerome = $x + 6$
Four years ago: Melvin = $x - 4$; Jerome = $x + 6 - 4 = x + 2$
$x + 2 = 3(x - 4)$; $x + 2 = 3x - 12$; $14 = 2x$; $x = 7$; Jerome is now 13 years old.

c No of children = x
No. of adults = $6x$
Fee for children = $x \times$ R100
Fee for adults = $6x \times$ R250
$\therefore (x \times \text{R}100) + (6x \times \text{R}250) = 960\ 000$
$\therefore 100x + 1\ 500x = 960\ 000$
$\therefore 1\ 600x = 960\ 000$
$\therefore x = \frac{960\ 000}{1\ 600}$
$\therefore x = 60$
No. children = 60
No. adults = 360

d Formula for perimeter = $2(l + b)$
Let the breadth = x
Length = $2x$
$\therefore 2(2x + x) = 104$ mm
$\therefore 4x + 2x = 104$ mm
$\therefore 6x = 104$ mm
$\therefore x = 17{,}3$ mm
$\therefore l = 34{,}6$ mm and b = 17,3 mm

Review Copy

CHAPTER

10 Geometry

Chapter overview

Learner's Book pages 170 to 228
Recommended pacing: 25 hours

This chapter focuses on the following:

Unit 1:	**Geometry of straight lines**	9 hours
	Angle relationships	
	Perpendicular lines and straight lines	
	Intersecting lines	
	Parallel lines and transversals	
	Identifying parallel lines	
	Problem-solving	
Unit 2:	**Constructing geometric figures**	4 hours
	Revision: Constructing angles	
	Bisecting line segments and angles	
	Constructing perpendicular lines	
	Constructing special angles without a protractor	
Unit 3:	**Properties of geometric figures**	4 hours
	Investigating the properties of triangles	
	Investigating the properties of quadrilaterals	
	Investigating the properties of polygons	
	Interior angles of polygons	
Unit 4:	**Geometry of 2D shapes: Triangles**	3 hours
	Revision: Properties and definitions of triangles	
	Revision: Classification of triangles	
	Solving triangles	
Unit 5:	**Geometry of 2D shapes: Quadrilaterals**	3 hours
	Revision: Classifying quadrilaterals	
	Solving quadrilaterals	
	Problem-solving	
Unit 6:	**Geometry of 2D shapes: Congruent and similar figures**	2 hours
	Congruent and similar figures	
	Investigate conditions for congruent triangles	
	Conditions for similarity	
	Problem-solving	
Chapter 10 Revision		

Review Copy

UNIT

1 Geometry of straight lines

Unit overview

Learner's Book page 171
Recommended pacing: 9 hours

This unit focuses on the following:

- revision: line, line segment and ray
- revision: perpendicular lines
- revision: parallel lines and a transversals
- intersecting lines and vertically opposite angles
- corresponding, alternating and co-interior angles

Resources: Learner's Book; exercise book; protractors; compasses; rulers

Mental Maths

(10 minutes per day)

In this unit learners will often need to calculate complements and supplements of angles. Encourage learners to calculate angles without the use of calculators. Learners may find question **1e** challenging. Ask: What was your thinking to find the answers for the previous questions? Use the same way of thinking to do this one.

Review copy

Suggested answers — Learner's Book page 171

1	**a**	70°	**b**	15°	**c**	56°
	d	22°	**e**	$90° - x°$	**f**	$90° - y°$
2	**a**	80°	**b**	60°	**c**	55°
	d	90°	**e**	$180° - 5x°$	**f**	$180° - 3y°$
3	**a**	270°	**b**	90°	**c**	315°
	d	225°	**e**	45°	**f**	$360° - 2y°$

Background information

- Napoleon Bonaparte allegedly said: "A good sketch is better than a long speech." This is also true when we do geometry.
- Any subject has its terminology: a set of technical terms that is related to the subject. Mathematics is no different.
- It is important to ensure that learners can give precise descriptions of the terminology we use in geometry.
- In Grade 7, learners studied the following:
  - Points, lines and angles
  - Complementary and supplementary angles (extension)
  - Identified and constructed parallel, perpendicular, horizontal and vertical lines
  - Identified the parts of angles

- Read, measured, classified and constructed angles
- Used rulers, compasses and protractors to measure and do constructions.

In this unit, learners revise the basic concepts learnt in Grade 7. They are introduced to vertically opposite angles, corresponding angles, alternate angles and co-interior angles (the latter is extension) and have to solve geometric problems to find unknown angles using these angle relationships (giving reasons).

Teaching guidelines

Revision of the basic concepts learnt in Grade 7 is very important since learners come from different primary schools and an assessment of their basic knowledge and understanding of the concepts needs to be made. Angle relationships should be introduced in the exploratory, investigative manner presented in the Learner's Book and as described below, to ensure that learners have a practical understanding of these relationships.

Angle relationships

Lines and points; Revision: What is an angle?; Revision: Classifying angles

Activity 1 **Revise words used in geometry**

Learner's Book page 171

Review Copy

Guidelines for implementing this activity

- Explain the meaning of the word terminology (as described in the Background information above), and the importance of using the correct terminology when doing Mathematics and specifically, Geometry.
- Activity 1 can be used to assess learners' prior knowledge of the basic terminology employed when working with lines, points and angles. Remember that learners in Grade 8 often come from different primary schools and therefore it is important to assess their understanding of the basic concepts.
- Learners should complete Activity 1 without assistance.

Remedial and extension

Spend some time revising the terminology of points, lines and angles if necessary. Learners can also work in pairs and test each other, using the table. Repeat these terms often when you teach geometry. Ask learners to explain the meaning of the words to you.

Suggested answers

1 f; iii **2** a; vi **3** d; v **4** c; ii **5** b; i **6** e; iv

Perpendicular lines and straight lines

Complementary angles; Supplementary angles

Activity 2 Understand complements and supplements

Learner's Book page 176

Guidelines for implementing this activity

- Revise angles, using a protractor to measure angles and classifying angles.
- Work carefully through the introduction to complementary and supplementary angles, as well as the examples in the Learner's Book. Although learners may have done this as extension in Grade 7, treat these concepts as though learners are doing them for the first time.
- Do additional, similar examples, that require that learners calculate complements and supplements, on the board before learners attempt the activity. Emphasise the importance of setting out the calculation correctly, in a step-by-step manner as it is set out in the Learner's Book.
- Note the difference in spelling: Somebody pays you a compliment if they say something positive about you. In this activity we work with a complement.
- Discuss how to number angles and encourage learners to do this in their own work.
- Learners should complete Activity 2 on their own.
- Questions **1** and **2** are applications of the mental maths.
- It is important that learners write out the answers as they are required to do in Geometry.

Doing arithmetic (discourage this)	Doing geometry (the correct way)
$90° - 27° = 63°$	$x + 27° = 90°$ (right angle)
$\therefore$ answer is $63°$	$x + 27° - 27° = 90° - 27°$ $\therefore\ x = 63°$

Review Copy

Remedial and extension

All the questions where learners need to find the value of x are opportunities to practise solving equations. Revise the concept of "adding/subtracting both sides". Do **not** say that learners have to "carry over and change the sign".

Suggested answers

1	**a** $48°$	**b** $72{,}7°$	**c** $9°$	**d** $90° - y°$
2	**a** $138°$	**b** $63°$	**c** $98{,}4°$	**d** $180° - 4p°$

3 Please note: we number angles from left to right (at the vertex where necessary).
   a $x + 27° = 90°$ (right angle) $\therefore\ x = 63°$
   b $x + 50° + x = 90°$ (right angle); $2x = 40°$ $\therefore\ x = 20°$
   c $131° + x = 180°$ (straight line) $\therefore\ x = 49°$
   d $3x + 120° = 180°$ (straight line); $3x = 60°$ $\therefore\ x = 20°$
   e $103° + 25° + x = 180°$ (straight angle) $\therefore\ x = 52°$
   f $2x + 90° + 3x = 180°$ (straight line); $5x = 90°$ $\therefore\ x = 18°$

Intersecting lines

Activity 3 Investigate intersecting lines

Learner's Book page 178

Guidelines for implementing this activity

- Work through the introduction and examples in the Learner's Book, carefully introducing the concept of vertically opposite angles.
- If necessary, ask learners to pair up and stand opposite each other in groups of four, and point to the persons vertically opposite them.
- Draw a diagram of this arrangement on the board, similar to the fence illustration. Let learners link their positions to the correct points on the diagram.
- Ensure that learners do not become confused between "vertically opposite angles" and "opposite angles". (In a parallelogram the opposite angles are equal.)
- Allow learners enough time to measure the angles and to draw their own conclusions. Discuss their answers with them.

Review Copy

Remedial and extension

Provide additional, similar examples for learners to practise identifying vertically opposite angles, as well as measuring them.

Suggested answers

1 a AD ∥ BE; BE ∥ CF; AD ∥ CF
   b CF ⊥ DF; AD ⊥ DF; EB ⊥ AC; etc.
   c Lines BF and CE intersect at point M.
2 a $B\hat{M}C = E\hat{M}F = 100°$
   b $B\hat{M}E = C\hat{M}F = 80°$
   c They are equal.
3 67°
4 The angles should measure the same. The angles of vertically opposite angles are equal.

Activity 4 Work with intersecting lines

Learner's Book page 179

Guidelines for implementing this activity

- In Activity 4 learners practise the following skills.
  - Interpreting the diagram.
  - Deciding on a strategy to solve the problem.
- Emphasise the importance of using the first line of the answer to write down a statement and a reason. Calculations follow in the second and third lines.

Remedial and extension

Remedial: Use the answers in the activity to ask more-interpretive questions, for example:

1. What is the supplement of 81°? (99°) Where is that angle on the diagram?
2. Classify the angle that you calculated. (obtuse angle)
3. How can you check your answers? (Total of angles should be 360°, because they form a revolution.)
4. Can you find the value for y first? (no)
5. What did you think when the value for x was not a natural number? (No problem – an angle can be divided into minutes and seconds. We work with decimal values.) What is the name for the line that divides an angle in two equal parts? (bisector)
6. Calculate the sizes of the two angles if $x = 25°$ (each 60°).

Review Copy

Suggested answers

1 a $x = 81°$ (vert opp ∠s)

b $2x + 15° = 137°$ (vert opp ∠s)
$2x = 122°$
$\therefore\ x = 61°$

c $x = 102°$ (vert opp ∠s)
$y = 41°$ (vert opp ∠s)
$x + y + z = 180°$ (straight line)
$102° + 41° + z = 180°$
$\therefore\ z = 37°$

2 a $3x + 12° = 120°$ (vert opp ∠s)
$3x = 108°$
$\therefore\ x = 36°$
$3y = x$ (vert opp ∠s)
$\therefore\ 3y = 36°$
$\therefore\ 3y = 12°$

b $2x = 65°$ (vert opp ∠s)
$\therefore\ x = 32{,}5°$
$y + 65° = 180°$ (straight line)
$\therefore\ y = 115°$
$z = y$ (vert opp ∠s)
$\therefore\ z = 115°$

c $3x - 15° = 2x + 10°$ (vert opp ∠s)
$\therefore\ x = 25°$

Parallel lines and transversals

Corresponding angles; Co-interior angles; Alternate angles

Activities 5–6 **Investigate angles formed by transversals; Identify angles formed by transversals**

Learner's Book pages 180–181

Guidelines for implementing these activities

- Work carefully through the examples and the definitions in the Learner's Book.
- Draw additional examples on the board that require learners to identify corresponding, co-interior and alternate angles. (Learners are not required to know about co-interior angles. This can therefore be taught as an extension.)
- Alternatively instruct learners, step-by-step, to construct two parallel lines using set squares and rulers, to draw a transversal and to mark corresponding, co-interior and alternate angles using dots, stars and ticks. Although they will do constructions in the next unit, this is an ideal "warm-up". Once they have completed the activity, they can revisit these constructions and measure the angles to further test the conclusions arrived at in Activity 5.

Remedial and extension

The "warm-up" construction activity in the guidelines above serves as practice and consolidation of the concepts of corresponding, co-interior and alternate angles. Provide additional homework tasks that require learners to construct parallel lines and transversals, label all the above-mentioned angles, measure them and write out conclusions about the relationships between these angles.

Review Copy

Suggested answers

Activity 5

1 a The angles are equal.
  b $A\hat{G}E$ and $C\hat{H}E$
2 a The angles add up to 180°.
  b $A\hat{G}F$ and $C\hat{H}E$
3 a The angles are equal.
  b $B\hat{G}F$ and $C\hat{H}E$

Activity 6

F = corresponding angles; U = co-interior angles; N = alternate angles

Identifying parallel lines

Activity 7 Use parallel lines

Learner's Book page 182

Guidelines for implementing this activity

- As mentioned previously, learners are not required to know about co-interior angles. This can therefore be taught as an extension. Even if learners do not know about co-interior angles, they can solve problems as in Example **1**.
- Explain the symbol (||) for parallel lines. (This is not new to learners, but it will help to revise it.)
- Learners measure the angles as required, and draw their own conclusions.
- They can use the letters F, U and N to remember the positions of the angles. (Mathematics is FUN!)
- Explain the difference between the following pairs of alternate angles.

Diagram 1

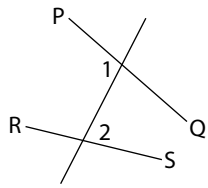
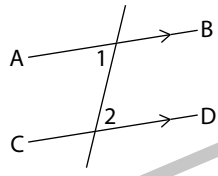


Diagram 2



Review Copy

- In Diagram 1 the two angles are alternate angles, but they are not equal.
- In Diagram 2 the two angles are alternate angles AND they are equal. They are equal, because the lines are parallel. Therefore it is essential that the two parallel lines should form part of the reason:
  $\hat{1} = \hat{2}$ (alt $\angle$s; AB || CD)
- Discuss the two worked examples.
- Learners work through Activity 5.
- Point out the difference in the two questions.
  - Question **1**: parallel lines given; angles to be calculated
  - Question **2**: equal angles given; parallel lines to be identified

Remedial and extension

Remedial: Use repetition. Learners need to become 100% at ease with the names and the properties of these pairs of angles. Pay special extra attention to the way learners present their answers.

Tip

Learners will memorise the names of parallel lines given horizontally. Teach learners to rotate the page through 90° if the parallel lines are given vertically; the diagram will again look like the one that they are familiar with.

Suggested answers

1 $a = 144°$ (alt ∠s; AB ∥ PQ)
$b = 144°$ (corresp ∠s; EF ∥ GH)
$c + 144° = 180°$ (co-int ∠s; XY ∥ RT); $c = 36°$

2 a CD ∥ EF (corresp ∠s are equal)
b XY ∥ PQ (alt ∠s are equal)
c PQ ∥ RS (There is an angle of 110° adjacent to the angle of 70° – straight line ∴ alt ∠s are equal)

Problem-solving

Activity 8 Solve problems

Learner's Book page 183

Guidelines for implementing this activity

- This is an activity in which everything is put together. Encourage learners to solve a problem in as many ways as they can, as long as they can give reasons for their thinking.

Tip

In Geometry you need to remember what you calculated in question **1** to be able to answer question **2**.

Review Copy

Suggested answers

1 a $p = 36°$ (corresp ∠s; MN ∥ BC)
$q = p$ (alt ∠s; MN ∥ BC) ∴ $q = 36°$
$r = q$ (corresp ∠s; MN ∥ BC) ∴ $r = 36°$
$s + r = 180°$ (str line); $s + 36° = 180°$ ∴ $s = 144°$
$t = 144°$ (revolution OR straight lines OR vert opp ∠s)
b $p = 72°$ (alt ∠s; AB ∥ EF)
$q = p$ (alt ∠s; AB ∥ CD) ∴ $q = 72°$
$r = 72°$ (vert opp ∠s) Please note: NO parallel lines used here!
$s + 72° = 180°$ (str line) ∴ $s = 108°$
$t = 108°$ (revolution OR straight lines OR vert opp ∠s OR corresp ∠s)

2 a $3x – 20° = 70°$ (alt ∠s; PQ ∥ RS); $3x = 90°$ ∴ $x = 30°$
b $2x – 30° = x – 10°$ (corresp ∠s; KL ∥ MN) ∴ $x = 20°$

3 a $3x + 2x + x = 180°$ (str line); $6x = 180°$ ∴ $x = 30°$
$y = 3x$ (alt ∠s; AB ∥ PR) ∴ $y = 3(30°) = 90°$
$z = (2x + 3x)$ (corresp ∠s; AB ∥ PR) ∴ $z = 5x = 5(30°) = 150°$
b $x = 27°$ (corresp ∠s; EB ∥ DC)
$x + 3y = 90°$ (right angle); $27° + 3y = 90°$ ∴ $y = 21°$
$z + 27° + 90° = 180°$ (int ∠s of △ DBC) ∴ $z = 63°$

4 1 – Mary; 2 – Lulama; 3 – Donovan; 4 – Lerato; 5 – Bongani; 6 – Byron; 7 – Palisa; 8 – Annette

UNIT

2 Constructing geometric figures

Unit overview

Learner's Book page 185
Recommended pacing: 4 hours

This unit focuses on the following:

- revising the classification of angles
- revising the measuring and drawing of angles with a protractor
- making accurate constructions using a compass:
  - bisecting lines and angles
  - bisecting angles of triangles
  - perpendicular lines at a given point or from a given point
- constructing angles of 30°, 45°, 60° and their multiples

Resources: Learner's Book; exercise book; rulers; protractors; compasses. For teacher: board compass; ruler; protractor

Mental Maths

(10 minutes per day)

Focus on revising the classification of angles from Grade 7. Emphasise the importance of the unit we measure angles in, namely degrees (°). Actively involve the learners by asking them to form angles with their arms, as shown in the Learner's Book. It is important for learners to have concrete experience of the size of an angle. This question is in preparation for the end of the unit where the learners will construct angles of 30°, 45° and 60° without the use of a protractor.

Review Copy

Suggested answers — Learner's Book page 185

1 a 90° b 180° c 180° d 360°
2 a acute b obtuse c reflex d reflex
3 a Learners stretch arms out horizontally.
b Learners make one arm vertical and the other horizontal.
c Learners make "half" of **b** above.
d Learners make "a third" of **b** above.
4 Protractor.

Background information

The word "compass" has more than one meaning. We refer to the Mathematics instrument as "a pair of compasses".

- In Grade 7, learners did the following:
  - worked with construction equipment: ruler, protractor and a pair of compasses
  - constructed and measured straight lines, angles, perpendicular lines, parallel lines, vertical and horizontal lines, triangles (extension), parallelograms (extension) and circles.

In Grade 8 learners are introduced to bisecting angles in a triangle and constructing special angles without a protractor.
In FET Mathematics, learners will often use special angles, especially in trigonometry. The important foundation is laid in Grade 8 Mathematics, when they construct these special angles.

Teaching guidelines

Constructions are a useful context of exploring the properties of all geometric figures and will therefore form part of all units dealing with Space and Shape (Geometry). Ensure that all construction equipment is available beforehand, and that learners use this accurately and safely at all times.

Revision: Constructing angles

Activity 1 **Measure and draw angles**

Learner's Book page 185

Guidelines for implementing this activity

- This is important revision as learners often come from different primary schools. This exercise therefore serves to assess their basic competency.
- Introduce the protractor again, revising how it is used, how we choose the correct scale, the importance of using it correctly for accurate measurement and so on.
- Introduce the word construct. Explain that when learners are asked to construct something, it means that they are required to draw using construction equipment such as a ruler, pair of compasses and/or a protractor.
- Work carefully through the steps in the Learner's Book.
- Do additional, similar examples on the board, pointing out each time how to use the protractor correctly. It is important that learners get sufficient practice in drawing angles.
- Ask learners to classify the angles that they draw each time (revision of Grade 7 content).
- Encourage learners to check their own work. This is something many learners might experience as being new, since at primary school the teacher does most of the checking of their work.

Review Copy

Remedial and extension

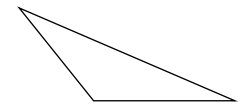
Remedial: Use the words clockwise and anticlockwise when employing a protractor. Informally introduce the concept of adjacent angles when learners construct a reflex angle.
Extension: After learners have completed Activity 1, provide similar, additional tasks if necessary.

Tip

Place eight copies of a protractor on an A4 sheet of paper. Copy this on to a transparency sheet and cut out the "protractors" for learners who do not have their own protractors.

Suggested answers

1 a Diagrams will differ, but there should be one obtuse angle in the triangle:

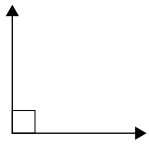


Triangle with one obtuse angle

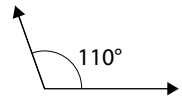
b Answers will depend on the diagram.
c If the total is 180°, it is an indication that the learner measured correctly.
d The interior angles of a triangle measure 180 °.

2 Both angles are correct. The size of an angle does not depend on the length of the arms.

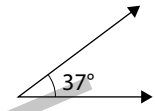
3 a



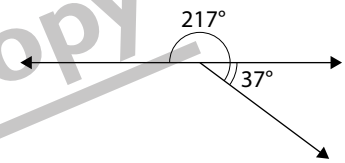
b



c



d A protractor has only 180° therefore it is necessary to make a calculation first: 217° = 180° + 37°. So, to construct an angle of 217°: draw a straight line and mark the vertex of the angle; then use one of the arms of the straight angle and construct an angle of 37°. Clearly mark the reflex angle on the construction.



e right angle (90°); obtuse angle (110°); acute angle (37°); reflex angle (217°)

Review Copy

Bisecting line segments and angles

Bisecting a line segment; Bisecting an angle

Activity 2 **Bisect line segments, angles and angles in triangles**

Learner's Book page 189

Guidelines for implementing this activity

- A group of Grade 8 learners using compasses can be a challenging situation. Keep a watchful eye on the learners to ensure that they use this potentially dangerous construction tool safely.
- Work through the steps in the two examples in the Learner's Book, and let learners practise bisecting line segments of different lengths and angles of different sizes.
- Introduce and repeat the new words (line segment; bisect; bisector; arc; midpoint) often while explaining and working through the practical exercises.
- Remind learners to check their work.
- Learners complete Activity 2 questions **1** to **4** as revision. Questions **5** and **6** focus on the new content.

Remedial and extension

Remedial: To allow for more practice, ask learners to divide a given angle into four equal parts, for example Activity 2 Question **4**: Divide the angle of 160° into four equal angles, each measuring 40°.
Extension: The point where the bisectors of the three angles of a triangle intersect is called the incentre of the circle. This point is the centre of a circle that can be drawn on the inside of the triangle to touch each side in one point. (See Learner's Book page 188: Activity 2 questions **5b** and **6b**.)
Learners can also be asked to bisect the three sides of a triangle. The point where the bisectors of the three sides of a triangle intersect is called the circumcentre of the circle. This point is the centre of a circle that can be drawn through the three vertices of the triangle.

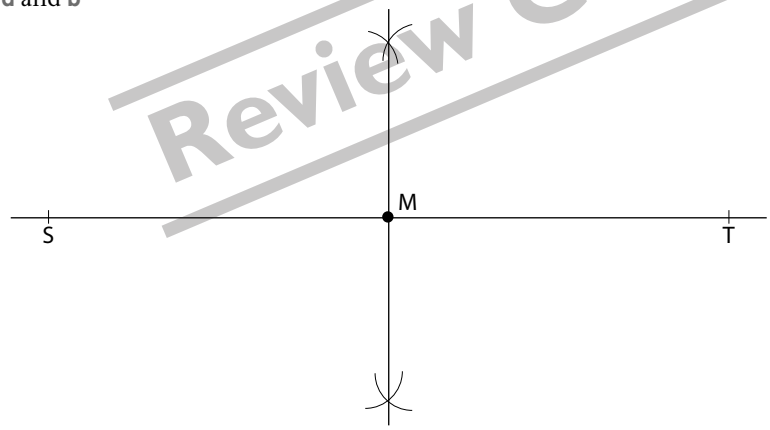
Tip

If learners are not comfortable using compasses at first, ask them to make patterns with circles to decorate the classroom, in order to practise using this construction tool.

Review Copy

Suggested answers

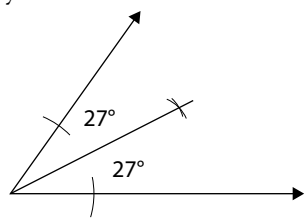
1 **a** and **b**



c SM = 4,5 cm and NT = 4,5 cm (or 45 mm)

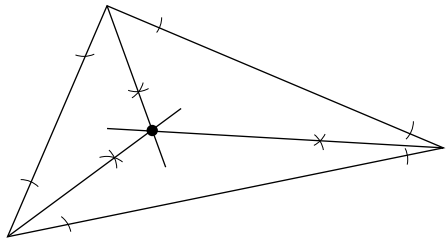
2 As in question **1**: Line segment to be divided into two equal parts of 63 mm each. Use this opportunity to revise conversion between cm and mm.

3 **a** and **b**



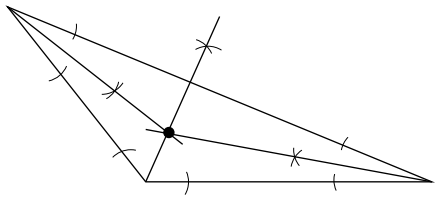
4 As in question 3: Angle to be divided into two equal parts of 80° each.

5 **a** and **b**



c The bisectors intersect in one point.

6 **a** and **b**



c The bisectors intersect in one point.

Constructing perpendicular lines

Review Copy

Activity 3 Construct perpendicular lines

Learner's Book page 190

Guidelines for implementing this activity

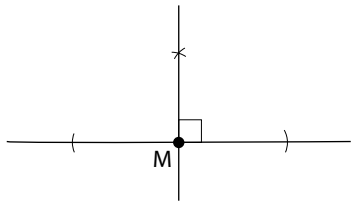
- Revise the concept of perpendicular lines. Point out the difference between parallel lines and perpendicular lines.
- Revise the symbols used to show perpendicular lines and parallel lines.
- Work through the steps in the Learner's Book.
- Ask learners to practise these two constructions as often as possible. Provide short homework tasks if necessary.
- Emphasise the importance of using the correct symbol for labelling perpendicular lines on their constructions.
- Learners do Activity 3. Remind them to check their constructions using a protractor.

Remedial and extension

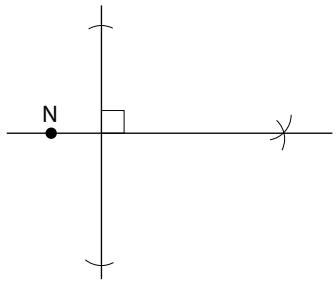
Regular, guided practice of these important constructions cannot be over-emphasised. It is the only way in which to demystify this important aspect of geometry.

Suggested answers

1



2



Constructing special angles without a protractor

Constructing angles of 60°, 30° and 45°

Activities 4–5 **Construct special angles with a compass**

Learner's Book page 190

Review Copy

Guidelines for implementing this activity

- All the work covered in this chapter comes together now. Learners need to analyse the questions and apply their knowledge and skills in these final constructions.
- Do the steps (contructing angles) in the example with the class. Encourage each learner to use a different length for the sides of the triangle.
- When leading learners through the conclusion, emphasise: "If the three sides of a triangle are of equal length, the three angles will each be 60°."
- Learners may remember the classification of triangles and will thus know that this is an equilateral triangle.
- Learners might find the last questions in the Activity 4 challenging. Help them by discussing different approaches to similar questions, for example an angle of 150° can be considered as 90° + 60° or it can be considered as 180° – 30°.
- Do not discuss the angles asked for in Activity 4. First give the learners the opportunity to apply their own thinking to questions **2** and **3**.
- Follow the same steps for Activity 5.

Remedial and extension

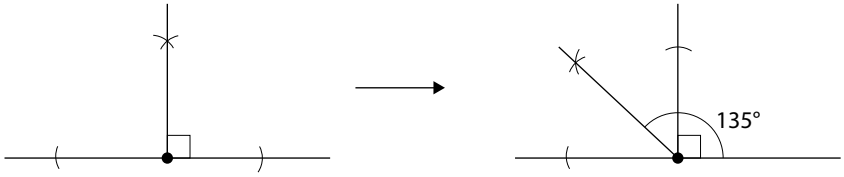
Discuss how to construct an angle of 240° (180° + 60°) or an angle of 300° (360° – 60°) and then mark the reflex angle. Practice, once again, is important. Provide as many opportunities as possible for learners to practise.

Suggested answers

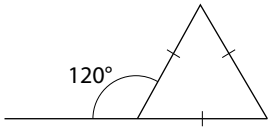
Activity 4

1 a Draw a straight line and place a point on (or above / below) the line. Make a construction as in Learner's Book page 188.
OR: Draw a straight line and indicate the vertex for the angle of 90°. Bisect the angle.
b Bisect an angle of 90°.
c Construct a triangle with three equal sides. Each angle will be equal to 60°.
d Bisect an angle of 60°.

2 Isaac can construct two adjacent angles of 90° each. Then he can bisect one of the right angles. The sum of two of the angles will be 135° (see on sketch) (135° = 90° + 45°).



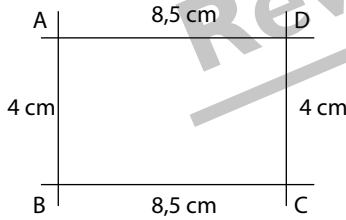
3 120° = 180° – 60° or 120° = 90° + 30°. The first construction will be easier for learners to do. Draw a straight line and construct a triangle with three equal sides on part of the line. The angle indicated in the diagram will be equal to 120°.



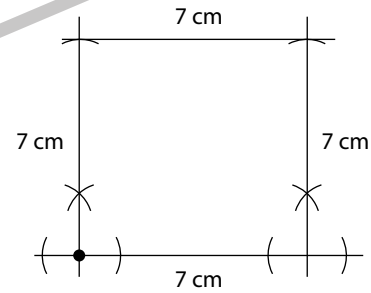
Review Copy

Activity 5

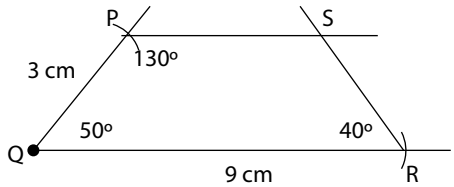
1



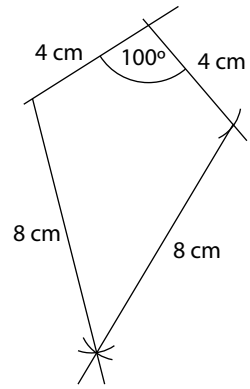
2



3



4



UNIT

3 Properties of geometric figures

Unit overview

Learner's Book page 193
Recommended pacing: 4 hours

This unit focuses on the following:

- the following properties of angles of triangles:
  - the sum of the interior angles of a triangle is equal to 180°
  - the exterior angle of a triangle is equal to the sum of the opposite interior angles.
- the following properties of quadrilaterals:
  - the properties of quadrilaterals regarding sides, angles and diagonals
  - the sum of the interior angles of a quadrilateral is equal to 360°.
- the following properties of polygons:
  - definitions of irregular and regular polygons
  - an investigation to find the sum of the interior angles of polygons.

Resources: Learners: Learner's Book; exercise book; rulers; protractors; sheet of paper and glue for the investigations. Teacher: cardboard; coloured pens to make a poster for the classroom on the properties of quadrilaterals

Review Copy

Mental Maths

(10 minutes per day)

At the end of the unit learners will need to know the names of different polygons. The Mental Maths exercise prepares learners for that. Also, the questions asked here will make learners aware that Mathematics is part of everyday life, for example, references to an octopus and the Pentagon in America.

Suggested answers — Learner's Book page 193

1 a 2 wheels b 3 wheels c 4 wheels
2 8 tentacles
3 8 notes (do, re, mi, etc.)
4 2 languages
5 3 people
6 5 sides
7 a 3 angles b 4 angles c 5 angles
d 6 angles e 7 angles f 8 angles

Background information

- In Grade 7, learners investigated the interior angles of triangles (as extension), and described, named, sorted and compared triangles.
- The emphasis in this unit is on investigating properties. In the next chapter learners will do problems in which they apply these properties.
- A triangle is one of the basic shapes that learners will study in many branches of mathematics, for example, Euclidean geometry, Trigonometry, Transformation geometry.

In FET Mathematics it is very, very important that learners know the properties of quadrilaterals. Learners in Grade 12 often cannot solve a problem because they cannot remember the properties of, for example, the diagonals of a rhombus.

Teaching guidelines

- At Grade 8 level it is important to allow learners to investigate and to experience Mathematics concretely as often as is possible. This will facilitate abstract thinking later on.

Investigating the properties of triangles

Activity 1–3 **Investigate the properties of triangles; Use the properties of triangles**

Learner's Book page 193

Guidelines for implementing this activity

- Learners have done similar investigations in Grade 7. They can therefore be expected to do these investigations and Activity 1 on their own, and to come to the required conclusions independently. Provide assistance only where a learner may need it.
- Learners need to use the correct terminology, for example "The exterior angle of a triangle is equal to the sum of the opposite interior angles" and **not** "Exterior angle is equal to interior angles".
- In Activity 1 Question **1**, learners repeat the properties they have investigated.
- In these examples only numbers are used. In a later unit, learners will solve equations to answer similar questions. They will also use abbreviations for the reasons, at a later stage.

Review Copy

Remedial and extension

Do additional, similar examples on the board to give learners enough opportunity to practise the two properties of the angles of triangles.

Tip

Vary the way in which you draw a triangle, for example do not always draw the exterior angle of a triangle in the same position. Encourage learners to rotate (turn) their books so that they can look at a given diagram from a different angle. This helps learners to see the diagram differently.

Suggested answers

1 a The sum of the three interior angles of a triangle is equal to 180°.
  b The exterior angle of a triangle is equal to the sum of the opposite two interior angles.
2 a $p = 57°$ (sum of the interior angles of a triangle is equal to 180°)
  b $q = 133°$ (exterior angle of a triangle is equal to the sum of the opposite interior angles)
  c $r = 70°$ (sum of the interior angles of a triangle is equal to 180°)

d $x = 40°$ (exterior angle of a triangle is equal to the sum of the opposite interior angles)
e $y = 60°$ (sum of the interior angles of a triangle is equal to 180°; all three angles equal)
f $a = 30°$ (sum of the interior angles of a triangle is equal to 180°)
$b = 34°$ (exterior angle of a triangle is equal to the sum of the opposite interior angles)

Investigating the properties of quadrilaterals

Activities 4–6 Name quadrilaterals; Investigate the properties of quadrilaterals; Use properties of quadrilaterals

Learner's Book pages 194–196

Guidelines for implementing these activities

- Ensure that the learners understand the basic terminology. Learners should not confuse opposite angles with vertically opposite angles.
- In a quadrilateral we have opposite angles.
- Learners do Activity 2 to assess if they know the names of quadrilaterals.
- Ask learners to name everyday objects that look like quadrilaterals, for example the classroom table is a rectangle, the tiles on the floor are squares, at a conference people sit around tables in the form of trapeziums, and so on.
- Learners follow the instructions for the investigation in Activity 3.
- Learners should use one page in their exercise books for a summary of the properties of quadrilaterals. Help the learners to plan the table so that they have enough space to mark the properties of the quadrilaterals. The summary encourages learners to "think in pictures".
- More than one example of a quadrilateral is given each time. We have to, at all times, consider more than one example in order to test a rule.
- Pay special attention to the following.
  - A rectangle is a special kind of parallelogram.
  - A rhombus is a special kind of parallelogram.
  - A square is a special kind of rectangle.
  - A square is a special kind of rhombus.
- Point out to learners that in some quadrilaterals (for example squares and rhombuses) the diagonals bisect the angles of the quadrilateral. In other cases (for example rectangles and parallelograms) this is not true. A kite is an interesting example in this regard: the vertical diagonal bisects the angles of the kite, but the horizontal diagonal does not bisect the angles of the kite.

Review Copy

Tip

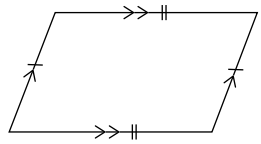
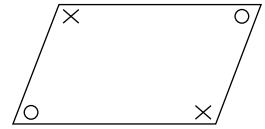
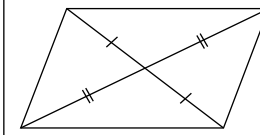
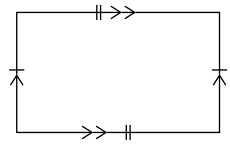
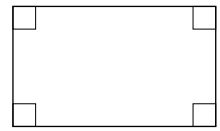
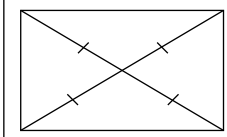
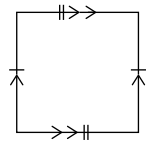
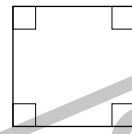
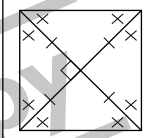
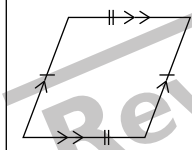
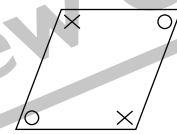
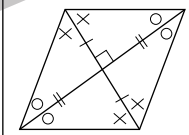
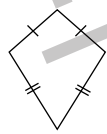
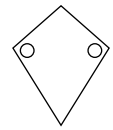
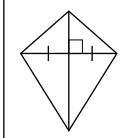
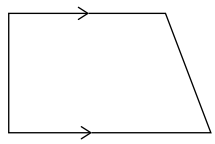
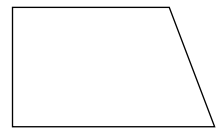
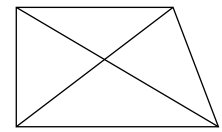
Make a poster for the classroom to show the classification of quadrilaterals. Learners can refer to the poster when they need to.

Suggested answers

Activity 2

1 c **2** e **3** b **4** d **5** f **6** b

Activity 3

Name	Sides	Angles	Diagonals
Parallelogram			
Rectangle			
Square			
Rhombus			
Kite			
Trapezium			

Review Copy

Activity 4

1 **a** True
b True
c False; only one pair
d True
e True
f False; only one pair
g False; A rhombus does not have right angles.
h True
i False, it has two pairs or opposite angles equal.
j False; only in a square, not in a rectangle

2 a A rectangle has four right angles; a parallelogram does not.
b A square has four equal sides; a rectangle has two pairs of equal sides.
c A square has four right angles; a rhombus does not.
d A rhombus has four equal sides; a parallelogram has two pairs of equal sides.

3 a kite b rectangle c rhombus
d parallelogram e trapezium f square

Investigating the properties of polygons

Revision: What are polygons?

Activity 7 Identify regular and irregular polygons

Learner's Book page 196

Guidelines for implementing this activity

- The new words taught here (for example hexagon, octagon, and so on), will be used again in the last term of the year when the learners study Platonic Solids. Refer to other words with the same prefixes to help the learners remember these words.
- Discuss the two definitions in the Learner's Book. Point out that polygon is a word to describe a collection of 2D shapes. Triangles and quadrilaterals are also polygons.
- Learners do Activity 5. It is important that they give reasons for their answers.

Review Copy

Suggested answers

1 a Yes b No – not a closed figure c Yes
d No – should have only lines, not a curve e Yes

2 a No – lengths of sides not all the same
b Yes – If the angles are all the same, then the sides will also be the same.
c No – lengths of sides not all the same
d Yes – all the sides and angles are the same.

3 a square
b Neither. The rhombus has equal side but the angles are not equal.

Interior angles of polygons

PoA | Investigation 1 Find the sum of the interior angles of polygons

Learner's book page 199

Guidelines for implementing this activity

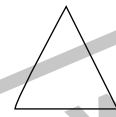
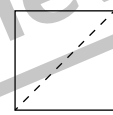
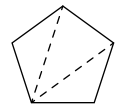
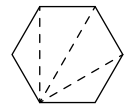
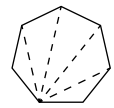
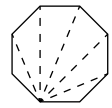
- Learners need to develop an understanding of the fact that there are many other shapes apart from triangles and quadrilaterals.
- In this investigation learners will investigate new shapes (with new names) and attempt to notice a pattern.
- Learners need to do this investigation on their own.
  - Explain when the investigation should be handed in.
  - Explain how the investigation should be presented, for example cover page (and what should be on it), introduction, method and conclusion.

- Read through the investigation with the learners and make sure that they understand each step. (This includes explaining what a conclusion is and how to write it.)
- Explain the rubric, making sure that they understand what they need to do in order to get full marks for each section.
- Do not teach learners a formula to calculate the sum of the interior angles of polygons. The aim is to teach learners to think so that they can solve similar problems on their own.

Remedial and extension

Remedial: Provide guidance and assistance where necessary, but it is important to encourage learners to do this task independently as far as possible.
Extension: Learners can be asked to explore the sum of the exterior angles of different polygons on their own.

Suggested answers

Polygon	No. of sides	Shape	Sum of interior angles	One angle in a regular polygon
Triangle	3		$1 \times 180° = 180°$	$180° \div 3 = 60°$
Quadrilateral	4		$2 \times 180° = 360°$	$360° \div 4 = 90°$
Pentagon	5		$3 \times 180° = 540°$	$540° \div 5 = 108°$
Hexagon	6		$4 \times 180° = 720°$	$720° \div 6 = 120°$
Heptagon	7		$5 \times 180° = 900°$	$900° \div 7 = 128{,}57°$
Octagon	8		$6 \times 180° = 1\,080°$	$1\,080° \div 5 = 135°$

Review Copy

Polygon	No. of sides	Sum of interior angles
Thus	n	$(n - 2) \times 180°$
	k	$(k - 2) \times 180°$
	t	$(t - 2) \times 180°$

Use the pattern to complete the last table.

Polygon	No. of sides	Sum of interior angles
Nonagon	9	$7 \times 180° = 1\ 260°$
Decagon	10	$8 \times 180° = 1\ 440°$
Dodecagon	12	$10 \times 180° = 1\ 800°$
Icosagon	20	$18 \times 180° = 3\ 240°$

Conclusion

If we divide the polygon into triangles, the number of triangles is always 2 less than the number of the sides of the polygon, e.g. a pentagon has 5 sides; it can be divided into $5 - 2 = 3$ triangles.

Use the following rubric to assess each learner's progress.

	4	3	2	1
Criterion	Outstanding/ Exceptional	Achieved	Partially achieved	Not achieved
Task	Completed as per instruction	Completed, but not according to all instructions	Incomplete, and not done according to instructions	Little or nothing attempted
Conclusion	Conclusion is correct and shows insight	Conclusion is incorrect, although shows some logic	Conclusion is incorrect and not related to task	No conclusion is drawn

Review Copy

UNIT 4

Geometry of 2D shapes: Triangles

Unit overview

Learner's Book page 202
Recommended pacing: 3 hours

This unit focuses on the following:
- classifying triangles in terms of sides
- classifying triangles in terms of angles
- solving triangles

Resources: Learner's Book; exercise book

Mental Maths

(10 minutes per day)

- The mental mathematics is revision of the properties of triangles and quadrilaterals. These properties will be applied in problems in the rest of this unit.
- Learners should use the correct words when they give the answers to Question **3** about the properties of quadrilaterals, for example refer to "opposite" or "adjacent" sides.
- This is also an opportunity to point out the similarities and the differences between quadrilaterals again.

Review Copy

Suggested answers — Learner's Book page 202

1 **a** 80° **b** 45° **c** 2°

2 **a** ... is equal to 180°.
b ... is equal to the sum of the opposite interior angles.
c ... is equal to 360°.

3 For sides:
a Two pairs of opposite sides are equal and parallel.
b Two pairs of opposite sides are equal and parallel.
c All four sides are equal; two pairs of opposite sides are parallel.
d All four sides are equal; two pairs of opposite sides are parallel.
e Two pairs of adjacent sides are equal.
f One pair of opposite sides are parallel.

For angles:
a Two pairs of opposite angles are equal.
b Four right angles.
c Four right angles.
d Two pairs of opposite angles are equal.
e One pair of opposite angles are equal.
f Each angle can have its own size; total 360°

Mental Maths

For diagonals:

a Diagonals bisect each other.
b Equal diagonals, bisect each other.
c Equal diagonals, bisect each other perpendicularly.
d Diagonals bisect each other perpendicularly.
e One diagonal bisects the other one perpendicularly.
f A trapezium has no fixed properties.

Background information

- Some of the words we use to classify triangles come from other languages:
  - equilateral: equi = equal in Latin; lateral = to do with sides
  - isosceles: iso = equal in Greek
- In Grade 7, learners did the following:
  - Classified triangles
  - Described, sorted, named and compared triangles
  - Investigated the interior angles of a triangles (extension)
  - Calculated the sizes of angles and constructed triangles

In Grade 8, learners will solve geometric problems for the first time.

Teaching guidelines

Solving geometric problems is an important focus of this unit, and it is extremely important that learners understand the difference between presenting solutions arithmetically, and presenting their thinking to a geometry problem using the methods that are required for this phase. While both are correct, it is extremely important that learners practise presenting their thinking using the appropriate methods required for this phase.

Review Copy

Example: Find $\hat{C}$ of $A\hat{B}C$ if $\hat{A} = 50°$ and $\hat{B} = 70°$.

Arithmetic (this method is not advised)	Geometry (the correct method)
$50° + 70° = 120°$ $180° - 120° = 60°$ Therefore $\hat{C}$ is 60°	$\hat{A} + \hat{B} + \hat{C} = 180°$ (3 $\angle$s of $\triangle ABC = 180°$) $50° + 70° + \hat{C} = 180°$ $\hat{C} = 60°$

The above requires practice and should be emphasised and expected at all times.

Revision: Properties and definitions of triangles

Activity 1 Work with triangles

Learner's Book page 203

Guidelines for implementing this activity

- Revise the definition of a triangle.
- Work through the practical exercise with learners.

- It is important that learners understand how a triangle is formed. If they understand that the smallest angle is opposite the shortest side, it will help them to realise whether an answer is possible or not.
- Pay attention to the use of correct notation. Learners should use capital letters for angles and small letters for sides.
- Learners complete Activity 1.

Suggested answers

1 biggest, shortest
2 a The side of 12 cm needs to be opposite the angle of 105°.
  b The side of 5 mm needs to be opposite the right angle and the side of 3 mm needs to be opposite the angle of 35°.
  c The two equal sides need to be opposite the two equal angles.

Revision: Classification of triangles

Activity 2 Classify triangles

Learner's Book page 203

Guidelines for implementing this activity

- Revise the classification of triangles in terms of sides and in terms of angles, covered in Grade 7.
- Ask learners what makes each type of triangle unique, for example all the sides and angles of an equilateral triangle are equal. Ensure that they can classify these triangles.
- Display a poster on the wall with the different types of triangle.
- Activity 2 question 2 is aimed at evaluating whether learners can read and interpret words, and present them graphically.

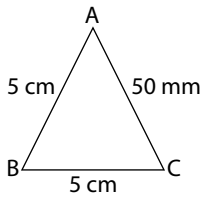
Review Copy

Tip

Learners can easily assume that all triangles are equilateral triangles, because that is the way we draw triangles most of the time. Vary the sketches of triangles that you draw on the board.

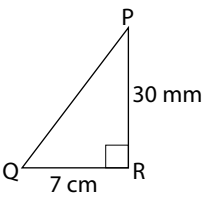
Suggested answers

1 a equilateral
  b two; sides; lengths
2 a



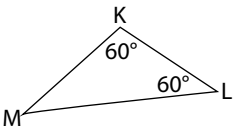
Acute-angled triangle
(Remember: 5 cm = 50 mm)

b



Right-angled triangle

c



Obtuse-angled triangle

3 a Equilateral triangle
b Scalene triangle (The side opposite the right angle needs to be the longest side)
c Isosceles triangle (Calculate $\hat{M}$ first)

Solving triangles

Activity 3 Solve triangles

Learner's Book page 207

Guidelines for implementing this activity

- **Note:** Learners will solve geometric problems for the first time.
- It is extremely important that they understand the difference between merely doing arithmetic and solving a geometry problem.

Example: Find $\hat{C}$ of $A\hat{B}C$ if $\hat{A} = 50°$ and $\hat{B} = 70°$.

Arithmetic (this method is not advised)	**Geometry (the correct method)**
$50° + 70° = 120°$ $180° - 120° = 60°$; Therefore $\hat{C}$ is 60°.	$\hat{A} + \hat{B} + \hat{C} = 180°$ (3 $\angle$s of $\triangle ABC = 180°$) $50° + 70° + \hat{C} = 180°$; $\hat{C} = 60°$

- Both methods will give the correct answer, but it is very important that learners practise presenting their thinking through the use of the second method.
- Ensure that the learners write only geometry in the first line, and not any numbers. If they can learn this habit from Grade 8, it will prevent many geometry problems in the FET phase!
- Spend enough time on the examples before learners do Activity 3.
- Do the examples on the board and discuss them step-by-step. This is the foundation for solving geometric problems.
- Carefully explain how to write down the reason correctly. For example, if you solve an isosceles triangle, name the sides or the angles.
- It is **not** acceptable to write "isosceles" or "base angles" as a reason. (There are three angles and three sides in a triangle. If you write "base angles", it is impossible to know which angles are being referred to.)
- These problems are an excellent opportunity to practise solving equations. Repeat and practise the principle of doing the same thing on both sides of the equation.

Review Copy

Remedial and extension

Remedial: Pay special attention to the fact that $132 = 84 + d$ is exactly the same as $84 + d = 132$. (Activity 3 Question **3b**) We are not solving the equation here; we are applying the commutative property.
Extension: Activity 3 Question **1e**: Ask the learners how many triangles they can see in the diagram. (There are 3: ACD; ABD and BCD) Learners may find it challenging if there is more than one triangle in a diagram (See Activity 3 Question **3**). Encourage them to cover parts of the diagram with their hands so that they can see only the triangle that they want to work with.

Tip

Always label geometric figures. It is nearly impossible to discuss the properties of a triangle if you cannot refer to a specific angle by saying, for example, "What do you notice about $\hat{A}$?" Teach learners to add their own labels to a diagram if labels are not given on a worksheet or in an assessment.

Suggested answers

1 a $\hat{A} + \hat{B} + \hat{C} = 180°$ (3 ∠s of △ABC = 180°)
$47° + 53° + a = 180°$
$a = 180° - 100°$
$a = 80°$

b $\hat{P} = \hat{Q}$ (RP = RQ)
$b = 24°$

c $\hat{Z} = \hat{Y}$ (XY = XZ)
$\hat{X} + \hat{Y} + \hat{Z} = 180°$ (3 ∠s of △XYZ = 180°)
$90 + c + c = 180°$
$c = 45°$

d ∠GEF = $\hat{D} + \hat{G}$ (Ext ∠ of △DEG = sum of opp int ∠s)
$x = 60° + 60°$ (DE = EG = DG) or (△DEG is equilateral)
$x = 120°$

e There is more than one triangle in the diagram, so it is even more important to say in which triangle you are working when you write down reasons.
$x = 60°$ (AC = CD = DA) or (△ACD is equilateral)
In △ABC: $\hat{A} + \hat{B} + \hat{D} = 180°$ (3 ∠s of △ABD = 180°)
$60° + 110° + y = 180°$
$y = 10°$
$y + z = 60°$ (AC = CD = DA) or (△ACD is equilateral)
$z = 50°$

f $a = 65°$ (RP = RQ)
$a + b = 90°$ ($Q\hat{P}S = 90°$)
$b = 25°$
△PRS = $\hat{Q}$ + △QPR (Ext ∠ of △PQR = sum of opp int ∠s)
$c = 65° + 65° = 130°$
In △PRS: $\hat{P} + \hat{R} + \hat{S} = 180°$ (3 ∠s of △PRS = 180°)
$25° + 130° + d = 180°$
$d = 25°$
△PRS is an obtuse-angled triangle ($c = 130°$).
△PRS is an isosceles triangle ($b = d = 25°$).

2 a $\hat{Z} = 60°$ (WY = WZ = YZ)
$3x = 60°$
$x = 20°$

b $\hat{D} + \hat{E} + \hat{F} = 180°$ (3 ∠s of △DEF = 180°)
$5x + 3x + 4x = 180°$
$12x = 180°$
$x = 15°$

Review Copy

c $\angle RSU = \hat{T} + \hat{U}$ (Ext $\angle$ of $\triangle STU$ = sum of opp int $\angle$s)
$85° = 4x + x$ (or it can be written: $= 4x + x = 85°$)
$5x = 85°$
$x = 17°$

d $\hat{A} = \hat{C}$ (BA = BC)
$\hat{A} + \hat{B} + \hat{C} = 180°$ (3 $\angle$s of $\triangle ABC = 180°$)
$x + x + 78° = 180°$
$x = 51°$

e $\angle KLN = \hat{N} + N\hat{M}L$ (Ext $\angle$ of $\triangle NLM$ = sum of opp int $\angle$s)
$6x + 15° = 63° + 3x$
$3x = 48°$
$x = 16°$

f $\hat{B} = \hat{A}$ (CA = CB)
$\hat{A} + \hat{B} = A\hat{C}D$ (Ext $\angle$ of $\triangle ABC$ = sum of opp int $\angle$s)
$5x + 5x = 128°$
$x = 12{,}8°$

3 There is more than one triangle in these diagrams, so it is important to say in which triangle you are working.

a Note the difference between A and a!
$\hat{A} + \hat{B} + \hat{C} = 180°$ (3 $\angle$s of $\triangle ABC = 180°$)
$50° + 90° + a = 180°$
$a = 40°$
$b = a$ (AC bisects $\triangle ACE$)
$b = 40°$
In $\triangle ACE$: $\hat{A} = \hat{E}$ (CA = CE)
$\hat{A} + \hat{C} + \hat{E} = 180°$ (3 $\angle$s of $\triangle ACE = 180°$)
$c + 40° + c = 180°$
$c = 70°$
$d = 60°$ (CE = ED = DC)

b $\hat{T} = \hat{S}$ (RT = RS)
$a = 69°$
$\hat{R} + \hat{S} + \hat{T} = 180°$ (3 $\angle$s of $\triangle RST = 180°$)
$b + 69° + 69° = 180°$
$b = 42°$
$c = 90°$ (Given)
$\angle PRS = \hat{P} + \hat{Q}$ (Ext $\angle$ of $\triangle PQR$ = sum of opp int $\angle$s)
$90° + 42° = 84° + d$
$d = 48°$

Review Copy

UNIT 5

Geometry of 2D shapes: Quadrilaterals

Unit overview

Learner's Book page 209
Recommended pacing: 3 hours

This unit focuses on the following:
- revising the properties of quadrilaterals
- solving word problems (triangles and quadrilaterals)

Resources: Learner's Book; exercise book

Background information

- In Grade 7, learners did the following:
  - classified quadrilaterals
  - described, sorted, named and compared quadrilaterals.

In Grade 8, learners will solve geometric problems for the first time.

Teaching guidelines

Computer programmers and engineers often use flow charts to plan their work. In a flow chart, there are only two possible answers to any question, namely "Yes" or "No". This system forces you to break down a complicated process into clear steps. This is exactly how we have to think when we do geometry.

Review Copy

Revision: Classifying quadrilaterals

Activity 1 Classify quadrilaterals

Learner's Book page 210

Guidelines for implementing this activity

- Before doing the revision at the beginning of this unit, instruct learners to do Activity 1.
- Activity 1 serves as revision, and learners should be expected to do this activity on their own.
- Go through this revision at the beginning of the unit only once learners have attempted the activity on their own, and for purposes of consolidation.

Suggested answers

1 square
2 rectangle
3 rhombus
4 parallelogram
5 kite
6 irregular quadrilateral
7 trapezium

Solving quadrilaterals

Activity 2 Solve quadrilaterals

Learner's Book page 210

Guidelines for implementing this activity

- Learners do Activity 2 to practise the properties of quadrilaterals once again.
- Reasons are important, because they compel learners to use clear and accurate definitions of quadrilaterals.
- Check that learners always provide reasons, and that they apply the correct geometrical breakdown in their thinking, as explained earlier.

Remedial and extension

Remedial: The diagrams in this activity can be used to revise conversion between mm and cm.
Extension: Question **6** challenges learners by asking something they have not been taught.

Tip

"Why?" This is the question you need to ask over and over again when you teach geometry!

Review Copy

Suggested answers

1 a AE = 6,5 cm (In rectangles diagonals bisect each other.)
  b DC = 5 cm = 50 mm (Opp sides of rectangle equal)
  c $\hat{C} = 90°$ (Property of rectangle)
  d Right-angled triangle
2 a PQ = 70 mm = 7 cm (Opp sides of square equal)
  b PT = QT = RT = ST (Diagonals of equal length, bisect each other)
  c PS ∥ QR
  d Isosceles triangle
3 a KL ∥ NM and KN ∥ LM (Definition of parm)
  b $\hat{N} = 57°$ (Opp angles equal in parm)
  c $\hat{K} + \hat{L} + \hat{M} + \hat{N} = 360°$ (∠s of quadrilateral)
  $\hat{K} + 57 + \hat{M} + 57 = 360°$
  $\hat{K} + \hat{M} = 246°$
  d $\hat{M} = 123°$ (Opp angles equal in parm)
  e Obtuse-angled triangle
4 a △QYZ (All four sides of rhombus are equal)
  b $\hat{Q} = 60°$ (XQ = QZ = ZX)
  c YP = PQ and XP = PZ (Diagonals bisect each other in rhombus)
  d XZ ⊥ YQ (Diagonals bisect each other perpendicularly in rhombus)

5 a In $\triangle DEG$:
$\hat{E} = \hat{G}$ (DE = DG; property of kite)
$\hat{D} + \hat{E} + \hat{G} = 180°$ (3 $\angle$s of $\triangle DEG = 180°$)
$100° + a + a = 180°$
$a = 40°$

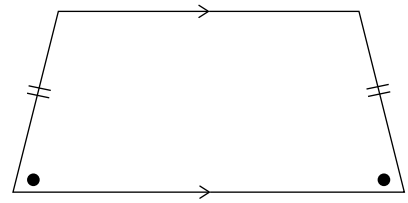
b In $\triangle EFG$:
$\hat{E} = \hat{G}$ (FE = FG; property of kite)
$\hat{E} + \hat{F} + \hat{G} = 180°$ (3 $\angle$s of $\triangle EFG = 180°$)
$65° + E\hat{F}G + 65° = 180°$
$E\hat{F}G = 50°$

c $\triangle DEF = \triangle DGF$ (property of kite)
$\triangle DEF = 40° + 65° = 105°$

d Scalene triangle

e $DF \perp EG$ (Diagonals perpendicular in kite)

6 In an isosceles trapezium the two non-parallel sides are equal. There will also be two equal angles, as shown on the sketch.



Problem-solving

Activity 3 Solve word problems

Learner's Book page 211

Review Copy

Guidelines for implementing this activity

- The secret for success in Mathematics (especially with word problems) can be summed up in three steps.
  1 READ
  2 THINK
  3 WRITE
- In these questions learners should draw their own diagrams. Do not give the learners diagrams too soon. They need to develop the skills to read, interpret and present information graphically.
- In question **4** no numbers are given. The idea is to concentrate on how to solve the problem and not on how to do the arithmetic!

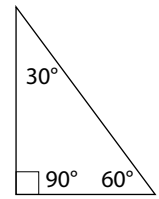
Remedial and extension

Challenge learners with the following question: How can Patricia measure the angle between the first two lines she drew if she does not have a protractor? See Activity 3 question **4**.

Suggested answers

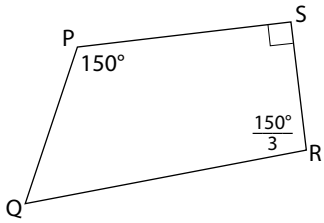
1 **a** 3 ∠s of △ = 180°
$p + 2p + 3p = 180°$
$6p = 180°$
$p = 30°$

b Angles: $p = 30°$;
$2p = 60°$
$3p = 90°$

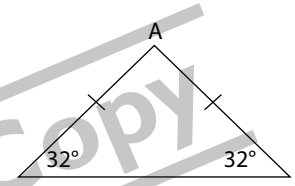


c Right-angled, scalene triangle.

2 $\hat{P} + \hat{Q} + \hat{R} + \hat{S} = 360°$ (∠s of Quadrilateral)
$150° + \hat{Q} + 50° + 90° = 360°$
$\hat{Q} = 70°$

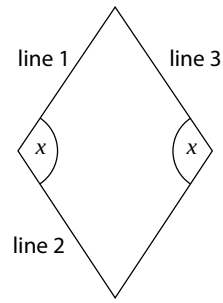


3 The roof forms an isosceles triangle. $\hat{A} = 116°$



Review Copy

4 Measure the angle between the first two lines. (x in diagram). In a rhombus the opposite angles should be equal (also x). If Patricia draws the rhombus on a piece of paper, she can calculate the angle between the first line and the third line if she keeps in mind that the sum of the interior angles of a rhombus should be 360° and that the opposite angles need to be equal.



UNIT 6

Geometry of 2D shapes: Congruent and similar figures

Unit overview

Learner's Book page 213
Recommended pacing: 2 hours

This unit focuses on the following:

- defining congruent and similar figures
- learning the four sets of conditions for congruent triangles
- finding conditions for similar triangles

Resources: Learner's book; exercise book; scissors; glue

Mental Maths

(10 minutes per day)

Learner's Book page 213

Learners are required to complete ratios. In similar figures, the sides are proportional. This means that the corresponding sides of the two shapes all have the same ratio. The mental mathematics exercise is preparation for doing calculations with similar triangles at the end of the unit.
Do not be satisfied with only the correct answers. Learners also need to know why they give the answers.

Review Copy

Suggested answers

1	a $\frac{2}{5} = \frac{4}{10}$	b $\frac{2}{5} = \frac{6}{15}$	c $\frac{2}{5} = \frac{8}{20}$	d $\frac{2}{5} = \frac{10}{25}$
2	a $\frac{3}{7} = \frac{6}{14}$	b $\frac{3}{7} = \frac{9}{21}$	c $\frac{3}{7} = \frac{12}{28}$	d $\frac{3}{7} = \frac{15}{35}$
3	a $\frac{6}{8} = \frac{3}{4}$	b $\frac{9}{12} = \frac{3}{4}$	c $\frac{12}{16} = \frac{3}{4}$	d $\frac{15}{20} = \frac{3}{4}$
4	a $\frac{14}{16} = \frac{7}{8}$	b $\frac{27}{30} = \frac{9}{10}$	c $\frac{16}{28} = \frac{4}{7}$	d $\frac{25}{45} = \frac{5}{9}$

Background information

In Grade 7, learners identified, matched and drew similar and congruent figures

Learning the conditions for congruency is new in Grade 8.

- The concepts of congruency and similarity are important in Mathematics.
- In transformation geometry learners will need to be clear about these two concepts. Many formal proofs in geometry are based on congruency, for example properties of parallelograms.
- Both congruency and similarity are used in solving geometric problems in FET Mathematics, so (as with everything else we do in Grade 8 Mathematics) it is once again important to lay a solid foundation.

Teaching guidelines

The investigations in this unit are very important. It is best when learners experience congruent triangles practically.

In Grade 8 learners can be challenged to construct the following:

- two triangles with equal angles and sides that are not proportional
- two triangles with proportional sides and angles that are not equal.

Congruent and similar figures

Congruent figures; Similar figures

Activity 1 **Work with congruency and similarity**

Learner's Book page 214

Guidelines for implementing this activity

- Quite often people think that congruency and similarity have to do with triangles only. This is not the case. Any two shapes can be congruent. Any two shapes can be similar.
- The first activity is to work with shapes other than triangles. Here the aim is that learners develop a "gut feeling" for the two concepts.
- Work carefully through the introduction, drawing frequently on learners' prior knowledge of these two concepts by asking questions such as "What do you remember …?"; "Are figures that are copies of each other similar or congruent?" "What is the difference between similarity and congruency?"
- Invite learners to make rough sketches of similar and congruent figures on the board, and explain/describe these.
- The rest of the unit focuses on congruency and similarity in triangles. Exact definitions will then be developed through construction and investigation.

Review Copy

Tip

Use practical everyday objects to develop the concepts of congruency and similarity, for example, the glass window panes in the classroom which may be congruent.

Suggested answers

a A, H and O; B and F; C, I and K; G, L and M; E and N

b Figures need to have the same shapes as J, P, D and Q, but different sizes.

Investigate conditions for congruent triangles

Investigation for first condition: SSS; Investigation for second condition: S∠S; Investigation for third condition: ∠∠S; Investigation for fourth condition: RHS

Activities 2–5 **Investigations of congruency**

Learner's Book pages 215–217

Guidelines for implementing these activities

- The investigations are very, very important. Learners have to concretely experience when triangles are congruent.
- Work through these investigations with the learners in order to ensure that they do them correctly. They should not draw incorrect conclusions.
- Work through the first example step-by-step with the learners.
- Spend time on the following: We use three parts of the triangles to find out if the triangles are congruent. If the triangles are congruent, we can assume that the other three parts (that we do not have information about) are also equal.
- Encourage learners to attempt Columns II, III and IV (Learner's Book pag 213), but do not leave them to complete these on their own. Discuss the meaning of an included angle (Column II), a corresponding side (Column III) and the fact that the type of congruency in Column IV is an exception to the rule in Column II.

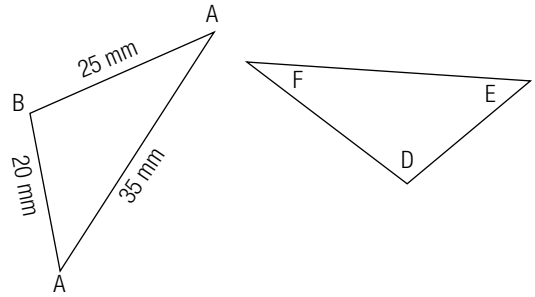
Tip

It really works to expose learners to congruency of triangles in a practical way. If it is possible, the four tables with triangles can be copied from the Learner's Book onto one page for the learners. Then they can actually cut out the given triangles and paste them into their exercise books, and cut out the correct matching triangle and paste it onto the given triangle.

Review Copy

Suggested answers

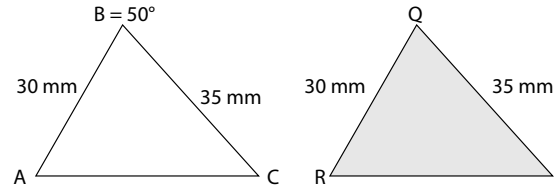
Column I



The learners write...	To point out...
In △ABC and △DEF	The order in which we write here does not matter.
1) AB = DF (25 mm) 2) BC = DE (20 mm) 3) CA = EF (35 mm)	Teach learners to number from 1 to 3. Then they will know that they should give **three** equal parts.

The learners write...	To point out...
$\triangle ABC \equiv \triangle FDE$ (SSS)	The order in which we write here, **has to** reflect which parts of the triangles are the same. Remember to add the condition (SSS).
$\hat{A} = \hat{F}$ $\hat{B} = \hat{D}$ $\hat{C} = \hat{E}$	Point out to the learners that this is the reason why we want to know if two triangles are congruent or not.

Column II
In $\triangle ABC$ and $\triangle PQR$:
1) AB = QR (30 mm)
2) $\hat{B} = \hat{Q}$ (50°)
3) BC = QP (35 mm)
$\triangle ABC \equiv \triangle RQP$ (S∠S)
$\hat{A} = \hat{R}$; $\hat{C} = \hat{P}$ and AC = RP



Column III
In $\triangle ABC$ and $\triangle XYZ$
1) AB = YZ (45 mm)
2) $\hat{A} = \hat{Y}$ (30°)
3) $\hat{C} = \hat{X}$ (110°)
$\triangle ABC \equiv \triangle YZX$ (∠∠S)
$\hat{B} = \hat{Z}$; AC = YX and BC = ZX

Column IV
In $\triangle ABC$ and $\triangle KLM$
1) $\hat{B} = \hat{L}$ (90°)
2) BC = LM (3 cm)
3) AC = KM (5 cm)
$\triangle ABC \equiv \triangle KLM$ (RHS)
$\hat{A} = \hat{K}$; $\hat{C} = \hat{M}$ and AB = KL

Review Copy

Activity 6 Summarise conditions for congruent triangles

Learner's Book page 218

Guidelines for implementing this activity

- The purpose of this activity is to sum up the conclusions of the investigations.
- Pay attention to the use of the correct words, for example including angle, corresponding side.
- Learners need to take note of the symbol for congruency (≡). Ensure that they use this symbol, and that they use it correctly.

Tip

Learners can think of the symbol for congruency as telling them that the two triangles are "extra equal". (The symbol for congruency looks like an = sign, with an extra line.)

Suggested answers

1 If the angles of two triangles are equal, the triangles are similar, but not congruent.
2 The angle is included between the two marked sides.
3 No, it is not necessary. But the sides should be opposite sides of the same length in the two different triangles.
4 a R: right angle (90°); H: hypotenuse (side opposite angle of 90°); S: side.
  b II. In this case we work with two sides and one angle that is not included, but because we use a right angle and the hypotenuse, it is okay to do so.

Activity 7 Identify congruent triangles

Learner's Book page 218

Guidelines for implementing this activity

- In this activity learners will apply the conditions for congruent triangles.
- Do additional, similar examples on the board to make sure all the learners are comfortable with the new concepts and the way that they have to present their answers.

Tip

In Activity 3 learners have to write down each answer as one line only. If learners are ready to move on to the next level, ask them to write out the complete proofs of congruency for one or two of the questions. Then their answers will be in the format of the investigation. This is how they will be expected to do it in Grade 9.

Review Copy

Suggested answers

1 Please note: if the triangles are **not** congruent, the order in which you write down the second triangle does not really matter.
  a $\triangle ABC \equiv \triangle RQP$ (SSS)
  b $\triangle ABC \equiv \triangle DEF$ (AAA – triangles are similar)
  c $\triangle ABC \not\equiv \triangle KLM$ (angle is not included in $\triangle KLM$)
  d $\triangle ABC \equiv \triangle KHG$ ($\angle\angle S$) The marked side is opposite the angle marked y in both triangles.)
  e $\triangle ABC \equiv \triangle NOM$ (RHS)
  f $\triangle ABC \equiv \triangle RST$ ($S\angle S$) The right angle is treated like any other angle here.
  g $\triangle ABC \not\equiv \triangle DEF$ (Side not corresponding.) The marked side is opposite the angle marked x in the one triangle, but opposite the side marked y in the second triangle.
  h $\triangle ABC \equiv \triangle QZY$ ($\angle\angle S$)
2 a If PR = XZ: SSS;
    If $\hat{Q} = \hat{Y}$: $S\angle S$
  b If PR = KM or if QR = LM or if PQ = KL: $\angle\angle S$
  c If PQ = DE: $S\angle S$
    If $\hat{R} = \hat{F}$ or if $\hat{Q} = \hat{E}$: $\angle\angle S$
  d If PQ = ST: RHS
    If QR = TU: $S\angle S$
3 Learners should use the properties of quadrilaterals to do this question.
  a $\triangle PQR \equiv \triangle RSP$ (SSS or RHS or $S\angle S$)
  b No congruent triangles

c $\triangle ABD \equiv \triangle ACD$ (S∠S or ∠∠S) If AB = AC, then $\hat{B} = \hat{C}$
d $\triangle VXZ \equiv \triangle YZX$ (SSS or S∠S)
e $\triangle KLN \equiv \triangle MLN$ (SSS or S∠S) or $\triangle KLM \equiv \triangle KNM$ (SSS or S∠S) OR $\triangle KLO \equiv \triangle MLO$ (RHS) or $\triangle KNO \equiv \triangle MNO$ (RHS) or $\triangle KLO \equiv \triangle MNO$ (RHS) or $\triangle MNO \equiv \triangle MLO$ (RHS)
f $\triangle ABD \equiv \triangle ACD$ (RHS or ∠∠S) If $\hat{B} = \hat{C}$, then AB = AC

4 a $\hat{L} = \hat{E} = 90°$
b $\hat{K} = \hat{D} = 23°$ (Sum of interior ∠s of △ = 180°)
c LM = EF = 5 cm
d The hypotenuse of △ KLM = DF = 13 cm

Conditions for similarity

Activities 8–9 **Use similarity of triangles; Use similarity and congruency in problems**

Learner's Book pages 221–223

Guidelines for implementing this activity

- Explain the following to learners.
  - Any two shapes can be similar.
  - If it is given that two shapes (for example, triangles) are similar, it means that the corresponding angles are equal and that the sides are in proportion.
  - If you are asked to prove that shapes are similar, there is a difference between the conditions for similar shapes in general and similar triangles:
- Two shapes are similar if their angles are the same and their sides are in proportion.
- Two triangles are similar if their angles are the same or if their sides are in proportion.
- Discuss the definition for shapes to be similar by studying the measurements of rectangles A, B and C. Point out that $\frac{2}{5} = \frac{4}{10}$ but $\frac{2}{5} \neq \frac{4}{7}$.
- Emphasise the fact that similarity in triangles work differently. Challenge learners to do the constructions. Award a prize for the best effort.
- Practise reading the definition for similar triangles from both directions.
  - From the left: If the angles of two triangles are the same, then the triangles are similar and therefore the sides of the triangles are in proportion.
  - From the right: If the sides of two triangles are in proportion, then the triangles are similar and therefore the angles of the two triangles are the same.
  - Point out the difference between the symbols for congruent triangles ("extra equal") and similar triangles (≡). In both cases the order in which we write down the triangles are important.
- Learners do Activity 4: basic but important questions about similar triangles.

Review Copy

Tip

Teach the learners to use the order of the letters to write down the proportional sides. For example, if ABC ··· PQR, always use the letters of the first triangle first and the letters of the second triangle next. Then $\frac{\text{first two letters}}{\text{first two letters}} = \frac{\text{last two letters}}{\text{last two letters}} = \frac{\text{first and last letter}}{\text{first and last letter}}$. Learners who work on computers are familiar with the concept of "zoom in" and "zoom out". This is another example of similarity.

Suggested answers

Activity 4

1 a Yes; because the corresponding angles are equal. $\triangle KML \cdots \triangle XZY$

2 Yes; because the sides are in proportion.

Longest sides: $\frac{XZ}{KM} = \frac{9}{36} = \frac{1}{4}$

Shortest sides: $\frac{XY}{KL} = \frac{7}{28} = \frac{1}{4}$

Third pair of sides: $\frac{YZ}{LM} = \frac{8}{32} = \frac{1}{4}$

All the ratios are equal to $\frac{1}{4}$.

XYZ ··· KLM

3 a Yes; the corresponding angles of the two triangles are equal. Rewrite and complete:

b $\hat{A} = \hat{R} = 30°$

c $\hat{C} = \hat{S} = 60°$

d $\triangle ABC \cdots \triangle RTS$

4 a Longest sides: $\frac{KM}{DF} = \frac{12}{25}$

b Shortest sides: $\frac{LK}{DE} = \frac{7}{14} = \frac{1}{2}$

c Third pair of sides: $\frac{LM}{EF} = \frac{10}{20} = \frac{1}{2}$

d The two triangles are not similar. The sides are not in proportion.

5 The triangles are similar. The sides are in proportion.

$\frac{JM}{DU} = \frac{ME}{UT}$ $\left(\text{or } \frac{JM}{DU} = \frac{JE}{DT}\right)$

$\frac{8}{DU} = \frac{6}{18} = \frac{1}{3}$

$DU = 8 \times 3 = 24$ cm

6 $\triangle ABC \cdots \triangle EDC$ The sides are in proportion.

a $\frac{AB}{ED} = \frac{BC}{DC}$

$\frac{15}{105} = \frac{BC}{49}$

$\frac{1}{7} = \frac{BC}{49}$

BC = 7 units

b $\frac{AB}{ED} = \frac{AC}{EC}$

$\frac{15}{105} = \frac{21}{EC}$

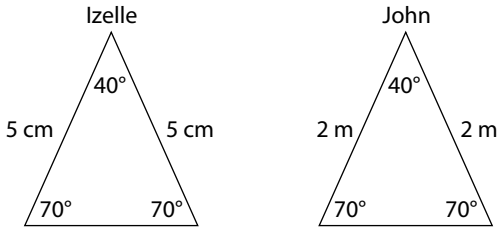
$\frac{1}{7} = \frac{21}{EC}$

$EC = 7 \times 21 = 147$ units

Review Copy

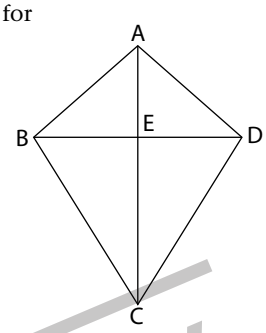
Activity 5

1 **a** and **b**



c The triangles are similar. AAA is never a reason for congruency of triangles.

2 ABE ≡ ADE (RHS)
CBE ≡ CDE (RHS)
ABC ≡ ADC (SSS or S∠S)
See the artwork on the right.



3 The triangles are similar. They are the same shape, but they have different sizes.

Chapter 10 Revision

Learner's Book page 224

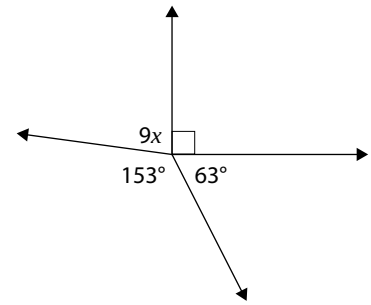
Review Copy

Encourage learners to review the content covered before attempting the revision activities. The activities should be used to assess learners' progress thus far, and to assess where remediation may be required.

Suggested answers

1 **a** True
b False; the sum of the interior angles of a quadrilateral is 360°.
c True
d False; a trapezium has one pair of parallel sides.

2 **a**, **b** and **c** the diagram on the right
d $90° + 63° + 153° + 9x = 360°$(revolution)
$9x = 360° - 306° = 54°$
$\therefore\ x = 6°$



3 $a = 64°$ (alt ∠s; PQ ∥ UV)
$a + b = 90°$ (right angle)
$64° + b = 90°$
$\therefore\ b = 26°$
$64° + c = 360°$ (revolution)
$\therefore\ c = 296°$
$d = 64°$ (corresp ∠s; WU ∥ PV)
$e = d$ (alt ∠s; RS ∥ UV)
$f = e$ (vert opp ∠s)
$\therefore\ f = 64°$
$\therefore\ e = 64°$

4 **a** $\hat{A} = \hat{B}$ (CA = CB)
$\therefore \hat{A} = \hat{B} = 42°$
$\hat{A} + \hat{B} + \hat{C} = 180°$ (int ∠s of △ABC)
$42° + 42° + 3x = 180°$
$3x = 96°$
$\therefore x = 32°$

b $x + 15° = 105°$ (vert opp ∠s)
$\therefore x = 90°$

c $\hat{C}ED = \hat{D} + \hat{F}$ (ext ∠ = sum of opp int ∠s)
$3x + 12° + x = 104°$
$4x = 92°$
$\therefore x = 23°$

5 **a** 90°

b BC = AD = 55 mm

c 7 cm (AE = half of AC and AC = BD)

6 **a** △KLM or △KNM

b △KPL or △KPN or △MPL or △MPN

c △NPM (RHS)

7 **a** △PQR ≡ △MLK (SSS)

b △PQR ≡ △ACB (S∠S)

c △ABC is not congruent to △DEF (side not opposite same angle)

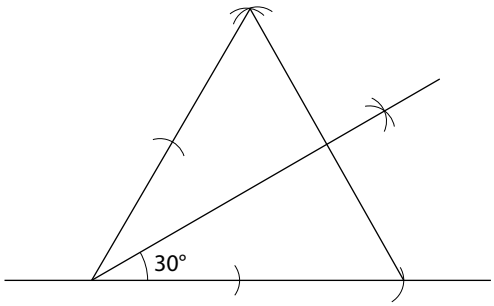
d △PQR is not congruent to △XYZ (equal angles never reason for congruent triangles; the triangles are similar).

8 **a** Angles in △PQR: 50°; 70° and 60° (sum of int ∠s of △ = 180°).
Angles in △ABC: 60°; 70° and 50° (sum of int ∠s of △ = 180°).
∴ △PQR ··· △BCA (AAA)

b Longest sides: $\frac{DE}{AC} = \frac{10}{15} = \frac{2}{3}$
Shortest sides: $\frac{EF}{CB} = \frac{6}{8} = \frac{3}{4}$
Third pair of sides: $\frac{DF}{AB} = \frac{8}{12} = \frac{3}{4}$
∴ Triangles are not similar (sides not in proportion).

9 **a** Construct accurately any equilateral triangle.

b Bisect any one of the angles.



Review Copy

Exam exemplar I (June): Memorandum

Tip

- Learners need to practise how to write an exam.
- Learners should be able to read the questions and answer them without any extra help.
- Learners also need to practise time management when writing a test or an exam. A question paper should show the time available for the test, as well as the marks allocated for each question.
- Do not allow learners extra time.
- It is a good idea to hand out the question paper as well as the memorandum to learners after the exam. They can use the memorandum to revise the questions they found challenging. They can also use it to prepare for the final exam of the year.

Section A: Algebra

Time: 50 minutes **Marks: 50**

Review Copy

1 $x + 2x + 1$✓ $= 20 + 2(20) + 1$✓ $= 61$ trees✓ (3)

2 $(20x - 5y) - (4x + 5y + 10y - 6x)$✓
$= (20x - 5y) - (-2x + 15y)$✓
$= (20x - 5y) + (2x - 15y)$✓
$= 22x - 20y$✓ (5)

3 **a** binomial✓ **b** monomial✓ (2)

4 **a** $-30pq$✓ (1)
b $2x^2$✓ $+ 1$✓ (2)
c $\frac{1}{6x^2y}$✓ $+ \frac{1}{xy^2}$✓ $- 6$✓ (3)
d $9x^3y^3$✓ $+ 40x^3y^3$✓ $= 49x^3y^3$ (3)
e $-2x^2y(-27x^{12}y^{15})$✓(coefficient) ✓(powers)
$= 54x^{14}y^{16}$✓ (3)
f $7xy^3$✓(A, coefficient) ✓(powers) (2)

5 $-2(1)^5 + 4(-3)^2$ ✓
$= -2$✓ $+ 4 \times 9$✓ $= 34$✓ (4)

6 **a** $3x = 39$✓; $x = 13$✓ (2)
b $x = \frac{1}{10}$✓ (1)
c $19x = 19$✓; $x = 1$✓ (2)
d $2x + 7 = -7x - 24$✓✓
$9x = -18$✓; $x = -2$✓ (4)
e $30 - 3x = -15$✓; $-3x = -45$✓; $x = 15$✓ (3)
f $x \in \mathbb{R}$✓ (1)
g $25 + 10 - 5x$✓ $= 8 - 32x$✓
$35 - 5x = 8 - 32x$✓; $27x = -27$✓; $x = -1$✓ (5)

7 NOW: Lauren = x; Tara = $5x$
IN 7 YEARS' TIME: Lauren = $x + 7$
Tara = $5x + 7$; $5x + 7 = 3(x + 7)$✓
$5x + 7 = 3x + 21$✓
$2x = 14$✓; $x = 7$
Lauren is now 7 years old✓ (4)

Section B: Geometry

Time: 50 minutes **Marks: 65**

1 $a = 55°$ (alt ∠s; AB ∥ CD)
$a + b = 90°$ (right angle)
$\therefore\ b = 35°$
$c = b$ (QD = QR)
$\therefore\ c = 35°$
$b + c + d = 180°$ (int ∠s of △QED)
$35° + 35° + d = 180°$
$\therefore\ d = 110°$
$e = d$ (vert opp ∠s)
$\therefore\ e = 110°$
$f = d$ (corresp ∠s; CD ∥ EK)
$\therefore\ f = 110°$
(2 marks each: 1 mark for answer; 1 mark for reason)
(12)

2 **a** $\hat{C}_2 = x + y$ (ext ∠ of △ABC = sum of opp int ∠s)
(2 marks: 1 mark for answer; 1 mark for reason)
b $x = 10°$ (1 mark for answer)
c $y = 100°$ (1 mark for answer)
d $\hat{C}_2 = x + y$
$= 10° + 100°$
$= 110°$
$\therefore\ \hat{C}_2$ is an obtuse angle (2 marks: 1 mark for 110° ; 1 mark for "obtuse angle")
(6)

3 **a** $p = 16°$ (vert opp ∠s) (1 mark for answer; 1 mark for reason)
$3q = 90°$ (vert opp ∠s)
$\therefore\ q = 30°$ (1 mark for answer; 1 mark for reason)
$90° + 16° + r = 180°$ (straight line)
$\therefore\ r = 74°$ (1 mark for answer; 1 mark for reason)
(6)

b $2p = 60°$ (AB = BC = CA)
$\therefore\ p = 30°$ (1 mark for answer; 1 mark for reason)
$q = 7$ cm (AB = AC = CA) (1 mark for answer; 1 mark for reason)
$A\hat{C}D = \hat{A} + \hat{B}$ (Ext ∠ of △ABC = sum of opp int ∠s)
$\frac{r}{3} = 60° + 60° = 120°$
$\therefore\ r = 360°$ (2 marks for answer; 1 mark for reason)
(7)

Review Copy

4 $2x + 10° = 56°$ (corresp ∠s; QP ∥ RT)
$2x = 46°$
$\therefore\ x = 23°$ (2 marks for first line: statement and reason; 2 marks for solving equation)

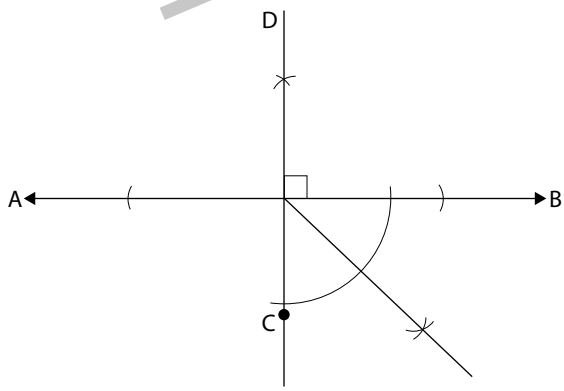
$3y - 20° = y + 40°$ (alt ∠s; QP ∥ RT)
$2y = 60°$
$\therefore\ y = 30°$ (2 marks for first line: statement and reason; 2 marks for solving equation)
(8)

5 **a** △ADC = 80° (opposite angles of parallelogram are equal) (1 mark for answer; 1 mark for reason)
b False: BC = 4,5 cm (1 mark for answer; 1 mark for reason)
c △ACD = △BAC (alt ∠s; BA ∥ CD) (1 mark for reason)
d acute-angled triangle (1 mark for answer)
e scalene triangle (1 mark for reason)
(7)

6 **a** △GJH ≡ △YWX (S∠S)
△KLM ≡ △UVT (∠∠S)
△NOP ≡ △SRQ (RHS)
b △ABC ··· △IZA (AAA) (2 marks each: one mark for △s in correct order; 1 mark for reason)
(8)

7 $\frac{PQ}{KL} = \frac{9}{15} = \frac{3}{5}$
$\frac{QR}{LM} = \frac{24}{x} = \frac{3}{5}$
$\therefore\ x = 40$ units (2 marks for two ratios; 1 mark for answer)
$\frac{PQ}{KL} = \frac{9}{15} = \frac{3}{5}$
$\frac{PR}{KM} = \frac{y}{30} = \frac{3}{5}$
$\therefore\ y = 18$ units (2 marks for two ratios; 1 mark for answer)
(6)

8 **a** Draw a perpendicular line from C to AB.
b Bisect one of the right angles.



(3 marks for perpendicular line; 2 marks for bisecting the angle)
(5)

Review Copy

Exam exemplar 1 (June): Additional

Time: 75 minutes **Marks: 75**

Question 1

1.1 Is $(-15) - 8 = 8 - (-15)$? Show calculations to prove your answer. (2)

1.2 True or false: $\sqrt{-100} = -10$? (1)

1.3 Which one is larger: 25 148 287 mm or 25 157 200 mm? (1)

1.4 Write the additive inverse of 250. (1)

1.5 Give the multiplicative inverse of $-\frac{1}{4}$. (1)

1.6 Rewrite the following numbers in ascending order:
$-35; -62; -8; 90; -21; -59; -33; -45$. (2)

1.7 Complete the following:
$(-20 + -1) + 5 = -20 + (-1 + \square) = \square + \square = \square$ (2)

[10]

Question 2

2.1 Find the LCM of 120 and 150. (2)

2.2 Write 448 as a product of its prime factors. (2)

2.3 Calculate.

2.3.1 $300 - 100 \div (-20) \times 2$ (3)

2.3.2 -9^2 (1)

2.4 Decrease R1 245 in the ratio 3 : 5. (3)

2.5 Sive wants to buy a double bed. The cash price of the bed is R5 700. She has to pay a 18% deposit and 24 monthly instalments of R389,50 each. How much will she pay for the bed in total? (3)

[14]

Review Copy

Question 3

3.1 Calculate the following without using a calculator. Show all the steps.

3.1.1 $9^2 - (-4)^2$ (3)

3.1.2 $\left(\sqrt[3]{37^3} + (-3)^3\right) \times \sqrt{16}$ (5)

3.2 Calculate without using a calculator.

3.2.1 $22x^2y \times -2x^5 \times 10y^3$ (2)

3.2.2 $\frac{-28x^2y^5}{-16y^{15}x^{15}}$ (2)

3.2.3 $4(-n^{12})^2$ (2)

3.3 Write 750 000 000 000 000 000 000 kg in scientific notation.
Round off to two decimal places. (1)

[15]

Question 4

4.1 Provide the rule (in words) to describe the relationship between the numbers in the sequence and then write the next four terms in the number sequence: 1; −2; 4; −8; ... (3)

4.2 Tatum makes bracelets. She has fixed costs of R50 per month and it costs her R10 to make one bracelet. The table below shows her total cost for making a certain amount of bracelets.

Number of bracelets (n)	1	2	3	200	x
Cost in rand (C)	60	70	80	?	780

4.2.1 Write out the pattern for the first two terms. (2)

4.2.2 Write the general rule algebraically. (1)

4.2.3 Find the 200th term. (1)

4.2.4 Draw a flow diagram to show how you will find the number of bracelets made if her total cost for the month is R780. (3)

[10]

Question 5

5.1 Write the following sentence in algebraic language: 13 increased by twice a number is 30 (1)

5.2 Give the coefficient of x in $-7x^3 + 5x + x^2 - 11$. (1)

5.3 Write the expression in the correct order.
$2 + m^2 + m^4 + 12m^3 - 3m$ (1)

5.4 Simplify where $x \neq 0$ and $y \neq 0$.

5.4.1 $\sqrt[3]{-64x^9y^{12}}$ (2)

5.4.2 $\dfrac{18x^{10}y^5 - 12x^5y^2 + 6x^3y}{6x^3y}$ (3)

5.5 Calculate the sum of $5x^2 + x - 6$; $3x^2 - 5x + 10$ and $x^2 - 9$. (3)

5.6 Solve for the variables. (None of the variables is equal to zero.)

5.6.1 $850 = 50p$ (1)

5.6.2 $k^2 + 12 = 61$ (2)

5.6.3 $20x - 13 = 35 - 4x$ (2)

5.6.4 $81 = 6(y - 1) + 15$ (3)

5.7 Calculate the value of $\sqrt{x^2 + 4y}$ if $x = -3$ and $y = \frac{7}{4}$. (4)

5.8 Nicolene is twice as old as Michael. Ten years ago Nicolene was 4 times as old as Michael. How old is Michael now? (3)

[26]

Review Copy

Exam exemplar 1 (June): Additional Memorandum

Question 1

1.1 No✓; $(-15) - 8 = -23$; $8 - (-15) = 23$✓(both) (2)
1.2 False✓ (1)
1.3 25 157 200 mm✓ (1)
1.4 -250✓ (1)
1.5 -4✓ (1)
1.6 -62; -59; -45; -35; -21; -8; 90✓✓(-1 per mistake) (2)
1.7 $(-20 + -1) + 5 = -20 + (-1 + 5) = -20 + 4$✓ $= -16$✓ (2)
[10]

Question 2

2.1 $M_{120} = \{120; 240; 360; 480; 600; \ldots\}$
$M_{150} = \{150; 300; 450; 600; \ldots\}$✓ (both multiples written out)
LCM = 600✓ (2)

2.2

2	448 ✓
2	224
2	112
2	56
2	28
2	14
7	7
	1

$448 = 2^6 \times 7$✓ (or $448 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 7$) (2)
2.3.1 $300 + 5 \times 2$✓ $= 300 + 10$✓ $= 310$✓ (3)
2.3.2 -81✓ (1)
2.4 $3 \div 5$✓ $\times 1\,245$✓ $=$ R747✓ (3)
2.5 $(0{,}18 \times 5\,700)$✓ $+ (24 \times \text{R}389{,}50)$✓ $=$ R10 374✓ (3)
[14]

Question 3

3.1.1 81✓ $- 16$✓ $= 65$✓ (3)
3.1.2 $(37$✓ $+ (-27)$✓$)4$✓$= 10 \times 4$✓ $= 40$✓ (5)
3.2.1 -440✓x^7y^4✓ (2)
3.2.2 $\frac{7}{4x^{13}y^{10}}$✓(coefficients)✓(variables) (2)
3.2.3 $4n^{24}$✓✓ (2)
3.3 $7{,}50 \times 10^{20}$✓ (1)
[15]

Question 4

4.1 I have to multiply each number by -2 to get the next number in the number sequence.✓
16; -32✓; 64; -128✓ (3)
4.2.1 $T_1 = 10(1) + 50 = 60$✓; $T_2 = 10(2) + 50 = 70$✓ (2)
4.2.2 $T_n = 10(n) + 50$✓ (1)

Review Copy

4.2.3 $T_{200} = 10(200) + 50 = 2\,050$✓ (1)

4.2.4 $\boxed{780} \rightarrow \boxed{-50} \rightarrow \boxed{\div 10} \rightarrow \boxed{73}$ (3)

[10]

Question 5

5.1 $13 + 2x = 30$✓ (1)

5.2 5✓ (1)

5.3 $m^4 + 12m^3 + m^2 - 3m + 2$✓ (1)

5.4.1 -4✓x^3y^4✓ (2)

5.4.2 $3x^7y^4$✓ $- 2x^2y$✓ $+ 1$✓ (3)

5.5 $9x^2$✓ $- 4x$✓ $- 5$✓ (3)

5.6.1 $p = 17$✓ (1)

5.6.2 $k^2 = 49$✓; $k = 7$✓ (2)

5.6.3 $24x = 48$✓; $x = 2$✓ (2)

5.6.4 $81 = 6y - 6$✓ $+ 15$; $6y = 72$✓; $y = 12$✓ (3)

5.7 $\sqrt{(-3)^2 + 4 \times \frac{7}{4}}$✓ $= \sqrt{9 + 7}$✓ $= \sqrt{16}$✓ $= 4$✓ (4)

5.8 Now: Michael = x years old; Nicolene = $2x$ years old
10 years ago: Michael = $x - 10$; Nicolene = $2x - 10$
$2x - 10 = 4(x - 10)$ ✓
$2x - 10 = 4x - 40$✓(any correct step)
$30 = 2x$
$x = 15$
Michael is now 15 years old✓ (3)

[26]

Review Copy

Exam exemplar 2 (June): Additional

Time: 75 minutes **Marks: 75**

Question 1

1.1 Choose the correct letter (A, B, C or D) in each case.

a $-x$ is a …

A constant **B** variable
C constant and a variable **D** none of the above (1)

b The correct expression according to mathematical conventions for $m \times 5 \times n$ is:

A mn^5 **B** $5nm$
C $5(mn)$ **D** $5mn$ (1)

c What do we call this expression: $18a + 12a \div 4 + a \times 2a^7$?

A monomial **B** binomial
C trinomial **D** polynomial (1)

d The $\sqrt[3]{-1}$ is …

A undefined **B** -1
C 1 **D** 1^3 (1)

e Which estimate is the best for $10\,775 \div 25$?

A $10\,000 \div 50 = 200$ **B** $20\,000 \div 50 = 400$
C $5\,000 \div 50 = 100$ **D** $6\,000 \div 50 = 120$ (1)

f The answer for $300 - 15 \times 4 + 1$ is:

A 290 **B** 225 **C** 241 **D** 236 (1)

1.2 Say in each case whether the statement is true or false.

a $3x^3$ is the unlike term in: $3x^2$; $3x^3$; $2x^2$.

b If $x = 5$, then both sides are equivalent in $5x - 3 = -(3x + 7)$.

c In $-5(mn)^3$ the base is $-5mn$.

d $x \in \mathbb{R}$ in $5x - 12 = 2x - 12 + 3x$. (4)

[10]

Question 2

2.1 Find the sum of 4 567,872 and 397,285. (2)

2.2 How many even multiples does 300 have? (1)

2.3 How many even factors does 300 have? (3)

2.4 Write 300 as a product of its prime factors. (2)

2.5 Find the LCM of 25 and 45. (2)

2.6 Jack has 400 songs on his MP3 player, Zola has 1 200 on hers and Neo 800 songs on his MP3 player. Write the number of songs each person has as a ratio and then simplify it. (2)

2.7 Study the following loan table.

Loan amount	**Repayment: 24 months**	**Repayment: 36 months**	**Repayment: 48 months**
R2 500	R160	R140	R120
R5 000	R230	R215	R200

Derrick takes out a loan for R2 500 and repays it over 36 months. How much money must he repay in total over this period? (2)

2.8 Parker visited London for two weeks. Upon his return he needed to exchange £956,75 to rand. How much money (in rand) does he get? Use the exchange rate of £1 = R12,9772. (2)

[16]

Review Copy

Question 3

3.1 Calculate:

a $-5 - 2 + (-1)$ (2)

b $130 + (3 - 6 \times -5)2$ (4)

c $5\ 587 \div 1\ 000$ (1)

d $(-7)^2 \div \sqrt{49^2}$ (3)

3.2 Write the next four numbers in the number sequence 15; 9; 3; … (2)

3.3 Calculate 89,85% of €3 277,29. Round off your answer to two decimal places. (2)

[14]

Question 4

4 Study the table and then answer the questions that follow.

x	1	2	3	4	10	f
y	18	17	16	15	e	–10

4.1 Find the rule and write the formula for finding y. (2)

4.2 Find the unknown output value e. (2)

4.3 Find the unknown input value f. (3)

[7]

Question 5

5.1 Expand $(3x)^2 \times (2y)^3$. (2)

5.2 Simplify.

- **a** $c^{12} \times c^5$ (1)
- **b** $6(-777a)^0$ (1)
- **c** $(5p - q)10$ (2)
- **d** $(7q^2 - 10q - 4) - (q^2 + 6q - 4)$ (3)
- **e** $\dfrac{30x^7y^3 + 2xy}{2xy}$ (2)
- **f** $-4pq^2(2q^3p^6)^2$ (3)

5.3 Write each number in normal notation and then determine which number is larger: $3{,}1 \times 10^6$ or 2×10^7. (2)

[16]

Question 6

6.1 Calculate the value of $p^2 + 6pq$ if $p = -5$ and $q = 2$. (3)

6.2 Solve for x in each case.

- **a** $x + 8 = 15$ (1)
- **b** $21 - x = 19x - 19$ (2)
- **c** $5x^3 + 12 = 4(x^3 + 5)$ (3)

6.3 How many pens does Zukisa have if Emily has $8m - 3$ pens, Jonathan has $5m + 12$ pens and they have $35m + 19$ pens altogether? (3)

[12]

Review Copy

Exam exemplar 2 (June): Additional Memorandum

Question 1

1.1 a C✓ b D✓ c C✓ d B✓ e B✓ f C✓ (6)

1.2 a T✓ b F✓ c F✓ d T✓ (4)

[10]

Question 2

2.1 "long addition" ✓; 4 965,157✓ (2)

2.2 an infinite amount✓ (1)

2.3 F_{300} = {1; 2; 3; 4; 5; 6; 10; 12; 15; 20; 25; 30; 50; 60; 75; 100; 150; 300}✓✓; 12 even factors✓ (3)

2.4

2	300
2	150
3	75
5	25
5	5
	1

$300 = 2^2 \times 3 \times 5^2$✓ (or $300 = 2 \times 2 \times 3 \times 5 \times 5$)

2.5 $M_{25} = \{25; 50; 75; 100; \ldots; 200; 225\}$
$M_{45} = \{45; 90; 135; 180; 225\}$✓
LCM = 225✓ (2)

2.6 400 : 1 200 : 800✓ = 1 : 3 : 2✓ (2)

2.7 R140 × 36✓ = R5 040✓ (2)

2.8 956,25 × 12,9772✓ ≈ R12 415,94✓ (2)

[16]

Question 3

3.1 **a** −7 + −1✓ = −8✓ (2)

b 130 + (3 + 30✓)2 = 130 + (33✓)2 = 130 + 66✓= 196✓ (4)

c 5,587✓ (1)

d 49✓ ÷ 49✓ = 1✓ (3)

3.2 −3; −9✓; −15; −21✓ (2)

3.3 0,8985 × 3 277,29✓ ≈ €2 944,65✓ (2)

[14]

Question 4

4.1 $y = -x + 19$✓✓ (2)

4.2 $e = -(10) + 19$✓ = 9✓ (2)

4.3 (−10 −19) = −29✓; −29 ÷ −1✓ = 29✓ (3)

[7]

Question 5

5.1 $3 \cdot 3 \cdot x \cdot x \cdot 2 \cdot 2 \cdot 2 \cdot y \cdot y \cdot y$✓ $= 72 \cdot x \cdot x \cdot y \cdot y \cdot y$✓ (1)

5.2 **a** c^{17}✓ (1)

b 6✓ (1)

c $50p$✓ − $10q$✓ (2)

d $7q^2 - 10q - 4 - q^2 - 6q + 4$✓ $= 6q^2$✓ $- 16q$✓ (3)

e $15x^6y^2$ ✓ + 1✓ (2)

f $-4pq^2 \times 4q^6p^{12}$✓ $= -16$✓$p^{13}q^8$✓ (3)

5.3 3 100 000 and 20 000 000✓; 3 100 000 < 20 000 000✓
(or $3{,}1 \times 10^6 < 2 \times 10^7$)

[16]

Question 6

6.1 $(-5)^2 + 6(-5)(2)$✓ = 25 + −60✓ = −35✓ (3)

6.2 **a** $x = 7$✓ (1)

b $21 + 19 = 19x + x$✓; $40 = 20x$✓; $x = 2$✓ (3)

c $5x^3 + 12 = 4x^3 + 20$✓; $x^3 = 8$✓; $x = 2$✓ (3)

6.3 $8m - 3 + 5m + 12 = 13m + 9$✓
$35m + 19 - (13m + 9)$ ✓ $= 35m + 19 - 13m - 9 = 22m + 10$✓ (3)

[12]

Review Copy

CHAPTER

11 Revision: Common fractions

Chapter overview

Learner's Book pages 229 to 248
Recommended pacing: 7 hours

This chapter focuses on the following:

Unit 1:	**Revision of common fractions**	45 minutes
	What are fractions?	
	Simplify fractions	
Unit 2:	**Adding and subtracting common fractions**	1 hour
	Converting fractions	
	Adding common fractions	
	Subtracting common fractions	
Unit 3:	**Multiplying and dividing common fractions**	3 hours
	Multiplying common fractions	
	Problem-solving	
	Dividing whole numbers by common fractions	
	Dividing common fractions	
	Squares and cubes of common fractions	
Unit 4:	**Percentages**	2 hours 15 minutes
	Percentages of whole numbers	
	Problem-solving	
	Percentages of parts of a whole	
	Percentage increase and decrease	
Chapter 11 Revision		

Review Copy

UNIT

1 Revision of common fractions

Unit overview

Learner's Book page 230
Recommended pacing: 45 minutes

This unit focuses on the following:

- defining numerators and denominators
- defining and identifying types of common fractions that are proper fractions, improper fractions and mixed numbers
- simplifying fractions by dividing by the Highest Common Factor

Resources: Learner's Book, exercise book; calculator (for checking answers)

Mental Maths

(10 minutes per day)

Suggested answers

Learner's Book page 230

1

	i	ii
a	15	16
b	18	7
c	25	30
d	87	120

2 a proper
b improper
c mixed number
d improper

3 a $\frac{1}{2}$ b 6 c $\frac{2}{5}$ d $\frac{9}{5}$

Review Copy

Background information

Learners have worked extensively with fractions in previous grades, where they did the following.

- the meaning of fractions
- comparing and ordering fractions; in Grade 7, comparing and ordering tenths and hundredths and extending these to thousandths were a specific focus
- simplifying fractions using knowledge of multiples and factors.

Teaching guidelines

In Grade 8 learners consolidate their basic understanding of the meaning of fractions, as well as comparing and ordering fractions, developed in Grade 7. The content in this unit is therefore revision, and serves to consolidate the basic skills and concepts learnt in previous grades. The revision can be fairly quick and if learners are struggling with any of the introductory concepts, allow them some time to do a few additional examples and assign some for homework. A good understanding of fractions will be necessary

when looking at algebraic fractions later, and especially when learners are required to do algebraic fractions in Grade 9. Work through the summary with the class as a form of revision, and encourage learners to refer to this summary when they need to.

What are fractions?

Activities 1–2 Identify fractions; Simplify fractions

Learner's Book pages 230–231

Guidelines for implementing these activities

- Revise numerators, denominators and common fractions (proper, improper and mixed numbers).
- Remind learners that they can simplify fractions by dividing the numerator and denominator by their HCF (the largest number that be divided into both the numerator and the denominator).
- Learners can do both activities in one lesson or for homework.

Remedial and extension

Remedial: Let learners work in pairs to find the HCF before they simplify fractions in Activity 2.
Extension: Activity 2 questions **16** to **20** are more challenging.

Review Copy

Suggested answers

Activity 1

1

	i	ii
a	1	2
b	14	13
c	8	15
d	3	10
e	17	20
f	98	65
g	203	300
h	99	100
i	315	250
j	713	625

2
- a proper
- b improper
- c mixed number
- d improper
- e proper
- f improper
- g mixed number
- h mixed number
- i mixed number
- j improper
- k proper
- l mixed number
- m improper
- n proper
- o proper

Activity 2

1 $\frac{1}{2}$	2 $\frac{1}{3}$	3 5	4 2
5 4	6 $\frac{4}{5}$	7 $\frac{5}{2}$ or $2\frac{1}{2}$	8 $\frac{22}{5}$ or $4\frac{2}{5}$
9 $\frac{25}{3}$ or $8\frac{1}{3}$	10 $\frac{7}{19}$	11 $\frac{3}{4}$	12 $\frac{11}{9}$ or $1\frac{2}{9}$
13 $\frac{3}{17}$	14 $\frac{5}{16}$	15 $\frac{78}{125}$	*16 50
*17 $\frac{1}{6}$	*18 $\frac{123}{200}$	*19 $\frac{24}{5}$ or $4\frac{4}{5}$	*20 $\frac{39}{10}$ or $3\frac{9}{10}$

UNIT 2

Adding and subtracting common fractions

Unit overview

Learner's Book page 232
Recommended pacing: 1 hour

This unit focuses on the following:

- converting mixed numbers to improper fractions
- converting improper fractions to mixed numbers
- finding the Lowest Common Denominator (LCD)
- adding and subtracting common fractions
- solving problems involving adding and subtracting common fractions

Resources: Learner's Book, exercise book; calculator (for checking answers)

Review Copy

Mental Maths

(10 minutes per day)

Suggested answers

Learner's Book page 232

1 a $\frac{49}{15}$ b $\frac{153}{10}$ c $\frac{2\,524}{25}$

2 a $2\frac{4}{7}$ b $9\frac{1}{5}$ c $6\frac{1}{30}$

3 a 24 b 105 c 200

4

	LCD	Answer (improper)	Answer (mixed)
a	24	$\frac{1}{2}$	
b	60	$\frac{43}{60}$	
c	80	$\frac{437}{80}$	$5\frac{37}{80}$

Background information

Learners have worked extensively with concepts covered in this unit in previous grades, where they did the following.

- converting mixed numbers to common fractions (and vice versa) in order to perform calculations with them
- calculations with fractions (addition and subtraction of fractions with the same denominators and denominators that are multiples of each other, as well as denominators that are not multiples of each other and addition and subtraction of mixed numbers)
- calculations with common fractions (addition and subtraction using knowledge of equivalent fractions).

Teaching guidelines

In Grade 8 learners consolidate their basic number knowledge and competency with calculation techniques, developed in Grade 7. The content in this unit is therefore revision and can be done fairly quickly. Many of the questions in the activities can be done as homework after learners have completed two or three questions in class. Learners can mark one another's work, or they can mark their own work against the answers posted up on the classroom walls. Moderate their work by doing quick checks of their overall progress, and signing it. Remember, a good understanding of fractions is an important precursor to working with algebraic fractions in Grade 9. It is therefore very important to ensure that learners have consolidated their understanding of fractions as a building block to algebraic fractions. The Think-Do examples crystallise what is happening at each step. This will help make it easier when learners have to transfer their numeric knowledge of fractions to Algebra in Grade 9.

Work through the summary with the class as a form of revision, and encourage learners to refer to this summary when they need to.

Review Copy

Converting fractions

Converting mixed numbers to improper fractions; Converting improper fractions to mixed numbers

Activity 1 **Convert fractions**

Learner's Book page 233

Guidelines for implementing this activity

- Do examples with the learners on the board. Learners understand better when they can see how they are done step-by-step. Learners can use their calculators to calculate larger numbers.
- Learners can also use the fraction buttons on their calculators afterwards to convert fractions and see whether they have done the calculations correctly or not.
- Ensure that learners write the mixed number correctly, for example when converting $\frac{29}{4}$ ($29 \div 4 = 7$ remainder 1), the number 7 is the whole number and the number 1 is the numerator of the fraction part of the mixed number.

Remedial and extension

Remedial: Ensure that learners work methodically and that they follow all the steps. They should re-do the questions until they get the correct answers, paying close attention to where they have gone wrong. When monitoring them, ask them to verbalise what they did wrong and how they should correct it.
Extension: Learners can also work in pairs and set up more challenging common fractions/mixed numbers for one another to convert. They can check their answers by using the fraction buttons on their calculators.

Suggested answers

1	a $\frac{7}{4}$	b $\frac{31}{12}$	c $\frac{29}{8}$	d $\frac{128}{25}$	e $\frac{153}{20}$	f $\frac{31}{2}$
	g $\frac{81}{4}$	h $\frac{504}{5}$	i $\frac{397}{100}$			
2	a $3\frac{1}{4}$	b $2\frac{7}{10}$	c $6\frac{3}{7}$	d $3\frac{23}{25}$	e $2\frac{1}{2}$	f $4\frac{2}{5}$
	g $12\frac{1}{2}$	h $2\frac{22}{25}$	i $3\frac{9}{10}$			

Adding common fractions

Activity 2 Find the LCD and add common fractions

Learner's Book page 234

Review Copy

Guidelines for implementing this activity

- Revise the LCD and how to find it.
  - Learners don't always have to write out the multiples of all the numbers to get the LCD.
  - A quicker method is for learners to write out the multiples of the larger number, choose the multiple that they consider to be the common multiple, and do the division by the denominator on their calculators. If they can do both steps mentally, it shows a good grasp of how the process works, and that they know their multiplication tables.
- Explain the steps for adding common fractions. Learners can also look back at the steps and examples while doing the activity.
- Emphasise that we convert fractions to equivalent fractions first before adding them (in order to make the denominators the same). This is therefore also revision of the meaning and use of equivalent fractions.
- Emphasise that only the numerators are added and that the denominator stays the same.
- An improper fraction and a mixed number are both in simplified form. Therefore both answers are correct and both are given in the suggested answers on the next page.

Remedial and extension

Remedial: Let learners work in pairs and/or re-do calculations with the help of a peer/teacher until they can answer them correctly on their own. When monitoring them, ask them to verbalise what they have done wrong and how they should correct it.
Extension: Questions **1e** to **1f** and **2i** to **2l** are more challenging.

Suggested answers

1 **a** 10 **b** 24 **c** 20 **d** 30 ***e** 200 ***f** 420

2

	LCD	Answer (improper)	Answer (mixed)
a	6	$\frac{5}{6}$	
b	15	$\frac{19}{15}$	$1\frac{4}{15}$
c	48	$\frac{31}{48}$	
d	105	$\frac{79}{35}$	$2\frac{9}{35}$
e	60	$\frac{457}{60}$	$7\frac{37}{60}$
f	24	$\frac{371}{24}$	$15\frac{11}{24}$
g	24	$\frac{101}{12}$	$8\frac{5}{12}$
h	60	$\frac{701}{60}$	$11\frac{41}{60}$
*i	12	$\frac{1}{2}$	
*j	60	$\frac{41}{20}$	$2\frac{1}{20}$
*k	40	$\frac{291}{40}$	$7\frac{11}{40}$
*l	60	$\frac{229}{15}$	$15\frac{4}{15}$

Review Copy

Subtracting common fractions

Activities 3–5 **Subtract common fractions; Add and subtract common fractions; Solve problems**

Learner's Book pages 235–236

Guidelines for implementing these activities

- Go through the worked example in the learner's Book, ensuring that learners are comfortable finding the LCD. Encourage them to try to find the LCD mentally as described in the previous activity.

- Emphasise that we convert fractions to equivalent fractions first before subtracting them (in order to make the denominators the same). Again, this is revision of the meaning and use of equivalent fractions.
- Learners can get negative answers. However, this is acceptable since they have already covered integers.
- Activities 3, 4 and 5 should be spread over two lessons.

Remedial and extension

Remedial: Let learners work in pairs and/or re-do the calculations with the help of a peer/teacher until they can answer them correctly on their own. Allow these learners to practise only the easier questions at first, and encourage them to do the more challenging questions as they build confidence. Ensure that they do all the questions, however, and also attempt the challenging ones, even if they do these for homework.
Extension: Activity 3 questions **9** to **12**, Activity 4 questions **9** to **12** and Activity 5 are more challenging.

Suggested answers

Activity 3

	LCD	Answer (improper)	Answer (mixed)
1	4	$\frac{1}{4}$	
2	15	$\frac{1}{5}$	
3	60	$-\frac{7}{30}$	
4	84	$-\frac{11}{42}$	
5	140	$\frac{64}{35}$	$1\frac{29}{35}$
6	24	$\frac{10}{3}$	$3\frac{1}{3}$
7	60	$\frac{25}{3}$	$8\frac{1}{3}$
8	60	$\frac{53}{10}$	$5\frac{3}{10}$
*9	144	$-\frac{11}{48}$	
*10	36	$\frac{25}{36}$	
*11	40	$\frac{67}{60}$	$1\frac{7}{60}$
*12	150	$\frac{209}{150}$	$1\frac{59}{150}$

Review Copy

Activity 4

	LCD	Answer (improper)	Answer (mixed)
1	18	$\frac{16}{9}$	$1\frac{7}{9}$
2	21	$\frac{2}{7}$	
3	35	$\frac{33}{35}$	
4	60	$\frac{13}{60}$	
5	20	$\frac{35}{4}$	$8\frac{3}{4}$
6	32	$\frac{93}{32}$	$2\frac{29}{32}$
7	60	$\frac{692}{60}$	$10\frac{29}{60}$
8	24	$\frac{95}{24}$	$3\frac{23}{24}$
*9	30	$\frac{13}{10}$	$1\frac{3}{10}$
*10	120	$\frac{17}{30}$	
*11	40	$\frac{191}{40}$	$4\frac{31}{40}$
*12	300	$\frac{1\,781}{300}$	$5\frac{281}{300}$

Review Copy

Activity 5

1 $\frac{3}{5} + \frac{1}{8} = \frac{24}{40} + \frac{5}{40} = \frac{29}{40}$; $\frac{40}{40} - \frac{29}{40} = \frac{11}{40}$

2 a $\frac{\text{R}25}{\text{R}80} = \frac{5}{16}$

b $\frac{16}{16} - \frac{5}{16} = \frac{11}{16}$

3 a $8\frac{1}{4} + 7\frac{13}{20} + 8\frac{93}{100} + 5\frac{2}{5} + 5 + 4\frac{1}{25}$; LCD = 100; Answer = $\frac{3\,927}{100}$ or $39\frac{27}{100}$

b 39 hours

c 34 hours × R20/h = R780

UNIT 3

Multiplying and dividing common fractions

Unit overview

Learner's Book page 237
Recommended pacing: 3 hours

This unit focuses on the following:

- multiplying and dividing common fractions, including dividing with whole numbers
- solving problems involving multiplying and dividing common fractions
- finding square and cube numbers of common fractions
- finding square and cube roots of common fractions

Resources: Learner's Book, exercise book; calculator (for checking answers)

Mental Maths

(10 minutes per day)

Suggested answers — Learner's Book page 237

1 36	**2** $\frac{3}{2}$	**3** $-\frac{32}{15}$	**4** $\frac{28}{3}$
5 48	**6** 100	**7** $\frac{9}{4}$ or $2\frac{1}{4}$	**8** $-\frac{2}{45}$
9 $-\frac{71}{27}$ or $-2\frac{17}{27}$	**10** $\frac{9}{4}$ or $2\frac{1}{4}$	**11** $-\frac{27}{100}$	**12** $-\frac{3}{2}$ or $-1\frac{1}{2}$

Review Copy

Background information

Learners have worked extensively with some concepts covered in this unit in previous grades, where they did the following:

- Multiplication of common fractions and mixed numbers (including multiplication of fractions where one denominator is not a multiple of the other)
- Finding fractions of whole numbers

Note: Learners divide by common fractions for the first time in Grade 8.

Teaching guidelines

The content in this unit is revision, except for division of fractions. Multiplication of fractions can therefore be done fairly quickly.

A good understanding of fractions is an important precursor to working with algebraic fractions in Grade 9. It is very important to ensure that learners have consolidated their understanding of fractions as a building block to algebraic fractions.

Once multiplication of fractions has been revised, remind learners that division by the fraction $\frac{a}{b}$ is the same as multiplying by the reciprocal, $\frac{b}{a}$.

Work through the summary with the class as a form of revision, and encourage learners to refer to this summary when they need to.

Multiplying common fractions

Activities 1–2 **Multiply common fractions; Solve multiplication problems**

Learner's Book pages 237–238

Guidelines for implementing these activities

- There are two methods that learners can use:
  - Method 1: As described in Learner's Book. Learners first simplify by dividing numerators and denominators by their HCFs. Then they multiply the numerators and denominators.
  - Method 2: Learners can first multiply the numerators and denominators, then simplify. However, it isn't always easy to multiply large numbers without a calculator since numbers that they will need to simplify will be very large. They should therefore be encouraged to choose the first method as described in the Learner's Book unless numerators and/or denominators do not have common factors, and therefore have to follow Method 2.
- Highlight the differences between adding/subtracting and multiplying common fractions.
- Do additional examples similar to the questions in the activity, on the board if necessary.
- Learners can complete Activities 1 and 2 in the same lesson. Activity 1 contains context-free calculations, while Activity 2 contains context-based calculations.

Review Copy

Remedial and extension

Remedial: Learners need to work systematically to find HCFs in order to simplify all the fractions where possible. Assist them to work their way to "seeing the HCF", which requires practice and guidance. While learners are busy, ask key questions such as "What is the highest factor that will divide into both numbers?" Use the word *factor* deliberately and continually to ensure that knowledge of what a *factor* is also consolidated. Learners should also recognise when numbers do not have common factors, and know how to multiply when they don't.
Extension: Activity 1 questions **9** to **12** and Activity 2 are more challenging.

Suggested answers

Activity 1

1 24	**2** 5	**3** 100	**4** $-\frac{1}{2}$
5 $-\frac{2}{3}$	**6** $\frac{3}{10}$	**7** $\frac{7}{5}$ or $1\frac{2}{5}$	**8** 60
***9** $\frac{3}{2}$	***10** -75	***11** $\frac{44\,100}{37}$	***12** $\frac{7\,953}{16\,384}$

Activity 2

1 $\frac{5}{8} \times 24 = 15$ green balls

2 $\frac{1}{4} \times 16 = 4$ chess pieces

3 $\frac{3}{4} \times 20 = 15$ apples

4 $18\frac{1}{3} \div 3\frac{3}{4} = 4\frac{8}{9}$; Chakalaka $= 18\frac{1}{3} - 4\frac{8}{9} = 13\frac{4}{9} \therefore$ 13 full bottles

5 $3\frac{3}{7} \times 1\frac{11}{12} = \frac{24}{7} \times \frac{23}{12} = \frac{46}{7} = 6\frac{4}{7}$ bags of potting soil

Dividing whole numbers by common fractions; Dividing common fractions

Activities 3–4 **Divide whole numbers by common fractions; Divide common fractions**

Learner's Book page 239

Guidelines for implementing these activities

- Once multiplication of fractions has been revised, show learners that division by the fraction $\frac{a}{b}$ is the same as multiplying by the reciprocal, $\frac{b}{a}$.
- This changes all division calculations to multiplication and therefore consolidates multiplication. Learners can now do this because they already know how to multiply.
- Work carefully through the example in the Learner's Book.

Review Copy

Remedial and extension

Remedial: Learners should re-do the questions until they get the answers correct, and always be encouraged to identify and verbalise their mistakes in order to reduce the risk of making the same mistakes again and/or to help isolate precisely where additional remediation may be required. For example, if "seeing" the HCF is challenging, then this step in the calculation should be practised with exercises such as: What is the HCF of 6 and 8?; What is the HCF of 12 and 100?; What is the HCF of 28 and 4?; What is the HCF of 3 and 7? Including these types of questions in mental maths activities at the beginning of lesson can also help to master this skill.
Extension: Learners may work in pairs and set up more challenging questions for each other to do.

Suggested answers

Activity 3

1 21	2 26	3 −22	4 −108
5 −12	6 −20	7 14	8 30
9 −100	10 −25	11 20	12 $\frac{40}{3}$ or $13\frac{1}{3}$

Activity 4

1 $\frac{6}{5}$	2 $\frac{1}{2}$	3 $-\frac{1}{48}$	4 $-\frac{2}{9}$
5 $\frac{2}{7}$	6 $\frac{3}{10}$	7 $\frac{3}{7}$	8 $-\frac{1}{30}$
9 $-\frac{1}{36}$	10 $-\frac{15}{16}$	11 $-\frac{10}{11}$	12 $-\frac{42}{23}$ or $-1\frac{19}{23}$

Squares, cubes and common fractions

Finding square and cube numbers of common fractions; Finding square and cube roots of common fractions

Activity 5 Work with squares and cubes of common fractions

Learner's Book page 240

Guidelines for implementing this activity

- It is useful for learners to learn the first 12 perfect squares and cubes and their roots.
- Work through the examples in the Learner's Book, and do a few additional examples on the board if necessary.

Remedial and extension

Remedial: Include simple exercises where learners have to find square and cube numbers of common fractions in mental maths sessions every day. Learners should also re-do questions until they get them correct, and always be encouraged to verbalise their mistakes in order to minimise the risk of repeating them.
Extension: Questions **9** to **12** are more challenging.

Review Copy

Suggested answers

1 $\frac{121}{81}$	2 $\frac{25}{49}$	3 $\frac{1}{64}$	4 $\frac{8}{125}$
5 $\frac{7}{8}$	6 $\frac{12}{5}$	7 $-\frac{2}{5}$	8 $\frac{10}{-1} = -10$
*9 $18 \times \frac{1}{9} = 2$	*10 $-15 \times \frac{1}{27} = -\frac{5}{9}$	*11 $25 \times -\frac{2}{2} = -25$	*12 $-45 \times \frac{2}{5} = -18$

UNIT

4 Percentages

Unit overview

Learner's Book page 241
Recommended pacing: 2 hours 15 minutes

This unit focuses on the following:

- calculating percentages of whole numbers
- finding percentages of part of a whole
- increasing or decreasing a quantity by a percentage
- calculating the percentage increase or decrease
- solving problems involving percentages

Resources: Learner's Book; exercise book; calculator

Mental Maths

(10 minutes per day)

Suggested answers — Learner's Book page 241

1 a 90 guests b 23 children

2 a 192,00 kg b R77,90

3 a $\frac{400\text{ g}}{2\,000\text{ g}} \times 100 = 20\%$ b $\frac{25\text{c}}{500\text{c}} \times 100 = 5\%$

4 107,5% of R18,50 ≈ R19,89

5 40% of 30 ml = 12 ml

6 $\frac{10{,}98 - 10{,}56}{10{,}56} \times 100 \approx 4\%$

7 $\frac{80\,000 - 65\,000}{80\,000} \times 100 \approx 18{,}75\%$

Review Copy

Background information

This unit focuses on revision of the basic techniques and strategies learnt in Grade 7, and some of the strategies learnt in the Financial Mathematics section in the first term, for example finding percentages. Ensure that learners master these basic techniques and strategies since they will need them throughout their lives.

Teaching guidelines

Focus on consolidating the basic skills and techniques, then introduce the use of a calculator once learners can do the basic techniques and strategies manually. Provide many additional, real-life examples throughout as short homework exercises, for example: Calculate 33% of R120; Calculate 25% of R80; What percentage is R40 of R120?; What was the percentage increase if train tickets increased from R100 to R115?; What is the percentage decrease if petrol came down from R12,10 per litre to R11,85 per litre? It is also useful for learners to bring their own examples to class and for the class to work through them. These can include pamphlets showing percentage discounts, news reports of increases and decreases in commodities such as petrol, milk, bread, and so on.

Percentages of whole numbers

Activities 1–2 Find a percentage of a whole number; Solve percentage problems

Learner's Book page 242

Guidelines for implementing these activities

- Work carefully through the examples in the Learner's Book, and do additional examples as described in the teaching guidelines above.
- In the first example (30% of the 95 women at a conference are from Cape Town…), rounding off the answer means we are approximating the answer in order to make it more meaningful (we do not get half a person). This is the difference between estimating an answer and approximating an answer. (We use estimation before we do a calculation in order to judge more or less what the answer will be, and we use approximation in order to make meaning of an answer once we have done the calculation.)
- In Activity 1 question **1** approximation is required, and should be explained to learners in simple terms but if they are confused, explain that we round off to the nearest whole number because we are working with people, books, plants, and so on (things that are whole).
- Remind learners to write down the units in their final answer.
- Allow learners to use calculators once they have mastered the basic skills to check their answers or when working with more complex calculations.
- Questions such as question **1d** also provide practice with multiplying (and dividing) decimal numbers which learners should be encouraged to practise first before using their calculators.

Review Copy

Remedial and extension

Remedial: Learners should re-do questions until they get the answers correct, and should be encouraged to identify and verbalise their mistakes in order to minimise the risk of repeating them. Verbalising also helps to crystallise their understanding.
Extension: Learners who understand the work and complete the activity quickly can help fellow learners who struggle.

Suggested answers

Activity 1

1	**a**	21 people	**b**	94 tables	**c**	482 children	**d**	4 kittens
2	**a**	400,00 kg	**b**	250,00 ml	**c**	675,00 kg	**d**	R66,65

Activity 2

1 50% of R750 = R375
2 18,5% of €975,23 ≈ €180,42
3 9,25% of 8 760 h ≈ 811 h
4 **a** 18,67% of ¥756 ≈ ¥141,15
b ¥756 – ¥141,15 = ¥614,85

Percentages of parts of a whole

Activity 3 Write parts of a whole as percentages

Learner's Book page 243

Guidelines for implementing this activity

- In this activity learners need to write a fraction and multiply it by 100 to get the percentage.
- The general formula is: $\frac{\text{part that is given}}{\text{total that is given}} \times \frac{100}{1}$. Do more examples on the board if necessary, until learners feel confident to attempt the activity on their own.
- Remind learners to write the units in their final answers.
- Learners may use their calculators once they have mastered the basic skills manually.

Remedial and extension

Remedial: Assist learners with the steps and provide additional examples as practice. Assist them with the steps on their calculators as well.
Extension: Questions **2** to **6** are in context and are therefore more challenging.

Review Copy

Suggested answers

1 **a** $\frac{20c}{320c} \times 100 = 6{,}25\%$ **b** $\frac{45\text{ cm}}{200\text{ cm}} \times 100 = 22{,}5\%$ **c** $\frac{240\text{ g}}{1\,000\text{ g}} \times 100 = 24\%$
d $\frac{375\text{ kg}}{1\,500\text{ kg}} \times 100 = 25\%$ **e** $\frac{1\,420\text{ ml}}{3\,000\text{ ml}} \times 100 = 47{,}3\%$

2 **a** $16 \div 20 \times 100 = 80\%$ **b** $100\% - 80\% = 20\%$

3 **a** $12 + 13 = 25$; $12 \div 25 \times 100 = 48\%$ **b** $100\% - 48\% = 52\%$

4 **a** $12 + 23 + 1 = 36$ animals; $12 \div 36 \times 100 = 33{,}3\%$
b $23 \div 36 \times 100 = 63{,}9\%$ **c** $1 \div 36 \times 100 = 2{,}8\%$

5 **a** $4 + 3 + 7 + 2 = 16$ balls of wool
b **i** $4 \div 16 \times 100 = 25\%$ **ii** $7 \div 16 \times 100 = 43{,}8\%$ **iii** $0 \div 16 \times 100 = 0\%$

6 **a** $4 + 5 + 3 + 9 = 21$ pieces of clothing **b** $4 \div 21 \times 100 = 19{,}0\%$
c $9 \div 21 \times 100 = 42{,}9\%$

Percentage increase and decrease

Calculating percentage increase; Percentage decrease; Using a formula to calculate percentage increase and decrease

Activities 4–5 Calculate percentage increase and decrease; Calculate percentage increase and decrease

Learner's Book pages 245–246

Guidelines for implementing these activities

- Work through the introduction in the Learner's Book and discuss real-life contexts where percentage increase are used. Allow learners to suggest their own examples.

- Before working through the example in the Learner's Book, let pairs of learners discuss a specific example such as the following: "The price of a loaf of bread has increased by 8% from R8. How would you go about working this out?" Prompt them with questions such as "What do you know? What does 8% mean? What information will your answer have?" Let them write down the facts that they know, for example: The cost of the bread was R8. And, 8% means $\frac{8}{100}$. So, 8% of R8 means $\frac{8}{100} \times$ R8. My answer must show by how many rand the price has increased and what the new price is. The unit I will therefore use is rand. (Learners often lose track of the latter, that is, what unit the answer has to be in, and neglect to include it in their answer. It is also helpful to refer them back to what the original question is asking, if necessary.)
- The above helps to break it down for them, and explain what the method actually means.
- Let them do an additional example or two in pairs. They do not have to do the actual calculation at first, only once you have worked through the methods in the Learner's Book with them.
- When working through the methods, learners first need to decide if the amount has increased or decreased.
  - If it has increased, the new percentage will be 100% + given %.
  - If it has decreased, the new percentage will be 100% – given %.
- Finding the new amount is the same as calculating a percentage of a whole number.
- Remind learners to write the units in their final answers.
- Learners may use their calculators when doing the more complex calculations.
- When they understand why the method works the way that it does and are asked to find the percentage increase or decrease, learners can now simply substitute the correct numbers into the given formula and calculate the answer.

Review Copy

Remedial and extension

Remedial: Pairs of learners may work through simple examples at first as explained in the teaching guidelines above. Include simple calculations in mental maths sessions every day, for example, 10% of R100; 5% of R10; 20% of R100; 10% of R20; etc.
Extension: Both activities are context-based and therefore more challenging.

Suggested answers

Activity 4

1 112 ÷ 100 × R4,50 ≈ R5
2 120 ÷ 100 × R99,99 ≈ R119,99
3 60 ÷ 100 × 90 ml = 54 ml
4 65 ÷ 100 × 67 days ≈ 44 days
5 110 ÷ 100 × R160 = R176
6 125 ÷ 100 × 15 g ≈ 19 g
7 60 ÷ 100 × R240 = R144
8 82 ÷ 100 × R468 = R383,76
9 95 ÷ 100 × R225 000 = R213 750
10 44 ÷ 100 × 237 = 104 items

Activity 5

1 $\frac{(1\ 150 - 980)}{980} \times 100 \approx 17\%$

2 $\frac{(2\ 800 - 2\ 500)}{2\ 500} \times 100 = 12\%$

3 $\frac{(11{,}06 - 10{,}72)}{11{,}06} \times 100 \approx 3\%$

4 $\frac{(5\ 020 - 4\ 500)}{4\ 500} \times 100 = 12\%$

5 $\frac{(3\ 150 - 2\ 500)}{2\ 500} \times 100 \approx 26\%$

6 $\frac{(80 - 50)}{80} \times 100 \approx 38\%$

7 $\frac{(2\ 000 - 1\ 690)}{2\ 000} \times 100 \approx 16\%$

8 $\frac{(8\ 965{,}30 - 8\ 725{,}35)}{8\ 725{,}35} \times 100 \approx 3\%$

9 $\frac{145\ 000 - 127\ 600}{145\ 000} \times 100 = 12\%$

Chapter 11 Revision

Learner's Book page 247

Encourage learners to review the content covered before attempting the revision activities. The revision activities should be used to assess learners' progress thus far, and to assess where remediation may be required.

Suggested answers

1

	i	ii
a	1	7
b	113	105
c	198	210

2 a proper
b mixed numbers
c improper

3 a $\frac{1}{5}$ b 12 c $\frac{7}{4}$ d $\frac{5}{26}$

4 a $\frac{101}{20}$ b $\frac{17}{4}$ c $\frac{207}{25}$

5 a $6\frac{1}{2}$ b $3\frac{1}{6}$ c $3\frac{15}{16}$

6 a 42 b 200

7 a LCD = 42; Answer = $\frac{25}{42}$ b LCD = 200; Answer = $\frac{27}{100}$
c LCD = 24; Answer = $\frac{51}{8}$ or $6\frac{3}{8}$ d R35
e $-\frac{1}{15}$ f −26 g $\frac{4}{5}$ h 3
i 144 j $-\frac{1}{5}$ k 4 l impossible

8 $\frac{1}{5} \times$ R1 490 = R298

9 a 35 men b 281 dogs

10 a 1 820 kg b R158,10

11 a R204,60 b R1 115,40

12 a 2% b 30%

13 a 14 balls b 36% c 7%
d They have a 7% chance of winning first prize. (Not a very good chance.)

14 105% of R8,50 ≈ R8,93

15 60% of 6 hours ≈ 3,6 h

16 Percentage increase = $\frac{10\ 250 - 9\ 500}{9\ 500} \times 100 = 8\%$

Review Copy

CHAPTER

12 Revision: Decimal fractions

Chapter overview

Learner's Book pages 249 to 266
Recommended pacing: 9 hours

This chapter focuses on the following:

Unit 1:	**Ordering and comparing decimal fractions**	2 hours
	Place value	
	Counting in decimals	
	Comparing and ordering decimal fractions and decimal numbers	
	Rounding off decimal fractions	
Unit 2:	**Adding and subtracting decimal fractions**	2 hours
	Adding decimal fractions	
	Subtracting decimal fractions	
Unit 3:	**Multiplying and dividing decimal fractions**	3 hours
	Multiplying decimal fractions	
	Dividing decimal fractions	
	Squares, cubes and decimal fractions	
Unit 4:	**Equivalent forms**	2 hours
	Common and decimal fractions	
	Common fractions, decimal fractions and percentages	
Chapter 12 Revision		

Review Copy

UNIT 1

Ordering and comparing decimal fractions

Unit overview

Learner's Book page 250
Recommended pacing: 2 hours

This unit focuses on the following:

- revising and working with place value
- counting forwards and backwards with decimal fractions
- comparing decimal fractions
- ordering decimal fractions in ascending and descending order
- rounding off decimal fractions
- solving problems that involve ordering and rounding off decimal fractions

Resources: Learner's Book; exercise book; calculator (for checking answers)

Mental Maths

(10 minutes per day)

Suggested answers — Learner's Book page 250

1 $9\,000 + 500 + 20 + 8 + \frac{1}{10} + \frac{2}{100} + \frac{5}{1\,000}$

2 nine thousand; five hundred; twenty; eight; one-tenth; two-hundredths; five-thousandths

3 a (+ 0,11) 2,405; 2,515; 2,625; 2,735; 2,845
b (–0,0098) 4,9604; 4,9506; 4,9408; 4,93; 4,9212

4 a 3,45 > 2,98 b 2,987,7 kg < 3,1 tonne

5 7,68; 7,3598; 7,304; 7,3; 7,256; 7,25

6 1,005; 1,0325; 1,1; 1,357; 1,68; 1,758

Review Copy

Background information

In this unit learners revise important basic skills that include working with place value, ordering and comparing decimal fractions and decimal numbers and rounding off decimal fractions.

Teaching guidelines

Since this is mostly revision, it is an opportunity to assess learners' prior knowledge and to consolidate the basics of decimal fractions and decimal numbers that still require attention. The worked examples and activities are aimed at crystallising learners' basic understanding and learners should be encouraged to work through these independently or in pairs while their progress and understanding are monitored and assessed. It is also important that learners are given opportunities to verbalise their thinking and understanding when having to work independently in this way.

Place value

Activity 1 Work with place value

Learner's Book page 251

Guidelines for implementing this activity

- Work through the examples in the Learner's Book or let learners work through these examples on their own or in pairs as described in the teaching guidelines on the previous page.
- Saying the different place values of the digits aloud (to a partner) as they work is also very useful and helps to crystallise their understanding of place value.
- Since learners are merely writing the digits in the same order as they appear in the columns, it is once again important to check that they know what the value of each digit is that they are positioning in the table. Letting them verbalise these values also encourages learners to think about what it is that they are doing, before they proceed to questions **2** and **3**.
- At this stage it is still important that learners position the values from the first whole number to the last decimal as opposed to a strategy commonly used that encourages them to work from the middle outwards.

Remedial and extension

Remedial: If learners need to build confidence in order to work independently, they can work in pairs or even small groups under close supervision and with targeted intervention. Working through the examples and activities in the manner described above serves as a remediation strategy on its own, that is positioning one digit at a time from beginning to end while verbalising the value of each digit.
Extension: Learners work independently through the examples and activities and/or are paired up with learners who require support, at your discretion.

Review Copy

Suggested answers

1

		Th	H	T	U	t	h	th
a	5 258,19	5	2	5	8	1	5	9
b	25,005			2	5	0	0	5
c	951,24		9	5	1	2	4	
d	2 861,289	2	8	6	1	2	8	9
e	4 561,02	4	5	6	1	0	2	

2 a $5\,000 + 200 + 50 + 8 + \frac{1}{10} + \frac{5}{100} + \frac{9}{1\,000}$

b $20 + 5 + \frac{5}{1\,000}$

c $900 + 50 + 1 + \frac{2}{10} + \frac{4}{100}$

d $2\,000 + 800 + 60 + 1 + \frac{2}{10} + \frac{8}{100} + \frac{9}{1\,000}$

e $4\,000 + 500 + 60 + 1 + \frac{2}{100}$

3 a five thousand; two hundred; fifty; eight; one-tenth; five-hundredths; nine-thousandths

b twenty; five; five-thousandths

c nine hundred; fifty; one; two-tenths; four-hundredths

d two thousand; eight hundred; sixty; one; two-tenths; eight-hundredths; nine-thousandths

e four thousand; five hundred; sixty; one; two-hundredths

Counting in decimals

Activity 2 Count decimals

Learner's Book page 251

Guidelines for implementing this activity

- Work through the example in the Learner's Book, or allow learners to work independently or in pairs as described in the teaching guidelines above. Instruct learners to find the number that is added or subtracted each time by calculating the difference between two consecutive numbers. (They can use calculators to do this.)

Review Copy

Remedial and extension

Remedial: Learners may find it difficult to count in decimals or figure out what numbers must be put onto the number line, chain diagram or number grid. Assist learners by providing clues but let them find the answers themselves.
Extension: Question **1f** is more challenging. Learners work independently through the examples and activities and/or are paired up with learners who require support, at your discretion.

Suggested answers

1 a (+ 0, 1) 0,415; 0,515; 0,615; 0,715; 0,815

b (+ 0,012) 1,586; 1,598; 1,61; 1,622; 1,63

c (– 0,025) 4; 3,975; 3,95; 3,925; 3,9

d (– 0,07) 3,789; 3,719; 3,649; 3,579; 3,509

e (+ 0,1111) 2,4567; 2,5678; 2,6789; 2,79; 2,9011

*f (– 0,0203) 5,5023; 5,482; 5,4617; 5,4414; 5,4211

2 (+ 0,005) A = 2,11; B = 2,115; C = 2,12; D = 2,125

3 (+ 0,002) A = 1,322; B = 1,326

4 1,5 → $\boxed{+1}$ → $\boxed{0,5}$ → $\boxed{+2}$ → $\boxed{2,5}$ → $\boxed{+3}$ → $\boxed{5,5}$

Comparing and ordering decimal fractions and decimal numbers

Activity 3 Compare and order decimals

Learner's Book page 252

Guidelines for implementing this activity

- Work through the examples in the Learner's Book or let learners work through these examples on their own or in pairs as described in the teaching guidelines on the previous page.
- Comparing is quite easy since learners can clearly see which number is larger if all the numbers have the same number of decimal places after the comma.
- Write zeros at the back of the number until all decimal fractions have the same number of decimal places, if learners need to see the numbers in this way, and encourage them to do this if they need to.
- For ordering decimal numbers it is useful to write all the numbers below one another, then add zeros to the back and lastly order them. For example, write 7,15; 7,165; 7,108; 7,144; 7,190 in descending order.

  Step 1: Write below one another.

  7,15
  7,165
  7,108
  7,144
  7,190

  Step 2: Add zeros.

  7,150
  7,165
  7,108
  7,144
  7,190

  Step 3: Order

7,150	3
7,165	2
7,108	5
7,144	4
7,190	1

  Step 4: Write the final answer.

  7,190; 7,165; 7,15; 7,144; 7,108

Review Copy

Remedial and extension

Remedial: Use the methods described in the teaching guidelines for ordering decimal fractions more easily.
Extension: Questions **1e**, **1f** and **5** are more challenging.

Suggested answers

1 a > b > c < d < e* < f* >
2 a 3,596; 3,578; 3,349; 3,278; 3,078; 3,027
b 9,2; 8,23; 6,54; 6,275; 4,25; 3,1547; 1,56
c 2,967; 2,7586; 2,4; 2,336; 2,3; 2,237; 2,15
3 a 7,159; 7,357; 7,418; 7,456; 7,529; 7,630
b 1,147; 3,4; 4,558; 5,78; 6,275; 7,1; 9,375
c 8,156; 8,249, 8,25; 8,3, 8,4537; 8,55; 8,972
4 5,65; 5,85; 5,95; 6,05
*5

0,1	0,01	–0,08	–0,17	–0,26
0,2	0,11	0,02	–0,07	–0,16
0,3	0,21	0,12	0,03	–0,06
0,4	0,31	0,22	0,13	0,04
0,5	0,41	0,32	0,23	0,14

Rounding off decimal fractions

Activity 4 Order and round off decimal fractions

Learner's Book page 253

Review Copy

Guidelines for implementing this activity

- Work through the examples in the Learner's Book or let learners work through these examples on their own or in pairs as described in the teaching guidelines on the previous page.
- Circling the digit that learners need to look at is helpful, for example if they have to round off to two decimal places, they can circle the third decimal digit.
  - Then ask learners if the circled digit is less than 5, or 5 and more than 5.
  - This will help learners to decide if the previous digit (second digit) should stay the same or change.
- Activity 4 can be completed by the learners in the one lesson.

Remedial and extension

Remedial: Learners use the circle method described in the teaching guidelines.
Extension: Activity 4 questions **1c**, **2c**, **3c**, **4c** and **5c** are more challenging.

Suggested answers

1 a 25 b 154 *c 2 897
2 a 75,3 b 358,1 *c 1 236,2
3 a 71,85 b 245,59 *c 9 541,57
4 a R10,57 b R10,98 *c R11,00
*5 a 1,5375 kg b 1,4659 kg c 1,6989 kg

6 a 1,098; 2,56; 3,45; 4,035; 4,56
b 1,1; 2,6; 3,5; 4,0; 4,6
7 a 2,65 m; 1,742 m; 1,258 m; 1,054 m; 0,63 m
b 2,65 m; 1,74 m; 1,26 m; 1,05 m; 0,63 m

UNIT 2

Adding and subtracting decimal fractions

Unit overview

Learner's Book page 254
Recommended pacing: 2 hours

This unit focuses on the following:

- adding and subtracting decimal fractions
- solving problems that involve adding and subtracting decimal fractions

Resources: Learner's Book; exercise book; calculator

Mental Maths

(10 minutes per day)

Suggested answers

Learner's Book page 254

1 R660,79
2 R178,32
3 643,2716 cm
4 1 130,7986 m
5 6 934,225 km
6 136,296 kg
7 R2 397,11
8 R306,58

Review Copy

Background information

Learners have been doing calculations with decimal fractions and decimal numbers since Grade 6, and should be competent at addition and subtraction of decimal fractions to at least three decimal places.

Teaching guidelines

Since this is mostly revision, it is an opportunity to assess learners' prior knowledge and to consolidate the basics of adding and subtracting decimal fractions and decimal numbers. The worked examples and activities are once again aimed at crystallising learners' basic understanding and learners should be encouraged to work through these independently or in pairs as revision while their progress and understanding are monitored and assessed. It is also important that learners are given opportunities to verbalise their thinking and understanding when having to work independently in this way. For calculations with larger numbers, calculators can be used since this is a skill learners will require throughout their lives.

Adding decimal fractions; Subtracting decimal fractions

Activities 1–3 **Add decimal fractions; Subtract decimal fractions; Solve problems**

Learner's Book pages 254–255

Guidelines for implementing these activities

- Work through the examples in the Learner's Book or let learners work through these examples on their own or in pairs as described in the teaching guidelines above.
- Remind learners to write their final answers with units.
- Monitor their progress closely, and provide additional exercises for homework if necessary.
- Allow learners to use a calculator for calculations with larger numbers, since this is a skill they will need throughout their lives.
- Use shopping pamphlets and let learners make lists of costs of items and add them.

Remedial and extension

Remedial: Learners should work methodically and redo questions if necessary, and verbalise their mistakes in order to become aware of what these mistakes are, and how to avoid them.
Extension: Activity 1 questions **9** to **12** are more challenging. Activity 2, questions **9** to **12** and Activity 3 are more challenging. Provide additional questions such as these if necessary, and if there is time.

Review Copy

Suggested answers

Activity 1

1 R37,79	**2** R578,21	**3** R80,48	**4** 10,765 cm
5 148,007 m	**6** 1 105,5 ml	**7** 7,625 m	**8** 10 457,661 cm
***9** 380,9122 kg	***10** 10 428,085 mg	***11** R15 355,60	***12** 533,7 cm

Activity 2

1 R215,35	**2** 1 141,1634 m	**3** R19,77
4 1 212,935 g	**5** 93,831 m	**6** R601,05
7 0,8362 cm	**8** 29,1134	***9** R1 065,17
***10** R9 988,47	***11** R196,54	***12** R3 045,84

Activity 3

1 51,703
2 5,325 m + 7,45 m = 12,775 m; 12,775 m – 2,16 m = 10,615 m
3 **a** R18 532,46 – R706,04 = R17 826,42
b R17 826,42 – R95,25 = R17 731,17

UNIT 3

Multiplying and dividing decimal fractions

Unit overview

Learner's Book page 256
Recommended pacing: 3 hours

This unit focuses on the following:

- multiplying and dividing decimal fractions
- solving problems that involve multiplying and dividing decimal fractions
- finding square and cube numbers of decimal fractions
- finding square and cube roots of decimal fractions

Resources: Learner's Book; exercise book; calculator

Mental Maths

(10 minutes per day)

Suggested answers — Learner's Book page 256

1 **a** 30 **b** 3 **c** 0,03
2 **a** 6 **b** 60 **c** 60
3 **a** Estimate: 520 × 10 ≈ 5 200
Calculate: 4 658,4
Check: 4 658,4 ÷ 9 = 517,6 (Do inverse operation: divide.)
b Estimate: 30 × 0,5 ≈ 15 × 1 ≈ 15
Calculate: 13,8915
c Estimate: 15 × 0,5 ≈ 7,5 × 1 ≈ 7,5
Calculate: 8,463

Review Copy

Background information

Learners have been doing calculations with decimal fractions and decimal numbers since Grade 6, and should be competent at multiplying and dividing decimal fractions to at least three decimal places. It is important to continue multiplication by decimal fractions not limited to one decimal place, and to extend division to division of decimal fractions by decimal fractions. Learners are also working with square numbers and cube numbers of decimal fractions for the first time.

Teaching guidelines

Since this is mostly revision with some extension, it is an opportunity to assess learners' prior knowledge and to consolidate the basics of multiplication and division of decimal fractions and decimal numbers. The worked examples and activities are once again aimed at crystallising learners' basic understanding and learners should be encouraged to work through these independently or in pairs while their progress and understanding are monitored and assessed. It is also important that learners are given opportunities to verbalise their thinking and understanding when having to work independently in this

way. For calculations with larger numbers, calculators can be used since this is a skill learners will require throughout their lives.

Learners should always estimate their answers before calculating, especially with multiplication by decimal fractions. They should be able to judge how reasonable answers are, and also check their answers. Multiplication by decimal fractions should start with familiar numbers that learners can calculate by inspection, so that they get a sense of how decimal places are affected by multiplication.

For division by decimal fractions without calculators, learners have to use their knowledge of multiplication by 10 or factors of 10 to make the divisor a whole number. Therefore learners start with familiar numbers that they can calculate by inspection so that they get a sense of how decimal places are affected by division. Examples such as these must be used:

42 ÷ 7 = 6

42 ÷ 0,7 = 420 ÷ 7 = 60 (Multiply both numbers by 10 to make the decimal fraction a whole number.)

42 ÷ 0,07 = 4 200 ÷ 7 = 600 (Multiply both numbers by 100 to make the decimal fraction a whole number.)

0,42 ÷ 0,07 = 42 ÷ 7 = 6 (Multiply both numbers by 100 to make the decimal fraction a whole number.)

The mental maths sessions at the beginning of every lesson are ideal opportunities to practise and consolidate the above.

Review Copy

Multiplying decimal fractions; Dividing decimal fractions

Activities 1–3 **Multiply decimal fractions; Divide decimal fractions; Solve problems**

Learner's Book pages 257–259

Guidelines for implementing these activities

- Work through the introduction and examples in the Learner's Book or let learners work through these examples on their own or in pairs as described in the teaching guidelines above.
- Ensure that learners first estimate their answers so that they can use these estimates to check their answers.
- When multiplying, make sure that learners write the amounts in the correct columns from the right-hand side, and that they set out their work neatly.
- They can also use calculators to check their answers by doing the inverse calculation (division for multiplication and multiplication for division).
- For long division with large numbers, learners can use calculators to assist them with the steps. However, they still need to show all their steps for long division.

- Activities 2 and 3 can be completed in the same lesson. (Learners can do Activity 3 for homework.) Activity 2 contains context-free calculations, while Activity 3 contains context-based calculations.

Remedial and extension

Remedial: Learners should redo the questions until they get them correct, and should be aware of exactly where they are making mistakes, and how to correct them so that they don't repeat them. They can also work in pairs.
Extension: Activity 2, question **2e** and Activity 3 are more challenging. Provide additional exercises that include calculator exercises with larger numbers.

Activity 1

1	a 12	b 1,2	c 0,12	d 0,012	e 0,0012
2	a 30	b 3	c 0,3	d 0,3	e 0,0003
3	a 1 250,4	b 1,2504	c 1,2089	d 120,82	e 16,5385
	f 165,385	g 104,55	h 10,455	i 88,4	j 8,84
	k 60,5604	l 60,5604	m 12,92741	n 12 927,41	o 5,8876
	p 5 887,6	q 15,677	r 1,5677	s 0,494208	t 4,94208

Activity 2

1	a 4	b 0,4	c 0,04	d 0,004	e 0,0004
2	a 25	b 250	c 2 500	d 25 000	*e 25
3	a 1,23	b 1,23	c 2,1	d 0,21	e 168
	f 0,168	g 0,32	h 3,2	i 486	j 0,00486
	k 1 109	l 1,109	m 0,432	n 4 320	o 303,74
	p 0,30374	q 348,5	r 3,485	s 0,685	t 6 850

Review Copy

Activity 3

1 R129,95 × 12 = R1 559,40
2 a 1,574 × 24 + 0,56 ≈ 1,5 × 24 + 0,5 ≈ 3 × 12 + 0,5 ≈ 36 + 0,5 ≈ 36,5
  b 1,574 × 24 + 0,56 = 37,776 + 0,56 = 38,336 ℓ
  c 38,336 ℓ − 0,56 = 37,776; 37,776 ÷ 24 = 1,574
3 3 000 ÷ 1,25 = 2 400 steps
4 a 8 × 11,85 = 94,8 m b 100 − 84,8 = 5,2 m

Squares, cubes and decimal fractions

Finding square and cube numbers of decimal fractions; Finding square and cube roots of decimal fractions

Activity 4 **Work with squares, cubes and decimal fractions**

Learner's Book page 260

Guidelines for implementing this activity

- Work very carefully through the introduction and the examples in the Learner's Book, and do additional examples on the board if necessary.

- The calculation strategy is somewhat trickier than for common fractions. When learners count the number of zeros they take away when shifting positions of decimal digits, they have to determine the square or cube root of the tens/hundreds/thousands that indicate the number of zeros. This is very important. Let them check their answers using calculators.

Remedial and extension

Remedial: Let learners work in pairs, and do as many additional exercises as possible in order for learners to practise the above strategies.
Extension: Activity 4 is challenging. If learners can do all the questions independently, they have done well. Provide additional exercises for homework.

Suggested answers

1	0,0025	2	0,000036	3	0,0121
4	0,027	5	0,000008	6	0,000064
7	0,8	8	0,01	9	0,4
10	0,05	11	0,01	*12	1,1

UNIT 4

Equivalent forms

Review Copy

Unit overview

Learner's Book page 261
Recommended pacing: 2 hours

This unit focuses on the following:
- converting common fractions to decimal fractions
- converting decimal fractions to common fractions
- writing common and decimal fractions as percentages
- writing percentages as common and decimal fractions
- solving problems that involve common fractions, decimal fractions and percentages

Resources: Learner's Book; exercise book; calculator

Mental Maths

(10 minutes per day)

Suggested answers

Learner's Book page 261

1 a 1,8 b 5,03 c 3,15

2 a $\frac{7}{10}$ b $\frac{575}{100} = \frac{23}{4}$ c $\frac{8\,345}{1\,000} = \frac{1\,669}{200}$ d $\frac{2\,009}{1\,000}$

3 a 80% b 85% c 45%
d 88% e 87,25% f 1,75%

4

a	$\frac{75}{100}$	$\frac{3}{4}$	0,75
b	$\frac{54,54}{100}$	$\frac{2\,727}{5\,000}$	0,5454
c	$\frac{2,5}{100}$	$\frac{1}{40}$	0,025

Background information

In Grade 7 learners revised the following, which were covered in Grade 6:

- common fraction and decimal fraction forms of the same number
- recognising equivalence between common fraction, decimal fraction and percentage forms of the same number.

In Grade 8, learners revise and consolidate this knowledge.

Review Copy

Teaching guidelines

Many learners struggle with the concepts of fractions, but are quite confident with decimal fractions and decimal numbers because of their almost intuitive understanding of money. Percentages are also often practised when learners get their marks for a test or an assessment.

Most of the work done in this unit is revision of Grade 7 concepts. Make the links between the three types of fraction very clear at the outset. Let learners practise with a table where they convert from one form to the other two forms. The calculations can be done without a calculator, but it is a good idea to give calculators to those that require them since this is a skill that will be used in everyday life.

Mental maths sessions are also ideal for practising conversions between these different equivalent forms.

Common fractions and decimal fractions; Common fractions, decimal fractions and percentages

Convert common fractions to decimal fractions; Converting decimal fractions to common fractions; Write fractions as percentages; Writing percentages as fractions

Activities 1–3 **Convert common fractions and decimal fractions; Convert between fractions and percentages; Work with common fractions, decimal fractions and percentages**

Learner's Book pages 262–264

Guidelines for implementing this activity

- Revise the meaning of finite decimal fractions, non-recurring decimal fractions and recurring decimal fractions as shown in the Learner's Book. Let learners do some examples that yield these three types of decimal fraction on their calculators, for example, 2 ÷ 5 (0,4 finite); 3 ÷ 4 (0,75 finite); 5 ÷ 7 (0,714285714 … recurring because there is a pattern); 5 ÷ 6 (0,8333 … recurring because there is a pattern); 8 ÷ 9 (0,88888 … recurring because there is a pattern); 2 ÷ 11 (0,18181818 … recurring because there is a pattern); 3 ÷ 79 (non-recurring because it does not end after the decimal and has no pattern) or let them do their own examples using digits of their choice saying whether they are finite, recurring or non-recurring. Ensure that they position the dot or line on the correct digits in recurring decimals.
- Work through the examples in the Learner's Book to explain converting fractions, decimal fractions and percentages and do additional examples on the board such as $\frac{2}{5}$ = 0,4 = 40%; $\frac{3}{4}$ = 0,75 = 75% and so on.
- Learners may use calculators to do calculations with larger numbers. They can also check if their answers are correct by using the fractions button on their calculators. (This is useful to check answers in tests and exams.)
- Activities 2 and 3 can be covered in the same lesson.

Review Copy

Remedial and extension

Consolidate the basic equivalent forms in the mental maths sessions.

Suggested answers

Activity 1

1	a 0,3	b 0,81	c 0,612	d 4,2
	e 5,19	f 2,008	g 0,6	h 0,9
	i 0,68	j 3,5	k 4,64	l 5,69
2	a $\frac{8}{10} = \frac{4}{5}$	b $\frac{85}{100} = \frac{17}{20}$	c $\frac{328}{1\,000} = \frac{41}{125}$	d $\frac{46}{10} = \frac{23}{5}$
	e $\frac{3\,674}{100} = \frac{1\,837}{50}$	f $\frac{19\,562}{1\,000} = \frac{9\,781}{500}$	g $\frac{34}{10} = \frac{17}{5}$	h $\frac{802}{100} = \frac{401}{50}$

i $\frac{6\,025}{1\,000} = \frac{241}{40}$ j $\frac{315\,055}{1\,000} = \frac{63\,011}{200}$ k $\frac{18\,006}{1\,000} = \frac{9\,003}{500}$ l $\frac{292\,929}{1\,000}$

Activity 2

1 a 40% b 85% c 83%
d 60% e 82% f 80%

2 a 13% b 74% c 99,8%
d 68,73% e 24,23% f 2,25%

3

	(i)	(ii)
a	$\frac{20}{100} = \frac{1}{5}$	0,2
b	$\frac{87}{100}$	0,87
c	$\frac{255}{100} = \frac{51}{20}$	2,55
d	$\frac{66,67}{100} = \frac{6\,667}{10\,000}$	0,6667
e	$\frac{25}{1\,000} = \frac{1}{40}$	0,025
f	$\frac{0,04}{100} = \frac{1}{2\,500}$	0,0004

4

	Simplified common fraction	Common fraction	Decimal fraction	Percentage
a	$\frac{21}{25}$	$\frac{84}{100}$	0,84	84%
b	$\frac{13}{20}$	$\frac{65}{100}$	0,65	65%
c	$\frac{17}{50}$	$\frac{34}{100}$	0,34	34%
d	$\frac{29}{200}$	$\frac{45}{100}$	0,45	14,5%
e	$\frac{4}{5}$	$\frac{80}{100}$	0,8	80%

Review Copy

Activity 3

1 a

Subject	Mark obtained	Common fraction	Decimal fraction	%
English	$\frac{35}{50}$	$\frac{70}{100}$	0,7	70%
Afrikaans	$\frac{17}{20}$	$\frac{85}{100}$	0,85	85%
Social Sciences	$\frac{28}{50}$	$\frac{56}{100}$	0,56	56%
Natural Sciences	$\frac{12}{25}$	$\frac{48}{100}$	0,48	48%
Mathematics	$\frac{87}{125}$	$\frac{696}{1\,000}$	0,696	69,6%

b Afrikaans

2 a $\frac{980}{2\,000} = \frac{4\,900}{10\,000} = 0{,}49$

b 49%

c 100% – 49% = 51%

3 a 12,5 m = 1 250 cm; 1 250 cm – 90 cm = 1 160 cm; $\frac{1\,160}{1\,250} = \frac{9\,280}{10\,000} = 0{,}9280$

b 92,80 %

Chapter 12 Revision

Learner's Book page 266

Encourage learners to review the content covered before attempting the revision activities. The activities should be used to assess learners' progress thus far, and to assess where remediation may be required.

Suggested answers

1 a $500 + 60 + 7 + \frac{9}{10} + \frac{2}{100} + \frac{7}{1\,000}$

b five hundred; sixty; seven; nine-tenths; two-hundredths; seven-thousandths

2 a (–0,014) 1,402; 1,388; 1,374; 1,36; 1,346

b (+0,015) 4,005; 4,02; 4,035; 4,05; 4,065

3 a >

b <

4 3,856; 3,786; 3,686; 3,4; 3,25; 3,24

5 1,2; 1,36; 1,638; 1,656; 1,789; 1,857

6 a 76 548 b 76 547,7 c 76 547,67 d 76 547,675

7 a R130,89 b R548,08 c 12,178 cm

d 9 038,4519 m e 12 028,151 mg f R4 672,59

8 a 2 912,5 b 694,278 c 0,297298

d 0,368 e 75,3 f 0,0581

9 2,024 × 25 = 50,6 ℓ

10 a 0,0081 b 0,064 c 0,1 d 0,5

11

	Simplified common fraction	Common fraction out of 100	Decimal fraction	Percentage
e.g.	$\frac{7}{10}$	$\frac{70}{100}$	0,7	70%
a	$\frac{2}{25}$	$\frac{8}{100}$	0,08	8%
b	$\frac{41}{50}$	$\frac{82}{100}$	0,82	82%
c	$\frac{163}{1\,000}$	$\frac{16{,}3}{100}$	0,163	16,3%
d	$\frac{19}{200}$	$\frac{9{,}5}{100}$	0,095	9,5%
e	$\frac{17}{20}$	$\frac{85}{100}$	0,85	85%

Review Copy

CHAPTER

13 Geometry of 2D and 3D

Chapter overview

Learner's Book pages 267 to 295
Recommended pacing: 15 hours

This chapter focuses on the following:

Unit 1: Theorem of Pythagoras 5 hours
- The lengths of the sides of triangles
- Using Pythagoras to prove right-angled triangles
- Use the Theorem of Pythagoras

Unit 2: Area and perimeter 2,5 hours
- Formulae to calculate perimeter and area
- Other applications of the formulae
- Perimeter and area of triangles
- Perimeter and area of polygons

Unit 3: Circles 2,5 hours
- Aspects of a circle
- Investigate the relationship between the diameter and the circumference of a circle
- Finding pi (π)
- Investigate why the formula for the area of a circle is $A = \pi r^2$

Unit 4: Surface area and volume 5 hours
- Prisms
- Volume
- The relationship between surface area and volume

Chapter 13 Revision
PoA Investigation 2. The relationship between the diameter and the circumfrence of a circle.
PoA Investigation 3. Why the formula for the area of the circle is $A = \pi r^2$

Review Copy

UNIT 1

Theorem of Pythagoras

Unit overview

Learner's Book page 268
Recommended pacing: 5 hours

This unit focuses on the following:

- investigating the relationship between the lengths of the sides of a right-angled triangle to develop the Theorem of Pythagoras
- determining if a triangle is a right-angled triangle or not, if the length of the three sides of the triangle are known
- using the Theorem of Pythagoras to calculate missing lengths in right-angled triangles (answers in surd form if necessary)

Resources: Learner's Book; exercise book

Mental Maths

(10 minutes per day)

Suggested answers — Learner's Book page 268

1	a 4	b 36	c 49	d 64
2	a 9	b 16	c 9 + 16 = 25	d 25
3	a 25	b 144	c 25 + 144 = 169	d 169
4	a 5	b 7	c 10	d 1
5	a 4	b 11	c 9	d $\sqrt{7}$

Review Copy

Background information

This is learners' first introduction to the Theorem of Pythagoras.
The Theorem of Pythagoras is probably the best-known theorem in geometry and there are many different proofs for the theorem. If learners have access to the Internet they can read more about Pythagoras and the habits of his group of followers.

Teaching guidelines

It is important that learners develop a sound grasp of the Theorem of Pythagoras because they will use it very often in Maths, for example, in Trigonometry, Euclidean geometry and Analytical geometry. This unit aims to introduce this theorem in a manner that encourages practical investigation, and it is important that all learners actively participate in these investigations.

Grade 8 learners do not work with irrational numbers. When they solve triangles with the use of the Theorem of Pythagoras, some of the answers can be irrational numbers. An irrational number is a real number that is not a rational number.

Make sure the learners know what perfect squares are. If their answer is not a perfect square, they should simply leave it in surd form (root form).

The questions asked in the mental maths exercises are the types of calculations that learners should be able to do quickly using the Theorem of Pythagoras.

Always make sure the learners understand *what* they are calculating. They need to be clear about the difference between the two questions:

- 4 squared (4^2) is equal to 4×4 and therefore equal to 16.
- the square root of 4 ($\sqrt{4}$) is equal to 2 (because $2 \times 2 = 4$).

Learners should pay extra attention to the answers to questions such as questions **2** and **3**, and see if they can notice the patterns on their own.

Questions such as question **5d** is an example to discuss: it is not a perfect square, because we cannot write it as the square of a number. We leave the answer in this form, and we refer to this as surd form.

Include examples in further mental maths sessions where decimal numbers are used, for example, $0{,}32 = 0{,}09$ and $\sqrt{0{,}16} = 0{,}4$.

The lengths of the sides of triangles

Review Copy

Activity 1 Investigate the sides of triangles

Learner's Book page 268

Guidelines for implementing this activity

- In the investigation, learners will count the smaller squares, point out that they are determining the areas of the bigger squares that are drawn on the sides of each triangle.
- When they memorise the words of the Theorem of Pythagoras, they need to see these squares in their minds.
- Revise the symbols < and >.
- The aim of the investigation is not to complete the table correctly, but for learners to develop insight into the theorem.
- It is important to emphasise what the lengths of the sides of the triangles are and to compare them to the areas of the squares drawn on the sides.

Remedial and extension

Remedial: It is important that learners work through the investigation in a practical and hands-on manner. Provide additional triangles (a combination of right-angled triangles and other triangles with given side lengths). Let learners draw the areas of the sides as shown in the triangles in the activity and record the information in a table as shown in order to further explore the Theorem of Pythagoras. Allow learners to verbalise their thinking at all times in order to crystallise their understanding.
Extension: The diagrams in the investigation can have other shapes added to them as well, for example, half-circles.

Suggested answers

1

Triangle	Type of triangle	p^2	q^2	$p^2 + q^2$	r^2
1	acute-angled	25	16	41	36
2	right-angled	16	9	25	25
3	obtuse-angled	9	4	13	16

2 Discuss any observations made by learners.
3 a $<$ b $=$ c $>$
4 Yes. Ask learners *why* they say "yes".

Review Copy

Using Pythagoras to prove right-angled triangles

Activity 2 Determine whether triangles are right-angled

Learner's Book page 270

Guidelines for implementing this activity

- This is a direct application of what was investigated in Activity 1.
- Work through the examples in the Learner's Book but let learners complete the activity on their own.
- Point out that in Example 2, the side opposite a vertex is named according to that vertex, for example, x is named after the vertex X directly opposite it.
- Do additional examples on the board if necessary, giving small groups each a different triangle to explore in this way. Allow groups to share their findings, and the rest of the class to evaluate these findings. Work your way through all groups.
- Encourage learners to make rough sketches when doing the activity.

Tip

Ask learners to say the words of the Theorem of Pythagoras out loud and then ask them to explain in their own words what they understand by this. The more they do this, the better.

Suggested answers

1 Square of the longest side = $9^2 = 81$
Sum of the squares of the other two sides = $6^2 + 8^2 = 36 + 64 = 100$
$81 \neq 100 \therefore \triangle ABC$ is not a right-angled triangle.
$\hat{A}$ is the largest angle; $\triangle ABC$ is an acute-angled triangle.

2 Square of the longest side = $13^2 = 169$
Sum of the squares of the other two sides = $5^2 + 12^2 = 25 + 144 = 169$
$k^2 = l^2 + n^2 \therefore \triangle KLN$ is a right-angled triangle.
$\hat{K}$ is the largest angle = 90°.

3 Square of the longest side = $10^2 = 100$
Sum of the squares of the other two sides = $4^2 + 7^2 = 16 + 49 = 65$
$100 \neq 65 \therefore \triangle PQR$ is not right-angled triangle.
$\hat{Q}$ is the largest angle; $\triangle PQR$ is an obtuse-angled triangle.

4 Square of the longest side = $0{,}5^2 = 0{,}25$
Sum of the squares of other two sides = $0{,}3^2 + 0{,}4^2 = 0{,}09 + 0{,}16 = 0{,}25$
$e^2 = d^2 + f^2 \therefore \triangle DEF$ is a right-angled triangle.
$\hat{E}$ is the largest angle = 90°.

Using the Theorem of Pythagoras

Activity 3 Use the Theorem of Pythagoras

Learner's Book page 271

Review Copy

Guidelines for implementing this activity

- Notice the difference in the examples.
  - Example 1: Calculate the hypotenuse if the right-angle sides are given.
  - Example 2: Find one of the right-angle sides if the hypotenuse and the other right-angle side are given.
  - Example 3: The triangle is tilted, but the calculation is similar to Example 1. Leave an answer in surd form if necessary.
- Teach learners to write down the theorem first. Teach them to always write the (hypotenuse)^2 first.
- Do not teach learners when to "add" (if hypotenuse is asked) or when to "subtract" (when hypotenuse is given) when using the Theorem of Pythagoras. If they do enough problems, they will discover the shortcut for themselves when they are ready for it.
- It is important for learners to make rough sketches if they are not given.
- Learners should use the correct units in the answers.

Remedial and extension

Remedial: It is useful to remember a few Pythagorean Triples. for example, (3, 4, 5); (5, 12, 13); (6, 8, 10); (7, 24, 25); (8, 15, 17) and (9, 40, 41). It is also very important that learners remember that the biggest number should be the hypotenuse.
Extension: Challenge learners to find more illustrations (or proofs) for the Theorem of Pythagoras.

Suggested answers

1 a $r^2 = p^2 + q^2$ (Pyth)
$= 8^2 + 6^2$
$= 100$
$r = \sqrt{100} = 10$ cm

b Convert first: 1,2 cm = 12 mm
$m^2 = k^2 + l^2$ (Pyth)
$= 12^2 + 9^2$
$= 225$
$m = \sqrt{225} = 15$ cm

c $c^2 = a^2 + b^2$ (Pyth)
$= 12^2 + 8^2$
$= 208$
$m = \sqrt{208}$ cm

d $p^2 = r^2 + q^2$ (Pyth)
$= 2^2 + 1^2$
$= 5$
$m = \sqrt{5}$ cm

2 a $r^2 = p^2 + q^2$ (Pyth)
$15^2 = 9^2 + q^2$
$q^2 = 225 - 81 = 144$
$q = \sqrt{144} = 12$ units

b In △ABC:
$c^2 = a^2 + b^2$ (Pyth)
$17^2 = 8^2 + b^2$
$b^2 = 289 - 64 = 225$
$b = \sqrt{225} = 15$ units

c In △ACD:
$a^2 = c^2 + d^2$ (Pyth)
$28^2 = c^2 + 15^2$
$c^2 = 784 - 225 = 559$
$b = \sqrt{559}$ units

3 a Corner should be equal to 90°. Thembakazi applied the Theorem of Pythagoras and noted:
Calculation 1: $BC^2 = 40^2 = 1\,600$.
Calculation 2: $AB^2 + AC^2 = 30^2 + 30^2 = 1\,800$.
∴ $\hat{A}$ is not a right angle and the shelf will not fit into the corner.

Review Copy

UNIT 2

Area and perimeter

Unit overview

Learner's Book page 272
Recommended pacing: 2,5 hours

This unit focuses on the following:

- using appropriate formulae to calculate perimeter and area of squares, rectangles and triangles
- calculating the area of polygons by decomposing them into rectangles and triangles
- using and converting between appropriate SI units.

Resources: Learner's Book; exercise book

Mental Maths

(10 minutes per day)

Suggested answers — Learner's Book page 272

1 area of a rectangle
2 area of a square
3 perimeter of a square
4 area of a rectangle (if her table has the shape of a rectangle)
5 perimeter of a rectangle
6 perimeter of a square (if the tiles in the class room are squares)
7 area of a rectangle (if the floor is a rectangle) and the area of a square (if the tiles are squares)

Review Copy

Background information

Learners have already encountered and worked with the concepts of area and perimeter in previous grades, and were formally introduced to the formulae in Grade 7.
The following are new in Grade 8:

- areas of polygons by decomposition
- circumference and area of a circle.

Formulae that learners should know and use are as follows:

- Perimeter of a square = $4s$
- Perimeter of a rectangle = $2(l + b)$ or $2l + 2b$
- Area of a square = s^2
- Area of a rectangle = $l \times b$
- Area of a triangle = $\frac{1}{2}(b \times h)$
- Diameter of a circle: $d = 2r$

- Circumference of circle: $C = \pi d$ or $2\pi r$
- Area of a circle: $A = \pi r^2$.

Teaching guidelines

Many learners in Grade 8 still struggle with the concepts of perimeter and area. Use examples from the environment of the classroom and school buildings to make it practical. The mental maths questions provide ideas of these kinds of examples.

The mental maths questions although not strictly speaking mental maths, aim to further crystallise learners' understanding of the differences between perimeter and area. Learners can also be asked to give the appropriate formula in each case. Their answers will give an indication of where remedial teaching and more revision are necessary.

Always bear in mind that learners are individuals and that their brains work differently. Some learners find information in a table easy to understand. Other learners prefer to look at a diagram. Use different approaches to help the learners understand the formulae. Learners need to memorise the formulae and know when and how to use them.

Formulae to calculate perimeter and area

Review Copy

Activity 1 Work with rectangles and squares

Learner's Book page 274

Guidelines for implementing this activity

- Spend time on developing and reinforcing the concepts of perimeter and area. If learners understand the concepts, they will not have difficulty using the correct units (cm or cm^2).
- Work through the examples in the Learner's Book.
- Revise the SI conversions if necessary.
- It helps to use the formula for the perimeter of a rectangle in the form where learners can see that they need to calculate a sum of the sides, in other words $P = 2l + 2b$
- When we calculate an area, we need to find a product, therefore use the formula for the area of a square in the form $A = s \times s$.
- Encourage learners to work systematically – as is shown in the worked examples in the Learner's Book.
- Learners do Activity 1 on their own. Questions 3 and 4 are included in the exercise to help learners think about the area of triangles.

Remedial and extension

Remedial: The activity is an opportunity to revise the properties of rectangles and squares. It helps if learners can form associations to see the difference between perimeter and area, for example "Am I an ant?" (perimeter) or "Am I a tiler?" (area).
Extension: Practise conversions of SI units as much as possible, for example, question **1b**. Use the Theorem of Pythagoras to find the length of the diagonal in question **1c**.

Suggested answers

1 a $P = 2l + 2b = 2(10) + 2(4) = 20 + 8 = 28$ cm
  b $P = 4s = 4(15) = 60$ mm (or 6 cm)
  c Convert mm to cm: 40 mm = 4 cm
  $P = 2l + 2b = 2(4) + 2(3) = 14$ cm
  d $P = 4s = 4(6) = 24$
2 a $A = l \times b = 10 \times 4 = 40$ cm^2
  b $A = s \times s = 15 \times 15 = 225$ mm^2
  c $A = l \times b = 4 \times 3 = 12$ cm^2
  d $A = s \times s = 6 \times 6 = 36$ m^2
  3 The area of △BCD is half the area of the rectangle.
  ∴ area of △BCD = 6 cm^2.
  4 The area of △PQR is half the area of the square.
  ∴ area of △PQR = 18 m^2.

Other applications of the formulae

Activity 2 Use formulae for perimeter and area

Learner's Book page 275

Guidelines for implementing this activity

- Do not teach learners new versions of a known formula since it might confuse them.
- Work through the examples in the Learner's book. Teach them to begin with the basic formula and then to think as if they are solving an equation.
- Learners already know how to solve equations. They need to realise that they can apply this skill even if the question does not say "Solve for x".

Review Copy

Tip

Encourage learners to make their own sketches if no sketches are given, for example, questions **2** and **3** of Activity 2.

Suggested answers

1 a $l \times b = A$
  $5 \times b = 10 \therefore b = \frac{10}{5} = 2$ cm
  b $s \times s = A$
  $s \times s = 81 \therefore s = 9$ mm
  c $l \times b = A$
  $8 \times b = 48 \therefore b = \frac{48}{8} = 6$ mm
  d $2l + 2b = P$
  $2(5) + 2b = 18$
  $2b = 8$
  $\therefore b = 4$ cm
  e $4 \times s = P$
  $4 \times s = 30 \therefore s = \frac{30}{4} = 7{,}5$ m

f Convert cm to mm: 2,1 cm = 21 mm
$2l + 2b = P$
$2l + 2(5) = 21$
$2l = 11 \therefore b = \frac{11}{2} = 5{,}5$ cm or 55 mm

2 $4 \times s$ = perimeter of tile
$4 \times s = 60$
$\therefore$ length of one side $s = \frac{60}{4} = 15$ cm
$A = s \times s$
$= 15 \times 15$
$= 225\ \text{cm}^2$

3 $l \times b$ = area of play card
$8 \times b = 40$
$\therefore b = 5$ cm
Area of card $= l \times b$
$8 \times 5 = 40\ \text{cm}^2$

Perimeter and area of triangles

Activity 3 Work with triangles

Learner's Book page 276

Review Copy

Guidelines for implementing this activity

- When learners calculate the perimeter of triangles, they should rely on their understanding of what perimeter is.
- Learners have already studied the properties of rectangles. Quickly revise these properties if necessary. They also worked with congruent triangles and a short revision of this concept might also be necessary. If they remember these basics, they should understand why the area of a triangle can be seen as half the area of a rectangle.
- Pay attention to the fact that any one of the three sides can be considered the base. The height should always be perpendicular to the base.
- Make sure that learners work correctly with the formula for the area of a triangle: $A = \frac{1}{2} \times b \times h$. It is simplification from left to right and not an application of the Distributive Law as some learners tend to think. This is very important, and should be carefully monitored when they do the activity.
- Use these questions to revise the *SI* conversions.
- Ensure that learners draw their own sketches when these are not given, for example, questions **3**, **4** and **5**.

Remedial and extension

Remedial: When calculating perimeter, let learners make a little dot next to one of the vertices of the triangle. Then they need to add the lengths of the sides until they are back at the dot again. This will help them with the calculations in this activity, but it is also very good preparation for the more complex figures to come.
Extension: Give learners some triangles with algebraic lengths, e.g. $a = 2x + 3$; $b = 2$; $c = x + 1$; $\therefore p = 2x + 3 + 2 + x + 1 = 3x + 6$.

Suggested answers

1 **a** $P = a + b + c$
$= 9 + 8 + 7$
$= 24$ m

b Convert mm to cm: 50 mm = 5 cm
$P = x + y + z$
$= 7{,}3 + 5 + 7{,}3$
$= 19{,}6$ cm or 196 mm

c $P = d + e + f$
$= 125 + 125 + 125$
$= 375$ mm or 37,5 cm

2 **a** $A = \frac{1}{2} \times b \times h$
$= \frac{1}{2} \times 36 \times 42$
$= 1\ 512\ \text{m}^2$

b $A = \frac{1}{2} \times b \times \text{h}$
$= \frac{1}{2} \times 8 \times 3$
$= 12\ \text{mm}^2$

c Convert cm to mm: 3 cm = 30 mm
$A = \frac{1}{2} \times b \times h$
$= \frac{1}{2} \times 74 \times 30$
$= 1\ 110\ \text{mm}^2$ or $11{,}1\ \text{cm}^2$

d $A = \frac{1}{2} \times b \times h$
$= \frac{1}{2} \times 5 \times 8 = 20\ \text{cm}^2$

3 $\frac{1}{2} \times b \times h = \text{A}$
$\frac{1}{2} \times b \times 9 = 36$
$b \times 9 = 72$
$b = \frac{72}{9} = 8$ cm

4 $\frac{1}{2} \times b \times h = \text{A}$
$\frac{1}{2} \times 15 \times h = 60$
$15 \times h = 120$
$\therefore h = \frac{120}{15} = 8$ mm

5 Find the height first:
$\frac{1}{2} \times b \times h = \text{A}$
$\frac{1}{2} \times 4 \times h = 6$
$\therefore \times 2$ on both sides:
$4 \times h = 12$
$\therefore h = \frac{12}{4} = 3$ cm
Use the Theorem of Pythagoras to calculate the third side:
$(\text{Third side})^2 = 3^2 + 4^2 = 25$
$\therefore$ Third side = 5 cm
Perimeter = 3 + 4 + 5 = 12 cm

Review Copy

Perimeter and area of polygons

Activity 4 Calculate perimeters and areas of polygons

Learner's Book page 277

Guidelines for implementing this activity

- This section is an opportunity for integration. Learners will use previous knowledge about regular polygons and the properties of quadrilaterals, as well as calculate perimeters and areas.
- Ask the following questions about the example in the Learner's Book: What do we know about the sides of a trapezium? Do we have a perpendicular height in the triangle?
- Learners should redraw the sketches in their exercise books and show how they divide the figures. Discuss all the possibilities, so that learners can see that there is often more than one solution to a question.

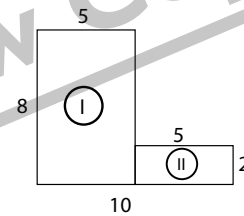
Remedial and extension

Remedial: Learners can also calculate the perimeter of the kite in question **2**.
Extension: Do revision of the names of regular polygons, as in questions **7a** and **b**.

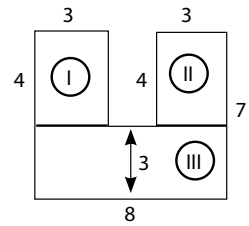
Review Copy

Suggested answers

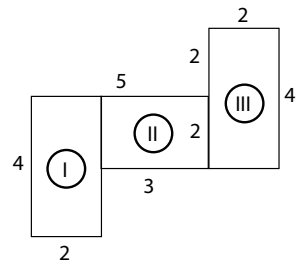
1 **a** Area I $= l \times b$
$= 8 \times 5$
$= 40\ \text{cm}^2$
Area II $= l \times b$
$= 5 \times 2$
$= 10\ \text{cm}^2$
$\therefore$ Total area $= 40 + 10 = 50\ \text{cm}^2$



b Area I $= l \times b$
$= 4 \times 3$
$= 12\ \text{cm}^2$
Area II $= l \times b$
$= 4 \times 3$
$= 12\ \text{cm}^2$
Area III $= l \times b$
$= 8 \times 3$
$= 24\ \text{cm}^2$
$\therefore$ Total area $= 12 + 12 + 24 = 48\ \text{cm}^2$



c Area I $= l \times b$
$= 4 \times 2$
$= 8\ \text{cm}^2$
Area II $= l \times b$
$= 3 \times 2$
$= 6\ \text{cm}^2$
Area III $= l \times b$
$= 4 \times 2$
$= 8\ \text{cm}^2$
$\therefore$ Total area $= 8 + 6 + 8 = 22\ \text{cm}^2$



d Area I $= \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 12 \times 4$

$= 24 \text{ cm}^2$

Area II $= l \times b$

$= 12 \times 6$

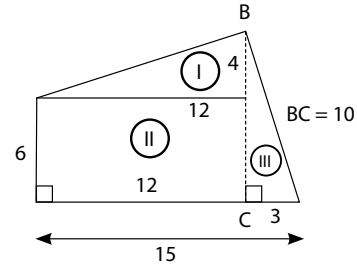
$= 72 \text{ cm}^2$

Area III $= \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 3 \times 10$

$= 15 \text{ cm}^2$

$\therefore$ Total area $= 24 + 72 + 15 = 111 \text{ cm}^2$



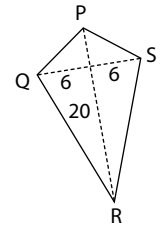
2 **a** Area PQR $= \frac{1}{2} \times b \times \text{h}$

$= \frac{1}{2} \times 20 \times 6$

$= 60 \text{ cm}^2$

PQR ≡ PSR (SSS)

$\therefore$ Total area $= 2 \times 60 = 120 \text{ cm}^2$



PR = 20; QS = 12

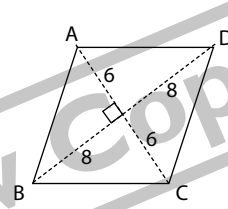
b Area ABC $= \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 12 \times 8$

$= 48 \text{ cm}^2$

ABC ≡ ADC (SSS)

$\therefore$ Total area $= 2 \times 48 = 96 \text{ cm}^2$



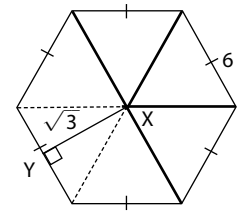
AC = 12; BD = 16

c Area of one $\triangle = \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 6 \times \sqrt{3}$

$= 3\sqrt{3} \text{ cm}^2$

Total area $= 6 \times 3\sqrt{3} = 18\sqrt{3} \text{ cm}^2$



XY = $\sqrt{3}$

3 Diagonals of rhombus bisect perpendicularly.

$\text{AB}^2 = \text{AE}^2 + \text{BE}^2$ (Pyth)

$= 6^2 + 8^2$

$= 100$

$\therefore$ AB = 10 cm

$\therefore$ Perimeter $= 10 \times 4 = 40$ cm

4 **a** Area of rectangle $= l \times b$

$= 10 \times 5$

$= 50 \text{ cm}^2$

Area of square $= s \times s$

$= 2 \times 2$

$= 4 \text{ cm}^2$

$\therefore$ Shaded area $= 50 - 4 - 4 = 42 \text{ cm}^2$; $42 \text{ cm}^2 = 4\,200 \text{ mm}^2$

Review Copy

b Area of square $= s \times s$
$= 120 \times 120$
$= 14\ 400\ \text{cm}^2$

Area of $\triangle = \frac{1}{2} \times b \times \text{h}$
$= \frac{1}{2} \times 40 \times 50$
$= 1\ 000\ \text{cm}^2$

$\therefore$ Shaded area $= 14\ 400 - 1\ 000 = 13\ 400\ \text{cm}^2$; $13\ 400\ \text{cm}^2 = 1{,}3\ \text{m}^2$

5 **a** Area of rectangle $= l \times b$
$= 10 \times 8$
$= 80\ \text{cm}^2$

b $P = 2l + 2b$
$= 2(10) + 2(8)$
$= 36\ \text{cm}$

c Area of rectangle $= l \times b$
$= 14 \times 12$
$= 168\ \text{cm}^2$

d $P = 2l + 2b$
$= 2(14) + 2(12)$
$= 52\ \text{cm}$

6 **a** Area ABCD $= 6 \times 5 = 30\ \text{cm}^2$

b Area EAD $= \frac{1}{2} \times 3 \times 6 = 9\ \text{cm}^2$

c Area DCF $= \frac{1}{2} \times 4 \times 5 = 10\ \text{cm}^2$

d Area FBE $= \frac{1}{2} \times 2 \times 2 = 2\ \text{cm}^2$

e The area outside $\triangle$DEF $= 9 + 10 + 2 = 21\ \text{cm}^2$

f $\triangle$DEF = Area ABCD − shaded area $= 30 - 21 = 9\ \text{cm}^2$

7 **a** $P = 5x$

b $P = 6y$

c $P = 4m + 2n$

Review Copy

UNIT 3

Circles

Unit overview

Learner's Book page 279
Recommended pacing: 2,5 hours

This unit focuses on the following:

- naming parts of a circle, for example, centre, radius, diameter, chord, sector
- investigating the relationship between the circumference and the diameter of a circle to develop a sense of where the irrational number pi (π) is derived from
- deriving a formula for the circumference of a circle
- investigating the relationship between the radius and the circumference of a circle to derive a formula for the area of a circle
- using appropriate formulae for the
  - diameter of a circle: $d = 2r$
  - circumference of a circle: $C = 2\pi r$ or $C = d\pi$
  - area of a circle: $A = \pi r^2$
- using and converting between appropriate SI units

Resources: Learner's Book; exercise book; compass; sheets of paper, compass, ruler, a piece of string (about 1 m long) and a calculator (for Investigation 1); two pieces of paper with 49 squares (each square 1 cm × 1 cm), compass, scissors and glue stick (for Investigation 2)

Review Copy

Mental Maths

(10 minutes per day)

Suggested answers — Learner's Book page 279

1 7; 14; 21; 28; 35; 42; 49; 56; 63; 70
2 1; 2; 11 and 22
3 a 22 b 220 c 88 d 66
4 a 31,4 mm b 8,8 cm c 4,4 cm^2 d 314 cm
e 0,7 m f 0,22 m^2 g 3 140 m h 0,33 km
i 22 million m^2 = 22 000 000 m^2 = 22 km^2

Background information

Learners have had a basic introduction to circles in previous grades. Using formulae and calculating the circumference and area of circles are new. Learners are also introduced to pi, the value of pi and its application in the formulae for circumference and area.

Teaching guidelines

This unit aims to make the introduction to circumference and area of circles as practical as possible, particularly through investigation. The content should be introduced carefully and systematically and learners' understanding of the differences between circumference and area should be monitored at every point.

The numbers used in the mental maths exercise are all numbers that will be used often in the calculations in the rest of the unit, and similar exercises must be given in the mental maths sessions throughout this unit, at the beginning of every lesson.
Revision of conversions between SI units should be an ongoing process.

Aspects of a circle

Activity 1 Identify the parts of a circle

Learner's Book page 280

Guidelines for implementing this activity

- Revise the parts of the circle. It is important to make sure that learner's are clear about the terminology.
- Use diagrams and words to explain the terminology; some learners think better in words and other learners think better in pictures.

Remedial and extension

Let learners draw their own circles showing the different parts, and swap them with a friend to label.

Review Copy

Suggested answers

1 a circle; centre; radius; AD; AB; sector
  b chord; ⊙R; diameter; *r*
2 b and c

PoA | Investigation 2 Investigate the relationship between the diameter and the circumference of a circle

Learner's Book page 280

Guidelines for implementing this activity

- When we calculate the perimeter of a circle, we use the term "circumference" of a circle. In Latin "circum" means "around". The word perimeter is generally used when the edge of the shape is formed by straight lines, for example, quadrilaterals and polygons.

- There are two investigations in this unit. Decide whether one or both will be used for formal assessment.
- Read the instructions with the learners in class. If there are no questions, they should do the investigation at home in their own time. Be very strict about the date that the investigation needs to be handed in.
- The total for the Investigation is 16 marks. If a total of 20 marks is more desirable, the learners can be asked to add the four answers of Activity 2 to the investigation.

Remedial and extension

Remedial: Re-explain the terminology to learners. Have them draw and write the meanings again.
Extension: Learners could be asked to collect four round objects (for example, saucer, empty can, and so on) and work with the outlines of these objects. The challenge here is then to find the diameter. If learners draw circles with compasses (as in the investigation) they will be able to construct the diameter accurately.

Suggested answers

Criteria	4	3	2	1
Task	Completed according to instructions Used 4 circles	Completed mostly according to instructions Used 2-3 circles	Instructions not followed. Used 1 circle.	No attempt made
Measurements	7–8 measurements were taken accurately and filled in	3–6 measurements were taken accurately and filled in	1–2 measurements were taken accurately and filled in	No measurements were taken or filled in
Calculations	All 4 calculations were accurate	2–3 calculations were accurate	1 calculation was accurate	No calculations were accurate
Conclusion	Conclusion is correct	Conclusion is not quite correct	Conclusion is incorrect	No conclusion provided

Review Copy

Finding pi (π)

Activity 2 Work with π

Learner's Book page 282

Guidelines for implementing this activity

- It is necessary to spend time on what the learners concluded in their investigation.
- Note the difference: $\frac{22}{7}$ is a rational number, because it can be written in the form $\frac{\text{whole number}}{\text{whole number} \neq 0}$
- π is an irrational number, because it cannot be written as the ratio of any two numbers.
- The four answers to this activity can be added to the investigation above to give a total of 20 marks.

Remedial and extension

Some Maths teachers celebrate "Pi Day" with their learners each year on the 14 March. They eat something round and do fun activities involving circles.

Review Copy

Suggested answers

1 Greek
2 circumference; diameter
3 irrational
4 circles

Activity 3 Calculate the circumference of a circle

Learner's Book page 282

Guidelines for implementing this activity

- Work through the examples in the Learner's Book; the formula can be used in any form ($C = d\pi$ or $C = 2\pi r$) as long as learners remember whether they are working with a radius or a diameter. Sometimes it is less confusing for learners if they work with only one form of a formula.
- Do additional examples in order to ensure that learners understand the formula and when and how to use it.
- If calculators are available, learners can use the π button on the calculator. It is preferred that once having been introduced and having calculated the value of pi, that learners remember it as: 3,14 or $\frac{22}{7}$.
- Pay attention to the correct rounding off of answers.

Remedial and extension

Remedial: For questions **1c** and **1d**, remind learners about the definition of circumference (perimeter in other shapes). They should begin at one point on the circumference and "walk" around the edge of the shape until they are back at the starting point.
For learners who require remedial as well as extension, provide many additional exercises on the use and application of the formula.

Suggested answers

1 **a** $C = 2\pi r$
$= 2(\pi)(4{,}2)$
$= 26{,}39$ cm

b $C = 2\pi r$
$= 2(\pi)(4{,}5)$
$= 28{,}27$ cm

c $C = 2\pi r$
$= 2(\pi)(30)$
$= 188{,}4955...$ mm for full circle

$\therefore \frac{1}{2}(188{,}4955...) = 94{,}25$ mm for semi-circle

$\therefore C = 94{,}25 + 60 = 154{,}25$ mm

d $C = 2\pi r$
$= 2(\pi)(14)$
$= 87{,}9645...$ cm for full circle

$\therefore \frac{3}{4}(87{,}9645...) = 65{,}97$ mm for $\frac{3}{4}$ of circle

$\therefore C = 65{,}97 + 14 + 14 = 93{,}97$ mm

2 **a** $2\pi r = C$
$2\pi r = 22$
$\pi r = 11$
$r = 11 \div \pi$
$r = 3{,}5$ mm
$d = 2 \times r = 2 \times 3{,}5 = 7$ mm

b $2\pi r = 36$
$\pi r = 18$
$r = 18 \div \pi$
$r = 5{,}73$ mm
$d = 2 \times r = 2 \times 5{,}73 = 11{,}46$ mm

Review Copy

PoA | Investigation 3 Why the formula for the area of a circle is $A = \pi r^2$

Learner's Book page 283

Guidelines for implementing this activity

- The area of a circle is the number of square units inside that circle. This is different from the circumference of a circle that measures the distance all the way around the circle.
- There are two investigations in this unit. Decide whether to use only one or both for formal assessment.
- Many learners find the fomulae for the circumference and the area of circles very confusing. The aim of this investigation is to concretely see why the formula for the area of a circle is $A = \pi r^2$.

- Read the instructions with the learners in class. If there are no questions, they should do the investigation at home in their own time. Be clear about the date on which the investigation needs to be handed in.

Tip

If you access to the Internet, create your own squared paper (or any other format graph paper or dotted paper) at http://www.incompetech.com/graphpaper/lite

Remedial and extension

Make a poster for the classroom. Write the formula for the circumference of the circle around the circle and write the formula for the area of the circle on the shaded part inside of a circle.

Suggested answers

Criteria	4	3	2	1
Task	Completed according to instructions Used 4 circles	Completed mostly according to instructions Used 2-3 circles	Instructions not followed. Used 1 circle.	No attempt made
Measurements	7–8 measurements were taken accurately and filled in	3–6 measurements were taken accurately and filled in	1–2 measurements were taken accurately and filled in	No measurements were taken or filled in
Calculations	All four calculations were accurate	2–3 calculations were accurate	One calculation was accurate	No calculations were accurate
Conclusion	Conclusion is correct	Conclusion is not quite correct	Conclusion is incorrect	No conclusion provided

Review Copy

5 **a** True

b Area of rectangle ABCD = AD × AB.
AD is half of (AD + BD) because AD = BC.
AB is the radius of the original circle.
∴ Area of the circle = AD × AB

$$= \frac{1}{2} \text{ of circumference} \times \text{radius}$$
$$= \frac{1}{2} \times 2\pi r \times r$$
$$= \pi r^2$$

Activity 4 Calculate the area of a circle

Learner's Book page 284

Guidelines for implementing this activity

- Work through the example in the Learner's Book. Point out that $\sqrt{49} = 7$ or -7. We use only the positive answer, because a radius is a length and a length cannot be negative.
- If calculators are available, learners can use the π button on the calculator. Otherwise they must remember what value to use: 3,14 or $\frac{22}{7}$
- Make sure that learners round their answers off correctly.

Remedial and extension

When using the formula for the area of a circle, sometimes say it as: "Area of a circle = π times r times r". This is to help learners understand that only the r is squared.

Suggested answers

1 **a** $A = \pi r^2$
$= \pi\,(4{,}2)^2$
$= 55{,}42\ \text{cm}^2$

b $A = \pi r^2$
$= \pi\,(4{,}5)^2$
$= 63{,}62\ \text{cm}^2$

c $A = \pi r^2$
$= \pi(30)^2$
$= 2\,827{,}4333...$ for full circle
$\therefore \frac{1}{2} \times 2\,827{,}4333...$
$= 1\,413{,}72\ \text{mm}^2$ for semi-circle

d $A = \pi r^2$
$= \pi(14)^2$
$= 615{,}752...$ for full circle
$\therefore \frac{3}{4} \times 615{,}752...$
$= 461{,}81\ \text{cm}^2$ for $\frac{3}{4}$ of circle

2 $A = \pi r^2 = 201{,}06$
$r^2 = 201{,}06 \div \pi$
$= 63{,}999... \approx 64$
$\therefore r = \sqrt{64} = 8$ cm
$d = 2r = 16$ cm

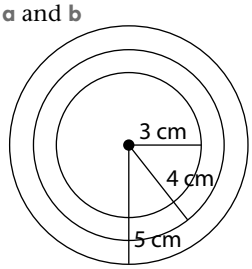
3 $A = \pi r^2$
$\pi r^2 = 78{,}55$
$r^2 = 78{,}55 \div \pi$
$= 24{,}987... \approx 25$
$\therefore r = \sqrt{25} = 5$ mm
$C = 2\pi r$
$= 2\pi(5)$
$= 31{,}42$ mm

4 **a** Convert cm to mm: 2 cm = 20 mm
Area to write on = Area of bigger circle − area of smaller circle
$= \pi(60)^2 - \pi(20)^2$
$= 10\,053{,}1\ \text{mm}^2$

b Convert mm to cm: 60 mm = 6 cm
or Area $= \pi(6)^2 - \pi(2)^2 = 100{,}53\ \text{cm}^2$

Review Copy

5 **a** and **b**



c $A = \pi r^2$
$= \pi(3)^2$
$\approx 28{,}3\ \text{cm}^2$
$A = \pi r^2$
$= \pi(4)^2$
$\approx 50{,}3\ \text{cm}^2$
$A = \pi r^2$
$= \pi(6)^2$
$\approx 113{,}1\ \text{cm}^2$

UNIT 4

Surface area and volume

Unit overview

Learner's Book page 286
Recommended pacing: 5 hours

This unit focuses on the following:

- using appropriate formulae to calculate the surface area, volume and capacity of cubes, rectangular prisms and triangular prisms
- describing the relationship between the surface area and volume of cubes, rectangular prisms and triangular prisms
- solving problems involving surface area, volume and capacity
- using and converting between appropriate SI units

Resources: Learner's Book; exercise book; boxes in the form of rectangular and triangular prisms and cubes

Review Copy

Mental Maths

(10 minutes per day)

Suggested answers — Learner's Book page 286

1 **a** 1×24; 2×12; 3×8; 4×6. Also 24×1; etc.
b 1×36; 2×18; 3×12; 4×9; 6×6. Also 36×1; etc.
c 1×64; 2×32; 4×16; 8×8. Also 64×1; etc.

2 **a** $2 \times 3 \times 4$; $2 \times 2 \times 6$
b $2 \times 2 \times 9$; $2 \times 3 \times 6$; $4 \times 3 \times 3$;
c $2 \times 2 \times 16$; $2 \times 4 \times 8$

3 **a** $A = l \times b$
b $P = 2l + 2b$
c $A = s \times s$
d $P = 4s$
e $A = \frac{1}{2} \times b \times h$
f $P =$ sum of the lengths of the sides
g $A = \pi r^2$
h $C = 2\pi r$ or $C = d\pi$

Background information

Learners have had a very basic introduction to surface area, volume and capacity in previous grades where the focus was on cubes and rectangular prisms. Learners revise some of the different formulae that they have already studied.
Calculating the volume of triangular prisms is new.

Teaching guidelines

It is important to spend time on the description and concept of what a prism is. Collect boxes or solid shapes beforehand (rectangular and triangular prisms and cubes) so that learners can see and handle these shapes.

Use the concept of congruency when explaining what a prism is. If I cut the prism like a loaf of bread, then all the slices will be congruent.

Many learners also find sketches of 3D objects difficult to interpret. Therefore it is extremely important that they work with nets.

When calculating surface area and volume, it is often necessary to calculate the product of two or three numbers. In the mental maths questions learners will practise these calculations.

Prisms

Investigate surface areas of prisms

Activity 1 **Use a net to calculate surface area**

Learner's Book page 287

Review Copy

Guidelines for implementing this activity

- Ideally each learner should have his or her own box to work with.
- Make sure that there are examples of the three different types of prisms as shown in question **1**.
- Ask learners to identify the prism shown in question **1b** (a cube). By making a net, learners will discover that a cube is formed by six congruent faces.
- Follow the instructions to cut the boxes up and calculate the area of each face (ignore the parts that overlap).

Tip

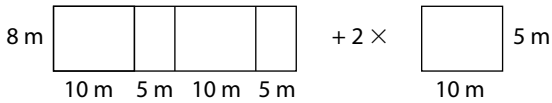
Keep a collection of prisms in the classroom.

Remedial and extension

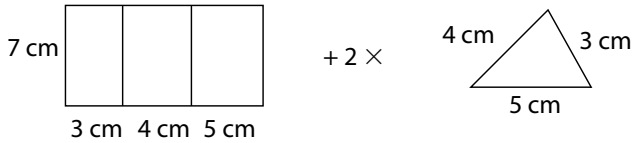
Remedial: Provide additional exercises in the form of nets, with the lengths of sides given. Let learners practise calculating the total surface area in small groups or in pairs.
Extension: Learners do the additional exercises as explained above. Ask learners how they will find the total surface area of a cylinder. Tell them that they will study cylinders in Grade 9.

Suggested answers

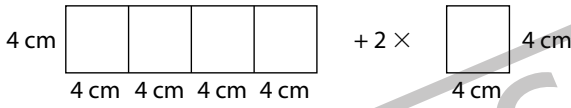
1 a



b



c



2 a Total surface area = $2(10 \times 8) + 2(5 \times 8) + 2(10 \times 5) = 340\ m^2$
b Total surface area = $(12 \times 35) + 2 \times (10 \times 7{,}5) = 570\ cm^2$
c Total surface area = $(16 \times 11) + 2 \times (4 \times 4) = 208\ cm^2$

Review Copy

Activity 2 Use a formula to calculate surface area

Learner's Book page 288

Guidelines for implementing this activity

- After learners have worked with nets they should understand the meaning of the formula and be able to use it in calculations.
- Ensure that learners always write down the formula first when calculating the total surface area. This is good practice throughout.

Remedial and extension

Remedial: Learners can calculate the areas of the different faces if they find it difficult to work with the product of the perimeter of the base and the height of the prism to find the area of the side faces.
Extension: Learners must be encouraged to work with the product of the perimeter of the base and the height of the prism to find the area of the side faces. Provide additional exercises as practice.

Suggested answers

1 a Total surface area of rectangular prism
= perimeter of base × height of prism + 2 × the area of the base
= (2 × 10 + 2 × 5) × 8 + 2 × (10 × 5)
= 340 m^2
Total surface area of cube
b SA = (2 × 10 + 2 × 7,5) × 12 + 2 × (10 × 7,5)
= 570 cm^2
c SA = (4 × 4) × 11 + 2 × (4 × 4)
= 208 cm^2

2 Total surface area of outside of box
= perimeter of base × height of prism + 1 × the area of the base
= (2 × 31 + 2 × 22) × 7,5 + (31 × 22)
= 1 477 cm^2

Volume

Volume, capacity and the SI system

Activity 3 Calculate volume

Learner's Book page 291

Review Copy

Guidelines for implementing this activity

- Spend time on the concept of volume. Learners have worked with volume and capacity using cubes and stacking them, in previous grades, and should be able to explain what volume is in their own words, as well as the difference between volume (the amount of 3D space taken up by a 3D object such as a prism) and capacity (the amount of space inside the prism). Spend some time unpacking this, if necessary. For example, a stack of cubes consisting of four cubes (1 cm^3) in each of three layers has a volume of 12 cm^3 (2 cm × 2 cm × 3 cm), and the capacity of a container into which this stack of cubes can fit is 12 cm^3.
- If they can only recite the formula for volume, they do not necessarily understand the concept. Check their understanding through application of the formula.
- Work through the examples in the Learner's Book.
- The origin of the SI system is very interesting and information about it can be found on the Internet.
- It is important not to teach learners that Volume = length × breadth × height. This is the formula for the volume of a rectangular prism only.
- In Grade 8 we start teaching learners the general formula:
  Volume of a prism = area of the base of the prism × height.
- Make an open box with measurements of 10 cm × 10 cm × 10 cm. Seal the edges as well as possible with sticky tape. Ask learners if they think that all the water from a one litre bottle can be poured into it. Then show them that it can. This will help learners to always remember the conversion between volume and capacity. 10 cm × 10 cm × 10 cm = 1 000 cm^3 = 1 litre. Then it is easy to know that 1 cm^3 = 1 millilitre.
- The conversion between cm^3 and millilitres will be practiced in question **3** of the activity.

Remedial and extension

Remedial: The concepts of volume and capacity can be quite confusing. Repetition will help learners to understand the concepts. Use many more examples to explain the concept and for learners to work through them.
Extension: Learners enjoy hearing stories about maths. Ask them to write a story about the origin of the SI system and link it to the French Revolution that they study in Social Sciences.

Suggested answers

1 a Area of base $= l \times b$
$= 6 \times 19$
$= 114\ \text{cm}^2$
$\therefore$ Volume = area of base $\times\ h$
$= 114 \times 8$
$= 912\ \text{cm}^3$

b Convert first: 20 mm = 2 cm
Area of base $= l \times b$
$= 30 \times 18$
$= 540\ \text{cm}^2$
$\therefore$ Volume = area of base $\times\ h$
$= 540 \times 2$
$= 1\ 080\ \text{cm}^3$

c Convert first: 0,5 m = 50 cm
Area of base $= l \times b$
$= 50 \times 7{,}5$
$= 375\ \text{cm}^2$
$\therefore$ Volume = area of base $\times\ h$
$= 375 \times 7{,}5$
$= 2\ 812{,}5\ \text{cm}^3$

d Area of base $= \frac{1}{2} \times b \times h$ of $\triangle$
$= \frac{1}{2} \times 2 \times 6$
$= 6\ \text{cm}^2$
$\therefore$ Volume = area of base $\times\ h$ of prism
$= 6 \times 9$
$= 54\ \text{cm}^3$

e Convert first: 150 mm = 15 cm
Area of base $= \frac{1}{2} \times b \times h$ of $\triangle$
$= \frac{1}{2} \times 10 \times 4$
$= 20\ \text{cm}^2$
$\therefore$ Volume = area of base $\times\ h$ of prism
$= 20 \times 15$
$= 300\ \text{cm}^3$

Review Copy

f Convert first: 50 mm = 5 cm
Area of base = $s \times s$
$= 5 \times 5$
$= 25\ cm^2$
$\therefore$ Volume = area of base $\times$ h of prism
$= 25 \times 5$
$= 125\ cm^3$

2 a $912\ cm^3 = 912\ ml = 0{,}912\ \ell$
b $1\ 080\ cm^3 = 1\ 080\ ml = 1{,}08\ \ell$
c $= 3\ 750\ cm^3 = 3\ 750\ ml = 3{,}75\ \ell$
d $54\ cm^3 = 54\ ml = 0{,}054\ \ell$
e $300\ cm^3 = 300\ ml = 0{,}3\ \ell$
f $125\ cm^3 = 125\ ml = 0{,}125\ \ell$

3 a Area of base $\times$ h = Volume
$40 \times h = 120$
$h = 120 \times 40$
$h = 3$ cm

b Convert first: 70 mm = 7 cm
Area of base $\times$ h = Volume
$A \times 7 = 343$
$\therefore A = 49\ cm^2$

c Area of base $\times$ h = Volume
$45 \times h = 135$
$\therefore h = 3$ m

d Convert first: $35\ ml = 35\ cm^3$
Area of base $\times$ h = Volume
$A \times 7 = 35$
$\therefore A = 5\ cm^2 = 500\ mm^2$

e Convert first: $64\ kl = 64\ m^3$
Area of base $\times$ h = Volume
But: $4 \times 4 \times 4 = 64$
$A = 16\ m^2$ and height of prism = 4 m

f Convert first: $690\ ml = 690\ cm^3$
Convert first: 230 mm = 23 cm
Area of base $\times$ h = Volume
$A \times 23 = 690$
$\therefore A = 30\ cm^2$

Review Copy

The relationship between surface area and volume

Activity 4 **Investigate relationships between surface area and volume of prisms**

Learner's Book page 292

Guidelines for implementing this activity

- Ask one question at a time and give learners the opportunity to investigate and formulate their own conclusions.
- For question 1 remind learners that a cube has six congruent squares. If they keep this in mind it will take less time to calculate the total surface.
- It is always risky to do Maths in compartments, for example, we calculate total surface areas of prisms and then we calculate volumes of prisms without thinking any further about it.

- In this investigation learners will have the opportunity to practse the two calculations, but then they will also be encouraged to explore the relationships between the total surface area and volume of different prisms.

Remedial and extension

Remedial: Ask learners to continue with the last column in question **1** (ratio SA : V) without doing any further calculations.
Extension: Learners can be asked to predict and verify the shape of the rectangular prism that will have the smallest surface area. (It will be a cube as in question **2** of the activity.)

Suggested answers

1 **a**

Length of one side of cube in m	Total surface area of cube in m^2 (SA)	Volume of cube in m^3 (V)	Ratio SA : V
1 m	$6 \times 1^2 = 6$	$1 \times 1 \times 1 = 1$	6 : 1
2 m	$6 \times 2^2 = 24$	$2 \times 2 \times 2 = 8$	24 : 8 = 6 : 2
3 m	$6 \times 3^2 = 54$	$3 \times 3 \times 3 = 27$	53 : 27 = 6 : 3
4 m	$6 \times 4^2 = 96$	$4 \times 4 \times 4 = 64$	96 : 64 = 6 : 4
5 m	$6 \times 5^2 = 150$	$5 \times 5 \times 5 = 125$	150 : 25 = 6 : 5
6 m	$6 \times 6^2 = 216$	$6 \times 6 \times 6 = 216$	216 : 216 = 6 : 6
7 m	$6 \times 7^2 = 294$	$7 \times 7 \times 7 = 343$	294 : 343 = 6 : 7
s	$6 \times s^2$	$s \times s \times s = s^3$	$6s^2 : s^3 = 6 : s$

Review Copy

b The ratio of Total Surface Area : Volume will always be 6 : s where s is the length of the side of the cube.

2 **a** Volume of A $= l \times b \times h$
$= 4 \times 2 \times 8$
$= 64\ cm^3$
Volume of B $= l \times b \times h$
$= 4 \times 4 \times 4$
$= 64\ cm^3$
Volume of C $= l \times b \times h$
$= 2 \times 2 \times 16$
$= 64\ cm^3$

b Answers will vary.

c Total surface area of A = perimeter of base × height of prism + 2 × the area of the base
$= (2 \times 4 + 2 \times 2) \times 8 + 2 \times (4 \times 2)$
$= 112\ cm^2$
Total surface area of B = 6 × the area of one face
$= 6 \times (4 \times 4)$
$= 96\ cm^2$

Total surface area of C = perimeter of base × height of prism + 2 × the area of the base
$= (4 \times 2) \times 16 + 2 \times (2 \times 2)$
$= 136\ cm^2$

d Answers will vary. There is no ratio as in question **1**.

e Yes, for the cube only 96 cm^2 is necessary.

f The cost will be less for the producer. The price will be better for the consumer. It is better for the environment as using less cardboard means that fewer trees have to be chopped down.

3 a We can do a similar investigation to the one in question 2. Draw sketches of triangular prisms with the same volume. Calculate the total surface area of each prism and see if there is a pattern.

b Learners make predictions.

4 Answers will vary.

Chapter 13 Revision

Learner's Book page 294

Encourage learners to review the content covered before attempting the revision activities. The revision activities should be used to assess learners' progress thus far, and to assess where remediation may be required.

Note: Learners should try to do as many of the questions on their own. Allow learners to make use of their Learner's Books and their exercise books. The learners were not taught how to calculate the perimeter of a rhombus as in question **2a**, but if they know the properties of a rhombus (four equal sides) and have a good concept of what perimeter is, they should be able to calculate the perimeter.

Review Copy

Suggested answers

1 a Area of square $= s \times s$
$= 4 \times 4$
$= 16\ cm^2$

b Area of rhombus: not enough information to calculate.

c Area of triangle $= \frac{1}{2} \times b \times h$
$= \frac{1}{2} \times 8 \times 4$
$= 16\ cm^2$

d Area of rectangle $= l \times b$
$= 8 \times 4$
$= 32\ cm^2$
∴ Square and triangle have equal areas.

2 a Diagonals of rhombus bisect each other perpendicularly.
∴ Length of one side of rhombus = 5 cm (Pythagoras)
$P = 4 \times s$
$= 4 \times 5$
$= 20$ cm

b P = sum of the sides
$= 6 + 7 + 11$
$= 24$ cm

c Perimeter of rectangle $= 2l + 2b$
$= 2(8) + 2(4)$
$= 24$ cm

d Not enough information to find perimeter of triangle.
$\therefore$ Triangle and rectangle have equal perimeters.

3 **a** $c^2 = a^2 + b^2$ (Pyth)
$= 5^2 + 12^2$
$= 169$
$\therefore$ AB = 169 = 13 cm

Radius = $\frac{1}{2}$ of diameter

$\therefore r = 6{,}5$ cm

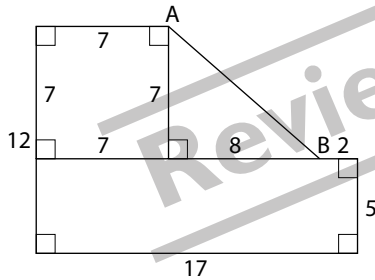
b Convert first: 6,5 cm = 65 mm
$C = 2\pi r$
$= 2\pi(65)$
$= 408{,}41$ mm

c $A = \pi r^2$
$= \pi(6{,}5)^2$
$= 132{,}7$ cm^2

4 **a** $61^2 = 3\ 721$
$60^2 + 11^2 = 3\ 600 + 121 = 3\ 721$
$\therefore 61^2 = 60^2 + 11^2$
It means that the Theorem of Pythagoras holds.
$\therefore$ AC $\perp$ BD.

b $AD^2 = AC^2 + CD^2$ (Pyth)
$= 60^2 + 60^2$
$= 7\ 200$
$\therefore$ AD = $\sqrt{7\ 200}$ m = $60\sqrt{2}$

5 **a**



b Area of rectangle = $l \times b$
$= 17 \times 5$
$= 85$ cm^2

Area of square = $s \times s$
$= 7 \times 7$
$= 49$ cm^2

Area of triangle = $\frac{1}{2} \times b \times h$
$= \frac{1}{2} \times 8 \times 7$
$= 28$ cm^2

$\therefore$ Total area = 85 + 49 + 28
= 162 cm^2

6 **a** Triangular prism: The two triangles are parallel to each other, therefore the base of the prism is a triangle. Also, if we "slice" the prism with a rectangle as the base, the slices will not be congruent (they will be similar). If we "slice" the prism with a triangle as the base, the slices will be congruent.

Review Copy

b Volume of prism = area of the base × height of the prism

Area of base $= \frac{1}{2} \times b \times h$ of triangle

$= \frac{1}{2} \times 15 \times 8$

$= 60 \text{ cm}^2$

$\therefore$ Volume $= 60 \times 10$

$= 600 \text{ cm}^3$

c Total surface area of prism = area of side faces + 2 × area of base

$= (10 \times 8) + (10 \times 15) + (10 \times 17) + 2 \times 60$

$= 520 \text{ cm}^2$

or

Total surface area of prism = perimeter of base × h of prism + 2 × area of base

$= (8 + 15 + 17) \times 10 + 2 \times 60$

$= 520 \text{ cm}^2$

7 Area of one face $= s \times s$

$= 3 \times 3$

$= 9 \text{ cm}^2$

The open box has 5 faces outside and 5 faces inside to paint.

$\therefore$ Area to paint $= 9 \times 10 = 90 \text{ cm}^2$

8 **a** Volume of prism = area of the base × height of the prism

Area of base $= l \times b$

$= 70 \times 50$

$= 3\,500 \text{ cm}^2$

$\therefore$ Volume $= 3\,500 \times 20$

$= 70\,000 \text{ cm}^3$

b $70\,000 \text{ cm}^3$: This answer tells us that the amount of 3D space this prism will take up is $70\,000 \text{ cm}^3$.

c $70\,000 \text{ cm}^3 = 70\,000 \text{ ml} = 70$ litre. This answer tells us that 70 litres is the maximum amount that can fit into this prism.

Review Copy

CHAPTER

14 Data handling

Chapter overview

Learner's Book pages 296 to 327
Recommended pacing: 10,5 hours

This chapter focuses on the following:

Unit 1: Collecting data 1,5 hours
- The data cycle
- Posing questions
- Sources for data collection
- Samples and populations
- Questionnaires

Unit 2: Organising and summarising data 2,5 hours
- The data cycle
- Tables
- Tallies
- Stem-and-leaf plots
- Measures of central tendency
- Measures of dispersion
- Grouped data

Unit 3: Representing data 3 hours
- The data cycle
- Bar graphs and double bar graphs
- Histograms
- Pie charts
- Broken-line graphs

Unit 4: Interpreting, analysing and reporting data 3,5 hours
- The data cycle
- Critically analyse data
- Comparing graphs
- Reporting data

PoA: Project: Data cycle: Task 1–4 – Collect data; Organise data; Represent data; Reporting on Data

Chapter 14 revision

Term 3 Test

Review Copy

UNIT 1

Collecting data

Unit overview

Learner's Book page 297
Recommended pacing: 1,5 hours

This unit focuses on the following:

- posing questions relating to social, economic and environmental issues
- selecting appropriate sources for the collection of data (including peers, family, newspapers, books and magazines)
- distinguishing between samples and populations
- suggesting appropriate samples for investigation
- designing and using simple questionnaires to answer questions with multiple choice responses

Resources: Learner's Book; exercise book; sheets of paper for questionnaires; examples of questionnaires from magazines or the Internet

Review Copy

Mental Maths

(10 minutes per day)

Note: In the process of collecting data, the questions will often include words like "during the past week/month/year".

Leap years: If the year is divisible by four it is a leap year, except if the year is divisible by 100 and not 400, in which case it is not a leap year. This means that 2000 and 2400 are leap years. 1800, 1900 and 2100 are not leap years.

Suggested answers — Learner's Book page 297

1	a 7 days	b 30 days	c 365 days
	d 31 days	e 366 days	
	f 29 days in leap year; 28 days in other years		
2	a 12 months	b 6 months	
3	a 52 weeks	b 13 weeks	
4	a 24 hours	b $24 \times 7 = 168$ hours	
5	a 12 eggs	b 12 dozen eggs = 144 eggs	

Background information

Learners should have developed most of the skills to be covered in this unit, in Grade 7. Posing questions, selecting sources, distinguishing between samples and populations and designing questionnaires are all skills that learners should be familiar with.

Teaching guidelines

This unit can be worked through fairly quickly, with more time spent on designing a sample questionnaire and evaluating it – a skill that learners will require for their project.

Ensure that while working through this unit learners understand the concepts covered since application of these is going to be important during the course of their project.

The data cycle; Posing questions

Activity 1 Ask good questions

Learner's Book page 298

Guidelines for implementing this activity

- Work through the introduction with the learners, focusing on the steps in the data cycle. Learners should be familiar with this cycle, and this part of the lesson could be conducted as revision. Ask key questions such as: Where does the data cycle start? Why are good sources important? What is the difference between a sample and a population? What purpose does a questionnaire serve? What goes into a questionnaire? What are ways in which we could organise data? What are ways in which we could represent data? and so on. These are the types of questions that should lead to dynamic discussion that revises and consolidates prior knowledge.
- Some of the questions in the examples given in the Learner's Book are about concrete things, while other questions are about what people think and believe. Explain to learners that collecting data requires that we respect the opinions and beliefs of other people and that the purpose of collecting data is to gather information related to the question being posed.
- Activity 1 helps learners formulate questions that can be investigated. Remind learners that the data collected is eventually represented in the form of a graph.
- Learners often find this aspect of the data handling cycle challenging: What are the key questions that we can ask that will generate the data that we require? For example, if the statement is: Teenagers spend too much time watching TV, questions can include: How many hours do you watch TV on weekdays? How many hours do you spend watching TV on weekends? Prompt learners by emphasising that this is about asking key questions that will generate information that can be used to reach a conclusion about the statement. Practise with a few additional examples before learners attempt the activity, focusing on no more than two simple questions at a time (as in the example in the Learner's Book).
- Questions asked should relate to social issues (How do teenagers communicate with one another?), economic issues (How do teenagers spend their pocket money?) or environmental issues (How do teenagers participate in recycling?) that are relevant to them.

Review Copy

Remedial and extension

Remedial: Let learners work in pairs or small groups at first, and formulate no more than two questions per statement.
Extension: Instruct learners to formulate three or more questions, and even to extend these questions (see suggested answers on the next page).

Suggested answers

1. How much time did you spend in front of the TV for each day of last week? Can be extended to ask: How much time did the adults in your house spend in front of the TV for each day of last week?
2. What is your favourite type of music to listen to? Can be extended to ask: To what music did our teachers/parents/grandparents listen when they were in Grade 8?
3. What percentage or fraction of each page is filled with advertisements? What is the total percentage of advertisements in the magazine?
4. Ask each teacher: What is the colour of your car? This question can be extended to asking a salesperson at a car dealer: What were the colours of the cars that you sold last year? What were the colours of other vehicles (bakkies, delivery vans, and so on) that you sold last month?
5. What pets do you have? How many pets do you have?
6. How many lights do you have in your house? How many of them are fitted with long life bulbs? How do you get rid of the long life bulbs?
7. How much time do you spend on your hair on a weekday? And on a weekend day? How much time do you spend doing Maths homework during the week? And over the weekend?
8. How much do you spend per day travelling to work? How much do you pay per month for a place to stay? These questions can be extended to include: In what age group are you?
9. What percentage of the newspaper is about sports? Economic news? Social commentary? News of the day? Advertisements? Compare these figures to the percentages of another newspaper.
10. Ask the boys: What chores in the house do you do every day? Ask the same questions to the girls. Also ask: In what age group are you?

Tip

Bring a newspaper or popular magazine to class. Ask the learners to formulate questions that can be investigated. based on statements made in the newspaper or magazine.

Review Copy

Sources for data collection

Activity 2 Identify sources for data collection

Learner's Book page 299

Guidelines for implementing this activity

- Let learners do the activity first. Then discuss each of the sources: How available are they? Can the information gained from them be trusted?
- Point out that we should always acknowledge our source/s: If we do an assignment in any subject, we need to list all the sources that we have used.

Remedial and extension

Remedial and extension: Let learners categorise the data into primary and secondary data sources. Also, let learners explain what sources they could use to answer the questions in Activity 1, and to categorise these sources.

Suggested answers

1 a family members; peers
b newspaper
c TV
d book
e article
f internet
g radio

2 a family members
b book with question papers or internet
c weather forecast on TV, in newspaper or on internet
d book in library or internet
e brochure from different banks or internet

3 If your source is not reliable, you will end up with incorrect or untrue conclusions.

4 Open discussion. If we want to investigate something that happened long ago, we need to make use of secondary data.

Samples and populations

Activity 3 Work with samples and populations

Learner's Book page 300

Guidelines for implementing this activity

- Discuss the difference between a population and a sample. Spend time on the example in the Learner's Book.
- A population is the complete set of all people in a country, town or any region (or just the number of such people).
- By extension, the term is used for the complete set of objects being researched, for example: all cars built by a particular company in the year 2001; all apples sold as Grade A by a particular supermarket; all students in a university; all smokers; and so on.
- A sample is a subset of a population usually chosen in such a way that it can be taken to represent the population with respect to some characteristic.
- If each member of the sample is selected by the equivalent of drawing the lots, the sample is a random sample.
- Ask learners to come up with examples of their own to see if they understand the difference between the two concepts. For example, the population of mothers and babies up to the age of 2 years at the clinic are all the mothers and babies on the clinic's records. A sample would be all the mothers and babies who visited the clinic between 9 a.m. and 1 p.m. on a specified morning. If necessary, prompt learners with examples such as these.

Review Copy

Remedial and extension

Let learners look again at Activity 1, and identify the population for each. Ask them how they would go about choosing a sample/what a representative sample would be.

Suggested answers

1 a Population: 100 soccer players; Sample: 25 players
  b Population: 12 buyers. No sample
2 a No. Learners who visit the swimming pool can mostly swim. Learners who cannot swim will rather stay away from the swimming pool.
  b Yes. His method was good. He did not select certain people or certain houses before the time.
3 If 60 out of 100 learners say they will vote for Cynthia, it can be expected that 60% of the 800 learners (480 learners) will vote for Cynthia. The accuracy of the projection will depend on how the sample of 100 learners was selected.

Questionnaires

Activity 4 Draw up a questionnaire

Learner's Book page 301

Guidelines for implementing this activity

- Remind learners that they are going to draw up their own questionnaires at the end of the unit.
- Discuss the tips in the Learner's Book. Prompt learners for additional tips.
- It will be good if there is an example of a questionnaire available on display in the class.
- Photocopy a blank questionnaire template such as the one provided in the Learner's Book (without the headings and information), and let learners practise drawing up one in pairs, using any of the statements and questions from Activity 1.
- Let pairs analyse/evaluate each other's questionnaires using given criteria decided upon by the class, and which are based on the tips in the Learner's Book.

Review Copy

Suggested answers

1 The second list is better. "Often" or "quite often" is too vague and can be interpreted differently by different people.
2 Question A. Question B is a "leading question", because it leads a person to agree with the statement.
3 a Only three options given as a response. There are learners who do not travel by train, bus or taxi.
  b Add a fourth option: "Other".
4 Open discussion in class.
  a Normally we do not ask for a person's name or address. People might hesitate to give their real opinion if they need to add their names.
  b Be careful to ask questions that can be embarrassing or offensive. This type of question can be asked, but then only in a private situation where there is a relationship of trust.

PoA | Project Data Cycle: Task 1 – Collect data

Learner's Book page 302

Guidelines for implementing this activity

- This is the first task in the process of doing one complete data cycle. The other tasks will be at the end of the subsequent units of this chapter.
- Discuss the types of question that need to be included in the questionnaire.
- Give clear deadlines: one date when the questionnaire should be completed and another date when the entire project should be completed.
- Decide whether the learners can work in pairs or as individuals (see tip below).

Remedial and extension

Remedial: If learners don't know where to start, guide them a little. Each learner should interview at least 20 respondents in order to generate sufficient data to represent in graphs. For this, they will need 20 copies of their questionnaires or they can prepare one data sheet to use. Remind them to adapt the headings in the questionnaire.

Suggested answers

Answers will differ. Use this table as a guide to record your answers.

Respondent	Question 1	Question 2	Question 3
1	M F	A B C D E _____	A B C D
2	M F	A B C D E _____	A B C D
3	M F	A B C D E _____	A B C D
...	M F	A B C D E _____	A B C D

Review Copy

Tip

If two learners work together and use the same questionnaire, they can combine their data for the rest of the project. It will be better to complete the rest of the data cycle based on a larger data set.

UNIT 2

Organising and summarising data

Unit overview

Learner's Book page 303
Recommended pacing: 2,5 hours

This unit focuses on the following:

- organising (including grouping where appropriate) and recording data using tallies, tables and stem-and-leaf displays
- grouping of data into intervals
- summarising data using measures of central tendency, including mean, median and mode
- summarising data using measures of dispersion, including range and extremes

Resources: Learner's Book; exercise book; calculators; a map of South Africa for Activity 1

Mental Maths

(10 minutes per day)

Learners practise some of the calculations that they will often do in the rest of the unit. Using their fingers will help them in question **1**!

Suggested answers — Learner's Book page 303

1 **a** 23 **b** 23,5 **c** 17
d 17,5 **e** 40,5 **f** 40

2 **a** 2 000 – 200 = 1 800 **b** 300 – 30 = 270
c 60 000 – 60 = 59 940 **d** 1 000 000 – 1 000 = 999 000
e 70 – 17 = 53 **f** 900 000 – 9 = 899 991

3 **a** 18 = 3 × 5 + 3 or 18 = 4 × 5 – 2
b 23 = 4 × 5 + 3 or 23 = 5 × 5 – 2
c 46 = 9 × 5 + 1 or 46 = 10 × 5 – 4
d 59 = 11 × 5 + 4 or 59 = 12 × 5 – 1
e 74 = 14 × 5 + 4 or 74 = 15 × 5 – 1
f 35 = 7 × 5

Review Copy

Background information

Learners should have developed most of the skills to be covered in this unit, in Grade 7. Using tally marks, tally tables, stem-and-leaf plots; grouping data into intervals, summarising data using measures of central tendency and measures of dispersion (excluding extremes) are all skills that learners are familiar with.

Teaching guidelines

This is an opportunity for learners to sharpen their knowledge and practise the basic skills they should have acquired in Grade 7 and previous grades, and apply these skills

to their project. Ensure that while working through this unit, learners understand the concepts covered.

The data cycle; Tables

Activity 1 Read information from a table

Learner's Book page 304

Guidelines for implementing this activity

- Introduce the lesson by zooming in on the next step in the data cycle, namely organising and summarising data.
- Use a map of South Africa if available.
- Begin by asking questions, not included in the activity, about the table, for example: What is the distance between Bloemfontein and Plettenberg Bay?
- Learners should do the activity on their own.

Remedial and extension

Remedial: Allow learners to work in pairs.
Extension: Ask learners to estimate the distance between two cities by looking at the map.

Review Copy

Suggested answers

1 769 km
2 1 351 km
3 Oudtshoorn
4 George and Plettenberg Bay
5 From East London to Pretoria: 1 040 km. From Cape Town to Bloemfontein: 1 004 km
∴ Statement is false.
6 a Knysna and Plettenberg Bay b 36 km
7 a Cape Town and Pretoria b 1 460 km
8 From Durban to Kimberley: 811 km
From Kimberley to Plettenberg Bay: 923 km
∴ 1 734 km
9 a Durban b 1 377 km

Tallies

Activity 2 Work with tallies

Learner's Book page 305

Guidelines for implementing this activity

- Learners should be familiar with tally marks and tally tables since they have worked extensively with these in previous grades. Work through the example in the Learner's Book.

- The idea of a tally table is to make the marks by working through the data from beginning to end. The idea is *not* to count how many times a certain item appears and then to make the corresponding number of tally marks. Ask learners to explain how Theodore made the marks in the example.
- Revise what an interval is, and why Mr Newman chose to divide the marks into these intervals (question **5**). When working through this question, it should become even clearer to learners why intervals were used. Check their understanding again.

Remedial and extension

Activity 2 provides good practice and extension (using tally marks).
Discuss every tally table that the learners have completed, for example, in question **1**: In most cars there are only one person. Why is that? What is the effect of this on traffic? The cost of transport? Question **2**: How many learners are in the class? Can the teacher be satisfied with the results?

Suggested answers

1

Number of people in car	Tally	Frequency
1	卌 卌 |||	13
2	卌 卌	10
3	卌 ||||	9
4	卌	5
5	||	2
6	|	1

Review Copy

2

Marks out of 20	Tally	Frequency
1 to 5	卌	5
6 to 10	卌 ||	7
11 to 15	卌 卌 ||||	14
16 to 20	卌 卌	10

3

Scary animals	Tally	Frequency
Crocodiles	卌 卌 |	11
Spiders	卌 卌 卌 卌	20
Snails	卌 ||||	9

Stem-and-leaf plots

Activity 3 Work with stem-and-leaf plots

Learner's Book page 307

Guidelines for implementing this activity

- Ask learners to give numbers smaller than 50 to form a data set. Explain what a data set is. Use these numbers and show learners how to draw a stem-and-leaf plot. It should become clearer to them what a data set is as they work through the activity. Check their understanding again.
- The idea of a stem-and-leaf plot is to once again work with the data set from the first number to the last number. This means that the stem-and-leaf plot will not be ordered. A second stem-and-leaf plot can be drawn to order the data.
- Emphasise the importance of the use of a key with a stem-and-leaf plot.
- Remind learners to keep in mind that there is a reason why data is collected (and then organised). Data should be user-friendly for the people who want to use it. It may be better to group the data, as in question **3** of the activity.

Remedial and extension

Discuss learners' stem-and-leaf plots: how they've organised the data in the data sets; why they have organised the data in this way; does it make it easier to work with/use?; is it more user-friendly than the initial data set?; and so on. This serves to revise the work that they have done, to help them look critically at what they have done and therefore crystallise their understanding of the process and the tools they have used.

Review Copy

Suggested answers

1 **a** The data in Set P is ordered. The data in Set Q is not ordered.

b

Stem	Leaves
2	4 6 6 9
3	5 7 9
4	0 1
5	5 8

Key: 2 | 4 = 2,4

c

Stem	Leaves
0	7
1	
2	4 9 2
3	3 0
4	9 8 6
5	1 4

Key: 5 | 1 = 51

d

Stem	Leaves
0	7
1	
2	2 4 9
3	0 3
4	6 8 9
5	1 4

Key: 5 | 1 = 51

2 **a** 11 **b** No **c** 495; 490; 493
d Three: 481 **e** 459

3 **a** First plot: Numbers from 10 to 19 in one row.
Second plot: Numbers from 10 to 19 in two rows.

b The first one is not easy to read. The second gives a better idea of how many jumps the learners in the class can do.

Stem	Leaves
1	2 4
1	5 5 6 7 8 8 9 9 9
2	0 1 1 1 2 2 3 3 3
2	5 5 5 5 6 6 6 6 7 7 7 7 8 8 8 9 9 9 9
3	0 0 1 2 3 3 4 4 4 4
3	5 5 5 5 6 6 7 7 7 7 7 8 8

Key: 1 | 2 = 12

Review Copy

Measures of central tendency

Activity 4 Calculate measures of central tendency

Learner's Book page 308

Guidelines for implementing this activity

- In real life data sets are very, very big. It is impossible to simulate that in a classroom. In the classroom we work with very small data sets.
- Spend time on the explanation of the concept "Measures of central tendency".
- Measure as a verb means to find the size or amount of something; central is in the middle; tendency is the direction in which something moves, or a trend. The measures of central tendency will show us how the data tend to cluster together.
- Do not assume that all the learners have worked with mean, median and mode before. Revise these concepts thoroughly.
- Be very strict if learners write calculations as follows:
  Mean = 91 + 140 + 140 + 160 + 163 + 170 + 179 ÷ 7 = 149
  Written like this, it means that only the 179 needs to be divided by 7.
  Correct calculation:
  Mean = (91 + 140 + 140 + 160 + 163 + 170 + 179) ÷ 7 = 149; or do the calculation in two steps as shown in the Learner's Book.
- The mean is often referred to as the arithmetic mean to distinguish it from other means than can be calculated, for example, geometric mean.

- Remind learners that the data needs to be ordered before the *median* can be calculated.
- Mode: If there is no *mode*, the answer is "No mode". Do *not* write 0 (zero).
- Encourage learners to check their answers, for example, in questions 2, 3 and 4.

Suggested answers

1 a Mean = 109 ÷ 9 = 12,1
Median = 12
Mode = 11

b Mean = 2 500 ÷ 10 = 250
Median = $\frac{1}{2}$ (200 + 300) = 250
Mode = 200 and 300

c Mean = 434 ÷ 7 = 62%
Median = 63%
No mode

d Mean = 41,1 ÷ 8 = 5,1
Median = $\frac{1}{2}$ (4,2 + 5,8) = 5
Mode = 8,6

2 a Mean = (31 + 14 + 18) ÷ 3
= 21

b $(x + 25) \div 2 = 21$
$\therefore x + 25 = 42$
$\therefore x = 17$

3 Mean of 50 after 4 matches: needs 200 in total.
Already has (53 + 46 + 49) = 148 runs
∴ should score 200 − 148 = 52 runs.

4 a Total for ten learners = 10 × 81 = 810

b Total for eight learners = 810 − 93 − 93 = 624
∴ Mean for eight learners = 624 ÷ 8 = 78%.

5 a Mode = 9 km

b Mean = 323 ÷ 21 = 15,4 km

c Median = 9 km

d Mean = 233 ÷ 21 = 11,1 km

e Median = 9 km

f Yes: the total of kilometres changed from 323 km to 233 km. The new mean is less than the first one.

g No: the middle number of the data set is still the same.

Review Copy

Measures of dispersion

Activity 5 Calculate measures of dispersion

Learner's Book page 310

Guidelines for implementing this activity

- Spend time on the explanation of the concept "Measures of central dispersion".
- Measure as a verb means to find the size or amount of something; disperse as a verb means to scatter something, to spread it out. Measures of dispersion will show us how scattered or spread out the values of a data set is.

- Work through the introduction and example in the Learner's Book, carefully explaining what minimum and maximum mean and checking that learners understand these.
- Tell learners that they will learn about more measures of dispersion in the years to come (for example, quartiles and standard deviation).

Suggested answers

1 151 − 62 = 89 km
2 a 68 − 47 = 21
  b Range = highest score − lowest score
  ∴ 19 = highest score − 58
  ∴ highest score was 77%
3 a Minimum is 28% and maximum is 79%.
  (range = 79 − 28 = 51%)
  b increase (range will be 79 − 18 = 61%)
  c stay the same (maximum still is 79%)

Tip

Use the words mean, median, mode and range when you hand out test results to the learners. Let learners use the scores as a data set and calculate the mean, median, mode and range.

Activity 6 Work with median, mode, mean and range

Learner's Book page 310

Review Copy

Guidelines for implementing this activity

- The purpose of the activity is to practise calculating and interpreting the measures of central tendency and the measures of dispersion.
- This is an opportunity for the learners to see how many of the questions they can do without any assistance.

Suggested answers

1 a Range = 65,1 − 47,5 = 17,6 kg
  b Mean = 597,9 ÷ 11 = 54,4 kg
  Median = 54,1 kg
  Mode = 47,5 kg
2 a Median = 15
  b 5 learners
  c Mean = (17 + 17 + 18 + 19 + 20) ÷ 5 = 18,2
3 a Range A = 25 − 10 = 15
  Range B = 25 − 10 = 15
  Both data sets have the same range.
  b Mean A = 98 ÷ 7 = 14
  Mean B = 163 ÷ 8 = 20,4
  Big difference in means – even though the sets have the same ranges.

c Median A = 12

Median B = $\frac{1}{2}(20 + 22) = 21$

The medians show that the central values of the data sets are very different.

d Mode A = 11
Mode B = 19 and 25
Data Set A has one mode and Data Set B has two modes: both of them higher than the mode of Data Set A.

4 a He calculated the range.
b Albert's dad only looked at the extremes of the marks. He did not look at the other marks on the reports.
c His dad should also calculate the mean of the codes.

5 Athini: The mean number on the cards is 126 ÷ 9 = 14.
Portia: The mode of the numbers is 36.
Zubair: The median number is 49.

Tip

Learners will interpret, analyse and report on data in Unit 4, but they can already begin to look at and interpret the answers they have calculated. Remind them repeatedly that data is always collected with a purpose in mind.

Grouped data

Activity 7 Work with grouped data

Learner's Book page 312

Review Copy

Guidelines for implementing this activity

- Discuss the worked example in the Learner's Book.
- Draw up a similar table and calculate the measures of central tendency again.
- In the activity the learners will go one step further by grouping the data in intervals and then using the midpoint of the interval for calculations.
- Learners should be careful when they write down the boundaries of intervals. "1 to 10" and "10 to 20" means all the values from 1 to 10 in the first interval and all the values from 10 to 20 in the second interval. A value of 10 is included in both intervals and this is incorrect. Rather work with intervals of "1 to 10" and "11 to 20".
- To find the mode of grouped data requires a formula that is beyond the level of Grade 8 learners.
- Learners in Grade 8 need only identify the interval in which the mode is, also referred to as the modal class.
- If we work with grouped data in a table, we do not have access to the values of the original data. It is therefore impossible to find the exact value of the range.

- Discuss the meaning of an answer "1,5 portions per day" or "5,1 books read". It is a statistical answer and we use it in this way. This does not mean that you can go into a shop and buy 1,5 apples or 5,1 books.

Suggested answers

1

No of portions (x)	Tally	Frequency (f)	$f \cdot x$
0	卌	5	$5 \times 0 = 0$
1	卌 ||||	9	$9 \times 1 = 9$
2	卌 |	6	$6 \times 2 = 12$
3	||||	4	$4 \times 3 = 12$
4		0	$0 \times 4 = 0$
5	|	1	$1 \times 5 = 5$
	Total = 25	Total = 38	

Mean = 38 ÷ 25 = 1,5 (rounded off to one decimal digit)
Mode = 1 fruit/vegetable per day
Median = 1 fruit/vegetable per day (Learner number 13)

2

Number of books	Frequency (f)	Midpoint of interval (x)	$f \cdot x$
1 to 3	68	2	$68 \times 2 = 136$
4 to 6	47	5	$47 \times 5 = 235$
7 to 9	34	8	$34 \times 8 = 272$
10 to 12	21	11	$21 \times 11 = 231$
Total = 170		Total = 874	

Modal class: Interval 1 to 3
Mean = 874 ÷ 170 = 25 = 5,1 book (rounded off to one decimal digit)
Median: In interval 4 to 6 (Median is the average of the number of books read by learner number 85 and learner number 86.)

Review Copy

PoA | Project The data cycle: Task 2–Organise and summarise data

Learner's Book page 313

Guidelines for implementing this activity

- Learners should now do the next task in completing their own data cycle.
- The example in the Learner's Book is based on the example about the use of cellphones as in Unit 1. Adapt it to the topics selected by the learners, for example in the given example the respondents are divided into two groups based on gender. It is possible that respondents can be divided according to age group.

- The data of the two tables will be used to draw graphs in the next unit.
- Learners need to know the date that they should hand in this task.

Suggested answers

Answers will depend on the data the learners collected.
Use the rubric on page 307 of the Teacher's Guide to assess the project.

UNIT

3 Representing data

Unit overview

Learner's Book page 314
Recommended pacing: 3 hours

This unit focuses on the following:

- drawing a variety of graphs by hand or using technology to display and interpret data, including:
  - bar graphs and double bar graphs
  - histograms with given and own intervals
  - pie charts
  - broken-line graphs

Resources: Learner's Book; exercise book; protractor; pair of compasses or round object to draw pie charts; grid paper to do graphs

Review Copy

Mental Maths

(10 minutes per day)

When drawing pie charts, learners will use calculations that involve fractions and percentages. The mental maths exercise prepares learners for that. Pay attention to the correct units in the answers.

Suggested answers — Learner's Book page 314

1	a	50%	b	25%	c	75%
	d	20%	e	60%	f	70%
2	a	180°	b	180°	c	90°
	d	90°	e	270°	f	270°
3	a	120	b	60	c	150
	d	12	e	100	f	2,5

Background information

Learners have worked extensively with bar graphs, histograms and pie charts in previous grades. Broken-line graphs are new in the context of data handling.

Teaching guidelines

It is important to keep the bigger picture in mind. Use the flow chart to help the learners understand where they are in the process and ensure that the following information is revised and consolidated about each type of graph covered in this unit.

Teaching bar graphs and double bar graphs

- A bar graph represents a number of observations in a given category, for example, the favourite colour of all the learners in the class.
- Bar graphs show the frequency of each data item.

A bar graph is used to represent discrete data. Data is discrete if it can only take certain numbers, for example, the number of sweets in a packet or the number of parents attending a meeting. (There cannot be half a parent.)

- If technology is available, learners can make use of that to further investigate bar graphs and double bar graphs. It is important, however, that they are able to do the graphs by hand.
- Focus on two aspects: the details of how to draw the graph and what information we can read from the graph.
- Always ask learners to interpret the graphs they have drawn.

Teaching histograms

- A histogram is used to represent data that is grouped into intervals.
- A histogram is used for continuous data. Continuous data is data that can be any value within a range, for example, the height of plants after three months or the amount of time it takes to open a can. (The answer can be a decimal number.)
- The size of each bar (rectangle) reflects the proportion of data points in that interval.

Review Copy

If technology is available, learners can make use of that to further investigate histograms. It is important, however, that they are able to do a histogram and the necessary calculations for it by hand.

- Focus on two aspects of histograms: the details of how to draw the graph and what information we can read from the graph.
- Always ask learners to interpret the histograms they have drawn.

Tip

The focus is on the representing (drawing) of the graph, but it is an opportunity to ask interpretive questions, for example:

- What is the difference between a bar graph and a histogram? (No spaces between the bars of a histogram.)
- Why is there a gap on the horizontal axis? (No values between 0 and 20.)
- How many learners scored between 31% and 40%? (15 learners)
- Can we tell what the highest mark in the class was? (No, we only know that it is between 71% and 80%)
- How many learners scored more than 50%? (25 + 10 + 5 = 40)
- What is the modal class? (41%–50%)

Teaching pie charts

- Pie charts are circles that are divided into a number of sectors ("slices").
- The circle represents the whole, for example, 100% or 1 unit.
- A pie chart is often used to show proportions, for example, how 100% of the expenses was divided or how the hours of the day were used.
- Do the necessary revision about fractions and percentages and the fact that there is 360° in a revolution (circle).
- Focus on two aspects: the details of how to draw the graph and what information we can read from the graph.
- If technology is available, learners can make use of that to further investigate pie charts. It is important, however, that they are able to do pie charts by hand.
- Always ask learners to interpret the pie chart they have drawn.

Broken-line graphs

- Broken-line graphs can show trends over time better than most other graphs, for example, the rainfall pattern over 12 months or the number of new HIV infections over six months.
- We can also use broken-line graphs to compare two similar situations, for example, the maximum temperatures of two different cities.
- Keep in mind that the broken-line graph may be new to the majority of the learners.
- Focus on two aspects: the details of how to draw the graph and what information we can read from the graph.
- If technology is available, learners can make use of that to further investigate broken-line graphs. It is important, however, that they are able to do the graph by hand.
- Always ask learners to interpret the graph they have drawn.

Review Copy

The data cycle; Bar graphs and double bar graphs; Histograms; Pie charts; Broken-line graphs

Activity 1 Decide which graph to use

Learner's Book page 318

Guidelines for implementing this activity

- Zoom in on this step of the data cycle, and introduce the different types of graph.
- The purpose of the activity is to give learners an overview of the characteristics of the different types of graph.
- Work carefully through the information and worked examples given on the different types of graph using the information provided in the Learner's Book to ask the key questions about their different features.

Suggested answers

1 C 2 E 3 D 4 B 5 A

Activity 2 Draw graphs

Learner's Book page 318

Guidelines for implementing this activity

- Learners should be able to draw the different types of graphs.
- Learners may require assistance calculating or deciding on appropriate scales for the axes.
- Emphasise the importance of titles, labels for the axes, appropriate scales and keys where necessary.
- Ask learners what information they can read from each graph.
- Questions to discuss are as follows.
  - Give possible reasons why there are fewer girls enrolled than boys. Why do so many learners drop out of school?
  - What is volunteer work? Where can you do volunteer work in your community?
  - Which places can be included in the category Other? Why do some learners bully other learners? Is there a system in place in our school to address bullying?
  - How can the killing of rhino be stopped? How many rhinos were killed since the table was drawn up? (Find updated information in the Internet.)

Tip

Questions on the vertical bar graph and horizontal bar graph in introduction
The focus is on representing (drawing) the graph, but it is an opportunity to ask interpretive questions, for example:

- Which graph is a vertical bar graph? (Craig's sales)
- Which graph is a horizontal bar graph? (Carlynne's sales)
- Who sold more large drinks? (Craig)
- Who sold more small size drinks? (Carlynne)
- Did the smaller kids buy from Carlynne or from Craig? (Probably from Carlynne)
- Which graph is easier for you to read?

Review Copy

Tip

Questions on the double bar graph in the introduction
The focus is on representing (drawing) the graph, but it is an opportunity to ask interpretive questions, for example:

- What is a museum? (It is a building in which cultural and historical objects are collected and preserved.)
- How do we know what each bar is showing us? (Look at the key)
- What is an adult? (Grown-up person)
- On what days were there more adults than children? (Wednesday)
- On what days were there more children than adults? (Monday and Tuesday)
- On what day where there an equal number of adults and children? (Thursday)
- What is the total of the children who visited the museum on the four days? (10 + 20 + 35 + 25 = 90 children)

Tip

Questions on the pie chart in the introduction
The focus is on representing (drawing) the graph, but it is an opportunity to ask interpretive questions, for example:

- Arrange the favourite pastimes in order from the most popular to the least popular. (Playstation; Xbox; Wii)
- True or false: Playstations are double as popular as Wii. (False)

Review Copy

Tip

Broken-line graphs in the introduction
The focus is on the representing (drawing) of the graph, but it is an opportunity to ask interpreting questions already, for example:
(Questions about temperature of patient)

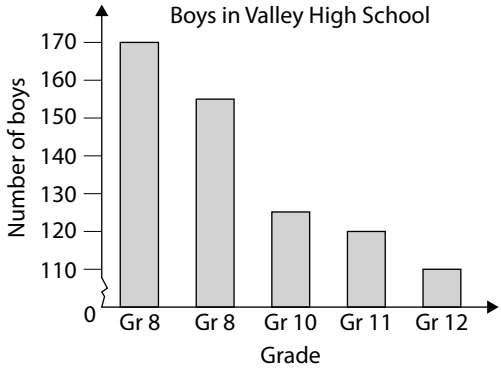
- When was the temperature the highest? (5 p.m.)
- What was the patient's temperature at 3 p.m.? (37,5 °C)
- Is the patient getting better or worse? (Better – the temperature is showing a downward trend.)

(Questions about the salespeople)

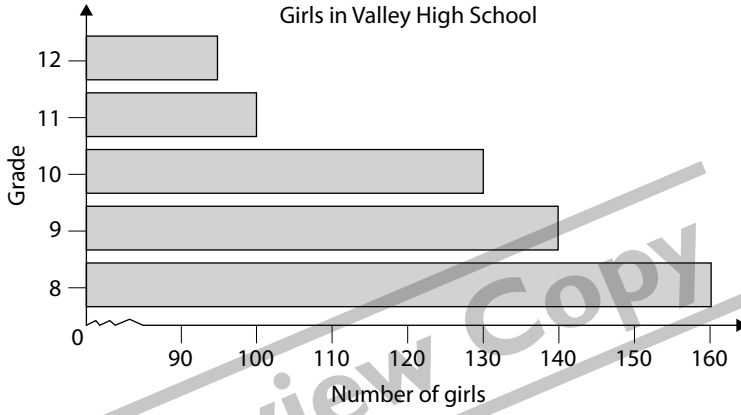
- What was Jabu's sales in April? (R2 500)
- In what month did Senzo have sales of R3 000? (March)
- What was the difference in the sales of the two salesmen in February? (R4 000 – R1 500 = R2 500)
- Describe Jabu's pattern of sales. (up and down)
- Describe Senzo's pattern of sales. (going upwards steadily)
- Who is the better salesperson? (discuss and allow different opinions)

Suggested answers

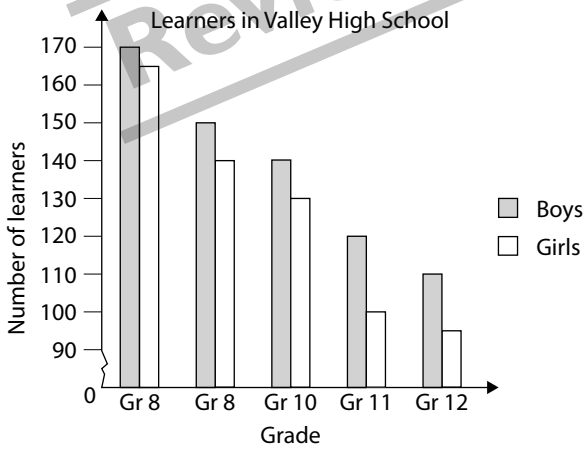
1 a



b

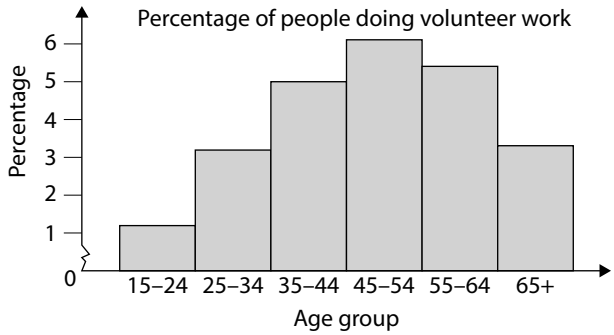


c



Review Copy

2

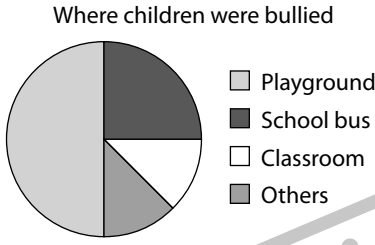


More information can be found on the Internet.

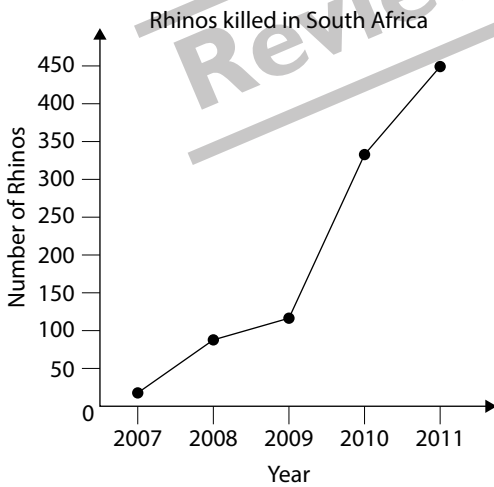
3 **a** Range = 270 – 90 = 180
∴ Width of interval = 180 ÷ 4 = 45
Intervals: 90 – 135; 136 – 180; 181 – 225; 226 – 270

b Range = 270 – 90 = 180
∴ Width of interval = 180 ÷ 5 = 36
Intervals: 90 to 126; 127 to 162; 163 to 198; 199 to 234; 235 to 270

4



5



Review Copy

Tip

Do not give learners graph paper with squares that are too small. Grid paper with squares of 1 cm × 1 cm works well. Design your own graph paper using helpful sites on the Internet.

PoA | Project **Data cycle: Task 3 – Represent data**

Learner's Book page 319

Guidelines for implementing this activity

- Learners should now do the third task in completing their own data cycle.
- The example in the Learner's Book is based on the example about the use of cellphones as in Units 1 and 2. Adapt to the topics selected.

Suggested answers

Answers will depend on the data that the learners collected.

UNIT 4

Interpreting, analysing and reporting data

Review Copy

Unit overview

Learner's Book page 320
Recommended pacing: 3,5 hours

This unit focuses on the following:

- critically reading and interpreting data represented in words, bar graphs, double bar graphs, pie charts, histograms and broken-line graphs
- critically analysing data by answering questions related to data categories (including data intervals), data sources and contexts, central tendencies (mean, mode and median), scales used on graphs, samples and populations, dispersion of data, error and bias in the data
- summarising (reporting) data in short paragraphs that include drawing conclusions about the data, making predictions based on the data, identifying sources of error and bias in the data, choosing appropriate summary statistics for the data and the role of extremes in the data

Resources: Learner's Book; exercise book

Mental Maths

(10 minutes per day)

To calculate the scale on the axis of a graph, learners need to find multiples quickly. They will practise this skill in the mental maths of this unit.

Suggested answers — Learner's Book page 320

1 a 20; 30; 40; 50; 60; 70; 80
  b 20; 25; 30; 35; 40; 45; 50; 55; 60; 65; 70; 75; 80
  c 20; 24; 28; 32; 36; 40; 44; 48; 52; 56; 60; 64; 68; 72; 76; 80
2 a 100; 90; 80; 70; 60; 50; 40
  b 100; 95; 90; 85; 80; 75; 70; 65; 60; 55; 50; 45; 40
  c 100; 93; 86; 79; 72; 65; 58; 51; 44
3 a 26 is not a perfect square.
  b 1 is not a prime number.
  c $\frac{2}{3}$ has a different value; all the others are equal.
  d 7% has a different value; all the others are equal.

Background information

- It is important to keep the bigger picture in mind.
- Use the flow chart to help the learners understand where they are in the process.
- This unit deals with the last task in the data cycle.

Teaching guidelines

The study of data handling in CAPS is intended to show learners how the statistical cycle helps in solving problems/questions in different situations (contexts).

Review Copy

The data cycle; Critically analyse data

Activities 1–2 Analyse data

Learner's Book pages 321–322

Guidelines for implementing this activity

- Discuss situations in everyday life where we have to collect and analyse data before we make a decision, for example before opening a bank account or before signing a contract.
- Data can be collected and represented in inaccurate ways. "Lies, damned lies, and statistics" is a well-known phrase to keep in mind. If learners are made aware of factors like context and bias, they will be empowered to critically analyse data and representation of data. Explore these two concepts with them and ask them to provide examples. Context can be explained as the situation in which the problem or question arises. For example, there are many problems/questions that require data collection, analysis, and interpretation within the HIV context, such as infection rates in a province, most affected age groups, and so on. Let learners look up the word bias in the dictionary, and allow them to offer their understanding of what bias means in data handling contexts.

- Explain the meaning of the word critical to the learners: we do not have to be aggressive or mean, to be critical. It means that we think logically and objectively, and ask good questions.
- Discuss the activity with learners after they have done it. The data in this table is an opportunity to enhance respect for one another.

Remedial and extension

Remedial: Ask learners to round off the figures given to the nearest percentage when they answer question **2**.

Suggested answers

Activity 1

1 **a** Gender **b** Race **c** Housing in cities or towns
d Housing in rural areas **e** Income

2 Have an open discussion in class on the effect of context on data.

Activity 2

1 **a** Double bar graph
b They wanted to compare their actual income to the target that they had set for themselves.
c False: Their target was R100 for the first 5 months and R150 for the last month.
d 5 × R100 + R150 = R650
e Month 1: R100 – R80 = R20; Month 5: R100 – R90 = R10
f Income for 5 months = R80 + R110 + R130 + R120 + R90 = R530
They need R650 – R530 = R120 in the last month
g Answers will vary.

2 **a** 10 + 19 + 12 + 16 + 8 = 65 learners **b** 12 + 16 = 28 learners
c More than 20 minutes = $\frac{8}{65} \times 100 = 12{,}3\%$
d The modal class is the interval with the highest number of learners: 6 to 10 minutes. There are 19 learners in this interval.
e Use the midpoint and frequency of each interval to find the total.
Total = ? = 810; Mean = 810 ÷ 65 = 12,5 minutes
f Median time is the time for learner number 33.
Learner number 33 will be in the interval 11 to 15 minutes.
g Answers will vary.

Review Copy

Comparing graphs

Activity 3 **Compare differences in graphs and data sets**

Learner's Book page 323

Guidelines for implementing this activity

- The aim of the activities is to develop critical thinking. Let learners study the two graphs in Activity 2 question **1** based on the questions: What is different about the two graphs? What is the same? Discuss their responses before they proceed to do question **1** and the rest of the activity on their own.

- Learners need to give reasons for their answers. It is possible that there is more than one interpretation.
- Pause and explain to learners what it means to comment on a statement and what is expected of them when they are asked to do so. They should consider the following when asked to comment:
  - Does this comment relate to the given data/representation?
  - Is this statement a fact or an opinion?
  - Is this statement true in relation to the given data/representation?
  - Do I agree/disagree with this statement?
  - Why do I agree/disagree?
  - If I agree/disagree, what evidence can I produce to support my view/opinion? (Use the given information to support a different view.)

Remedial and extension

Work with small groups of learners. Clarify the questions with them. This might mean having to rephrase questions, checking what they understand by the questions, letting them rephrase the questions themselves, and so on. Often learners know what to do but need support in understanding what the question is asking. If they need support with knowing how to answer the questions, zoom in on this by referring them back/doing quick revision.

Review Copy

Suggested answers

1 a He changed the scale on the vertical axis. He changed the width of the bars.
  b Conclusion 1: Mr Scrooge will use Graph I if he tells the bank manager that the sales was good and therefore he will be able to pay back a loan easily.
  Conclusion 2: Mr Scrooge will use Graph II if he tells his workers that the sales were bad and therefore he cannot pay bonuses.

2 a No scale on vertical axis and dates on horizontal axis from 2012 to 2010: it seems as if the investment has grown, but it has not.
  b Did not show all the subjects: the code for maths is missing.

3 On the bar graph it is clear to see that there are 11 players in a hockey team and that no one wears size 6 shoes. The pie chart does not show the information as clearly.

4 a i There are learners in secondary schools who are older than 17 years.
  ii Learners have different opinions of what is far from school, therefore the researchers had to give a clear definition for "far from school". The definition was based on the time it takes to travel to school and not the distance in km.
  b Percentages differ: Highest in Eastern Cape (23,7%) and lowest in Northern Cape (12%).
  c Provinces that showed improvement from 2002 to 2010 (percentages are lower): Eastern Cape; Free State; Gauteng; North West; Northern Cape; Western Cape. The percentages in the other provinces are higher, which means that more learners now are far from their schools than in 2002.
  d Range for 2002 = 31,5 – 12 = 19,5%
  Range for 2010 = 34,4 – 8,8 = 25,6%
  e The range is bigger: it means the differences between the provinces is bigger. In both cases the maximum value was from KwaZulu-Natal and the minimum value was from Northern Cape.

f Mean for 2002 = 178,5 ÷ 9 = 19,8%
Mean for 2010 = 144,2 ÷ 9 = 16%
Median for 2002 = 18,7%
Median for 2010 = 19,1%
No mode for 2002
No mode for 2010

g Although the range became wider from 2002 to 2010, the mean was lower in 2010 than in 2002. This is good news.
The median was lower in 2002 than in 2010. This is partly because of the wider range in 2010.
There is no mode – not even if you look at the data in both sets.

h The mean: it reflects the fact that there was an improvement in six of the nine provinces.

Reporting data

Activity 4 Report on data

Learner's Book page 325

Guidelines for implementing this activity

- The aim of reporting is to explain the trends and patterns in the data that was collected.
- Explain to learners that a report should include predictions or recommendations. The people (or institution) who asked for the study can then use the report to make decisions. This is the full data cycle.
- Discuss the example in the Learner's Book.
- Make sure the learners understand the different words used: a conclusion is made after you have done an investigation; a prediction is about the future; and a recommendation is a suggestion.
- Again, when asked to comment on a statement (question 2) the following should be considered:
- Pause and explain to learners what it means to comment on a statement and what is expected of them when they are asked to do so. They should consider the following when asked to comment:
  - Does this comment relate to the given data/representation?
  - Is this statement a fact or an opinion?
  - Is this statement true in relation to the given data/representation?
  - Do I agree/disagree with this statement?
  - Why do I agree/disagree?
  - If I agree/disagree, what evidence can I produce to support my view/opinion? (Use the given information to support a different view.)
- The above will become increasingly important as learners progress to subsequent grades.

Review Copy

Remedial and extension

Work with small groups of learners. Clarify the questions and the structure of the report with them. This might mean having to rephrase questions, checking what they understand by the questions, letting them rephrase the questions themselves, and so on. Often learners know what to do but need support in understanding what the question is asking or how to phrase sentences in a report. If they need support with knowing how to answer the questions, zoom in on this by referring them back/doing quick revision. If they need support with formulating their ideas/sentences in their report, let them verbalise it (using you as a sounding board), check the relevance/suitability of their thinking/idea (with prompting) before writing them down. This is important for when they have to write their reports for their project.

Suggested answers

1 Report on world record in 20 km walk
Date: 24 May 2012.
Source: The internet. (http://www.athletix.org)
Analysis: The record times to walk 20 km become less and less for men and women. The women improve at a faster rate than the men.
Prediction: It is possible that the records for men and women can be the same in near future, or the record time for women can be faster than the record time for men.

2 a The learners of School A had the highest number of distinctions, but their pass rate was the lowest of all the schools. Only 50% of the learners passed Maths. School A was not the best if you look at the bigger picture.
b School B is a smaller school. The 16 learners that passed Maths represent 80% of the learners who took Maths. So it was a good year.
c It will be good if the learners of School C can achieve distinctions next year. Their pass rate for this year was very good; at 96% it was the best pass rate of the three schools that were compared.
d School A

Review Copy

Tip

Collect graphs and reports from newspapers and magazines to discuss with the learners.

PoA | Activity Data cycle: Task 4 – Reporting on data

Learner's Book page 326

Guidelines for implementing this activity

- Learners should now complete their project according to the instructions in the Learner's Book.
- Give the date on which the project needs to be handed in. Deduct marks for late submissions.

Suggested answers

Answers will depend on the data the learners collected.

Criteria	Outstanding 4	Achieved 3	Partially achieved 2	Not achieved 1
Task 1 Collecting of data				
Formulation of topic	Very clear	Fairly clear	Unclear	Weak
Format of questionnaire	Form very neat and clear	Neat and fairly clear	Average with few mistakes	Untidy; many mistakes
Question-naire	3 questions as specified; good responses	3 questions as specified	2 questions as specified	1 question as specified
Task 2 Organising and summarising of data				
Table 1	Complete and very neat	Complete and fairly neat	Incomplete	Incomplete; untidy
Table 2	Complete and very neat	Complete and fairly neat	Incomplete	Incomplete; untidy
Calculations	3 correct calculations	2 correct calculations	1 correct calculation	Attempted all calculations
Task 3 Presenting data				
Double bar graph	Correct graph; title and labels	Correct graph; No title and/or labels	Fairly accurate graph	Many mistakes
Pie chart	Correct graph; title and key	Correct graph; No title and/ or key	Fairly accurate graph	Many mistakes
Task 4 Interpreting, analysing and reporting of data				
Interpret and analyse	Source; context and good analysis.	Source or context given; reasonable analysis	No source or context given; Poor analysis	Incorrect analysis
Report	Conclusion; prediction; Shows good insight	Cconclusion or pre-diction; Shows insight	No con-clusion or prediction; Less than 50 words.	Not related to data

Review Copy

Chapter 14 Revision

Learner's Book page 327

Encourage learners to review the content covered before attempting the revision activities. The revision activities should be used to assess learners' progress thus far, and to assess where remediation may be required.

Learners should see how many of these questions they can answer without any help. It will help the learners if they are given the headings for the Frequency table asked for in question **3b**.

Suggested answers

1 **a** 240 learners

b $\frac{24}{240} \times 100\% = 10\%$

c 3 out of 24 learners sleep less than 6 hours per night
$\therefore$ it can be expected that 30 out of 240 learners sleep less than 6 hours per night.

2 Median is incorrect: Median $= \frac{1}{2}(54 + 57) = 55{,}5$
Mode is correct
Mean: Incorrect notation
Sum of numbers $= 42 + 62 + 40 + 54 + 57 + 41 + 65 + 53 + 54 + 63 + 58 + 67$
$= 656$
$\therefore$ Mean $= 656 \div 12 = 54{,}7$ (NOT 45,7)
OR use brackets:
Mean $= (42 + 62 + 40 + 54 + 57 + 41 + 65 + 53 + 54 + 63 + 58 + 67) \div 12$
$= 54{,}7$

b

Stem	Leaves
4	1 2 0
5	3 4 4 7 8
6	2 3 5 7

Key: 4 | 1 = 41

3 **a** Range $= 270 - 72$
$= 198$
Width of interval $= 198 \div 5$
$= 39{,}6$
≈ 40
$\therefore$ Intervals: 70 to 110; 111 to 150; 151 to 190; 191 to 230; 231 to 270

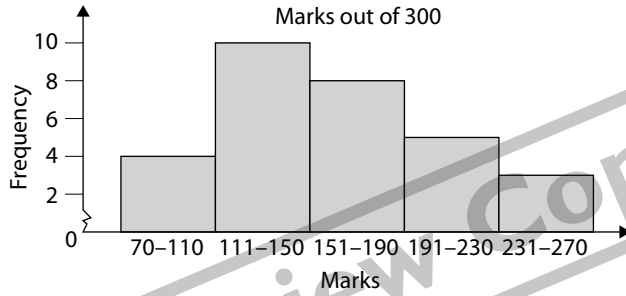
Review Copy

b

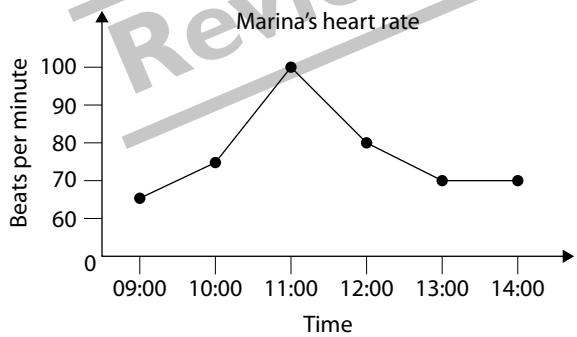
Marks out of 300	Tally	Frequency (f)	Midpoint of interval (x)	$f.x$
70 to 110	||||	4	90	$4 \times 90 = 360$
111 to 150	卌 卌	10	130	$10 \times 130 = 1\ 300$
151 to 190	卌 |||	8	170	$8 \times 170 = 1\ 360$
191 to 230	卌	5	210	$5 \times 210 = 1\ 050$
231 to 270	|||	3	250	$3 \times 250 = 750$
		Total = 30		Total = 4 820

c Mean = 4 820 ÷ 30 = 160,7
Modal class: the interval 111 to 150
Median mark will be in the interval 151 to 190

d



4 a



b Marina's heart rate started off at 65 beats per minute and then increased till it reached a peak at 11 o'clock (100 beats per minute). After that it came down and stabilised at 70 beats per minute for the last two hours.

c A broken-line graph is the best form to show a trend.

Review Copy

Term 3 Test

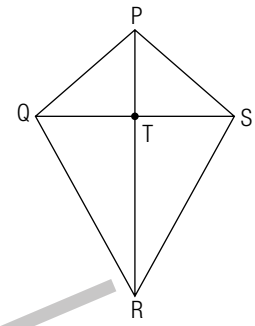
This test can be used as a controlled test at the end of Term 3. **Marks: 65**

1 True or false? (Give a reason if false.)

- **a** The hypotenuse is the longest side of a right-angled triangle.
- **b** The length of the radius of a circle is equal to half the length of the diameter of the circle.
- **c** The formula for the total surface area of a prism = perimeter of base × h + 2 × area of the base (3)

2 PQRS is a kite with QT = 4 cm; PT = 30 mm and QR = 8 cm.

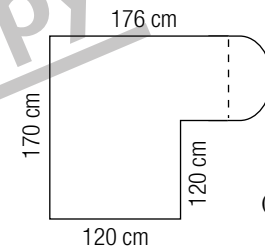
- **a** Find the length of PQ.
- **b** Find the length of TR (leave the answer in surd form).
- **c** Find the area of △PQS.
- **d** Find the perimeter of the kite in metres. (13)



3 In △ABC a = 12 cm; b = 15 cm and AB = 200 mm.
Choose the correct word: △ABC is (an acute-angled/a right-angled/an obtuse-angled) triangle. (Show all calculations.) (5)

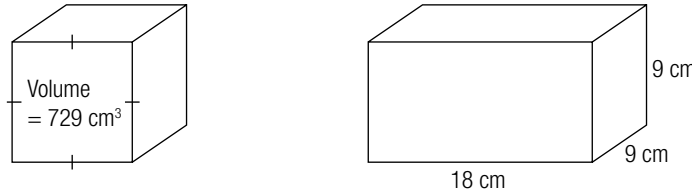
4 Karelien wants a new kitchen counter. The plan of the kitchen counter is shown below.

- **a** Redraw the plan of the kitchen counter. Divide the plan into three shapes and fill in all the missing measurements.
- **b** Determine the area of the kitchen counter $\left(\text{Use } \pi \approx \frac{22}{7}\right)$.
- **c** Determine the perimeter of the kitchen counter $\left(\text{Use } \pi \approx \frac{22}{7}\right)$. (20)



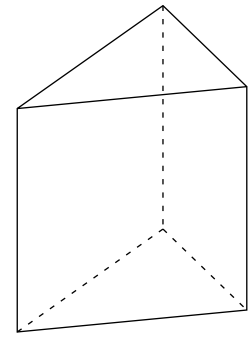
Review Copy

5 The volume of the cube is 729 cm^3.



- **a** Find the length of a side of the cube.
- **b** Isaac says the volume of the rectangular prism must be twice the volume of the cube. Why does Isaac say this?
- **c** Find the volume of the rectangular prism to decide whether he is correct.
- **d** Calculate the total surface area of the rectangular prism. (18)

6 The area of the base of the prism is 55 cm^2.
The volume of the prism is 605 cm^3.
a Find the height of the prism in mm.
b Give the capacity of the prism in litres. (6)



Suggested answers

1 a True b True c True (3)

2 a In △PQT:
$t^2 = p^2 + q^2$ (Pyth; diagonals of kite ⊥)
$= 4^2 + 3^2$ (1 mark for theorem; 1 mark for substitution;
$= 25$ (1 mark for theorem; 1 mark for substitution;
∴ PQ = 25 = 5 cm 1 mark for answer)
(3)

b In △QTR:
$t^2 = r^2 + q^2$ (Pyth; diagonals of kite ⊥)
$8^2 = 4^2 + q^2$ (1 mark for theorem; 1 mark for substitution;
$\therefore q^2 = 48$ 1 mark for answer)
$\therefore TR = \sqrt{48}$ cm (3)

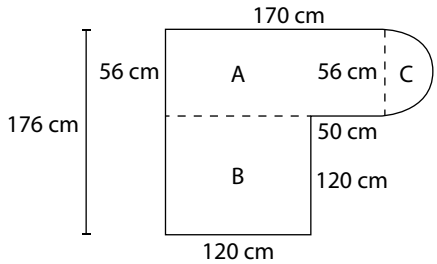
c Area of △PQS $= \frac{1}{2} \times b \times h$
$= \frac{1}{2} \times 8 \times 3$
$= 12\ cm^2$ (1 mark for formula; 1 mark for substitution;
1 mark for answer)
(3)

d Perimeter of kite = PQ + QR + RS + PS
= 5 + 8 + 8 + 5
= 26 cm
= 0,26 m (1 mark for formula; 1 mark for substitution;
1 mark for answer; 1 mark
for conversion from cm to m)
(4)

3 Longest side^2 = 20^2 = 400
Sum of squares of other sides:
$12^2 + 15^2 = 144 + 225 = 369$
400 > 369
∴ △ABC is an obtuse-angled triangle (3 marks for calculations;
1 mark for inequality; 1 mark for answer)
(5)

Review Copy

4 **a**



(1 mark for 56 cm; 1 mark for 50 cm)
(2)

b Area of rectangle $A = l \times b$
$= 170 \times 56$
$= 9\ 520\ \text{cm}^2$ (1 mark for formula; 1 mark for substitution; 1 mark for answer)
(3)

Area of square $= s \times s$
$= 120 \times 120$
$= 14\ 400\ \text{cm}^2$ (1 mark for formula; 1 mark for substitution; 1 mark for answer)
(3)

Area of full circle $= \pi \times r^2$
$= \frac{22}{7} \times 28 \times 28$
$= 2\ 464\ \text{cm}^2$
$\therefore$ Area for semi-circle $= 1\ 232\ \text{cm}^2$ (1 mark for formula; 1 mark for substitution; 1 mark for area of circle; 1 mark for area of semi-circle)
(4)

Total area $= 9\ 520 + 14\ 400 + 1\ 232$
$= 25\ 152\ \text{cm}^2$ (2)

c Perimeter of full circle $= \pi \times d$
$= \frac{22}{7} \times 56$
$= 176$ cm
$\therefore$ Perimeter for semi-circle $= 88$ cm (1 mark for formula; 1 mark for substitution; 1 mark for perimeter of circle; 1 mark for perimeter of semi-circle)
(4)

For perimeter: add lengths of all the sides and the perimeter of the semi-circle.
Total perimeter $= 170 + 88 + 50 + 120 + 120 + 176$
$= 724$ cm (2)

5 a Volume of cube $= s \times s \times s$
$\therefore$ if volume of cube $= 729\ \text{cm}^3$, then $s = 9$ cm
$(9 \times 9 \times 9 = 729)$ (1 mark for formula; 1 mark for substitution; 1 mark for answer)
(3)

b Isaac noticed the measurements of the rectangular prism: $18 \times 9 \times 9$
It can be rewritten as $2 \times 9 \times 9 \times 9$ (1)

Review Copy

c Volume of rectangular prism = area of the base × h of prism
$= l \times b \times h$
$= 18 \times 9 \times 9$
$= 1\ 458\ cm^3$
Isaac is correct: $1\ 458 = 2 \times 729$ (1 mark for formula; 1 mark for substitution; 1 mark for answer; 1 mark for conclusion)
(4)

d Total surface area = area of faces + 2 × area of base
Area of faces = perimeter of base × h of prism
Perimeter of base $= 2l + 2b$
$= 2(18) + 2(9)$
$= 54$ cm
∴ Area of faces $= 54 \times 9$
$= 486\ cm^2$
Area of base $= l \times b$
$= 18 \times 9$
$= 162\ cm^2$
∴ Total surface area $= 486 + 2 \times 162$
$= 810\ cm^2$ (1 mark for first formula; 4 marks for area of faces; 3 marks for area of base; 2 marks for final answer)
(10)

6 a Volume of prism = area of base × h of prism
$\therefore 605 = 55 \times h$
$\therefore h = 605 \div 55$
$= 11$ cm
$= 110$ mm (1 mark for formula; 1 mark for substitution; 1 mark for answer in cm; 1 mark for answer in mm)
(4)

b Volume $= 605\ cm^3$
∴ Capacity = 605 ml
= 0,605 litres (2)

Total marks: 65

Review Copy

CHAPTER

15 More algebraic expressions

Chapter overview

Learner's Book pages 328 to 338
Recommended pacing: 3 hours and 20 minutes

This chapter focuses on the following:

Unit 1:	**Input and output values**	1 hour
	Flow diagrams and tables	
	Formulae and equations	
Unit 2:	**Equivalent forms**	1 hour
	Representing equivalent forms	
Unit 3:	**Problem-solving with substitution**	1 hour 20 minutes
	Using substitution to generate ordered pairs	

Chapter 15 Revision

PoA: Assignment 3

Review Copy

UNIT

1 Input and ouput values

Unit overview

Learner's Book page 329
Recommended pacing: 1 hour

This unit focuses on the following:

- working with input and output values
- working with flow diagrams, tables, formulae and equations

Resources: Learner's Book; exercise book; calculator

Mental Maths

(10 minutes per day)

Suggested answers — Learner's Book page 329

1 90; 20; $\frac{205}{4}$; $\frac{325}{8}$

2 a $y = -\frac{1}{3}(n) + 6\frac{1}{3}$

b $a = -\frac{1}{3}(10) + 6\frac{1}{3} = 3$; $-\frac{1}{3} - 6\frac{1}{3} = -6\frac{2}{3}$; $b = -6\frac{2}{3} \div -\frac{1}{3} = -\frac{20}{3} \times -3 = 20$

Background information

All the concepts in this unit have been covered in previous chapters. This section therefore serves as revision of the basic concepts covered previously.

Teaching guidelines

Since this is revision of concepts allow learners, as far as possible, to work through the examples and activities on their own in class and monitor their progress carefully. Zoom in on problem areas and revise these intensively in small groups or pairs or, if they are a general challenge, with the class by working through the worked examples with them and pausing at and remediating the steps that they still find challenging. Provide learners who have mastered this section with opportunities to "teach" their peers in order to help manage the remediation process and reach everyone who require assistance. Alternatively, follow the guidelines per activity below.

Flow diagrams and tables

Activity 1 **Find output values**

Learner's Book page 330

Guidelines for implementing this activity

- Revise input and output values by working through the introduction (Remember) in the Learner's Book.
- Work through the example in the Learner's Book and observe learners carefully when they do Activity 1. Ensure that they write the corresponding input and output values below each other in the table.
- Ask questions such as: What is the T_n for $n = 8$? What is the T_n for $n = 0$? and so on (while working through the example). It is important that learners continue to hear the terminology used in order to consolidate their grasp.
- Remind them that we usually write input values in ascending order.

Review Copy

Remedial and extension

Remedial: Allow learners to work in mixed-ability pairs and encourage them to verbalise the process to each other, for example: My input value n is 1; I now have to multiply this value by 6. I now subtract 4 from my answer and then divide this answer by 2. My output value is 1. This is the first T_n in my table. (If necessary, let them write each answer to each step in pencil on the right-hand side of the page in their exercise book so that they do not lose track.)

Suggested answers

1

n	1	2	4	11
T_n	1	4	10	31

2

n	−10	−5	−2	0
T_n	−36	−11	4	14

3

n	−4	−1	3	10
T_n	31	1	−39	−109

4

n	−12	−1	0	$\frac{3}{5}$
T_n	6	−16	−18	$-19\frac{1}{5}$

5

n	$\frac{3}{4}$	$1\frac{1}{2}$	$-\frac{1}{5}$	$-2\frac{5}{8}$
T_n	6	$10\frac{1}{2}$	$\frac{3}{10}$	$-14\frac{1}{4}$

Formulae and equations

Activity 2 Find input values, output values and formulae

Learner's Book page 332

Review Copy

Guidelines for implementing this activity

- Work through the example in the Learner's Book with the learners. Learners should follow the steps for finding the formula carefully and methodically. Remind them to work forwards to find output values, and backwards to find input values.

Remedial and extension

Remedial: Encourage learners to draw flow diagrams to help them find input and output values.
Extension: Questions **7** and **8** are more challenging.

Suggested answers

1 a $y = 2(n) - 3$

b $a = 2(5) - 3 = 7$

$29 + 3 = 32$; $b = \frac{32}{2} = 16$

2 a $y = 4(n) - 14$

b $c = 4(6) - 14 = 10$

$30 + 14 = 44$; $d = \frac{44}{4} = 11$

3 a $y = -5(n) + 13$

b $e = -5(8) + 13 = -27$

$-32 - 13 = -45$; $f = \frac{-45}{-5} = 9$

4 a $y = -(n) + 6$

b $f = -(7) + 6 = -1$

$10 - 6 = 4$; $g = \frac{4}{-1} = -4$

5 a $y = -10(n) + 60$

b $h = -10(10) + 60 = -40$

$210 - 60 = 150$; $j = \frac{150}{-10} = -15$

6 a $y = \frac{1}{3}(n)$

b $k = \frac{1}{3}(12) = 4$

$m = 7\frac{2}{3} \div \frac{1}{3} = \frac{23}{3} \times \frac{3}{1} = 23$

*7 a $y = -\frac{1}{2}(n) + 4\frac{1}{2}$

b $n = -\frac{1}{2}(8) + 4\frac{1}{2} = \frac{1}{2}$

$-\frac{1}{2} - 4\frac{1}{2} = -5$; $p = \frac{-5}{-\frac{1}{2}} = 10$

*8 a $y = -\frac{1}{4}(n) + 5\frac{1}{4}$

b $q = -\frac{1}{4}(9) + 5\frac{1}{4} = 3$

$\frac{1}{4} - 5\frac{1}{4} = -5$; $r = \frac{-5}{-\frac{1}{4}} = 20$

UNIT

2 Equivalent forms

Unit overview

Learner's Book page 333
Recommended pacing: 1 hour

This unit focuses on the following:

- representing equivalent forms through describing in words, formulae, flow diagrams and tables
- solving problems, that is, finding output values

Resources: Learner's Book; exercise book; calculator

Review Copy

Mental Maths

(10 minutes per day)

Suggested answers

Learner's Book page 333

1 $a = b \times l = 5 \times 9 = 45\ \text{cm}^2$

2 Multiplied the breadth by the length.

3 A → $\div l$ → b

7 → $\div 3{,}5$ → 2

Background information

There are no new concepts introduced in this unit. Learners simply need to practise all the skills that they have already acquired. However, the questions are put into context, that is they are based on real-life problems.

Teaching guidelines

Since this is revision of concepts already covered in previous chapters ,allow learners, as far as possible, to work through the examples and activities on their own in class and

monitor their progress carefully. Zoom in on problem areas and revise these intensively in small groups or pairs or, if it is a general challenge, with the class by working through the worked examples with them and pausing at and remediating the steps that they still find challenging. Provide learners who have mastered this section with opportunities to "teach" their peers in order to help manage the remediation process and reach everyone who require assistance. Alternatively, follow the guidelines per activity below.

Representing equivalent forms

Activity 1 Work with equivalent forms

Learner's Book page 334

Guidelines for implementing this activity

- Learners should be familiar with the formulae used to calculate the area of polygons, as well as their application. Do a quick revision of these formulae in the form of a short quiz.
- Work through the example in the Learner's Book, particularly **b**. It might work even better to let learners work through the example in pairs and offer their understanding and feedback to the class. Facilitate some discussion of key steps in the process, particularly of working backwards using a flow diagram to find an unknown dimension if the area is given.
- Practise with two or three additional examples if necessary, by letting learners do the calculations quickly in pairs and then compare answers with the class.
- Allow learners, as far as possible, to attempt Activity 1 on their own and provide specific, targeted guidance only where absolutely necessary.

Review Copy

Remedial and extension

Remedial: Learners can work is pairs or small groups, if necessary, but monitor their progress carefully and provide specific, targeted guidance where necessary.
Extension: This is a challenging activity, and provides a good indication of learners' mastery of the concepts covered.

Suggested answers

1 a $A = \frac{1}{2} \times b \times \perp h = \frac{1}{2} \times 10 \times 6 = 30\text{ cm}^2$ b $b = 42 \div 8 \div \frac{1}{2} = \frac{21}{4} \times \frac{2}{1} = 10\frac{1}{2}$

c

Base (cm)	1	2	3	4	5	6
Height (cm)	7	5	4	8	10	12
Area (cm²)	3,5	5	6	16	25	36

2 a $b \rightarrow \boxed{+l} \rightarrow \boxed{\times 2} \rightarrow p$

b $\frac{1}{2} \rightarrow \boxed{+4} \rightarrow \boxed{\times 2} \rightarrow q$; $q = 9$

c Add the length and breadth. Multiply the answer by 2.

d $p \rightarrow \boxed{\div 2} \rightarrow \boxed{-1} \rightarrow b$ b $15 \rightarrow \boxed{\div 2} \rightarrow \boxed{-3\frac{1}{4}} \rightarrow 4\frac{1}{4}$

3 a Multiply the number of cell-phones fixed by 25 and then add 200.

b $C = 25x + 200$

c

Number of cell-phones fixed	0	10	20	25	50
Total cost (rand)	200	450	700	825	1 450

UNIT 3

Problem-solving with substitution

Unit overview

Learner's Book page 335
Recommended pacing: 1 hour 20 minutes

This unit focuses on the following:

- using substitution to generate and write ordered pairs

Resources: Learner's Book; exercise book

Review Copy

Mental Maths

(10 minutes per day)

Suggested answers Learner's Book page 335

1 $y = -3x - 10$

x	−3	−1	0	2	5
y	−1	−7	−10	−16	−25

(−3; −1); (−1; −7); (0; −10); (2; −16); (5; −25)

2 $y = -2x^2 + 6$

x	−3	−1	0	2	5
y	−12	4	6	−2	−44

(−3; −12); (−1; −4); (0; 6); (2; −2); (5; −44)

Background information

This unit provides further practice at finding output values by substituting input values into formulae, as well as writing corresponding input and output values in a table and then using the values in the table to write ordered pairs.

Teaching guidelines

Since this is revision of concepts, allow learners, as far as possible, to work through the examples and activities on their own in class and monitor their progress carefully. Zoom in on problem areas and revise these intensively in small groups or pairs or, if they are a general challenge, with the class by working through the worked examples with them and pausing at and remediating the steps that they still find challenging. Provide learners who have mastered this section with opportunities to "teach" their peers in order to help manage the remediation process and reach everyone who require assistance. Alternatively, follow the guidelines per activity below.

Using substitution to generate ordered pairs

Activity 1 Find and write ordered pairs

Learner's Book page 336

Guidelines for implementing this activity

- Work through the example in the Learner's Book and let learners practise with a few additional examples in their exercise books before attempting the activity on their own. Alternatively, allow a learner to work through the example with the class on the board.

Remedial and extension

Remedial: Learners can work in pairs through the activity, if necessary. However, limit paired work by encouraging learners to work through at least half of the activity on their own (up to question **4**). If necessary, let them work in pairs again through questions **5** and **6** since these questions are more challenging.
Extension: Questions **5** and **6** are more challenging.

Review Copy

Suggested answers

1 $y = -4x + 1$

x	–3	–1	0	2	5
y	13	5	1	–7	–19

(–3; 13); (–1; 5); (0; 1); (2; –7); (5; –19)

2 $y = -x - 3$

x	–3	–1	0	2	5
y	0	–2	–3	–5	–8

(–3; 0); (–1; –2); (0; –3); (2; –5); (5; –8)

3 $y = x^2 + 5$

x	–3	–1	0	2	5
y	14	6	5	9	30

(–3; 14); (–1; 6); (0; 5); (2; 9); (5; 30)

4 $y = x^3 - 2$

x	–3	–1	0	2	5
y	–29	–3	–2	6	123

(–3; –29); (–1; –3); (0; –2); (2; 6); (5; 123)

5 $y = 4x^2 - 3$

x	–3	–1	0	2	5
y	33	1	–3	13	97

(–3; 33); (–1; 1); (0; –3); (2; 13); (5; 97)

6 $y = -2x^3 + 1$

x	–3	–1	0	2	5
y	55	3	1	–15	–249

(–3; 55); (–1; 3); (0; 1); (2; –15); (5; –249)

Chapter 15 Revision

Learner's Book page 337

Encourage learners to review the content covered before attempting the revision activities. The revision activities should be used to assess learners' progress thus far, and to assess where remediation may be required.

Suggested answers

1

n	–2	–4	$\frac{3}{5}$	$\frac{1}{7}$	$-2\frac{1}{4}$
T_n	6	–6	$\frac{72}{5}$	$\frac{120}{7}$	$59\frac{1}{2}$

2 a $y = 7(n) - 14$; $a = 7(6) - 14 = 28$; $b = \frac{(-42 + 14)}{7} = \frac{-28}{7} = -4$

b $y = -\frac{1}{3}(n) + 2\frac{1}{3}$; $c = -\frac{1}{3}(8) + 2\frac{1}{3} = -\frac{1}{3}$; $d = \frac{(-2\frac{1}{3} + 2\frac{1}{3})}{-\frac{1}{3}} = 0$

3 a $A = \frac{1}{2} \times 8 \times 4\frac{1}{5} = 16\frac{4}{5}$ cm

b $B = 42{,}4 \div 0{,}5 \div 10{,}6 = 8$ cm

c

Base (cm)	3	1	2	14	$\frac{3}{4}$
Height (cm)	4	4	6	$\frac{1}{2}$	8
Area (cm²)	6	2	6	$3\frac{1}{2}$	3

4 a $y = 5x - 6$

x	–3	–1	0	2	5
y	–21	–11	–6	4	19

(–3; –21); (–1; –11); (0; –6); (2; 4); (5; 19)

b $y = -x^3 + 2$

x	–3	–1	0	2	5
y	29	3	2	–6	–123

(–3; 29); (–1; 3); (0; 2); (2; –6); (5; –123)

Review Copy

PoA: Assignment 3: Memorandum

Learner's Book page 338

Time: 50 minutes **Total: 50 marks**

1 **a**

x	0	6	–2	3	$\frac{1}{4}$
y	5	17	1	11	$5\frac{11}{2}$

✓✓✓✓✓(each correct answer)
✓✓(rows in table correct)
(7)

b

x	1	0	–2	$\frac{3}{4}$	$-\frac{7}{5}$
y	–27	–37	–57	$-29\frac{1}{2}$	–51

✓✓✓✓✓(each correct answer)
✓✓(rows in table correct)
(7)

2 **i** **a** $y = 2x - 11$✓✓ (2)

b $m = 2(8) - 11$✓ $= 5$✓ (2)

$n = (15 + 11)$ ✓ $\div 2$✓ $= 13$✓ (3)

ii **a** $y = -\frac{1}{3}x + 3\frac{1}{3}$ ✓✓ (2)

b $p = -\frac{1}{3}(9) + 3\frac{1}{3}$ ✓ $= -\frac{1}{3}$ ✓ (2)

$q = (-1 - 3\frac{1}{3})$ ✓ $\div -\frac{1}{3}$ ✓ $= 13$✓ (3)

3 **a** $y = -2x + 3$

x	–3	–1	0	2	5
y	9✓	5✓	3✓	–1✓	–7✓

(–3; 9); (–1; 5); (0; 3); (2; –1); (5; –7) ✓

b $y = 5x^2 - 1$

x	–3	–1	0	2
y	44✓	4✓	–1✓	19✓

(–3; 44); (–1; 4); (0; –1); (2; 19) ✓ (11)

4 **a** $V = 3{,}142 \times 10^2 \times 6$✓ $= 1\ 885{,}2\ \text{cm}^3$✓ (2)

b $157\ 100 = 3{,}142 \times 252 \times H$✓

$H = 157\ 100 \div 3{,}142 \div 25^2$✓ $= 80\ \text{mm}$✓ (3)

c $A = 3{,}142 \times 2^2 \times 7$✓ $= 87{,}976\ \text{cm}^3$✓

$B = 628{,}4 \div 3{,}142 \div 5^2$✓ $= 8\ \text{cm}$✓

$C = \sqrt{904{,}896 \div 3{,}142 \div 2}$✓ $= \sqrt{144} = 12\ \text{cm}$✓ (6)

Review Copy

CHAPTER

16 Graphs

Chapter overview

Learner's Book pages 339 to 352
Recommended pacing: 9 hours

This chapter focuses on the following:

Unit 1: Interpreting graphs 1, 5 hours
- Features of graphs
- Discrete and continuous data

Unit 2: Understanding and drawing graphs 7,5 hours
- The relationship between distance, speed and time
- The relationship between time and temperature
- The Cartesian plane
- Draw graphs by plotting points

Chapter 16 revision

Review Copy

UNIT

1 Interpreting graphs

Unit overview

Learner's Book page 340
Recommended pacing: 1,5 hours

This unit focuses on the following:

- analysing and interpreting global graphs of problem situations, with a special focus on the following trends and features:
  - linear or non-linear
  - constant, increasing or decreasing
  - maximum or minimum
  - discrete or continuous

Resources: Learner's Book; exercise book; block or graph paper

Mental Maths

(10 minutes per day)

The aim of this exercise is to practise substitution skills. This skill will be necessary at the end of the chapter when learners have to draw up tables and plot graphs.

Suggested answers — Learner's Book page 340

1	**a**	15	**b**	−21	**c**	0	**d**	$\frac{1}{3}$	**e** 2,4
2	**a**	−32	**b**	8	**c**	0	**d**	$-\frac{1}{2}$	**e** −0,12
3	**a**	34	**b**	−21	**c**	−1	**d**	0	**e** 2
4	**a**	−99	**b**	31	**c**	1	**d**	−4	**e** −8

Background information

Learners were introduced to basic linear and non-linear graphs in Grade 7. The focus in this grade was on drawing, analysing and interpreting global graphs only. Learners did not have to plot points to draw graphs. They focused only on the features of the global relationship shown in the graph.

Examples of contexts for global graphs included:
- the relationship between time and distance travelled
- the relationship between temperature and time over which it is measured
- the relationship between rainfall and time over which it is measured.

Unit 1 is aimed at revising the above content in order to progress learners to a deeper understanding of graphs, as well as prepare them for the drawing of graphs.
New features of global graphs dealt with in this unit include: maximum and minimum; discrete and continuous.

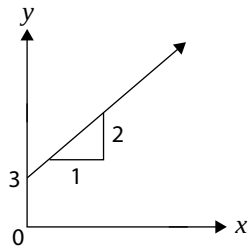
Review Copy

Teaching guidelines

Although this unit serves as revision, time should be taken to ensure that learners understand the concepts revised and new concepts introduced here.

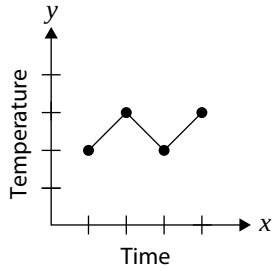
When teaching graphs in Grade 8, do not confuse graphs of linear functions with broken-line graphs.

- A linear function has the equation $y = mx + c$. Learners will plot examples of linear functions at the end of this chapter.



$y = 2x + 3;\ x \geq 0$

- A broken-line graph is a line graph where the different points are joined by straight lines. Learners worked with broken-line graphs in a previous chapter.



Features of graphs

Activity 1 Identify features of graphs

Learner's Book page 341

Guidelines for implementing this activity

- Revise the features of graphs by working through the introduction in the Learner's Book.
- Take note of the following.
  - Increasing and decreasing values: Point out that the part of the line showing an increase in values is steeper than the part showing a decrease in values.
  - Minimum/maximum: We can indicate the minimum/maximum value on the graph for the interval of the graph that we have in front of us. It is possible that (if the graph is extended) there might be another minimum or maximum.
- Draw similar graphs on the board and discuss their features with the learners.

Review Copy

Remedial and extension

Remedial: Provide many additional examples of graphs for homework to ensure that learners understand their basic features. Learners may also attempt the additional activities mentioned in the Extension exercise below.
Extension: Since learners have worked with data measures in a previous chapter, they can be asked to calculate the range, median, mean and mode in question **2**.

Suggested answers

1 **a** non-linear **b** linear **c** non-linear **d** linear

2 Average rainfall:
a 30 mm **b** March **c** 0 mm **d** September
e July and August
f From January to March; from September to November
g From March to July; from November to December

3 Average night-time temperature:
a 14 °C **b** January and February

c 0 °C
d July
e January and February
f From July to December
g From February to July

Tip

Carnarvon is the site for the Square Kilometre Array (SKA) project. Find out more about this project on the Internet and discuss the details and importance of this project for South Africa.

Discrete and continuous data

Activity 2 Work with discrete and continuous data

Learner's Book page 343

Guidelines for implementing this activity

- The learners need to know the difference between discrete and continuous data and this difference must therefore to be explained very carefully by working through the graphs and the information in the introduction to this section.
- Ask learners to think of more examples of discrete and continuous data, for example single-unit items such as children; grocery items such as canned foods, toiletries, and so, on are discrete units. Items such as meat (that can be any quantity in terms of kilograms); fluids such as juice, water, and so on are all examples of continuous data.
- Be sure to work carefully through the notes in the Remember box to ensure that learners grasp the difference between the two types of graph before they attempt Activity 2.
- Work through the summary in the Learner's Book with learners to consolidate the concepts covered here.

Review copy

Suggested answers

1 a discrete b continuous c discrete d continuous
2 a 3, 4 or 5 (considered normal); discrete data
b Decimal value on a Brix scale (values between 18 and 26 considered good sugar percentages to harvest); continuous data. (Source: http://nanaimowinemakers.org/Steps/PickingGrapes.htm)
c 5, 6, 7, and so on; discrete data
d 10 to 12, and so on; discrete data
e Decimal value on stop watch (world record for men: 43,18 seconds by Michael Johnson in 1999; world record for women: 47,6 seconds by Marita Koch in 1988); continuous data. Source: http://records.unitarium.com/400m
f Decimal number of units of Joule (1 Joule is considered the energy needed to lift one small apple to a height of 1 metre); continuous data. (Source: http://en.wikipedia.org/wiki/Joule)
3 Answers will vary. Possible answers are number of learners, number of desks and number of calculators for discrete data; and temperature in the classroom, time it takes for learners to open their book and heights of learners for continuous data.

UNIT 2

Understanding and drawing graphs

Unit overview

Learner's Book page 344
Recommended pacing: 7,5 hours

This unit focuses on the following:

- drawing global graphs from given descriptions of a problem situation and identifying features as in Unit 1
- examples of contexts for problem situations:
  - relationship between time and distance travelled
  - relationship between temperature and time over which it is measured
  - relationship between rainfall and time over which it is measured
- using tables of ordered pairs to plot points and draw graphs on the Cartesian plane

Resources: Learner's Book, exercise book; squared paper to draw graphs

Review Copy

Mental Maths

(10 minutes per day)

Learners will have to interpret and draw graphs about the relationship between time and the distance travelled. The aim of the Mental Maths is to help them develop an understanding of the relationship between: distance (km travelled); speed (km per hour) and time (hours).

Suggested answers — Learner's Book page 344

1	a	100 km	b	300 km	c 500 km
	d	900 km	e	50 km	f 150 km
2	a	50 km	b	200 km	c 300 km
	d	500 km	e	25 km	f 75 km
3	a	80 km	b	160 km	c 560 km
	d	40 km	e	20 km	f $80 \times x = 80x$ km
4	True				

Background information

In this unit, learners' understanding of graphs is extended to include the following new content and skills: plotting points to draw graphs.

Examples of contexts for global graphs include:

- the relationship between time and distance travelled

- the relationship between temperature and time over which it is measured
- the relationship between rainfall and time over which it is measured.

Teaching guidelines

Most learners find time/distance graphs challenging to understand, therefore it is important to spend enough time on this topic.

The relationship between distance, speed and time

Activity 1 Work with time/distance on graphs

Learner's Book page 345

Guidelines for implementing this activity

- The formula (distance = speed × time) was developed in the Mental Maths questions. It is not necessary to teach learners three different formulae; they can always use the basic formula (as in the Think/Do examples in the Learner's Book).
- Work through Activity 1 step-by-step with the learners so that they understand the three conclusions (questions **1f**, **2g** and **3g**).

Remedial and extension

Remedial: Learners do not have to do Activity 1 on their own. Working step-by-step with the learners through Activity 1 is important to help with their understanding of graphs, and to deal with the challenges that they may experience hands-on.
Extension: Learners can attempt question **3** on their own.

Review Copy

Suggested answers

1 a A: 1 hour; B: 4 hours b 3 hours c A: 20 km; B: 20 km
d 0 km e No
f There was no movement, the car stopped.

2 a 2 hours b 200 km
c $D = s \times t$; $200 = s \times 2 \therefore s = 200 \div 2 = 100$ km/h
d 5 hours
e 500 km
f $D = s \times t$; $500 = s \times 5 \therefore s = 500 \div 5 = 100$ km/h
g The car travels at a constant speed.

3 a A: 3 hours b 60 km
c $D = s \times t$; $60 = s \times 3 \therefore s = 60 \div 3 = 20$ km/h
d B: 3 hours e 300 km
f $D = s \times t$; $300 = s \times 3 \therefore s = 300 \div 3 = 100$ km/h
g The car travelled faster. 100 km/h is faster than 20 km/h

Activity 2 Read and draw time/distance graphs

Learner's Book page 346

Guidelines for implementing this activity

- Ask learners to do question **1** of Activity 2 without any further explanation to see how much they understand.
- Discuss the answers to question **1**. Address difficulties they may have by working through these with the learners, and by constantly taking them back to the basics.
- Ask learners to complete the rest of the activity on their own. Monitor the progress of learners who still require support, and provide individual assistance where required.

Tip

Take note of the slopes of the line segments on time/distance graphs: A negative (downward) slope (as in the graph of Mrs Mokoena) indicates that the person is travelling back to the original starting point. Xolani travelled in only one direction (from his house to the school) and therefore the different line segments all have positive (upward) slopes.

Remedial and extension

Remedial: Working through the first question with the learners in the form of a discussion and monitoring their progress with subsequent questions provide opportunities to iron out all difficulties they may have. Ask questions about the graph that the learners did in question **3**, for example: Explain the difference in the steepness of the lines.
Extension: Allow learners to do questions **2** to **4** on their own.

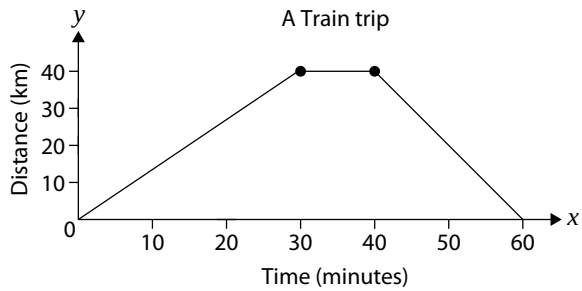
Review Copy

Suggested answers

1 **a** 20 minutes **b** 30 minutes **c** 50 – 30 = 20 km
d 25 minutes **e** 50 km **f** 35 minutes

2 **a** 12 minutes **b** 4,5 km **c** 1 minute
d 2 km **e** CD **f** 2 km
g 4,5 – 2,5 = 2 km

3



4 Learners draw a graph about a story.

The relationship between time and temperature

Activity 3 Draw graphs of time and temperature

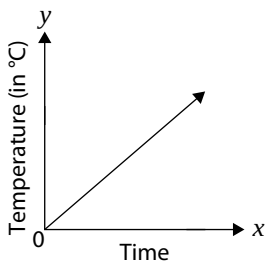
Learner's Book page 347

Guidelines for implementing this activity

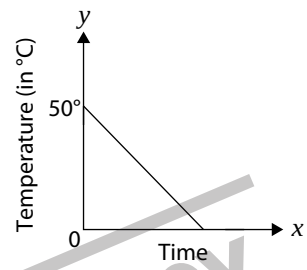
- Discuss examples of temperature in everyday life, for example the temperature outside today; the temperature of a sick person.
- Do Activity 3 question **1** with the learners.
- Learners complete the rest of the activity on their own.

Suggested answers

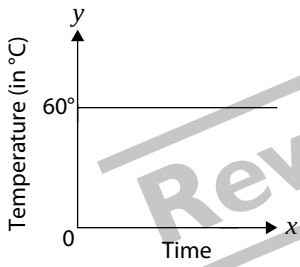
1 a



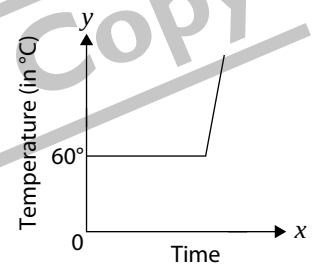
b



2 a



b



Review Copy

3 a The temperature rises steadily for a while until it reaches 70 °C and then drops very quickly to 0 °C.

b The temperature rises very quickly for a while, stays constant for a while and then rises steadily again.

c The temperature stays constant at 10° C for a while and then drops steadily until it is lower than 0 °C.

The Cartesian plane

Activity 4 Work with the Cartesian plane

Learner's Book page 349

Guidelines for implementing this activity

- Explain the features of the Cartesian plane to the learners:
  - who developed it
  - the four quadrants

- the origin on the Cartesian plane
- how many quadrants it has
- what a quadrant is
- how to plot points on a Cartesian plane.

- Work through the example in the Learner's Book. Make sure that learners use the correct notation for a point: round brackets and a semicolon.
- Pay special attention to the coordinates of points that are on the axes, for example (1; 0) and (0; –2).
- Learners do Activity 4 on their own. However, provide some guidance where necessary.

Remedial and extension

Remedial: Ask learners further questions about the answers to question **2**, for example:

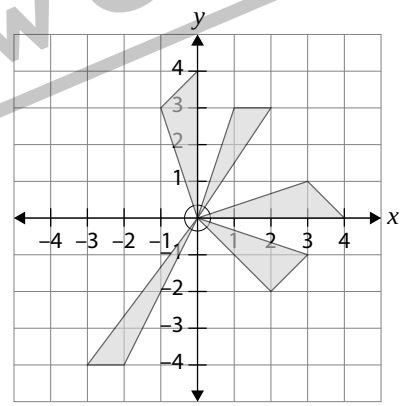
- What do they notice about all the points on the *x*-axis? (*y*-coordinate always equal to 0.)
- What do they notice about all the points on the *y*-axis? (*x*-coordinate always equal to 0.)
- What do they notice about the points that are on the same horizontal line? (These have the same *y*-coordinate.)
- What do they notice about the points that are on the same vertical line? (These have the same *x*-coordinate.)

Review Copy

Suggested answers

1 A(1; 4) B(–3; 3)
C(3; 3) D(0; 2)
E(–4; 0) F(3; 0)
G(–2; –1) H(–2; –2)
I(0; –2) J(4; –2)
K(–3; –3) L(2; –3)
O(0; 0)

2



3 **a** E(8; 7) **b** H(0; 3) **c** A(–4; 2) **d** C(–12; 0)
e D(–5; –5) **f** B(0; –12) **g** F(1; –6) **h** G(3; 0)

4 **a** True **b** False **c** False **d** True **e** True

Tip

Learners can think about an aeroplane when plotting a point: first they go horizontally and then up (or down).

Draw graphs by plotting points

Activity 5 Draw graphs

Learner's Book page 350

Guidelines for implementing this activity

- The skill that learners will develop in this activity is one that they will use over and over again in FET Maths. If we are given an equation and we want to see what the graph represented by the equation looks like, we draw up a table, calculate the coordinates and plot the points.
- Discuss and work through the worked example in the Learner's Book.
- Make sure that learners use brackets when they substitute values into an equation.
- Do additional, similar examples on the board.
- Point out that we join the points because we are working with continuous data. We can work with an x-value of 2,5 or 3,125 and find the corresponding y-value.
- Point out that we can use any x-values. (The values do not always have to be integers from –3 to 3.) It is always a good idea to use some negative numbers, zero and some positive numbers.

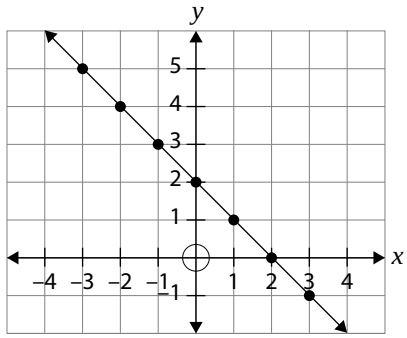
Remedial and extension

Remedial: Provide support and guidance while learners are busy doing Activity 5, should they need it, by sitting with them in small groups or pairs and guiding them through the questions and what they are expected to do. Gradually reduce the level of support though and make it an expectation for them to do some parts of the activity on their own without any support. Assess their progress and provide additional examples for them to do for homework if necessary.
Extension: Encourage learners to do the activity on their own, without any help.

Review Copy

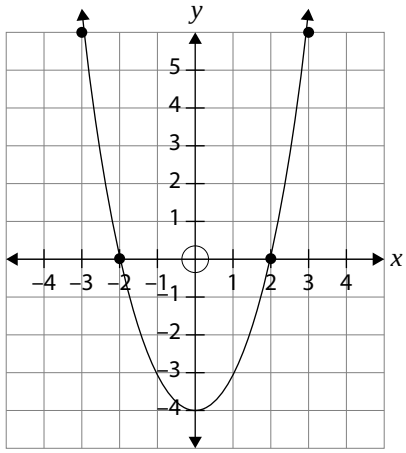
Suggested answers

1 a



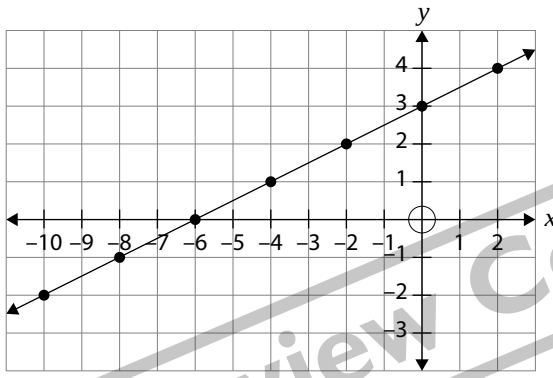
x	–3	–2	–1	0	1	2	3
$y = -x + 2$	5	4	3	2	1	0	–1

b



x	–3	–2	–1	0	1	2	3
$y = x^2 - 4$	5	0	–3	–4	–3	0	5

c



x	–3	–2	–1	0	1	2	3
$y = \frac{1}{2}x + 3$	–2	–1	0	1	2	3	4

Review Copy

2 a

x	–3	–2	–1	0	1	2	3
$y = x - 1$	–4	–3	–2	–1	0	1	2

b

x	–3	–2	–1	0	1	2	3
$y = x^2 - 1$	8	3	0	–1	0	3	8

c Graph in **2a** is linear; graph in **2b** is non-linear.

d Linear graph: x in equation; non-linear graph: x^2 in equation.

Tip

Point out that the values of the y-coordinates in these examples form a pattern. If learners are aware of this, they will be able to spot mistakes in their calculations quickly.

Chapter 16 Revision

Learner's Book page 352

Encourage learners to review the content covered before attempting the revision activities. The revision activities should be used to assess learners' progress thus far, and to assess where remediation may be required.

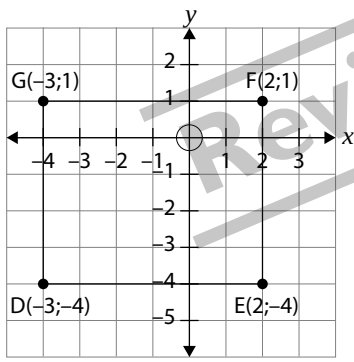
Suggested answers

1 a 7°C
b Saturday
c From Sunday to Monday
d From Wednesday to Thursday
e Continuous; it is possible to have a temperature of any value between e.g. 7 °C and 14 °C.
f Non-linear; it is a broken-line graph.

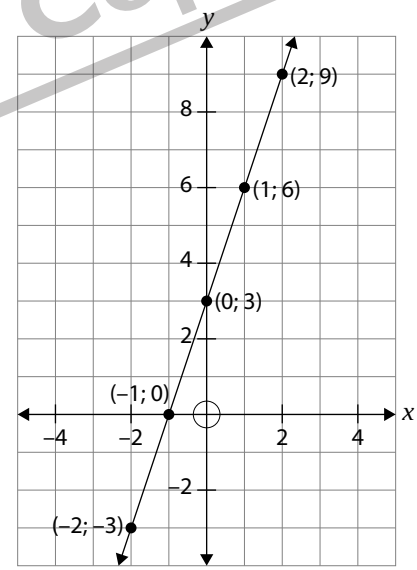
2 a $D = s \times t$; $D = 63{,}5 \times 2 = 127$ km
b $D = s \times t$; $560 = s \times 8{,}6 \therefore s = 560 \div 8{,}6 = 65{,}1$ km/h

3 a P: 100 m; Q: 70 m. P covered a longer distance than Q in the same time; which means P ran faster than Q.
b P: 40 – 30 = 10 m; Q: 40 m. P started 30 metres ahead of Q, but P ran much slower than Q.
c P: 40 – 20 = 20 m; Q: 20 m. P started 20 metres ahead of Q, but they ran at the same speed.
d P: 100 m; Q: 100 m. P ran at a constant speed while Q ran fast, stopped, and started running again.

4



5



x	–2	–1	0	1	2
$y = 3x + 3$	–3	0	3	6	9

Review Copy

CHAPTER

17 Transformation geometry

Chapter overview

Learner's Book pages 353 to 368
Recommended pacing: 6 hours

This chapter focuses on the following:

Unit 1: Transformations 3 hours
- Three types of transformation
- Transforming a point
- Transforming a triangle

Unit 2: Enlargements and reductions 3 hours
- Similar figures are in proportion
- Enlargement and reduction

Chapter 17 Revision

PoA: Investigation 4: Investigate enlargement and reduction

Review Copy

UNIT

1 Transformations

Unit overview

Learner's Book page 354
Recommended pacing: 3 hours

This unit focuses on the following:

- recognising, describing and performing transformations with points on a coordinate plane, focusing on:
  - translating a point within and across quadrants
  - reflecting a point in the x-axis or y-axis
  - rotating a point around the origin
- recognising, describing and performing transformations with triangles on a coordinate plane, focusing on the coordinates of the vertices when:
  - translating a triangle within and across quadrants
  - reflecting a triangle in the x-axis or y-axis
  - rotating a triangle around the origin
- recognising that translations, reflections and rotations:
  - only change the position of the figure and not its shape or size
  - produce congruent figures

Resources: Learner's Book; exercise book; squared paper

Mental Maths

(10 minutes per day)

Learners need to be comfortable with the Cartesian plane if they want to do transformations on the Cartesian plane. The Mental Maths exercise aims to revise the features of the Cartesian plane. Learners also need to realise that points can have decimal numbers or fairly big numbers as coordinates.

Suggested answers — Learner's Book page 354

1 a Quadrant II
b positive part of y-axis
c positive part of x-axis
d Quadrant I
e the origin
f Quadrant IV
g negative part of x-axis
h Quadrant III
i negative part of y-axis

2 a (6; 3) b (1; 3) c (3; 7)
d (3; 2) e (4; 2) f (0; 5)

Background information

In Grade 7, learners used squared paper to do transformations, which allowed them to perform transformations more accurately and to compare the shape and size of figures. They learnt and did the following:

- Translations, reflections and rotations only change the position of the figure, and not its shape or size and that these transformations produced congruent figures.
- Enlargements and reductions change the size of figures by increasing or decreasing the length of sides, but keeping the angles the same, producing similar rather than congruent figures.
- Worked out the factor of enlargement or reduction.

Review Copy

In Grade 8, transformations are done on a coordinate plane.

Teaching guidelines

Doing transformations on the coordinate plane is an opportunity to practise plotting points with ordered pairs, and links up with drawing algebraic graphs.

- Learners have to learn how to plot points on the coordinate plane and read the coordinates of points off the x-axis and y-axis. This is also done with algebraic graphs. Point out the differences between the axes in the four quadrants. Doing transformations on a coordinate plane focuses attention on the coordinates of points and vertices of shapes.
- Learners should continue to recognise that translations, reflections and rotations change only the position of the figure, and not its shape or size.
- Learners should also recognise that the above transformations produce congruent figures.
- Learners do not have to learn general rules for the transformations at this stage, but should explore the way the coordinates of points change when performing different transformations with lines or shapes.

Three types of transformation; Transforming a point

Translating a point on a Cartesian plane; Reflecting a point on a Cartesian plane; Rotating a point around the origin

Activity 1 Transform a point

Learner's Book page 354

Guidelines for implementing this activity

- Learners will do transformations with a point first and then move on to transforming triangles in the next activity.
- Work through the first example in the Learner's Book.
- Do not teach learners formulae for the different transformations. At this stage they need to explore the different transformations by counting the units (steps) on a Cartesian plane.

When doing translations

- Pay attention to the correct notation, for example P(2; 1) → P′(3; 5) and explain the word prime.
- Emphasise the following:
  - A transformation does not alter the shape or size of a shape or point in this case – only its position changes.
  - We use the vocabulary of direction to explain a translation, namely, up, down, right and left.
  - An image of the original shape is produced. Pause to explain the notation of this image again (using a prime).
- Provide examples of translations for learners to do on graph or block paper. Provide instructions differently each time, such as:
  - Mark point E(3; 1). Translate this point to a new position E′(5; 4). Describe the translation. (Point E was moved 2 units to the right and 3 units up.)
  - Mark point G(5; 4). Move it 3 units to the left and 2 units down. Write down its new position. Do not forget write the image using the correct notation: G′(2; 2).
  - The point C(2; 1) was moved 1 unit to the right and 3 units up. Write down the translation C(2; 1) → C′(3; 4).
- Do as many translations as possible in the above ways before learners attempt the activity, so that they become completely comfortable with it.
- Learners do Activity 1 question **1**.

When doing reflections

- Work through the second example in the Learner's Book.
- Pay attention once again to the correct notation, for example P(2; 1) → P′(3; 5) and emphasise the importance of the prime.

Review Copy

- Emphasise the following:
  - In a reflection every point is the same distance from a central line called a mirror line.
  - Points can be reflected in the x-axis or in the y-axis (as shown in the examples).
- Provide examples of reflections for learners to do on graph or block paper. Provide instructions differently each time, similar to the way described in the guidelines above. Be sure to include examples that require learners to reflect points in the x-axis as well as the y-axis.
- Learners do Activity 1 question **2**.

When doing rotations

- Revise the meanings of clockwise and anticlockwise. Use an actual clock if necessary to explain these meanings.
- Follow the same pattern as described for the above transformations, starting by working through the third example in the Learner's Book.
- Learners do the rest of Activity 1.

Remedial and extension

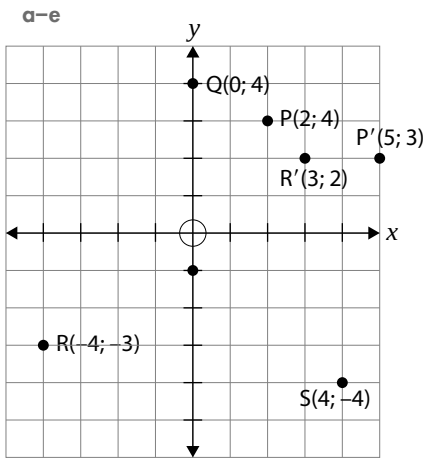
Remedial: Do not assume that all learners in Grade 8 are familiar with transformations. If the concept is new to them, it is a good idea to have little flags (or other shapes) available for them to practically explore the three types of transformations that are dealt with in this unit. In this instance, do the basic transformation first (on grid paper without the use of a Cartesian plane). Let them do as many of these as possible for homework in order to consolidate the basics. Introduce the Cartesian plane after you have covered some basics with them.

Extension: Allow learners to work in mixed-ability groups where possible, so that they can assist one another. It is also very important that learners verbalise the various transformations as they do them. This will help to crystallise their understanding.

Review Copy

Suggested answers

1 **a–e**



2 **a, b** and **c**

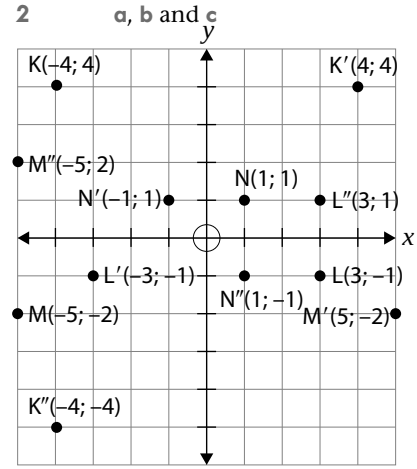
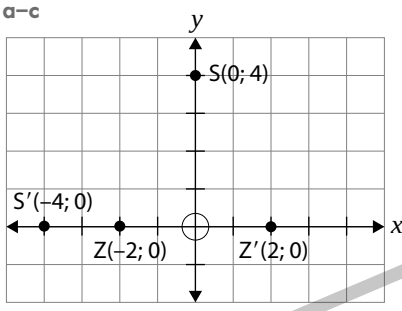


Table I	
Point	**Image**
K(–4; 4)	K′(4; 4)
L(3; –1)	L′(–3; –1)
M(–5; –2)	M′(5; –2)
N(1; 1)	N′(–1 ; 1)

Table II	
Point	**Image**
K(–4; 4)	K″(–4; –4)
L(3; –1)	L″(3; 1)
M(–5; –2)	M″(–5; 2)
N(1; 1)	N″(1; –1)

d Reflection in y-axis: the y-coordinates stayed the same; the sign of the x-coordinate changed.
Reflection in x-axis: the x-coordinates stayed the same; the sign of the y-coordinate changed.

3 **a–c**



4 **a** A(0;2) → A′(2; 0); rotation of 90° around the origin in a clockwise direction or translation of two units to the right and two units down.
b B(3; –2) → B′(–3; –2); reflection in y-axis.
c C(–4; 1) → C′(–4; –1); reflection in x-axis.
d D(2; 2) → D′(4; 3); translation of 3 units right and 2 units up.
e E(1; –1) → E′(–1; –4); translation of 2 units left and 3 units down.

Review Copy

Tip

When describing a translation there is no difference between the following two statements:
- "Point A moved 3 units up and 4 units to the right."
- "Point A moved 4 units to the right and 3 units up."

It is a good idea to always describe the horizontal translation first and the vertical translation second. In doing this, the convention of writing the x-coordinate first and the y-coordinate second is reinforced.

Transforming a triangle

Activity 2 Transform a triangle

Learner's Book page 360

Guidelines for implementing this activity

- Grade 8 learners focus on transformation of points and triangles, but it is good to keep in mind that any figure (for example, a quadrilateral or circle) can be transformed.
- It is important to emphasise that we work with one point (vertex of triangle) at a time.
- Teach learners to check their work: the image of the triangle should be congruent to the original triangle.
- Work through the examples in the Learner's Book step-by-step and do an additional, similar example of each type of transformation on the board.
- Let learners practise using another example of each type of transformation on graph or block paper as you lead them step-by-step through each transformation. Structure the questions and instructions differently each time, as with the previous transformations in the guidelines for Activity 1.
- The learners who are interested in cars will enjoy question **5** of the activity.

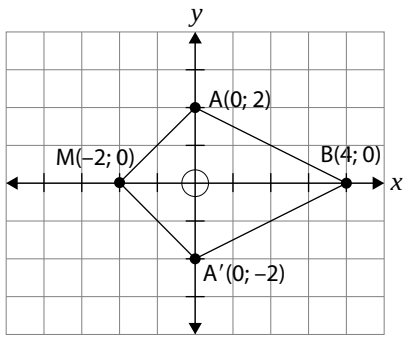
Remedial and extension

Remedial: Allow learners to work in mixed-ability groups where possible, so that they can assist one another. It is also very important that learners verbalise the various transformations as they do them. This will help to crystallise their understanding. In question ***5*** learners can also give the names of the cars represented by the logos.
Extension: Challenge learners to notice other examples of transformations around them – in the classroom and in their everyday lives.

Review Copy

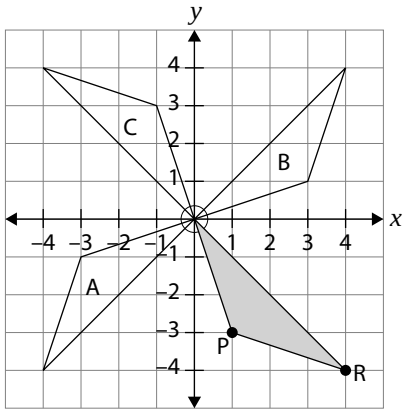
Suggested answers

1 **a** and **b**



c Kite
d false

2 **a** to **d**

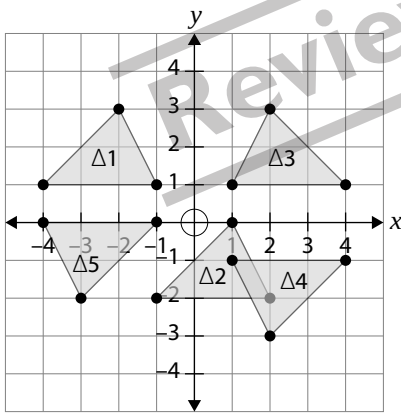


e Congruent

3 **a** △A to △D: reflection in *y*-axis.
b △A to △E: reflection in *x*-axis.
c △B to △C: rotation of 90° around origin in clockwise direction.
d △B to △F: reflection in *x*-axis.
e △C to △D: translation of 2 units right and 2 units up.
f △C to △G: reflection in *x*-axis.
g △C to △H: translation 2 units right and 4 units down.
h △D to △H: translation 6 units down.
i △H to △D: translation 6 units up.
j △D to △E: rotation of 180° around origin.
k △F to △G: rotation of 90° around origin in anticlockwise direction.
l △G to △F: rotation of 90° around origin in clockwise direction.

Review Copy

4



b Coordinates for △2: (1; 0); (2; –2) and (–1; –2)
c Coordinates for △3: (1; 1); (4; 1) and (2; 3)
d Coordinates for △4: (1; –1); (4; –1) and (2; –3)
e Coordinates for △5: (–4; 0); (–3; –2) and (–1; 0)

5 **a** translation
b reflection in *y*-axis
c rotation
d rotation
e translation
f reflection in *y*-axis or reflection in *x*-axis or rotation.

UNIT 2

Enlargements and reductions

Unit overview

Learner's Book page 362
Recommended pacing: 3 hours

This unit focuses on the following:

- using proportion to describe the effect of enlargement or reduction on the area and perimeter of geometric figures
- working out the factor of enlargement or reduction of a figure
- recognising that enlargements or reductions:
  - change the lengths of the sides of the figures, but keep the angles of the figures the same
  - produce similar figures

Resources: Learner's Book; exercise book; squared paper

Mental Maths

(10 minutes per day)

Suggested answers — Learner's Book page 362

1 a Times 2 each time b Times 3 each time
c Divided by 3 each time d Divided by 2 each time
e Times 2 each time

2 a 8 cm b 16 cm c 32 cm

3 a 81 cm^2 b 9 cm^2 c 1 cm^2

Review Copy

Background information

Learners worked with enlargements and reductions in Grade 7 by drawing enlargements and reductions of geometric figures on squared paper and comparing them in terms of shape and size.

They have been expected to recognise that enlargements and reductions change the size of figures by increasing or decreasing the length of sides, but keeping the angles the same, producing similar rather than congruent figures. Learners have also been expected to work out the factor of enlargement or reduction.

In Grade 8, learners have to use proportion to describe the effect of enlargement or reduction on area and perimeter of geometric figures. They also revise that enlargement and reduction change the size of figures by increasing or decreasing the length of sides and that the angles stay the same, producing similar rather than congruent figures. Learners continue to work out the factor of enlargement or reduction of a figure.

Teaching guidelines

It is important to remain practical in the approach to the content in this unit, and to have a hands-on approach in areas that learners still find challenging. Have lots of grid paper handy for learners to practise drawing enlargements and reductions at Grade 7 level if necessary in order to facilitate the leap to a more abstract understanding of the concepts, especially their understanding of proportion as well as working with enlargement and reduction in the context of area and perimeter. Always encourage learners to draw rough sketches of figures when doing problem-solving.

Similar figures are in proportion; Enlargement and reduction

Activity 1 Work with proportion

Learner's Book page 363

Guidelines for implementing this activity

- Learners worked with similar triangles previously. This unit is an opportunity to do revision and also to look at similarity in pictures (for example, clowns) and other geometric figures. Discuss the introduction in the Learner's Book.
- Pay attention to notation: $\frac{AB}{BC} = \frac{PQ}{QR}$ can also be written as AB : BC = PQ : QR.
- Work carefully through the different features of the two similar trapeziums in the example in the Learner's Book, focusing specifically on calculating the proportions and the scale factor. Provide additional examples for learners to do before they attempt the activity. Ensure that they use the correct vocabulary and that they understand what proportion means.
- Encourage learners to create their own rough drawings of similar figures with lengths of sides indicated (not to scale). Give them the opportunity to present their drawings to the class, and to explain why they are similar. (Listen to the vocabulary that they use.)

Review Copy

Tip

Dilation is a collective word that is used to describe the resizing of a geometric figure. If the scale factor is greater than 1, the dilation is an enlargement. If the scale factor is less than 1, the dilation is a reduction. Sometimes people prefer to only use the term enlargement. The symbol k is used as an enlargement factor. If $k > 1$, the image of the original figure will be an enlargement. If $k < 1$, the image of the original figure will be a reduction.

Suggested answers

1 a Enlargement factor = 4 b Enlargement factor = 2; (Pyth: 3-4-5 : 6-8-10)
2 a Reduction factor = 5 b Reduction factor = 3
3 C and G; B and E
4 a $\hat{G} = \hat{M}$ b $\hat{K} = \hat{E}$ c EF : KL = 5 : 15
d KLMN is an enlargement of EFGH with $k = 3$.
e EH = 4 cm f MN = 3 cm g LM = 24 cm

PoA | Investigation 4 Investigate enlargement and reduction

Learner's Book page 364

Guidelines for implementing this activity

- Do not teach learners any formulae. The aim of the investigation is for them to reach their own conclusions.
- **Task 1:** The diagram is given.
- **Task 2:** They do their own diagram on a Cartesian plane to ensure that their rectangles are to scale.
- **Task 3:** Only rough sketches are necessary.
- Give clear instructions about the date when the investigations should be completed and handed in.

Remedial and extension

Remedial: Discuss the conclusions with the learners after they have handed in their investigations. Point out that:

- Perimeter has only one dimension ∴ new P = old $P \times k$.
- Area has two dimensions (for example, length and breadth) ∴ new A = old $A \times k \times k$
- Volume has three dimensions ∴ new V = old $V \times k \times k \times k$.

Review Copy

Suggested answers: Memorandum

Task 1

1 an enlargement (1 mark for each answer)

2 a reduction (2)

3

Square A	Square B	Square C
(1; 1)	(3; 3)	(9; 9)
(2; 1)	(6; 3)	(18; 9)
(2; 2)	(6; 6)	(18; 18)
(1; 2)	(3; 6)	(9; 18)
$(x; y)$	$(3x; 3y)$	$(9x; 9y)$

(1 mark for each pair of coordinates) (8)

4 $k = 3$ (1)

5 It is possible to draw three straight lines from the origin through all the corresponding corners of the square. (1)

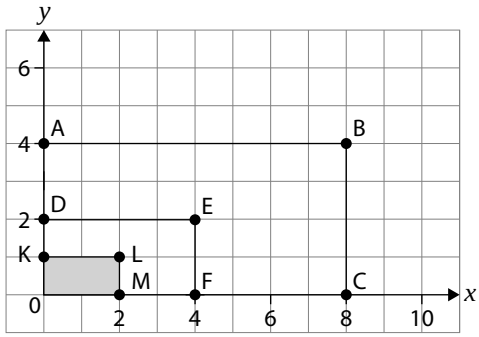
6

	Square A	Square B	Square C
Length of side	1 cm	3 cm	9 cm
Perimeter	4 cm	12 cm	36 cm
Area	1 cm^2	9 cm^2	81 cm^2

(1 mark for each answer) (6)

Task 2

1, **2** and **3**



(1 mark for KOML; 2 marks for DOFE; 2 marks for AOCB) (5)

4

	KOML	DOFE	AOCB
Length	2 cm	4 cm	8 cm
Breadth	1 cm	2 cm	4 cm
Perimeter	6 cm	12 cm	24 cm
Area	2 cm^2	8 cm^2	32 cm^2

(1 mark for each answer) (8)

5 $k = 2$ (1)

Task 3

1

	Circle D	Circle E	Circle F
Radius	25 cm	5 cm	1 cm
Circumference	50π cm	10π cm	2π cm
Area	625π cm^2	25π cm^2	π cm^2

(2 marks for each answer) (8)

2 $k = 5$ (1)

Review Copy

Conclusion

1 Effect of an enlargement on the perimeter (or circumference):
If $k = 3$, then the perimeter of the larger figure will be three times the perimeter of the smaller figure. (2)

2 Effect of an enlargement on the area:
If $k = 3$, then the area of the larger figure will be the area of the smaller figure $\times$ 9. (2)

Total marks: 45

Activity 2 Work with enlargements and reductions

Learner's Book page 367

Guidelines for implementing this activity

- The activity gives learners the opportunity to put everything (including conversion of SI units) together.

- If learners cannot apply the conclusions of the investigation directly, they can make use of sketches and calculate the perimeters (or areas) of the given figure as well as of the figure obtained after the enlargement or reduction.

Suggested answers

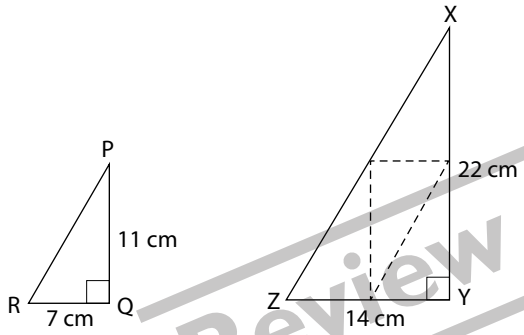
1 a Area = 16 cm^2 b Area = $16 \times 3 \times 3 = 144$ cm^2
c Perimeter = 160 mm = 16 cm d Perimeter = 16 mm = 1,6 cm
2 a Perimeter = $24 \times 3 = 72$ cm b Perimeter = $24 \div 2 = 12$ cm
3 a Area = $48 \times 2 \times 2 = 192$ m^2 b Area = $48 \div 4 \div 4 = 13$ m^2
4 a $144 \div 36 = 4 \therefore k = 2$ b $324 \div 36 = 9 \therefore k = 3$
c $4 \div 36 = \frac{1}{9} \therefore k = \frac{1}{3}$

Note: In question **4c** the new square will be a reduction of the given square.

5 a $60 \div 20 = 3 \therefore k = 3$
b Conversion: 800 mm = 80 cm; $80 \div 20 = 4 \therefore k = 4$
c $4 \div 20 = \frac{1}{5} \therefore k = \frac{1}{5}$

Note: In question **5c** the new pentagon will be a reduction of the given pentagon.

6 a and b



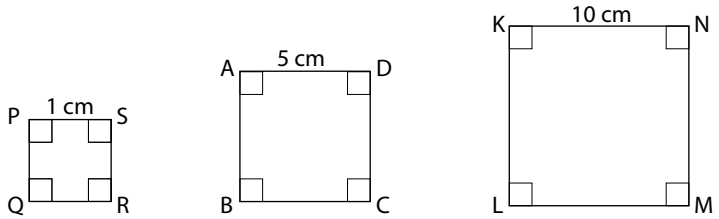
Review Copy

c Four
d Area △PQR : Area △XYZ = 1 : 4
e Area △XYZ : Area △PQR = 4 : 1
f Area △PQR = $\frac{1}{2} \times 7 \times 11 = 38{,}5$ cm^2

Area △XYZ = $\frac{1}{2} \times 14 \times 22 = 154$ cm^2

$154 = 4 \times 38{,}5 \therefore$ previous calculations were correct.

7 a, b and c



d $SQ^2 = 1^2 + 1^2 = 2$ (Pyth) $\therefore SQ = \sqrt{2}$
$BD^2 = 5^2 + 5^2 = 50$ (Pyth) $\therefore BD = \sqrt{50}$
$LN^2 = 10^2 + 10^2 = 200$ (Pyth) $\therefore LN = \sqrt{200}$
e $\sqrt{50} = \sqrt{25 \times 2} = 5 \times \sqrt{2}$; $\sqrt{200} = \sqrt{4 \times 50} = \sqrt{2} \times \sqrt{50}$
The lengths of the diagonals changed in the same ratio as the lengths of the sides.

Chapter 17 Revision

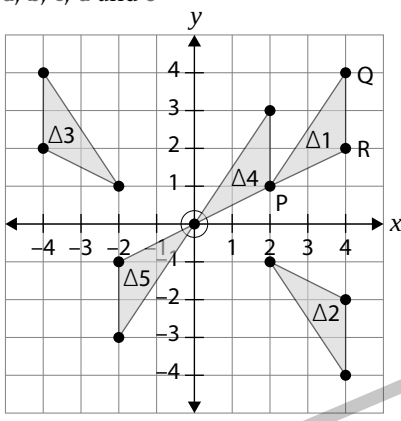
Learner's Book page 368

Encourage learners to review the content covered before attempting the revision activities. The revision activities should be used to assess learners' progress thus far, and to assess where remediation may be required.

Suggested answers

1 **a** T′(0; –4) **b** T′(2; 4) **c** T′(2; 0) **d** T′(–2; –4) **e** T′(5; –5)

2 **a**, **b**, **c**, **d** and **e**



f In △2: R′(4; –2)
In △3: R′(–4; 2)
In △5: R′(–2; –1)

3 **a** Enlargement through the origin with $k = 2$.
b Rotation 90° clockwise around the origin.
c Rotation 90° anticlockwise around the origin.
d Rotate △3 90° clockwise around the origin *or* rotate △4 90° anticlockwise around the origin *or* rotate △1 180° around the origin.
e △1, △2, △3 and △4 are congruent.
f △1 and △2 are similar.

4 New perimeter = 126 × 1,5 = 189 mm
∴ length of B′C′ = 189 ÷ 3 = 63 mm

5 New area = 135 ÷ 3 ÷ 3 = 15 cm^2

Review Copy

CHAPTER

18 Geometry of 3D objects

Chapter overview

Learner's Book pages 369 to 381
Recommended pacing: 7 hours

This chapter focuses on the following:

Unit 1:	**Classifying 3D objects**	1,5 hours
	Classifying polygons	
	Classifying polyhedra	
Unit 2:	**Platonic solids**	3 hours
	Recognising and designing nets	
	Platonic solids	
	Euler's theorem for polyhedra	
Unit 3:	**Building 3D models**	2,5 hours
	Nets	
Chapter 18 Revision		

Review Copy

UNIT

1 Classifying 3D objects

Unit overview

Learner's Book page 370
Recommended pacing: 1,5 hours

This unit focuses on the following:

- revising and expanding the classification of polygons or (2D shapes)
- classifying polyhedra (3D shapes)
- describing the differences between prisms, pyramids and Platonic solids

Resources: Learner's Book; exercise book; examples of prisms and pyramids for the learners to handle; nets of prisms and pyramids

Mental Maths

(10 minutes per day)

Suggested answers — Learner's Book page 370

1 a flat shape; it needs to be closed; all the edges are straight lines.

2 a triangle b quadrilateral c pentagon
d hexagon e heptagon f octagon
g nonagon h decagon i dodecagon
j icosagon

Background information

In Grade 7, learners consolidated the basic concepts covered in Grade 6. The focus was on grouping and categorising of:

- cubes (only square faces)
- rectangular prisms (only rectangular faces)
- triangular prisms (only triangular and rectangular faces)
- pyramids (square and triangular faces)
- cylinders (circular faces).

Learners also focused on:

- using and constructing nets as contexts for exploring and consolidating properties of polyhedra
- recognising, using and constructing nets of different solids
- drawing sketches of the nets using their knowledge of shape and number of faces of the solids, before drawing and cutting out the nets to build models.
- the construction of nets in Grade 7 was based on the number and shape of faces of the solids, and did not require measuring of internal angles of polygons.
- working out the relative position of the faces of the nets, and use trial-and-error to match up the edges and vertices, in order to build the 3D object.

Review Copy

In this unit, the focus is on right prisms and right pyramids.

- In a right prism the faces are perpendicular to the base and therefore they will be rectangles.
- In a right pyramid the apex is directly above the centre of the base of the pyramid.

If a prism or a pyramid is not a right prism, it is oblique. In an oblique prism the faces are not perpendicular to the base and therefore the faces can be parallelograms. (For an overview of solids, go to http://www.korthalsaltes.com/index.html. On this website there are examples of Platonic solids, Archimedean solids, Kepler-Poinsot polyhedra, other uniform polyhedra, compounds, pyramids, concave pyramids, truncated pyramids, dipyramids, other pyramids, prisms, antiprisms, concave prisms, concave antiprisms, twisted prisms, other prisms, other polyhedra and kaleidocycles.)

Teaching guidelines

Before commencing with the teaching of the content in this chapter, make prisms and pyramids for the learners by printing nets from this website: http://www.senteacher.org/Worksheet/12/Nets.xhtml. On this site there are free nets to print for a number of polyhedra.

Learners need to be comfortable with the terminology we use to describe polygons, because polygons form the basis of 3D objects (polyhedra). The aim of this mental maths in this section is to revise previous knowledge and terminology. Revise this every day at the start of the lesson.

Ensure that learners' knowledge of polygons is well consolidated before starting with the lessons on polyhedra, since this knowledge forms the basis of their understanding of how polyhedra are formed. Learners need to practise looking at a diagram (that is flat; net) and being able to see that it represents a 3D object.

Classifying polygons

Activity 1 **Classify polygons**

Learner's Book page 371

Guidelines for implementing this activity

- It is essential that learners understand and are comfortable with polygons (2D shapes) before they move on to polyhedra (3D objects).
- Revise polygons with the learners, making sure that they understand the following:
  - A polygon is a flat, closed 2-dimensional (2D) figure.
  - Polygons can be classified as regular and irregular.
  - Polygons can be convex or concave.
  - A polygon is named after the number of sides it has.
- Let learners name the (regular) polygons that they have encountered in previous grades; draw them underneath the headings Regular polygons in their exercise books and draw irregular versions of these underneath the heading Irregular polygons.
- Explain the difference between concave and convex and let learners further classify the polygons that they have drawn into these two categories. They may need to draw concave polygons if they have not included drawings of these in the irregular category above.

Review Copy

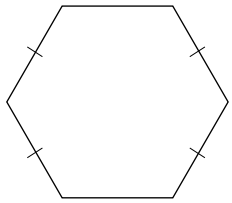
Tip

Use the new words regular, irregular, concave and convex repeatedly so that learners become familiar with them. It is a good idea to make a poster with the names and the shapes of regular polygons for the classroom.

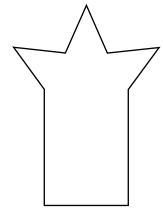
Suggested answers

1 a convex b concave c convex d concave
2 a scalene triangle b square
c equilateral triangle d kite
e regular pentagon f irregular hexagon
g regular heptagon h irregular octagon
3 a Shape with six sides of equal lengths; no interior angle greater than 180°.
b Any shape with nine sides of unequal lengths, at least one interior angle greater than 180°. For example:

A



B



Classifying polyhedra

Activity 2 Classify polyhedra

Learner's Book page 373

Review Copy

Guidelines for implementing this activity

- The learners should now proceed from 2D shapes to 3D objects.
- Spend enough time on the descriptions (definitions) in the table that classify polyhedra.
- Revise the names and features (base; no. of faces; no. of edges; no. of vertices) of the different polyhedra (prisms and pyramids) covered in Grade 7:
  - Cube; Rectangular prism; Triangular prism; Pentagon-based prism; Hexagon-based prism; Heptagon-based prism; Octagon-based prism
  - Triangular pyramid; Tetrahedron; Square-based pyramid; Hexagon-based pyramid; Heptagon-based pyramid; Octagon-based pyramid

Tip

Help learners to develop the ability to "see" 3D by having shapes in the form of pyramids and prisms available in the classroom. If learners handle the shapes and physically count the faces, edges and vertices, they will find the concept of 3D easier to understand.

Suggested answers

1 a pyramid b prism c pyramid d prism
e prism f prism g pyramid h pyramid
i prism
2 a hexagonal pyramid b hexagonal prism
c square-based pyramid d triangular prism

e rectangular prism (or cuboid)
f pentagonal prism
g pentagonal pyramid
h triangular pyramid
i squared-based prism (or cube)

3 a square-based prism
b rectangular prism
c If we look from the side: possibly a prism with a trapezium as a base.
d Hexagonal prism (or cylinder if curved – then it is not a polyhedron).

4 a True: base is a square; faces are rectangles.
b True: curved object.
c False: curved objects are not prisms.
d False: curved objects are not pyramids.
e True

5

	Name of polyhedron	Number of faces	Number of edges	Number of vertices
a	Square-based pyramid	5	8	5
b	Square-based prism	6	12	8
c	Hexagonal pyramid	7	12	7
d	Triangular prism	5	9	6

UNIT 2

Platonic solids

Review Copy

Unit overview

Learner's Book page 375
Recommended pacing: 3 hours

This unit focuses on the following:

- the five Platonic solids by describing, naming and comparing them in terms of:
  - the shape of the faces
  - the number of faces
  - the number of vertices
  - the number of edges
- investigating the relationships between the number of faces, vertices and edges of Platonic solids

Resources: Learner's Book; exercise book; cardboard to make templates, paper to make Platonic solids; sticky tape to paste Platonic solids together; plastic straws; Plasticine™; toothpicks; Jelly tots™

Mental Maths

(10 minutes per day)

Suggested answers

Learner's Book page 375

1 prism	2 pyramid	3 flat shape
4 pyramid	5 prism	6 pyramid
7 prism	8 flat shape	9 flat shape
10 prism or pyramid	11 flat shape	12 prism

Background information

In this unit, learners build on their knowledge of polyhedra. Platonic solids is new in Grade 8.

Teaching guidelines

Once learners have been introduced to Platonic solids, it is important that their knowledge is consolidated with practical constructions and handling of actual objects. As far as possible, let learners use the objects that they have constructed to explore and investigate the properties of Platonic solids in order to ensure a more meaningful learning experience. In addition to using paper cut-out to construct nets, learners can also use straw and Plasticine™ or toothpicks and Jelly tots™ to construct models of Platonic solids.

Review Copy

Recognising and designing nets

Platonic solids

Activity 1 **Construct Platonic solids**

Learner's Book page 376

Guidelines for implementing this activity

- The example of the net in the Learner's Book will produce a cube.
- Discuss the definition of Platonic solids and the introduction to Platonic solids in the Learner's Book.
- Learners follow the steps to make accurate constructions of the templates to use in nets for the Platonic solids.
- It will save time if learners can do this activity in groups. Do not use more than one lesson for this activity. Learners should complete it in their own time.
- Learners need the models that they have made in Activity 1, to do Activity 2.
- If learners have access to the internet, they will find a wealth of information on the topic of Platonic solids.
- Euler is pronounced "Oy-ler" (as in oyster).

Tip

The five Platonic solids make wonderful mobiles to hang from the ceiling of the classroom.

Activity 2 Investigate properties of Platonic solids

Learner's Book page 377

Guidelines for implementing this activity

- Use the models built in Activity 1 to do the investigation.

Tip

Why are there only five Platonic solids?
The interior angles of the polygons that meet at a vertex of a polyhedron should have a degree value smaller than 360°. If the degree value is more than 360°, it is of course not a 3D figure.
Angles of three triangles = 3 × 60° = 180° (Tetrahedron)
Angles of four triangles = 4 × 60° = 240° (Octahedron)
Angles of five triangles = 5 × 60° = 300° (Icosahedron)
But: Angles of six triangles = 6 × 60° = 360° which is a flat (2D) surface.

Suggested answers

Platonic solid	Shape of face	Vertices (V)	–	Edges (E)	+	Faces (F)	=	2
Tetrahedron	Triangle	4	–	6	+	4	=	2
Cube	Square	8	–	12	+	6	=	2
Octahedron	Triangle	6	–	12	+	8	=	2
Dodecahedron	Pentagon	20	–	30	+	12	=	2
Icosahedron	Triangle	12	–	30	+	20	=	2

Review Copy

Tip

It is possible to print nets of the Platonic solids and make copies for the learners but then they do not experience the use of accurate constructions.

Euler's theorem for polyhedra

Activity 3 Work with Platonic solids

Learner's Book page 378

Guidelines for implementing this activity

- Learners do the activity to conclude this unit.

Suggested answers

1 a True
  b False: there are only five Platonic solids.
  c False: a cube has squares as faces and a dodecahedron has pentagons as faces.
  d True
  e True
  f False: an octahedron has eight faces.
2 a A very small organism that causes diseases, e.g. HIV.
  b Polio is a disease caused by a virus and can lead to temporary or permanent paralysis. All children should receive a vaccine against the disease.
  c The virus looks like a Platonic solid with 20 faces.

UNIT 3

Building 3D models

Unit overview

Learner's Book page 379
Recommended pacing: 2,5 hours

This unit focuses on the following:

- recognising, constructing and using nets to make models of geometric solids, including:
  - cubes
  - prisms
  - pyramids

Resources: Learner's Book; exercise book; instruments to do constructions

Review Copy

Mental Maths

(10 minutes per day)

Remind learners of the long calculations that Euler could do in his mind (Unit 2). Learners do the first calculation and then use that answer to do the next step. If they work correctly, their final answer will be the same number as the one that they started with.

Suggested answers — Learner's Book page 379

1 x
2 a $5x$ b $5x + 8$ c $10x + 16$ d $10x + 12$
  e $5x + 6$ f $5x$ g x
3 x

Background information

Learners should have worked extensively with nets in previous grades and even if some learners have had more exposure than others, all learners should be familiar with what a net is even if their practical experience of working with nets differs. Learners should be working more and more independently and creatively with nets in Grade 8.

Teaching guidelines

Before commencing with this unit, revise matching nets and their polyhedra that learners have encountered thus far. It is also a good idea to give learners exercises that require that they do rough sketches of nets in their exercise books. For example, show them a polyhedron such as a rectangular prism or an octagonal pyramid or name the polyhedron and let them sketch the nets.

Nets

Activity 1 Recognise nets

Learner's Book page 379

Guidelines for implementing this activity

- Learners can work through Activity 1 without any further explanation. They can refer to the Learner's Book or their exercise books for information.

Review Copy

Suggested answers

1 **b**, **d** and **e**

2 **a** one hexagon and six isosceles triangles
  b heptagonal prism
  c the shape of the side-faces: a prism has rectangles as faces; a pyramid has isosceles triangles as faces.
  d tetrahedron (triangular pyramid)
  e two triangles and three rectangles

3 **a** –C – 2 **b** –A – 3 **c** –D – 1 **d** –B – 4

Tip

The learners need to use the examples in the table of Question **3** in Activity 2 when they design nets.

Activity 2 Construct nets

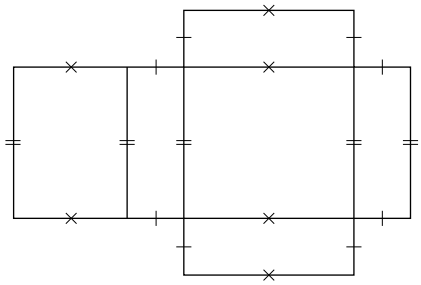
Learner's Book page 380

Guidelines for implementing this activity

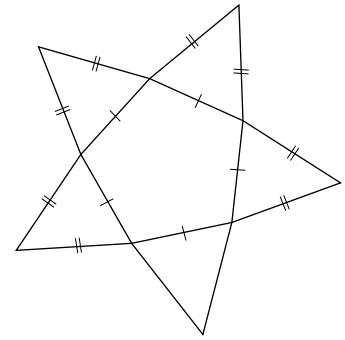
- It is important that learners do the constructions and build the models. Concrete experience is always good.

Suggested answers

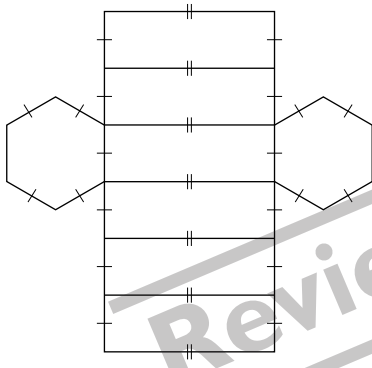
1 **a** All angles need to be right angles.



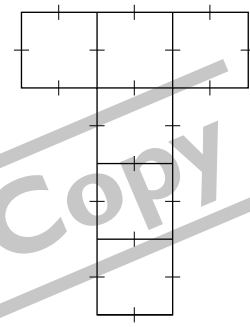
b Each interior angle of regular pentagon is equal to 108°. Height of triangles should not be too small.



c Each interior angle of a regular hexagon is equal to 120°.

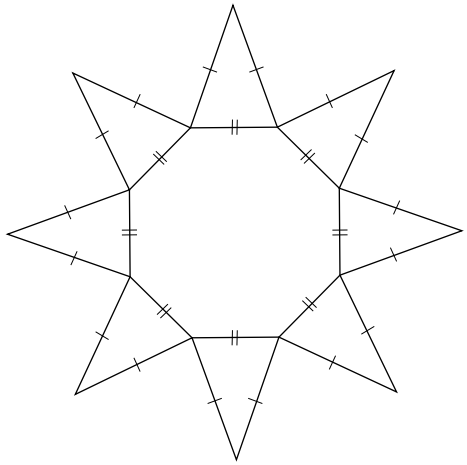


d All angles are right angles.



Review Copy

e Each interior angle of a regular octagon is equal to 112,5°. Height of triangles should not be too small.



2, **3** and **4** Answers will vary.

Tip

Building polyhedra out of toothpicks and Jelly tots™ is a very fun activity to do with learners, if possible.

Chapter 18 Revision

Learner's Book page 381

Encourage learners to review the content covered before attempting the revision activities. The revision activities should be used to assess learners' progress thus far, and to assess where remediation may be required.

Suggested answers

1 a False: polygon is 2D and polyhedron is 3D.
  b False: concave polygon
  c True
  d True
  e False: only one decagon as base
  f True
  g False: 10 vertices
  h True
2 a Square-based pyramid
  b Open box in the form of a cube
3 a $V - E + F = 2$
  b V = vertices (where edges meet)
  E = edges (where faces meet)
  F = faces (flat shapes in the form of squares)
  A cube has 8 vertices, 12 edges and 6 faces.
  $\therefore V - E + F = 8 - 12 + 6 = 2$
4 Cube C

Review Copy

CHAPTER

19 Probability

Chapter overview

Learner's Book pages 382 to 395
Recommended pacing: 4 hours

This chapter focuses on the following:

Unit 1: Theoretical probability 2 hours
- What is probability?
- Possible outcomes
- Probability rules

Unit 2: Relative frequency 2 hours
- Formula for relative frequency
- Comparing relative frequency and probability
- Using relative frequency

Chapter 19 Revision

Review Copy

UNIT

1 Probability: Theoretical probability

Unit overview

Learner's Book page 383
Recommended pacing: 2 hours

This unit focuses on the following:
- defining probability
- listing all the possible outcomes of a simple situation with equally likely outcomes
- determining the probability of each outcome using the definition of probability
- recording the outcomes

Resources: Learner's Book; exercise book; spinner; dice; deck of cards

Mental Maths

(10 minutes per day)

Probabilities can be given in terms of percentages, decimal fractions or common fractions. It is helpful if learners can easily convert between the different forms.

Suggested answers — Learner's Book page 383

1	a 0,75	b 0,5	c 0,25	d 1
2	a 50%	b 75%	c 25%	d 100%
3	a $\frac{1}{4}$	b $\frac{3}{5}$	c $\frac{3}{4}$	d $\frac{1}{2}$
	e $\frac{3}{4}$	f $\frac{1}{5}$	g $\frac{4}{5}$	h $\frac{1}{10}$

Background information

Probability is a branch of Mathematics. Learners will continue to study this topic in FET Maths. It is important for learners to realise that this branch of Mathematics developed because of a practical need to indicate the possibility (probability) of events occurring. Unit 1 deals with theoretical probabilities.

In the Intermediate Phase and Grade 7, learners did probability experiments with coins, dice and spinners. In Grade 8, doing actual trials of experiments becomes less important as learners start to consider the probability of hypothetical events such as choosing an Ace of Spades from a deck of cards, or throwing a 6 on a dice.

Review Copy

Teaching guidelines

It is important to use the correct words when teaching probability, and to use these words throughout the course of the lessons in order to consolidate their meaning in context.

- If you roll a dice a number of times, you are doing an experiment.
- Each time that you roll the dice, is a trial in the experiment.
- You can roll a 1, 2, 3, 4, 5 or 6: these numbers are the possible outcomes. Each number has the same chance to be rolled in a single trial of the experiment; therefore we refer to "equally likely outcomes".
- Before you roll the dice, you decide what you are looking for. If you want to roll even numbers {2; 4; 6}, this is referred to as the event. An event can be one or more outcomes of the experiment.
- Probability is the chance that you have to achieve the event (roll even numbers) that you are interested in. P(even numbers) = 3 out of 6 = $\frac{1}{2}$ = 0,5 = 50%.

What is probability?

Activity 1 Work with probability

Learner's Book page 384

Guidelines for implementing this activity

- Discuss the probability scale with learners, as well as the examples. Ask learners to come up with their own examples, such as: It is my birthday tomorrow; My dog will jump on me when I get home; It is going to rain tomorrow; and so on. Ask learners to add a value to their statements based on the probability scale in the Learner's Book (define their probabilities), and to explain their decision.
- Learners should do the activity on their own.
- Question **1** serves as background to the topic of this chapter.
- Do not spend too much time on this activity as it is only the introduction to the rest of the chapter.

Tip

Provide learners with as many opportunities as possible to give statements that express likelihoods, and to define these probabilities using the probability scale. They can do these types of exercises in pairs as well. It may be interesting to see how similarly or differently learners define probabilities, and why. Encourage learners to realise that Maths develops (even today!) because of the need in real-life situations to find answers, to calculate something or to find a rule or formula to apply.

Review copy

Suggested answers

1 Blaise Pascal was born in **France** in 1623. His **mother** diced when he was three years old and he was raised by his father who was a **judge**. He was interested in Maths from a very **young** age. Before he turned 20, he developed a calculating **machine** to help his father with **tax** calculations. One day in 1653 he received a **visit** from Chevalier de Méré. De Méré asked for help with a problem: Suppose two players agree to play a certain number of games, say a best of seven, and are interrupted before they can finish. How should the stakes be **divided**? Pascal wrote a letter to his friend, Pierre de Fermat. De Fermat was a lawyer who was also very interested in Maths. The two friends wrote a number of **letters** to each other and from these letters developed the theory of **probability** as we do use in Maths today.

2 a Impossible b Likely c Even
d Unlikely e Certain

Possible outcomes

Activity 2 List outcomes

Learner's Book page 385

Guidelines for implementing this activity

- Work through the introduction to this section in the Learner's Book.
- Revise and consolidate terminology such as outcome; experiment; equally likely and sample space.
- Learners should do the activity on their own.

Remedial and extension

Extension: Questions **3** and **4** are a good challenge for the learners.

Suggested answers

1 a {heads; tails}
b {red; blue; green; yellow}
c {1; 2; 3; 4; 5; 6}
d {rough; smooth}
e {M; I; R; A}
f {3; 6; 9; 12; 15; 18; 21}

2 a {Sunday; Monday, Tuesday; Wednesday; Thursday; Friday; Saturday}
b {April; June; September; November}
c Answers will vary, e.g. {Coke; Fanta; Sprite; etc}

3 {HH; TT; HT; TH}

4 {H2; T2; H3; T3; H4; T4; H5; T5; H6; T6}

Review Copy

Probability rules

Activity 3 Determine probabilities

Learner's Book page 386

Guidelines for implementing this activity

- Probability rules form part of Theoretical probability (as opposed to Experimental probability). This is a thinking exercise that we do on paper. We are not doing the actual experiment.
- Discuss the probability formula with the learners.
- Do a few examples and then give the learners time to do the activity on their own.

Suggested answers

1 a $P(H) = \frac{1}{2}$
b $P(B) = \frac{1}{4}$
c $P\left(\frac{1}{2}\right) = 0$
d $P(R) = \frac{1}{2}$
e Answers will vary.
f $P(S) = \frac{2}{7}$
g $P(\text{square no}) = \frac{3}{10}$
h $P(\text{ending on y}) = \frac{4}{12} = \frac{1}{3}$
i $P(\text{prime no}) = \frac{4}{10} = \frac{2}{5}$
j $P(W) = \frac{1}{2}$

2 a P(green) = $\frac{3}{12} = \frac{1}{4}$ b P(yellow) = $\frac{4}{12} = \frac{1}{3}$

c P(blue) = $\frac{5}{12}$ d P(green or yellow or blue) = $\frac{12}{12} = 1$

e The probabilities of all the colours add up to 1. f P(orange) = 0

g P(green or yellow) = $\frac{7}{12}$ h P(not green) = $\frac{5}{12}$

3 a $x = 30$ b 15 silver marbles c P(not red) = $1 - \frac{7}{30} = \frac{23}{30}$

4 a P(apple juice) = $\frac{18}{60} = \frac{3}{10}$

b P(orange juice) = $\frac{30}{60} = \frac{1}{2}$

P(guava juice) = $\frac{12}{60} = \frac{1}{5}$

$\therefore$ difference = $\frac{1}{2} - \frac{1}{5} = \frac{3}{10}$

5 a 52 b 4 c black d ace; 10 e queen; king f jokers

6 a P(R) = $\frac{26}{52} = \frac{1}{2}$ b P(S) = $\frac{13}{52} = \frac{1}{4}$

c P(J or Q or K) = $\frac{12}{52} = \frac{3}{13}$ d P(5) = $\frac{4}{52} = \frac{1}{13}$

e P(Black 5) = $\frac{2}{52} = \frac{1}{26}$ f P(Red club) = $\frac{0}{52} = 0$

7 P(5) on Spinner A = 0

P(5) on Spinner B = $\frac{1}{3}$

P(5) on Spinner C = $\frac{2}{5}$

$\therefore$ best chance with Spinner C

Review Copy

UNIT 2

Relative frequency

Unit overview

Learner's Book page 389
Recommended pacing: 1,5 hours

This unit focuses on the following:

- predicting, with reasons, the relative frequency of the possible outcomes for a series of trials based on probability
- comparing relative frequency with probability
- explaining possible differences between relative frequency and probability

Resources: Learner's Book; exercise book; spinners and other useful maths tools (for example: number grids, number lines). The following website is useful: http://illuminations.nctm.org/ActivityDetail.aspx?ID=205

Mental Maths

(10 minutes per day)
Learner's Book page 389

Learners need to compare relative frequency to probability at the end of this unit. They will save time if they can compare fractions and percentages easily.

Suggested answers

1	**a** $\frac{5}{17}$	**b** $\frac{7}{11}$	**c** $\frac{11}{13}$	**d** $\frac{81}{100}$
2	**a** $\frac{1}{3}$	**b** $\frac{1}{2}$	**c** $\frac{1}{5}$	**d** $\frac{1}{2}$
3	**a** 3 out of 10	**b** 17 out of 20	**c** 50%	**d** 23 out of 50

Background information

Learners are introduced to the concept of relative frequency for the first time in Grade 8. The relative frequency is the observed number of successful outcomes for a finite sample of trials. For example, if you toss a coin 60 times and the results are 32 *heads* and 28 *tails*, you can define *heads* as a successful outcome. The relative frequency of *heads* is: $\frac{32}{60}$ = 53,33%.
The probability of *heads* is 50% (one of two likely outcomes). The difference between the relative frequency of 53,33% and the probability of 50% is due to the small *sample size*.

The more trials you do, the closer the relative frequency gets to the probability. This can be compared in class by combining results from trials done in groups or pairs.

Review Copy

Formula for relative frequency

Activity 1 Determine relative frequency

Learner's Book page 391

Guidelines for implementing this activity

- Explain the concept of relative frequency by working through the example and steps in the Learner's Book.
- Ensure that learners understand the difference between frequency and relative frequency.
- Learners should not confuse the formula to calculate relative frequency with the formula to calculate probability. They need to realise that we calculate probability before we do an experiment or if there is no experiment. Relative frequency can be calculated only after something practical takes place.
- Question 6 is in preparation for the issue addressed in Activity 2.

Remedial and extension

Remedial: For question **6**, ask learners if they notice a pattern in the three tables. It is possible that they notice that the percentages come closer to 50–50% if they toss the coin more times.

Suggested answers

1 **a** $\frac{4}{25}$ **b** $\frac{21}{25}$

2 **a** $\frac{7}{14}$ **b** $\frac{6}{14}$ **c** $\frac{4}{14}$ **d** $\frac{8}{14} = \frac{4}{7}$

e 0

f All the customers buy bread and/or milk and/or sweets. The owner needs to make sure that these items are always in stock.

3 **a** Relative frequency of faulty parts:

Machine A $= \frac{3}{50}$

Machine B $= \frac{4}{96}$

b $\frac{3}{50} = 6\%$ and $\frac{4}{96} = 4\%$

This means that Machine B is better, because the percentage faulty parts is lower.

4 Relative frequency of Japanese cars $= \frac{18}{40}$

$\therefore$ Relative frequency of car not from Japan is $\frac{22}{40} = \frac{11}{20}$.

b Impossible to say: it is a different situation.

5 **a** 50 chips

b frequency

c In this packet 12 out of 50 chips were broken: that is much less than half the chips. Half the chips is 25 out of 50.

d The 29 means that almost 60% of the chips in this packet were acceptable. It is not a very high total. The company should check more packets of chips and then decide what to do.

e No. Relative frequency only gives an indication of what to expect.

Review Copy

6 I

Outcomes	Frequency	Relative frequency
Heads	7	$\frac{7}{10} = 70\%$
Tails	3	$\frac{3}{10} = 30\%$
Total	10	1

II

Outcomes	Frequency	Relative frequency
Heads	56	$\frac{56}{100} = \frac{14}{25} = 56\%$
Tails	44	$\frac{44}{100} = \frac{11}{25} = 44\%$
Total	100	1

III

Outcomes	Frequency	Relative frequency
Heads	491	$\frac{491}{1\,000} = 49{,}1\%$
Tails	509	$\frac{509}{1\,000} = 50{,}9\%$
Total	1 000	1

Review Copy

a A coin was tossed 10 times in Table I; 100 times in Table II and 1 000 times in Table III.
b {head; tail}
c P(H) and P(T) $= \frac{1}{2} = 50\%$
d Table I, Head : Tail = 70 : 30
Table II, Head : Tail = 56 : 44
Table III, Head : Tail = 49,1 : 50,9
e Probability is a theoretical calculation; relative frequency is based on actual actions in real life.

Tip

Encourage learners to develop their entrepreneurial skills – as in Eddice's hot dog sales.

Comparing relative frequency and probability

Activity 2 Compare relative frequency and probability

Learner's Book page 393

Guidelines for implementing this activity

- The aim of the activity is for the learners to find out for themselves what the difference is between probability and relative frequency.

Tips

a Fill in totals in Column E so that the relative frequency of each number on the dice will be equal to the calculated probability of each number on the dice (1 out of 6).
b Will it be possible to obtain the above results for Column E?

Suggested answers

1

Number on dice	A	B	A + B	C	D	A + B + C + D	A + B + C + D
1	3	5	8	4	2	14	Answers will vary
2	4	5	9	3	4	16	
3	4	3	7	4	4	15	
4	6	1	7	4	6	17	
5	1	1	2	6	6	14	
6	6	9	15	3	2	20	

2 a Yes – total is 24. b $P(1) = \frac{3}{24}$ c 3 times
d $P(2) = \frac{4}{24}$; 4 rolls; $P(3) = \frac{4}{24}$; 4 rolls; $P(4) = \frac{6}{24}$; 6 rolls
$P(5) = \frac{1}{24}$; 1 roll; $P(6) = \frac{6}{24}$; 6 rolls
3 a 8 b and c In table
4 In table
5 a Answers will vary. b Answers will vary.
6 The more times we roll the dice, the nearer the relative frequency will come to the calculated probability of 1 out of 6 per number on the dice.

Review Copy

Using relative frequency

Activity 3 Using relative frequency

Learner's Book page 394

Guidelines for implementing this activity

- Learners may ask what the use of all the calculations is. Point out that relative frequencies are used to make predictions.
- Ask learners to give more examples, for example, the number of previous wins by a sports team gives an indication of what to expect from the next match; and so on.

Suggested answers

1 It is highly unlikely (nearly impossible) to roll a dice and obtain these results. The table shows the answer to calculating the probability of rolling each number.

2 a Relative frequency for red roses $= \frac{16}{40} = \frac{2}{5}$

Relative frequency for white roses $= \frac{8}{40} = \frac{1}{5}$

Relative frequency for Barberton daisies $= \frac{3}{40}$

Relative frequency for lilies $= \frac{6}{40}$

Relative frequency for carnations $= \frac{7}{40}$

b $\frac{8}{40} = \frac{16}{80}$ ∴ expect to sell 16 white roses.

c $\frac{3}{40} = \frac{9}{120}$ ∴ expect to sell 21 Barberton daisies.

d $\frac{6}{40} = \frac{3}{20}$ ∴ expect to sell 3 lilies.

e Expects to sell $16 \times 5 = 80$ red roses.

$\frac{16}{40} = \frac{80}{200}$ ∴ expect to sell 200 flowers in total.

Chapter 19 Revision

Learner's Book page 395

Encourage learners to review the content covered before attempting the revision activities. The revision activities should be used to assess learners' progress thus far, and to assess where remediation may be required.

Review Copy

Suggested answers

1 a Probability scale b 1

c An impossible event, e.g. he will grow an extra nose today.

2 There are only two outcomes, green and yellow.

P(green) $= \frac{4}{9}$ and this is not equal to 50%.

3 a {S; U; Z; I; E} b equally likely outcomes

c i P(N) = 0 ii P(S) $= \frac{1}{5}$ iii P(E) $= \frac{1}{5}$

d Frequency of drawing an E = 18% of 150 = 27

e P(E) $= \frac{1}{5}$ = 20%. The difference between 18% and 20% is because the sample size is too small.

4 a

Height in cm	Frequency	Relative frequency
Less than 140 cm	16	$\frac{16}{100} = 16\%$
140 to 159 cm	27	$\frac{27}{100} = 27\%$
160 to 179 cm	54	$\frac{54}{100} = 54\%$
More than 180 cm	3	$\frac{3}{100} = 3\%$

b 54% c $54 + 3 = 57$ ∴ 57%

d $\frac{16}{100} = \frac{x}{350}$. This means $x = 56$ ∴ expect 56 learners out of 350 to be shorter than 140 cm

e $\frac{3}{100} = \frac{1}{y}$. This means $y = 33\frac{1}{3}$ ∴ Expect 33 learners in the class.

Exam exemplar 2 (November): Paper 1 Memorandum

Time: 1 hour 40 minutes **Total: 100 marks**

1.1 **a** F✓ **b** T✓ **c** T✓ **d** F✓ **e** T✓ (5)
1.2 **a** A✓ **b** A✓ **c** D✓ **d** A✓ **e** C✓ (5)

2.1 $\mathbb{Z} = \left\{-4; 18; 93; \frac{-65}{-65}; 0\right\}$✓✓(−1 per mistake) (2)

2.2 **a** $F_{240} = \{1; 2; 3; 4; 5; 6; 8; 10; 12; 15; 16; 20; 24; 30; 40; 48; 60; 80; 120; 240\}$
✓✓✓(−1 per mistake) (3)
b $PF_{240} = \{2; 3; 5\}$✓ (1)
2.3 $240 = 2 \times 2 \times 2 \times 2 \times 3 \times 5$
2.4 28✓ (1)
2.5 **a** "long subtraction"✓; 292✓ (2)
b "long multiplication"✓; 23 380✓ (2)
c undefined✓ (1)
d $15 + 20 + 8 = 35 + 8$✓ $= 43$✓ (2)
e 25✓ $-15(-1)$✓ $= 25 + 15$✓ $= 40$✓ (4)
2.6 $2 + 1 + 5 = 5$ parts✓; 1 part $= 10 \div 5 = 2$ goals✓; $2 \times 2 = 4$ goals✓ (3)
2.7 $7 \div 8$✓ $\times$ 4 000✓ = R3 500✓ (3)
2.8 R12 000 × 0,065✓ = R780✓; R780 × 3 = R2 340✓ (3)
3.1 **a** −5✓ **b** 1✓ (2)
3.2 **a** 8✓a^{15}✓ **b** 4^3✓ $= 64$✓ **c** -150✓$p^{11}q^{15}$✓ (6)
3.3 54×10^{24}✓ $= 5{,}4 \times 10^{25}$✓ (2)
4.1 **a** $T_1 = 4(1) - 24 = -20$✓; $T_2 = 4(2) - 24 = -16$✓
$T_n = 4(n) - 24$✓ (3)
b $T_{200} = 4(200) - 24 = 776$✓ (1)
4.2 **a** $y = \frac{1}{4}n$✓ (1)
b $m = \frac{1}{4} \times 12 = 3$✓; $n = 5\frac{1}{4} \div \frac{1}{4} = \frac{21}{4} \times \frac{1}{4} = 21$✓ (2)
4.3 $y = 2(-3)^3 - 5 = -59$✓(SF); $y = 2(4)^3 - 5 = 123$; $(-3; -59)$✓; $(4; 123)$✓ (3)
5.1 $x + x + 1$✓ (1)
5.2 **a** $6x$✓ $+ 18y$✓ **b** 5✓x^2y^6✓ (4)
c $\frac{12x^3y}{4x^2y} - \frac{14x^2y^5}{4x^2y} = 3x$✓ $- \frac{7}{2}y^4$✓ (4)
5.3 **a** $x = 0$✓ **b** $4k = 36$✓; $k = 9$✓ (3)
c $\frac{30}{x} = \frac{1}{2}$; $x = 60$✓ **d** $24 - 12 + 2x = 0$✓; $-2x = 12$✓; $x = -6$✓ (4)
5.4 $\frac{4(-3)^2(-1)}{5(4)}$✓ $= -\frac{9}{5}$ ✓ (3)
5.5 $2(x + 5) + 2(x - 3) = 44$✓
$2x + 10 + 2x - 6 = 44$✓; $4x + 4 = 44$✓; $4x = 40$; $x = 10$✓ (4)
6.1 R7,99✓ (1)

Review Copy

6.2 **a** $\frac{1}{144}$✓✓ (2)

b 0,000064✓ (1)

c $\frac{17}{3}$✓ $- \frac{13}{4} + \frac{4}{5} = \frac{340}{60} - \frac{195}{60} + \frac{48}{60}$✓ $= \frac{193}{60}$✓ (3)

d $-\frac{65}{16}$✓ $\times \frac{1}{5}$ ✓ $= -\frac{13}{16}$✓ (3)

e "long subtraction" ✓; R690,65✓; "long addition" ✓; R3 504,39✓ (4)

f 20,65 – 29,9907✓(long ×)✓ = 90,6583✓(long –)✓ (4)

6.3 100 % – 24,5 % = 75,5 %; 0,755 × 207✓ ≈ 156 days✓
(or 0,245 × 207 ≈ 51✓; 207 – 51 = 156 days✓) (2)

Exam exemplar 2 (November): Paper 2 Memorandum

Time: 120 **Total: 100 marks**

1 **a**

Stem	Leaves
3	1 9
4	2 3
5	2 3 3 5 6 7 8
6	2 5 8
7	3 4 0

Key: 3 | 9 = 39

(1 mark for stem; 2 marks for leaves; 1 mark for key) (4)

b Mean = 951 ÷ 17 = 55,9 (1 mark for total; 1 mark for answer) (2)
Mode = 53 (1)
Median = 56 (1)

c There is only one data set. Use a double bar graph to compare two sets of data. (1)

2 **a** 2011 (1)

b Maximum number for both birds counted in 2010 (1)

c Range = 27 – 9 = 18 (2)

d False: Between 2009 and 2012 the counts of both birds were higher than in 2012. (1 mark for reason) (1)

3 **a** {20c; 50c; R1; R2} **b** P(20c) = $\frac{4}{10} = \frac{2}{5}$ (2)

c P(R1 or R2) = $\frac{5}{10} = \frac{1}{2}$ **d** P(R5) = 0 (2)

4 **a** P(3) = $\frac{1}{6}$ **b** Relative frequency = $\frac{2}{30} = \frac{1}{15}$ (2)

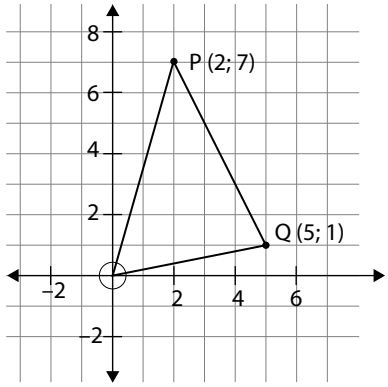
c The relative frequency is different from the probability because the sample size was too small. (1)

5 **a** S′(–3; –5) (1 mark for each coordinate) (2)

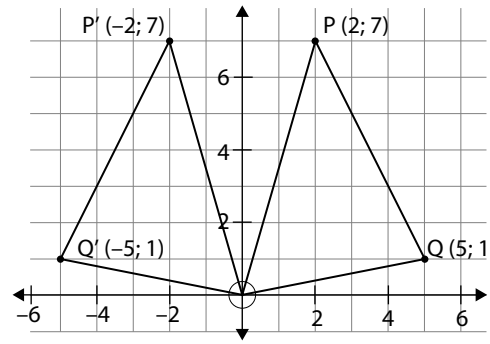
b S′(1; 0) (1 mark for each coordinate) (2)

Review Copy

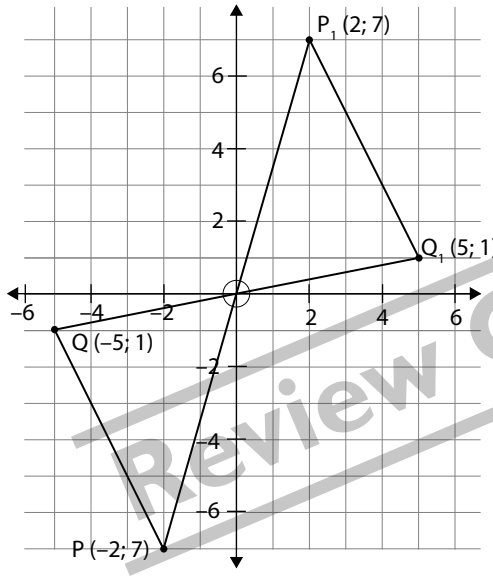
6 **a**



b



c



Review Copy

(3 marks for a; 2 marks for b; 2 marks for c) (7)

7 **a** $x = 72°$ (vertically opp ∠s) (1 mark for answer, 1 mark for reason) (2)
$y = 72°$ (alternate ∠s; EF ∥ GH) (1 mark for reason) (1)
$z = 72°$ (corresponding ∠s; EF ∥ GH) (1 mark for reason) (1)

b $2x = 138°$ (vertically opp ∠s)
$\therefore x = 69°$ (1 mark for $2x = 138°$; (1 mark for reason, 1 mark for answer) (3)
$138° + 3y = 180°$ (angles on straight line)
$3y = 42°$
$\therefore y = 14°$ (3)
(1 mark for statement and reason in first line; 1 mark for $3y = 42°$; 1 mark for answer) (3)

c $102° + 48° + x = 180°$ (3 ∠s of △STQ)
$\therefore x = 30°$
(1 mark for statement in first line; 1 mark for reason, 1 mark for answer) (3)
$y = 60°$ (PR = RT = TP) (1 mark for reason, 1 mark for answer) (2)

d $2x + 10° + x = 106°$ (ext ∠ = sum of opp int ∠s)
$3x = 96°$
$\therefore x = 32°$ (1 mark for statement in first line; 1 mark for reason, 1 mark for answer) (3)

e No: $\hat{R} = 60°$ and $x = 32°$. Therefore the corresponding angles are not equal.
(1 mark for calculation shown; 1 mark for reason) (2)

8 a $\triangle ABC \not\equiv \triangle DEF$ (angle not included) (1 mark for answer; 1 mark for reason) (2)

b $\triangle PQR \equiv \triangle SUT$ ($\angle\angle S$) (1 mark for correct order of triangles; 1 mark for reason) (2)

c $\triangle PQS \equiv \triangle RSQ$ (RHS or other possibility, e.g. SSS)
(1 mark for correct order of triangles; 1 mark for reason) (2)

9 $x = 7$ cm, $y = 8$ cm and $z = 14$ cm
(Longest side)$^2 = 14^2 = 196$
$7^2 + 8^2 = 49 + 64 = 113$
$196 \neq 113$
$\therefore \triangle XYZ$ is not right-angled
(1 mark for values of sides in cm; 2 marks for calculations; 1 mark for conclusion) (4)

10 a prism (1)

b $b^2 = a^2 + c^2$ (Pyth)
$= 5^2 + 12^2 = 169$
$\therefore AC = 13$ cm
(1 mark for statement and reason; 1 mark for substitution; 1 mark for 169;
1 mark for answer) (4)

c Area of $\triangle ABC = \frac{1}{2} \times b \times h$
$= \frac{1}{2} \times 5 \times 12$
$= 30$ cm^2
(1 mark for formula; 1 mark for substitution; 1 mark for answer) (3)

d Area of the faces = perimeter of $\triangle ABC \times h$
$= (5 + 12 + 13) \times 20$
$= 600$ cm^2 (1 mark for formula; 1 mark for substitution; 1 mark for answer) (3)
$\therefore$ Total surface area $= 600 + 2 \times$ Area of $\triangle ABC$
$= 600 + 2 \times 30$
$= 660$ cm^2 (1 mark for formula; 1 mark for answer) (2)

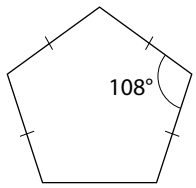
11 a Volume = area of base $\times h$
$\therefore 1\,280$ = area of the base $\times 8$
$\therefore$ Area of the base $= 160$ cm^2
(1 mark for formula; 1 mark for substitution; 1 mark for answer) (3)

b $1\,280$ cm$^3 = 1\,280$ millilitre
$1\,280$ millilitre $= 1{,}28$ litre
(1 mark for conversion to ml; 1 mark for answer) (2)

12 Perimeter of full circle $= 2\pi r$
$= 2 \times \pi \times 1{,}4$
$= 2 \times \frac{22}{7} \times 1{,}4$
$= 8{,}8$ cm
(1 mark for formula; 1 mark for substitution; 1 mark for answer) (3)

$\therefore$ perimeter of $\frac{3}{4}$ of circle $+ 2 \times 1{,}4 = \frac{3}{4} \times 8{,}8 + 2 \times 1{,}4 = 9{,}4$ cm
(1 mark for formula; 1 mark for answer) (2)

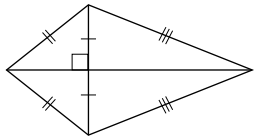
13 a



(1 mark for 5 equal sides; 1 mark for correct angles) (2)

Review Copy

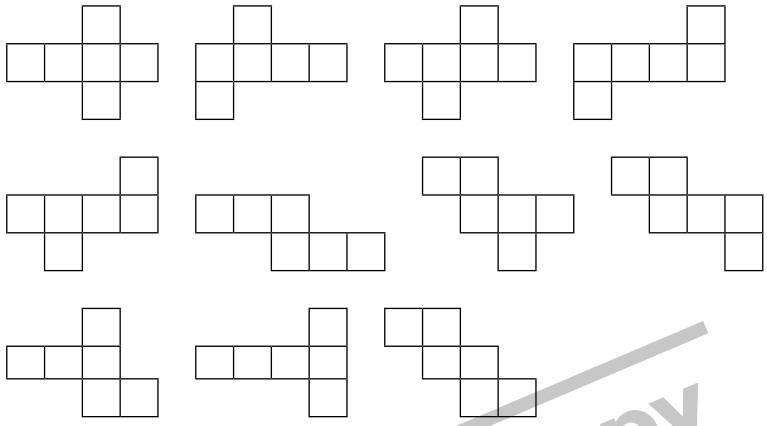
b



(1 mark for lengths of sides; 1 mark for perpendicular diagonals;
1 mark for diagonal that is bisected) (3)

---

c All 11 nets are correct.



(1 mark for using squares; 1 mark for correct net) (2)

d False: A cube can be classified as a prism or as a Platonic solid.
(1 mark for explanation) (1)

14 a TR = 6 m
TS = 10 m (2)

b QR = 9 m (1)

c Area of square $= s \times s$
$= 9 \times 9$
$= 81\ m^2$ (1 mark for formula; 1 mark for answer) (2)

d New area $= 81 \times \frac{1}{3} \times \frac{1}{3} = 9\ m^2$ (1 mark for ratio; 1 mark for answer) (2)

Review Copy

Exam exemplar 1 (November): Additional Paper 1

Time: 2 hour 30 minutes **Marks: 125**

Question 1 **[7]**

1 The following set of numbers is given: {0; 1; 2; 3; 5; 8; 15; 16; 24}. Use only the numbers in the given set to list:

1.1 the factors of 16

1.2 the prime factors of 15

1.3 the square of 4

1.4 multiples of 5

1.5 all possible values of a if $a \in \mathbb{N}$ and $a \leq$ the cube root of 27

1.6 lowest common multiple of 6 and 8

1.7 highest common factor of 6 and 8.

Question 2 **[1]**

2 Write down the number of the statement that is NOT true.

A $\frac{56}{56} = 1$ **B** $56 \times 1 = 56$ **C** $\frac{56}{0} = 0$ **D** $56 \times 0 = 0$

Question 3 **[4]**

3 Rewrite the following and replace the * with =; < or >

3.1 $3^2 * 2^3$

3.2 $5^3 \times 5^4 * 25^7$

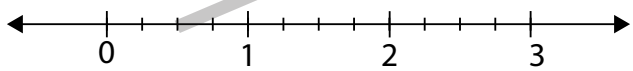
3.3 $(-210)^9 * (-210)^6$

3.4 $7\frac{8}{11}\left(3\frac{1}{2} - \frac{4}{7}\right) * \frac{85}{11} \times \frac{7}{2} - \frac{85}{11} \div \frac{7}{4}$

Review Copy

Question 4 **[3]**

4.1 Redraw this number line in your book.



4.2 Simplify the following and fill them in on the number line.

$2{,}5 \times 10^0$ $\sqrt{0{,}25}$ $1 + (0{,}5)^2$

Question 5 **[3]**

5 Rewrite and complete: $\frac{4}{?} = \frac{8}{14} = \frac{?}{21a} = \frac{?}{0{,}07}$

Question 6 **[12]**

6 Simplify without the use of a calculator.

6.1 $2\frac{3}{8} - 4\frac{5}{12}$ (3)

6.2 $(3\sqrt{9} + \sqrt{50 - 1}) \div 2$ (3)

6.3 $\sqrt[3]{11^2 + 2^2}$ (3)

6.4 $\sqrt[3]{-2\frac{10}{27}}$ (3)

Question 7 **[4]**

7 Given: $4a^4 - 3a^3b + 6a^2b^2 \div 3 - \frac{2}{5}a \times b^3$

7.1 How many terms are in the expression?

7.2 Write down the exponent of a in the last term.

7.3 Write down the value of the expression if $a = \frac{1}{2}$ and $b = 0$

Question 8 **[34]**

8 Simplify showing all your work:

8.1 $3 \cdot a \cdot a + (2a)^2 - \frac{5a^3}{a}$ (3)

8.2 $(2x^2y)(-3x^3)(\sqrt{9}x^4y)$ (3)

8.3 $(4m + 5m) \div 3 + 2m - 16m \div 2$ (3)

8.4 $\frac{2}{3}p(6p^2)^3$ (3)

8.5 $\frac{-18m^2n^4r}{6m^2n^2r^3}$ (3)

8.6 $\frac{4p^2q - 2pq + 8p}{-2pq}$ (3)

8.7 $\frac{17y^6 - y^6}{2y^3} - 2y^3 \cdot 2y$ (3)

8.8 Multiply $(5x^2 - 15x + 20)$ by $-\frac{1}{5}x^2y$ (3)

8.9 Subtract $7a + 4b - 3c$ from $8a - 3b + 4c$ (3)

8.10 $\frac{q}{2} + \frac{3q}{5} - \frac{2q}{3}$ (4)

8.11 $\frac{-y^7z^5}{4} \div \frac{z^3}{8y^2}$ (3)

Question 9 **[11]**

9 Solve for the unknown:

9.1 $5a - 1{,}2 = 4{,}8$ (2)

9.2 $(2x + 5)3 - 5 = 2(x - 5) - (3x + 1)$ (7)

9.3 $2y - 4 = \frac{1}{2}(4y - 8)$ (2)

Question 10 **[3]**

10 Given: $y = -2$ and $x + y + 3 = 0$. Find the value of $3xy^2$.

Question 11 **[6]**

11 Nelly has written down three secret numbers. The first number is ten more than the second number and the third number is twice the second number. The three numbers add up to 310. Write down a suitable equation and find the first number.

Question 12 **[4]**

12 Simplify the following ratios:

12.1 $\frac{15 \text{ minutes}}{2 \text{ hours}}$

12.2 2 litres : 250 ml

Review Copy

Question 13 **[5]**

13 Mr Smith and Mr Ebrahim invested money in the ratio 4 : 5 to begin a business. At the end of the first month there is a profit of R1 800.

13.1 How much money will Mr Ebrahim receive if the profit is divided in the same ratio as the investment? (2)

13.2 Mr Smith says he wants 45% of the total profit deposited in his bank account. Will Mr Ebrahim be happy with this request? (3)

Question 14 **[5]**

14 Rachel buys a fridge of R2 000 on hire purchase. She pays a deposit of 10% and must repay R95 per month for 2 years.

14.1 How much does Rachel pay for the fridge? (3)

14.2 Write the interest as a percentage of the original loan amount. (2)

Question 15 **[3]**

15 Luvo invests R2 250 at Best Bank at 12,5% simple interest per year. How much interest will Luvo earn over a period of 4 years?

Question 16 **[4]**

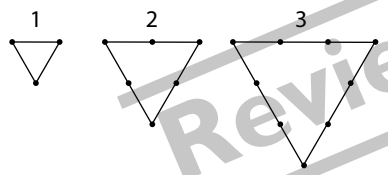
16 Rewrite and write down the next 3 numbers in the pattern.

16.1 $-2; -7; -12; \ldots; \ldots; \ldots$

16.2 $\frac{2}{5}; \frac{3}{7}; \frac{4}{9}; \ldots; \ldots; \ldots$

Question 17 **[17]**

17.1 Study the given pattern and complete the table.



Pattern number	1	2	3	4	x
Number of dots on side of triangle	2	3		5	
Total number of dots on triangle	3	6	9		

(4)

17.2 Complete the following rules for the pattern in words.

a The number of dots on one side of a triangle = the pattern number …

b The total number of dots on a triangle = the pattern number … (2)

17.3 How many dots will be on one side of a triangle if the total number of dots on a triangle is 69? (1)

17.4 What is the total number of dots on a triangle if one side of a triangle has 8 dots? (1)

17.5 Use the values for the first four pattern numbers in the table. Plot the points for the equation $y = 3x$ on a Cartesian plane. (7)

17.6 Given: [linear/non-linear/discrete/continuous]
Choose two words from the list to describe the graph. Explain why you chose those two words. (2)

Review Copy

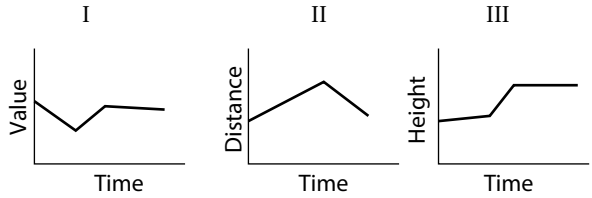
Question 18 **[3]**

18 Write down the number of the graph that fits each description.

A The balloon stayed at a certain height, flew higher and then stayed at that height.

B The value of the shares decreased, increased and then stayed constant.

C Mercia walked towards the tree and then turned away from it.



Exam exemplar 1 (November): Additional Paper 1 Memorandum

Question 1 **[7]**

1.1 {1; 2; 8; 16} **1.2** {3; 5} **1.3** {16} **1.4** {5; 15}
1.5 {1; 2; 3} **1.6** {24} **1.7** {2}

Question 2 **[1]**

2 Statement C: Undefined

Question 3 **[4]**

3.1 $3^2 > 2^3$

3.2 $5^3 \times 5^4 < 25^7$

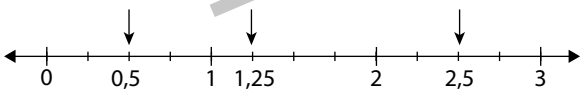
3.3 $(-210)^9 < (-210)^6$

3.4 $7\frac{8}{11}\left(3\frac{1}{2} - \frac{4}{7}\right) = \frac{85}{11} \times \frac{7}{2} - \frac{85}{11} \div \frac{7}{4}$

Review Copy

Question 4 **[3]**

$2{,}5 \times 10^0 = 2{,}5$ $\sqrt{0{,}25} = 0{,}5$ $1 + (0{,}5)^2 = 1{,}25$



(1 mark for each correct value on number line)

Question 5 **[3]**

$\frac{4}{7} = \frac{8}{14} = \frac{12a}{21a} = \frac{0{,}04}{0{,}07}$

Question 6 **[3]**

6.1 $2\frac{3}{8} - 4\frac{5}{12}$

$= \frac{19}{8} - \frac{53}{12}$ ✓

$= \frac{57 - 106}{24}$ ✓

$= -\frac{49}{24}$

$= 2\frac{1}{24}$ ✓ (3)

6.2 $(3\sqrt{9} + \sqrt{50 - 1}) \div 2$

$= (3 \times 3 + \sqrt{49}) \div 2$ ✓
$= (9 + 7) \div 2$ ✓
$= 8$ ✓ (3)

6.3 $\sqrt[3]{11^2 + 2^2}$
$= \sqrt[3]{121 + 4}$ ✓
$= \sqrt[3]{125}$ ✓
$= 5$ ✓ (3)

6.4 $\sqrt[3]{-2\frac{10}{27}}$

$= \sqrt[3]{-\frac{64}{27}}$ ✓

$= -\frac{4}{3}$ ✓

$= -1\frac{1}{3}$ ✓ (3)

Question 7 **[4]**

7.1 4 terms ✓
7.2 1 ✓
7.3 $4\left(\frac{1}{2}\right)^4 = \frac{1}{4}$ ✓✓

Question 8 **[34]**

8.1 $3a^2 + (2a)^2 - \frac{5a^3}{a}$
$= 3a^2 + 4a^2 - 5a^2$ ✓✓
$= 2a^2$ ✓ (3)

8.2 $(2x^2y)(-3x^3)(\sqrt{9}x^4y)$ ✓✓
$= -18x^9y^2$ ✓ (3)

8.3 $(4m + 5m) \div 3 + 2m - 16m \div 2$
$= \frac{9m}{3} + 2m - \frac{16m}{2}$ ✓
$= 3m + 2m - 8m$ ✓
$= -3m$ ✓ (3)

8.4 $\frac{2}{3}p(6p^2)^3$

$= \frac{2}{3} \times p \times 6p^2 \times 6p^2 \times 6p^2$ ✓

$= 144p^7$ ✓✓ (3)

8.5 $\frac{-18m^2n^4r}{6m^2n^2r^3}$ ✓

$= -\frac{3n^2}{r^2}$ ✓✓ (3)

8.6 $\frac{4p^2q - 2pq + 8p}{-2pq}$

$= -2p + 1 - \frac{4}{q}$ (3)

8.7 $\frac{17y^6 - y^6}{2y^3} - 2y^3 \cdot 2y$

$= \frac{16y^6}{2y^3} - 4y^4$ ✓✓

$= 8y^3 - 4y^4$ ✓ (3)

Review Copy

8.8 $-\frac{1}{5}x^2y(5x^2 - 15x + 20)$
$= -x^4y + 3x^3y - 4x^2y$ (3)

8.9 $8a - 3b + 4c - (7a + 4b - 3c)$
$= 8a - 3b + 4c - 7a - 4b + 3c$
$= a - 7b + 7c$ (3)

8.10 $\frac{q}{2} + \frac{3q}{5} - \frac{2q}{3}$
$= \frac{15q + 18q - 20q}{30}$ ✓✓✓
$= \frac{13q}{30}$ ✓ (4)

8.11 $\frac{-y^7z^5}{4} \div \frac{z^3}{8y^2}$
$= \frac{-y^7z^5}{4} \times \frac{8y^2}{z^3}$ ✓
$= -2y^9z^2$ ✓✓ (3)

Question 9 **[11]**

9.1 $5a - 1{,}2 = 4{,}8$
$5a = 6$ ✓
$a = \frac{6}{5} = 1{,}2$ ✓ (2)

9.2 $(2x + 5)3 - 5 = 2(x - 5) - (3x + 1)$
$6x + 15 - 5 = 2x - 10 - 3x - 1$ ✓✓✓
$6x + 10 = -x - 11$ ✓
$6x + x = -11 - 10$ ✓
$7x = -21$ ✓
$x = -3$ ✓ (7)

9.3 $2y - 4 = \frac{1}{2}(4y - 8)$
$2y - 4 = 2y - 4$ ✓
$y \in \mathbb{R}$ ✓ (2)

Question 10 **[3]**

Given: $y = -2 \therefore x + (-2) + 3 = 0$
$x = -1$ ✓
$\therefore 3xy^2 = 3(-1)(-2)^2 = -12$ ✓✓

Question 11 **[6]**

Let the second number be x ✓
Then $(x + 10) + x + 2x = 310$ ✓✓
$4x + 10 = 310$ ✓
$4x = 300$
$x = 75$ ✓
$\therefore$ first number is 85 ✓

Question 12 **[4]**

12.1 $\frac{15 \text{ minutes}}{2 \text{ hours}} = \frac{15}{120} = \frac{1}{8}$

12.2 2 litres : 250 ml = 2 000 : 250 = 8 : 1
(1 mark for same unit; 1 mark for answer in each question)

Review Copy

Question 13 **[5]**

13.1 Mr Ebrahim receives $\frac{5}{9} \times$ R1 800
= R1 000 (2)

13.2 45% of R1 800 = $\frac{45}{100} \times$ R1 800 = R810
Mr Ebrahim will not be happy: it is R10 more than Mr Smith's portion of the profit. (3)

Question 14 **[5]**

14.1 Deposit + 24 instalments
= R200 + R95 × 24
= R2 480 (3)

14.2 $\frac{480}{2\,000} \times 100\% = 24\%$ (2)

Question 15 **[3]**

Interest for one year = $\frac{12{,}5}{100} \times$ R2 250
= R281,25 ✓✓
Interest for 4 years = R281,25 × 4
= R1 125 ✓

Question 16 **[4]**

16.1 −2; −7; −12; −17; −22; −27 (2)

16.2 $\frac{2}{5}; \frac{3}{7}; \frac{4}{9}; \frac{5}{11}; \frac{6}{13}; \frac{7}{15}$ (2)

Question 17 **[17]**

17.1

Pattern number	1	2	3	4	x
Number of dots on side of triangle	2	3	4	5	$x + 1$
Total number of dots on triangle	3	6	9	12	$3x$

(4)

(1 mark for 4 and 12; 1 mark for $x + 1$; 1 mark for $3x$)

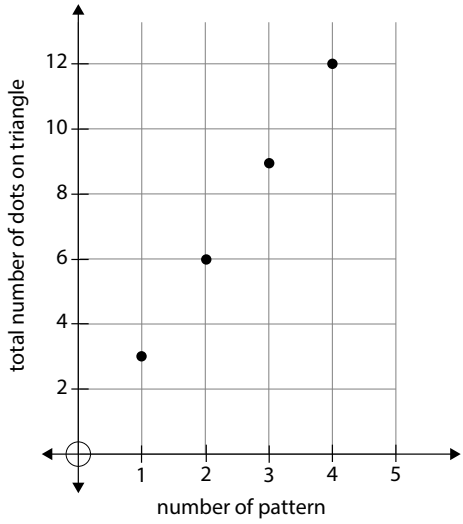
17.2.1 The number of dots on one side of a triangle = the pattern number plus one

17.2.2 The total number of dots on a triangle= the pattern number times three (2)

17.3 23 dots (1)

17.4 24 dots (1)

17.5 Triangle pattern



(7)

Review Copy

17.6 Linear: the points lie in a straight line
Discrete: a triangle cannot have $2\frac{1}{2}$ dots. Will not join points on graph. (2)

Question 18 **[3]**
Write down the number of the graph that fits each description:
A : III
B : I
C : II

Exam exemplar 2 (November) Additional Paper 1

Time: 1 hour 40 minutes **Marks: 100**

Question 1 **[5]**
True or false?
1.1 $-4\ 065$ is larger than $-6\ 540$.
1.2 $5x^4 + x^3 - 7x^2 + 2x - 16$ is written in the correct order.
1.3 There are twelve multiples of 40 in 480.
1.4 The additive inverse of 16 is $\frac{1}{16}$
1.5 The coefficient of x is 9 in this expression: $2x - x^3 + 9x^2 - 1$. (5)

Question 2 **[20]**
2.1 Estimate the answer of 875×99 without using a calculator. (2)
2.2 Find the HCF of 60 and 90. (3)
2.3 Calculate the following without using a calculator:
2.3.1 $-4 + -7$ (1)
2.3.2 $-12 \times 4 \times -2$ (2)
2.3.3 $11 - (24 - 7 \times 3)5$ (4)
2.3.4 $92\ 050 \div 25$ (2)
2.4 It takes Veronique 5 hours to drive to her parents' house if she drives at a average speed of 120 km/h. How far does she have to drive? Distance (km) = speed (km/h) × time (h) (2)
2.5 Mr du Toit buys a television on hire-purchase. The cash price of the television is R3 275. He pays a 12,25% deposit and 24 monthly payments of R275 each.
2.5.1 How much will he pay for the television in total? (3)
2.5.2 How much interest will he pay? (1)

Question 3 **[15]**
3.1 Expand $8a^3(2b)^2$. (3)
3.2 Write in exponential notation:
$35 \times 35 \times 35 \times 35 \times m \times 35 \times 35 \times m \times m$ (2)
3.3 Simplify.
3.3.1 $75^{13} \times 75^{20}$ (1)
3.3.2 $-20(-365v^{50}w)^0$ (2)
3.3.3 $\sqrt{\frac{16}{81}}$ (2)
3.3.4 $\frac{48m^{15}n^{22}}{-24n^{22}m^{10}}$ (2)

Review Copy

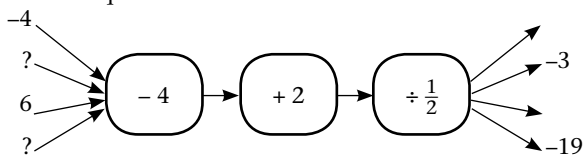
3.4 Complete the following number sentence:
16 million = ... = 1,6 × 10 (2)

3.5 Write 3,8534 × 10^6 kg in normal notation. (1)

Question 4 **[13]**

4.1 Provide the rule to describe the relationship between the numbers in the number sequence 12; 11; 9; 6; … and the write the next two terms in the number sequence. (2)

4.2



Find all the unknown input and output values. (4)

4.3 Study the table below and then answer the questions that follow.

x	1	2	3	4	10	q
y	3	6	11	18	p	146

4.3.1 What pattern do you notice? How do you calculate the output value? Write your answer as a sentence. (1)

4.3.2 Write a formula to calculate the output value: y = … (2)

4.3.3 Calculate the values of p and q. (4)

Question 5 **[21]**

5.1 Round 1,60598248 mg to 4 decimal places. (1)

5.2 Write the next 3 numbers in the number sequence: 3,998; 3,748; 3,498; … (1)

5.3 Convert $\frac{27}{25}$ to a decimal fraction. (2)

5.4 Calculate without using a calculator:

54.1 $\frac{1}{3} - \frac{1}{4} + \frac{1}{5}$ (3)

5.4.2 $\frac{2}{3}$ of 36 ÷ $-\frac{4}{5}$ (4)

5.4.3 $10\sqrt{\frac{49}{25}}$ (3)

5.4.4 582,2 m × 9,2 (2)

5.5 Geraldine saw a pair of jeans that she would like to buy. Last week they cost R490. However, this week the shop has a "−12,5% sale". How much does the pair of jeans cost now? (2)

5.6 Tyrone spent R895 on medication in January and R1 020 in February. Calculate the percentage increase in terms of the money he spent on medication. You may use a calculator for this calculation. (3)

Question 6 **[26]**

6.1 Calculate the difference between $9m^4 + 3m^2 - 2$ and $3m^2 + 1$. (3)

6.2 Simplify. (None of the variables is equal to zero.)

6.2.1 $16x - x + 13x + 2x$ (1)

6.2.2 $\sqrt[3]{y^{12}}$ (1)

6.2.3 $\frac{-28x^5 + 7x^2}{7x^2}$ (2)

6.2.4 $3x \times 2y^4 + 5xy \times 5y^3$ (3)

Review Copy

6.2.5 $3(-x^2 + 7xy) - (21xy - 14x^2)$ (2)

6.2.6 $\frac{35x^{10}y^9 + 10x^8y^7 - 5x^2y^3}{5x^2y^2}$ (3)

6.3 Calculate the value of $\sqrt{a^2 - b}$ if $a = -2$ and $b = 3$. (2)

6.4 Solve for x:

6.4.1 $25 - x = 16$ (1)

6.4.2 $\frac{0}{16} = 27x$ (1)

6.4.3 $10 + x - (19 + 3x) = -5x$ (4)

6.5 Three types of ticket were sold to spectators who wanted to watch swimming during the Olympic Games 2012: Category A tickets for £30 each, Category B tickets for £20 each and Category C tickets for £10 each. There were 250 Category A tickets sold and twice as many Category B tickets than Category C tickets. How many people bought Category C tickets if the ticket sales totaled £27 500? (3)

Exam exemplar 2 (November): Additional Paper 2 Memorandum

Review Copy

Question 1 **[5]**

1.1 True ✓ **1.2** True ✓ **1.3** True ✓ **1.4** False ✓ **1.5** False ✓

Question 2 **[20]**

2.1 $875 \times 99 = 900 \times 100$✓ $= 90\,000$✓ (2)

2.2 F60 = {1; 2; 3; 4; 5; 6; 10; 12; 15; 20; 30; 60}✓
F90 = {1; 2; 3; 5; 6; 9; 10; 15; 18; 30; 45; 90}✓
HCF = 30✓ (3)

2.3.1 -11✓ (1)

2.3.2 -48✓ $\times -2 = 96$✓ (2)

2.3.3 $11 - (24 - 21)5$✓ $= 11 - (3)5$✓ $= 11 - 15$✓ $= -4$✓ (4)

2.3.4 "long division"✓; 3 682✓ (2)

2.4 Distance = 120 km/h × 5 h✓ = 600 km✓ (2)

2.5.1 (0,1225 × R3 275)✓ + (24 × R275)✓ = R7 001,19✓ (3)

2.5.2 7 001,19 – 3 275 = R3 726,19✓ (1)

Question 3 **[15]**

3.1 $8 \cdot a \cdot a \cdot a \cdot 2 \cdot b \cdot 2 \cdot b$✓ $= 32$✓$\cdot a \cdot a \cdot a \cdot b \cdot b$✓ (3)

3.2 35^6✓m^3✓ (2)

3.3.1 75^{38}✓ (1)

3.3.2 -20×1✓ $= -20$✓ (2)

3.3.3 $\frac{4}{9}$✓✓ (2)

3.3.4 -2✓m^5✓ (2)

3.4 16 000 000✓ $= 1{,}6 \times 10^7$✓ (2)

3.5 3 853 400✓ (1)

Question 4 **[13]**

4.1 I have to subtract✓ consecutive numbers✓ each time, i.e. subtract 1, then subtract 2, then subtract 3, etc. Next two terms are 2; –3. (2)

4.2 Output values: -12✓; 8✓
Input values: 2; 10✓ (4)

4.3.1 It is the square of the input value plus two.✓ (1)

4.3.2 $y = x^2 + 2$✓✓ (2)

4.3.3 $p = 10^2 + 2$✓ $= 100 + 2 = 102$✓

$146 - 2 = 144$✓; $q = \text{R}144$✓ $= 12$✓ (4)

Question 5 **[21]**

5.1 1,6060✓ (1)

5.2 3,248; 2,998; 2,748; …✓ (1)

5.3 $\frac{108}{100}$✓ $= 1{,}08$✓ (2)

5.4.a $\frac{20}{60} - \frac{15}{60} + \frac{12}{60}$✓✓ $= \frac{17}{60}$✓ (3)

b $\frac{2}{3} \times \frac{36}{1}$✓ $\times - \frac{5}{4}$✓ $= \frac{24}{1}$✓ $\times - \frac{5}{4} = -30$✓ (4)

c $10 \times \frac{7}{5}$✓✓ $= 14$✓ (3)

d "long multiplication" ✓; 5 356,24✓ (2)

5.5 $100 - 12{,}5 = 87{,}5\%$; $0{,}875 \times 490$✓ $= \text{R}428{,}75$✓ (2)

5.6 $\frac{1\,020 - 895}{895} \times 100$✓✓ $\approx 13{,}97\ \%$✓ (3)

Question 6 **[26]**

6.1 $(9m^4 + 3m^2 - 2) - (3m^2 + 1)$ ✓ $= 9m^4 + 3m^2 - 2 - 3m^2 - 1$✓ $= 9m^4 - 3$✓ (3)

6.2.a $30x$✓ (1)

b y^4✓ (1)

c $-4x^3$✓ $+ 1$✓ (2)

d $6xy^4$✓ $+ 25xy^4$✓ $= 31xy^4$✓ (3)

e $-3x^2 + 21xy - 21xy + 14x^2$✓ $= 11x^2$✓ (2)

f $7x^8y^7$✓ $+ 2x^6y^5$✓ $- y$✓ (3)

6.3 $\sqrt{(-2)^2 - 3}$✓ $= \sqrt{4 - 3} = \sqrt{1} = 1$✓ (2)

6.4.1 $x = 9$ (1)

$x = 0$ (1)

6.4.3 $10 + x - 19 - 3x = -5x$✓

$\therefore -2x + 5x = 19 - 10$✓

$\therefore 3x = 9$✓

$\therefore x = 3$✓ (4)

6.5 Category A: £30 each; 250

Category B: £20 each; $2x$

Category C: £10 each; x

$x(10) + 2x(20) + 250(30) = 27\ 500$✓

$\therefore 10x + 40x + 7\ 500 = 27\ 500$✓

$\therefore 50x = 20\ 000 \therefore x = 400$

400 Category C tickets were sold.✓ (3)

Review Copy

Exam exemplar 2 (November): Additional Paper 3

Time: 2 hours 30 minutes **Total:** 125 marks

Question 1 **[10]**

The length of time (in minutes) that patients spent in a doctor's waiting room was recorded in the below table.

11	15	9	25	23	30	8	14	18
13	18	24	23	31	a	13	17	

1.1 Find the value of a, if the average (mean) time spent waiting is exactly 18 minutes. (3)

1.2 How many modes are in this data set? (1)

1.3 Draw a stem-and-leaf plot to display the data. (5)

1.4 Draw circles on the stem-and-leaf plot to show where the median number of minutes can be read off. (1)

Question 2 **[8]**

The table shows the average monthly rainfall of a city for six months.

Month	January	February	March	April	May	June
mm	10	50	80	150	250	300

2.1 Draw a broken-line graph of the data. (5)

2.2 Write down the range of the data. (2)

2.3 Between which two months was the increase in average monthly rainfall the greatest? (1)

Review Copy

Question 3 **[3]**

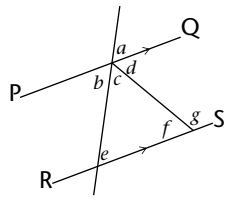
George wants to find out what the primary school children in his family eat for breakfast. There are seven primary school children in his family and he asked each one of them the following question: "Do you also think that it is unhealthy to eat marshmallows for breakfast?"

3.1 Were the seven children a sample or a population? Give a reason for your answer. (2)

3.2 Why can the question be considered biased? (1)

Question 4 **[6]**

Use the diagram to give a reason for each of the given statements on the next page.



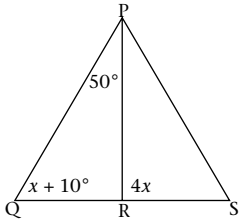
4.1 $b + c + d = 180°$

4.2 $e + c + f = 180°$

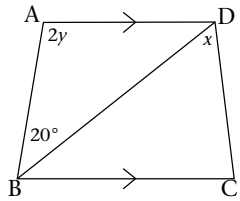
4.3 $a = b$

4.4 $b = e$
4.5 $a = e$
4.6 $g = c + e$

Question 5 **[4]**
Find, with reasons, the value of x.



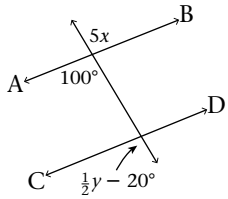
Question 6 **[7]**
ABCD is a trapezium and DC ⊥ BC.



6.1 Write down the value of x if x is the complement of 40°. (1)
6.2 Find, with reasons, the value of y. (6)

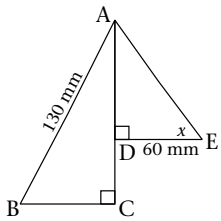
Review Copy

Question 7 **[6]**



7.1 Find, with a reason, the value of x. (2)
7.2 For which value of y will AB ··· BC? Give reasons. (4)

Question 8 **[9]**
Leave answers in this question in surd form if necessary.



8.1 Find the length of AE if $D\hat{A}E = \hat{E}$ and $AD \perp DE$.
8.2 Find the length of DC if $BC = \frac{5}{6}DE$.

Question 9 **[6]**

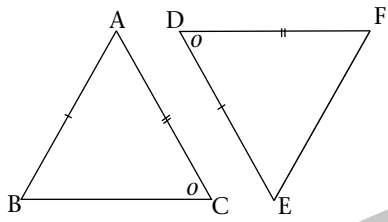
Rewrite and complete the following table.

Properties of diagonals of quadrilaterals			
Name of quadrilateral	**Parallelogram**	**Kite**	**.... or ....**
Diagonals are perpendicular to each other			Yes
Diagonals bisect each other			Yes

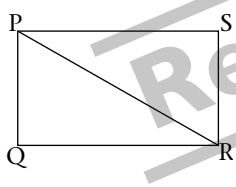
Question 10 **[8]**

In each question 2 triangles are given. State if the2 triangles next to each number are congruent or not. Give a reason for your answer.

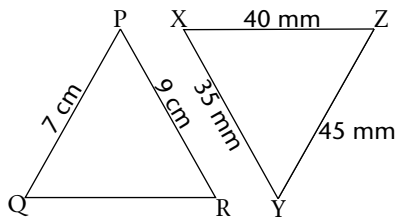
10.1



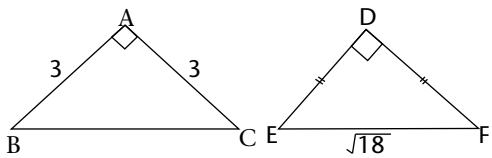
10.2 PQRS is a rectangle.



10.3



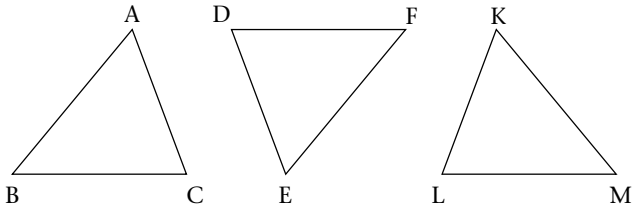
10.4



Review Copy

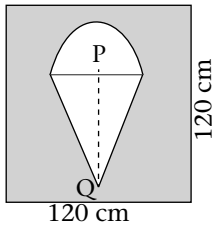
Question 11 **[2]**

11 Which of the following triangles are similar? Give a reason for your answer.



Question 12 **[13]**

Chris used a square with side length 120 cm to make a poster to advertise ice-creams. The background of the poster is painted blue, the cone is painted yellow and the ice-cream is painted white. The length of the cone (PQ) is 60 cm and the diameter of the ice-cream is 70 cm.



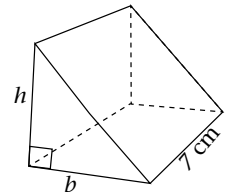
12.1 Find the area that must be painted blue. Calculate the answer in cm^2 and then convert it to m^2. (9)

12.2 Find the perimeter of the area that must be painted white. (4)

Review Copy

Question 13 **[9]**

The volume of the triangular prism is 280 cm^3.



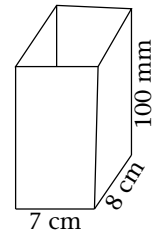
13.1 Find the area of the base of the prism. (3)

13.2 Write down one set of possible values for h and b. (4)

13.3 Give the capacity of the prism in litres. (2)

Question 14 **[7]**

14 Sam drew a plan for an open box with measurements as given on the diagram. How much cardboard does Sam need to make the box?



Question 15 **[6]**

Plot the point A(–1; 1) on a Cartesian plane. Do the following transformations and write down the coordinates of each point.

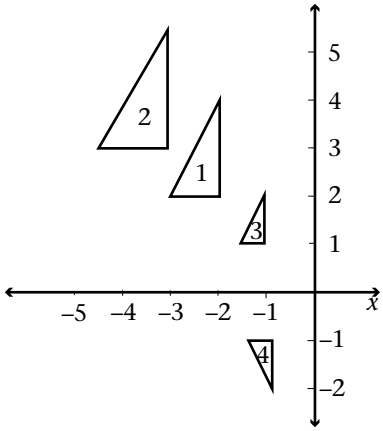
15.1 Reflect point A in the y-axis to find point A′.

15.2 Translate point A′ 3 units down and 1 unit to the right to find point A″.

15.3 Rotate point A″ 90° in a clockwise direction through the origin to find A‴.

Question 16 **[6]**

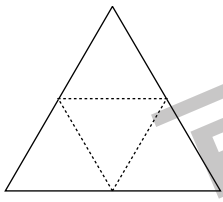
Use the diagram to describe precisely the transformation that took place.



16.1 from △1 to △2?
16.2 from △1 to △3?
16.3 from △3 to △4?

Question 17 **[8]**

The given net consists of 4 equilateral triangles.



Review Copy

17.1 Choose the correct word:
- **17.1.1** The diagram shows the net of a (prism/pyramid).
- **17.1.2** The net will form a (tetrahedron/icosahedron).
- **17.1.3** The 3D object is an example of a Platonic solid with (4/6) edges. (3)

17.2 Make an accurate construction of 1 of the equilateral triangles if the total length of one side of the net is 5 cm. (Do not use a protractor in this construction.) (3)

17.3 Bisect one of the angles in the construction. (2)

Question 18 **[2]**

A spinner is in the shape of a regular pentagon. The last letters of the alphabet are written on the spinner. List all the possible outcomes.

Question 19 **[3]**

There are 20 CDs in a box: 6 with hip-hop music; 3 with house music; 4 with jazz music, 2 with country music and 5 with classical music. Write down the probability of randomly selecting a CD with:

19.1 hip-hop music

19.2 house music or jazz music
19.3 not country music.

Question 20 **[2]**
A coin was flipped 17 times. It landed on tails 7 times.
20.1 Write down the relative frequency of the coin landing on heads.
20.2 Explain why the answer in question 20.1 is not equal to 50%.

Exam exemplar 2 (November): Additional Paper 3 Memorandum

Time: 2 hours 30 minutes **Marks: 125**

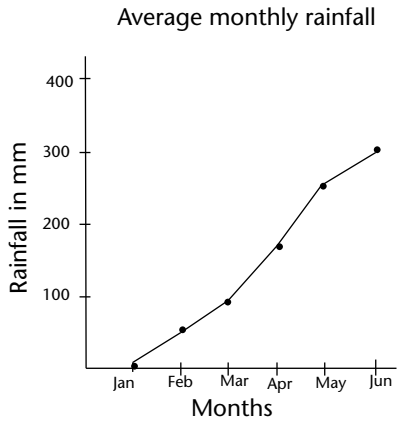
Question 1 **[10]**
1.1 Total number of minutes ÷ number of patients = mean
$\frac{292 + a}{17} = 18$✓
$292 + a = 18 \times 17 = 306$✓
$a = 14$ minutes✓ (3)
1.2 Mode: 13; 14; 18 and 23 each occur twice
Therefore 4 modes. ✓ (1)
1.3

Stem	Leaves								
0	8	9							
①	1	3	3	4	4	5	⑦	8	8
2	3	3	4	5					
3	0	1							

Key: 3 | 0 means 30 (4 marks for diagram; 1 mark for key) (5)
1.4 Median: 17 (1 mark for circles on diagram) (1)

Review Copy

Question 2 **[8]**
2.1



(2 marks for labels and heading; 3 marks for plotting points) (5)

2.2 Range = 300 − 10 = 290 mm (2)
2.3 Between April and May. (1)

Question 3 **[3]**
3.1 Population: ✓ There are only 7 primary school children in his family and he asked the question to all of them. ✓ (2)
3.2 The question leads the children to agree with the statement. ✓ (1)

Question 4 **[6]**
4.1 $b + c + d = 180°$ (straight line)
4.2 $e + c + f = 180°$ (sum of interior angles of △)
4.3 $a = b$ (vertically opposite ∠s)
4.4 $b = e$ (alternate ∠s, PQ ∥ RS)
4.5 $a = e$ (corresponding ∠s, PQ ∥ RS)
4.6 $g = c + e$ (exterior ∠ of △ = sum of opp interior ∠s) (1 mark each)

Question 5 **[4]**
5 $4x = x + 10° + 50°$ ✓ (exterior ∠ of △= sum of opp interior ∠s) ✓
$3x = 60°$✓
$x = 20°$✓

Question 6 **[7]**
6.1 $x = 50°$✓ (1)
6.2 In △BDC: $\hat{B} = 180° - (90° + 50°)$ (sum of interior angles of △)✓
$\hat{B} = 40°$✓
$A\hat{D}B = \hat{B}$ (alternate ∠s, AD ∥ BC) ✓
$A\hat{D}B = 40°$✓
In △ABD: $40° + 20° + 2y = 180°$ (sum of interior angles of △)✓
$2y = 120°$; $y = 60°$✓ (6)

Question 7 **[6]**
7.1 $5x = 100°$ (vertically opposite ∠s) ✓
$x = 20°$✓ (2)
7.2 AB ∥ BC if corresponding angles are equal. ✓
if $\frac{1}{2}y - 20° = 100°$✓
$\frac{1}{2}y = 120°$✓
$y = 240°$✓ (4)

Question 8 **[9]**
8.1 AD = DE ($D\hat{A}E = \hat{E}$) ✓
$AE^2 = AD^2 + DE^2$ (Pyth) ✓
$= 60^2 + 60^2 = 3\,600 + 3\,600$
$= 7\,200$ ✓
$AE = \sqrt{200}$ cm ✓
8.2 $BC = 60 \times \frac{5}{6} = 5$ cm; AB = 130 mm = 13 cm ✓
$BC^2 + AC^2 = AB^2$ (Pyth) ✓
$5^2 + AC^2 = 13^2$
$AC^2 = 169 - 25 = 144$ ✓
AC = 12 cm✓
DC = 12 − 6 = 6 cm✓

Review Copy

Question 9 **[6]**

Properties of diagonals of quadrilaterals			
Name of quadrilateral	**Parallelogram**	**Kite**	**Rhombus or square** ✓✓
Diagonals are perpendicular to each other	No ✓	Yes ✓	Yes
Diagonals bisect each other	Yes ✓	No ✓	Yes

Question 10 **[8]**

10.1 △ABC is NOT congruent to △ DEF ✓ (angle is not included). ✓

10.2 △PQR ≡ △RSP ✓ (RHS) ✓

10.3 △PQR is not congruent to △YXZ ✓ (sides are not equal – they are in proportion.) ✓

10.4 △ABC ≡ △ DEF ✓ (S∠S OR SSS OR RHS) ✓

Explanation: BC = $\sqrt{3^2 + 3^2} = \sqrt{18}$ and it is given that DE = DF.

Question 11 **[2]**

All 3 triangles are similar: ✓

Each triangle has angles of 50°, 60° and 70°. ✓

Question 12 **[13]**

12.1 Blue area = Total area – cone – ice cream

Total area = $s \times s$

$= 120 \times 120$

$= 14\ 400\ \text{cm}^2$ ✓✓

Area of cone = $\frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 70 \times 60$

$= 2\ 100\ \text{cm}^2$ ✓✓

Area of ice-cream = $\frac{1}{2}(\pi \times r^2)$ ✓

$= \frac{1}{2}\left(\frac{22}{7} \times 35^2\right)$

$= 1\ 925\ \text{cm}^2$ ✓✓

Blue area = 14 400 – 2 100 – 1 925

$= 10\ 375\ \text{cm}^2$ ✓

$10\ 375\ \text{cm}^2 = 1{,}0375\ \text{m}^2$ ✓ (9)

12.2 Perimeter = $\frac{1}{2}(2 \times \frac{22}{7} \times r)$ + perimeter of cone ✓

$= \frac{1}{2}(2 \times \frac{22}{7} \times 35) + 2\sqrt{60^2 + 35^2}$ ✓

$= 110 + \sqrt{19\ 300}$ ✓

$= 248{,}92$ cm ✓ (4)

Question 13 **[9]**

13.1 Area of base × height = volume ✓

area of base × 7 = 280 ✓

area of base = 40 cm² ✓ (Correct unit!) (3)

13.2 Area of base: $\frac{1}{2} \times b \times h = 40$

one set of possible values can be $h = 8$ cm ✓✓ and $b = 10$ cm ✓✓

or $h = 16$ cm and $b = 5$ cm, etc. (4)

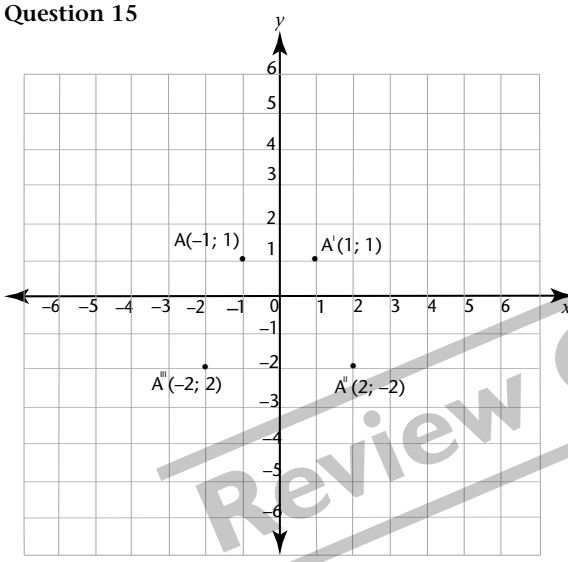
Review Copy

13.3 280 cm^3 = 280 ml ✓
280 ml = 0,28 litre ✓ (2)

Question 14 **[7]**

14 Total surface area = area of one base + area of faces ✓
Area of base = $l \times b$ ✓
= 8×7
= 56 cm^2 ✓
Area of faces = perimeter of base × height of prism ✓
= $(7 + 8 + 7 + 8) \times 10$ ✓
= 300 cm^2 ✓
Sam needs 356 cm^2 of cardboard to make the box. ✓

Question 15 **[6]**



Review Copy

(2 marks for each correct transformation WITH coordinates)

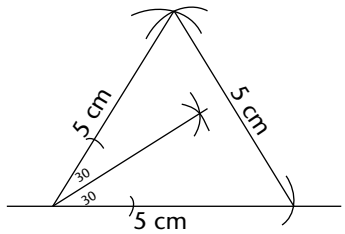
Question 16 **[6]**

16.1 Enlargement ✓ through the origin with enlargement factor $k = \frac{6}{5} = 1\frac{1}{5} = 1{,}2$ ✓
16.2 Reduction ✓ through the origin with reduction factor $k = 2$.✓
OR Enlargement through the origin with enlargement factor $k = \frac{1}{2}$.
16.3 Reflection ✓ in the x-axis. ✓

Question 17 **[8]**

17.1.1 pyramid ✓
17.1.2 tetrahedron ✓
17.1.3 6 edges ✓ (3)

17.2–17.3



(3 marks for triangle; 2 marks for bisecting angle) (5)

Question 18 **[2]**

{V; W; X; Y; Z}

Question 19 **[3]**

19.1 P(hip-hop music) $= \frac{6}{20} = \frac{3}{10}$ ✓

19.2 P(house music or jazz music) $= \frac{3}{20} + \frac{4}{20} = \frac{7}{20}$ ✓

19.3 P(not country music) $= \frac{18}{20} = \frac{9}{10}$ ✓

Question 20 **[2]**

20.1 Relative frequency of heads $= \frac{10}{17}$ ✓

20.2 The experiment was not done enough times for the theoretical probability to be equal to the experimental probability. ✓

Review Copy

Resources

Question words: Verbs to introduce questions/activities

Verbs to address skills to be assessed at the various levels, from the lower to the higher cognitive levels. Below is a list of such verbs.

Recall of knowledge:

Choose	Identify	Provide
Define	Indicate	Select
Describe	Label	State
Fill in	Mention	Supply
Give	Name	Write notes on

Comprehension/insight:

Arrange	Estimate	Interpret
Change	Expand	Justify
Compare	Explain	Predict
Deduce	Generalise	Rewrite
Distinguish between	Indicate	Tabulate

Application:

Apply	Deduce	Formulate
Calculate	Differentiate	Integrate
Change	Discover	Solve
Construct	Discuss critically	Suggest

Analysis:

Break down	Discriminate	Indicate
Deduce	Distinguish between	Select
Differentiate	Illustrate	Separate

Synthesis:

Categorise	Compose	Form
Classify	Create	Rewrite
Combine	Design	Sum up
Compile	Estimate	

Evaluation:

Compare	Distinguish between	Support
Contrast	Interpret	Value
Criticise	Justify	Verify
Deduce	Sum up	

Review Copy

1 cm-Grid paper

Review Copy

0,5 cm-Grid paper

Review Copy