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Successful

Mathematics

TEACHER'S GUIDE

GRADE 7

WITH EXAM INFO AND EXEMPLAR PAPERS

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Contents

Term 3

Term 4

POA

How this course works

This series meets the requirements of the National Curriculum and Assessment Policy Statement (CAPS) for the Senior Phase.
In Grade 7, this series consists of two core components: a Learner's Book and a Teacher's Guide.

The Learner's Book

The Learner's Book provides content and subject knowledge as well as activities for learners to develop, practise and consolidate their mathematical knowledge and skills. Written texts are supported by diagrams and illustrations that help to explain content. All examples, exercises and illustrations are representative of all cultural groups. Exercises steadily become more challenging so that learners progressively develop their understanding of concepts.

The Teacher's Guide

The Teacher's Guide provides you, the teacher, with all the planning, teaching and assessment tools. Teachers receive guidance on how to teach important concepts and advice on how to teach each exercise.

How to use this Teacher's Guide

The Teacher's Guide covers all Mathematics content and provides rich resources to ensure complete curriculum coverage and the successful development of mathematical concepts and skills in Grade 7.
The Teacher's Guide supports you by providing support and information on how to teach the subject. Some of the features you will find in the Teacher's Guide include the following:

Defining the subject, CAPS and teaching terminology for the teacher.

The CAPS for Mathematics

Each CAPS document provides:
- an overview of topics and content areas for its subject (see below)
- the weighting prescribed for each content area (see below)
- a teaching plan for the subject (see Section C – Planning and assessment).

The following content areas comprise the Senior Phase Mathematics curriculum:
- Numbers, Operations and Relationships
- Patterns, Functions and Algebra
- Space and Shape (Geometry)
- Measurement
- Data Handling.

Each content area has a prescribed weighting to ensure complete curriculum coverage.

Content Area	Grade 7	Grade 8	Grade 9
Numbers, Operations and Relationships	30%	25%	15%
Patterns, Functions and Algebra	25%	30%	35%
Space and Shape (Geometry)	25%	25%	30%
Measurement	10%	10%	10%
Data Handling (Statistics)	10%	10%	10%
Total	100%	100%	100%

Chapter 20 Programme of Assessment

This Chapter provides all the resources you need to ensure your learners meet the requirements for promotion.
- It includes two options for each of the required formal programme of assessment tasks
- Exemplar exams for learners to use to practise for their exams. These exemplars are in the Learner's book with the memoranda supplied in this section of the Teacher's Guide.
- Control tests for you to use as part of the POA, along with the memoranda.
- A June and a December exam paper, with memoranda.

The following table lays out the programme for you. The shaded cells are only in the teacher's guide to ensure the tests and exams are unseen by the learners.

Term	Task		Learner's Book page	Teacher's Guide page
1	Assignment	Option 1 Financial maths	371	245
		Option 2 Constructions	373	246
	Control Test 1			247
2	Investigation	Option 1 Functions and relationships	375	250
		Option 2 Relationship between volume and surface area	377	251
	Control Test 2			253
	Exemplar June exam	Exemplar paper for revision purposes	392	255
	June Exam			258
3	Assignment	Option 1 Patterns	379	265
		Option 2 Algebra	381	267
	Project	Option 1 Functions relationships and graphs	382	268
		Option 2 Transformations	384	269
	Control Test 3			270
4	Assignment	Option 1 Integers	386	272
		Option 2 Algebra	387	273
	Investigation	Option 1 Data	388	274
		Option 2 Probability	390	276
	Exemplar December exam	Exemplar paper for revision purposes	395	277
	December exam			279

We suggest that in order to prepare learners adequately for formal assessment you use the allocated Revision time prescribed in the CAPS for revising work.
This Series advises that you use the Consolidation exercises and Summaries at the end of each chapter to revise. The Consolidation exercises have mark allocations to enable informal assessment of how learners are managing the specific content area

244 **Chapter 20:** Programme of Assessment

Providing Formal Assessment Tasks as required by the CAPS.

SECTION 2 Planning and assessment

Planning

Types of planning tools

The following planning tools are provided:
- A teaching plan
- A sample lesson plan

Teaching plan for Mathematics Grade 7

This teaching plan shows:
- the pacing of the topics for the course by term
- where to find the relevant content and activities in the Learner's Book
- when Formal Assessment takes place, cross-referenced to suitable activities in the Learner's Book.

This teaching plan follows the time allocations as set out in the CAPS for Mathematics. It assumes six hours of teaching per week.

Term	Content/topics (as per CAPS)	Learner's Book	LB pp.	TG pp.	Time allocation	Assessment
1	Whole numbers Ratio and finance	Chapter 1	11-44	28-49	9 hrs	Informal in class assessment OR Option 1 Assignment PoA
	Exponents	Chapter 2	45-60	50-60	9 hrs	Informal in class assessment
	Constructions	Chapter 3	61-81	61-68	10 hrs	Informal in class assessment OR Option 2 Assignment PoA
	Geometry of 2D shapes	Chapter 4	82-113	69-81	10 hrs	Informal in class assessment
	Revision				4 hrs	Informal in class assessment
	End of term test			247	1 hr	Formal assessment Test 1

Structuring the course into units, with advice on pacing content according to the CAPS.

The National Curriculum and Assessment Policy Statements

This series is based on the National Curriculum Statement Grades R–12 (NCS, January 2012), which is the policy document for learning and teaching in South Africa. The NCS consists of three documents, namely:

- Curriculum and Assessment Policy Statements (CAPS) for all approved subjects for Grades R–12
- National Policy pertaining to the Programme and Promotion Requirements of the National Curriculum Statement Grades R–12
- National Protocol for Assessment Grades R–12 (January 2012).

Each CAPS document has four sections:

- Section 1: Introduction to the Curriculum and Assessment Policy Statements for the specific subject
- Section 2: The specific subject's aims, time allocations and requirements to offer it as a subject
- Section 3: Overview of topics and teaching plan for the specific subject
- Section 4: Assessment in the specific subject.

Sections 2, 3 and 4 of the CAPS documents, together with the National Policy pertaining to the Programme and Promotion Requirements of the NCS, represent the norms and standards of the National Curriculum Statement Grades R–12. Together these documents are the basis for determining minimum outcomes, processes and procedures for the assessment of learner achievement in public and independent schools.

Instructional time allocation

The instructional time in the Senior Phase is as follows:

Subject	Teaching hours per week	Total hours per term
Home Language	5	50
First Additional Language	4	40
Mathematics	4,5	45
Natural Science	3	30
Social Sciences	3	30
Technology	2	20
Economic Management Sciences	2	20
Life Orientation	2	20
Creative Arts	2	20
Total	27,5	275

The CAPS for Mathematics

Each CAPS document provides:
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- the weighting prescribed for each content area (see below)
- a teaching plan for the subject (see Section C – Planning and assessment).

The following content areas comprise the Senior Phase Mathematics curriculum:
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- Patterns, Functions and Algebra
- Space and Shape (Geometry)
- Measurement
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Space and Shape (Geometry)	25%	25%	30%
Measurement	10%	10%	10%
Data Handling (Statistics)	10%	10%	10%
Total	100%	100%	100%

Topic overview

	Grade 7	Grade 8	Grade 9
Term 1	• Mental calculations • Order and compare whole numbers (9 digits) • Properties of whole numbers • Calculations with whole numbers • Addition and subtraction (6 digits) • Multiplication and division (4-digit by 2-digit) • Multiples and factors (of 2- and 3-digit whole numbers) • Prime factors • LCM and HCF (3-digit whole numbers) • Solve problems (ratio and rate; percentages, decimal fractions; financial context) • Exponents • Measure angles • Construct geometric figures • Classify 2D shapes • Similar and congruent 2D shapes • Solve problems	• Order and compare whole numbers (prime numbers to 100) • Properties of whole numbers • Calculations with whole numbers • Multiples and factors • Solve problems (ratio and rate; percentages, decimal fractions; financial context) • Count, order and compare integers • Calculations with integers • Properties of integers • Solve problems • Represent numbers in exponential form • Calculations in exponential form • Laws of exponents • Numeric and geometric patterns • Input and output values or rules for patterns and relationships • Equivalent forms • Algebraic language • Expand and simplify algebraic expressions • Set up equations and solve by inspection	• Properties of whole numbers • Calculations with whole numbers • Multiples and factors • Solve problems (ratio and rate; direct and indirect proportion; percentages, decimal fractions; financial context) • Calculations with integers • Properties of integers • Solve problems • Common fractions • Decimal fractions • Exponents • Calculations in exponential form • Solve problems • Numeric and geometric patterns • Input and output values or rules for patterns and relationships • Equivalent forms • Algebraic language • Expand and simplify algebraic expressions • Equations (use factorisation of the form where a product of factors = 0)

Term 2	• Common fractions • Percentages • Decimal fractions • Equivalent forms • Solve problems • Input and output values for patterns and relationships • Equivalent forms (verbal, flow diagrams, tables, formulae, number sentences) • Area and perimeter of 2D shapes • Convert SI units • Surface area and volume of 3D objects	• Algebraic language • Expand and simplify algebraic expressions • Set up equations and solve by using additive and multiplicative inverses • Construct and investigate geometric figures • Classify 2D shapes • Similar and congruent triangles • Angle relationships • Solve problems	• Investigate properties of geometric figures by construction • Classify 2D shapes • Similar and congruent triangles • Solve problems • Angle relationships • Use the Theorem of Pythagoras • Area and perimeter of 2D shapes (polygons and circles)
Term 3	• Numeric and geometric patterns • Input and output values for patterns and relationships • Equivalent forms • Algebraic language • Number sentences • Interpret and draw graphs • Transformations • Classify 3D objects • Build 3D models	• Common fractions • Percentages • Decimal fractions • The Theorem of Pythagoras • Area and perimeter of 2D shapes • Surface area and volume of 3D objects • Solve problems • Data handling	• Input and output values or rules for patterns and relationships • Algebraic language • Expand and simplify algebraic expressions • Factorise algebraic expressions • Equations • Draw and interpret graphs • Draw linear graphs from given equations • Surface area and volume of 3D objects (include cylinders)
Term 4	• Integers • Numeric and geometric patterns • Input and output values for patterns and relationships • Algebraic language • Number sentences • Data handling • Probability	• Input and output values or rules for patterns and relationships • Equivalent forms • Solve algebraic equations • Interpret and draw graphs • Transformations • Enlargements and reductions • Classify 3D objects • Build 3D models • Probability	• Transformations • Enlargements and reductions • Classify 3D objects • Build 3D models • Data handling • Probability

Planning

Types of planning tools

The following planning tools are provided:
- a teaching plan
- a sample lesson plan

Teaching plan for Mathematics Grade 7

This teaching plan shows:
- the pacing of the topics for the course by term
- where to find the relevant content and activities in the Learner's Book
- when Formal Assessment takes place, cross-referenced to suitable activities in the Learner's Book.

This teaching plan follows the time allocations as set out in the CAPS for Mathematics. It assumes six hours of teaching per week.

Term	Content/topics (as per CAPS)	Learner's Book	LB pp.	TG pp.	Time allocation	Assessment
1	Whole numbers Ratio and finance	Chapter 1	11-45	28-49	9 hrs	Informal in class assessment OR Option 1 Assignment PoA
	Exponents	Chapter 2	46-61	50-60	9 hrs	Informal in class assessment
	Constructions	Chapter 3	62-82	61-68	10 hrs	Informal in class assessment OR Option 2 Assignment PoA
	Geometry of straight lines	Chapter 4 Unit 1	83-86	70-71	2 hrs	Informal in class assessment
	Geometry of 2D shapes	Chapter 4 Unit 2-4	87-117	72-82	10 hrs	
	Revision				4 hrs	Informal in class assessment
	End of term test			247	1 hr	Formal assessment Test 1

Term	Content/topics (as per CAPS)	Learner's Book	LB pp.	TG pp.	Time allocation	Assessment
2	Common fractions	Chapter 5	118-149	83-100	9 hrs	Informal in class assessment
	Decimal fractions	Chapter 6	150-167	101-116	9 hrs	Informal in class assessment
	Functions and relationships	Chapter 7	168-183	117-126	3 hrs	Informal in class assessment OR Option 1 Investigation PoA
	Test			253	1 hr	Formal assessment Test 2
	Area and perimeter	Chapter 8	184-199	127-135	7 hrs	Informal in class assessment
	Surface area and volume	Chapter 9	200-212	136-143	8 hrs	Informal in class assessment OR Option 2 Investigation PoA
	Revision		392	255	5 hrs	Informal in class assessment
	Mid Year Exam			258	2 hrs	Formal assessment

Term	Content/topics (as per CAPS)	Learner's Book	LB pp.	TG pp.	Time Allocation	Assessment
3	Numeric and geometric patterns	Chapter 10	213-224	144-150	6 hrs	Informal in class assessment
	Functions and relationships	Chapter 11	225-235	151-159	3 hrs	Informal in class assessment Assignment option for PoA
	Algebraic expressions	Chapter 12 Unit 1	236-241	160-164	3 hrs	
	Algebraic equations	Chapter 12 Unit 2	242-249	164-168	3 hrs	Informal in class assessment Assignment and Project option for PoA
	Graphs	Chapter 13	250-264	169-176	6 hrs	Informal in class assessment Project option for PoA
	Transformation geometry	Chapter 14	265-281	177-184	9 hrs	Informal in class assessment
	Geometry of 3D objects	Chapter 15	282-298	185-191	9 hrs	Formal assessment
	Revision				4 hrs	Informal in class assessment
	End of term test			270	1 hr	Formal assessment Test 3

Term	Content/topics (as per CAPS)	Learner's Book	LB pp.	TG pp.	Time Allocation	Assessment
4	Integers	Chapter 16	299-319	192-203	9 hrs	Informal in class assessment OR Option 1 Assignment PoA
	Numeric and geometric patterns	Chapter 17 Units 1 - 2	320-325	204-208	3 hrs	Informal in class assessment
	Functions and relationships	Chapter 17 Units 3 – 4	326-333	209-213	3 hrs	Informal in class assessment
	Algebraic expressions	Chapter 18 Unit 1	334-336	214-216	3 hrs	Informal in class assessment OR Option 2 Assignment PoA
	Algebraic equations	Chapter 18 Unit 2	337-341	217-219	4 hrs	
	Collect, organize and summarize data	Chapter 19 Units 1 - 2	342-354	220-231	4 hrs	Informal in class assessment OR Investigation Option 1 for PoA
	Represent data	Chapter 19 Unit 3	355-359	232-235	3 hrs	
	Analyse, interpret and report data	Chapter 19 Unit 4	360-365	235-240	3,5 hrs	
	Probability	Chapter 19 Unit 5	366-369	240-243	4,5 hrs	Informal in class assessment OR Investigation Option 2 for PoA
	Revision		395	277	10 hrs	Informal in class assessment
	End of year exam			279	2 hrs	Formal assessment

Example of a lesson plan

Some may find daily lesson plans useful, although these are not a formal policy requirement. An example of how to complete a lesson plan is below.

<table>
<tr><td>Date:</td><td>Grade: 7</td><td>Term: 1</td></tr>
<tr><td>Chapter: 1</td><td>Unit: 1</td><td>Contact time: 1 hour</td></tr>
<tr><td>Content/concept: Ordering and comparing whole numbers</td><td>Exercise: 1 and 2</td><td>Resources required:</td></tr>
<tr><td colspan="3"></td></tr>
<tr><td colspan="3">Links with previous knowledge or exercise: Learners should already be able to:
• do mental calculations with whole numbers including multiplication facts up to 12 × 12
• round off numbers to the nearest 10
• understand basic place value up to 9 digits</td></tr>
<tr><td colspan="3">Links with next activity: This unit helps learners revise mental calculations and the necessary concepts of ordering and comparing numbers in preparation for working with the properties of whole numbers and calculating with whole numbers.</td></tr>
<tr><td colspan="3">Teaching plan
• Spend time assessing learner’s knowledge of the basic skills, as listed above, before doing the exercises in this unit. Learners may struggle if this revision is not done.
• Revise multiplication tables by dividing the class into groups and having time trials.
• Prescribe exercise 1. Learners complete the exercises on their own.
• Mark the work in class, by writing the answers on the board, and learners mark their own work.
• Revise the various terms used in this unit. Such as: prime, whole numbers, counting numbers and factors.
• Ensure learners can distinguish between the different kinds of numbers.
• Do examples together as a class, using the board, on place value of 9 digit numbers, ordering and comparing whole numbers and rounding off whole numbers.
• Prescribe Exercise 2.
• Walk round the class and observe how learners coped with the exercise.
• Identify learners who experienced difficulty and assign similar practice examples for homework.</td></tr>
<tr><td colspan="3">Assessment: Informal self-assessment with teacher supervision.</td></tr>
<tr><td colspan="3">Teacher reflection:</td></tr>
</table>

Assessment

Assessment is the planned process of identifying, gathering and interpreting information about learners' performance on an on going basis. Assessment should be both informal and formal, and a variety of assessment tasks should be used. Learners should timeously receive feedback on both informal and formal assessment.

The four steps of assessment

1 Generating and collecting evidence of achievement.
2 Evaluating the evidence.
3 Recording the findings.
4 Using the findings to guide future learning and teaching.

Types of assessment

Type of assessment	Description
Baseline assessment	Establishes whether learners meet basic skills and knowledge level required. Helps teacher plan for the year and for each learner. Is administered at the beginning of the year and before a particular topic. Results are used as a guide for teaching and not for promotion purposes.
Diagnostic assessment	Informs the teacher about certain specific problem areas that may hinder performance. May help determine whether a learner's problems are content or psycho-social based. Appropriate interventions should follow on from diagnostic assessment. Results should inform interventions and not be used for promotion purposes.
Formative assessment	Used to aid the learning process and not for promotion purposes. Usually informal, to provide the teacher and learner with a more frequent account of where the learner is at. Teachers can use this form of assessment to modify and adapt their own teaching.
Summative evaluation	Carried out after completion of a topic or cluster of topics. Is an assessment of learning that has taken place. Recorded and used for promotion. This is usually a formal assessment, making up the Formal Programme of Assessment.

Informal or daily assessment

Informal assessment is a daily monitoring of learners' progress. This is done through observations, discussions, practical demonstrations, learner-teacher conferences, and informal classroom interactions. All daily exercises completed in the mathematics classroom can be used for informal assessment.
CAPS tells us that informal assessment should be used to provide feedback to the learners and to inform planning for teaching, but need not be recorded or taken into account for promotion. It should not be seen as separate from learning activities taking place in the classroom. Learners or teachers can mark these assessment tasks.

Formal assessment

Certain tasks make up the Formal Programme of Assessment for the year. Formal assessment tasks are marked and formally recorded by the teacher for progression and certification purposes. All formal assessment tasks are subject to moderation for the purpose of quality assurance and to ensure that appropriate standards are maintained. Formal assessment provides teachers with a systematic way of evaluating how well learners are progressing in a grade and in a particular subject. Examples of formal assessments include tests, examinations, projects, assignments and investigations. Formal assessment tasks form part of a year-long formal programme of assessment in each grade and subject.

Formal Assessment

Formal assessment requirements of Mathematics

The forms of assessment used should be appropriate for the learners' ages and developmental levels. Learners must complete formal assessments each term. Formal assessments include formally assessed tasks, along with projects and examinations. The Formal Programme of Assessment as prescribed by the CAPS is shown below. This Programme of Assessment is generic across the three grades in the Senior Phase, and lists the types of formal assessments required each term.

Minimum requirements for formal assessment

	Forms of assessment	Minimum requirements per term				Number of tasks per year	Weighting
		Term 1	Term 2	Term 3	Term 4		
SBA (School-based Assessment)	Tests	1	1	1		3	40%
	Examination		1			1	
	Assignment	1		1	1	3	
	Investigation		1		1	2	
	Project			1		1	
	Total	2	3	3	2	10*	
Final examination		End of year				1	60%

* To be completed before the final examination at the end of the year

Types of formal assessment for Mathematics

Tests and examinations

These are individual assessment tasks. Tests and examinations for formal assessment should cover a substantial amount of content. Tests and examinations must be completed under strictly controlled conditions.

Each test and examination must cater for a range of cognitive levels in the correct allocation (see the table below).

Cognitive level	Description of skill to be demonstrated
Knowledge ≈25%	• Estimation and appropriate rounding of numbers • Straight recall • Identification and direct use of correct formula • Use of mathematical facts • Appropriate use of mathematical vocabulary
Routine procedures ≈45%	• Performing of well-known procedures • Simple applications and calculations which might involve many steps • Derivation from given information may be involved • Identification and use (after changing the subject) of correct formulae generally similar to those encountered in class
Complex procedures ≈20%	• Problems involving complex calculations and/or higher-order reasoning • Investigate elementary axioms to generalise them into proofs for straight line geometry, congruence and similarity • No obvious route to the solution • Problems not necessarily based on real-world contexts • Making significant connections between different representations • Require conceptual understanding
Problemsolving ≈10%	• Unseen non-routine problems (which are not necessarily difficult) • Higher-order understanding and processes are often involved • Might require the ability to break the problem down into its constituent parts

Projects

Learners complete *one* project in Mathematics in each grade. Projects can be used to test a range of skills and competencies. It is prescribed that learners complete a project in Term 3 of each grade. Projects must provide learners with the ability to demonstrate their understanding of a mathematical concept and apply it to a real-life situation. Be wary of prescribing projects that are beyond the cognitive level of the learners, or that will simply involve duplicated facts and data from reference material.

Assignments

An assignment is also an individual task, similar to tests and examinations. However, the assignment should be an extended piece of work with a focus on more demanding work than that covered in class. *Three a*ssignments per year are required by the CAPS. The assignment can include past questions, but should also include more challenging aspects encouraging the learner to use additional material to help them. The assignment can be completed at home.

Investigations

An investigation should be used to discover rules or concepts. It is recommended that learners should conduct investigations in class as much as possible, and that the final written task definitely be done in class. Rubrics are used to assess investigations. *Two* investigations per year are required by the CAPS.
The skills involved in investigations include:

- organising and recording ideas and discoveries in tables and diagrams
- explaining ideas in appropriate forms
- showing clear understanding of concepts and procedures through calculations
- generalising and drawing conclusions.

Guidelines for Assessment Tasks

Tasks should be designed to cover the content and concepts of the subject and include a variety of activities selected to assess the identified aims and skills.
Before handing out an assessment task to learners, teachers should ensure that they are able to answer all the questions themselves. When teachers set an assessment task, they should draw up a memorandum of answers and/or a rubric for the assessment. Refer to the seven-point rating code or scale of achievement when constructing a rubric.
Feedback should acknowledge strengths and identify areas of weakness for learners' developmental needs. Action plans on how learners will be supported should accompany this feedback. It is important that the feedback provided to learners encourages them to do better, and builds their self-confidence.

Planning for assessment

We have provided a full assessment plan for you to use.

Programme of Assessment

Term	Task	Topics	Learner's Book page	Teacher's Guide page
1	Assignment	Option 1: Financial maths	375	245
		Option 2: Constructions	377	246
	Test	Chapters 1 – 4		247-249
2	Investigation	Option 1: Functions and relationships	379	250
		Option 2: Volume and surface area	381	251
	Test	Chapters 5 - 7		253-254
	Exam	Chapters 1 - 9		258-264
3	Assignment	Option 1: Patterns	383	265
		Option 2: Algebra 1	385	267
	Project	Option 1: functions, relationships and graphs	386	268
		Option 2: Transformations	388	269
	Test	Chapters 10 - 15		270-271
4	Assignment	Option 1: Integers	390	272
		Option 2: Algebra 2	391	273
	Investigation	Option 1: Data	392	274
		Option 2: Probability	394	276
	Exam	Chapters 1 - 19		279-285

Inclusive assessment

Teachers need to develop adaptive and alternative methods to assess learners with barriers to learning, so that learners are given opportunities to demonstrate competence in ways that suit their needs. Here are some examples of how to assess these learners while still maintaining the validity of the assessment.

- Some learners may need concrete apparatus for a longer time than their peers.
- Assessments tasks, especially written tasks, may have to be broken up into smaller sections for learners who cannot concentrate or work for a long time, or they may be given short breaks during the tasks. Learners can also be given extra time to complete tasks.
- Some learners may need to do their assessment tasks in a separate venue to limit distractions.
- A variety of assessment instruments should be used, as a learner may find that a particular assessment instrument does not allow him/her to show what they can do.

- Learners who cannot read can have tasks read to them and they can orally dictate answers. Assessment can also include a practical component in which learners can demonstrate their competence without having to use language.
- A sign language interpreter can be used.
- Assessment tasks could be available in Braille or enlarged with bolded text.
- Assessment can include the use of dictaphones or computers with voice synthesisers.
- The forms of assessment used should be age and developmental level appropriate. The design of these tasks should cover the content of the subject and include a variety of tasks designed to achieve the objectives of the subject.

Recording and reporting assessment

- Recording: Recording documents the level of a learner's performance in a specific assessment task. It indicates learner progress towards the achievement of the knowledge as prescribed in the curriculum. Records of learner performance should be used to verify the progress made by teachers and learners in the teaching and learning process.
- Reporting: Learners' performance can be reported in a number of ways. These include report cards, parents' meetings, school visitation days, parent-teacher conferences, phone calls, letters, class or school newsletters, and so on. Teachers in all grades report in percentages against the subject. The various achievement levels and their corresponding percentage bands are as shown in the table below.

Rating code	Description of competence	Marks
7	Outstanding achievement	80–100
6	Meritorious achievement	70–79
5	Substantial achievement	60–69
4	Adequate achievement	50–59
3	Moderate achievement	40–49
2	Elementary achievement	30–39
1	Not achieved	20–29

Metacognitive Strategies

What are metacognitive strategies and how can I use them?

Metacognition is the process of thinking about how you think. Adults often do this automatically. Before taking on something new, we may ask ourselves: What do I already know about this? What will help me understand it better? How is it structured? As we engage with a text or action, we may ask ourselves: Did I understand that? Why do I think that? How does this connect with what I already know? How could I apply this in my life? Then we evaluate what we have learnt or done by asking questions like: Did I understand that well?

What strategies helped and what strategies didn't help? What should I do the next time I take on a task like this?

Learners, however, are often unaware of how they think and engage with learning material. You help learners to learn independently by explicitly guiding them to plan, monitor, and evaluate their reading and learning strategies. This is particularly effective for those learning in English as a second language and for learners who are struggling. It can dramatically improve their performance.

You teach metacognitive skills by asking learners to explain what they are thinking and what strategies they are using to understand material. This is best done in small groups. You can also use 'think aloud' strategies when engaging with texts and images. 'Think-alouds' are often effective when reading texts to learners, and during small-group and pair reading exercises. Here is an example of how to teach metacognitive strategies using a 'think aloud':

1. Choose a short piece of text and note where you will stop during reading to model your thought processes.
2. Things to include in this planning stage could be:
   - reading the text title and the table of contents
   - looking at the images and predicting what the text may be about
   - skim-reading the text looking for headings, words in bold, and summaries. As you skim read, think about what you already know about the subject and what more you would like to know.
3. In class, explain what you will be doing to the learners. Start by explaining how you planned before reading the text.
4. To monitor understanding during reading, you can explain where you stopped to ask yourself whether you understood the content. If the text has a long or complex sentence, describe how you divided it up to understand it. Find places where you could ask questions such as:
   - Why would this ...?
   - Is this similar to ...?
   - How can I figure out what this new word means?
   - What does the writer want me to know?
   - What do I think will happen next? Why do I think that?
   - Do I need to re-read this for detailed information?
5. Now show learners how to evaluate their metacognitive strategies by asking and answering questions such as:
   - Did I read and understand this well?
   - What helped me to understand? What didn't help?
   - What should I do next time I read about this topic?
   - What will help me remember what I read?

By engaging with how learners think, you can better prepare them for their lives and learning in the future.

What is Mathematics?

Mathematics is a language. It uses symbols and notations to describe relationships. Mathematics is a human activity that involves observing, representing and investigating patterns and relationships in both the physical and social dimensions. Mathematics helps develop key mental processes such as logical and critical thinking, accuracy and problem-solving. All of these processes contribute to a learner's decision-making ability.

The specific aims of Mathematics

The aims of Mathematics are to develop:
- a critical awareness of mathematical relationships
- confidence and competence in Mathematics without fear of the subject
- curiosity and a love of Mathematics
- appreciation for the beauty and elegance of Mathematics
- recognition of the subject as a creative art
- the deep conceptual understanding required to understand Mathematics
- the acquisition of specific skills in order to apply Mathematics
- the study of related subject matter and to study further in Mathematics.

Specific skills for Mathematics

In order to develop the essential skills, the learner should:
- develop correct mathematical language use
- develop number vocabulary, number concept and application skills
- learn to listen, communicate, think, reason logically and apply the knowledge gained
- learn to investigate, analyse, represent and interpret information
- learn to pose and solve problems
- be aware that Mathematics plays a key role in real-life situations.

Content area focus in the Senior Phase

Content area	General focus	Specific focus
Numbers, Operations and Relationships	Meaning of different kinds of numbers Relationships between different kinds of numbers Relative sizes of different numbers Representations of numbers in various ways Operations with numbers Estimation and checking solutions	Represent numbers in a variety of ways and move flexibly between representations Recognise and use properties of operations with different number systems Solve a variety of problems, use an increased range of numbers and are able to perform multiple operations correctly and fluently
Patterns, Functions and Algebra	Achieve efficient manipulation skills which will carry over into other domains of the subject Describe patterns and relationships through the use of symbolic expressions, graphs and tables Identify and analyse regularities and change in patterns Make predictions and solve problems	Investigate numerical and geometric patterns to establish the relationships between the variables Express rules governing patterns in algebraic language or symbols Develop algebraic manipulative skills to recognise the equivalence between different representations of the same relationship Analyse situations in a variety of contexts Use of different and equivalent representations – algebraic language, formulae, expressions, equations and graphs
Space and Shape (Geometry)	Properties of shapes and objects Relationships between these properties Orientations, positions and transformations of two-dimensional shapes and three-dimensional objects	Draw and construct a wide range of geometric figures and solids using appropriate instruments Appreciate the use of constructions to investigate the properties of geometric figures and solids Develop clear and precise descriptions and classification categories of geometric figures and solids Solve geometric problems drawing on known properties of geometric figures and solids
Measurement	Select and use appropriate units, instruments and formulae Make sensible estimates Be aware of sensibleness and reasonableness of measurement and results	Use formulae for measuring area, perimeter, surface area and volume of geometric figures and solids Select and convert between units of measurement Use the Theorem of Pythagoras to solve problems involving right-angled triangles
Data Handling	Ask questions and find answers in order to describe events and the social, technological and economic environment Collect, organise, represent, analyse, interpret and report data Enabled to make informed predictions by the study of probability Describe randomness and uncertainty	Pose questions for investigation Collect, summarise, represent and critically analyse data Interpret, report and make predictions about situations Probability – include single and compound events and their relative frequency in simple experiments

Teaching Mathematics in the Senior Phase

The Senior Phase can be a difficult phase to teach. Each grade brings with it an added challenge. In Grade 7 learners are in the final year of Primary School and preparing for the transition to High School. In Grade 8 learners are adjusting to a new school, new peers and usually a new way of teaching and learning. In Grade 9 learners need to select their subjects for matric, and ultimately start preparing for their career and a life outside of school. Each of these periods add additional pressure and stress to learners in this phase.

Senior Phase learners do Mathematics every day. In this phase learners move from mathematics with a primarily arithmetic focus to that of a more formal and abstract approach. Essentially learners start doing 'real' maths. In order to cope with this shift, learners must be challenged to think abstractly and critically, and not to merely copy formulae and do substitutions.

The writing of formal tests and exams becomes even more important. The Mathematics teacher must spend time developing exam techniques which include unpacking terminology used in exams, such as: determine, identify, deduce, predict, present, summarise, expand, suggest, illustrate, and so on. The Learner's Book provides many built-in opportunities for learners to engage with these. Presenting answers, time management, and exam stress management are all important areas in which learners must receive constant coaching. Mathematics teachers should work very closely with Life Orientation teachers in order to support learners with these issues.

The volume and depth of material that learners are expected to engage with in this phase are higher. Learners are expected to start marking their own work (from the board) and this is new to many Grade 8 learners. They will need support and instruction as they learn how to manage this form of responsibility.

Grade 9 is a crucial year in the teaching of Mathematics: learners are required to make a choice between Mathematics and Mathematical Literacy in Grade 10. This choice will be based on their experience and level of success achieved in Grade 9. For learners who have some idea of their future career, the choice between these two might be somewhat more straightforward.
It is of the utmost importance that teachers lay a good foundation for basic algebra and geometry in Grades 8 and 9 in order to facilitate learners who wish to pursue further Mathematics in Grade 10.

Inclusive teaching

What is inclusive teaching?

In the Senior Phase, it is crucial that learners find themselves in an environment where they can develop an interest in learning and the belief that they can learn. Inclusive Education is defined as a learning environment that promotes the full personal, academic and professional development of all learners irrespective of race, class, gender, disability, religion, culture, sexual preference, learning styles and language.

Inclusion is about acknowledging and respecting:
- that all children have the right to learn
- that all children can learn
- that all learners need support
- that all learners are unique and have different, but equally valued, learning needs
- that all learners need the opportunity to build on their own unique strengths
- that the learner is the centre of the teaching and learning process
- that there are differences in learners, such as age, gender, language, culture, learning styles, disabilities, HIV status and so on.

Inclusion is also about:
- enabling educational structures, systems and learning methodologies to meet the needs of all learners
- more than just formal schooling – it embraces learning that occurs in the home, community and so on
- changing attitudes, behaviour, methodologies and environments to meet the needs of all learners
- ensuring maximum participation of all learners in the culture and curriculum of all educational institutions
- identifying and minimising barriers to learning that can occur at any level of the system.

Some of the learners in your class may already suffer from exclusion or think negatively about education. There is no reason for their exclusion from class activities. It is the responsibility of the teacher to ensure the inclusion of these learners. This means adapting activities to suit their needs and capabilities. It is equally important that the class is not divided because of this. Rather, learners with these challenges should be accepted and helped where possible by their peers. Learners should at all times be discouraged from teasing, bullying or ignoring learners with special needs. When these attitudes are directed towards a learner they create in that learner a barrier to learning.

Practical guidelines for inclusive teaching

- Have a true understanding of each learner's background, strengths, unique abilities, needs and barriers. Then use this information to inform your planning and give a clearer focus.
- Remember that the teacher is a facilitator of learning.
- Keep the content and material as relevant as possible.
- Break down learning into small, manageable and logical steps. Keep instructions clear and short (plan beforehand).
- Grade activities according to the different levels and abilities of learners. Try to ensure that learners remain challenged enough without undue stress.
- Develop a balance between individual, peer tutoring, cooperative learning and whole class teaching.
- Use learners to help one another in the form of group types, peer assisted learning, buddy systems and so on. Ensure that learners feel included and supported in the classroom by both the teacher and their peers.
- Set up pairs and groups of learners where members can have different tasks according to strengths and abilities. Promote self-management skills and responsibility through group roles and the types of tasks you set.
- Motivate learners and affirm their efforts and individual progress. Build confidence. Encourage questioning, reasoning, experimentation with ideas and risking opinions.
- Determine the learner's Zone of Proximal Development (ZPD) and use it for effective teaching and learning. Vygotsky described the ZPD as the distance between what the learner already knows and understands and what he/she can understand with adult support. Learning is thus a social interaction as the teacher mediates and supports the learner as he/she understands a new concept.
- Spend time on consolidating new learning. Use different ways to do this until all learners understand the concept. Make time to go back to tasks so that learners can learn from their own and others' experiences and methods.
- Use and develop effective language skills (expressive and receptive, verbal and non-verbal).
- Experiment with a variety of teaching methods and strategies to keep learners interested and to cater for and develop different learning styles. Use games, co-operative group work, brainstorming, problem-solving, debates, presentations, and so on.

Learners with barriers to learning

A barrier to learning is anything that prevents a learner from participating fully and learning effectively. This includes learners who were formerly disadvantaged and excluded from education because of the historical, political, cultural and health challenges facing South Africans. Some other examples of barriers to learning may be learners who are visually or hearing impaired; learners who are left handed or learners who are intellectually challenged. Barriers to learning cover a wide range of possibilities and learners may often experience more than one barrier. Some barriers, therefore, require more than one adaptation in the classroom and varying types and levels of support.

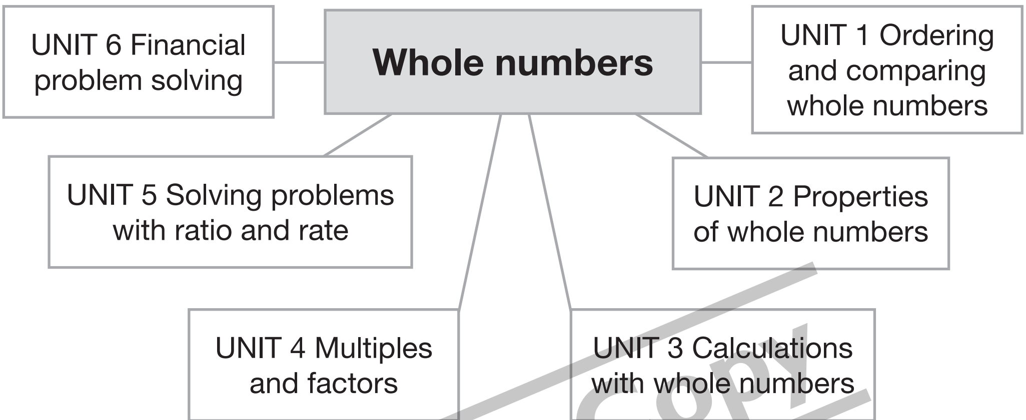
These learners may require and should be granted more time for:

- completing tasks
- acquiring thinking skills (own strategies)
- assessment activities.

Teachers need to adapt the number of activities to be completed without interfering with the learners gaining the required language skills.

Chapter 1 Whole numbers

Overview of concepts



Content		Time allocations	LB page
Unit 1	Ordering and comparing whole numbers	1,5 hours	12
Unit 2	Properties of whole numbers	1 hour	17
Unit 3	Calculations with whole numbers	2 hours	20
Unit 4	Multiples and factors	1 hour	27
Unit 5	Solving problems with ratio and rate	1,5 hours	30
Unit 6	Financial problem solving	2 hours	34

Background information on whole numbers

Revise the basics of whole numbers and how to work with them. It is not possible to learn prime numbers, multiples or any other more complex sequences of numbers without understanding how to count or how to use the basic operations with whole numbers.

Revise terminology and concepts such as:

- Addend plus addend equals sum.
- Multiplier times multiplicand = product.
- Dividend divided by divisor = quotient.
- Subtrahend minus minuend = difference.
- Whole numbers are all positive numbers including 0. For example, 0; 1; 2; 3…
- Natural numbers are generally regarded as all positive numbers excluding 0. For example, 1; 2; 3; 4…

- Integers are all whole numbers including 0 as well as all negative numbers. For example, ... –3; –2; –1; 0; 1; 2; 3… The further right you go on the number line, the bigger the value of the number. The further left you go on the number line, the smaller the value of the number.

Generic teaching guidelines for teaching whole numbers

When introducing whole numbers, have learners count in 1s, 2s, 3s and 4s as far as possible and revise times tables up to 12 × 12. Play games and have class competitions to encourage learners to participate and remember these core concepts. Set the class up in teams and reward correct answers with points. Learners can also play ball games and skipping games outside. They have to see how many bounces or skips they can make while counting or repeating the times tables.

Revise the four basic operations of addition, subtraction, multiplication and division. Do examples as a class on the board. Have learners come up to the board and complete the sums. Revise the inverse operation between addition and subtraction, and multiplication and division. Show how the inverse operations work.

Revise multiplication facts for Units and Tens by: multiples of 10; multiples of 100; multiples of 1 000; multiples of 10 000. Revise division facts as well.

Resources

Number cards, HTU charts up to millions, cardboard, colour pens for creating flash cards, comparison cards (cards with the < > or = signs), fraction and number cards, 100 number blocks, number grids, blank number lines, grid paper for learners to use with column calculations. Each learner should have their own calculator.

Unit 1 Ordering and comparing whole numbers

Learner’s Book page 12

Unit focus

- revise various mental calculations
- revise comparing whole numbers
- revise ordering whole numbers.

Background information on ordering and comparing whole numbers

Learners should have a thorough sense of the size of whole numbers by now. Have an HTU chart that goes up to ten million available for learners to refer to.
Mental calculations are important and become increasingly heavily relied upon in later grades as the work becomes more complex. The sooner learners can integrate the mental calculations into their working, the quicker their maths work will become.

Exercise 1 — Learner's Book page 13

Guidelines on how to implement this activity

Revise times tables and multiplying by powers of 10. Use class competitions to stimulate learners. Encourage all learners to participate and have prizes for learners that manage tricky examples. Ensure learners can work with inverse operations and understand how one operation can undo another.

Suggested answers

1.1	36	**1.2**	21	**1.3**	48	**1.4**	99	**1.5**	50
1.6	56	**1.7**	12	**1.8**	24	**1.9**	44	**1.10**	54
1.11	84	**1.12**	42	**1.13**	27	**1.14**	40	**1.15**	77

2.1	6	**2.2**	9	**2.3**	7	**2.4**	9	**2.5**	6
2.6	5	**2.7**	8	**2.8**	8	**2.9**	9	**2.10**	9
2.11	8	**2.12**	9	**2.13**	10	**2.14**	12	**2.15**	8

3.1 76 450	**3.2** 500	**3.3** 7 040 000
3.4 31 200	**3.5** 456 700	**3.6** 7 800 900
3.7 343 000	**3.8** 3 789 000	**3.9** 12 010 000

4.1	1 800	**4.2**	220	**4.3**	4 800	**4.4**	2 100	**4.5**	720
4.6	3 000	**4.7**	24 000	**4.8**	240	**4.9**	44	**4.10**	54
4.11	49 000	**4.12**	210 000	**4.13**	28 000	**4.14**	40 000	**4.15**	1 700

5.1 $4 \times 12 = 48$ then $48 \div 4 = 12$ (or $48 \div 12 = 4$)
5.2 $27 \div 3 = 9$ then $3 \times 9 = 27$
5.3 $12 \times 3 = 36$ then $36 \div 3 = 12$
5.4 $81 \div 3 = 27$ then $27 \times 3 = 81$ (or $3 \times 27 = 81$)

Remedial

If learners are struggling with their times tables, provide flashcards and homework based on the times tables. Test learners every week on times tables, offering prizes of 5 minutes of extra break time for learners scoring full marks. Test learners every week until they are proficient.

Exercise 2 — Learner's Book page 15

Guidelines on how to implement this activity

Discuss the value of 9 digit numbers. Have an HTU chart available and discuss the value of each digit in the table. Discuss how we use place value to compare numbers. Revise the signs for comparing numbers – greater than, less than and equal to. Do examples together as a class where you compare large numbers. Have learners come up to the board to fill in the correct comparison sign between two numbers.

Revise rounding off to 5, 10, 100 and 1 000. Do some examples together as a class. Show learners how using the place value chart can help them round off effectively.
Revise prime numbers. Ensure learners understand that a prime number has only two factors. Put lists of numbers on the board, and ask learners to identify as quickly as they can all the prime numbers.
Learners should complete the exercise on their own.

Suggested answers

1.1 2 227; 2 236; 2 245; 2 254; 2 263; 2 272; 2 281; 2 290; 2 299; 2 308 (+ 9 each time)
1.2 7; 11; 13; 17; 19; 23; 29 (prime numbers)
1.3 7; 9; 11; 13; 15; 17; 19; 21 (+ 2 each time)

2.1	True	**2.2**	False	**2.3**	False

3.1	<	**3.2**	>	**3.3**	=	**3.4**	<
3.5	=	**3.6**	>	**3.7**	<	**3.8**	<
3.9	<	**3.10**	=	**3.11**	=	**3.12**	<

4.1 46 988; 56 789; 65 467; 80 450; 89 764
4.2 145 678; 154 987; 178 000; 187 000; 210 000; 300 211
5 No, 0 is a whole number but not a natural number.
6.1 (i) 30, (ii) 350, (iii) 8 880, (iv) 54 650
6.2 (i) 0, (ii) 300, (iii) 8 900, (iv) 54 700
6.3 (i) 0, (ii) 0, (iii) 9 000 (iv) 55 000
6.4 (i) 30, (ii) 350, (iii) 8 880 (iv) 54 655

7.1	7 000 000 ÷ 10 = 700 000	**7.2**	50 × 10 = 500
	700 000 ÷ 10 = 70 000		500 × 10 = 5 000
	70 000 ÷ 10 = 7 000		5 000 × 10 = 50 000
	7 000 ÷ 10 = 700		50 000 × 10 = 500 000
	700 ÷ 10 = 70		500 000 × 10 = 5 000 000
	70 ÷ 10 = 7		5 000 000 × 10 = 50 000 000

8.1	50	**8.2**	170	**8.3**	1 490
8.4	800	**8.5**	2 300	**8.6**	7 960
8.7	1 234 000	**8.8**	5	**8.9**	57
8.10	843	**8.11**	9	**8.12**	73

9 Learners own work

Remedial

All the work covered in this exercise and unit should be revision, and should be done with limited calculations. Learners should be completing these types of sums mentally. Encourage this at all times. If learners have experienced problems completing any of the material in this exercise, provide Grade 6 material for learners to practise and revise. Identify learners who have difficulty in doing mental calculations and provide additional homework with the appropriate Grade 6 material until learners can cope at the desired Grade 7 level.

Extension

Learners can extend their times tables up to 14. Encourage learners to complete calculations mentally whenever they can.

Unit 2 Properties of whole numbers

Learner's Book page 17

Unit focus

- recognise the properties involved when adding numbers
- recognise the properties involved when multiplying numbers
- recognise and use the properties of zero (0)
- recognise and use the properties of 1.

Background information on properties of whole numbers

- The communicative property allows us to change the order of numbers when adding or multiplying. For example, $2 + 3 = 3 + 2$ and $2 \times 3 = 3 \times 2$. It often is easier to calculate when we can change the order.
- The communicative property does not allow us to change the order of numbers when subtracting or dividing. For example, $3 – 2 \neq 2 – 3$ and $4 \div 2 \neq 2 \div 4$.
- The associative property allows us to group numbers differently when we add or multiply. For example, $(2 + 3) + 4 = 2 + (3 + 4)$ and $(2 \times 3) \times 4 = 2 \times (3 \times 4)$.
- Because the associative property allows us to group numbers differently, we can group them so that they are easier to calculate. For example, $(93 + 75) + 25$ is easier to do if we group it this way $93 + (75 + 25)$.
- The associative property does not allow us to group numbers differently when subtracting or dividing. For example, $4 – 3 – 2 \neq 4 – 2 – 3$ and $4 \div 2 \div 2 \neq 2 \div 2 \div 4$.
- The distributive property allows us to multiply or divide the sum of 2 numbers or the difference between 2 numbers by spreading the multiplier or divisor over the operation. For example, $2(3 + 4)$ is the same as $2 \times 7 = 14$, but with the setting $2(3 + 4)$ we can say $2 \times 3 + 2 \times 4 = 6 + 8 = 14$. This is helpful when multiplying bigger numbers like 25×43. This can be $25 (40 + 3) = 25 \times 40 + 25 \times 3 = 1000 + 75 = 1\ 075$.
- The distributive property allows us to multiply or divide the difference between 2 numbers by spreading the multiplier or divisor over the operation.
  For example, $(6 – 4) \div 2 = 6 \div 2 – 4 \div 2 = 3 – 2 = 1$.

Revision exercise

Learner's Book page 17

Guidelines on how to implement this activity

Learners should remember the basic properties of numbers. Encourage learners to do this activity before revising the properties of numbers. This will enable you to see how much of the properties of numbers learners remember. You can use this exercise to identify the average prior knowledge learners have, and use that information to guide the level of revision required.

Suggested answers

1	T	**2**	F	**3**	T
4	F	**5**	T	**6**	F
7	T	**8**	T	**9**	T
10	T				

Exercise 1

Learner's Book page 18

Guidelines on how to implement this activity

Revise the properties of whole numbers by working through the table in the Learner's Book. For each property provide an example and show on the board how the property applies to numbers. Ensure learners understand why the property applies to some operations and not to others. Complete the working out to show learners conclusively the properties in action. Pay special attention to the distributive property and multiplying out brackets, as this becomes very important when learners start working with algebra.
Revise the concept of variables, and what a variable in a statement means and represents. This will help prepare learners for questions involving letters instead of numbers in the exercise.

Suggested answers

1.1 27 + 45 = 72
1.2 23 + 234 + 2 345 = 2 602
1.3 2 222 + 3 333 + 48 = 5 603
1.4 79 + 87 + 693 = 859
2

■	8	5	9
7	■	6	■
2	6	0	2
■	■	3	■

3.1 6 × 8 = 48
3.2 10 × 7 = 70
3.3 12 × 11 = 132
3.4 26 × 42 = 1 092
3.5 132 × 716 = 94 512
4.1 4 × (2 × 7) = 4 × 14 = 56; (4 × 2) × 7 = 8 × 7 = 56
4.2 7 × (3 × 5) = 7 × 15 = 105; (7 × 3) × 5 = 21 × 5 = 105
4.3 (8 × 6) × 10 = 48 × 10 = 480; 8(6 × 10) = 8 × 60 = 480
4.4 (12 × 11) × 20 = 132 × 20 = 2 640; 12 × (11 × 20) = 12 × 220 = 2 640
4.5 24 × (32 × 43) = 24 × 1 376 = 33 024; (24 × 32) × 43 = 768 × 43 = 33 024
4.6 (68 × 73) × 42 = 4 964 × 42 = 208 488; 68 × (73 × 42) = 68 × 3 066 = 208 488
5.1 3(4 + 6) = 3 × 10 = 30; 3 × 4 + 3 × 6 = 12 + 18 = 30
5.2 4(15 + 24) = 4 × 39 = 156; 4 × 15 + 4 × 24 = 60 + 96 = 156
6.1 16(15 + 29) = 16 × 44 = 704; 16 × 15 + 16 × 29 = 240 + 464 = 704
6.2 28(37 + 12) = 28 × 49 = 1 372; 28 × 37 + 28 × 12 = 1 036 + 336 = 1 372
6.3 10(19 + 11) = 10 × 30 = 300; 10 × 19 + 10 × 11 = 190 + 110 = 300
6.4 24(16 + 45) = 24 × 61 = 1 464; 24 × 16 + 24 × 45 = 384 + 1 080 = 1 464
6.5 9(24 – 19) = 9 × 5 = 45; 9 × 24 – 9 × 19 = 216 – 171 = 45
6.6 15(87 – 35) = 15 × 52 = 780; 15 × 87 – 15 × 35 = 1 305 – 525 = 780
6.7 32(274 – 169) = 32 × 105 = 3 360; 32 × 274 – 32 × 169 = 8 768 – 5 408 = 3 360
6.8 45(356 – 57) = 45 × 299 = 13 455; 45 × 356 – 45 × 57 = 16 020 – 2 565 = 13 455
7.1 Integers **7.2** Integers **7.3** Multiples of 5
8.1 1 (one) **8.2** 0 (zero) **8.3** 1 (one) **8.4** 0 (zero)
9.1 1 (one) **9.2** 0 (zero) **9.3** 1 (one)
10 (13 + 13 + 13 + 13 + 13) + (14 + 14 + 14) = 5 × 13 + 3 × 14 = 65 + 42 = 107

Remedial

Always give many simple examples to learners who are struggling and make sure that they have the first couple correct to increase confidence. Examples are:
Commutative property: $2 + 3 = \ldots + 2$; $3 \times 5 = \ldots \times \ldots$; and so on.
Sometimes it is good to let a learner with problems work in a pair with a friend who can help, but not in a group; in a group he or she may be ignored or mocked.
Encourage learners to play a variety of number games based on the properties. For example, divide the class into 2 teams. Start with one learner from each team at the board and say, 10 plus 5 equals 5 plus what?' As soon as they have written the answer, they sit down and the next learner goes up. As soon as the first learner has written the CORRECT answer, you read the next question. Read each question clearly and only once.
Give practise worksheets with very simple examples of the properties for those learners who struggle to do the calculations.

Extension

Learners who have a good grasp of the concept can be given 2 or 3 'pupils' to teach. Put them in groups and give the teacher an 'easy' worksheet to work from.
Ask learners who have a good grasp of the concept to make number cards to play 'property games': cards with single digits (1; 2; 3; ...), cards with double digits (10; 11; 12; …) and cards with operation signs (+; –; ×; ÷). Let them play in pairs. Each learner draws 5 number cards and 2 operation cards. The first one uses the cards to make an operation. For example, $4 \times (59 + 76)$. Both have paper and pencils to work out the answer. Use calculators to check the answers. Work out a method of scoring.

Unit 3 Calculations with whole numbers

Learner's Book page 20

Unit focus

- revise calculation strategies covered in earlier grades
- add and subtract with whole numbers
- multiply and divide with whole numbers
- round off numbers
- estimate answers
- use calculators where appropriate.

Background information on calculations with whole numbers

Learners have performed these operations in depth in the intermediate phase.
In Grade 7 the emphasis is less on performing the actual calculation itself - learners increasingly use calculators to do calculations - but on judging the reasonableness of the solution they acquire on a calculator. However, in order to be able to understand the operation and calculation, learners need to revise these calculations at the beginning of the year to ensure they can perform them correctly.

In order to judge the reasonableness of a solution, learners need to be able to estimate effectively, and round off numbers. Learners have also covered this in depth in earlier grades. However, these skills are still in great demand, and need to be practised and honed in Grade 7.

Revision exercise

Learner's Book page 20

Guidelines on how to implement this activity

Encourage learners to complete this activity on their own, and use their answers to inform your planning going forward. This exercise will help you determine a learner's prior knowledge and identify which learners struggle with the operations and identifying the most reasonable answer. Learners should complete this activity on their own. Learners must show their workings.

Suggested answers

1 1.1 **2** 2.2 **3** 3.2 **4** 4.1

Exercise 1

Learner's Book page 22

Guidelines on how to implement this activity

Start with adding whole numbers. Revise using rounding off and compensating as a means for adding whole numbers. Discuss what it means to compensate and how rounding off is used to estimate. Work through the notes in the Learner's book for adding and subtracting using compensation. Discuss how using rounding off and compensating to add is different to subtracting. Focus in particular on when to subtract and when to add the compensated values from the estimated total. Discuss place value and the importance of setting out the sum correctly with the place values aligned correctly underneath each other for each number. Revise carrying and how we present this when adding in columns. Remind learners that we always start with the units and work our way across to the tens, hundreds, etc. Do a few examples together as a class and show the working out clearly on the board.
Revise subtracting in columns and remind learners to use the place value correctly and to align numbers neatly underneath each other in corresponding place value. Revise borrowing and the concept of taking from the larger place value and adding this to the lower place value. Ensure learners can do this correctly. Show how we present the carrying when working in columns. Do a few examples together as a class. Learners should complete this exercise on their own.

Suggested answers

1.1

				1	1	
	HTh	TTh	Th	H	T	U
		1	1	3	5	6
+		2	5	4	6	9
		3	6	8	2	5

1.2

				1	1	
	HTh	TTh	Th	H	T	U
		4	1	4	6	9
+		1	7	3	5	6
		5	8	8	2	5

1.3

	HTh	TTh	Th	H	T	U
			1			
		3	3	9	2	3
+		1	1	7	6	2
		4	5	6	8	5

1.4

	HTh	TTh	Th	H	T	U
			1			
	2	3	4	7	6	2
+	3	2	1	9	2	3
	5	5	6	6	8	5

2.1

	HTh	TTh	Th	H	T	U
			11	8	12	15
		1	1	9	3	5
–			9	1	8	7
			2	7	4	8

2.2

	HTh	TTh	Th	H	T	U
			6	9	9	10
	6	7	7	0	0	0
–	1	1	3	6	0	4
	5	6	3	3	9	6

2.3

	HTh	TTh	Th	H	T	U
				6	15	15
		9	8	7	6	5
–		1	8	3	9	7
		8	0	3	6	8

2.4

	HTh	TTh	Th	H	T	U
			8	15	14	14
	7	8	9	6	5	4
–	6	5	4	7	8	9
	1	3	4	8	6	5

3.1 4 786 + 4 541 ≈ 4 800 + 4 500 ≈ 9 300
4 800 – 4 786 = 14
4 541 – 4 500 = 41
9 300 – 14 + 41 = 9 327

3.2 7 984 + 11 996 ≈ 8 000 + 12 000 ≈ 20 000
8 000 – 7 984 = 16
12 000 – 11 996 = 4
20 000 – 16 – 4 = 19 980

3.3 7 999 – 4 328 ≈ 8 000 – 4 300 ≈ 3 700
8 000 – 7 999 = 1
4 328 – 4 300 = 28
3 700 – 1 – 28 = 3 671

3.4 12 765 – 3 489 ≈ 12 800 – 3 500 ≈ 9 300
12 800 – 12 765 = 35
3 500 – 3 489 = 11
9 300 – 35 + 11 = 9 276

4.1 School A: 43 734 – 29 876 = 13 858 School B: 37 003 – 27 835 = 9 168

4.2 School A: 13 858 – 9168 = 4 690

Remedial

If learners have problems with adding and subtracting in columns, set aside some additional time, and work through additional examples with learners. These skills should have been developed and mastered in Grade 6, and if they are still lacking, this needs to be corrected. Work through the methods again, starting with simple examples involving 3 digit numbers only and one carry or borrow.

Gently build up the complexity until learners can cope with 6 digits and complex borrowing and carrying. Provide Grade 6 material on adding and subtracting in columns for homework until learners can cope with the desired complexity.

Extension

Encourage learners to extend the number range to 9 and 10 digit numbers.

Exercise 2

Learner's Book page 24

Guidelines on how to implement this activity

Work through the methods for multiplying whole numbers. Learners must be able to multiply 4 digit by 3 digit numbers. Discuss the column method, how the columns work and what each column represents. Do examples together as a class. Start with 3 digit by 2 digit and work up to the desired complexity of 4 digit by 3 digit.
Introduce division. Discuss methods with which learners are familiar.
Revise long division and how it works. Discuss remainders and what these mean. Discuss the method using an example on the board. Do additional examples as a class, until learners are able to tackle the calculations on their own. Learners should be able to divide 4 and 5 digit numbers by 2 and 3 digit numbers. Learners should complete this exercise on their own.

Suggested answers

1.1

	HTh	TTh	Th	H	T	U
			6	7	5	3
×					4	2
		1	3	5	0	6
				1	2	0
			2	0	0	0
		2	8	0	0	0
+	2	4	0	0	0	0
	2	8	3	6	2	6

1.2

		HTh	TTh	Th	H	T	U
			2	8	3	2	1
×						5	3
			8	4	9	6	3
						5	0
				1	0	0	0
			1	5	0	0	0
		4	0	0	0	0	0
+	1	0	0	0	0	0	0
	1	5	0	1	0	1	3

1.3

	HTh	TTh	Th	H	T	U
		1	3	4	5	6
×					1	7
		9	4	1	9	2
					6	0
				5	0	0
			4	0	0	0
		3	0	0	0	0
+	1	0	0	0	0	0
	2	2	8	07	5	2

1.4

		HTh	TTh	Th	H	T	U
			4	5	0	0	0
×						3	4
		1	8	0	0	0	0
		1	5	0	0	0	0
+	1	2	0	0	0	0	0
	1	5	3	0	0	0	0

1.5

	*	*	4	2
75	3	1	5	0
–	3	0	0	
		1	5	
		1	5	0

1.6

	*	*	8	7
33	2	8	7	1
–	2	6	4	
		2	3	
		2	3	0

2.1 Round 4 590 to the nearest 100 and divide by 20, that is 4 600 ÷ 20 = 230

2.2 Round 45 987 to the nearest 1 000 and multiply with 21, that is 46 000 × 21 = 966 000

2.3 Round 6 787 to the nearest 100 and multiply with 75, that is 6 800 × 75 = 510 000

2.4 Round 57 900 to the nearest 1000 and divide by 55, that is 58 000 ÷ 55 = 1 054,55

3.1 87 642 ÷ 27 = 3 246

3.2 87 642 ÷ 36 = 2 434,5; 3 246 – 2 434,5 = 811,5

Remedial

Learners should manage this exercise. However, many learners experience problems with long division. If necessary, spend additional time on long division, revising the steps involved. Provide additional division examples, from Grade 6 material, for learners to practise with at home. Continue to provide additional material until learners become proficient in the work.

Extension

Encourage learners to try 5 and 6 digit numbers for multiplication and division.

Exercise 3

Learner's Book page 25

Guidelines on how to implement this activity

Learners need to master using a calculator accurately. Spend time teaching basic calculator skills. Show them the memory function and how to use it effectively. Walk through the class and ensure each learner can use their calculator correctly. Have speed trials by asking learners to perform sums on the calculator and award points for learners who finish correctly first. Let learners complete this exercise in groups and facilitate as necessary.

Suggested answers

3.1 5 + 10

3.2 5 + 10 = 15

3.3 25

3.4 adds 10 each time

3.5 nothing happens

3.6 888 + 2

3.7 type [6];[.];[5]

3.8 adds 5 each time

3.9 12

3.10 372

4

		Estimate (without using a calculator)	Check using a calculator
4.1	9 + 21	10 + 20 = 30	30
4.2	379 + 421	380 + 420 = 800	800
4.3	2 379 + 3 321	2 400 + 3 300 = 5 700	5 700
4.4	32 579 + 45 221	33 000 + 45 000 = 78 000	77 800
4.5	31 – 12	30 – 10 = 20	19
4.6	531 – 412	530 – 410 = 120	119
4.7	6 331 – 2 231	6 300 – 2 200 = 4 100	4 100

5.1 68 **5.2** 64 **5.3** 768 **5.4** 12
5.5 1 443 **5.6** 345 **5.7** 214 776 **5.8** 321
5.9 476 + (2 374 – 1987) of 2 – 99 = 476 + 387 × 2 – 99 = 476 + 774 – 99 = 1151
6.1 49; 50 **6.2** 0; 57
6.3 869; 868 **6.4** –5; –34; –769
6.5 0; 42 **6.6** 0; 679
6.7 54; 1 **6.8** $\frac{1}{3}$; $\frac{1}{5}$; $\frac{1}{8}$
6.9 0; 0; 0 **6.10** 87 × 0 + 1 = 0 + 1 = 1
7 On a calculator, you will get an error message.
8 (12 ÷ 4) × 12,5 + 12,5 = 3 × 12,5 + 12,5 = 37,5 + 12,5 = 50
9 3 × 3 × 4 = 9 × 4 = 36

Remedial

Let learners who battle to master the use of a calculator practise on one when they have a free minute in class.

Extension

Encourage learners who complete their calculation tasks faster than the rest of the class to check their answers on a calculator.

Unit 4 Multiples and factors

Learner's Book page 27

Unit focus

- revise multiples of numbers
- revise factors of numbers
- revise prime factors
- learn about the highest common factor
- learn about the lowest common multiple.

Background information on multiples and factors

Learners have worked with multiples and factors in Grade 6. Learners should be able to recognise multiples and factors, as well as determine them. In Grade 7 the work progresses to finding prime factors, finding the highest common factor (HCF) and the lowest common multiple (LCM), and finding factors of larger numbers. Learners will need their times table skills here in order to quickly determine multiples and factors.

Revision exercise
Learner's Book page 27

Guidelines on how to implement this activity

Revise what a factor is and what a multiple is. Do some examples in the class where you find factors and multiples of numbers. Revise a prime factor. If necessary, remind learners what a prime number is. Assign learners to pairs and have them complete this revision exercise.

Suggested answers
Learners' answers will differ depending on the numbers they chose.

Exercise 1
Learner's Book page 29

Guidelines on how to implement this activity

Revise prime factorising. Show learners how to first find all the factors and then identify the prime numbers in the factors. Do a few examples together as a class. Introduce the concept of highest common factor. Show learners by means of a worked example how to identify the highest common factor. Introduce the concept of lowest common multiple. Show learners by means of a worked example how to find the lowest common multiple. Learners should complete this exercise on their own.

Suggested answers

1 For example: 12; 15

1.1 First 10 multiples of 12: 12; 24; 36; 48; 60; 72; 84; 96; 108; 120
First 10 multiples of 15: 15; 30; 45; 60; 75; 90; 105; 120; 135; 150

1.2 Factors of 12: 1; 2; 3; 4; 6; 12 Factors of 15: 1; 3; 5; 15 HCF is 3

1.3 $12 \times 15 = 180$
Multiples of 12: 12; 24; 36; 48; 60; 72; 84; 96; 108; 120; 132; 144; 156; 168; 180
Multiples of 15: 15; 30; 45; 60; 75; 90; 105; 120; 135; 150; 165; 180
LCM is 60

2.1 1; 2; 4; 8

2.2 1; 2; 3; 4; 6; 12

2.3 1; 2; 3; 4; 6; 8; 9; 12; 18; 24; 36; 72

2.4 1; 2; 4; 5; 10; 20; 25; 50; 100

2.5 1; 2; 3; 4; 6; 11; 12; 22; 33; 44; 66; 132

2.6 1; 2; 7; 11; 14; 22; 77; 154

3.1 The answer is 8 every time you multiply the two factors.

3.2 The answer is 12 every time you multiply the two factors.

3.3 The answer is 72 every time you multiply the two factors.

3.4 The answer is 100 every time you multiply the two factors.

3.5 The answer is 132 every time you multiply the two factors.

3.6 The answer is 154 every time you multiply the two factors.
We observe that you get the same number again as answer every time you multiply the two factors.

4 Factors of 100: 1; 2; 4; 5; 10; 20; 25; 50; 100
Factors of 132: 1; 2; 3; 4; 6; 11; 12; 22; 33; 44; 66; 132
Common factors: 1; 2; 4 HCF is 4

5 Factors of 105: 1; 3; 5; 7; 15; 21; 35; 105 Prime factors of 105 : 3; 5; 7
Factors of 420: 1; 2; 3; 4; 5; 6; 7; 10; 12; 14; 15; 28; 30; 35; 42; 60; 70; 84; 105; 140; 210; 420
Prime factors of 420: 2; 3; 5; 7
Factors of 770: 1; 2; 5; 7; 10; 11; 14; 22; 35; 55; 70; 110; 154; 385; 770
Prime factors of 770: 2; 5; 7; 11

Remedial

Provide additional, easier examples for learners to work with in identifying the HCF and LCM. Build learners' confidence in their ability and gradually increase the level of complexity until learners can manage the required activity.

Extension

Provide more complex and larger numbers for learners to work with.

Unit 5 Solving problems with ratio and rate

Learner's Book page 30

Unit focus

- compare two or more quantities of the same kind (ratio)
- compare two quantities of different kinds (rate)
- solve problems involving rate and ratio.

Background information on ratio and rate

Learners have worked with ratio and rate in earlier grades. Ratio involves comparing two quantities of the same kind, while rate involves comparing two quantities of different kinds. Learners must know that every day relationships such as speed (km/h) and rands per kilogram, are all rates. It is important that learners can identify the use of ratio and rate in everyday life.

Exercise 1

Learner's Book page 31

Guidelines on how to implement this activity

Introduce the concept of ratio. Learners must understand that ratio is a relationship between two quantities. Show learners that this relationship can be represented using a fraction or using a colon. Discuss how ratio can be used to solve problems. Work through the example with the learners. Do additional examples such as sharing money or goods in a set ratio between 2 or more learners. If possible, have concrete apparatus to use, such as sweets or blocks, to make the initial examples more tangible for learners.

Suggested answers

1.1 $\frac{5}{9}$ **1.2** $\frac{15}{27} = \frac{5}{9}$ **1.3** $\frac{35}{63} = \frac{5}{9}$ **1.4** $\frac{50}{90} = \frac{5}{9}$

2 $1 + 2 + 5 = 8$
$\frac{1}{8} \times 24 = 3$; $\frac{2}{8} \times 24 = 6$; $\frac{5}{8} \times 24 = 15$
The answer is 3 : 6 : 15

3 $3 + 4 + 1 = 8$
$\frac{3}{8} \times 4\,000 = 1\,500$; $\frac{4}{8} \times 4\,000 = 2\,000$; $\frac{1}{8} \times 4\,000 = 500$
The answer is 1 500 : 2 000 : 500

4 $1 + 2 = 3$
Nandie: $\frac{1}{3} \times 15 = 5$ and Thabo: $\frac{2}{3} \times 15 = 10$

5 $5 + 3 + 4 = 12$
Joshua: $\frac{5}{12} \times 288 = 120$; Katlego: $\frac{3}{12} \times 288 = 72$; Pavashnee: $\frac{4}{12} \times 288 = 96$

Remedial

Learners who have problems with fractions, may struggle with ratios. If necessary, have some fraction problems at hand to help learners revise how to work with fractions.

Extension

Provide more complex examples for stronger learners. Let the problems involve decimals.

Exercise 2

Learner's Book page 32

Guidelines on how to implement this activity

Rate has a strong presence in everyday life and it is important for learners to observe and note this. If possible have real life examples for learners to work from. Discuss the concept of rate and show learners its use in everyday life – show learners cell phone rate charts, electricity rate charts, and pricing charts from a supermarket for fruit and vegetables sold by weight. Do a few examples together with the learners calculating rate. Encourage learners to try this exercise on their own.

Suggested answers

1 1 kg of flour costs $\frac{15}{2}$ = 7,5 rand, therefore 4 kg cost 4 × 7,5 = R30.

2 1 litre of milk weighs $\frac{3,5}{5}$ = 0,7 kg, therefore 2 litres weigh 2 × 0,7 = 1,4 kg.

3 In 1 hour he ran 3 × 1 = 3 km, therefore he will run 3 × 3 = 9 km in 3 hours.

4 To heat 1 litre of water we need $\frac{60}{100}$ = 0,6 kW of electricity, therefore to heat 75 litres of water, we need 75 × 0,6 = 45 kW of electricity.

5 1 banana costs $\frac{10}{5}$ = R2, therefore 3 bananas cost 3 × 2 = R6.

Remedial

Allow learners to work in groups, grouping stronger learners with weaker learners for support.

Exercise 3

Learner's Book page 32

Guidelines on how to implement this activity

This exercise extends learners' knowledge of rate and ratio. Encourage learners to do this exercise on their own, and use it to assess learners ability to cope with work covered in this unit.

Suggested answers

1 Local : long distance : international = 2 : 3 : 5

2.1 $\frac{2}{3}$ **2.2** $\frac{2}{5}$ **2.3** $\frac{2}{10}$

3

Local calls (per second)	Long distance calls (per second)	International calls (per second)
$\frac{1,65}{45} = 0,036$	$\frac{2,68}{25} = 0,1072$	$\frac{3,69}{10} = 0,369$

Local : long distance : international = 0,036 : 0,1072 : 0,369

4 2 + 3 + 5 = 10. International calls: $\frac{5}{10} \times 30 = 15$

5 1 minute = 60 seconds. Cost per minute: 60 × 0,369 = R22,14
Total cost = 15 × 22,14 = R332,10

6 The ratio of calls received from Tracy is 1 : 2, therefore Tasneem received $\frac{1}{3} \times 12 = 4$ calls from Tracy over the weekend.

7.1 distance, time **7.2** beats, time

7.3 roses, price **7.4** HIV infections, humans

8.1 Gouda cheese per kg: R33 cheddar cheese per kg: $\frac{60}{2} = \text{R}30$
Judith should choose the cheddar cheese. It is cheaper per kilogram and therefore the best value for money.

8.2 One apple, from the bag of apples, costs $\frac{9,80}{8} = \text{R}1,225$. Judith should buy individual apples at R0,80 per apple. It is the cheaper option.

8.3 There are 3 × 12 = 36 bananas in 3 packets, which will cost 2 × 7 = R14, since one packet is free. The price then for one banana is $\frac{14}{36} = \text{R}0,38$. This is the cheaper option, since an individual banana cost R0,70.

9.1 The rand : euro ratio is 7,80 : 1. Thus you get R7, 80 for one euro. For 700 euro you get 700 × 7,80 = R5 460.

9.2 The rand : pound ratio is 9,70 : 1. Thus you get R9,70 for one pound. For 60 pounds you get 60 × 9,70 = R582.

9.3 The rand : dollar ratio is 6,98 : 1. Thus you get R6,98 for one dollar. For 5000 dollars you get 5000 × 6,98 = R34 900.

Remedial

If learners have experienced problems with this exercise, they may require additional intervention to work effectively with ratio and rate. Provide additional problems for learners to work with at home. Provide Grade 5 and 6 examples of working with ratio and rate, and then gradually increase the complexity until learners can cope with the prescribed level.

Extension

Learners can create a poster showing all the different ways rate and ratio are used in everyday life. This poster can be used to decorate the classroom.

Unit 6 Financial problem solving

Learner's Book page 34

Unit focus

- revise the concept of percentages
- revise decimals
- learn about profit, loss and discount
- create budgets
- learn about accounts, loans and interest.

Background information on financial problem solving

Learners have been doing problem solving since the Foundation Phase. However, as soon as problems are presented in word form, many learners struggle. It is important that learners revise the steps for solving problems effectively. Learners should follow the following steps:

1 Read the question carefully.
2 Identify what they are being asked to find.
3 Identify what information they have been given.
4 Develop a number sentence to find what is being asked.
5 Substitute the correct numbers into the number sentence.
6 Solve the number sentence.
7 Check the reasonableness of their solution.

Financial problems work best with a relevant context for learners. Learners are aware of the world around them, and financial mathematics helps them make sense of a lot of the information they have seen around them. This involves discount, tax, hire purchase, accounts and loans. Keep the information as relevant to learners as possible. Help them to relate it to events they are familiar with.

Learners will need to use percentages and fractions, so some additional revision of these concepts may be necessary for learners to cope with the exercises in this unit.

Revision exercise

Learner's Book page 34

Guidelines on how to implement this activity

This exercise will help you to identify a learner's prior knowledge. Encourage learners to complete this exercise on their own.

Suggested answers

1	T	**2**	F	**3**	T
4	F	**5**	T		

Remedial

If learners struggle to answer these questions correctly, refer them to the relevant unit they have covered in this chapter and have them revise the necessary concepts.

Exercise 1

Learner's Book page 35

Guidelines on how to implement this activity

Revise equivalent representations of percentages and fractions. Percentages are fractions out of 100. Practise some examples of converting fractions to percentages, and vice versa. Do a few examples together as a class and have learners complete this exercise on their own.

Suggested answers

1.1 $\frac{99}{100}$ **1.2** $\frac{41}{100}$

1.3 $\frac{23}{100}$ **1.4** $\frac{70}{100} = \frac{7}{10}$

1.5 $\frac{30}{100} = \frac{3}{10}$ **1.6** $\frac{625}{1000}$

2.1 0,99 **2.2** 0,41 **2.3** 0,23
2.4 0,7 **2.5** 0,3 **2.6** 0,625

3.1 $\frac{1}{2} \times \frac{2}{2} = \frac{2}{4}$; $\frac{1}{2} \times \frac{3}{3} = \frac{3}{6}$; $\frac{1}{2} \times \frac{5}{5} = \frac{5}{10}$; $\frac{1}{2} \times \frac{10}{10} = \frac{10}{20}$; $\frac{1}{2} \times \frac{50}{50} = \frac{50}{100}$

3.2 $\frac{3}{4} \times \frac{2}{2} = \frac{6}{8}$; $\frac{3}{4} \times \frac{4}{4} = \frac{12}{16}$; $\frac{3}{4} \times \frac{5}{5} = \frac{15}{20}$; $\frac{3}{4} \times \frac{10}{10} = \frac{30}{40}$; $\frac{3}{4} \times \frac{25}{25} = \frac{75}{100}$

3.3 $\frac{3}{5} \times \frac{2}{2} = \frac{6}{10}$; $\frac{3}{5} \times \frac{3}{3} = \frac{9}{15}$; $\frac{3}{5} \times \frac{8}{8} = \frac{24}{40}$; $\frac{3}{5} \times \frac{12}{12} = \frac{36}{60}$; $\frac{3}{5} \times \frac{20}{20} = \frac{60}{100}$

4.1 $50\% = \frac{50}{100} = \frac{5}{10} = 0{,}5$ **4.2** $25\% = \frac{25}{100} = 0{,}25$

4.3 $75\% = \frac{75}{100} = 0{,}75$ **4.4** $40\% = \frac{40}{100} = \frac{2}{5} = 0{,}4$

4.5 $90\% = \frac{90}{100} = \frac{9}{10} = 0{,}9$ **4.6** $88\% = \frac{88}{100} = \frac{22}{25} = 0{,}88$

Remedial

Some learners may require additional assistance with percentage and converting to its fraction form. Provide additional examples if necessary.

Exercise 2

Learner's Book page 39

Guidelines on how to implement this activity

Explain the concept of percentage increase and decrease. Work through the examples showing how to calculate percentage increase and decrease.

Discuss the concept of profit and loss. Explain that when companies, or individuals, make more money than they spend, i.e. their income exceeds their expenses, they have made a profit. When companies, or individuals, spend more than they earn, i.e. their expenses are larger than their income, they make a loss. Show learners how to determine profit or loss and then to write this as a percentage. Learners should do this exercise on their own.

Suggested answers

1.1 $\frac{4\ 256{,}48}{2} = 2\ 128{,}24$

1.2 $\frac{36}{100} \times 8\ 674{,}20 = 3\ 122{,}712$

1.3 $\frac{7}{8} \times 3\ 000 = 2\ 625$

2.1 $\frac{12}{100} \times 784 = 94{,}08$

2.2 $4 \times 94{,}08 = 376{,}32$

3.1 8 923 815,22 – 7 598 736,93 = 1 327 078,29.
Therefore the company makes a profit of R1 327 078,29.

3.2 8 923 815,22 – 9 000 123,45 = –76 308,23
Therefore the company makes a loss of R76 308,23.

4.1 $\frac{1\ 327\ 078{,}29}{8\ 923\ 815{,}22} \times \frac{100}{1} = 14{,}87\%$ (rounded to two decimal places)

4.2 $\frac{76\ 308{,}23}{8\ 923\ 815{,}22} \times \frac{100}{1} = 0{,}86\%$ (rounded to two decimal places)

5 100% – 21% = 79%

5.1 sofa: $\frac{79}{100} \times \frac{3\ 999}{1} = R3\ 159{,}21$ coffee table: $\frac{79}{100} \times \frac{1\ 179}{1} = R931{,}41$

plasma TV: $\frac{79}{100} \times \frac{8\ 995}{1} = R7\ 106{,}05$ dishwasher: $\frac{79}{100} \times \frac{4\ 325}{1} = R3\ 416{,}75$

5.2 sofa: 3 999 – 1 599 = 2 400; $\frac{2\ 400}{1\ 599} \times \frac{100}{1} = 150{,}09\%$

coffee table: 1 179 – 829 = 350; $\frac{350}{829} \times \frac{100}{1} = 42{,}22\%$

plasma TV: 8 995 – 4 950 = 4 045; $\frac{4\ 045}{4\ 950} \times \frac{100}{1} = 81{,}72\%$

dishwasher: 4 325 – 2 115 = 2 210; $\frac{2\ 210}{2\ 115} \times \frac{100}{1} = 104{,}49\%$

5.3 sofa: 3 159,21 – 1 599 = 1 560,21; $\frac{1\ 560{,}21}{1\ 599} \times \frac{100}{1} = 97{,}57\%$

coffee table: 931,41 – 829 = 102,41; $\frac{102{,}41}{829} \times \frac{100}{1} = 12{,}35\%$

plasma TV: 7 106,05 – 4 950 = 2 156,05; $\frac{2\ 156{,}05}{4\ 950} \times \frac{100}{1} = 43{,}56\%$

dishwasher: 3 416,75 – 2 115 = 1 301,75; $\frac{1\ 301{,}75}{2\ 115} \times \frac{100}{1} = 61{,}55\%$

Remedial

Provide simpler numbers for learners to work with if they are not coping with this exercise. This will help learners focus on the calculation and not get confused by the complex calculation. Continue with simple examples until learners are clear on the concept, and then move them on to more complex numbers.

Exercise 3

Learner's Book page 41

Guidelines on how to implement this activity

Discuss what a budget is. Have learners inform you on their understanding of a budget. Do an example as a class of having a class picnic, but having a budget of R500. What would we buy? Discuss household budgets, and why they are necessary. Work through the example with the learners on the board. Learners can complete this exercise in pairs.

Suggested answers

1

Item	Expense	Income
Rent	R1 300	
Transport	R300	
School fees	R160	
Prepaid electricity	R150	
TV license	R23	
Total expenses	**R1 933**	
Income from Marie		R3 800
Total income		**R3 800**
Total remaining for retirement	R1 867	

2

Item	Expense	Income
Mortgage bond	R10 000	
Vehicle instalments	R4 500	
Insurance	R1 100	
School fees	R1 360	
Groceries	R3 400	
DSTV	R546	
Entertainment	R1 300	
Retirement	R1 500	
Electricity	R1 100	
Rates	R876	
Total expenses	**R25 682**	
Income from Mrs Dlamini		R19 800
Income from Mr Dlamini		R10 500
Total income	**R30 300**	
Total remaining for luxuries or savings	R4 618	

Extension

Set an informal project on budgeting. Tell learners they have R5 000 to redecorate their bedroom. Learners must visit shops and get real prices of objects, furniture and appliances they would want in their room. Learners must put together a budget for their room, and show what they would buy.

Exercise 4 Learner's Book page 43

Guidelines on how to implement this activity

Introduce the concept of a loan. Loans are given by banks and other financial institutions such as cash loan businesses which are found in most shopping areas. Banks and small loan businesses make money from loans by charging interest. Discuss what the interest rate is. Show learners examples of bank adverts showing the interest rate. Show learners how to work out interest using the simple interest formula. Remind learners that the amount we pay back to the bank or a cash loan business in full is the total loan amount plus the total interest amount. Work through the example with learners. Do some additional examples if you feel they are necessary. Encourage learners to do this exercise on their own.

Suggested answers

1.1 550,66 + 30,50 = 581,16

1.2 550,66 + 581,16 = 1 131,82

1.3 $\frac{30,50}{550,66} \times \frac{100}{1}$ = 5,54%. No, it is incorrect to say, since she only budgets 5,54%.

($\frac{8}{100} \times \frac{550,66}{1}$ = 44,05. No, it is incorrect to say, since she only budgets R30,50.)

2.1 12 × 273,69 = 3 284,28

2.2 3 284,28 – 3 000 = 284,28

2.3 $\frac{284,28}{3\ 000} \times \frac{100}{1}$ = 9,48%

3.1 $\frac{2\ 448}{12}$ = 204

3.2 100% + 9% = 109%; $\frac{109}{100} \times \frac{2\ 448}{1}$ = 2 668,32

$\frac{9}{100} \times \frac{2\ 448}{1}$ = 220,32; 2448 + 220,32 = 2 668,32)

Monthly payment over 12 months: $\frac{2\ 668,32}{12}$ = 222,36

4 100% – 5% = 95%

4.1 $\frac{95}{100} \times \frac{379,80}{1}$ = 360,81

4.2 $\frac{95}{100} \times \frac{6\ 748,40}{1}$ = 6 410,98

4.3 $\frac{95}{100} \times \frac{457,89}{1}$ = 435,00

5.1 $\frac{15}{100} \times \frac{499,80}{1}$ = 74,97; 499,80 + 74,97 = 574,77

5.2 $\frac{15}{100} \times \frac{5\ 760,60}{1}$ = 864,09; 5 760,60 + 864,09 = 6 624,69

5.3 $\frac{15}{100} \times \frac{1\ 688,40}{1}$ = 253,26; 1688,4 + 253,26 = 1 941,66

6 574,77 + 6 624,69 + 1 941,66 = 9 141,12
(499,80 + 5 760,60 + 1 688,40 = 7 948,80; $\frac{115}{100} \times \frac{7\ 948,80}{1}$ = 9 141,12)

Remedial

Learners may need assistance on how to use their calculators effectively for this exercise. Be available to guide them and help them as necessary.

Extension

Have learners research banking and interest rates. Learners can collect different product packages from banks and compare them in a large sheet in the class. It is important for learners to be exposed to real world applications of mathematics in this way.

Consolidation

Learner's Book page 45

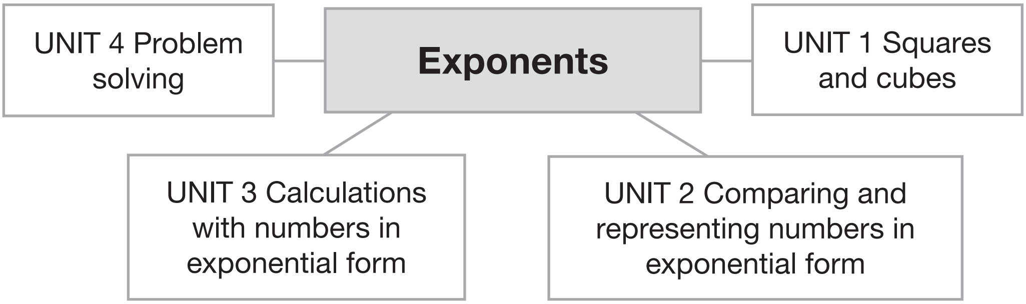
Before doing this consolidation exercise, encourage learners to review the work covered in this chapter. Advise learners to use the summary and to revise their work. This exercise can be used as an informal assessment task for you to track how learners are coping with the chapter and the concepts covered.

Suggested answers

1.1	18 + 33 = 33 + 18 = 51	
1.2	49 + 76 = 76 + 49 = 125	
1.3	283 + 371 + 542 = 283 + 542 + 371 = 371 + 283 + 542 = 371 + 542 + 283 = 542 + 283 + 371 = 542 + 371 + 283 = 1196	(3)
2.1	15 × 43 = 43 × 15 = 645	
2.2	27 × 36 = 36 × 27 = 972	
2.3	238 × 832 = 832 × 238 = 198 016	(3)
3.1	23 + (46 + 51) = 23 + 97 = 120; (23 + 46) + 51 = 69 + 51 = 120	
3.2	(34 + 65) + 89 = 99 + 89 = 188; 34 + (65 + 89) = 34 + 154 = 188	
3.3	324 + (465 + 333) = 324 + 798 = 1 122; (324 + 465) + 333 = 789 + 333 = 1 122	(3)
4.1	15 × (12 × 11) = 15 × 132 = 1 980; (15 × 12) × 11 = 180 × 11 = 1 980	
4.2	(23 × 32) × 41 = 736 × 41 = 30 176	
4.3	(251 × 152) × 10 = 38 152 × 10 = 381 520	(3)
5.1	23(32 + 41) = 23(73) = 23 × 73 = 1 679	
5.2	100(100 – 99) = 100(1) = 100 × 1 = 100	(4)
6.1	10; 40; 460; 2000	(5)
6.2	0; 0; 500; 700; 2000	(5)
6.3	0; 0; 0; 1000; 2000	(5)
7.1	Tom: R29 000; Thabo: R45 000; Jasmine: R52 000	(3)
7.2	29 000 + 45 000 + 52 000 = 126 000	(2)
7.3	28 999 + 45 299 + 51 870 = 126 168	(2)
7.4	126 168 – 126 000 = 168	(1)
7.5	Tom: $\frac{28\,999}{126\,168} \times 100 = 22{,}98\%$	
	Thabo: $\frac{45\,299}{126\,168} \times 100 = 35{,}9\%$	
	Jasmine: $\frac{51\,870}{126\,168} \times 100 = 41{,}11\%$	(6)
		[45]

Chapter 2 Exponents

Overview of concepts



Content		Time allocations	LB page
Unit 1	Squares and cubes	2 hours	47
Unit 2	Comparing and representing numbers in exponential form	2 hours	50
Unit 3	Calculations with numbers in exponential form	2 hours	53
Unit 4	Problem solving	3 hours	57

Background information on exponents

It is important that learners are familiar with the following information on exponents:

- 5^2 can be read as 5 to the power of 2 or 5 squared.
- 5^3 can be read as 5 to the power of 3 or 5 cubed.
- The number 5 is called the base and the small 2 and 3 are exponents or indices.
- 5^2 means 5×5 and it equals 25. 25 is a square number.
- 5^3 means $5 \times 5 \times 5$ and it equals 125. 125 is a cube number.
- Know square numbers to at least $12^{2.}$
- Know cube numbers to at least $12^{3.}$
- Know square roots. For example, $3 \times 3 = 3^2 = 9$ and $\sqrt{9} = 3$.
- Know cube roots. For example, $3 \times 3 \times 3 = 3^3 = 27$ and $\sqrt[3]{27} = 3$.

The first evidence we have of a square root can be found on the Yale Babylonian Collection YBC 7289 clay tablet, which was created between 1800 BC and 1600 BC. This tablet shows $\sqrt{2}$ and $30\sqrt{2}$ as 1;24,51,10 and 42;25,35. We have further evidence of the Egyptians extracting square roots from about 1650 BC.
The method for finding the cube of a number having many digits was developed by an Indian mathematician-astronomer Aryabhata, as long ago as 499 CE.

Generic teaching guidelines for teaching exponents

Remind learners of the following when introducing exponents:

- When a series of the same of number are added together they can be expressed as multiplication. For example, $3 + 3 + 3 + 3$ can be written 4×3.
- Multiplication can also be written in a shorter way. For example, $3 \times 3 \times 3 \times 3$ can be written 3^4. The 3 is called the base and the 4 the exponent. We can then say, 3 to the power of 4.
- When there are only 2 numbers, for example 439×439 we can write it 439^2. Here we can say 439 squared or to the power of 2.
- Exponents are important in writing square and cube numbers. A square, 5 m by 5 m is $5\text{ m} \times 5\text{ m} = 25\text{ m}^2$. A cube, $5\text{ m} \times 5\text{ m} \times 5\text{ m} = 125\text{ m}^3$.

Resources

Multiplication tables, blank number lines, number cards and comparison cards. Each learner should have their own calculator.

Unit 1 Squares and cubes

Learner's Book page 47

Unit focus

- revising square numbers and square roots learnt in Grade 6
- revising cube numbers and cube roots learnt in Grade 6.

Background information on squares and cubes

Ensure leaners are familiar with the following information on squares and cubes:

- Revise the squaring of natural numbers. For example, $1^2 = 1 \times 1 = 1$; $2^2 = 2 \times 2 = 4$; $3^2 = 3 \times 3 = 9$.
- Revise the cubing of natural numbers. For example, $1^3 = 1 \times 1 \times 1 = 1$; $2^3 = 2 \times 2 \times 2 = 8$; $3^3 = 3 \times 3 \times 3 = 27$; $4^3 = 4 \times 4 \times 4 = 64$.
- Square numbers are: 1; 4; 9; 16; 25; 36; 49; 64; 81; 100; ...
- Cube numbers are: 1; 8; 27; 64; 125; 216; 343; 512; ...
- Revise finding square roots. For example, what is the square root of 144? Find the prime factors of 144: $144 \div 2 = 72$ and $72 \div 2 = 36$
$36 \div 2 = 18$ and $18 \div 2 = 9$
$9 \div 3 = 3$ and $3 \div 3 = 1$
The prime factors of 144 are $2 \times 2 \times 2 \times 2 \times 3 \times 3$
Divided into 2 equal parts $2 \times 2 \times 3$ and $2 \times 2 \times 3$
12 and 12
12 is the square root of 144.
- The sign for square roots is $\sqrt{144} = 12$.
- The sign for cube roots is $\sqrt[3]{1} = 1$ $(1 \times 1 \times 1 = 1)$.

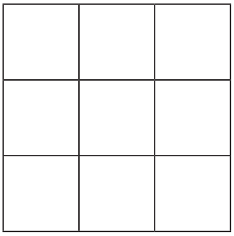
Exercise 1

Learner's Book page 48

Guidelines on how to implement this activity

Before tackling this activity revise square numbers. For example, $4 \times 4 = 4^2 = 16$ which is a square number. Use verbal exercises to practice saying square numbers.

Draw a square on the board and divide the sides each into 3 equal parts. Join the lines, thus dividing the square into 9 equal squares. Show learners that a square with sides of 3 units has an area (length × breadth) of $3 \times 3 = 3^2$ (3 squared) = 9 square units.



Let learners experiment with squares with sides of 2 units, 4 units and 5 units. Learners can work through this exercise in pairs.

Suggested answers

Number	Expanded notation	Exponential notation	Square number	Square root notation	Square root
1	1×1	1^2	1	$\sqrt{1}$	1
2	2×2	2^2	4	$\sqrt{4}$	2
3	3×3	3^2	9	$\sqrt{9}$	3
4	4×4	4^2	16	$\sqrt{16}$	4
5	5×5	5^2	25	$\sqrt{25}$	5
6	6×6	6^2	36	$\sqrt{36}$	6
7	7×7	7^2	49	$\sqrt{49}$	7
8	8×8	8^2	64	$\sqrt{64}$	8
9	9×9	9^2	81	$\sqrt{81}$	9
10	10×10	10^2	100	$\sqrt{100}$	10
11	11×11	11^2	121	$\sqrt{121}$	11
12	12×12	12^2	144	$\sqrt{144}$	12

Remedial

For learners struggling with square numbers, have them work through the following additional activities:

- List natural numbers up to 20, square numbers up to 12 squared and 12 cubed.
- Fill in missing numbers on number lines:

  $\longleftarrow$ 1 4 9 … … … 49 $\longrightarrow$

- What is: $\sqrt{4}$ $\sqrt{16}$ $\sqrt{100}$ $\sqrt{64}$

Extension

Learners can draw square posters and divide them into smaller squares. Let them then paint or colour the posters so that the square root is one colour. For example, if the square is divided into 5 × 5 squares equalling 25 squares, then 5 must be coloured or painted the same colour. These can be anywhere on the square, not necessarily all together. Then create black and white patterns on the rest of the squares. Use this as a background on which to glue cut out letters for a poster.

Exercise 2

Learner's Book page 49

Guidelines on how to implement this activity

- Revise cube numbers. For example, $4 \times 4 \times 4 = 4^3 = 64$ which is a cube number.
- Use verbal exercises to practice saying square and cube numbers.
- Discuss what the result would be if you had a cube with sides each 3 units long. A cube has length, breadth and height so you would have to say: $3 \times 3 \times 3 = 3^3$ (3 cubed) = 27 cube units.
- Let learners experiment with cubes with sides of 2 units, 4 units and 5 units.

Suggested answers

Number	Expanded notation	Exponential notation	Cubed number	Cube root notation	Cube root
1	$1 \times 1 \times 1$	1^3	1	$\sqrt[3]{1}$	1
2	$2 \times 2 \times 2$	2^3	8	$\sqrt[3]{8}$	2
3	$3 \times 3 \times 3$	3^3	27	$\sqrt[3]{27}$	3
4	$4 \times 4 \times 4$	4^3	64	$\sqrt[3]{64}$	4
5	$5 \times 5 \times 5$	5^3	125	$\sqrt[3]{125}$	5
6	$6 \times 6 \times 6$	6^3	216	$\sqrt[3]{216}$	6

Remedial

For learners struggling with cube numbers, provide the following additional activities:

- List natural numbers up to 20, and cube numbers up to 12 cubed.
- Fill in missing numbers on number lines:

  1 8 27 … … … 343

- What is: $\sqrt[3]{64}$ $\sqrt[3]{27}$ $\sqrt[3]{1\,000}$ $\sqrt[3]{8}$
- Allow learners to create number lines with square of cube numbers. They must then exchange with another learner and complete the partner's number line. Change back and mark the answers. Make sure you check this work carefully.

Extension

Encourage learners to work in groups to create work sheets and answer sheets based on square and cube numbers and square and cube roots. Distribute these worksheets around the class, so that no group has their own worksheet.

Unit 2 Comparing and representing numbers in exponential form

Learner's Book page 50

Unit focus

- writing numbers in exponential form
- comparing numbers in exponential form.

Background information on comparing and representing numbers in exponential form

Ensure learners are familiar with the following:

- Any number written in exponential form has a base number and an exponent or index number. For example, $7^4 = 7 \times 7 \times 7 \times 7$. 7 is the base and 4 is the exponent.
- Writing in exponential form: $1 \times 1 \times 1 \times 1 \times 1 \times 1 = 1^6$ and $2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$.

Revision exercise

Learner's Book page 50

Encourage learners to do this activity on their own and use it to assess leaners' prior knowledge before starting the rest of the unit.

Suggested answers

1 4; 16; 25; 49; 64; 81
2 8; 27; 64
3 2; 3; 16
4 2; 3; 64

Exercise 1

Learner's Book page 51

Guidelines on how to implement this activity

Provide numerous examples on the board, and plenty of practice of representing numbers in exponential form. Have learners come up to the board and write numbers in exponential and in expanded form. Learners should complete this activity on their own.

Suggested answers

1.1	5^4	**1.2**	3^3	**1.3**	10^5	**1.4**	23^3
1.5	4^1	**1.6**	4^5	**1.7**	47^6	**1.8**	$6^3 \times 7^3$
1.9	$3^4 \times 8^2$	**1.10**	45^1				

2.1 $50 \times 50 \times 50 \times 50$
2.2 $7 \times 7 \times 7 \times 7 \times 7$
2.3 23
2.4 $6 \times 6 \times 6$
2.5 $0 \times 0 \times 0 \times 0 \times 0$
2.6 $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$
2.7 $3 \times 3 \times 3 \times 3$
2.8 $5 \times 5 \times 5 \times 5 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

Remedial

For learners struggling with this activity revise the relationship between addition and multiplication. For example, $2 + 2 + 2 = 3 \times 2$. Give many simple examples for learners to say or write in exponential form. For example, 2×2; 3×3; 4×4; $2 \times 2 \times 2$; $5 \times 5 \times 5$. Remind learners that exponential numbers can be written: six squared; 6 to the power of 2; 6×6 or 6^2.

Exercise 2 Learner's Book page 52

Guidelines on how to implement this activity

Start by comparing the following numbers by writing them on the board:
5^2 and 3^3 $5^2 = 5 \times 5 = 25$ and $3^3 = 3 \times 3 \times 3 = 27$.
By multiplying these numbers out show learners how $5^2 < 3^3$.
Do additional examples until learners have grasped the concept. Encourage learners to complete this exercise on their own.

Suggested answers

1 >, since $5^3 = 125$ and $8^2 = 64$
2 <, since $3^3 = 27$ and $9^2 = 81$
3 >, since $6^2 = 36$ and $2^5 = 32$
4 <, since $10^2 = 100$ and $12 \times 10 = 120$
5 >, since $2 \times 4^2 = 2 \times 16 = 32$ and $3 \times 2^3 = 3 \times 8 = 24$
6 <, since $6^6 = 46\ 656$
7 =, since $4 \times 5^2 = 4 \times 25 = 100$ and $10^2 = 100$
8 <, since $4^1 = 4$ and $2^4 = 16$
9 <, since $1^{87} = 1$ and $87^1 = 87$
10 >, since $6^2 + 5^3 = 36 + 125 = 161$ and $2^6 + 3^2 = 64 + 9 = 73$

Remedial

Learners who do not know the times tables will not manage exponents. Make sure all learners know their multiplication tables.

Extension

Create a number pattern using exponential numbers. For example, they might start with a number squared. For example, $2^2 = 4$. Or a number cubed. For example, $2^3 = 8$. A number to the power of 4, for example even $2^4 = 16$ and so on.
The pattern is then 4; 8; 16; …
Draw a diagram to show the pattern:
* * * *
* * * * * * * *
* * * * * * * * * * * * * * * *
Use exponential numbers in a number puzzle. For example, make a number square with certain numbers. You could start with:

1	4	4

Unit 3 Calculations with numbers in exponential form

Learner's Book page 53

Unit focus

- add and subtract with exponents and roots
- multiply and divide with exponents and roots.

Background information on calculation with numbers in exponential form

Learners have not yet learnt the formal laws of exponents. When they calculate with exponents they need to write in expanded form and then find the values. Learners should only be working with numbers raised to powers at this stage when finding a finite value.

Revision exercise

Learner's Book page 53

Guidelines on how to implement this activity

Encourage learners to do this activity on their own and use it to assess leaners' prior knowledge before starting the rest of the unit

Suggested answers

1	1.1	**2**	2.2	**3**	3.3
4	4.1	**5**	5.3	**6**	6.1

Exercise 1

Learner's Book page 55

Guidelines on how to implement this activity

Before starting this exercise ensure learners can confidently work with squares, cubes and exponents. Work together as a class through the examples in the Learner's Book, and if necessary do additional examples until learners are confident with the material. Learners should complete this exercise on their own.

Suggested answers

1.1 1; 4; 9; 16; 25; 36; 49; 64; 81; 100

1.2 1; 8; 27; 64; 125

1.3 10; 100; 1 000; 10 000; 100 000; 1 000 000

2 Square numbers

3.1 $2^2 = 2 \times 2 = 4$ **3.2** $3^2 = 3 \times 3 = 9$ **3.3** $4 + 9 = 13$

4. $2^3 - 2^1 = 8 - 2 = 6$

5.1 $3^2 + 5^2 = 9 + 25 = 34$

5.2 $5^2 - 2^3 = 25 - 8 = 17$

5.3 $10^2 + 10^3 + 10^4 + 10^5 = 100 + 1\ 000 + 10\ 000 + 100\ 000 = 111\ 100$

5.4 $3^2 + 3^2 + 3^2 + 3^2 = 4 \times 3^2 = 4 \times 9 = 36$

5.5 $8^2 - (3^2 + 5^2) = 64 - (9 + 25) = 64 - 34 = 30$

6.1 $2^3 \times 2^3 = 2^{(3+3)} = 2^6 = 64$

6.2 $3^2 \times 3^3 = 3^{(2+3)} = 3^5 = 243$

6.3 $5^3 \times 5^2 = 5^{(3+2)} = 5^5 = 312$

6.4 $2^5 \div 2^2 = 2^{(5-2)} = 2^3 = 8$

6.5 $3^5 \div 3^4 = 3^{(5-4)} = 3^1 = 3$

6.6 $5^5 \div 5^2 = 5^{(5-2)} = 5^3 = 125$

6.7 $9^2 \div 3^2 = 81 \div 9 = 9$

6.8 $2^2 \times 6^2 = 4 \times 36 =144$

6.9 $2^3 \times 3^2 = 8 \times 9 = 72$

6.10 $2^2 + 6^2 \div 3^2 = 4 + 36 \div 9 = 4 + 4 = 8$

6.11 $3^3 \times 4^2 - 5^3 = 9 \times 16 - 125 = 144 - 125 = 19$

6.12 $\sqrt{25} - \sqrt[3]{8} = 5 - 2 = 3$

7.1 Five times ten squared plus two times ten cubed.
$(5 \times 10^2) + (2 \times 10^3) = (5 \times 100) + (2 \times 1\ 000) = 500 + 1\ 000 = 1\ 500$

7.2 Seven times two to the power of five minus two times ten squared.
$(7 \times 2^5) - (2 \times 10^2) = (7 \times 32) - (2 \times 100) = 224 - 200 = 24$

7.3 Four times ten to the power of six plus three times ten to the power of five plus seven times ten to the power of four plus six times ten squared.
$(4 \times 10^6) + (3 \times 10^5) + (7 \times 10^4) + (6 \times 10^2) = (4 \times 1\ 000\ 000) + (3 \times 100\ 000) + (7 \times 10\ 000) + (6 \times 100) = 4\ 000\ 000 + 300\ 000 + 70\ 000 + 600 = 4\ 370\ 600$

8.1 25; 36

8.2 Square numbers

8.3 Area = 3 cm × 3 cm = 9 cm^2

9 $\sqrt{1} = 1$; $\sqrt{4} = 2$; $\sqrt{9} = 3$; $\sqrt{16} = 4$; $\sqrt{25} = 5$; $\sqrt{36} = 6$; $\sqrt{49} = 7$

10.1 We know that $\sqrt{9} = 3$, hence 3 would be the closest number to $\sqrt{10}$.

10.2 We know that $\sqrt{36} = 6$, hence 6 would be the closest number to $\sqrt{35}$.

11.1 $(3^2)^2 = 9^2 = 81$; $(3^2)^2 = 3^2 \times 3^2 = 9 \times 9 = 81$

11.2 $3(3^3)^2 = 3 \times 27^2 = 3 \times 729 = 2\ 187$

11.3 $(4^4)^4 = 256^4 = 4\ 294\ 967\ 296$; $(4^4)^4 = 4^4 \times 4^4 \times 4^4 \times 4^4 = 4^{16} = 4\ 294\ 967\ 296$

12 12.2

13 125; 216; 343

14 Cube numbers

15 $1^3 = 1$; $2^3 = 8$; $3^3 = 27$

16 $7^3 = 7 \times 7 \times 7 = 343$; $8^3 = 8 \times 8 \times 8 = 512$

Remedial

If learners are experiencing difficulty, do additional examples of calculations using exponential numbers. For example, $2^2 \times 2^3 = 2^{2+3} = 2^5 = 2 \times 2 \times 2 \times 2 \times 2$ and $(2 \times 2 \times 2 \times 2) \div (2 \times 2 \times 2) = 16 \div 8 = 2$. Try $3^4 \div 3^2$ and $2^3 \div 2^2$ and $3^3 \div 3^2$ and $4^3 \div 4^{2.}$

Extension

Encourage learners who are confident about this concept to experiment with bigger exponents. For example, 2 to the power of 7, to the power of 8, to the power of 9. What happens to the numbers in each case? Try multiplication of numbers with bigger bases. For example, $10^2 \times 10^3$.
Try division of numbers with bigger exponents and with bigger bases.

Unit 4 Problem solving

Learner's Book page 57

Unit focus

- revise operations with exponents
- solve problems with exponents.

Background information on problem solving

Learners have been solving problems since the Foundation Phase. The focus in Grade 7 is to start formalising the process and using algebra and variables to help make the problems more manageable. At this early stage in Grade 7, and with the introduction of exponents as a concept, focus on how exponents are used in everyday life and strategies for tackling them.

Revision exercise

Learner's Book page 57

Guidelines on how to implement this activity

Remind learners that the solution to problem solving lies in reading the question carefully. Learners must concentrate on what information they are given in the problem, and what they are asked to find. Inform learners that problems using exponential numbers need the same approach. Allow learners to tackle these problems to see how they tackle problems with an exponential component.

Suggested answers

1.1 Cubic capacity: 2 cm ×2 cm × 2 cm = 8 cm^3

1.2 Total capacity: 5 × 8 cm^3 = 40 cm^3

2.1 4^2

2.2 4

2.3 2

2.4 $4^2 = 4 \times 4 = 16$

2.5 $\sqrt{16} = 4$

Exercise 1

Learner's Book page 58

Guidelines on how to implement this activity

Provide the following steps for learners when problem solving:

- Read the question carefully.
- What information have you been given?
- What question have you been asked?
- In which unit will the answer be?

Make sure there is an understanding of exponential numbers before starting this activity.
Use the following example:
Mary has 2^2 pairs of school socks. Martin has $\sqrt{25}$ pairs. Who has more pairs?
If the socks cost R8,00 a pair what will they cost altogether?
Help learners to identify the information given: Mary has 22 pairs. Martin has $\sqrt{25}$ pairs. They cost R8 a pair; and what they have been asked to find: Who has more pairs? Unit of answer – pairs of socks, and what will they cost? Unit of answer – Rand.

Suggested answers

1.1 $10^7 + 7^6 = 10\,000\,000 + 117\,649 = 10\,117\,649$

1.2 $(10^7 + 7^6) - 7^8 = 10\,117\,649 - 5\,764\,801 = 4\,352\,848$

2 117 – ? = "a perfect square"
We know that $9^2 = 81$ and $10^2 = 100$. Therefore, the smallest possible whole number must be 17, because 117 – 17 = 100 (100 is a perfect square).

3.1 $21^2 = 21 \times 21 = 441$; $\sqrt{250\,000} = 500$. Therefore, Megan bought more bottles.

3.2 $4 \times (21^2 + \sqrt{25\,000}) = 4 \times (441 + 500) = 4 \times 941 = 3\,764$

4.1 10 × 30 = 300; It would take her 300 minutes (5 hours).

4.2 1 m = 100 cm; 1 m × 1 m = 100 cm × 100 cm = 10 000 cm^2
Number of squares = $\frac{10\,000}{5 \times 5} = 400$

4.3 30 × 400 = 12 000; It would take her 12 000 minutes (200 hours).

5 $10^6 - 5^8 = 1\,000\,000 - 390\,625 = 609\,375$

6 Jabu should take 3^4.
For example, $4^3 = 4 \times 4 \times 4 = 64$; $3^4 = 3 \times 3 \times 3 \times 3 = 81$

7.1 $10^6 = 1\,000\,000$; $20^5 = 3\,200\,000$
The reptilian exhibit should come first, since it is 20^5 years old, and the plant life exhibit second.

7.2 $20^5 - 10^6 = 3\,200\,000 - 1\,000\,000 = 2\,200\,000$

Remedial

For learners experiencing any difficulties with this problem solving exercise provide some additional simple examples of problems. For example, is 22 < or > 23?
Which number is closer to 100, 92 or 112 ?
Ann has 23 marbles and Tom has 32 marbles. Who has more marbles?
Allow learners to work in pairs to solve problems.Try to team up learners of mixed abilities. Encourage learners to discuss the problems and to explain their reasoning and methods.

Extension

- Let learners work in pairs to create a 'problem' chart. Start by choosing a theme. For example, a picnic, a trip, a day at school etc. Next work out about 4 problems related to the theme and to exponential numbers. Write and illustrate the problems on the chart to make a wall poster. Decorate the class with posters.
- Encourage some learners to start looking at the relationship of the Earth, sun, moon and so on. Let them note the distances between planets in the solar system.
- Encourage some learners to work out the square root of the year they were born, such as $\sqrt{2\,000}$ or to square or cube their ages, such as 12^2 or 12^3

Consolidation

Learner's Book page 61

Before doing this consolidation exercise, encourage learners to review the work covered in this chapter. Advise learners to use the summary and to revise their work. This exercise can be used as an informal assessment task for you to track how learners are coping with the chapter and the concepts covered.

Suggested answers

1.1	10^5	(1)
1.2	$10 \times 10 \times 10 = 1\ 000$	(1)
1.3	$2 \times 2 \times 2 \times 2 \times 2 = 32$	(1)
2.1	10^5	(1)
2.2	$100\ 000$	(1)
2.3	3^4	(1)
2.4	81	(1)
3.1	$6^3 = 6 \times 6 \times 6 = 216$	(1)
3.2	$3^4 = 3 \times 3 \times 3 \times 3 = 81$	(1)
3.3	$7^3 = 7 \times 7 \times 7 = 343$	(1)
4.1	$3^2 + (8^2 - 2^3) \times 4^3 = 9 + (64 - 8) \times 64 = 9 + 56 \times 64 = 9 + 3\ 584 = 3\ 593$	(2)
4.2	$10^6 - (10^2 - 10^3) \div 10^2 = 1\ 000\ 000 - (100 - 1\ 000) \div 100 = 1\ 000\ 000 - (-900) \div 100 = 1\ 000\ 000 - (-9) = 1\ 000\ 009$	(2)
5.1	$16 + 25 = 41$	(1)
5.2	$36 - 27 = 9$	(1)
5.3	$64 \div 16 = 4$	(1)
5.4	$9 \times 25 = 225$	(1)
5.5	$100 + 1\ 000 + 100\ 000 + 1\ 000\ 000 = 1\ 101\ 100$	(1)
5.6	$4 \times 2^2 = 4 \times 4 = 16$	(1)
6.1	$21^2 = 21 \times 21 = 441$	(2)
6.2	$2 \times 441 = 882$	(2)
6.3	$\sqrt{10\ 000} + \sqrt{100} + \sqrt{81} + \sqrt{144} = 100 + 10 + 9 + 12 = 131$	(2)
6.4	$441 - 131 = 310$	(2)
6.5	$3 \times 441 - 882 = 1\ 323 - 882 = 441$	(3)
7.1	$2 \times 2 = 4$	(1)
7.2	2^2	(1)
7.3	$2 \times 2 \times 2 = 8$	(1)
7.4	2^3	(1)

[35]

Chapter 3 Construction of geometric figures

Overview of concepts

Construction of geometric figures

UNIT 3 Parallel and perpendicular lines

UNIT 1 Drawing and measuring angles

UNIT 2 Constructing 2D shapes

	Content	Time allocations	LB page
Unit 1	Drawing and measuring angles	3 hours	63
Unit 2	Constructing 2D shapes	4 hours	69
Unit 3	Parallel and perpendicular lines	3 hours	75

Background information on constructions

Learners have done some construction in Grade 6, however it is a relatively new concept to learners. Learners need to appreciate the necessity of construction and its use for investigating properties of angles, lines and shapes. Learners should have the necessary equipment with them: a protractor, set square, ruler and a sharp pencil.

In order to place constructions in context, show learners pictures of ancient buildings like the pyramids or Acropolis. Use these buildings to demonstrate that ancient Egyptians and Greeks worked with angles in order to build such geometrically proportional buildings. Discuss why accurate measurement is important in creating buildings with architectural integrity, which are pleasing to the eye.

It is also relevant to note that the ancients in various parts of the world recognised that the sun followed a circular path, which took about 360 days. It is thought that this is why a circle is divided into 360 degrees.

Generic teaching guidelines for teaching constructions

Ensure that learners have maths sets and that they are familiar with using compasses, rulers, protractors and so on. Allow learners to experiment by drawing a variety of shapes with their geometry sets. Focus learners' constructions by asking learners to design a space ship using geometric shapes.

Play a game recognising right angles in the classroom. For example, corners of books, paper and doors. Count the number of angles found. Always try to integrate counting with other games. Use the protractor to measure acute angles in the classroom. For example, letters of the alphabet on a chart, hands of a clock and pages of books.

Resources

Each learner should have a pair of compasses, a protractor, a ruler, a set square and a sharp pencil. Pictures of Ancient Egyptian pyramids, cardboard and colour pens for learners to decorate their constructions. A board size protractor and ruler to show examples on the board. Each learner should have their own calculator.

Unit 1 Drawing and measuring angles

Learner’s Book page 63

Unit focus

- revise types of angles
- practise using geometry implements accurately
- construct and measure angles.

Background information on drawing and measuring angles

Learners need to be able to define and recognise the following angles:

- An acute angle is less than 90°
- A right angle is 90°
- An obtuse angle is more than 90° and less than 180°
- A straight angle is 180°
- A reflex angle is more than 180° and less than 360°
- A revolution is 360°

Exercise 1

Learner’s Book page 65

Guidelines on how to implement this activity

Explain that an angle is formed where 2 straight lines cut. The point where the lines meet is called the vertex. Angles are named using letters of the alphabet. Let's examine the angle $X\hat{Y}Z$. The vertex is at Y. The lines are XY and YZ. Show learners a variety of angles and let learners say whether they are acute, right, obtuse, reflex and so on. Discuss where we might find different angles and identify different sized angles in the classroom.

Revise how to measure angles using a protractor. Discuss how to hold the protractor accurately, how to position it on the line of the angle, and which scale on the protractor to use. Have two learners come up to work on the board and draw a freehand angle of about 45°. Ask another 2 learners to measure the angles drawn to see which is closer to the required 45°. Continue with a variety of angles. Learners should complete this exercise on their own.

Suggested answers

1 $A\hat{B}C$; $D\hat{E}F$; $G\hat{H}I$; $J\hat{K}L$; $M\hat{N}O$; $P\hat{Q}R$; $S\hat{T}U$; $V\hat{W}X$

2 acute angle; right angle; straight angle; reflex angle; obtuse angle; acute angle; acute angle; obtuse angle

3 $A\hat{B}C = 25°$; $D\hat{E}F = 90°$; $G\hat{H}I = 180°$; $J\hat{K}L = 250°$; $M\hat{N}O = 100°$; $P\hat{Q}R = 60°$; $S\hat{T}U = 45°$; $V\hat{W}X = 160°$

Remedial

Learners may require hands-on assistance and guidance as to how to measure using their protractor. Walk around the class and observe learners as they measure. Check that learners are doing it correctly and step in and help where necessary.
Encourage informal estimation of angles in the classroom. For example, write down 5 places where you see right angles; 5 places where you see acute angles.

Exercise 2

Learner's Book page 66

Guidelines on how to implement this activity

When drawing angles it is important that learners work neatly and have a sharp pencil. Practice placing the protractor accurately and marking and drawing angles of given sizes. Show learners how to place their protractor correctly and to measure off the necessary angle. Do a few examples on the board if you have a board sized protractor, or demonstrate to small groups at a time on paper.

Suggested answers

1 – 7 Learners' own constructions

8 acute angle; obtuse angle; acute angle; obtuse angle; obtuse angle

Extension

Have learners work in pairs. Learners take turns deciding on the size of the angle and drawing the required angle.

Exercise 3

Learner's Book page 67

Guidelines on how to implement this activity

This exercise brings together the skills covered in this unit. Learners to put together what they know about angles in estimating the size of angles you draw on the board. For example, is this an acute, obtuse, right angle? What do you think it measures? Measure the angle under discussion to see how accurate estimates are.

Allow learners to draw carefully measured lines as described in the last lesson. Mark in a vertex. Measure the base line. Mark in the size of the angle. Draw the angle. Ask learners to investigate if the length of the arms influence the size of the angle. Draw a 40° angle with arms 3 cm long. Increase the length of the arms to 5 cm. Measure the angle. It still measures 40°. Learners should complete this exercise on their own.

Suggested answers

1.1 obtuse angle
1.2 right angle
1.3 b
2

Angle	Kind	Estimated size	Measured size
$A\hat{B}C$	obtuse		120°
$C\hat{D}E$	straight		180°
$F\hat{G}H$	acute		45°
$J\hat{K}L$	reflex		230°

3 Learners' own constructions
4.1 straight angle
4.2 right angle
4.3 reflex angle
4.4 full rotation
5 – 7 Learners' own constructions

Remedial

Observe learners as they construct the angles. Ensure learners are measuring correctly with their protractor. Provide additional angles for learners to draw, if any learners are having problems.

Unit 2 Constructing 2D shapes

Learner's Book page 69

Unit focus

- construct accurate straight lines
- construct shapes using your knowledge of lines of angles
- construct circles using a ruler, pencil and a pair of compasses.

Background information on constructing 2D shapes

Learners should have mastered drawing angles correctly. In this unit learners need to be able to draw accurate constructions of 2D shapes. This involves being able to draw accurate angles and line segments. Learners need to be able to use a ruler and a compass to draw accurate line segments.

Learners need to be able to hold and manipulate a pair of compasses correctly. Ensure learners have enough time to adequately learn and master the skill. Have learners practise measuring set lengths of their compass against a ruler.

When working with circles it is important learners know the following terminology associated with circles: The perimeter of a circle is called the circumference. A straight line passing through the centre of a circle and touching the circumference on both sides is called the diameter. Half the diameter, or the straight line from the centre to the perimeter is called the radius. The diameter is 2 times the length of the radius.

Exercise 1

Learner's Book page 70

Guidelines on how to implement this activity

Learners are taught how to draw accurate straight lines using a compass and a ruler. Practice drawing lines of specific length. For example, draw a line 3 cm long, 5 cm long using a ruler. Then explain how to use a compass to draw a line of a specific length. Draw a slightly longer than required horizontal line. Mark point Y. Put the compass point at 0 on the ruler and open compasses to required length. Put the compass point at Y. Draw an arc to cut the line at Z. Work through the example in the Leaner's Book, and then do additional examples together as a class. Ensure learners hold their compass correctly.

Suggested answers

1 Learners' own constructions

2.1 5,5 cm **2.2** 9,1 cm **2.3** 7,9 cm

Remedial

Provide additional examples until you are satisfied that all learners can draw a line segment accurately.

Exercise 2

Learner's Book page 72

Guidelines on how to implement this activity

Learners use their knowledge and skill for constructing angles and line segments by constructing triangles. Learners need to be guided through the process by means of a worked example. Show learners how the skills need to be integrated. Do a few examples together as a class, and then provide additional examples for learners to do in class. Observe learners as they construct the triangles. Watch out for any errors. Remind learners to always have a sharp pencil, and that in construction it is necessary to work accurately (to the nearest mm) and neatly.

Suggested answers

1 – 3 Learners' own constructions

Extension

Learners can experiment drawing different triangles:

- triangles with the same angles but different length sides
- triangles with the same lengths, but different angles
- triangles with one angle the same but 2 sides the same length.
  Encourage learners to create geometric art. Show them prints of art works by Cubists like Braque, Leger and Picasso. Then ask them to create paintings based on geometric shapes.

Exercise 3 Learner's Book page 73

Guidelines on how to implement this activity

Introduce circles to learners. Ask learners to identify real life examples of circles. Define the parts of a circle with learners. Learners must be able to recognise and identify these parts. Focus on the radius. Show learners that we use the radius to draw circles. Show how we use a compass against a ruler and measure up the length of the radius. Demonstrate to learners how we draw a circle using a compass. Encourage learners to draw a variety of circles with varying radii.

Suggested answers

1 - 2 Learners' own constructions
3.1 The third circle **3.2** The first circle
3.3 The third circle **3.4** The first circle
3.5 The circle size increases with the increase in the length of the radius.
4.1 6 cm
4.2 The diameter is bigger than the radius. It is twice as big as the radius.
5 Circle A
Circle C
Circle B
6.1 2,5 cm **6.2** 1,25 cm **6.3** 1,7 cm

Remedial

Ensure learners are holding the compass correctly when drawing circles. Provide hands on additional help for learners who may be experiencing difficulties.
Tell learners to make a dot as the centre of the circle.
Do many practice runs to ensure:

- The point of the compass stays in the same place on the marked dot.
- The compass arms remain at the same angle.
- The pencil is firmly in the holder and does not slide.

Extension

Ask learners to draw house plans which include circular, triangular and rectangular rooms. All angles must be accurately measured. The 'plan' can then be coloured and turned into an art work.

Unit 3 Parallel and perpendicular lines

Learner's Book page 75

Unit focus

- learn about parallel lines
- learn how to construct parallel lines
- learn about perpendicular lines
- learn how to bisect line segments
- learn how to construct perpendicular lines.

Background information on parallel and perpendicular lines

Lines are basic to all geometric constructions. There are two special types of lines that are integral to geometry. These are parallel and perpendicular lines. Learners need to be able to define them, recognise them and know the correct notation for parallel and perpendicular lines. Parallel lines and perpendicular lines are used in architecture. Demonstrate the importance of these lines on appearance and stability of structure.

Revision exercise

Learner's Book page 75

Guidelines on how to implement this activity

Encourage learners to do this activity on their own to assess their understanding of the work covered so far.

Suggested answers

1 Learners' own constructions
2.1 Radius of A = 1,25 cm
2.2 Radius of B = 0,8 cm
2.3 Radius of C = 1,9 cm
3 Length of AB = 4 cm; Length of CD = 3,5 cm; Length of FG = 5,2 cm
4 Learners' own constructions

Remedial

If learners experience difficulty, provide remedial activities to help learners understand the necessary work.

Exercise 1

Learner's Book page 77

Guidelines on how to implement this activity

Discuss the concept of parallel lines. Have a picture of a railway track on the board to help learners visualise the idea of two lines never touching or getting any nearer together. Ask learners to identify other examples of parallel lines in nature or construction. Work through the example on how to draw parallel lines in the Learner's Book. Learners should attempt this exercise on their own.

Suggested answers

1-4 Learners' own constructions and answers
5 diagram B

Extension

Learners can create interesting artwork using only parallel lines. Assign learners to groups. Let them create posters to put up in the classroom.

Exercise 2 Learner's Book page 80

Guidelines on how to implement this activity

Show learners an example of lines meeting at 90°. Inform learners that we call lines that meet at 90° perpendicular. Ask learners to identify perpendicular lines in the classroom. Encourage learners to measure possible right angles with their protractor or set square to test if they really are right angles.
Show learners how to draw perpendicular lines, and a perpendicular bisector for a line. Work carefully through the method. Provide additional examples for learners to work through. Observe learners as they complete the construction, provide assistance if necessary. Learners should complete this exercise on their own.

Suggested answers

1.4 PX = XR = 4 cm
1.5 $P\hat{X}M = M\hat{X}R = P\hat{X}N = N\hat{X}R = 90°$
2.2 AY = YB = 4,8 cm
2.3 $A\hat{Y}C = B\hat{Y}C = A\hat{Y}D = B\hat{Y}D = 90°$
3.3 $A\hat{R}P = P\hat{R}B = 90°$; $A\hat{R}B = 180°$

Remedial

Learners may need additional assistance and guidance with constructions. Attend to individual needs as they arise.

Extension

Encourage learners to use their geometry equipment as much as possible to practise working with these tools. Always encourage neatness and accuracy.

Consolidation Learner's Book page 82

Before doing this consolidation exercise, encourage learners to review the work covered in this chapter. Advise learners to use the summary and to revise their work. This exercise can be used as an informal assessment task for you to track how learners are coping with the chapter and the concepts covered.

Suggested answers

1.1	AC, DC and CB are radii	(3)
1.2	Diameter	(1)
1.3	AB is an arc	(1)
1.4	Circumference	(1)
1.5	The turn or rotation from point A to point B on the circle.	(2)
2	Bisecting means the two lines intersect at their midpoints.	(3)
3 – 4	Learners' own constructions	(5)
5	Perpendicular: AD ⊥ CD; DC ⊥ BC; AB ⊥ CB; BA ⊥ DA; PS ⊥ RS; SR ⊥ QR; RQ ⊥ PQ; QP ⊥ SP; MN ⊥ NO Parallel: AB || CD; AD || BC; PQ || RS; PS || QR	(4)
		[20]

Chapter 4 Geometry of lines and 2D shapes

Overview of concepts

Geometry of lines and 2D shapes

UNIT 1 Geometry of straight lines

UNIT 2 Triangles

UNIT 3 Quadrilaterals and circles

UNIT 4 Similar and congruent figures

Content		Time allocations	LB page
Unit 1	Geometry of straight lines	2 hours	84
Unit 2	Triangles	3,5 hours	87
Unit 3	Quadrilaterals and circles	3,5 hours	97
Unit 4	Similar and congruent figures	3 hours	107

Background information on geometry of lines and 2D shapes

Learners have explored 2D shapes by means of construction in Chapter 3. This chapter explores the properties of 2D shapes in a more theoretical way. Learners need to know these properties for further work with geometry in mathematics. This is the learners' first introduction to formal geometry and it is recommended that you spend time working through the properties and explain how to set out geometry problems in much detail.

Generic teaching guidelines for teaching geometry of lines and 2D shapes

Ensure that learners are confident with geometric equipment, that they measure lines and angles accurately and that they understand instructions and demonstrations. Discuss triangles (triangle means 3 angles) according to their angles. For example, right angled triangle.

Encourage learners to draw and cut out triangles, squares and rectangles. Test which shapes have right angles, which have acute or obtuse angles, which have sides equal and so on. Play with shapes. Encourage learners to devise a game with the shapes they have cut out. In ancient Greece, tessellated tiles, glass and stones were used to make mosaics that still exist today. Get learners to colour the shapes they have cut out and try to tessellate some of them to build a pattern.

Resources

Each learner should have a pair of compasses, a protractor, a ruler, a set square and a sharp pencil. A board size protractor and ruler to show examples on the board. Each learner should have their own calculator. Have card and colour pens available for learners to make posters of the properties of triangles and put them up around the class.

Unit 1 Geometry of straight lines

Learner's Book page 84

Unit focus

- define a line segment
- learn about rays
- learn more about straight lines
- define parallel lines
- define perpendicular lines

Background information on geometry of straight lines

Learners have worked with lines before, and in the previous chapter have even worked with parallel and perpendicular lines. This unit serves to help lock down the knowledge learners already have about lines and consolidate it by means of formal definitions for the concepts of straight lines. It is important that learners understand and learn these formal definitions and are able to distinguish between lines, line segments and rays. Notation is very important with all forms of geometry so ensure learners are familiar and able to work with the correct notation when working with straight lines. This includes the symbols for parallel and perpendicular lines.

Exercise 1

Learner's Book page 86

Guidelines on how to implement this activity

Spend time asking learners about lines around them. Encourage learners to identify the different types of lines they can see in the class room. Encourage learners to reflect on any special types of lines they worked with in Chapter 7, namely parallel and perpendicular lines. Discuss the difference between a line, a line segment and a ray. Use the notes from the Learner's Book to help you. Have learners write the formal definition of each, line, line segment and ray, in their exercise books, and to supply a sketch for each type of line to help them identify these lines.
Discuss parallel and perpendicular lines. Revise the properties they learnt from Chapter 7. Have learners identify these lines in the classroom. Discuss how to identify these lines in geometry diagrams. Do a few examples together on the board as a class. Learners do part of this exercise together in pairs, namely identifying the line types in the class, and the rest on their own.

Suggested answers

1 Learners answers will differ, but should be in a table such as below.

Lines in my classroom	Example 1	Example 2	Example 3
Straight lines			
Curved lines			
Parallel lines			
Perpendicular lines			

2.1 False. Perpendicular lines meet one another at 90°.
2.2 True
2.3 False. Parallel lines are always straight lines.
2.4 False. A ray is a line with a definite starting point and no end point.
2.5 False. A line is made up of small points, very close together.
3.1 DE ∥ FG **3.2** JK ∥ YZ **3.3** AC ∥ DF
4.1 AD ⊥ CF **4.2** JM ⊥ KN
4.3 EC ⊥ BG and FI ⊥ BG

Unit 2 Triangles

Learner's Book page 87

Unit focus

- describe triangles according to their angles and sides
- learn the basic properties of different types of triangles
- learn different types of triangles
- classify triangles according to their angles and sides.

Background information on triangles

Learners should know the following information on triangles:

- A right angled triangle has 1 angle = 90°.
- An acute angled triangle has 3 angles all less than 90°.
- An obtuse angled triangle has 1 angle > 90°.
- How to sketch each type of triangle.
- Equilateral triangles have 3 equal sides.
- Scalene triangles have all sides unequal.
- Isosceles triangles have 2 sides equal.

Revision exercise

Learner's Book page 87

Guidelines on how to implement this activity

Revise with learners how to construct triangles. Revise how to measure lines and angles. Work through this first investigation with the learners. Ensure learners reach the desired observations: namely that this triangle has a right angle, and that the interior angles add up to 180°.

Suggested answers

1 - 4 Learners' own constructions

5 right angle **6** 90° **7** ≈ 37°

8 90° + 37° + 53° = 180°

Remedial

If learners struggle to construct a triangle, send additional time revising how to construct. Revise using the compass, ruler and protractor.

Extension

Learners can create additional triangles and tessellate them to create a pattern.

Investigation 1

Learner's Book page 88

Guidelines on how to implement this activity

Encourage learners to do this investigation on their own. Observe learners as they do the constructions. Provide assistance and guidance. Ensure learners are measuring correctly. Once learners have completed the investigation ask learners to share what they found. Summarise that learners should have found that all sides were equal, all angles were equal, and that all the angles inside the triangle added up to 180. If learners did not reach these observations investigate where they went wrong and encourage them to try the investigation again. Show learners how to identify equilateral triangles from a sketch.

Suggested answers

1 - 6 Learners' own constructions

7 PQ = 5 cm; PR = 5 cm; QR = 5 cm

8 The line segments all have the same length

9 $P\hat{Q}R = 60°$; $Q\hat{R}P = 60°$; $R\hat{P}Q = 60°$

10 All the angles have the same size

11 60° + 60° + 60° = 180°

Remedial

Learners might struggle with manipulation of the geometry tools. Provide assistance as required. Remind learners to work neatly, with a sharp pencil, in order for their constructions to be accurate.

Extension

Prescribe the challenge activity for learners.

Challenge

Learner's Book page 88

Guidelines on how to implement this activity

Remind learners to work neatly and accurately when they draw a triangle with three 60° angles.

Suggested answers
Yes, this is true in general.

Investigation 2 Learner's Book page 89

Guidelines on how to implement this activity

Learners should also attempt this investigation on their own.
Be available to assist where necessary. Have learners discuss their findings in groups and then present them to the class. Consolidate the findings by ensuring that all learners determined that for an isosceles triangle two sides are equal, two of the angles are equal and all the interior angles add up to 180°. Show learners how to identify isosceles triangles from a sketch.

Suggested answers

1-6	Learners' own constructions
7	DE = 6 cm; EF = 4,5 cm; DF = 6 cm
8	Sides DE and DF have the same length
9	$D\hat{E}F \approx 68°$; $E\hat{F}D \approx 68°$; $F\hat{D}E = 44°$
10	$D\hat{E}F$ and $E\hat{F}D$ have the same size
11	68° + 68° + 44° = 180°

Remedial

If learners did not make the correct observations, encourage them to redraw the triangle using a sharp pencil and working accurately. Help learners measure the sides and angles and prove the observations for themselves. It is important that learners develop these properties of triangles for themselves as it helps make the properties more concrete for the learner.

Extension

Assign learners who were able to easily observe the properties the challenge exercise.

Challenge Learner's Book page 89

Guidelines on how to implement this activity

Arrange learners in pairs and have them work together to prove or disprove this statement. Learners must provide at least five examples showing that this hypothesis is mostly correct.

Suggested answers
Yes, this is true in general.

Investigation 3 Learner's Book page 90

Guidelines on how to implement this activity

In this exercise learners do not do the construction themselves, but measure the given angles. Learners must determine that no matter what a triangle looks like, its interior angles will always add up to 180°.

Ask learners whether they think its possible for all triangles to have interior angles of 180° or not. Learners should complete this exercise on their own.

Suggested answers

1 26°; 37°; 117° (26° + 37° + 117° = 180°)
2 45°; 62°; 73° (45° + 62° + 73° = 180°)
3 18°; 21°; 141° (18° + 21° + 141° = 180°)
4 30°; 42°; 108° (30° + 42° + 108° = 180°)
5 In every triangle, the three angles add up to 180°.

Remedial

Ensure learners see that all triangles will have interior angles that add up to 180°. Have learners copy the table from the Learner's Book that summarises the properties of triagles. Encourage learners to memorise these properties.

Exercise 1

Learner's Book page 91

Guidelines on how to implement this activity

Discuss with learners how you would identify the various triangles. How would you know when sides are equal? How would you know when angles are equal? Discuss notation and how it can be shown on a sketch. Have learners come up to the board and demonstrate. Do examples of identifying different triangles together as a class. Revise constructing triangles, and how knowing the type of triangle required can help us in drawing the triangle. Learners should complete this exercise on their own.

Suggested answers

1 A: equilateral triangle
B: right angle triangle
C: scalene triangle
D: isosceles triangle
E: scalene triangle
2 Learners' own constructions

Extension

Have learners create posters for the classroom of the different types of triangles and their properties. Encourage learners to make the poster as bright and interesting as possible.

Exercise 2

Learner's Book page 96

Guidelines on how to implement this activity

This is the first time learners will be using the properties of a figure to help find missing sides and angles. Revise identifying different triangles and discussing how knowing the properties can help us. Work through all the examples in the Learner's Book. Initially focus on finding missing sides using the properties, before progressing to finding missing angles using the properties. Remind learners that these diagrams are only sketches and the dimensions provided are not accurate.

When working through the examples be sure to explain to learners how to set out their thinking and providing a reason for their thinking. Explain how we can use the problem solving steps here. Show learners how you identify what is being asked – mark this on the diagram. Then identify the information that is given. Create a number sentence or equation to solve the problem. Substitute in the necessary values and then solve. Remind learners to check their answer by assessing the reasonableness of their answer.

Suggested answers

1 BA = 74 mm; $B\hat{C}A = 48°$; $A\hat{B}C = 180° - 48° - 48° = 84°$

2 DE = 9 cm; $E\hat{D}F = E\hat{F}D = (180° - 90°) \div 2 = 45°$

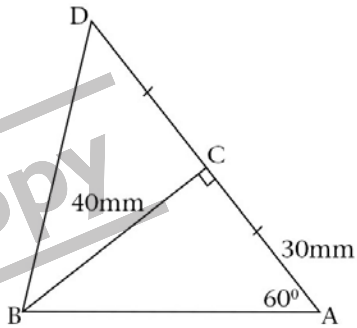
3 XY = YZ = 4 cm; $X\hat{Z}Y = 180° - 60° - 60° = 60°$

4 $Q\hat{P}R = 180° - 35° - 14° = 131°$

5 $J\hat{M}K = J\hat{K}M = M\hat{J}K = 60°$; MJ = JK = MK = 3 cm; $K\hat{M}L = K\hat{L}M = (180° - 90°) \div 2 = 45°$

6 AC = CD = 30 mm; $B\hat{C}A = 90°$; CB = 40 mm

Find: AB; $C\hat{B}A$; $D\hat{C}B$; DB; $B\hat{D}A$; $D\hat{B}C$; $D\hat{B}A$



$AB = \sqrt{BC^2 + AC^2}$

$AB = \sqrt{40^2 + 30^2}$

$AB = \sqrt{1600 + 900} = \sqrt{2500} = 50$ mm

(Pythagoras's Theorem)

$C\hat{B}A=180°-(B\hat{C}A+C\hat{A}B)$

$C\hat{B}A=180°-(90°+60°)=30°$ (Sum $\angle$'s Δ)

$D\hat{C}B=180-90°=90°$ ($\angle$'s on a straight line)

DB= √(BC^2+DC^2)

$DB= \sqrt{40^2 + 30^2}$

$DB= \sqrt{1600 + 900} = \sqrt{2500} = 50$mm (Pythagoras's Theorem)

AB=DB=50mm $\therefore$ ΔDBA is isos $\therefore B\hat{D}A=C\hat{A}B=60°$ (Base $\angle$'s isos. Δ)

$D\hat{B}C=180°-(B\hat{D}C+D\hat{C}B)$

$D\hat{B}C=180°-(90°+60°)=30°$ (Sum $\angle$'s Δ)

$D\hat{B}A = 30° + 30° = 60°$

Remedial

Work with small groups to help learners experiencing difficulty. Provide tips and hints as they complete the exercises.

Unit 3 Quadrilaterals and circles

Learner's Book page 97

Unit focus

- describe, sort, name and compare quadrilaterals in terms of:
  - length of sides
  - parallel and perpendicular sides
  - size of angles (right angled or not)
- describe and name the parts of a circle.

Background information on quadrilaterals and circles

The following properties of quadrilaterals are important. Learners should be able to identify and tabulate the following properties of quadrilaterals.

- Squares have 4 sides equal, 4 right angles, opposite sides parallel.
- Rectangles have opposite sides equal, 4 right angles, opposite sides parallel.
- Parallelograms have opposite sides equal, opposite angles equal and opposite sides parallel.

Investigation 1 Learner's Book page 98

Guidelines on how to implement this activity

Introduce quadrilaterals to learners. Discuss what feature unites all quadrilaterals. Discuss how quadrilaterals got their name. Show learners that the polygons are named according to their number of sides. Discuss some of the quads that learners may know of. Introduce the square and ask learners what is special about a square. Have learners complete this exercise on their own. Guide learners to the following observations: all sides of a square are equal; all four sides perpendicular; opposite sides are parallel and the sum of the interior angles are equal to 360°.

Suggested answers

1 AB = BC = CD = AD = 6 cm
2 The sides are all equal in length
3 AB and CD are parallel
4 BC and AD are parallel
5 $A\hat{B}C = B\hat{C}D = C\hat{D}A = D\hat{A}B = 90°$
6 All the angles are equal in size; they are right angles
7 $90° + 90° + 90° + 90° = 4 \times 90° = 360°$
8 AB and BC are perpendicular
9 BC and CD are perpendicular

Remedial

Ensure learners measure correctly. Observe how learners work with the geometry tools and provide assistance as necessary. If learners did not observe the necessary properties of a square encourage learners to do the exercise again, measuring more carefully.

Investigation 2 Learner's Book page 99

Guidelines on how to implement this activity

Show learners a rectangle. What do learners notice about a rectangle compared to a square? Ask learners to comment on which properties they think will be the same as a square and which properties they think will be different. Learners do the investigation on their own. On completion of the exercise ensure that learners know the properties of the rectangle.

Suggested answers

1 PQ = RS = 6 cm; QR = PS = 4 cm
2 PQ and RS are equal in length; QR and PS are equal in length
3 PQ and RS are parallel

4 QR and PS are parallel
5 $P\hat{Q}R = Q\hat{R}S = R\hat{S}P = S\hat{P}Q = 90°$
6 All the angles are equal in size; they are right angles
7 $90° + 90° + 90° + 90° = 4 \times 90° = 360°$
8 PQ and QR are perpendicular
9 RS and PS are perpendicular

Extension

Record learners' hypotheses about which properties will be the same and which will be different and discuss who was correct in their hypotheses.

Investigation 3

Learner's Book page 100

Guidelines on how to implement this activity

Show learners a parallelogram. What do learners notice about a parallelogram compared to a rectangle? Learners can again comment on which properties they think will be the same, this time as the rectangle, and which properties they think will be different. Learners do the investigation on their own. On completion of the exercise ensure that learners know the properties of a parallelogram and how it differs from a rectangle.

Suggested answers

1 DE = FG = 6 cm; EF = GD = 4 cm
2 DE and FG are equal in length; EF and GD are equal in length
3 DE and FG are parallel
4 EF and GD are parallel
5 Learners measure their angles
6 $G\hat{D}E = E\hat{F}G$; $D\hat{E}F = F\hat{G}D$
7 The sum of the interior angles equals 360°

Investigation 4

Learner's Book page 101

Guidelines on how to implement this activity

Show learners a rhombus. What do learners notice about a rhombus compared to a parallelogram and compared to a square? Learners can again comment on which properties they think will be the same, to a parallelogram and a square, and which properties they think will be different. Learners do the investigation on their own. On completion of the exercise ensure that learners know the properties of a rhombus and how it differs from a parallelogram and a square.

Suggested answers

1 HI = IJ = JK = KH = 6 cm
2 The sides all have the same length
3 HI and JK are parallel
4 IJ and HK are parallel
5 Learners measure their angles
6 Opposite angles are equal in size
7 Adjacent angles add up to 180°
8 The sum of the interior angles equals 360°

Investigation 5

Learner's Book page 102

Guidelines on how to implement this activity

Show learners a kite. What do learners notice about a kite? Is it similar to any previous quadrilaterals? Learners can comment on which properties they think will be the same as any of the quads covered previously, and which properties they think will be different. Learners do the investigation on their own. On completion of the exercise ensure that learners know the properties of a kite.

Suggested answers

1 LM = MN = 2 cm; NO = OL = 5 cm

2 LM and MN are equal in length; NO and OL are equal in length.

3 Learners measure their angles

4 $M\hat{L}O = O\hat{N}M$

5 $O\hat{N}M \neq N\hat{M}L$

6 The sum of the interior angles equals 360°

Investigation 6

Learner's Book page 102

Guidelines on how to implement this activity

Show learners a trapezium. What do learners notice about a trapezium? Is it similar to any previous quadrilaterals? Learners can comment on which properties they think will be the same as any of the quads covered previously, and which properties they think will be different. Learners do the investigation on their own. On completion of the exercise ensure that learners know the properties of a trapezium. Learners should also be able to identify properties that are universal across all quadrilaterals.

Suggested answers

1 TU = 4 cm; UV = 3 cm; VW = 6 cm; WT = 3 cm

2 There are 2 sides equal in length, i.e. UV = VT

3 TU and VW are parallel

4 UV and WT are not parallel

5 Learners measure their angles

6 None of the angles are equal.

7 The sum of the interior angles equals 360°

Exercise 1

Learner's Book page 104

Guidelines on how to implement this activity

Remind learners how they used the properties of triangles to solve for missing sides and angles in Unit 1. Explain that we can do the same using the properties of quadrilaterals. Work through the examples in the Learner's Book and use the properties to solve the problems. Do additional examples if required to demonstrate how we use the properties. Again revise problem solving techniques: -show learners how you identify what is being asked – mark this on the diagram.

Then identify the information that is given. Create a number sentence or equation to solve the problem. Substitute in the necessary values and then solve. Remind learners to check their answer by assessing the reasonableness of their answer. Learners can work in pairs, but each learners must show all the working involved.

Suggested answers

1.1	False	**1.2**	True	**1.3**	True
1.4	True	**1.5**	False	**1.6**	True
2.1	rhombus	**2.2**	trapezium	**2.3**	rectangle

3.1 BC = CD = DA = 3 cm; $A\hat{B}C = B\hat{C}D = C\hat{D}A = D\hat{A}B = 90°$

3.2 $L\hat{M}J = 60°$; $K\hat{J}M = 180° - 60° = 120°$; $K\hat{L}M = 120°$

3.3.1 FG = 8 cm; GH = 13 cm **3.3.2** $F\hat{E}H = F\hat{G}H$

3.4 KN = 6 cm; KL = NM = $\frac{1}{3} \times 6 = 2$ cm

4 trapezium

5 The two shorter sides have same length (5,5 m). The two longer sides have same length (2 × 5,5 m = 11 m).

Remedial

Some learners may find it difficult remembering all the different properties of the quadrilaterals. Encourage learners to create a table in their exercise books where they record all the properties of each quadrilateral, and mark those that are common across the quads. Learners can then refer to this when they complete the exercise.

Extension

Learners can create bingo boards listing the properties of various quads. You can then call out various properties and when learners have enough properties describing a particular quad they call out Bingo.

Exercise 2

Learner's Book page 106

Guidelines on how to implement this activity

Revise the parts of the circle. Ensure learners can identify each part. Practise these words and have learners point them out to you. Learners can complete this exercise on their own.

Suggested answers

1	segment	**2**	chord	**3**	sector
4	radius	**5**	circumference	**6**	diameter

Unit 4 Similar and congruent figures

Learner's Book page 107

Unit focus

- learn to recognise and describe similar shapes
- learn to recognise and describe congruent shapes.

Background information on similar and congruent figures

Learners need to be able to recognise the difference between similar figures and congruent figures. This is a new concept in Grade 7 and learners should do as much drawing and physical comparison as possible to help them assimilate the concepts into their mathematical thinking.

Similar figures have the same shape and proportion, but not the same size. For example a right angle triangle ABC with AB and BC equal to 5 cm and the angle at B equal to 90° is similar to a right angle triangle PQR with PQ and QR equal to 3 cm and the angle at Q equal to 90°.

Congruent figures are exactly the same shape and size. For example a right angle triangle ABC with AB and BC equal to 3 cm and the angle at B equal to 90° is congruent to a right angle triangle PQR with PQ and QR equal to 3 cm and the angle at Q equal to 90°.

Exercise 1

Learner's Book page 109

Guidelines on how to implement this activity

Discuss similarity as a concept. Emphasise that similar does not mean exactly the same but rather a larger or smaller copy of. Discuss the two conditions for similarity. Demonstrate the notation and how it is necessary for the corresponding vertices to be written in order. Show learners how to identify similarity in triangles and how to prove similarity. Progress to showing learners similarity in quadrilaterals and how to recognise and prove similarity – make sure learners note that in quadrilaterals the proportion of the sides are a necessary condition for similarity. Encourage learners to work on their own to complete this exercise, but allow learners to work in pairs if necessary.

Suggested answers

1.1 False **1.2** True **1.3** False **1.4** False

2.1 They are similar as all the angles are the same.

2.2 They are similar because they are both rhombuses whose angles are the same.

2.3 They are similar because they are both rectangles with their sides in proportion.

2.4 They are similar because all the angles are the same.

2.5 They are similar because all the angles are the same.

2.6 They are both similar because they are squares.

2.7 They are not similar, as their angles are not the same.

2.8 They are not similar, as their sides are not in proportions.

3 ΔABC ||| ΔEDF (AAA)
STUV ||| EFGH (AAAA) and all sides proportional
DEGF ||| JKLM rectangle with sides proportional
ΔKLM ||| ΔDFE (AAA)
ΔXYZ ||| ΔNML (AAA)

Remedial

If learners struggle with the concept of similarity show learners pictures of objects. When the objects are the same but not the same size, we would call them similar.

Exercise 2

Learner's Book page 113

Guidelines on how to implement this activity

Introduce learners to the concept of congruency. Explain that congruency means identical – and that corresponding angles and sides of the two triangles must all be identical. Demonstrate the notation and how it is necessary for the corresponding vertices to be written in order. Show learners how to identify congruency in triangles and how to prove that two triangles are congruent. Progress to showing learners congruency in quadrilaterals and how to recognise and prove that the quadrilaterals are congruent. Encourage learners to work on their own to complete this exercise, but allow learners to work in pairs if necessary.

Suggested answers

1.1	congruent	
1.2	congruent	
1.3	congruent	
1.4	congruent	
1.5	not congruent	
2.1	$B\hat{A}C = C\hat{D}E = 70°$	
	$A\hat{B}C = C\hat{E}D = 49°$	
	$A\hat{C}B = E\hat{C}D = 61°$ AC = DC = 6	
	AB = ED = 5	BC = EC = 7
	$B\hat{C}E = 180° - 2(61)$	Angles on a straight line
	$B\hat{C}E = 58°$	
	$C\hat{B}E = B\hat{E}C$	Base angles isosceles ΔCBE
	$C\hat{B}E = (180° - 58) \div 2 = 61°$	Interior angles of triangle CBE
	$B\hat{E}C = C\hat{B}E = 61°$	
2.2	KLMP is a square	All sides are equal and has a 90° angle
	In ΔKPQ	
	$P\hat{K}Q = N\hat{M}P = 90°$	$\Delta MNP \equiv \Delta KPQ$ given
	KP = KQ	Given
	$K\hat{P}Q = K\hat{Q}P$	Base angles isosceles triangle
	$K\hat{P}Q = (180° - 90) \div 2 = 45°$	KQP = 45°
	In ΔMNP $M\hat{N}P = K\hat{P}Q = 45°$	$\Delta MNP \equiv \Delta KPQ$ given
	$M\hat{P}N = K\hat{Q}P = 45°$	$\Delta MNP \equiv \Delta KPQ$ given
2.3	$B\hat{C}D = D\hat{C}F = 135°$	ABCD ≡ EDCF given
	$B\hat{C}F = 360° - 135° - 135° = 90°$	Angles round a point
	$C\hat{B}F = C\hat{F}B$	Base angles isosceles ΔBCF
	In ΔBCF	
	$C\hat{B}F = (180° - B\hat{C}F) \div 2$	
	$= (180° - 90°) \div 2 = 45°$	
	$C\hat{F}B = 45°$	

2.4 In ΔBAC and ΔCDE

AB = CD	given
BC = DE	given
AC = CE	sides of square ACEG
ΔABC ≡ ΔCDE	(SSS)
In Δ CDE and Δ EFG	
CD = EF	given
DE = FG	given
CE = EG	sides of square ACEG
ΔCDE ≡ ΔEFG	(SSS)
Therefore ΔABC ≡ ΔCDE ≡ ΔEFG	

Extension

Learners can investigate the minimum conditions for triangles to be congruent.

Consolidation

Learner's Book page 117

Before doing this consolidation exercise, encourage learners to review the work covered in this chapter. Advise learners to use the summary and to revise their work. This exercise can be used as an informal assessment task for you to track how learners are coping with the chapter and the concepts covered.

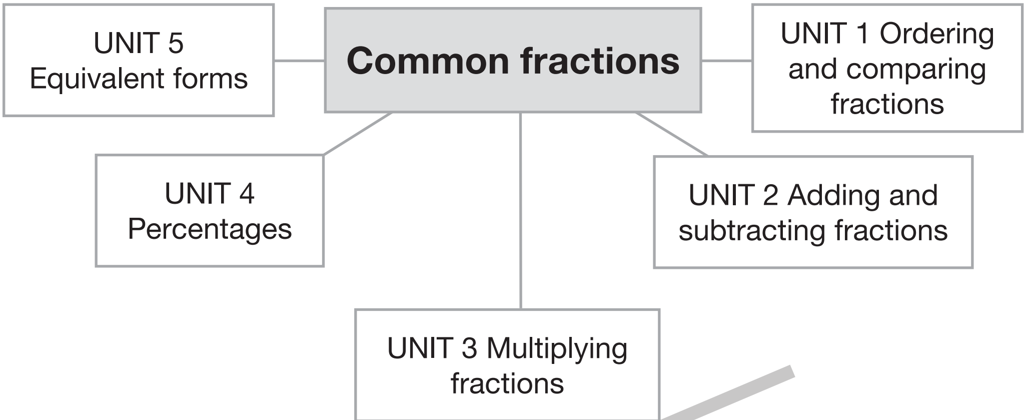
Suggested answers

1.1 – 1.3	Learners' own constructions	(9)
1.4.1	equilateral triangle	
1.4.2	right-angled triangle	
1.4.3	isosceles triangle	(3)
1.5.1	180°	
1.5.2	180°	
1.5.3	180°	(3)
2	Learners' own constructions	(5)
3	Learners' own constructions	(3)
4.1 – 4.3	Learners' own constructions	(9)
4.4.1	360°	
4.4.2	360°	
4.4.3	360°	(3)

[35]

Chapter 5 Common fractions

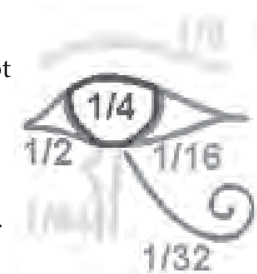
Overview of concepts



Content		Time allocations	LB page
Unit 1	Ordering and comparing fractions	1 hour	119
Unit 2	Adding and subtracting fractions	3 hours	124
Unit 3	Multiplying fractions	2 hours	133
Unit 4	Percentages	1,5 hours	138
Unit 5	Equivalent forms	1,5 hours	144

Background information on common fractions

Ancient Egyptians worked with fractions about 4 000 years ago. They used the image of Horus' eye (Horus was an Egyptian God) to represent fractions. Each part of the eye represented a fraction, and by adding up the parts you could represent any fraction up $\frac{1}{2}$ to $\frac{63}{64}$.



The ancient Greeks also used fractions but their notation was not always clear, and it was necessary to know the exact context in order to accurately read the fraction.

Neither used the methods we use today for working with fractions. Fractions are often a problem area for learners. This may be because the concept of having one number over another that sometimes have to be worked with as one number, and other times separately can be confusing. It is important that learners build up a strong foundation for working with fractions to help prepare learners for working with complex algebraic fractions in future grades.

Generic teaching guidelines for teaching common fractions

Remind learners that a fraction is a 'part of'. When you divide a number by another number you get a fraction. For example, 10 ÷ 2 = 5 and 5 is $\frac{5}{10}$ or $\frac{1}{2}$ of 10.
A fraction can also be part of a whole. For example $\frac{4}{5}$ is $\frac{4}{5}$ of 1 whole.
Remind learners that the top number in a fraction is the numerator and the bottom number is the denominator. In a proper fraction the numerator is smaller than the denominator. For example, $\frac{1}{3}$. In an improper fraction the numerator is bigger than the denominator. For example, $\frac{4}{3}$, which can be written as a mixed number: $1\frac{1}{3}$.
Before teaching the operations of addition, subtraction, multiplication and division of fractions, ensure that learners understand the basic concepts.

Resources

Fraction walls, number cards, fraction cards, blank number lines, cardboard and colour pens for creating posters and creating flashcards, comparison cards, counters and fraction apparatus for learners requiring more concrete interaction.

Unit 1 Ordering and comparing fractions

Learner's Book page 119

Unit focus

- revise fractions
- compare and order common fractions
- work with tenths, hundredths and thousandths
- simplify fractions.

Background information on ordering and comparing fractions

Learners should be familiar with comparing and ordering fractions. However, as fractions are generally a problem area for learners, it is worth revising these. Wherever possible use concrete apparatus to help learners visualise and concretise the process of working with fractions. Fraction walls are helpful tools to help learners compare basic fractions.
Proper fractions can be converted to their simplest form by dividing or multiplying them by 1.
Remember that $\frac{2}{2}$, $\frac{3}{3}$, $\frac{4}{4}$ ... all = 1
So: $\frac{5}{10} \div \frac{5}{5}$ (5 is the common factor of 5 and 10) = $\frac{1}{2}$
Fractions having the same value, like $\frac{5}{10}$ and $\frac{1}{2}$, are called equivalent fractions.

Revision exercise

Learner's Book page 119

Guidelines on how to implement this activity

To re-introduce fractions ask learners to cut a sheet of paper into 2 equal pieces. 1 sheet gives 1 divided by 2 which is $1 \div 2$ or $\frac{1}{2}$. How many halves make 1? $\frac{2}{2} = 1$. Next cut each half into 2 equal parts. There are now four equal parts. $1 = \frac{4}{4}$. $\frac{1}{2} = \frac{2}{4}$. $1 = \frac{2}{2} = \frac{4}{4}$. Continue with eighths and sixteenths.

Suggested answers

1 $1 \div 4\frac{1}{4}$; $1 \div 7\frac{1}{7}$; $1 \div 10\frac{1}{10}$; $1 \div 100\frac{1}{100}$; $1 \div 1000\frac{1}{1000}$

2 $\frac{1}{5}$; $\frac{3}{10}$; $\frac{2}{5}$; $\frac{1}{2}$; $\frac{3}{5}$; $\frac{4}{5}$; $\frac{9}{10}$

Exercise 1

Learner's Book page 121

Guidelines on how to implement this activity

Do some comparisons of fractions using a fraction wall. Ask learners how they would go about comparing fractions that were not on the fraction wall. Introduce learners to another method of comparing denominators which is to make the denominators the same. When we make the denominators the same we are using equivalent fractions. Revise with learners how to work with equivalent fractions and remind them that whatever we do to the denominator we have to do to the numerator. Work through some examples with the learners.

Suggested answers

1.1 $\frac{1}{3} \times \frac{2}{2} = \frac{2}{6}$; $\frac{1}{3} \times \frac{3}{3} = \frac{3}{9}$; $\frac{1}{3} \times \frac{4}{4} = \frac{4}{12}$

1.2 $\frac{4}{7} \times \frac{2}{2} = \frac{8}{14}$; $\frac{4}{7} \times \frac{3}{3} = \frac{12}{21}$; $\frac{4}{7} \times \frac{4}{4} = \frac{16}{28}$

1.3 $\frac{6}{10} \div \frac{2}{2} = \frac{3}{5}$; $\frac{6}{10} \times \frac{2}{2} = \frac{12}{20}$; $\frac{6}{10} \div \frac{3}{3} = \frac{18}{30}$

1.4 $\frac{2}{5} \times \frac{2}{2} = \frac{4}{10}$; $\frac{2}{5} \times \frac{3}{3} = \frac{6}{15}$; $\frac{2}{5} \times \frac{4}{4} = \frac{8}{20}$

1.5 $\frac{5}{6} \times \frac{2}{2} = \frac{10}{12}$; $\frac{5}{6} \times \frac{3}{3} = \frac{15}{18}$; $\frac{5}{6} \times \frac{4}{4} = \frac{20}{24}$

1.6 $\frac{3}{8} \times \frac{2}{2} = \frac{6}{16}$; $\frac{3}{8} \times \frac{3}{3} = \frac{9}{24}$; $\frac{3}{8} \times \frac{4}{4} = \frac{12}{32}$

1.7 $\frac{2}{3} \times \frac{2}{2} = \frac{4}{6}$; $\frac{2}{3} \times \frac{3}{3} = \frac{6}{9}$; $\frac{2}{3} \times \frac{4}{4} = \frac{8}{12}$

1.8 $\frac{8}{10} \div \frac{2}{2} = \frac{4}{5}$; $\frac{8}{10} \times \frac{2}{2} = \frac{16}{20}$; $\frac{8}{10} \div \frac{3}{3} = \frac{24}{30}$

2.1 $\frac{1}{2} = \frac{2}{4} = \frac{3}{8} = \frac{4}{16} = \frac{8}{32} = \frac{50}{100} = \frac{500}{1\,000}$

2.2 $\frac{3}{5} = \frac{6}{10} = \frac{15}{25} = \frac{24}{40} = \frac{60}{100} = \frac{600}{1\,000}$

2.3 $\frac{5}{8} = \frac{25}{40} = \frac{50}{80} = \frac{625}{1\,000} = \frac{6\,250}{10\,000}$

2.4 $\frac{3}{4} = \frac{9}{12} = \frac{30}{40} = \frac{75}{100} = \frac{750}{1\,000}$

3.1 $\frac{25}{30} \div 1 = \frac{25}{30} \div \frac{5}{5} = \frac{5}{6}$

3.2 $\frac{24}{30} \div 1 = \frac{24}{30} \div \frac{6}{6} = \frac{4}{5}$

3.3 $\frac{75}{100} \div 1 = \frac{75}{100} \div \frac{25}{25} = \frac{3}{4}$

3.4 $\frac{700}{1\,000} \div 1 = \frac{700}{1\,000} \div \frac{100}{100} = \frac{7}{10}$

Exercise 2

Learner's Book page 123

Guidelines on how to implement this activity

If learners understand that fractions are part of a whole, it is easier to grasp their value. For example, $\frac{1}{2} > \frac{1}{4}$. Demonstrate with a diagram on the board. Demonstrate on a number line that a proper fraction is always less than 1 and an improper fraction is always more than 1. For example, $\frac{4}{5} < \frac{5}{5}$ which = 1 and $\frac{6}{5} > \frac{5}{5}$ which = 1.

Ask learners to arrange a series of fractions, proper and improper in ascending and descending order: ascending: $\frac{9}{4}$; $\frac{15}{8}$; $\frac{3}{4}$; $\frac{1}{2}$ and Descending: $\frac{1}{2}$; $\frac{3}{4}$; $\frac{15}{8}$; $\frac{9}{4}$. Compare proper fractions, improper fractions and mixed numbers, $2\frac{1}{3} > \frac{5}{3} > \frac{2}{3}$ and $\frac{2}{5} < \frac{11}{10} < 2\frac{1}{5} < \frac{19}{5}$.
Do an example with learners showing how easy it becomes to compare fractions when their denominators are the same. Remind learners that their answer must contain the original fraction.

Suggested answers

1.1 $\frac{6}{12} \div \frac{3}{3} = \frac{2}{4}$, therefore $\frac{6}{12} < \frac{3}{4}$

1.2 $\frac{3}{4} \times \frac{3}{3} = \frac{9}{12}$, therefore $\frac{3}{4} > \frac{8}{12}$

1.3 multiples of 9: 9; 18; 27; 36; 45; 63; 72
multiples of 8: 8; 16; 24; 32; 40; 48; 56; 64; 72
$\frac{6}{9} \times \frac{8}{8} = \frac{48}{72}$; $\frac{5}{8} \times \frac{9}{9} = \frac{45}{72}$, therefore $\frac{6}{9} > \frac{5}{8}$

1.4 $\frac{6}{8} \div \frac{2}{2} = \frac{3}{4}$, therefore $\frac{6}{8} = \frac{3}{4}$

1.5 $\frac{3}{4} > \frac{8}{12}$

1.6 multiples of 15: 15; 30
multiples of 6: 6; 12; 18; 24; 30
$\frac{9}{15} \times \frac{2}{2} = \frac{18}{30}$; $\frac{5}{6} \times \frac{5}{5} = \frac{25}{30}$, therefore $\frac{9}{15} < \frac{5}{6}$

1.7 multiples of 3: 3; 6; 9; 12; 15; 18; 21; 24; 27; 30
multiples of 10: 10; 20; 30
$\frac{2}{3} \times \frac{10}{10} = \frac{20}{30}$; $\frac{7}{10} \times \frac{3}{3} = \frac{21}{30}$, therefore $\frac{2}{3} < \frac{7}{10}$

1.8 $\frac{44}{50} \times \frac{2}{2} = \frac{88}{100}$, therefore $\frac{89}{100} > \frac{44}{50}$

1.9 $\frac{32}{40} \times \frac{25}{25} = \frac{800}{1\,000}$, therefore $\frac{32}{49} > \frac{760}{1\,000}$

1.10 $\frac{5}{8} \times \frac{125}{125} = \frac{625}{1\,000}$, therefore $\frac{5}{8} < \frac{650}{1\,000}$

2.1 $\frac{2}{5} \times \frac{20}{20} = \frac{40}{100}$; $\frac{3}{10} \times \frac{10}{10} = \frac{30}{100}$ ascending order: $\frac{3}{10}$; $\frac{2}{5}$; $\frac{56}{100}$

2.2 multiples of 7: 7; 14; 21; 28; 35; 42; 49; 56; 63; 70; 77; 84; 91; 98; 105; 112; 119; 126
multiples of 9: 9; 18; 27; 36; 45; 54; 63; 72; 81; 90; 99; 108; 117; 126
multiples of 6: 6; 12; 18; 24; 30; 36; 42; 48; 54; 60; 66; 72; 78; 84; 90; 96; 102; 108; 114; 120; 126
$\frac{6}{7} \times \frac{18}{18} = \frac{108}{126}$; $\frac{7}{9} \times \frac{14}{14} = \frac{98}{126}$; $\frac{5}{6} \times \frac{21}{21} = \frac{105}{126}$ ascending order: $\frac{7}{9}$; $\frac{5}{6}$; $\frac{6}{7}$

2.3 $\frac{3}{4} \times \frac{10}{10} = \frac{30}{40}$; $\frac{12}{20} \times \frac{2}{2} = \frac{24}{40}$ ascending order: $\frac{23}{40}$; $\frac{12}{20}$; $\frac{3}{4}$

2.4 $\frac{67}{100} \times \frac{10}{10} = \frac{670}{1\,000}$; $\frac{23}{50} \times \frac{20}{20} = \frac{460}{1\,000}$ ascending order: $\frac{23}{50}$; $\frac{665}{1\,000}$; $\frac{67}{100}$

2.5 multiples of 80: 80; 160; 240; 320; 400
multiples of 50: 50; 100; 150; 200; 250; 300; 350; 400
multiples of 40: 40; 80; 120; 160; 200; 240; 280; 320; 400
$\frac{65}{80} \times \frac{5}{5} = \frac{325}{400}$; $\frac{45}{50} \times \frac{8}{8} = \frac{360}{400}$; $\frac{32}{40} \times \frac{10}{10} = \frac{320}{400}$ ascending order: $\frac{32}{40}$; $\frac{65}{80}$; $\frac{45}{50}$

2.6 $\frac{456}{500} \times \frac{2}{2} = \frac{912}{1\,000}$; $\frac{76}{100} \times \frac{10}{10} = \frac{760}{1\,000}$ ascending order: $\frac{76}{100}$; $\frac{876}{1\,000}$; $\frac{456}{500}$

3.1 False, $(\frac{1}{2} + \frac{2}{4}) \times \frac{3}{4} = (\frac{2}{4} + \frac{2}{4}) \times \frac{3}{4} = \frac{4}{4} \times \frac{3}{4} = \frac{3}{4} = \frac{9}{12}$; $\frac{2}{6} \times \frac{4}{8} = \frac{8}{48} = \frac{1}{6} = \frac{2}{12}$

3.2 True, $(\frac{2}{8} \times \frac{1}{6}) + \frac{5}{8} = \frac{2}{48} + \frac{5}{8} = \frac{2}{48} + \frac{30}{48} = \frac{32}{48} = \frac{2}{3}$

3.3 False, $1 = \frac{12}{12}$

Remedial

Provide concrete apparatus for learners to work with, including fraction charts, to help learners compare and order fractions. Provide additional material from Grade 6 curriculum for learners to practise with at home.

Unit 2 Adding and subtracting fractions

Learner's Book page 124

Unit focus

- revise adding and subtracting fractions with denominators that are multiples of each other
- revise making sure our answers are in the simplest form
- revise finding fractions of whole numbers
- learn how to add and subtract fractions with denominators that are not multiples of each other
- work with mixed numbers.

Background information on adding and subtracting fractions

Learners have worked with adding and subtracting fractions in Grade 6. However, as fractions are a problematic content area it is wise to revise the following core concepts: Fractions have to have the same denominator in order to add and subtract them. When they are supplied with the same denominator it is easy to add or subtract them. For example, $\frac{3}{5} + \frac{1}{5} = \frac{4}{5}$ and $\frac{3}{5} - \frac{1}{5} = \frac{2}{5}$. Remind learners that we do not add or subtract the denominators, only the numerators.
When one denominator is a multiple of the other, we use equivalent fractions to make the denominators the same: $\frac{3}{5} + \frac{1}{10} = \frac{6}{10} + \frac{1}{10} = \frac{7}{10}$ and $\frac{3}{5} - \frac{1}{10} = \frac{6}{10} - \frac{1}{10} = \frac{5}{10} = \frac{1}{2}$.
This needs to be extended to adding or subtracting fractions where one denominator is not a multiple of the other as in $\frac{1}{3} + \frac{1}{5}$. We need to find the LCM of 3 and 5. It is 15, so: $\frac{1}{3} \times \frac{5}{5} = \frac{5}{15}$ and $\frac{1}{5} \times \frac{3}{3} = \frac{3}{15}$. $\frac{5}{15} + \frac{3}{15} = \frac{8}{15}$ or $\frac{5}{15} - \frac{3}{15} = \frac{2}{15}$.

Exercise 1

Learner's Book page 126

Guidelines on how to implement this activity

Revise adding and subtracting fractions that have the same denominators and denominators that are multiples of each other. In order to make the denominators the same, we need to use equivalent fractions. Do a few examples of equivalence before moving onto examples of adding and subtracting fractions. Do these examples as a class. If necessary remind learners that we do not add or subtract the denominators only the numerators. The denominator remains constant. Learners must provide their answer in the simplest form. This requires working with equivalent fractions again. Do a few of examples of this as well. Discuss finding fractions of whole numbers. Do a few examples together. Explain to learners the real life application of this and do some 'real life' problems together. Learners should complete this exercise on their own.

Suggested answers

1.1 $\frac{1}{6} + \frac{4}{6} = \frac{5}{6}$

1.2 $\frac{3}{7} + \frac{2}{7} = \frac{5}{7}$

1.3 $\frac{3}{10} + \frac{4}{10} = \frac{7}{10}$

1.4 $\frac{1}{5} + \frac{3}{10} = \frac{2}{10} + \frac{3}{10} = \frac{5}{10} = \frac{1}{2}$

1.5 $\frac{17}{100} + \frac{17}{50} = \frac{17}{100} + \frac{34}{100} = \frac{51}{100}$

1.6 $\frac{3}{25} + \frac{3}{100} = \frac{12}{100} + \frac{3}{100} = \frac{15}{100} = \frac{3}{20}$

2.1 $\frac{5}{6} - \frac{4}{6} = \frac{1}{6}$

2.2 $\frac{3}{7} - \frac{2}{7} = \frac{1}{7}$

2.3 $\frac{9}{10} - \frac{4}{10} = \frac{5}{10} = \frac{1}{2}$

2.4 $\frac{3}{5} - \frac{3}{10} = \frac{6}{10} - \frac{3}{10} = \frac{3}{10}$

2.5 $\frac{17}{50} - \frac{1}{100} = \frac{34}{100} - \frac{1}{100} = \frac{33}{100}$

2.6 $\frac{3}{25} - \frac{3}{100} = \frac{12}{100} - \frac{3}{100} = \frac{9}{100}$

3.1 $\frac{1}{2} \times \frac{10}{1} = \frac{10}{2} = 5$

3.2 $\frac{3}{4} \times \frac{12}{1} = \frac{36}{4} = 9$

3.3 $\frac{4}{10} \times \frac{20}{1} = \frac{80}{10} = 8$

4. $\frac{1}{3} \times \frac{42\,863\,748}{1} = \frac{42\,863\,748}{3} = 14\,287\,916$

5.1 $\frac{1}{5} + \frac{3}{5} = \frac{4}{5}$

5.2 $\frac{4}{7} + \frac{2}{7} = \frac{6}{7}$

5.3 $\frac{4}{5} - \frac{1}{5} = \frac{3}{5}$

5.4 $\frac{6}{7} - \frac{2}{7} = \frac{4}{7}$

Remedial

Provide simple Grade 5 and 6 work on adding and subtracting fractions for learners experiencing difficulties with the exercise. Additional practise should help them master the skill of working with fractions.

Extension

Encourage learners to add more than two fractions together. Provide examples of adding three or even four fractions.

Exercise 2

Learner's Book page 128

Guidelines on how to implement this activity

Revise finding the lowest common multiple (LCM) and highest common factor (HCF) with learners. This will help them find the lowest common denominator (LCD) and simplify fractions more easily. Do a few examples of adding and subtracting fractions with denominators that are not multiples of each other. Learners should complete this exercise on their own.

Suggested answers

1 LCD = 10, $\frac{2}{5} \times \frac{2}{2} + \frac{1}{2} \times \frac{5}{5} = \frac{4}{10} + \frac{5}{10} = \frac{9}{10}$

2 LCD = 12, $\frac{1}{4} \times \frac{3}{3} + \frac{2}{3} \times \frac{4}{4} = \frac{3}{12} + \frac{8}{12} = \frac{11}{12}$

3 LCD = 10, $\frac{9}{10} - \frac{1}{2} \times \frac{5}{5} = \frac{9}{10} - \frac{5}{10} = \frac{4}{10} = \frac{2}{5}$

4 LCD = 12, $\frac{11}{12} - \frac{2}{3} = \frac{11}{12} - \frac{2}{3} \times \frac{4}{4} = \frac{11}{12} - \frac{8}{12} = \frac{3}{12} = \frac{1}{4}$

5 LCD = 12, $\frac{5}{6} - \frac{3}{4} = \frac{5}{6} \times \frac{2}{2} - \frac{3}{4} \times \frac{3}{3} = \frac{10}{12} - \frac{9}{12} = \frac{1}{12}$

6 LCD = 20, $\frac{17}{20} - \frac{3}{4} = \frac{17}{20} - \frac{3}{4} \times \frac{5}{5} = \frac{17}{20} - \frac{15}{20} = \frac{2}{20} = \frac{1}{10}$

Remedial

If learners are struggling to find the LCD, encourage learners to check if their LCD is correct either with yourself or with a classmate, before making the equivalent fraction and calculating the sum.

Exercise 3 Learner's Book page 129

Guidelines on how to implement this activity

Revise the concept of mixed numbers. Learners should remember that a mixed number is always larger than 1. Revise converting between mixed numbers and common fractions. Do a few examples together as a class before learners tackle the activity on their own.

Suggested answers

1.1 $\frac{4}{3}$ **1.2** $\frac{7}{5}$ **1.3** $\frac{9}{4}$

1.4 $\frac{69}{9} = \frac{23}{3}$ **1.5** $\frac{17}{3}$ **1.6** $\frac{84}{10} = \frac{42}{5}$

2.1 $1\frac{2}{3}$ **2.2** $1\frac{2}{5}$ **2.3** $2\frac{1}{4}$

2.4 $6\frac{4}{8} = 6\frac{1}{2}$ **2.5** $9\frac{6}{9} = 9\frac{2}{3}$ **2.6** $6\frac{2}{5}$

3.1 $\frac{2}{3} + \frac{4}{5} = \frac{2}{3} \times \frac{5}{5} + \frac{4}{5} \times \frac{3}{3} = \frac{10}{15} + \frac{12}{15} = \frac{22}{15} = 1\frac{7}{15}$

3.2 $\frac{7}{8} + \frac{5}{6} = \frac{7}{8} \times \frac{3}{3} + \frac{5}{6} \times \frac{4}{4} = \frac{21}{24} + \frac{20}{24} = \frac{41}{24} = 1\frac{17}{24}$

3.3 $\frac{3}{4} + \frac{3}{5} = \frac{3}{4} \times \frac{5}{5} + \frac{3}{5} \times \frac{4}{4} = \frac{15}{20} + \frac{12}{20} = \frac{27}{20} = 1\frac{7}{20}$

Remedial

Provide additional conversion examples for learners who require more practise. Ensure learners can manage conversions before starting the next exercise.

Exercise 4 Learner's Book page 130

Guidelines on how to implement this activity

When operating with mixed numbers, learners should start by adding or subtracting the whole numbers first. When they work with the fractions, they may need to add to the whole number or borrow from the whole number as required. Work through a few examples together as a class. Ensure you include examples of carrying over to the whole number when adding, and borrowing from the whole number when subtracting. Encourage learners to work on this exercise on their own.

Suggested answers

1.1 $1 + 2 = 3;\ 3\frac{1}{5} + \frac{2}{5} = 3\frac{3}{5}$

1.2 $3 + 2 = 5;\ 5\frac{1}{2} + \frac{1}{5} = 5\frac{5}{10} + \frac{2}{10} = 5\frac{7}{10}$

1.3 $4 + 6 = 10;\ 10\frac{19}{100} + \frac{28}{100} = 10\frac{47}{100}$

1.4 $3 - 2 = 1;\ 1\frac{2}{5} - \frac{1}{5} = 1\frac{1}{5}$

1.5 $3 + 2 = 5;\ 5\frac{1}{2} - \frac{1}{5} = 5\frac{5}{10} - \frac{2}{10} = 5\frac{3}{10}$

1.6 $8 - 6 = 2;\ 2\frac{19}{100} - \frac{3}{25} = 2\frac{19}{100} - \frac{12}{100} = 2\frac{7}{100}$

2.1 $5\frac{1}{4}$

2.2 $\frac{25}{5} - \frac{3}{5} = \frac{22}{5} = 4\frac{2}{5}$

2.3 $1 + 2 = 3;\ 3\frac{1}{7} + \frac{8}{9} = 3\frac{9}{63} + \frac{56}{63} = 3\frac{65}{63}$

2.4 $5 - 4 = 1;\ 1\frac{4}{5} - \frac{2}{6} = 1\frac{24}{30} - \frac{10}{30} = 1\frac{14}{30} = 1\frac{7}{15}$

2.5 $5 + 3 = 8;\ 8\frac{2}{3} + \frac{3}{4} = 8\frac{8}{12} + \frac{9}{12} = 8\frac{17}{12}$

2.6 $5 - 2 = 3;\ 3\frac{6}{7} - \frac{5}{6} = 3\frac{36}{42} - \frac{35}{42} = 3\frac{1}{42}$

Remedial

Permit struggling learners to work alongside a stronger learner to provide support.

Extension

Provide more complex mixed numbers, with more complex denominators for learners who are coping easily with the prescribed material.

Exercise 5

Learner's Book page 131

Guidelines on how to implement this activity

When problem solving always remind learners to:
- identify what is being asked
- identify what is given
- create a number sentence
- substitute the relevant values
- calculate and check the relevance of the answer.

Do a few examples as a class, being sure to refer to each step in the process as you work through the problem. Allow learners to work in pairs and discuss the problems and their thinking processes.

Suggested answers

1 $\frac{1}{5} + \frac{2}{5} = \frac{3}{5}\ ;\ \frac{3}{5} \times \frac{75}{1} = \frac{141}{5} = 28\frac{1}{5}$

2 $\frac{3}{3} - \frac{1}{3} = \frac{2}{3}\ ;\ \frac{2}{3} \times \frac{36}{1} = \frac{72}{3} = 24$

3 $1 - (\frac{1}{2} + \frac{1}{5} + \frac{1}{5}) = 1 - (\frac{5}{10} + \frac{2}{10} + \frac{2}{10}) = \frac{10}{10} - \frac{9}{10} = \frac{1}{10}\ ;\ \frac{1}{10} \times \frac{40}{1} = 4$

4 $21\frac{1}{2} + 15\frac{3}{4} + 26\frac{1}{3} = 21\frac{6}{12} + 15\frac{9}{12} + 26\frac{4}{12} = 62\frac{19}{12} = 63\frac{7}{12}$

5.1 9 **5.2** 3 **5.3** $\frac{3}{9} = \frac{1}{3}$

5.4 2 **5.5** $\frac{2}{9}$ **5.6** $\frac{3}{9} + \frac{2}{9} = \frac{5}{9}$

6 Learners' own answers

7.1 1 year = 12 months, $\frac{1}{12}$ **7.2** $\frac{1}{12} \times \frac{60\,000}{1} = \frac{60\,000}{12} = 5\,000$

7.3 $\frac{1}{4} \times \frac{5\,000}{1} = \frac{5\,000}{4} = 1\,250$ **7.4** $\frac{3}{10} \times \frac{5\,000}{1} = \frac{15\,000}{10} = 1\,500$

7.5 $\frac{1}{5} \times \frac{5\,000}{1} = \frac{5\,000}{5} = 1\,000$

7.6 $1 - (\frac{1}{4} + \frac{3}{10} + \frac{1}{5}) = 1 - (\frac{5}{20} + \frac{6}{20} + \frac{4}{20}) = \frac{20}{20} - \frac{15}{20} = \frac{5}{20} = \frac{1}{4}$

7.7 $1 - \frac{1}{4} = \frac{4}{4} - \frac{1}{4} = \frac{3}{4}$

8 $8\frac{2}{3} + 9\frac{1}{4} + 10\frac{5}{6} = 8\frac{8}{12} + 9\frac{3}{12} + 10\frac{10}{12} = 27\frac{21}{12} = 28\frac{9}{12} = 28\frac{3}{4}$

Remedial

Provide additional support for learners by working through the initial stages of the problem with them. Help them to identify what is being asked in the problem, identify the information given, and provide support as they compile a number sentence. Allow learners to complete the calculation on their own. Encourage learners to discuss the reasonableness of their answer in terms of what was asked. If learners continue to struggle provide additional material to learners to have additional practise.

Extension

Provide more complex word problems with complex calculations for stronger learners.

Unit 3 Multiplying fractions

Learner's Book page 133

Unit focus

- revise finding fractions of whole numbers
- multiply common fractions
- multiply mixed numbers
- do problem solving by multiplying fractions.

Background information on multiplying fractions

- In order to multiply fractions learners must grasp that 'of' means times. For example, $\frac{1}{2}$ of $\frac{1}{2} = \frac{1}{2} \times \frac{1}{2}$. Learners should also know that brackets also mean times. For example $\frac{1}{2}$ ($\frac{1}{2}$) means $\frac{1}{2} \times \frac{1}{2}$.
- It is also important that when multiplying a whole number by a fraction, for example $3 \times \frac{1}{3}$, learners know to write the 3 as a fraction: $\frac{3}{1}$. $\frac{3}{1} \times \frac{1}{3} = 1$
- Multiplicative inverses are important and learners should know what these are and how they are used. For example, for $\frac{2}{3}$ the inverse is $\frac{3}{2}$. $\frac{2}{3} \times \frac{3}{2} = 1$. The product of any number and its multiplicative inverse is 1.

Revision exercise

Learner's Book page 133

Guidelines on how to implement this activity

Use this exercise to assess how learners have managed so far with the work in this chapter. Provide immediate remediation before starting the rest of the chapter.

Suggested answers

1.1 $\frac{1}{2} \times \frac{8}{1} = \frac{8}{2} = 4$

1.2 $\frac{2}{3} \times \frac{12}{1} = \frac{24}{3} = 8$

2.1 $3\frac{1}{3}$

2.2 $18\frac{6}{8} = 18\frac{3}{4}$

2.3 $277\frac{9}{12} = 277\frac{3}{4}$

3.1 $\frac{2 \times 3+1}{3} = \frac{7}{3}$

3.2 $\frac{5 \times 5+3}{3} = \frac{28}{5}$

3.3 $\frac{12 \times 12+7}{12} = \frac{151}{12}$

Exercise 1 — Learner's Book page 135

Guidelines on how to implement this activity

When multiplying fractions by whole numbers, we write the whole number as a numerator over a denominator of 1. For example, 4 as $\frac{4}{1}$. We then multiply as we would for regular proper fractions.
Inform learners that to multiply fractions, you multiply the numerator by the numerator and the denominator by the denominator (This is different to adding and subtracting when the denominator remained constant).
Use examples such as $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$. Ask: Is half of a half equal to a quarter? Do a diagram to illustrate this. Provide more examples such as $\frac{1}{3}$ of $\frac{1}{3} = \frac{1}{9}$ or $\frac{1}{4}$ of $\frac{1}{3} = \frac{1}{12}$. Wherever possible have diagrams to show the multiplication.
Show learners that when multiplying with mixed numbers we have to convert to improper fractions. We cannot multiply the whole numbers. Do a few examples as a class. Encourage learners to manage this exercise on their own.

Suggested answers

1.1	$\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$	**1.2**	$\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$
1.3	$\frac{7}{1} \times \frac{3}{8} = \frac{21}{8} = 2\frac{5}{8}$	**1.4**	$\frac{2}{3} \times \frac{9}{1} = \frac{2}{1} \times \frac{3}{1} = 6$
1.5	$\frac{3}{4} \times \frac{8}{1} = \frac{3}{1} \times \frac{2}{1} = 6$	**1.6**	$\frac{8}{12} \times \frac{4}{5} = \frac{8}{3} \times \frac{1}{3} = \frac{8}{9}$
1.7	$\frac{1}{3} \times \frac{6}{7} = \frac{1}{1} \times \frac{2}{7} = \frac{2}{7}$	**1.8**	$\frac{2}{3} \times \frac{6}{7} = \frac{2}{1} \times \frac{2}{7} = \frac{4}{7}$
1.9	$\frac{3}{10} \times \frac{100}{121} = \frac{3}{1} \times \frac{10}{121} = \frac{30}{121}$	**1.10**	$\frac{5}{8} \times \frac{1\,600}{1\,650} = \frac{1}{1} \times \frac{200}{330} = \frac{20}{33}$
1.11	$\frac{3}{4} \times \frac{1}{5} \times \frac{3}{10} = \frac{9}{200}$	**1.12**	$\frac{1}{2} \times \frac{1}{5} \times \frac{3}{10} = \frac{3}{100}$
1.13	$\frac{3}{8} \times \frac{24}{1} = \frac{3}{1} \times \frac{3}{1} = 9$	**1.14**	$\frac{37}{50} \times \frac{100}{740} = \frac{1}{1} \times \frac{2}{20} = \frac{1}{10}$
2.1	$\frac{3}{1} \times \frac{3}{2} = \frac{9}{2} = 4\frac{1}{2}$	**2.2**	$\frac{10}{1} \times \frac{69}{4} = \frac{5}{1} \times \frac{69}{2} = \frac{345}{2} = 172\frac{1}{2}$
2.3	$\frac{18}{5} \times \frac{17}{4} = \frac{9}{5} \times \frac{17}{2} = \frac{153}{10} = 15\frac{3}{10}$	**2.4**	$\frac{16}{3} \times \frac{19}{8} = \frac{2}{3} \times \frac{19}{1} = \frac{38}{3} = 12\frac{2}{3}$
2.5	$\frac{39}{11} \times \frac{7}{3} = \frac{13}{11} \times \frac{7}{1} = \frac{91}{11} = 8\frac{3}{11}$	**2.6**	$\frac{53}{9} \times \frac{15}{4} = \frac{53}{3} \times \frac{5}{4} = \frac{265}{12} = 22\frac{1}{12}$

Remedial

Learners experiencing any problems may require additional practise. Provide additional homework activities based on Grade 6 material.

Exercise 2 — Learner's Book page 136

Guidelines on how to implement this activity

When working with mixed operations, it is vital that learners remember BODMAS and the order of operations. Revise how to use BODMAS, especially when working with fractions. Do a few examples together as a class. Learners should manage this exercise on their own.

Suggested answers

1.1 $\frac{7}{1} \times \frac{1}{2} \times \frac{1}{2} = \frac{7}{4} = 1\ \frac{3}{4}$

1.2 $\frac{5}{6} - (\frac{3}{12} + \frac{4}{12}) = \frac{10}{12} - \frac{7}{12} = \frac{3}{12} = \frac{1}{4}$

1.3 $\frac{1}{2} \times \frac{3}{4} + (\frac{8}{10} - \frac{5}{10}) = \frac{1}{2} \times \frac{3}{4} + \frac{3}{10} = \frac{3}{8} \times \frac{3}{10} = \frac{9}{80}$

2.1 $\frac{1}{2} + (\frac{2}{10} + \frac{5}{10})$ of $\frac{15}{16} = \frac{1}{2} + \frac{7}{10} \times \frac{15}{16} = \frac{1}{2} + \frac{7}{2} \times \frac{3}{16} = \frac{16}{32} + \frac{21}{32} = \frac{37}{32} = 1\ \frac{5}{32}$

2.2 $\frac{29}{4} - \frac{1}{2} \times \frac{21}{2} = \frac{29}{4} - \frac{21}{4} = \frac{8}{4} = 2$

3.1 $\frac{2}{3}$

3.2 $\frac{1}{6}$

3.3 $(\frac{1}{2} + \frac{1}{2}) \times 1 = 1 \times 1 = 1$

Remedial

Learners who have struggled with the operations individually may really struggle when putting missed operations into one problem. Work with these learners showing them how to use BODMAS to carefully break up the calculation into each operation. Show learners how to use BODMAS to systematically work through each operation, how to set out the sum, and ultimately find the correct answer. This may require sitting in a small group while the rest of the class completes the exercise.

Exercise 3

Learner's Book page 136

Guidelines on how to implement this activity

When solving problems, learners should:

- identify what is being asked
- identify what is given
- create a number sentence
- substitute the relevant values
- calculate and check the relevance of the answer.

Do a few examples as a class and refer to each step in the process as you work through the problem. Allow learners to work in pairs and discuss the problems and their reasoning.

Suggested answers

1 $\frac{7}{8} \times \frac{3}{4} \times \frac{160}{1} = \frac{7}{8} \times \frac{3}{1} \times \frac{40}{1} = \frac{7}{1} \times \frac{3}{1} \times \frac{5}{1} = \text{R}105$

2 $\frac{5}{6} \times \frac{3}{1} \times \frac{18}{1} = \frac{5}{1} \times \frac{3}{1} \times \frac{3}{1} = 45$ kg

3 $192 + (\frac{1}{2} \times \frac{192}{1}) + \frac{5}{12} \times (\frac{1}{2} \times \frac{192}{1}) = 192 + 96 + \frac{5}{12} \times \frac{96}{1} = 192 + 96 + 40 = 296$

4 $\frac{1}{4} \times \frac{4}{7} \times \frac{35}{1} = \frac{1}{1} \times \frac{1}{1} \times \frac{5}{1} = 5$

5 $35 \times 52 = 1\,820$; $\frac{7}{2} \times \frac{13}{3} = \frac{91}{6}$ $\frac{1\,820}{1} \div \frac{91}{6} = \frac{1\,820}{1} \times \frac{6}{91} = \frac{20}{1} \times \frac{6}{1} = 120$

6.1 $\frac{2}{5} \times \frac{1\,450}{1} = \frac{2}{1} \times \frac{290}{1} = 580$

6.2 $\frac{9}{10} \times \frac{1\,450}{1} = \frac{9}{1} \times \frac{145}{1} = 1\,305$; $1\,305 - 580 = 725$ $\frac{3}{5} \times \frac{1\,450}{1} = \frac{3}{1} \times \frac{290}{1} = 870$ $\frac{725}{870} = \frac{5}{6}$

6.3 $\frac{1}{1} - \frac{9}{10} = \frac{10}{10} - \frac{9}{10} = \frac{1}{10}$

7 $\frac{11}{3} + \frac{11}{4} = \frac{44}{12} + \frac{33}{12} = \frac{77}{12}$; $\frac{6}{1} \times \frac{77}{12} = \frac{1}{1} \times \frac{77}{2} = 38\ \frac{1}{2}$

8.1 kitchen and bedrooms: $\frac{9}{2} \times \frac{15}{4} = \frac{135}{8} = 16\ \frac{7}{8}\ \text{m}^2$ lounge: $\frac{9}{2} \times \frac{9}{2} = \frac{81}{4} = 20\ \frac{1}{4}\ \text{m}^2$

8.2 $\frac{4}{1} \times \frac{135}{8} + \frac{81}{4} = \frac{135}{2} + \frac{81}{4} = \frac{270}{4} + \frac{81}{4} = \frac{351}{4} = 87\,\frac{3}{4}\text{ m}^2$

8.3 $\frac{3}{1} \times \frac{135}{8} = \frac{405}{8} = 50\,\frac{5}{8}\text{ m}^2$

8.4 $87\,\frac{3}{4} - 50\,\frac{5}{8} = 87\,\frac{6}{8} - 50\,\frac{5}{8} = 37\,\frac{6}{8} - \frac{5}{8} = 37\,\frac{1}{8}\text{ m}^2$

9.1-9.3 Learners' own work

9.4 $4 \times (\frac{16}{3} \times \frac{17}{4}) + 2 \times (\frac{17}{4} \times \frac{23}{4}) + (\frac{16}{3} \times \frac{7}{2}) = 4 \times (\frac{4}{3} \times \frac{17}{1}) + 2 \times (\frac{17}{4} \times \frac{23}{4}) + (\frac{8}{3} \times \frac{7}{1}) = \frac{4}{1} \times \frac{68}{3} + \frac{2}{1} \times \frac{391}{16} + \frac{56}{3} = \frac{272}{3} + \frac{391}{8} + \frac{56}{3} = \frac{2\,176}{24} + \frac{1\,173}{24} + \frac{448}{24} = \frac{3\,797}{24}$
$= 158\,\frac{5}{24}\text{ m}^2$

9.5 $\frac{3\,797}{24} - \frac{391}{16} = \frac{7\,594}{48} - \frac{1\,173}{48} = \frac{6\,421}{48} = 133\,\frac{37}{48}\text{ m}^2$

Remedial

Provide additional support for learners by working through the initial stages of the problem with them. Help them to identify what is being asked in the problem, identify the information given, and provide support as they compile a number sentence. Allow learners to complete the calculation on their own. Encourage learners to discuss the reasonableness of their answer in terms of what was asked. If learners continue to struggle provide additional material to learners to have additional practise.

Extension

Provide more complex word problems with complex calculations for stronger learners.

Unit 4 Percentages

Learner's Book page 138

Unit focus

- revise percentages of whole numbers
- calculate percentages of parts of a whole
- calculate percentage increase and decrease
- solve problems using percentage

Background information on percentages

Learners have worked with percentage in Grade 6, however they may require some revision. It important that learners note that a percentage is a number expressed as a part of 100. Per cent means 'per 100.' Fractions can be written as percentages.

For example, $\frac{4}{5} = \frac{80}{100} = 80\%$.
Percentages are used all around us and in business to calculate salary increases, discounts, and other general increases.

Revision exercise

Learner's Book page 138

Guidelines on how to implement this activity

Use this exercise as a means to assess learners' prior knowledge before launching into the chapter. Identify any learners requiring remediation, and perform the necessary remediation before continuing with the rest of the unit.

Suggested answers

1.1 $\frac{25}{100} = \frac{1}{4}$ **1.2** $\frac{88}{100} = \frac{22}{25}$ **1.3** $\frac{150}{100} = \frac{3}{2}$

2.1 $\frac{5}{10} \times \frac{100}{1} = \frac{500}{10}$ =50% **2.2** $\frac{99}{100} \times \frac{100}{1}$ =99% **2.3** $\frac{3}{4} \times \frac{100}{1}$ =75%

3.1 10 × 10 = 100 **3.2** 52% **3.3** $\frac{52}{100}$

3.4 $\frac{52}{100} = \frac{13}{25}$ **3.5** 100% – 52% = 48%

Exercise 1

Learner's Book page 139

Guidelines on how to implement this activity

Revise the concept of percentages. Learners must know that per cent means out of 100. If you get 90% for a test it means 90 marks out of a possible 100. In this exercise learners focus on finding percentages of whole numbers. Remind learners that when we work with whole numbers, we can write them as a fraction over a denominator of 1. This makes the multiplication easier. Do a few examples as a class, and encourage learners to manage this exercise on their own.

Suggested answers

1.1 $\frac{10}{100} \times \frac{80}{1} = \frac{80}{10} = 8$ **1.2** $\frac{25}{100} \times \frac{400}{1} = \frac{25}{1} \times \frac{4}{1} = 100$

1.3 $\frac{73}{100} \times \frac{2\,000}{1} = \frac{73}{1} \times \frac{20}{1} = 1\,460$ **1.4** $\frac{24}{100} \times \frac{150}{1} = \frac{24}{2} \times \frac{3}{1} = 36$

1.5 $\frac{19}{100} \times \frac{56}{1} = \frac{19}{25} \times \frac{14}{1} = \frac{266}{25} = 10{,}64$ **1.6** $\frac{79}{100} \times \frac{167}{1} = \frac{13\,193}{100} = 131{,}93$

2.1 $\frac{60}{100} \times \frac{80}{1} = \frac{60}{5} \times \frac{4}{1} = 48$ **2.2** $\frac{12}{100} \times \frac{10}{1} = \frac{12}{10} \times \frac{1}{1} = 1{,}2$

2.3 $\frac{5}{100} \times \frac{4\,000\,000}{1} = \frac{5}{1} \times \frac{40\,000}{1} = 200\,000$ **2.4** $\frac{120}{100} \times \frac{500}{1} = \frac{120}{1} \times \frac{5}{1} = 600$

Remedial

If learners struggled with multiplying fractions they will struggle here. Encourage these learners to revise their times tables and to simplify the numbers as much as possible. If learners require additional practise provide examples with simple calculations so that learners can focus on the concept and not the calculation. Once they can manage the concept increase the examples in complexity until they can manage at the prescribed level

Extension

Encourage learners to find examples of percentage use in magazines and newspapers. Learners can create a poster of percentages around us, and then infer the relevance of learning about percentage.

Exercise 2

Learner's Book page 140

Guidelines on how to implement this activity

Discuss with learners instances of percentage increase and decrease in real life. Use examples such as, 'Salaries to increase by 12,5%' or 'All goods marked down by 30%'. These are examples of percentage increase and percentage decrease.

Work through some examples of finding the amount if given the percentage increase or decrease together as a class. In other instances learners are required to find the percentage increase or decrease. To do this learners need to calculate the actual increase or decrease and then determine this as a percentage of the original amount. Do some examples of determining the percentage increase or decrease before learners do this activity in pairs.

Suggested answers

1.1 $\frac{7}{21} \times 100 = 33{,}34\%$

1.2 $\frac{8}{42} \times 100 = 19{,}05\%$

1.3 $\frac{20}{45} \times 100 = 44{,}45\%$

1.4 $\frac{513}{987} \times 100 = 51{,}98\%$

1.5 $\frac{135}{865} \times 100 = 15{,}61\%$

1.6 $\frac{360}{5\ 640} \times 100 = 6{,}38\%$

2 $\frac{14}{100} \times \frac{32}{1} = \frac{14}{25} \times \frac{8}{1} = 4{,}48;\ 32 - 4{,}48 = 27{,}52$

3 $500 - (\frac{25}{100} \times 500) = 500 - 125 = 375$

4 $\frac{75}{100} \times \frac{1\ 500}{1} = \frac{75}{1} \times \frac{15}{1} = 1\ 125;\ 1\ 500 + 1\ 125 = 2\ 625$

5 $\frac{70}{100} \times \frac{10}{1} = \frac{70}{10} \times \frac{1}{1} = 7;\ 10 + 7 = 17$

6 $\frac{40}{100} \times \frac{800}{1} = \frac{40}{1} \times \frac{8}{1} = 320;\ 800 - 320 = 480$

7 $189 - 156 = 33;\ \frac{33}{156} \times \frac{100}{1} = \frac{33}{39} \times \frac{25}{1} = 21{,}15\%$

8 $1\ 789 - 1\ 500 = 289;\ \frac{289}{1\ 789} \times \frac{100}{1} = 16{,}15\%$

9 $890 - 650 = 240;\ \frac{240}{650} \times \frac{100}{1} = \frac{24}{13} \times \frac{20}{1} = 36{,}92\%$

10 $1\ 700 - 1\ 090 = 610;\ \frac{610}{1\ 700} \times \frac{100}{1} = \frac{610}{17} \times \frac{1}{1} = 35{,}88\%$

11 $\frac{50}{1\ 000} \times \frac{100}{1} = \frac{50}{10} = 5\%$

Remedial

Learners may need help inputting the values into their calculators. Ensure each learner inputs the values correctly into their calculators.

Exercise 3

Learner's Book page 142

Guidelines on how to implement this activity

When problem solving, always remind learners to:

- identify what is being asked
- identify what is given
- create a number sentence
- substitute the relevant values
- calculate and check the relevance of the answer.

Do a few examples as a class and refer to each step in the process as you work through the problem. Allow learners to work in pairs and discuss the problems and their reasoning.

Suggested answers

1.1 $\frac{16\ 000\ 000}{2\ 000\ 000\ 000} \times \frac{100}{1} = \frac{16}{20} = 0{,}8\%$

1.2 $\frac{26}{100} \times \frac{16\ 000\ 000}{1} = \frac{26}{1} \times \frac{160\ 000}{1} = 4\ 160\ 000$

1.3 $\frac{10}{100} \times \frac{25\ 800}{1} = \frac{10}{1} \times \frac{258}{1} = 2\ 580$

1.4 $\frac{2}{5} \times \frac{100}{1} = \frac{2}{1} \times \frac{20}{1} = 40\%$; $\frac{40}{100} \times \frac{2\ 580}{1} = \frac{4}{1} \times \frac{258}{1} = 1\ 032$

1.5 $\frac{13}{100} \times \frac{16\ 000\ 000}{1} = \frac{13}{1} \times \frac{160\ 000}{1} = 2\ 080\ 000$; $16\ 000\ 000 + 2\ 080\ 000 = 18\ 080\ 000$

2. Toyota: $\frac{25}{100} \times \frac{15\ 760}{1} = 3\ 940$; $15\ 760 + 3\ 940 = 19\ 700$

Mercedes: $\frac{25}{100} \times \frac{76\ 500}{1} = 19\ 125$; $76\ 500 + 19\ 125 = 95\ 625$

Daihatsu: $\frac{25}{100} \times \frac{35\ 870}{1} = 8\ 967{,}50$; $35\ 870 + 8\ 967{,}50 = 44\ 837{,}50$

Corsa bakkie: $\frac{25}{100} \times \frac{45\ 600}{1} = 11\ 400$; $45\ 600 + 11\ 400 = 57\ 000$

3.1 $\frac{1}{8} \times \frac{100}{1} = 12{,}5\%$

3.2 $\frac{95}{2} = 47{,}5$ or $\frac{12{,}5}{100} \times \frac{4}{1} \times \frac{95}{1} = 47{,}5$

3.3 $\frac{48}{8} \times 47{,}5 = 285$

4.1 March: $20\ 654 - 16\ 505 = 4\ 149$ (profit); $\frac{4\ 149}{16\ 505} \times \frac{100}{1} = 25{,}14\%$

April: $13\ 765 - 17\ 654 = -3\ 889$ (loss); $\frac{3\ 889}{17\ 654} \times \frac{100}{1} = 22{,}29\%$

May: $22\ 765 - 18\ 999 = 3\ 766$ (profit); $\frac{3\ 766}{18\ 999} \times \frac{100}{1} = 19{,}82\%$

4.2 Total income: $20\ 654 + 13\ 765 + 22\ 765 = 57\ 184$

Total costs: $16\ 505 + 17\ 654 + 18\ 999 = 53\ 158$

$57\ 184 - 53\ 158 = 4\ 026$ (profit); $\frac{4\ 026}{53\ 158} \times \frac{100}{1} = 7{,}57\%$

5.1 $\frac{500}{2\ 500} \times \frac{100}{1} = 20\%$

5.2 $\frac{1}{3} \times \frac{100}{1} = 33{,}3\%$

5.3 $5 \times \frac{4}{100} \times \frac{34\ 860}{1} = 6\ 972$; $\frac{11\ 620 - 6\ 972}{5} = \frac{4\ 648}{5} = 929{,}6$l

5.4 $\frac{5}{80} \times \frac{100}{1} = 6{,}25\%$

Remedial

Provide additional support for learners by working through the initial stages of the problem with them. Help them to identify what is being asked in the problem, identify the information given, and provide support as they compile a number sentence. Allow learners to complete the calculation on their own. Encourage learners to discuss the reasonableness of their answer in terms of what was asked. If learners continue to struggle provide additional material to learners to have additional practise.

Extension

Provide more complex word problems with complex calculations for stronger learners.

Unit 5 Equivalent forms

Learner's Book page 144

Unit focus

- revise equivalent fractions
- recognise equivalence between common and decimal fractions
- recognise equivalence between common fractions, decimal fractions and percentages.

Background information on equivalent forms

- other words for equivalent are 'alikeness', 'equality' and 'similarity'
- equivalent fractions all have the same value. For example, $\frac{2}{5} = \frac{4}{10} = \frac{40}{100}$
- common fractions, decimal fractions and percentages are different forms of writing the same information.

Exercise 1 — Learner's Book page 144

Guidelines on how to implement this activity

This exercise revises equivalent fractions. Learners have worked with equivalence in Unit 1. They should manage this exercise easily and not have any problems. Revise what an equivalent fraction is and how to work with them. Learners should complete this exercise on their own.

Suggested answers

1.1 $\frac{1}{3} \times \frac{2}{2} = \frac{2}{6}$; $\frac{1}{3} \times \frac{3}{3} = \frac{3}{9}$; $\frac{1}{3} \times \frac{4}{4} = \frac{4}{12}$

1.2 $\frac{2}{7} \times \frac{2}{2} = \frac{4}{14}$; $\frac{2}{7} \times \frac{3}{3} = \frac{6}{21}$; $\frac{2}{7} \times \frac{4}{4} = \frac{8}{28}$

1.3 $\frac{8}{15} \times \frac{2}{2} = \frac{16}{30}$; $\frac{8}{15} \times \frac{3}{3} = \frac{24}{45}$; $\frac{8}{15} \times \frac{4}{4} = \frac{31}{60}$

1.4 $\frac{5}{60} \div \frac{5}{5} = \frac{1}{12}$; $\frac{5}{60} \times \frac{2}{2} = \frac{10}{120}$; $\frac{5}{60} \times \frac{3}{3} = \frac{15}{180}$

1.5 $\frac{4}{9} \times \frac{2}{2} = \frac{8}{18}$; $\frac{4}{9} \times \frac{3}{3} = \frac{12}{27}$; $\frac{4}{9} \times \frac{4}{4} = \frac{16}{36}$

1.6 $\frac{13}{24} \times \frac{2}{2} = \frac{26}{48}$; $\frac{13}{24} \times \frac{3}{3} = \frac{39}{72}$; $\frac{13}{24} \times \frac{4}{4} = \frac{52}{96}$

1.7 $\frac{8}{120} \div \frac{2}{2} = \frac{4}{60}$; $\frac{8}{120} \div \frac{4}{4} = \frac{2}{30}$; $\frac{8}{120} \div \frac{8}{8} = \frac{1}{15}$

1.8 $\frac{16}{64} \div \frac{2}{2} = \frac{8}{32}$; $\frac{16}{64} \div \frac{4}{4} = \frac{4}{16}$; $\frac{16}{64} \div \frac{8}{8} = \frac{2}{8}$

2.1 $\frac{4}{5} \times \frac{8}{8} = \frac{32}{40}$; $\frac{7}{8} \times \frac{5}{5} = \frac{35}{40}$; $\frac{4}{5} < \frac{7}{8}$

2.2 $\frac{12}{25} \times \frac{2}{2} = \frac{24}{50}$; $\frac{23}{50} < \frac{12}{25}$

2.3 $\frac{14}{18} \times \frac{2}{2} = \frac{28}{36}$; $\frac{14}{18} < \frac{30}{36}$

2.4 $\frac{67}{80} \times \frac{15}{15} = \frac{1\,005}{1\,200}$; $\frac{100}{120} \times \frac{10}{10} = \frac{1\,000}{1\,200}$; $\frac{67}{80} > \frac{100}{120}$

3.1 $\frac{1}{2} \times \frac{2}{2} = \frac{2}{4}$; $\frac{1}{2} \times \frac{3}{3} = \frac{3}{6}$; $\frac{1}{2} \times \frac{4}{4} = \frac{4}{8}$; $\frac{1}{2} \times \frac{5}{5} = \frac{5}{10}$; $\frac{1}{2} \times \frac{6}{6} = \frac{6}{12}$

3.2 $\frac{1}{4} \times \frac{2}{2} = \frac{2}{8}$; $\frac{1}{4} \times \frac{3}{3} = \frac{3}{12}$

3.3 $\frac{1}{3} \times \frac{2}{2} = \frac{2}{6}$; $\frac{1}{3} \times \frac{4}{4} = \frac{4}{12}$

3.4 $\frac{1}{5} \times \frac{2}{2} = \frac{2}{10}$

3.5 $\frac{3}{4} \times \frac{2}{2} = \frac{6}{8}$; $\frac{3}{4} \times \frac{3}{3} = \frac{9}{12}$

3.6 $\frac{4}{5} \times \frac{2}{2} = \frac{8}{10}$

3.7 $1 \times \frac{2}{2} = \frac{2}{2}$; $1 \times \frac{3}{3} = \frac{3}{3}$; $1 \times \frac{5}{5} = \frac{5}{5}$; $1 \times \frac{6}{6} = \frac{6}{6}$; $1 \times \frac{8}{8} = \frac{8}{8}$; $1 \times \frac{10}{10} = \frac{10}{10}$; $1 \times \frac{12}{12} = \frac{12}{12}$

Remedial

If learners had problems with this exercise refer them back to unit 1 and have them rework through all the exercises and work on equivalent fractions.

Exercise 2 — Learner's Book page 146

Guidelines on how to implement this activity

Use a table as a workable way of teaching equivalence:

Common fraction	Conversion	Decimal fraction	Conversion	Percentage	Conversion to common fraction
$\frac{7}{10}$	$7 \div 10$	0,7	$\frac{7}{10} \times \frac{100}{1}$	70 %	$\frac{70}{100} \div \frac{10}{10}$

- To convert a common fraction to a decimal fraction: Divide the numerator by the denominator.
- Find an equivalent fraction with a denominator of 10 or a power of 10 and write as H T U, t h
- To convert a decimal fraction to a percentage: Know the value of columns so 3 in the tenths column is $\frac{3}{10} = \frac{30}{100} = 30\%$ or multiply $\frac{3}{10}$ by $\frac{100}{1} = 30\%$.
- To convert a percentage to a common fraction: Write the percentage as a fraction. For example, $70\% = \frac{70}{100}$ then simplify $\frac{7}{10}$.

Do additional examples using the table on the board. This helps show the learners what we are doing and provide a means of recording their working.

Suggested answers

1 $\frac{3}{5} = \frac{6}{10} = \frac{60}{100} = \frac{600}{1\,000}$

2.1 9 tenths = 0,9
2.2 79 hundredths = 0,79
2.3 197 thousandths= 0,197
2.4 5 tenths = 0,5
2.5 14 hundredths = 0,14
2.6 1 hundredths = 0,01
2.7 52 hundredths = 0,52
2.8 624 thousandths = 0,624
2.9 58 hundredths = 0,58

3.1 $\frac{17}{100}$ **3.2** $\frac{39}{100}$ **3.3** $\frac{87}{100}$
3.4 $\frac{53}{100}$ **3.5** $\frac{88}{100} = \frac{22}{25}$ **3.6** $\frac{567}{1\,000}$

4.1 17 hundredths = 0,17
4.2 39 hundredths = 0,39
4.3 87 hundredths = 0,87
4.4 53 hundredths = 0,53
4.5 88 hundredths = 0,88
4.6 567 thousandths = 0,567

5.1 $\frac{25}{100}$ **5.2** $\frac{64}{100}$ **5.3** $\frac{5}{100}$
5.4 $\frac{564}{1\,000}$ **5.5** $\frac{125}{1\,000}$ **5.6** $\frac{3\,333}{10\,000}$

6.1 $\frac{1}{4}$ **6.2** $\frac{16}{25}$ **6.3** $\frac{1}{20}$
6.4 $\frac{141}{250}$ **6.5** $\frac{1}{8}$ **6.6** $\frac{3\,333}{10\,000}$

7.1 25% **7.2** 64% **7.3** 5%
7.4 56,4% **7.5** 12,5% **7.6** 33,33%

8

Common fractions	Decimal fractions	Percentages
$\frac{3}{4}$	0,75	75%
$\frac{9}{20}$	0,45	45%
$\frac{21}{25}$	0,84	84%
$\frac{11}{20}$	0,55	55%
$\frac{19}{20}$	0,95	95%
$\frac{6}{5}$	1,2	120%

9.1 $\frac{540}{900} \div \frac{90}{90} = \frac{6}{10}$
9.2 $900 - 540 = 360$; $\frac{360}{900} \div \frac{90}{90} = \frac{4}{10}$
9.3 6 tenths = 0,6
9.4 4 tenths = 0,4
9.5 $\frac{6}{10} \times \frac{10}{10} = \frac{60}{100} = 60\%$
9.6 $\frac{4}{10} \times \frac{10}{10} = \frac{40}{100} = 40\%$

Remedial

Encourage learners to use a table in their exercise books to keep track of their working. Have learners create a table of commonly used conversions, and have them memorise these. This will help them in their work.

Consolidation

Learner's Book page 149

Guidelines on how to implement this activity

Before doing this consolidation exercise, encourage learners to review the work covered in this chapter. Advise learners to use the summary and to revise their work. This exercise can be used as an informal assessment task for you to track how learners are coping with the chapter and the concepts covered.

Suggested answers

1 $\frac{1}{5} = \frac{200}{1\,000}$; $\frac{99}{100} = \frac{990}{1\,000}$; $\frac{4}{5} = \frac{800}{1\,000}$; $\frac{1}{4} = \frac{250}{1\,000}$; $\frac{1}{2} = \frac{500}{1\,000}$; $\frac{9}{10} = \frac{900}{1\,000}$

descending order: $\frac{99}{199}$; $\frac{9}{10}$; $\frac{798}{1\,000}$; $\frac{4}{5}$; $\frac{1}{2}$; $\frac{1}{4}$; $\frac{1}{5}$; $\frac{150}{1\,000}$ (2)

2.1 $\frac{8}{10} = \frac{4}{5}$ **2.2** $\frac{900}{1\,000} = \frac{9}{10}$ **2.3** $\frac{8}{16} = \frac{1}{2}$

2.4 $\frac{30}{100} + \frac{17}{100} = \frac{47}{100}$ **2.5** $\frac{379}{1\,000} + \frac{110}{1\,000} = \frac{489}{1\,000}$

2.6 $\frac{4}{20} + \frac{7}{20} = \frac{11}{20}$ **2.7** $2 + 3 = 5;\ 5\,\frac{1}{5} + \frac{2}{5} = 5\,\frac{3}{5}$

2.8 $1 + 3 = 4;\ 4\,\frac{2}{4} + \frac{1}{4} = 4\,\frac{3}{4}$ **2.9** $4 + 2 = 6;\ 6\,\frac{2}{6} + \frac{1}{6} = 6\,\frac{3}{6} = 6\,\frac{1}{2}$ (18)

3.1 $\frac{1}{10} \times \frac{1}{750} = 75$ **3.2** $\frac{7}{10} \times \frac{540}{1} = 7 \times 54 = 378$

3.3 $\frac{17}{1\,000} \times \frac{3\,000}{1} = 17 \times 3 = 51$ (6)

4.1 $\frac{8}{30} + \frac{5}{30} = \frac{13}{30}$ **4.2** $\frac{42}{48} - \frac{8}{48} = \frac{34}{48} = \frac{17}{24}$ **4.3** $\frac{20}{36} - \frac{15}{36} = \frac{5}{36}$ (6)

5.1 $\frac{6}{21} = \frac{2}{7}$ **5.2** $\frac{1}{2} \times \frac{3}{5} = \frac{3}{10}$

5.3 $\frac{2}{3} \times \frac{15}{4} = \frac{1}{1} \times \frac{5}{2} = \frac{5}{2} = 2\,\frac{1}{2}$ (4)

6.1 $\frac{(3\times4+3)}{4} = \frac{15}{4}$ **6.2** $\frac{(5\times100+79)}{4} = \frac{579}{4}$ **6.3** $\frac{(7\times11+8)}{11} = \frac{85}{11}$ (6)

7.1 $\frac{17}{100} \times \frac{300}{1} = 7 \times 3 = 21$ **7.2** $\frac{55}{100} \times \frac{3\,500}{1} = 55 \times 35 = 1\,925$

7.3 $\frac{99}{100} \times \frac{500}{1} = 99 \times 5 = 495$ (6)

8

Common fractions	Decimal fractions	Percentages
$\frac{7}{10}$	0,7	7%
$\frac{77}{100}$	0,77	77%
$\frac{49}{100}$	0,49	49%
$\frac{750}{1\,000}$	0,75	75%
$\frac{155}{100}$	1,55	155%

(10)

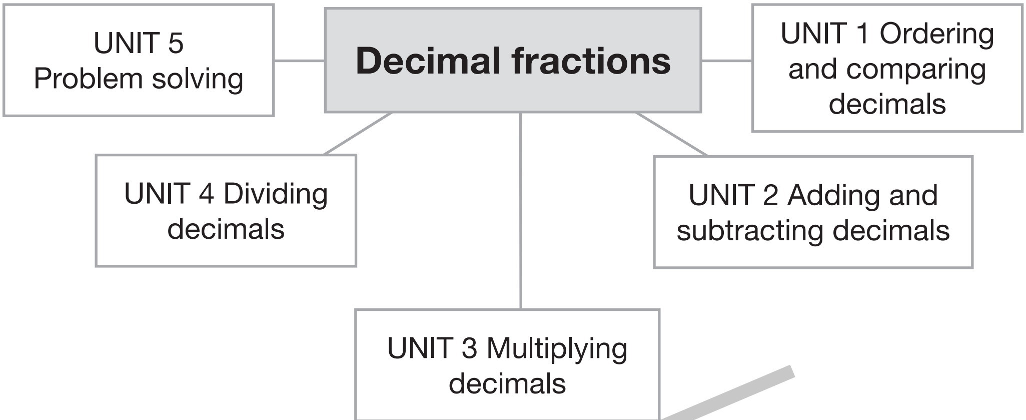
9.1 $\frac{20}{100} \times \frac{100}{1} = 20,\ 100 - 20 = 80;\ \frac{20}{100} \times \frac{500}{1} = 100,\ 500 - 100 = 400;$
$\frac{20}{100} \times \frac{450}{1} = 90,\ 450 - 90 = 360;\ \frac{20}{100} \times \frac{15}{1} = 3{,}15 - 3 = 12$ (5)

9.2 $\frac{1}{2} \times \frac{42}{1} = 21;\ \frac{1}{2} \times \frac{27{,}66}{1} = 13{,}83;\ \frac{1}{2} \times \frac{130}{1} = 65$ (5)

[70]

Chapter 6 Decimal fractions

Overview of concepts



Content		Time allocations	LB page
Unit 1	Ordering and comparing decimals	1 hour	151
Unit 2	Adding and subtracting decimals	2 hours	154
Unit 3	Multiplying decimals	3 hours	157
Unit 4	Dividing decimals	2 hours	161
Unit 5	Problem solving	1 hour	164

Background information on decimal fractions

Decimal fractions are based on 10 or a power of 10. This means, written as common fractions, they would look like this: $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1\,000}$ ... Decimal fractions are not usually written like common fractions. They follow the same pattern as natural numbers:

TTh	Th	H	T	U ,	t	h	th	tth
1	1	1	1	1 ,	1	1	1	1

Each digit to the right of the comma is a fraction, ten times less than the digit to its left.

$0{,}1 = \frac{1}{10}$ $0{,}01 = \frac{1}{100}$ $0{,}001 = \frac{1}{1\,000}$

It is thought that the Chinese first used the decimal system and that it then spread to the Middle East to Greece, Rome and Egypt.

Generic teaching guidelines for teaching decimal fractions

To teach decimals it is essential to ensure that learners understand the 10s system. Our number system is based on 10s and powers of 10 for example, 100 and 1 000.

Learners should know that decimal fractions, common fractions and percentages can be written as equivalent fractions. For example, $0,9 = \frac{9}{10} = 90\%$.

Revise that it is possible to convert common fractions to decimal fractions by division. For example, divide the numerator by the denominator, so $\frac{4}{5} = 4 \div 5 = 0,8$.

Ensure that learners are aware that some decimal fractions have more than 1 digit after the decimal point. For example, $\frac{1}{8} = 1 \div 8 = 0,125$. This means 1 tenth, 2 hundredths and 5 thousands or 125 thousandths.

Revise the concept of non-terminating or recurring decimal fractions.

For example, $\frac{1}{3} = 1 \div 3 = 0,3333\ldots$. The 3s will go on forever so we usually stop at one 3 and put a dot over it to show it is a recurring number, $0,\dot{3}$.

Resources

Decimal fraction wall, a HTU chart extending to tenths, hundredths and thousandths; grid paper for learners to do column calculations on; comparison chart for fractions, decimals and percentages, blank number lines, cardboard and colour pens for creating posters, creating flashcards and comparison cards. Each learner should have their own calculator.

Unit 1 Ordering and comparing decimals

Learner's Book page 151

Unit focus

- revise place value of decimals
- count in decimals
- compare and order decimals
- round off decimals.

Background information on ordering and comparing decimals

To order and compare decimals it is necessary to recognise the place value of the numbers. For example, in the number 123 the 1 is worth 100, the 2 is worth 20 and the 3 is 3.

Revise the Th H T U , t h th table to ensure that place value is familiar.

Number lines are a good way to show place value. For example, ask learners to fill in the missing units on the following line:

23 2… 2… 2… 27

Asking learners to arrange numbers in ascending or descending order helps to familiarise the concept of number value.

Exercise 1

Learner's Book page 151

Guidelines on how to implement this activity

Revise counting in decimals. Ask learners to do this exercise as revision. Learners need to be able to identify the interval they are counting in and then complete the number lines. Learners should complete this exercise on their own.

Suggested answers

1 24,6; 25,2; 25,8; 26,4; 30; 30,6; 31,2; 31,8; 32,4 (+ 0,6)
2 56,45; 55,34; 54,23; 53,12; 52,01; 50,9; 49,79; 48,68 (– 1,11)
3 0,21; 0,53; 0,85; 1,17; 1,49; 1,81; 2,13; 2,45; 2,77 (+ 0,21)
4 4,16; 4,18; 4,2; 4,22; 4,24; 4,26; 4,28; 4,3 (+ 0,02)
5 12,8; 11,7; 10,6; 9,5; 8,4; 7,3; 6,2 (– 1,1)
6 6,50; 6,16; 5,82; 5,48; 5,14; 4,8; 4,46; 4,12; 3,78 (– 0,34)
7 16,765; 15,764; 14,763; 13,762; 12,761; 11,76; 10,759; 9,758 (– 1,001)
8 2,854; 2,756; 2,658; 2,56; 2,462; 2,364; 2,266; 2,168 (– 0,098)
9 100,001; 101,002; 102,003; 103,004; 104,005; 105,006 (+ 1,001)
10 0,704; 0,725; 0,746; 0,767; 0,788; 0,809;0,83 (+ 0,021)

Remedial

Learners may need to revise how to calculate the counting interval. Be on hand to remind them.

Extension

Learners can extend the number lines to an additional ten numbers in each direction.

Exercise 2

Learner's Book page 153

Guidelines on how to implement this activity

Draw an HTU chart on the board, including decimal places up to ten thousandths. Explain each column and ensure learners understand that each place value is ten times less than the column before it. Show learners how to compare decimals using the HTU chart. The place value is important here and learners must understand that 2 200 is larger than 999 because of place value. Revise rounding off decimals to two decimal places. Again use the HTU chart to show learners what value to look for in the thousandths column. Do examples of each of the above concepts together as a class, and then allow learners to complete this exercise in pairs.

Suggested answers

1.1 2,9
1.2 5,3
1.3 12,3
1.4 2,73
1.5 54,69
1.6 3,333

2.1 81; 49; 27; 9

H	T	U	,	t	h	th
	2	7				
	8	1				
	4	9				
		9				

2.2 1,67; 1,57; 1,49; 1,32

H	T	U	,	t	h	th
		1	,	5	7	
		1	,	4	9	
		1	,	6	7	
		1	,	3	2	

2.3 23,9; 23,5; 23,4; 23,2

H	T	U	,	t	h	th
	2	3	,	5		
	2	3	,	2		
	2	3	,	9		
	2	3	,	4		

2.4 89,26; 87,39; 84,36; 71,45

H	T	U	,	t	h	th
	8	4	,	3	6	
	8	9	,	2	6	
	7	1	,	4	5	
	8	7	,	3	9	

2.5 176,1; 176,029; 176,01; 176,001

H	T	U	,	t	h	th
1	7	6	,	0	1	
1	7	6	,	1		
1	7	6	,	0	0	1
1	7	6	,	0	2	9

3.1 0;17; 19; 42; 137

H	T	U	,	t	h	th
		0				
	4	2				
	1	7				
1	3	7				
	1	9				

3.2 9,29; 9,32; 9,89; 9,9

H	T	U	,	t	h	th
		9	,	3	2	
		9	,	8	9	
		9	,	2	9	
		9	,	9		

3.3 5,76; 21,99; 43,79; 81,29

H	T	U	,	t	h	th
	4	3	,	7	9	
	8	1	,	2	9	
	2	1	,	9	9	
		5	,	7	6	

4.1 10 **4.2** 10 **4.3** 10
5.1 0,2 **5.2** 0,1 **5.3** 132,1
6.1 45,12 **6.2** 2,88 **6.3** 0,12
6.4 5,01 **6.5** 45,75 **6.6** 8,71

Remedial

Learners may need help rounding off whole numbers before doing decimals. Revise this with learners, especially the rules for rounding up and down.

Unit 2 Adding and subtracting decimals

Learner's Book page 154

Unit focus

- revise adding and subtracting decimals with two decimal places
- add and subtract decimals with three decimal places.

Background information on adding and subtracting decimals

When adding and subtracting decimal numbers it is important for learners to be aware of place value. Tens are added to Tens, Units to Units, Tenths to tenths etc. They must ensure that the commas are in a vertical line so that the columns will be correct. When adding or subtracting numbers always start on the right hand side at the numbers with the smallest value.

Th	H	T	U	,	t	h	th
2	3	9	5	,	8	1	4
3	6	4	2	,	3	2	9
6	0	3	8	,	1	4	3
1	1		1			1	

When adding in columns 4 + 9 = 13. The 3 thousandths is written in the th column, and the 1 hundredth is carried to the h column, written underneath (or above) and added to the hundredths. 1 + 2 + 1 = 4.

Exercise 1

Learner's Book page 155

Guidelines on how to implement this activity

Revise adding and subtracting whole numbers in columns. Do an example of each on the board. Show learners how we use the column method for adding and subtracting decimals. Draw an HTU chart on the board, and fill in the decimal values into the chart when adding and subtracting. This helps learners to be aware of the place value while they complete the calculation. Explain that the same aspects of carrying and borrowing apply to decimals as to whole numbers. Do a few examples together as a class on the board. Encourage learners to come up to the board and do some of the calculation. Learners should complete this exercise on their own.

Suggested answers

1.1 x = 100 000; y = 10 000; z = 1 000; a = 100; b = 10; c = 1

1.2 p = 0,1; r = 0,01

1.3 r = 7 000; s = 70; t = 7; v = 0,7

2.1

	M	HTH	TTH	TH	H	T	U
				2	5	6	7
			2	3	5	6	7
+		2	3	4	5	6	7
		2	6	0	7	0	1
			1	1	2	2	

2.2

	M	HTH	TTH	TH	H	T	U
	2	1	2	3	3	2	1
–	1	4	5	6	6	5	4
		6	6	6	6	6	7

3.1

M	HTH	TTH	TH	H	T	U	,	t	h	th
5	0	0	3	1	5	7	,	1	4	7

3.2 Five million, three thousand, one hundred fifty seven and one hundred forty seven thousandths

4.1

	TH	H	T	U
			2	7
+		1	2	7
+	2	1	2	7
	2	2	8	1

4.2

	TTH	TH	H	T	U
	2	3	9	8	7
–	2	1	7	8	9
		2	1	9	8

4.3

	U	,	t
	1	,	2
+	2	,	1
	3	,	3

4.4

	U	,	t
	9	,	7
–	7	,	9
	1	,	8

4.5

	H	T	U	,	t	h
		2	9	,	0	8
+	4	1	2	,	9	
	4	4	1	,	9	8

4.6

	H	T	U	,	t	h
	1	0	0	,	0	2
–		9	9	,	5	
			0	,	5	2

5.1 2 765,4 + 47 832,9 + 4 609,0 = 55 207,3

	TTH	TH	H	T	U	,	t	h	th
		2	7	6	5	,	3	9	
+	4	7	8	3	2	,	9	4	
+		4	6	0	9	,	0	1	9
	5	5	2	0	7	,	3	4	9

5.2 8 142,1 – 4 228,4 = 3 913,7

	TH	H	T	U	,	t	h
	8	1	4	2	,	0	6
–	4	2	2	8	,	3	7
	3	9	1	3	,	6	9

5.3 4 567,53 + 489,57 = 5 057,1

	TH	H	T	U	,	t	h	th
	4	5	6	7	,	5	2	8
+		4	8	9	,	5	7	4
	5	0	5	7	,	1	0	2

5.4 9 783,24 – 4 567,67 = 5 215,57

	TH	H	T	U	,	t	h	th
	9	7	8	3	,	2	3	9
–	4	5	6	7	,	6	7	2
	5	2	1	5	,	5	6	7

6.1

	HTH	TTH	TH	H	T	U	,	t	h
	2	7	6	4	5	3	,	8	4
+		6	4	8	9	2	,	7	3
+			7	8	9	3	,	6	5
	3	4	9	2	4	0	,	2	2

6.2

	HTH	TTH	TH	H	T	U	,	t	h
	8	7	2	4	2	1	,	5	5
−	6	9	3	2	5	4	,	6	7
	1	7	9	1	6	6	,	8	8

7

	T	U	,	t	h	th
	8	4				
+		3	,	6	7	
+		0	,	0	0	3
	8	7	,	6	7	3

8.1

	TH	H	T	U	,	t	h
	7	8	9	3	,	4	2
+		9	9	9	,	0	7
+			6	5	,	9	7
	8	9	5	8	,	4	6

8.2

	TH	H	T	U	,	t	h
	1	0	0	0	,	4	2
−		4	5	3	,	8	8
		5	4	6	,	5	4

9.1

	T	U	,	t	h
		5	,	3	4
+		7	,	6	
+	1	0	,	5	
	2	3	,	4	4

9.2

	TH	H	T	U	,	t	h	th
	2	1	2	3	,	5		
−		7	6	1	,	2	7	5
	1	3	6	2	,	2	2	5

10.1

	T	U	,	t	h	th
	7	0	,	8	7	9
−	6	5	,	4	9	7
		5	,	3	8	2

10.2

	H	T	U	,	t	h	th
		2	5	,	9		
+		1	7	,	6	6	5
+		8	3	,	7	8	9
	1	2	7	,	3	5	4

11.1 78,7 + 657,07 + 4 567,007

	TH	H	T	U	,	t	h	th
			7	8	,	7		
+		6	5	7	,	0	7	
+	4	5	6	7	,	0	0	7
	5	3	0	2	,	7	7	7

11.2 758,005 – 289,88

	H	T	U	,	t	h	th
	7	5	8	,	0	0	5
–	2	8	9	,	8	8	
	4	6	8	,	1	2	5

11.3 88,5 + 89,75 + 89,777

	H	T	U	,	t	h	th	th
		8	8	,	5			
+		8	9	,	7	5		5
+		8	9	,	7	7	7	5
	2	6	8	,	0	2	7	

11.4 73,2 – 18,125

	T	U	,	t	h	th
	7	3	,	2		
–	1	8	,	1	2	5
	5	5	,	0	7	5

12.1 0,99 + 0,89 + 0,63

	U	,	t	h
	0	,	9	9
+	0	,	8	9
+	0	,	6	3
	2	,	5	1

12.2 0,72 – 0,49

	U	,	t	h
	0	,	7	2
–	0	,	4	9
	0	,	2	3

12.3 0,568 – 0,329

	U	,	t	h	th
	0	,	5	6	8
–	0	,	3	2	9
	0	,	2	3	9

12.4 0,995 – 0,3499

	U	,	t	h	th	tth
	0	,	9	9	5	
–	0	,	3	4	9	9
	0	,	6	4	5	1

Remedial

Learners should be encouraged to always set the work out in their exercise book with and HTU chart. This helps them keep track of the place value. Encourage learners to check their answers with a calculator. Learners are not allowed to use a calculator to calculate the answer as they have to show all their working.

Unit 3 Multiplying decimals

Learner's Book page 157

Unit focus

- multiply decimals by whole numbers
- multiply decimals by decimals.

Background information on multiplying decimals

When multiplying decimal fractions, it is just as important as with other operations to know the place value of digits. It is important that learners grasp the concept of multiplying by 10 and powers of 10. For example: 3 × 10 = 30, which means the 3 moves to the tens columns and a 0 placeholder takes the place of the units.
3 × 100 = 300 which means the 3 moves to the hundreds column and 0 placeholders take the places of the tens and units.
When multiplying a number with 1 decimal place by a whole number, the answer should have 1 decimal place. For example, 12,4 × 3 = 37,2. When multiplying 2 numbers each with 1 digit after the decimal point the answer should have 2 digits after the decimal place. For example, 1,2 × 1,2 = 1,44.

Revision exercise

Learner's Book page 157

Guidelines on how to implement this activity

This exercise serves as revision of multiplication of powers of 10 and of decimals. Use this exercise to assess learners' prior knowledge and to identify if any remediation needs to take place before starting the rest of the chapter.

Suggested answers

1.1	30	**1.2**	110	**1.3**	4 560	**1.4**	72 230
1.5	3	**1.6**	9	**1.7**	11	**1.8**	456
2.1	500	**2.2**	700	**2.3**	6 700	**2.4**	2 930
2.5	130	**2.6**	40	**2.7**	13	**2.8**	25,7
3.1	0,6	**3.2**	2,8	**3.3**	7,2		
3.4	0,69	**3.5**	4,23	**3.6**	37,44		

Remedial

If learners experience problems with this chapter, revise multiplying by 10 and powers of 10. Revise basic decimal multiplication by working through the exercise together as a class.

Exercise 1

Learner's Book page 159

Guidelines on how to implement this activity

Introduce learners to multiplication with decimals by revising multiplying whole numbers by 10.

Use an HTU chart to show how the multiplication by 10 changes the place value, first with whole numbers and then with decimals. Extend learners to multiplying decimals by powers and multiples of 10, again using the HTU chart. Do a few examples together as a class. Introduce learners to multiplying decimals by whole numbers. Show them how to keep track of the number of decimal places and how these decimal places must be in the answer. Explain why the decimal places work this way, by showing how we multiply out by 10 or 100 to remove the decimals, and then divide back by 10 or 100 to ensure the answer is correct. Do as many examples together as class as you feel is necessary for learners to fully understand the concept. Learners should do this exercise on their own.

Suggested answers

1.1 It moves one space (column) to the left.

1.2 It would move 2 spaces (columns) to the left, i.e. 0,3 × 100 = 3
It would move 3 spaces (columns) to the left, i.e. 0,3 × 1 000 = 300

2.1 74 **2.2** 25

2.3 46 **2.4** 124

2.5 437,3 **2.6** 123

2.7 378 **2.8** 895

3.1 80; 830; 8 760; 784 530

3.2 3; 17; 7, 6; 434, 3

3.3 8; 30; 7 600; 845 300

3.4 30; 170; 76; 4 343

3.5 8 000; 83 000; 876 000; 78 453 000

3.6 300; 1 700; 760; 43 430

4.1 3,2 × (10 × 3) = 32 × 3 = 96; 4,05 × (10 × 5) = 40,5 × 5 = 202,5;
6,023 × (100 × 3) = 602,3 × 3 = 1 806,9

4.2

$$\begin{array}{rrrrrrr} & & 1 & 2 & , & 2 & 6 \\ \times & & 3 & 1 & & & \\ \hline & 3 & 8 & 0 & , & 0 & 6 \end{array} \qquad \begin{array}{rrrrrrrrr} & & & 8 & 9 & , & 2 & 9 & 1 \\ \times & & & 4 & 7 & & & & \\ \hline & 4 & 1 & 9 & 6 & , & 6 & 7 & 7 \end{array}$$

$$\begin{array}{rrrrrrrr} & & & 7 & , & 6 & 2 & 3 \\ \times & & 5 & 4 & & & & \\ \hline & 4 & 1 & 1 & , & 6 & 4 & 2 \end{array}$$

5 When multiplying a number x by a multiple of ten, rewrite the multiple of ten as a product of ten multiplied by a whole number. Then first multiply x with ten and then multiply that answer with the whole number.
When multiplying a number with a power of ten, you shift the digits of that number to the left. The exponent tells you how many spaces you should shift.

6.1 2,3 × (10 × 2) = 23 × 2 = 64; 3,1 × (10 × 3) = 31 × 3 = 93; 2,4 × (10 × 4) = 24 × 4 = 96; 2,3 × (100 × 2) = 230 × 2 = 640; 3,1 × (100 × 3) = 310 × 3 = 930; 2,4 × (100 × 4) = 240 × 4 = 960

6.2 0,4 × 1 000 = 400; 0,012 × 1 000 = 12; 1,2 × (20 × 20) = 1,2 × (10 × 2) × (10 × 2) = 12 × 2 × (10 × 2) = 24 × (10 × 2) = 480

Remedial

Do additional examples of multiplying decimals by whole numbers. Work in a small groups with learners experiencing problems, and work in a very simple step by step way explaining each step in the process. Allow the learners to work with you through each step and encourage them to ask questions whenever they do not understand.

Exercise 2

Learner's Book page 160

Guidelines on how to implement this activity

When multiplying decimals by decimals it is important to keep track of the decimal places. Learners can work with the fact that the total number of decimal places need to be reflected in the answer, but they also need to be shown why this is. Demonstrate by means of an example that we multiply by powers of ten to remove the decimals, and that we have to divide our answer by the same powers of 10 to get an accurate answer at the end. Learners can use the column method of multiplication when calculating. Work through the worked examples in the Learner's Book, on the board, and explain the process as you work though each problem. Learners should complete this exercise on their own.

Suggested answers

1.1

$$\begin{array}{r} 1{,}69 \\ \times \quad 4{,}2 \\ \hline 7{,}098 \end{array}$$

1.2

$$\begin{array}{r} 7{,}3 \\ \times \quad 32{,}58 \\ \hline 237{,}104 \end{array}$$

1.3

$$\begin{array}{r} 901{,}34 \\ \times \quad 5{,}3 \\ \hline 4777{,}102 \end{array}$$

1.4

$$\begin{array}{r} 2{,}567 \\ \times \quad 8{,}34 \\ \hline 21{,}40878 \end{array}$$

1.5

$$\begin{array}{r} 4{,}662 \\ \times \quad 2{,}34 \\ \hline 10{,}90908 \end{array}$$

1.6

$$\begin{array}{r} 2{,}195 \\ \times \quad 4{,}23 \\ \hline 9{,}28485 \end{array}$$

2.1

$$\begin{array}{r} 24 \\ \times \quad 3{,}6 \\ \hline 86{,}4 \end{array}$$

2.2

$$\begin{array}{r} 0{,}46 \\ \times \quad 34 \\ \hline 15{,}64 \end{array}$$

2.3

$$\begin{array}{r} 1{,}023 \\ \times \quad 87 \\ \hline 89{,}001 \end{array}$$

2.4

$$\begin{array}{r} 24{,}56 \\ \times \quad 21 \\ \hline 515{,}76 \end{array}$$

2.5

$$\begin{array}{r} 1{,}04 \\ \times \quad 12 \\ \hline 12{,}48 \end{array}$$

3.1

$$\begin{array}{r} 2,4 \\ \times\ 3,6 \\ \hline 8,64 \end{array}$$

3.2

$$\begin{array}{r} 0,46 \\ \times\ 3,4 \\ \hline 1,564 \end{array}$$

3.3

$$\begin{array}{r} 1,023 \\ \times\ 8,7 \\ \hline 8,9001 \end{array}$$

3.4

$$\begin{array}{r} 24,56 \\ \times\ 2,1 \\ \hline 51,576 \end{array}$$

4.1

$$\begin{array}{r} 1,12 \\ \times\ 2,12 \\ \hline 2,3744 \end{array}$$

4.2

$$\begin{array}{r} 4,33 \\ \times\ 0,12 \\ \hline 0,5196 \end{array}$$

4.3

$$\begin{array}{r} 4,13 \\ \times\ 5,21 \\ \hline 21,5173 \end{array}$$

Remedial

It may be necessary to revise the column method for multiplication before working with the decimals. Revise the structure and how we set it out and how it works. Do a few whole number examples before progressing to multiplying the decimals.

Unit 4 Dividing decimals

Learner's Book page 161

Unit focus

- divide decimals.

Background information on dividing decimals

When dividing decimal numbers it is important that learners remain aware of the place value of digits. Dividing a decimal fraction by 10 or a power of 10, means the place value drops and the number moves to the right. For example, 20,3 ÷ 10 = 2,03 (remember: 20 ÷ 10 = 2). The 2 has moved from the Tens column to the Units column. It is important that learners estimate roughly what the answer will be before dividing. This helps learners assess the reasonableness of the answers.

Revision exercise

Learner's Book page161

Guidelines on how to implement this activity

Before learners divide decimals, it is important that they are able to divide whole numbers and that they can round off correctly. Use this exercise to assess learners' ability and provide the necessary remediation if required.

Suggested answers

1.1 12 **1.2** 282 **1.3** 187
1.4 1,2 **1.5** 28,2 **1.6** 1,87
2.1 20 **2.2** 30
2.3 290 **2.4** 1 700
3.1 1,2 **3.2** 78,9
3.3 452,1 **3.4** 8776,0

Remedial

If learners struggle with this exercise provide appropriate Grade 6 material for learners to practise the relevant skill.

Exercise 1

Learner's Book page 163

Guidelines on how to implement this activity

Discuss division as the inverse operation to multiplication. Using the HTU chart show how dividing by 10 means the place value shifts one column to right and we use 0 as place holders. Show learners how to divide decimals by whole numbers that do not have a remainder. Show learners how to use short and long division. Do a few examples as a class, have learners come up to the board to fill in various parts of the calculation. Next show learners what to do when there should be a remainder. This can be confusing for learners so provide a few examples. Assign this exercise in steps and work through the answers together before continuing with the next question.

Suggested answers

1.1 0,26 **1.2** 3,263 **1.3** 0,9734
1.4 0,576 **1.5** 0,500987 **1.6** 0,000876

2.1
$$\begin{array}{r|l} 3 & 5{,}19 \\ \hline & 1{,}73 \end{array}$$

2.2
$$\begin{array}{r|l} 7 & 50{,}82 \\ \hline & 7{,}26 \end{array}$$

2.3
$$\begin{array}{r|l} 9 & 88{,}38 \\ \hline & 9{,}82 \end{array}$$

2.4
$$\begin{array}{r|l} 12 & 43{,}322 \\ \hline & 3{,}611 \end{array}$$

2.5
$$\begin{array}{r|l} 25 & 807{,}925 \\ \hline & 32{,}317 \end{array}$$

2.6
$$\begin{array}{r|l} 42 & 26{,}544 \\ \hline & 0{,}632 \end{array}$$

2.7
$$\begin{array}{r|l} 15 & 0{,}675 \\ \hline & 0{,}045 \end{array}$$

2.8
$$\begin{array}{r|l} 15 & 21{,}675 \\ \hline & 1{,}445 \end{array}$$

2.9
$$\begin{array}{r|l} 32 & 76{,}544 \\ \hline & 2{,}392 \end{array}$$

3.1 $34 \div 9 = 4$
3.2 $36 \div 4 = 9$
3.3 $1\,000 \div 50 = 20$
3.4 $60 \div 20 = 3$

4.1

$$\begin{array}{r|l} 9 & 3\,4\,,\,2 \\ \cline{2-2} & \;\;3\,,\,8 \end{array}$$

4.2

$$\begin{array}{r|l} 4 & 3\,6\,,\,4 \\ \cline{2-2} & \;\;9\,,\,1 \end{array}$$

4.3

$$\begin{array}{r|l} 5\,0 & 9\,9\,9\,,\,5\,3\,0 \\ \cline{2-2} & \;\;1\,9\,,\,9\,9\,0\,6 \end{array}$$

4.4

$$\begin{array}{r|l} 2\,0 & 6\,2\,,\,1\,2\,4 \\ \cline{2-2} & \;\;3\,,\,1\,0\,6\,2 \end{array}$$

5 Help learners who do not know how to use their calculators.

Remedial

Spend additional time revising short and long division of whole numbers. Learners who struggle with mathematics usually find division particularly problematic, and will require revision of whole number division before attempting division with decimals.

Unit 5 Problem solving

Learner's Book page 164

Unit focus

- solve problems involving adding and subtracting decimals
- solve problems involving multiplying decimals
- solve problems involving dividing decimals by whole numbers.

Background information on problem solving

Ensure learners use the steps for problem solving:
- identify what is being asked and Identify what is given.
- create a number sentence to find what is required.
- estimate your answer.
- substitute in values and calculate.
- check the reasonableness of your answer.

Many problems relating to decimal numbers will be based on money, mass, length and other forms of measurement. Rands and cents are always written as decimal numbers for exampe R5,89. The 5 represents ones or Units. It also means the number of Rand. The 8 represents tenths of a Rand (100 cents) which means it's less than a Rand, so it is a fraction. $\frac{8}{10}$ of 100c = 80 cents. The 9 represents hundredths of Rands which means it's less than a Rand, so it is a fraction. $\frac{9}{100}$ = 9 cents. Problem solving is an important aspect of life for everyone. How to deal with everyday problems can be learned from learning problem solving skills at school.

Exercise 1

Learner's Book page 165

Guidelines on how to implement this activity

Ask learners to identify where decimals are used in everyday life.

Encourage learners to provide as many examples as possible, but direct learners to examples with money, measurement, and financial percentages such as interest rates. Relate these real life experiences of decimals to problem solving. Discuss possible problems that can occur with decimals. Work through the example in the Learner's Book. Discuss the steps for problem solving and highlight which step you are doing for learners as you complete each step in the process. Encourage learners to refer back to the operations they performed on decimals to help them complete this exercise. Learners can discuss the approach to each problem in the exercise in groups, but should record their workings on their own.

Suggested answers

1 1,2 + 0,5 = 1,7 (estimate) 1,23 + 0,45 = 1,68
2 45,5 – 5 = 40,5 (estimate) 45,65 – 4,77 = 40,88
3 (490 ÷ 4) × 3 = 122,5 × 3 = 367,5 (estimate) (488,88 ÷ 4) × 3 = 122,22 × 3 = 366,66
4 (456,7 + 34,3) – (25 × 12) = 491 – 300 = 191 (estimate)
(456,782 + 34,34) – (24,7 × 12) = 491,112 – 296,4 = 194,712
5.1 Bloemfontein **5.2** Durban
5.3 1 391 – 399 = 392 (estimate) 1 390,789 – 998,768 = 392,021
5.4 1 045 + 1 606 + 999 + 1 391 = 5 041 (estimate)
1 045,39 + 1 605,999 + 998,768 + 1 390,789 = 5 040,946
e. 2 × 999 = 1 998 (estimate) 2 × 998,768 = 1 997,536
5.5 1 391 – 1 045 = 346 (estimate) 1 390,789 – 1 045,39 = 345,399
5.6 345,399
6 70 – 15 – 7 – 46 = 2 (estimate)
70,45 – (14,5 + 7,25 + 45,69) = 70,45 – 67,44 = 3,01 (extra money)
7 46 – 45,5 = 0,5 (estimate) 46,25 – 45,55 = 0,7
8 7 900 ÷ 5 = 1 580 (estimate) 7 894,95 ÷ 5 = 1 578,99
9 (60 + 60) – (60 + 55) = 120 – 115 = 5 (estimate)
(60 + 59,58) – (60 + 55,45) = 119,58 – 116,45 = 4,13

Remedial

Learners often struggle to create the correct number sentence for problems, but are able to perform the calculation. Ask learners who are experiencing problems to ask you to check their number sentence before they progress to the calculation. If learners are struggling to formulate the number sentence, provide specific guidance and pointers to help them.

Extension

As a class discuss the answers from this exercise, and ask learners to create additional problems using decimals from newspaper and magazines.

Consolidation

Learner's Book page 167

Guidelines on how to implement this activity

Before doing this consolidation exercise, encourage learners to review the work covered in this chapter. Advise learners to use the summary and to revise their work. This exercise can be used as an informal assessment task for you to track how learners are coping with the chapter and the concepts covered.

Suggested answers

1 3 176,029; 376,1; 376,011; 376,001

TH	H	T	U	,	t	h	th
	3	7	6	,	0	1	1
	3	7	6	,	1		
	3	7	6	,	0	0	1
3	1	7	6	,	0	2	9

2 25,999; 45,54; 45,716; 51,25

T	U	,	t	h	th
4	5	,	5	4	
5	1	,	2	5	
2	5	,	9	9	9
4	5	,	7	1	6

(2)(2)

3.1

$$\begin{array}{r} 379{,}05 \\ +\ 492{,}2 \\ \hline 871{,}25 \end{array}$$

3.2

$$\begin{array}{r} 300{,}011 \\ -\ 97{,}5 \\ \hline 202{,}511 \end{array}$$

(4)

4.1 0.6 **4.2** 0,3 **4.3** 192,1 (3)

5.1

$$\begin{array}{r} 0{,}33 \\ \times\ \phantom{0{,}3}3 \\ \hline 0{,}99 \end{array}$$

5.2

$$\begin{array}{r} 2{,}52 \\ \times\ \phantom{2{,}5}3 \\ \hline 7{,}56 \end{array}$$

5.3

$$\begin{array}{r} 7{,}26 \\ \times\ \phantom{7{,}2}6 \\ \hline 56{,}56 \end{array}$$

(3)

6.1

$$\begin{array}{r} 1{,}23 \\ \times\ 4{,}2 \\ \hline 5{,}166 \end{array}$$

6.2

$$\begin{array}{r} 32{,}12 \\ \times\ 2{,}3 \\ \hline 73{,}876 \end{array}$$

6.3

$$\begin{array}{r} 904{,}34 \\ \times\ 6{,}3 \\ \hline 5697{,}342 \end{array}$$

(3)

7.1

$$\begin{array}{r|l} 4 & 5{,}16 \\ \hline & 1{,}29 \end{array}$$

7.2

$$\begin{array}{r|l} 5 & 51{,}85 \\ \hline & 10{,}37 \end{array}$$

7.3

$$\begin{array}{r|l} 12 & 43{,}32 \\ \hline & 3{,}61 \end{array}$$

(3)

8 106 – 55 – 9 – 46 = –4 (estimate) (3)

105,63 – (54,50 + 9,25 + 45,69) = 105,63 – 109,44 = –3,81

9

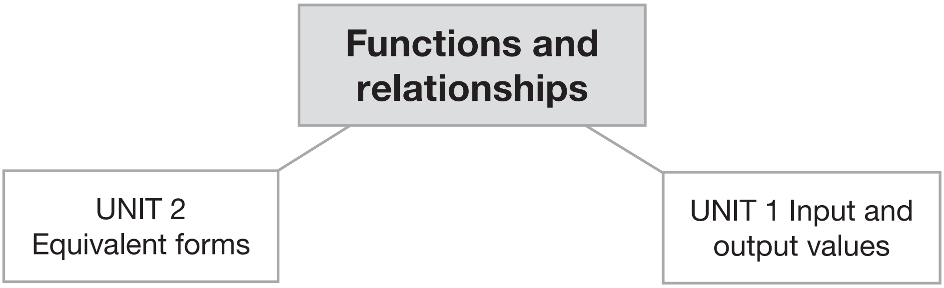
Decimal fraction	Common fraction	Percentage
0,76	$\frac{76}{100}$	76%
0,125	$\frac{125}{1\,000}$	12,5%
0,72	$\frac{18}{25}$	72%
0,0375	$\frac{375}{1\,000}$	3,75%
0,94	$\frac{94}{100}$	94%
1,04	$\frac{104}{100}$	104%

(12)

[35]

Chapter 7 Functions and relationships

Overview of concepts



Content		Time allocations	LB page
Unit 1	Input and output values	1,5 hours	169
Unit 2	Equivalent forms	1,5 hours	177

Background information on functions and relationships

In mathematics patterns, functions and relationships form the basis of problem solving. Problem solving requires rules which are based on structures. For example, patterns. Finding a variable or unknown requires understanding of functions and relationships. All number patterns, including ordinary counting numbers, are solved by discovering or using the rules which control their relationship. For example, if the input number is 5, the rule is plus 1 what is the output number?

5 → + 1 → *b*
5 → + 1 → 6 $b = 6$

Generic teaching guidelines for teaching functions and relationships

When teaching patterns it is always useful to:

- Draw a flow diagram on the board like this:

Input		Rule		Output
x	→	+ 7	→	10

Then investigate how the flow diagram works by substituting a number. For example, $x = 3$.
Be sure to explain that the ‘rule’ is the application of one or more mathematical operations. For example, + or × or – or ÷ .

- Transfer the information on the flow diagram to a table. Demonstrate on the board.

Input	x
Rule	+ 7
Output	10

Discuss how the flow diagram and table both represent the same relationship.

Resources

Blank flow diagrams and tables, number cards, and arrow cards. Each learner should have their own calculator.

Unit 1 Input and output values

Learner's Book page 169

Unit focus

- learn about and work with rules
- learn about the different forms used to represent relationships
- determine input and output values
- determine the function or rule.

Background information on input and output values

Flow diagrams are a way to structure a problem. They allow one to see the question clearly. For example: Tom has 3 more marbles than Thabo, who has 5 marbles. How many marbles has Tom?

Set the information out in a flow diagram:

- We know Thabo has 5 marbles, so this 5 the input value.
- We know Tom has 3 more marbles than Thabo, so the rule is + 3.
- We need to find the output value, the number of marbles Tom has.

Input (Thabo's marbles)		Rule		Output (Tom's marbles)
5	→	+ 3	→	y

The output (Tom's marbles) $y = 8$

Exercise 1

Learner's Book page 170

Guidelines on how to implement this activity

Learners should be familiar with working with flow diagrams and input and output values. This exercise serves to revise these concepts by asking learners to identify input and output values. Do a few examples as a class. Learners should complete this exercise on their own.

Suggested answers

Given	Input	Rule	Output
2; 6; × 3	2	× 3	6
+ 5; 3; 8	3	+ 5	8
28; × 7; 4	4	× 7	28
93; 87; + 6	87	+ 6	93
240; × 12; 20	20	× 12	240
999; 899; + 100	899	+ 100	999
1 879; – 21; 1 900	1 900	– 21	1 879
× 5; 857; 4 285	857	× 5	4 285
+ 499; 1 706; 1 207	1 207	+ 499	1 706

2.1 $z = 2 \times 5 = 10$
2.2 $z = 3 \times 5 = 15$
2.3 2 and 3
2.4 10 and 15
2.5 $y \times 5 = z$

Remedial

Learners should not have any problem identifying the inputs and outputs, but if they struggle provide blank flow diagrams for learners to use. These will help learners arrange the number and make identifying the values easier.
Then create black and white patterns on the rest of the squares. Use this as a background on which to glue and cut out letters for a poster.

Exercise 2

Learner's Book page 173

Guidelines on how to implement this activity

Learners need to be able to find input and output values from flow diagrams, tables and formulae. Do examples of each, finding input and output values, but pay special attention to finding the input values. This is trickier and involves working with the flow diagram or formula backwards. Before starting the activity allow learners to try some examples in small groups or pairs. Learners should complete this exercise on their own.

Suggested answers

1.1 $5 \times 7 + 6 = 41$; $6 \times 7 + 6 = 48$; $7 \times 7 + 6 = 55$; $8 \times 7 + 6 = 62$
1.2 $40 \times 10 - 4 = 396$; $42 \times 10 - 4 = 416$; $44 \times 10 - 4 = 436$; $46 \times 10 - 4 = 456$
2.1 $(x \div 5) \times 2 = y$; $(15 \div 5) \times 2 = 3 \times 2 = 6$
2.2 $(x \div 7) \times 294 = \text{y}$; $(49 \div 7) \times 294 = 7 \times 294 = 2\,058$
2.3 $(x + 8) \div 8 = \text{y}$; $(128 + 8) \div 8 = 136 \div 8 = 17$
3.1 $p = 1 \times 6 = 6$; $q = 3 \times 6 = 18$; $r = 4 \times 6 = 24$
3.2 $s = 5 + 7 = 12$; $t = 10 + 7 = 17$; $u = 20 + 7 = 27$
3.3 $f = (9 \times 3) + 2 = 27 + 2 = 29$; $g = (25 \times 3) + 2 = 75 + 3 = 77$;
$h = (32 \times 3) + 2 = 96 + 2 = 98$
3.4 $m = (8 + 5) \times 2 = 13 \times 2 = 26$; $j = (34 \div 2) - 5 = 17 - 5 = 12$; $k = (58 \div 2) - 5 = 24$;
$n = (32 + 5) \times 2 = 37 \times 2 = 74$

4.1 Input Rules Output

$5 \rightarrow \boxed{\times 3} \rightarrow \boxed{-4} \rightarrow y$

4.2 $y = (5 \times 3) - 4 = 15 - 4 = 11$

5.1 Input Rules Output

$27 \rightarrow \boxed{\times 12} \rightarrow \boxed{+3} \rightarrow z$

$z = (27 \times 12) + 3 = 324 + 3 = 327$

5.2 Input Rules Output

$27 \rightarrow \boxed{+53} \rightarrow \boxed{\times 2} \rightarrow z$

$y = (39 + 53) \times 2 = 92 \times 2 = 184$

5.3 Input Rules Output

$342 \rightarrow \boxed{-275} \rightarrow \boxed{\times 21} \rightarrow w$

$w = (342 - 275) \times 21 = 67 \times 21 - 1\ 407$

5.4 Input Rules Output

$42 \rightarrow \boxed{\times 9} \rightarrow \boxed{-53} \rightarrow v$

$v = (42 \times 9) - 53 = 378 - 53 = 325$

5.5 Input Rules Output

$p \rightarrow \boxed{\times 12} \rightarrow \boxed{+5} \rightarrow 77$

$p = (77 - 5) \div 12 = 72 \div 12 = 6$

6 $z = (46\ 137 \times 2) \div 13 - 5\ 309 = 92\ 274 \div 13 - 5\ 309 = 7\ 098 - 5\ 309 = 1\ 789$

7.1 $\frac{2}{6}$ **7.2** $\frac{5}{15}$ **7.3** $\frac{12}{36}$

8.1

Input	1	2	3	4	5	6	7	8	9
Output Rule: × 5; + 4	9	14	19	24	29	34	39	44	49

8.2

Input	5	6	7	8	9	10	15	20	100
Output Rule: – 3; × 3	6	9	12	15	18	21	36	51	291

8.3

Input	1	3	5	10	15	20	30	50	100
Output Rule: + 1; × 7	14	28	42	77	112	147	217	357	707

9.1 $A = \frac{1}{2} \times 6 \times 12 = 3 \times 12 = 36$

9.2 $S = \frac{60}{5} = 12$

9.3 $60 = u + (10 \times 5); 60 = u + 50; u=10$

9.4 $f = (50 \times 2 - 8) \div 4 = \frac{92}{4} = 23$

Remedial

Provide additional support for learners when substituting values in the formula to try find the input value as this is fairly new to learners.

Extension

Encourage learners to create their own patterns and flow diagrams, and to determine different input and output values.

Exercise 3

Learner's Book page 176

Guidelines on how to implement this activity

In order to find the function performed on the input value to obtain the output value, learners need to try trial and error by proposing a possible function and substituting in the input value. If learners obtain the correct output value then they know their function is correct. However, it is important to show learners that the function must work for all given values, not just one value.
For example, given the table:

Input	1	2	3	4
Output	2	5	10	17

If we only used the first input and output value we would think the rule was + 1, but we can see that this does not work for the other input and output values. Learner must test their function for all given values to see if it is valid. Learners can complete this exercise in small groups or pairs. Encourage learners to share their thinking processes with one another.

Suggested answers

1.1 Divide by 3, multiply 10
1.2 Divide by 99, plus 3
1.3 Subtract 7
1.4 Plus 23
1.5 Multiply 5
1.6 Divide by 3
1.7 Divide by 8, plus 54
1.8 Subtract 13
1.9 Multiply 2
2 Yes
2.1 Plus 56
2.2 Subtract 781
2.3 Divide by 12, plus 81
2.4 Subtract 78, multiply 2
2.5 Plus 80
3 Rule: Number of booklets × 42,20

Number of booklets	1	2	3	4	8	16	20
Price	R42,20	R84,40	R126,60	R168,80	R337,60	R675,20	R844

4 Rule: (Input × 60) + 5

Input	1	2	3	4	5	6	7
Output	65	125	185	245	305	365	425

5 Rule: × 3
4 × 3 = 12; 5 × 3 = 15

Remedial

Provide additional time if required for learners to work through this exercise. Determining functions becomes very important in later grades and the necessary groundwork should be given the appropriate time to be explored.

Unit 2 Equivalent forms

Learner's Book page 177

Unit focus

- determine and interpret the different forms functions and relationships can be presented in
- justify and show the equivalence between the different forms.

Background information on equivalent forms

Learners know that relationships between numbers can be represented in different ways. The different forms learners need to know in Grade 7 are: verbal descriptions; flow diagrams; tables; formulae and number sentences. This unit focuses on learners being able to translate between the different representations. In later grades the emphasis is on translating equations to graphs, however as learners have yet to cover graphs, focus on translating between equations and tables as an interim step to later grades.

Revision exercise

Learner's Book page 177

Guidelines on how to implement this activity

This exercise revises completing flow diagrams and tables. Learners should be able to manage on their own as they have revised the necessary concepts in the previous unit.

Suggested answers

1 $a = (9 + 6) \div 3 = 15 \div 3 = 5$; $b = (7 - 2) \times 7 = 5 \times 7 = 35$; $c = (5 \times 12) + 3 = 60 + 3 = 63$

2

Number of learners	1	2	3	4	5	10	15
Number of family members	3	7	11	15	19	39	59

Remedial

If learners have problems completing this exercise, revise the previous unit's work with them.

Exercise 1

Learner's Book page 178

Guidelines on how to implement this activity

Learners need to be able to translate fluidly between the different representations. It must be made clear that despite being in different forms, each form is equivalent. Discuss each form and the pros and cons of depicting the relationship in each form. Do some examples of representing a relationship in each of the different ways. Allow learners to do some of the initial examples in small groups, but encourage them to work on their own by the end of the exercise.

Suggested answers

1.1 2 + 2 = 4

1.2

Number of days	1	2	3	4	5	6	7	8	9	10	11
Number of SMSs	4	8	12	16	20	24	28	32	36	40	44

1.3 Number of days

1.4 Number of SMSs

1.5 Input Rule Output

Number of days → $\boxed{\times 4}$ → Number of SMSs

2.1 7 + 4 = 13

2.2

SMS	1	2	3	4	5	6	7	8	9	10
Number of characters in each SMS	7	11	15	19	23	27	31	35	39	43

2.3 Number of characters = (4 × SMS) + 3

3.1 6 × 430,50 = 2 583

3.2

Day	1	2	3	4	5	6
Total profit made	R430,50	R861	R1 291,50	R1 722	R2 152,50	R2 583

3.3

Week	1	2	3	4	5
Total profit made	R2 583	R5 166	R7 749	R10 332	R12 915

3.4 52 × 2 583 = 134 361

4.1

Input odd numbers	5	7	9	11
Rule	× 7	× 7	× 7	× 7
Output	35	49	63	77

4.2 Yes, because it is an equivalent way of representing the same information.

5

Input	5	10	15	20	40
Rule 1	+ 7	+ 7	+ 7	+ 7	+ 7
Rule 2	× 7	× 7	× 7	× 7	× 7
Output	84	119	154	189	329

6

Number of Pine trees	9	45	b	c	99	32
Rule	× 12	× 12	× 12	× 12	× 12	× 12
Total litres of water	y	z	84	3 288	1 188	3 84

6.1 $y = 108$

6.2 $z = 48$

6.3 $b = 84 \div 12 = 7$

6.4 $a = 3\ 288 \div 12 = 274$

6.5 × 12

6.6 × 12

7

Input	3	8	15	x	y
Rule 1	× 5	× 5	× 5	× 5	× 5
Rule 2	+ a	+ a	+ a	+ a	+ a
Output	b	c	87	112	612

$(15 \times 5) + a = 87$; $75 + a = 87$; $a = 12$

$b = (3 \times 5) + 12 = 15 + 12 = 27$;

$c = (8 \times 5) + 12 = 40 + 12 = 52$

$x = (112 - 12) \div 5 = 100 \div 5 = 20$;

y = $(612 - 12) \div 5 = 600 \div 5 = 120$

8 $p = (6 \times 7) - 5 = 42 - 5 = 37$; $q = (9 \times 7) - 5 = 63 - 5 = 58$; $r = (12 \times 7) - 5 = 84 - 5 = 79$

8.1 The numbers on the far left in the flow diagram.

8.2 × 7 and – 5

8.3 $p = 37$

8.4 $q = 58$

8.5 $r = 79$

9.1

Input	3	6	9	12
Output	16	32	48	64

9.2 No, they do not.

9.3 Yes, it does.

10.1 Output = Input × 4

$3 \times 4 = 12$; $4 \times 4 = 16$

10.2 Output = Input × 7

$6 \times 7 = 42$; $8 \times 7 = 56$

10.3 Output = (Input × 2) + 5

$(4 \times 2) + 5 = 13$; $(5 \times 2) + 5 = 15$; $(6 \times 2) + 5 = 17$; $(7 \times 2) + 5 = 19$;
$(8 \times 2) + 5 = 21$; $(9 \times 2) + 5 = 23$

10.4 Output = (Input × 3) – 1
(4 × 3) – 1 = 11; (5 × 3) – 1 = 14; (6 × 3) – 1 = 17; (7 × 3) – 1 = 20
(8 × 3) – 1 = 23; (9 × 3) – 1 = 26

11.1 Yes, it can be.

11.2 Input Rules Output

$a \rightarrow \boxed{\times 5} \rightarrow \boxed{+ 2} \rightarrow b$

11.3 The output is equal to five times the input plus two.

12.1

Input	1	2	3
Rule 1	× 7	× 7	× 7
Rule 2	– 3	– 3	– 3
Output	4	11	18

12.2 Output = (Input × 7) – 3

Remedial

If learners experience any difficulty with translating between the different forms, be on hand to provide hints and reminders. Provide simpler examples for learners to revise the necessary concepts, and then increase the complexity until the required level.

Extension

Provide additional examples for learners to translate between different forms.

CHAPTER 7

Consolidation

Learner's Book page 183

Before doing this consolidation exercise, encourage learners to review the work covered in this chapter. Advise learners to use the summary and to revise their work. This exercise can be used as an informal assessment task for you to track how learners are coping with the chapter and the concepts covered.

Suggested answers

1.1 144; 132; 120; 108; 69; 84; 72

1.2 1; 3; 6; 10; 15

1.3 15; 24; 33; 42; 51; 60 (3)

2

Input	11,5	49	79	104
Rule	+ 6 ÷ 5	+ 6 ÷ 5	+ 6 ÷ 5	+ 6 ÷ 5
Output	3,5	11	17	22

(4)

3.1 Let n represent the number of loaves and R represents the revenue and p the price of a loaf of bread, $R = p \times n$
$R = 9{,}50 \times 30 = 285$ (2)

3.2 Let m presents how often Tom runs in a week, D the total distance per week and d the distance Tom runs per day.
$D = 7 \times 3 = 21$ (2)

3.3 Let P represent the pocket money per week, p the pocket money per month and k the number of weeks in a month, $P = \frac{p}{k}$
$P = \frac{24}{4} = 6$ (2)

3.4 Let S represent the number of sweets each friend gets, s the total number of sweets and n the number of friends: $S = \frac{s}{n}$ (2)
$S = \frac{49}{7} = 7$

4.1 $2(17 - 9) = 2 \times 8 = 16$; $4 \times 4 = 16$; $a = 4$ (2)

4.2 $(15 + 15 + 15) \div 9 = 45 \div 9 = 5$; $3 \times 4 - 7 = 5$; $b = 4$ (3)

4.3 $2 \times 9 = 18$; $6 \times 3 = 18$; $c = 3$ (2)

4.4 $9(12 - 4) = 9 \times 8 = 72$; $d = 9$ (3)

5.1 Input — Rules — Output

$56 \rightarrow \boxed{+\ 44} \rightarrow \boxed{\times\ 3} \rightarrow z$

$z = (56 + 44) \times 3 = 1\ 003 = 300$ (2)

5.2 Input — Rules — Output

$y \rightarrow \boxed{\times\ 15} \rightarrow \boxed{+\ 38} \rightarrow 98$

$y = (98 - 38) \div 15 = 60 \div 15 = 4$ (3)

[30]

Chapter 8 Area and perimeter

Overview of concepts

Area and perimeter

UNIT 1
Regular polygons

UNIT 2
Irregular polygons

UNIT 3
Problem solving

Content		Time allocations	LB page
Unit 1	Regular polygons	2 hours	185
Unit 2	Irregular polygons	2 hours	191
Unit 3	Problem solving	3 hours	196

Background information on area and perimeter

In Grade 6 learners did not have to use formulae to calculate area and perimeter. In Grade 7 they have to learn and use the following formulae to calculate perimeter and area respectively:

- Perimeter of a square = $4s$.
- Perimeter of a rectangle = $2(l + b)$ or $2l + 2b$.
- Area of a square = $l \times l = l^2$.
- Area of a rectangle = $l \times b$.
- Area of a triangle = $\frac{1}{2} \times (\text{base} \times \text{height}) = \frac{1}{2} \times (b \times h)$.

The main objectives in this chapter are for the learners to:

- calculate the perimeter of regular and irregular polygons
- use formulae to calculate perimeter and area of squares, rectangles and triangles
- solve problems that involve perimeter and area.

Generic teaching guidelines for teaching area and perimeter

When teaching these formulae in the different units, remind learners what each of the letters l, b, h and s stand for: length, breadth, height and side. Therefore, the area of a square can also be written as $A = s \times s = s^2$, where s is the side.
The horizontal side of a triangle is the base (b). The height (h) of a triangle is the perpendicular line segment that connects the base to the vertex.
The topic of measurement lends itself to some fun practical examples.

Learners must be reminded that when they work with perimeter and area they work with flat surfaces or in two dimensions (2D). Teach the definitions of new words such as perimeter, perpendicular and vertex:

- Perimeter is the measured length or distance around a polygon.
- Perpendicular means at a 90°angle or right angle.
- Vertex means the top or highest point.

Resources

Have examples of the different polygons available to show learners, have flash cards with the formulae on to display around the classroom, cardboard and colour pens to create posters, and rulers. Have a conversion table showing the conversion between relevant SI units. Each learner should have their own calculator.

Unit 1 Regular polygons

Learner's Book page 185

Unit focus

- revise regular polygons
- use formulae to calculate the perimeter and area of regular polygons.

Background information on regular polygons

A polygon is a two-dimensional (2D) closed figure made up of straight lines. A polygon is named according to the number of sides. If all sides have the same length and all angles are equal, it is a *regular* polygon. Use a square and an equilateral triangle to show learners, as these are two common examples of regular polygons.

Revision exercise

Learner's Book page 185

Guidelines on how to implement this activity

This exercise can be used to assess learners' prior knowledge of polygons and converting between units of length. Use this exercise to identify any problem areas you need to remediate before progressing with the exercise.

Suggested answers

1 pentagon; hexagon; heptagon; octagon

2 Each polygon is regular.

3

		1 cm	= 10 mm
	1 m	= 100 cm	= 1 000 mm
1 km	= 1 000 m		

Remedial

Work through the naming of various polygons up to decagon, using the number of sides. Revise converting between units of length.

Exercise 1 Learner's Book page 187

Guidelines on how to implement this activity

To introduce the topic of perimeter, let learners use their rulers or bring at least one tape measure so that they can measure a few things in the class. Revise what perimeter is. It is the measurement of the sides of the shape. Let learners work in small groups to measure things inside the class room, such as the perimeter of a desk or a book. If possible take learners outside in their groups so that they can measure the perimeter of a suitable area or a few of the classroom windows. Encourage learners to write down all their measurements so that they can compare their answers with the other groups' answers.
Ensure that learners are aware of units of measurement. When they take measurements inside, they will probably use rulers and work in mm and cm, but when they work outside with larger measurements, these can be done in m.
Work through the worked examples in the Learner's Book and introduce the formula for calculating perimeter. Do some examples on the board using the formula.
Encourage learners to complete the exercise on their own.

Suggested answers

1.1 $3 \times s = 3 \times 6{,}5 = 19{,}5$
1.2 $5 \times s = 5 \times 3{,}45 = 17{,}25$
1.3 $8 \times s = 8 \times 11{,}4 = 91{,}2$
1.4 $4 \times s = 4 \times 4{,}2 = 16{,}8$
2 $s = \frac{54}{4} = 13{,}5$
3.1 $4 \times s = 4 \times 4{,}5 = 18$
3.2 $18 - 0{,}9 = 17{,}2$
3.3 $4{,}5 + 1{,}2 = 5{,}7$; $4 \times 5{,}7 = 22{,}8$
3.4 $22{,}8 - 0{,}9 = 21{,}9$
3.5 $s = \frac{32}{4} = 8$
3.6 $s = 8$

Remedial

Even if tape measures or rulers are not available to everyone, learners can measure perimeter by pacing and counting their steps or paces, or using hand spans. It is important for learners to practically measure as this helps them to conceptualise.

Exercise 2 Learner's Book page 189

Guidelines on how to implement this activity

Revise the concept of area with learners. Area is the amount of space that a 2D object takes up. Revise how learners worked out area in Grade 6 – using grid paper and counting the number of blocks. Show learners using a large square on grid paper how the number of squares on the grid that make up the rectangle can be calculated by multiplying the length and the breadth of the square. Explain to learners that this is the formula for calculating the area of a rectangle.

Show learners the formula for the area of a square and a triangle. Do some examples using the formulae for finding the area of rectangles, squares and triangles. Discuss the units of measurement and that area is measured in cm^2, mm^2 or m^2. Revise converting between these units of measurement. Do at least one example of a complex shape, involving at least two shapes put together. Show learners how to deconstruct the shape and determine its area.

Suggested answers

1.1 $A = l \times l = 5{,}4 \times 5{,}4 = 29{,}16$ ($29{,}2$ m^2)

1.2 1 m = 100 cm; $29{,}2 \times (100 \text{ cm})^2 = 29{,}2 \times 10\,000 \text{ cm}^2 = 291\,600 \text{ cm}^2$

2.1 $l = \frac{64}{4} = 16$; $A = l \times l = 16 \times 16 = 256 \text{ m}^2$

2.2 $2 \times 256 = 512$

2.3 $A = \frac{1}{2} \text{ m}^2 = 0{,}5 \text{ m}^2$

2.4 We work out the square root of the area of the tile to calculate the side length, then we multiply that value by four.

3.1 30 cm = 0,3 m
Perimeter = $4 \times (2 \times 0{,}3 \text{ m}) = 2{,}4$ m
Area of one triangle = $\frac{1}{2} \times b \times h = \frac{1}{2} \times 0{,}3 \text{ m} \times 0{,}26 \text{ m} = 0{,}039 \text{ m}^2$
Area of the square = $l \times l = 0{,}3 \text{ m} \times 0{,}3 \text{ m} = 0{,}09\text{m}^2$
Total area = $(4 \times 0{,}039 \text{ m}^2) + 0{,}09 \text{ m}^2 = 0{,}156 \text{ m}^2 + 0{,}09 \text{ m}^2 = 0{,}246 \text{ m}^2$

3.2 8 cm = 80 mm
Perimeter = 12×80 mm =960 mm
Area of one triangle = $\frac{1}{2} \times b \times h = \frac{1}{2} \times 80 \text{ mm} \times 70 \text{ mm} = 2\,800 \text{ mm}^2$
Area of one square = $l \times l = 80 \times 80 = 6\,400\text{mm}^2$
Total area = $(4 \times 2\,800 \text{ mm}^2) + (4 \times 6\,400 \text{ mm}^2) = 11\,200 \text{ mm}^2 + 25\,600 \text{ mm}^2 = 36\,800 \text{ mm}^2$

3.3 4 000 mm = 400 cm; 3,5 m = 350 cm
Perimeter = $6 \times s = 6 \times 400$ cm = 2 400 cm
Area of one triangle = $\frac{1}{2} \times b \times h = \frac{1}{2} \times 400 \text{ cm} \times 350 \text{ cm} = 70\,000 \text{ cm}^2$
Total area = $6 \times 70\,000 \text{ cm}^2 = 420\,000 \text{ cm}^2$

Remedial

Allow learners who are experiencing difficulty to work in pairs. Learners may need to revise substitution in a formula and converting between different units of measurement.

Extension

Provide additional complex shapes made up of triangles, rectangles and squares, for learners to work with.

Unit 2 Irregular polygons

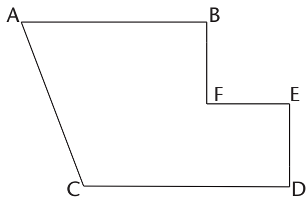
Learner's Book page 191

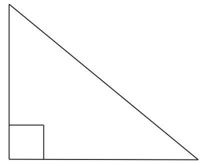
Unit focus

- revise irregular polygons
- use formulae to calculate the perimeter and area of irregular polygons.

Background information on irregular polygons

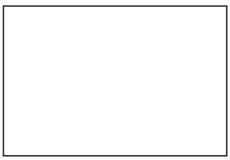
A polygon whose sides are not all equal is called **irregular**.
Introduce the topic by pointing out that it is sometimes easy to see when a polygon is irregular. For example, polygon ABCDEF. In the case of a right-angled triangle it is less obvious.



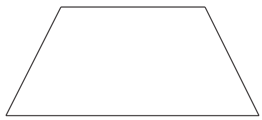


Right-angled triangle

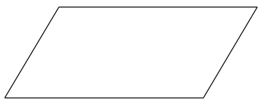
More examples of irregular polygons



Quadrilateral (4 sides)



Trapezium



Parallelogram

Exercise 1

Learner's Book page 192

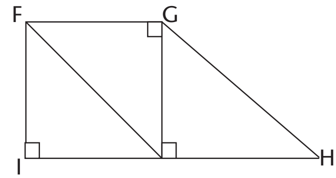
Guidelines on how to implement this activity

Discuss what makes a polygon irregular. Provide examples of shapes for learners to identify as regular or irregular. Introduce the formula for the perimeter of a rectangle. Ask learners why a rectangle is an irregular shape . Do an example using the formula to find the perimeter of a rectangle. Will this formula work for other shapes? Discuss with learners how to calculate the perimeter of other irregular shapes. Show learners how by knowing the perimeter of a shape, we can find a missing side. Do some examples demonstrating this to learners. Learners should complete this exercise on their own.

Suggested answers

1.1 AD = 29,5 cm – (5 cm + 2 × 5 cm + $\frac{1}{2}$ × 5 cm) = 29,5 cm – 17,5 cm = 12 cm
1.2 Learners' own work
1.3 Learners' own work
1.4 DE = 10 cm
1.5 Perimeter = $2l + 2b$ = 2 × 10 cm + 2 × 2,5 cm = 25 cm
1.6 AE = 5 cm – 2,5 cm = 2,5 cm Perimeter = 12 cm + 10 cm + 2,5 cm = 24,5 cm
1.7 Right angle triangle; irregular polygon, since not all the sides have the same length.
2.1 I: rectangle; II: parallelogram
2.2 No
2.3 Perimeter of I = $2l + 2b$ = 2 × 4,5 cm + 2 × 4,2 cm = 17,4 cm
2.4 Perimeter of II = $2l + 2b$ = 2 × 4,5 cm + 2 × 6,2 cm = 21,4 cm
2.5 The perimeter of polygon II is greater than that of polygon I.

2.6 For example, Trapezium FGHI



2.7 Perimeter = FG + GH + HI + IF = 4,5 cm + 6,2 cm + 2 × 4,5 cm + 4,2 cm = 23,9 cm

3 Because the triangle is isosceles, the other two sides have the same length; denote the length by the symbol *a*.
Perimeter = 2 × *a* + 14,3, that is, 27,9 cm = (2 × *a*) cm + 14,3 cm.
27,9 cm – 14,3 cm = (2 × *a*) cm
13,6 cm = (2 × *a*) cm
$a = \frac{13,6 \text{ cm}}{2} = 6,8$ cm

Remedial

Performing mixed operations on decimals may require some additional revision. Allow learners experiencing difficulty to revise this chapter, and to use their calculators to check their working with decimals, before continuing with the calculation.

Extension

Encourage learners to draw odd shapes, with all straight sides, for one another. Learners can swap these shapes with a partner and measure the perimeter of the shapes.

Exercise 2

Learner's Book page 194

Guidelines on how to implement this activity

Revise the formula for the area of a square and a triangle. Ask learners how we measured the area of the square – *s* multiplied by *s*. And with the square the sides were equal. Discuss with rectangles we multiply side by side, but he sides are not equal. We thus name them length and breadth. Do examples showing how to use the formula to find the area. Do examples of mixed shapes, that learners need to breakdown into identifiable shapes in order to calculate the area.

Suggested answers

1.1 A = $l \times b$ = 1,9 m × 0,6 m = 1,14 m^2
1.2 b = 0,2 m + 0,6 m + 0,2 m = 1 m
1.3 A = $\frac{1}{2} \times b \times h = \frac{1}{2}$ × 1 m × 0,9 m = 0,45 m^2
1.4 A = area of rectangle + area of triangle = 1,14 m^2 + 0,45 m^2 = 1,59 m^2
1.5 1 m = 100 cm
1,59 × (100 cm)^2 = 1,59 × 10 000 cm^2 = 15 900 cm^2
2.1 Perimeter = 5 cm + 3 cm + 5 cm + 2 cm + 2 cm + 3 cm = 20 cm
Area = (5 × 3) cm^2 + (2 × 2) cm^2 = 15 + 4 = 19 cm^2
2.2 Perimeter = 4 cm + 5 cm + 8 cm + 6 cm = 23 cm
Area = (4 × 5) cm^2 + ($\frac{1}{2}$ × 4 × 5) cm^2 = 30 cm^2
2.3 Perimeter = 6 + 4 + 6 + 3 + 5 + 6 + 5 + 3 + 4 = 36 cm
Area = (6 × 8) cm^2 + 2($\frac{1}{2}$ × 3 × 4) cm^2 = 60 cm^2

2.4 Perimeter = 3 cm + 4 cm + 4,2 cm + 3 cm + 4,2 cm + 4 cm = 22,4 cm

Area = (3×4) cm^2 + $2 \times (\frac{1}{2} \times \sqrt{3^2 - 2{,}3^2}) \times 2{,}3)$ cm^2 + $[(4{,}2 - \sqrt{3^2 - 2{,}3^2}) \times 2{,}3$ cm^2

= 12 cm^2 + $2 \times (\frac{1}{2} \times 1{,}93 \times 2{,}3)$ cm^2 + $[(4{,}2 - 1{,}93) \times 2{,}3]$ cm^2 = 12 cm^2 +

$(2 \times 2{,}2195)$ cm^2 + $(2{,}27 \times 2{,}3)$ cm^2 = 12 cm^2 + 4,439 cm^2 + 5,221 cm^2 = 21,66 cm^2

3.1 Area = 4,2 m × 2,4 m = 10,08 m^2

3.2 Perimeter = 2 × 4,2 m + 2 × 2,4 m = 13,2 m

Remedial

Encourage learners to work in pairs if they are experiencing any difficulty.

Unit 3 Problem solving

Learner's Book page 196

Unit focus

- solve problems involving the perimeter and area of polygons.

Background information on problem solving

In the case of solving problems with polygons where learners must either calculate perimeter or area or both, it is important to firstly correctly identify the polygon. Once they can do that, they will know which formula to use.

The use of formulae provides opportunities for learners to practise solving equations by inspection. For example, if the perimeter of a square is 32 cm, what is the length of each side? Solving by inspection means that learners can write it as 4*s* = 32 and ask themselves: 4 times what will be 32? The answer is 8, so the length of each side, *s* = 8 cm. To be able to solve by inspection, learners must know their multiplication tables. As often as time allows, use 2 minutes at the beginning of the class to do some revision of tables and let them write a short table test which they can mark themselves.

Exercise 1

Learner's Book page 196

Guidelines on how to implement this activity

Discuss with learners how we solve problems in mathematics. Revise the steps for problem solving as set out in the Learner's Book. Discuss each step and ensure learners understand it. Explain to learners that when solving problems with polygons it is necessary to identify the polygon and then identify the formula to use. Do an example together as a class. Identify each step in the problem solving chart. Allow learners to use calculators where necessary, and suggest they round off their answer to 2 decimal places.

Suggested answers

1.1 $A = l \times l$

1.2 $1\ 125\ cm^2 = l \times l$; $l = \sqrt{1\ 125}$ cm = 33,54 cm (rounded to 2 decimal places)

1.3 Rectangle

1.4 $A = l \times b = 135\ cm \times 200\ cm = 27\ 000\ cm^2$

1.5 24 squares; $\frac{27\ 000}{1\ 125} = 24$

1.6 Area of bed = $l \times b = 90\ cm \times 190\ cm = 17\ 100\ cm^2$

$27\ 000\ cm^2 - 17\ 100\ cm^2 = 9\ 900\ cm^2$

2.1 The second polygon

2.2 Perimeter of polygon 1 = 6 × 3 cm = 24 cm

Perimeter of polygon 2 = 12 × 2,5 cm = 30 cm

2.3 $A = \frac{1}{2} \times b \times h$

2.4 $A = \frac{1}{2} \times 7,5\ cm \times 6,5\ cm = 24,375\ cm^2$

2.5 $b = 2,5$ cm; $A = \frac{1}{2} \times 2,5\ cm \times 2,2\ cm = 2,75\ cm^2$

2.6 Total area = $3 \times 2,75\ cm^2 + 24,375\ cm^2 = 8,25\ cm^2 + 24,375\ cm^2 = 32,625\ cm^2$

2.7 1 cm = 10 mm

$32,625 \times 10\ mm^2 = 32,625 \times 100\ mm^2 = 3\ 262,5\ mm^2$

3.1 l = 115 ÷ 100 = 1,15 m; $b = 50 \div 100 = 0,5$ m

$A = l \times b = 1,15\ m \times 0,5\ m$

3.2 $A = 0,575\ m^2$

3.3 Area of the marble = $l \times b = 1,8\ m \times 0,75\ m = 1,35\ m^2$

$1,35\ m^2 - 0,575\ m^2 = 0,775\ m^2$

4.1 Quadrilateral

4.2 Perimeter = 5 cm + 18 cm + 7 cm + 21 cm = 51 cm

4.3 Area of $\Delta AFB = \frac{1}{2} \times b \times h = \frac{1}{2} \times AF \times FB = \frac{1}{2} \times \sqrt{5^2 - 4,5^2}\ cm \times 4,5\ cm =$

$\frac{1}{2} \times 2,18\ cm \times 4,5\ cm = 4,905\ cm^2$

Area of $\Delta BFD = \frac{1}{2} \times b \times h = \frac{1}{2} \times FD \times FB = \frac{1}{2} \times (22\ cm - 2,18\ cm) \times 4,5\ cm =$

$\frac{1}{2} \times 19,82\ cm \times 4,5\ cm = 44,595\ cm^2$

Area of $\Delta CED = \frac{1}{2} \times b \times h = \frac{1}{2} \times ED \times CE = \frac{1}{2} \times \sqrt{7^2 - 6^2}\ cm \times 6\ cm =$

$\frac{1}{2} \times 3,61\ cm \times 6\ cm = 10,83\ cm^2$

Area of $\Delta ACE = \frac{1}{2} \times b \times h = \frac{1}{2} \times AE \times CE = \frac{1}{2} \times (22\ cm - 3,61\ cm) \times 6\ cm =$

$\frac{1}{2} \times 18,39\ cm \times 6\ cm = 55,17\ cm^2$

Area of polygon ABDC = $4,905\ cm^2 + 44,595\ cm^2 + 10,83\ cm^2 + 55,17\ cm^2 =$
$115,6\ cm^2$

4.4 Total area = $18 \times 115,6\ cm^2 = 2\ 080,8\ cm^2$

Remedial

Many learners struggle with problem solving. This is why it is necessary to revise the steps in the process. Creating the equation or number sentence is often the hardest for learners. Allow learners to work in groups, and have them check their number sentence with you before commencing with the calculation.

Consolidation

Learner's book page 199

Before doing this consolidation exercise, encourage learners to review the work covered in this chapter. Advise learners to use the summary and to revise their work. This exercise can be used as an informal assessment task for you to track how learners are coping with the chapter and the concepts covered.

Suggested answers

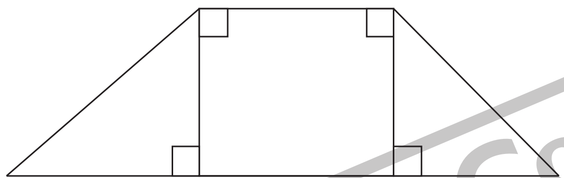
1.1 All sides have the same length and all angles have the same size.
Octagon and equilateral triangle (4)

1.2 Perimeter (octagon) = 8 × *s* = 8 × 3 cm = 24 cm
Perimeter (equilateral triangle) = 3 × *s* = 3 × 5,2 cm = 15,6 cm (2)

1.3 XY; XZ and YZ (3)

2 Area of a triangle = $\frac{1}{2} \times b \times h = \frac{1}{2} \times 0{,}4\text{ m} \times 0{,}35\text{ m} = 0{,}07\text{ m}^2$
Area of square = $l \times l = 0{,}4\text{ m} \times 0{,}4\text{ m} = 0{,}16\text{ m}^2$
Total area = 4 × Area of a triangle + Area of square = $4 \times 0{,}07\text{ m}^2 + 0{,}16\text{ m}^2 = 0{,}44\text{ m}^2$ (4)

3.1



(2)

3.2 Perimeter = 2 × 250 mm + 2 × 320 mm + 2 × 200 mm = 1 290 mm (4)

3.3 1 mm = 0,1 cm
1 290 × 0,1 cm = 129 cm (1)

[20]

CHAPTER 8

Chapter 9 Surface area and volume

Overview of concepts

Surface area and volume

UNIT 1 Surface area of cubes and rectangular prisms

UNIT 2 Volume and capacity of cubes and rectangular prisms

UNIT 3 Problem solving

Content		Time allocations	LB page
Unit 1	Surface area of cubes and rectangular prisms	3 hours	201
Unit 2	Volume and capacity of cubes and rectangular prisms	3 hours	204
Unit 3	Problem solving	2 hours	208

Background information on surface area and volume

In Grade 6 learners did not have to use formulae to calculate surface area and volume. In Grade 7 they have to learn and use the following formulae to calculate surface area and volume respectively:

- Surface area of a prism = the sum of the areas of all its faces.
- Volume of a prism = the area of the base × the height.
- Volume of a rectangular prism = $l \times b \times h$.
- Volume of a cube = $l \times l \times l = l^3$.

From Chapter 8, which deals with perimeter and area calculations, learners should know the formulae to calculate the areas of squares and rectangles. They will use this knowledge to calculate the surface area of a prism.

Learners must be reminded that when they worked with perimeter and area in Chapter 8 they worked with flat surfaces or in two dimensions (2D).

In Chapter 9 they will work in 3 dimensions because each prism or 3D object has 3 dimensions: length (l), breadth (b) and height (h).

Generic teaching guidelines for teaching surface area and volume

The topic of surface area and volume lends itself to some very nice practical demonstrations. The world around us is filled with 3D objects.

- Introduce the topic by letting the learners identify all the objects around them that are cubes or rectangular prisms. Cardboard boxes of different shapes and sizes are easy to identify and are very good examples.

- Use a box to explain the 3 dimensions of length (l), breadth (b) and height (h) of a 3D object or prism to the learners.
- Use a cube such as a die from a board game or a box in the form of a cube to show that a cube is a prism where $l = b = h$.
- Use a rectangular box to explain that a rectangular prism is a 3-D object where $l \neq b \neq h$.
- As a last fun exercise, ask learners to collect and bring some empty toilet rolls. First ask them how they will go about to calculate the surface area. Now, let them cut it open along one side. It will be a rectangle, which might come as a surprise to some. Let them measure the length and breadth and ask them which formula they can use to calculate the area in mm^2 or cm^2.

Resources

Boxes and prisms to show learners the difference between are and surface area, and volume and capacity. Use rulers and tape measures to physically measure objects. Learners can also use boxes to open and display nets. Empty toilet rolls, cardboard, colour pens can be used. Each learner should have their own calculator.

Unit 1 Surface area of cubes and rectangular prisms

Learner's Book page 201

Unit focus

- work with nets
- calculate the surface area of cubes and rectangular prisms.

Background information on the surface area of cubes and rectangular prisms

A net is a 2D flat diagram of a prism:
Remind learners of the example of the cut-open toilet roll. That was also a net.

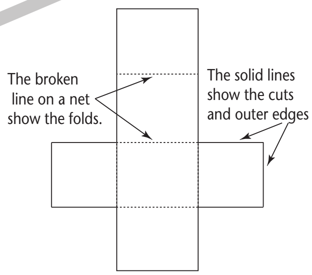


Figure 1: A net

Revision exercise

Learner's Book page 201

Guidelines on how to implement this activity

Provide learners with cardboard boxes of different sizes and shapes. Ideally ask learners the week before to start bringing in various boxes into the class. A small box like a Smartie box or match box is best to open up and paste onto an A4 size paper or thin cardboard. Encourage learners to count the number of edges on the box, the number of faces, and the number of corners. Have learners cut the box open, along one of the edges and to examine the net. Learners trace round the edges and identify the fold lines. Explain that the net is the 2D shape that when folded correctly creates the 3D object.

Suggested answers

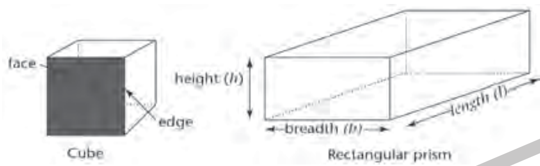
1. Cube: The length, width and height of a cube are equal. (The volume is a cube number.) Rectangular prism: All faces are rectangles.
2. 6 faces
3. 12 edges

Remedial

Provide as many different boxes as possible. Learners can draw the box on large card, and then trace around the net. By doing this learners can see how the net becomes the object.

Extension

Make a poster of the features of a prism to put up in class, as shown in the diagram. Point out the flat sides of a prism (called the faces), and the edges (where the faces meet).



Investigation 1

Learner's Book page 202

Guidelines on how to implement this activity

Ensure learners have the necessary equipment. Observe learners as they draw the net for a cube. Encourage learners to think why using the net can help determine surface area. Help learners discover how adding each of the areas of each face together adds up to the total surface area.
Ensure learners grasp this concept before moving on to the next exercise.

Exercise 1

Learner's Book page 203

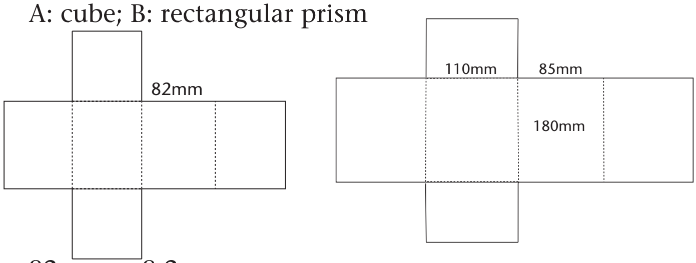
Guidelines on how to implement this activity

Revise the concept of area that was covered in Chapter 8. Revise the formula for a square and a rectangle. Ask learners how they could measure the area of the surface of a cube. Try guide learners to providing you with the answer that you would need to find the area of each face and then add them together. Explain to learners that this is called surface area. Do an example together as a class and then have learners calculate the surface area of cubes and prisms, by measuring the physical forms. Encourage learners to draw the net of the shape in their exercise books and to fill in the values to help them calculate surface area.

Suggested answers

1.1 A: cube; B: rectangular prism

1.2



1.3 82 mm = 8,2 cm
85 mm = 8,5 cm; 110 mm =11 cm; 180 mm = 18 cm

1.4 Area = $l \times l$ = 8,5 cm × 8,5 cm = 72,25 cm²

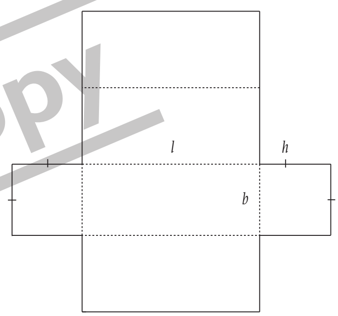
1.5 Total surface area = 6 × area of one face = 6 × 72,25 cm² = 433 cm²

1.6 Area of face 1 = $l \times b$ = 8,5 cm × 11 cm = 93,5 cm²
Area of face 2 = $l \times b$ = 8,5 cm × 18 cm = 153 cm²
Area of face 3 = $l \times b$ = 11 cm ×18 cm = 198 cm²

1.7 Total surface area = 2 × 93,5 cm² + 2 × 153 cm² + 2 × 198 cm² =
187 cm² + 306 cm² + 396 cm² = 889 cm²

2.1 Learners' own work

2.2 Rectangular prism



2.3 Area of face 1 = $l \times b$ = 0,5 m × 0,25 m = 0,125 m²
Area of face 2 = $h \times b$ = 0,25 m × 0,25 m =
0,0625 m²
Area of face 3 = $l \times h$ = 0,5 m × 0,25 m = 0,125 m²

2.4 Total surface area = 2 × 0,125 m² + 2 × 0,0625 m²
+ 2 × 0,125 m² = 0,25 m² + 0,125 m² + 0,25 m² =
0,625 m²

2.5 cube

2.6 Area of one face = $l \times l$ = 0,25 m × 0,25 m =
0,0625 m²
Total surface area = 6 × 0,0625 m² = 0,375 m²

2.7 1 m² = 10 000 cm²
(0,375 × 10 000) cm² = 3 750 cm²

Extension

Have learners investigate more about SI units and other possible disasters occurring due to inaccurate measurement and measurement conversions.

Unit 2 Volume and capacity of cubes and rectangular prisms

Learner's Book page 204

Unit focus

- learn the difference between volume and capacity
- use formulae to calculate the volume of cubes and rectangular prisms.

Background information on volume and capacity of cubes and rectangular prisms

When working with capacity and volume, it is important that learners know the difference between the two concepts:

- The amount of space occupied by a prism is called its volume (mm^3, cm^3, m^3).
- The amount of space inside a prism is called its capacity (litres, millilitres).
- To explain the relationship between volume and capacity, you can teach them the following:
  - An object with a volume of 1 cm^3 will displace exactly 1 m*l* of water. Therefore 1 cm^3 = 1 m*l*.
  - An object with a volume of 1 m^3 will displace exactly 1 k*l* of water, therefore 1 m^3 = 1 k*l*.

Exercise 1

Learner's Book page 205

Guidelines on how to implement this activity

Discuss the concept of volume. Show learners two cubes, and have learners identify which will have the larger volume. Explain that the formula for volume involves the area of the base and the height of the prism. Show learners the formula for volume. Do a few examples of finding the volume of various prisms. Revise how to convert between various SI units. Ensure learners are able to do this, as they may have to convert their answers from one unit to another. Allow learners to use their calculators, but remind them to always check the reasonableness of their answer. Encourage learners to complete this exercise on their own.

Suggested answers

1 16 mm = 1,6 cm
Volume = $l \times l \times l$ = 1,6 cm × 1,6 cm × 1,6 cm = 4,096 cm^3

2 Volume = $l \times l \times l$ = 16 mm × 16 mm × 16 mm = 4 096 mm^3

3 1 mm^3 = $(0,1 \text{ cm})^3$ = 0,001 cm^3
(4 096 × 0,001) cm^3 = 4,096 cm^3

Remedial

Some learners may require you to revise area. Ask these learners to write the area formulae in pencil in the top right hand corner of their exercise book for them to refer to while finding the volume.

Extension

Provide additional examples for learners to practise. Provide triangular prisms and even more complex objects for learners to try.

Exercise 2

Learner's Book page 207

Guidelines on how to implement this activity

Explain the concept of capacity and how it differs from volume. Show learners by means of empty boxes the difference in a physical and concrete form.

Spend time explaining the capacity of an object and how it is calculated using the volume. Learners often find the conversion to litres confusing. If necessary revise converting m^3 to cm^3 in order to help learners to provide their answers in litres. Work through a few examples as a class of using the volume of an object to determine its capacity. Allow learners to discuss the problems in Exercise 2 in pairs, but ensure each learner records all the working out and calculation.

Suggested answers

1 $l = 2$ m; $b = 1$ m

2 $1\ m^3 = 1\ 000\ 000\ cm^3$; $2\ m^3 = 2\ 000\ 000\ cm^3$
Capacity = 2 000 000 ÷ 1 000 = 2 000 ℓ

3 Capacity of one box = $\frac{2\ 000\ ℓ}{20}$ = 100 ℓ

4 100 ℓ = (100 × 1 000) cm^3 = 100 000 cm^3
(100 000 ÷ 1 000 000) m^3 = 0,1 m^3

5 Total volume = 20 × 0,1 m^3 =2 m^3

6 Total capacity = 10 × 2 000 ℓ = 20 000 ℓ

Remedial

If learners are experiencing difficulty with conversions, help the learners to create a chart to help them. The chart should include the equivalent between cm^3, m^3 and litres. Learners should keep this with them at all times while working with volume. Encourage learners to memorise the chart and test them on this.

Extension

Encourage learners to calculate the capacity of triangular prisms. Provide example with dimensions for learners to work with. Keep dimensions as realistic as possible.

Challenge

Learner's Book page 207

Guidelines on how to implement this activity

Prescribe this exercise only if learners have managed easily with the rest of the material in this chapter.

Suggested answers

Volume = area of base × height = ($\frac{1}{2}$ × 28 mm × 16 mm) × 62 mm =
224 mm^2 × 62 mm = 13 888 mm^3

Unit 3 Problem solving

Learner's Book page 208

Unit focus

- solve problems by calculating surface area, volume and capacity.

Background information on problem solving

Many learners find problem solving difficult, however, it is a very important skill for them to learn. As in the case of Chapter 8 Unit 3 on problem solving, learners need to READ and understand each problem. Before they immediately try and solve it, they must get into a habit to first identify what is given and what is asked.

Revision exercise

Learner's Book page 208

Guidelines on how to implement this activity

Revise once again the difference between volume and capacity. Learners will use equivalence between units when solving problems: 1 cm^3 = 1 m*l* and 1 m^3 = 1 k*l*, so revise these as well. Have learners write out each formula that they plan to use in symbol form using the correct symbols or variables depending on the formula, for example *b*, *l*, *h* or *s*. They should not immediately just substitute the values in.

Suggested answers

1 1 cm = 10 mm; 1cm^3 = 1 000 mm^3

2 1 m = 100 cm; 1 m^3 =1 000 000 cm^3

3 1 cm^3 = 1 m*l* **4** 1 m^3 = 1 k*l*

Remedial

Use this exercise to assess if learners have grasped all the concepts covered so far. If learners are having difficulty prescribe some remediation exercises for learners to work with.

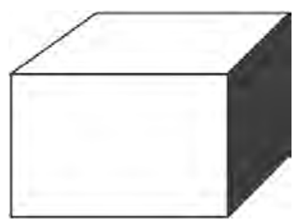
Exercise 1

Learner's Book page 209

Guidelines on how to implement this activity

Revise the steps for problem solving. Discuss with the learners how to create a number sentence that meets the problem. Ask some learners to provide tips that help them. Do some examples together as a class.

Suggested answers



1.1 →

1.2 Volume = 480 ℓ ÷ 1 000 = 0,48 k*l* = 0,48 m^3

1.3 Rectangular prism

1.4 Volume = $l \times b \times h = l \times b^2$

1.5 100 cm = 1 m

$0{,}48 = 1 \times b^2$

$b^2 = 0{,}48$

$b = \sqrt{0{,}48} = 0{,}69$ (rounded to two decimal places)

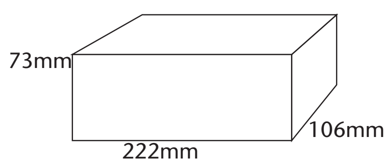
1.6 Table breadth, $b = \frac{0{,}85 \text{ m}^2}{1{,}1 \text{ m}} = 0{,}77$ m

The table has length 1,1 m compared to the fish tank's 1 m, and the table has breadth 0,77 m compared to the fish tank's 0,69 m. Yes, the fish tank will fit on the table.

2.1 Area = $l \times b$ = 3 m × 4 m = 12 m^2

2.2 Learners' own work

2.3 Area = $l \times h$ = 222 mm ×73 mm = 16 206 mm^2



2.4 $12\ m^2 = 12\ 000\ 000\ mm^2$
Number of bricks = $\frac{12\ 000\ 000}{16\ 206}$ = 740 (rounded to nearest unit)
3.1 Volume = $l \times b \times h$ = 30 cm × 30 cm × 6 cm = 5 400 cm^3
3.2 5 400 m*l* **3.3** (5 400 ÷ 1 000) ℓ = 5,4 ℓ
3.4 Volume = $l \times b \times h$ = 30 cm ×30 cm × 4 cm = 3 600 cm^3
Capacity = (3 600 ÷ 1 000) ℓ = 3,6 ℓ
3.5 $\frac{36}{3,6} = 10$ **3.6** $\frac{10}{2} = 5$
3.7 250 m*l* = 0,25 ℓ
Milk needed for cake mixture: 0,25 ℓ × 10 = 2,5 ℓ
Milk available: 1 + 1,5 = 2,5 ℓ
Yes, Trudy has just enough milk.

Remedial

Encourage learners to work on their own, but allow learners who are having problems to work together in pairs or small groups. Revise the steps for problem solving again. Instruct these learners to write each step and identify what they are asked to find and what they are given. Have these learners check their number sentence with you before they embark on the calculation.

Extension

Have learners compare the volume of various packaging to its capacity. For example, measure the dimensions of a rectangular milk carton, and determine if it is holding 1 litre.

Consolidation

Learner's book page 212

Before doing this consolidation exercise, encourage learners to review the work covered in this chapter. Advise learners to use the summary and to revise their work. This exercise can be used as an informal assessment task for you to track how learners are coping with the chapter and the concepts covered.

Suggested answers
1.1 Area = $l \times l$ = 1,2 m × 1,2 m = 1,44 m^2 (3)
1.2 1 m^2 = 10 000 cm^2
(1,44 × 10 000) cm^2 = 14 400 cm^2 (2)
1.3 Total surface area = 6 × 1,44 m^2 = 8,64 m^2 (3)
2.1 1 000 cm^3 = 1
Volume = (5 × 1 000) cm^3 = 5 000 cm^3 (2)
2.2 Volume = $l \times b \times h$ (3)
2.3 5 000 = 20 × b × 25
5 000 = 500 × b
$\frac{5\ 000}{500} = b$
10 = b (3)
2.4 76 ÷ 5 = 15,2 (For example 15 containers) (3)
2.5 Volume of crate = $l \times b \times h$ = 40 cm × 110 cm × 30 cm = 132 000 cm^3
Capacity of crate = 132 000 ÷ 1 000 = 132.
Yes, 15 containers would fit into the crate. (4)
2.6 2 rows and 10 columns (1 column of space left open for sawdust) (2)
[25]

Chapter 10 Numeric and geometric patterns

Overview of concepts

Numeric and geometric patterns

UNIT 2
Find the rule

UNIT 1
Patterns

Content		Time allocations	LB page
Unit 1	Patterns	3 hours	214
Unit 2	Find the rule	3 hours	219

Background information on numeric and geometric patterns

Patterns are used in every culture and have been since early man first made marks on cave walls. Sometimes patterns are strictly for decoration, and sometimes they are a form of communication. In mathematics we learn the art of making a variety of types of patterns, for example: repeat patterns, regular and irregular patterns, mirror images, and tessellations.

Patterns and symbols are based on rules which are based on mathematical principles. Ask learners to design a number pattern using multiplication(×) and subtraction (–). For example, $1 \times 3 - 2 = 1$; $2 \times 3 - 2 = 4$; $3 \times 3 - 2 = 7$ etc.

$\times\ 3 - 2$ is the rule on which the pattern is based.

Generic teaching guidelines for teaching numeric and geometric patterns

- Encourage learners to see patterns in everyday things, for example: floor tiles, brick work, on plates, mugs, mats, or curtains. Learners can also find examples in nature such as on butterfly wings, tree bark or leopard spots.
- Make patterns using the letters in their names. They can enlarge, reduce, or reverse some of the letters and rearrange the order.
- Number patterns do not have to be based on a regular difference.
  For example: 1, 4, 9, 16, 25 which is the pattern of square numbers 1^2; 2^2; 3^2 and so on.
- Number patterns can be described by giving the rule according to which the numbers are arranged.

- Geometric patterns are patterns made up of geometric shapes such as tessellated floor tiles, wallpaper, curtaining and so on.

Resources

Matchsticks, Jelly Tots, toothpicks or counters to create and manipulate as patterns. Pictures of examples of patterns in nature. Blank tables for learners to use to represent their patterns. Cardboard and colour pens. Each learner should have their own calculator.

Unit 1 Patterns

Learner's Book page 214

Unit focus

- revising extending number patterns
- revising extending geometric patterns
- looking for the relationship between terms in a pattern
- representing patterns in tables.

Background information on patterns

Patterns can be found in numbers, letters of the alphabet, geometric shapes, buildings, plants, and animals. Patterns in nature are often fairly irregular and have organic shapes, for example: patterns on leaves, flowers, birds feathers, or sea shells. Patterns using letters of the alphabet can be regular or irregular, for example: ЕШЕШЕШЕ or ABCDEFGHIJK. Patterns using numbers can be regular or irregular. For example: 2; 4; 6; 8; 10; 12…(even numbers) or 1; 3; 6; 10; 15….(triangular numbers). Patterns using geometric shapes can be regular or irregular. For example: border patterns using geometric shapes like this:  or an all over pattern using a variety of tessellated shapes.

Exercise 1

Learner's Book page 215

Guidelines on how to implement this activity

Discuss patterns with learners. Ask them about patterns in nature and around them. Revise simple number patterns on the board. Write out an example of a pattern, and ask learners to extend the pattern. Ask learners to invent their own patterns. Show learners how to use a table to help them analyse and extend patterns.

Suggested answers

1.1	18; 21; 24; 27	**1.2**	9; 3; 1; $\frac{1}{3}$
1.3	30; 32; 34; 36	**1.4**	32; 64; 128; 256
1.5	26; 27; 28; 29	**1.6**	$7\frac{1}{2}$; $8\frac{1}{4}$; 9; $9\frac{3}{4}$
1.7	731; 720; 709; 698	**1.8**	85,1; 84,3; 83,5; 82,7
2.1	add three	**2.2**	divide by three
2.3	add two	**2.4.**	times two
2.5	add one	**2.6.**	add three quarters
2.7	subtract eleven	**2.8**	subtract eight tenths

3.1

Position	1	2	3	4	5	6	7	8	9	10
Term	4	9	14	19	24	29	34	39	44	49

3.2

Position	1	2	3	4	5	6	7	8	9	10
Term	97,5	95	92,5	90	87,5	85	82,5	80	77,5	75

3.3

Position	1	2	3	4	5	6	7	8	9	10
Term	$34\frac{1}{4}$	$34\frac{3}{4}$	$35\frac{1}{4}$	$35\frac{3}{4}$	$36\frac{1}{4}$	$36\frac{3}{4}$	$37\frac{1}{4}$	$37\frac{3}{4}$	$38\frac{1}{4}$	$38\frac{3}{4}$

Remedial

Learners are sometimes able to extend patterns, but are not able to verbalise how they extended the pattern. Help them by asking them to not only analyse the difference between subsequent terms, but the relationship between the term number and the term value.

Exercise 2

Learner's Book page 216

Guidelines on how to implement this activity

Review the patterns learners worked on in Exercise 1. Explain to learners that each of the patterns they have dealt with were patterns that increased by a constant sum or a constant difference. Learners need to know that this is not the only type of pattern there is. Explore patterns, without a constant difference or ratio, as a class. Encourage learners to create their own patterns like this. When learners do Exercise 2, allow learners to discuss their thinking in pairs or small groups.

Suggested answers

1.1 Previous term value plus the difference between the two previous consecutive term values, plus two or square of position number.

1.2 Previous term value plus the difference between the two previous consecutive term values, plus one.

1.3 The cube of the position number.

1.4 Previous term value plus the difference between the two previous consecutive term values, plus one tenth.

2.1 36; 49; 64

2.2 16; 22; 29

2.3 64; 125; 216

2.4 3,1; 2,7; 2,4

Remedial

Make working with patterns enjoyable. Encourage learners to find working out the patterns a challenge and to try different methods and trial and error to extend them.

Exercise 3

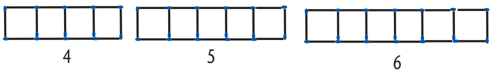
Learner's Book page 217

Guidelines on how to implement this activity

Use the pattern 1; 3; 6; 10 and draw the triangular dot pattern to show learners how a number pattern can be represented in a diagrammatic form. Ask learners to come up to the board and extend the diagrammatic pattern. Show learners how it is sometimes easier to extend the pattern diagrammatically rather than work out what needs to be added. Show learners how to translate the geometric pattern to a table. Do additional examples until learners feel confident with working with geometric patterns. Remind learners that the top row of the table represents the term number with regard to its position in the number sequence. Allow learners to work through this exercise in pairs or small groups.

Suggested answers

1.1



1.2 Added three more matchsticks to complete another square.

1.3

Position	1	2	3	4	5	6	7	8	9
Number of matches	4	7	10	13	16	19	22	25	28

1.4 31 matches

1.5 Number of matches = (Position number × 3) + 1

The 50th term would have (50 × 3) + 1 = 150 + 1 = 151 matches.

2

Position	1	2	3	4	5	6	7	8	9
Number of matches	3	5	7	9	11	13	15	17	19

3.1 14 dots

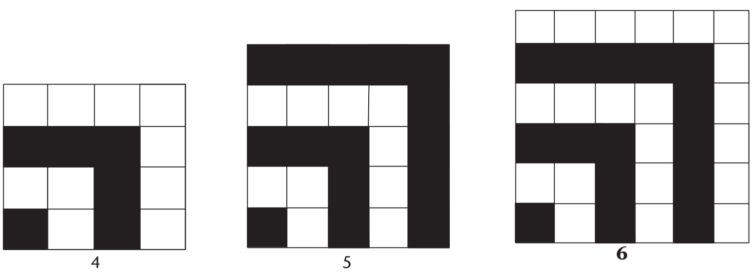
3.2 26 dots

3.3

Position	1	2	3	4	5	6	7	8	10
Number of dots	6	10	14	18	22	26	30	34	38

Term position multiplied by 4 plus 2.

4.1



4.2

Position	1	2	3	4	5	6	7	8
Total squares	1	4	9	16	25	36	49	64
White squares	0	3	3	10	10	21	21	36
Dark squares	1	1	6	6	15	15	28	28

4.3 The total number of squares are all square numbers.

Remedial

Encourage learners to always draw the pattern in their exercise books.

Unit 2 Find the rule

Learner's Book page 219

Unit focus

- revising using tables to show patterns
- learning how to find the rule for patterns
- using the rule to find the value of any term in a sequence.

Background information on finding the rule

When numbers or shapes form a regular pattern, the difference between them is controlled by a rule. When there is a regular number pattern like 10; 20; 30; 50; 60 … the rule applied is + 10. With an irregular pattern like 1; 4; 9; 16 (square numbers) the rule is to the power of 2: 1^2; 2^2; 3^2;… Flow diagrams show the input number, the rule and the output number. An example is:

Input	Rule	Output
5	z	15

To find the rule you have to decide what has to be done to 5 to make it 15. There are a number of possibilities: + 10 or × 3 are the most obvious.

Exercise 1

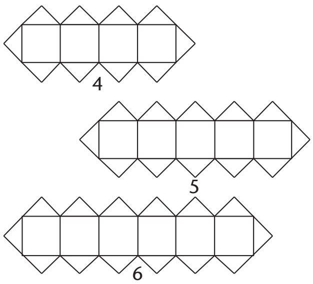
Learner's Book page 221

Guidelines on how to implement this activity

Discuss with learners how they managed to extend the patterns in the previous unit. Did they use a rule to determine what the next term would be? Explain to learners that the general rule looks at the relationship between the term value and term number (which is its position in the sequence). The general rule is the rule that allows us to determine the term value of any term in the sequence, or used in reverse allows us to determine the term number from its value. Show learners how tables help us to do this. Work through the worked examples as a class. Discuss any alternate methods learners may have. Explain to learners that they have to trial their rule on at least 3 other terms in the sequence. The rule must work for all terms to be valid. Allow learners to work in pairs or small groups, but each learner must record their own working.

Suggested answers

1.1.1 Term value = Term number × 15
1.1.2 Term value = (Term number × 5) – 1
1.1.3 Term value = (Term number × 7) + 14
1.1.4 Term value = (Term number × 2) + 3
1.2.1 25 × 15 = 375
1.2.2 (25 × 5) – 1 = 124
1.2.3 (25 × 7) + 14 = 189
1.2.4 (Term Number × 2) + 3 = 53
2.1



2.2

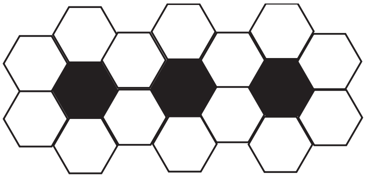
Term number	1	2	3	4	5	10	15	20
Number of squares	1	2	3	4	5	10	15	20
Number of triangles	4	6	8	10	12	22	32	42

2.3 Number of triangles = (Term number × 2) + 2
2.4 100 squares
2.5 Number of triangles in term 27 = (27 × 2) + 2 = 56
3.1 8; 12; 16;… (number of small squares)
(1 × 4) + 4 = 8; (2 × 4) + 4 = 12; (3 × 4) + 4 = 16
Term value = (Term number × 4) + 4
Value of 15th term = (15 × 4) + 4 = 64
3.2 6; 11; 16;… (number of sides)
(1 × 5) + 1 = 6; (2 × 5) + 1 = 11; (3 × 5) + 1 = 16
Term value = (Term number × 5) + 1
Value of 15th term = (15 × 5) + 1 = 76
3.3 3; 6; 10;… (number of dots)

Term number	1	2	3	4	5	6	7	8	10
Number of dots	3	6	10	15	21	28	36	45	66

Term value = $\frac{1}{2}$ × (Term number + 1) × (Term number + 2)
Number of dots in 15th term = $\frac{1}{2}$ × (15 + 1) × (15 + 2) = $\frac{1}{2}$ × 16 × 17 = 136

4.1



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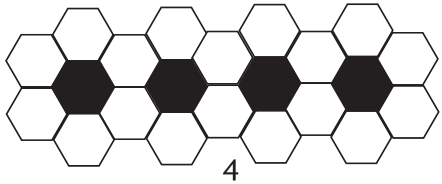
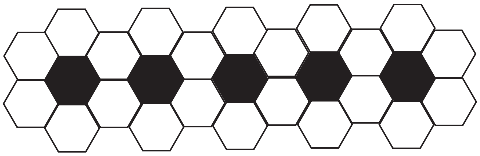


4



5

4.2 6; 10; 14; 18;…
$(1 \times 4) + 2 = 6$; $(2 \times 4) + 2 = 10$; $(3 \times 4) + 2 = 14$; $(4 \times 4) + 2 = 18$
Number of slabs = (Term number × 4) + 2
Number of slabs in 10^{th} term = $(10 \times 4) + 2 = 42$

Remedial

If learners are experiencing difficulty revise substitution and generating the formula, as this is where many learners may experience problems. Have these learners start with simpler patterns and have learners gradually try more complex patterns. Provide encouragement and tips for learners as they work on patterns.

Consolidation

Learner's book page 224

Before doing this consolidation exercise, encourage learners to review the work covered in this chapter. Advise learners to use the summary to revise their work. This exercise can be used as an informal assessment task for you to track how learners are coping with the chapter and the concepts covered.

Suggested answers

1.1	square	(1)
1.2	equilateral triangle	(1)
1.3	rhombus	(1)
1.4	parallelogram	(1)
2.1	4; 8; 12; 16; 20	(3)
2.2	This is a regular pattern. The regular difference is 4.	(2)
2.3	81; 64; 49; 36; 25; 16; 9; 4; 1	(6)
2.4	Irregular pattern	(2)
2.5	1; 3; 6; 10; 15; 21	(3)
2.6	Irregular pattern	(2)
3	17; 42; 67; 92; 117	(5)

[27]

Chapter 11 Functions and relationships

Overview of concepts

Functions and relationships

UNIT 1 Input and output values of formulae

UNIT 2 Equivalent forms

UNIT 3 Problem solving

Content		Time allocations	LB page
Unit 1	Input and output values of formulae	1 hour	226
Unit 2	Equivalent forms	1 hour	229
Unit 3	Problem solving	1 hour	231

Background information on functions and relationships

Learners have worked with functions and relationships in grade 6. They are able to read and interpret flow diagrams and tables, however it is always useful to start this chapter by revising these representations. Learners have worked with patterns in the previous chapter and learnt how to translate patterns into tables.

Flow diagrams can show the input and output value of equations. They show relationships. Tables also show relationships between values. Equations will become the dominant form for representing relationships and functions in later grades, and thus learners should be asked to write relationships in equations as much as possible to help them prepare for this.

Generic teaching guidelines for teaching functions and relationships

Show learners that it is helpful to use a flow diagram or a table to solve problems with variables or unknowns. For example, a word sentence might say 'If x plus seven is equal to twelve, what is the value of x? This can be written as a number sentence: $x + 7 = 12$.

It can be shown as a flow diagram:

Input		Rule		Output
x	$\longrightarrow$	$\boxed{+7}$	$\longrightarrow$	12

This information can then be translated to a table. Demonstrate on the board.

Input	x
Rules	+ 7
Output	12

Encourage learners to work in pairs to create examples to give others to do. Show them how to set out flow diagrams and how to transfer the information to tables. Give a few minutes to make the examples, and then allow them to swap with another pair and solve the questions.

Resources

Blank flow diagrams, tables and value grids for learners to use when filling in input and output values. Each learner should have their own calculator.

Unit 1 Input and output values of formulae

Learner's Book page 226

Unit focus

- revising input and output values,
- revising tables and flow diagrams,
- using a formula to find output values, and
- using a formula to find input values.

Background information on input and output values of formulae

Input and output values are central to solving equations.
If you have an equation, for example, $x + 4 = y$, the input values are whatever numbers you choose for x, and the output is y, the answer you get after performing the operation + 4.
For example if $x = 2$ then $x + 4 = 2 + 4 = 6$. This means $y = 6$.
The input number is 2 (x) and the output number is 6 (y). We call [2,6] the **ordered pair**.
To solve an equation with only one variable means the value of that variable is fixed so that you can't substitute a variety of numbers for the variable. For example: $x + 6 = 7$. To find the input value (x) you have to subtract 6 from the left hand side. In an equation, what you do to the LHS you must also do to the right hand side (RHS). So: $x + 6 - 6 = 7 - 6$ and $x = 1$.

Revision exercise

Learner's Book page 226

Guidelines on how to implement this activity

Use this exercise to assess learners' prior knowledge of working with flow diagrams and tables. Learners should be able to find input and output values easily and shouldn't experience any difficulty here.

Suggested answers

1 $c = (7 \times 4) \div 2 = 28 \div 2 = 14$
$d = (9 \times 4) \div 2 = 36 \div 2 = 18$
$e = (11 \times 4) \div 2 = 44 \div 2 = 22$
$a = (26 \times 2) \div 4 = 13$
$b = (30 \times 2) \div 4 = 15$

2 $a = (22 + 5) \div 3 = 27 \div 3 = 9$
$b = (20 - 2) \times 7 = 18 \times 7 = 126$
$c = (18 \times 12) + 3 = 216 + 3 = 219$

3

Farmer	1	2	3	4	5	6	7
Number of cows	11	15	20	26	33	41	50

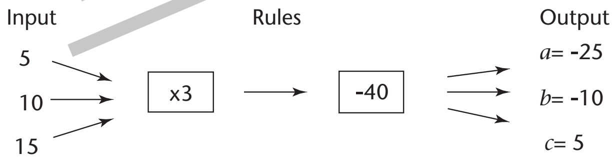
Remedial

If learners struggle, provide immediate remediation of problem areas. Refer learners to Grade 6 work on this concept and provide simple manipulation of equations.

Exercise 1

Learner's Book page 228

Guidelines on how to implement this activity

Revise the following key points with learners:
- An input is the number we are given.
- The rule is what we do to the input.
- The output is the result of applying the rule to the input.
- Using a flow diagram is an uncomplicated way of teaching input and output values. Apply the rules to each of the input values to arrive at the output values.
- 

- Transfer the information from the flow diagram to a table:

Input	5	10	15
Rules	× 3	x 3	x 3
Rules	– 40	– 40	– 40
Output	*a*	*b*	*c*

- Write the problem as a word sentence.
  For example, 'Five times three minus forty is equal to *a*.'
- Write the problem as an equation. For example, $(5 \times 3) - 40 = a$.

Focus on writing relationships and functions as equations or formulae with variables. Do a few examples as a class on how to do this, and encourage learners to complete this exercise in pairs.

Suggested answers

1.1 $b = (32 \div 4) + 9$; $b = 8 + 9 = 17$

1.2 $c = (40 \div 4) + 9$; $c = 10 + 9 = 19$

1.3 $d = (48 \div 4) + 9$; $d = 12 + 9 = 21$

1.4 next input number = 48 + 8 = 56
Output number = (56 ÷ 4) + 9 = 14 + 9 = 23

2

Input	32	40	48	56
Rules	÷ 4 and + 9	÷ 4 and + 9	÷ 4 and + 9	÷ 4 and + 9
Output	17	19	21	23

3.1 $y = (27 + 3) - 20 = 30 - 20 = 10$ (inverse operations)

3.2 $z = (27 + 3) - 22 = 30 - 22 = 8$ (inverse operations)

3.3 $c = (27 + 3) - 24 = 30 - 24 = 6$ (inverse operations)

4.1 $t = (36 \times 1) - 14 = 22$

4.2 $t = (36 \times 5) - 14 = 166$

4.3 $t = (36 \times 10) - 14 = 346$

4.4 $t = (36 \times 25) - 14 = 886$

5 First make a the subject of the equation: $a = 2b - 5$

5.1 $a = 2(1) - 5 = -3$

5.2 $a = 2(7) - 5 = 9$

5.3 $a = 2(17) - 5 = 29$

5.4 $a = 2(99) - 5 = 193$

6.1 $x = (2 \times y) + 9$
$77 = 2y + 9$
$68 = 2y$
$y = 34$

6.2 $x = (2 \times y) + 9$
$x = (2 \times 100) + 9$
$x = 209$

Remedial

- Remind learners of basic number concepts they learned in lower grades. In grades 1 and 2 they used a square or triangle to represent an unknown. Now they use a letter of the alphabet:

0	+	*a*	=	5
1	+	*b*	=	5
2	+	*c*	=	5
3	+	*d*	=	5
4	+	*e*	=	5
5	+	*f*	=	5

- The above is a similar concept to a flow diagram:
- Input Rules Output

Input		Rules		Output
7	→	$\boxed{+3}$	→	*g*
h	→	$\boxed{\times 4}$	→	12

- These are also similar to equations where we say: $j - 3 = 5$ and work with the LHS and RHS. So: $j - 3 + 3 = 5 + 3$ which means $j = 8$
- Remember, whatever is done to the LHS must be done to the RHS.

Extension

Input and output values are regularly used in financial matters. For example, if a person invests R5 000,00 at 10% interest, what will he make? This can be done:

Input		Rule		Output
R5 000,00	$\longrightarrow$	$\boxed{+10\%}$	$\longrightarrow$	z

Calculate interest at 10% : $\frac{\text{R5 000}}{1} \times \frac{10}{100}$ = R500,00 interest
Total = R5 500,00 = z
Ask learners to create a number of financial calculations and to work them out using input and output values. Ask learners to experiment with flow diagrams, trying various operations and testing the variables in input, rule and output columns.

Unit 2 Equivalent forms

Learner's Book page 229

Unit focus

- revising how to use tables to represent formulae,
- revising how to use flow diagrams to represent formulae, and
- revising finding input and output values from formulae.

Background information on equivalent forms

Learners need to be able to work with the following forms of representing relationships: verbal descriptions, flow diagrams, tables, formulae and equations. Learners have had some exposure to representing these in Unit 1 and in the previous chapter.

Ensure learners practise using all these forms to represent relationships, and that they understand that each form is equivalent. Learners need to be able to substitute values in formulae and equations in order to manage this unit.

Exercise 1

Learner's Book page 230

Guidelines on how to implement this activity

Provide a verbal description of a function. Ask a learner to come up to the board and write it as a flow diagram. Test the flow diagram with a few input values. Have another learner come up to the board and to write it as a table. Again test the flow diagram using a variety of input values. Together as a class create an equation or formula to represent the equivalent relationship. Once you have developed the formula test it with the same values you used for the table and check if the equation is valid. Repeat with a few examples until learners are able to fluidly work between the different but equivalent forms. Learners can complete this exercise in pairs, as long as each learner records their own working.

Suggested answers

1.1 $a = 17 - 5 = 12$ **1.2** $b = 23 + 7 = 30$

1.3 $c = 43 + 22 - 54 = 11$ **1.4** $d = 17 + 6 + 37 = 60$

2.1 $y = 40$ **2.2** $z = 38$ **2.3** $c = 36$

3

x	y	$x + y$
0	7	7
1	6	7
2	5	7
3	4	7
4	3	7
5	2	7
6	1	7
7	0	7

4 $a = 5$ and $b = 3$

5 For f to be positive $5a < 24$. $5a$ is less than 24 for $a = \{0; 1; 2; 3; 4\}$.
If $a = 5$, $5a = 25 > 24$.

6 $k = -2$ and $j = 3$

7.1 $v = 20 - 7 = 13$ **7.2** $w = 14 - 9 = 5$

7.3 $x = 23 + 8 - 15 - 3 = 13$ **7.4** $x = 23 - 12 = 11$

7.5 $y = 5 + 2 = 7$ **7.6** $z = 6 \times 2 = 12$

7.7 $a = \frac{27}{9} = 3$ **7.8** $b = \frac{55 - 5}{2} = 25$

Remedial

Work with learners experiencing difficulty in a separate small group. Discuss the equivalent forms and do additional examples representing the same function in different ways. Start with simple patterns and increase the complexity until the prescribed Grade 7 level.

Extension

Encourage learners to develop their own patterns and to represent them in as many of the forms as possible.

Unit 3 Problem solving

Learner's Book page 231

Unit focus

- revising using formulae, tables and flow diagrams
- applying what you have learned to solving problems.

Background information on problem solving

In solving any word problem remind learners to always identify what they are given and what they are expected to find. When working with variables finding the value of the variable is always at the heart of the problem. For example, if Thandi has y sweets and Nandi has $4y$ sweets. How many sweets does each girl have if together they have 25 sweets? To solve this problem, you have to know the value of y.

Revision exercise

Learner's Book page 231

Guidelines on how to implement this activity

Use this exercise to assess if learners have coped with the chapter so far. Observe learners as they complete this exercise and identify which learners need remediation. Identify where the problem areas are and offer immediate remediation before continuing with the unit.

Suggested answers

1.1 $y = (60 + 3) \div 9 = 63 \div 9 = 7$

1.2 $z = (27 + 22) - 13 = 49 - 13 = 36$

2

x	+	y	=	5
0	+	5	=	5
1	+	4	=	5
2	+	3	=	5
3	+	2	=	5
4	+	1	=	5
5	+	0	=	5

Remedial

Provide any necessary remediation.

Exercise 1

Learner's Book page 232

Guidelines on how to implement this activity

Remind learners that when problem solving it is very important to read the question carefully. Learners must then identify what they have been given and what they have been asked to find. When working with patterns and functions suggest to learners that they should put the information in a table or flow diagram to clarify the idea. Revise how flow diagrams and tables work. Encourage learners to always check their answers and to write a closing sentence that answers the given question.

Suggested answers

1 $15 + y = 32$
$y = 32 - 15 = 17$

2 $5 + z = 9$
$z = 9 - 5 = 4$

3 $250 + x = 520$
$x = 520 - 250 = 270$

4 $c + 3c = 36$
$4c = 36$
$c = 36 \div 4 = 9$
Anita: 9 marbles;
Jane: $3 \times 9 = 27$ marbles

5 $d + 2d = 30$
$3d = 30$
$d = 30 \div 3 = 10$
Faisel: 10 books;
Manie: $2 \times 10 = 20$ books

6.1 Brown: $(a + 12) - 3 = 109$
Green: $(b + 12) - 5 = 57$
Yellow: $(c + 12) - 17 = 95$
White: $(d + 12) - 29 = 33$
Blue: $(e + 12) - 11 = 101$

6.2 Brown: $3 \times 10 = 30$
Green: $5 \times 11 = 55$
Yellow: $17 \times 12 = 204$
White: $29 \times 13 = 377$
Blue: $11 \times 14 = 154$

6.3 Number of bears bought $= 12 \times 5 = 60$
6.4 Number of bears sold $= 3 + 5 + 17 + 29 + 11 = 65$
6.5 Number of bears in stock $= 109 + 57 + 95 + 33 + 101 = 395$
6.6 Total profit $= 30 + 55 + 204 + 377 + 154 = 820$
7.1 $z + x = 18$
7.2

z	+	*x*	=	18
10	+	8	=	18
11	+	7	=	18
12	+	6	=	18
13	+	5	=	18
14	+	4	=	18
15	+	3	=	18
16	+	2	=	18
17	+	1	=	18
18	+	0	=	18

z	+	*x*	=	18
0	+	18	=	18
1	+	17	=	18
2	+	16	=	18
3	+	15	=	18
4	+	14	=	18
5	+	13	=	18
6	+	12	=	18
7	+	11	=	18
8	+	10	=	18
9	+	9	=	18

7.3 $z + 2z = 3z = 18$
$z = 18 \div 3 = 6$
Xolani: 6 goals; Thabo: $2 \times 6 = 12$ goals
8 $a = \{(24 \div 3) + 7\} - 5 = \{8 + 7\} - 5 = 10$
$b = \{(27 \div 3) + 7\} - 5 = \{9 + 7\} - 5 = 11$
$c = \{(30 \div 3) + 7\} - 5 = \{10 + 7\} - 5 = 12$
$d = \{(33 \div 3) + 7\} - 5 = \{11 + 7\} - 5 = 13$
$e = \{(36 \div 3) + 7\} - 5 = \{12 + 7\} - 5 = 14$

Remedial

Learners who have difficulty with word problems invariably have difficulty interpreting the question. This might be a reading problem, or it might be a number problem. Read through the examples with the learners ensuring they understand what they are being asked to do. Allow learners experiencing problems to work in pairs or small groups to discuss what they understand. Sometimes you could allow an advanced learner to be with the group experiencing difficulty and to help them. Be careful who you choose for this task.

Extension

Pairs or small groups of more advanced learners could create and illustrate short word problem stories. For example, they could choose characters, decide on a plot (perhaps a mystery), describe where and when their story happens, and then make sure they include word problems in the theme.

Consolidation

Learner's book page 235

Before doing this consolidation exercise, encourage learners to review the work covered in this chapter. Advise learners to use the summary to revise their work. This exercise can be used as an informal assessment task for you to track how learners are coping with the chapter and the concepts covered.

Suggested answers

1 $a = (35 + 5) \div 5 = 40 \div 5 = 8$
$b = (40 - 26) \times 3 = 14 \times 3 = 42$
$c = (45 \times 10) + 3 = 450 + 3 = 453$

Input Rules Output

$35 \longrightarrow \boxed{+\ 5} \longrightarrow \boxed{\div\ 5} \longrightarrow 8$

Input Rules Output

$40 \longrightarrow \boxed{-\ 26} \longrightarrow \boxed{\times\ 3} \longrightarrow 42$

Input Rules Output

$45 \longrightarrow \boxed{\times\ 10} \longrightarrow \boxed{+\ 3} \longrightarrow 453$ (2)

2

Teachers	1	2	3	4	5	6	7
Number of learners in all classes	24	30	36	42	48	54	60

(8)

3.1 $z + 3z = 24$ (3)
3.2 $4z = 24$; $z = 24 \div 4 = 6$
Danie: 6 goals (2)
3.3 Thabo: $3 \times 6 = 18$ goals (2)
4 $78 + x = 123$
$x = 123 - 78 = 45$ (3)
[20]

Chapter 12 Algebra 1

Overview of concepts

Algebra 1

UNIT 2 Number sentences and equations

UNIT 1 Algebraic language

Content		Time allocations	LB page
Unit 1	Algebraic language	3 hours	237
Unit 2	Number sentences and equations	3 hours	242

Background information on algebra 1

The word algebra comes from the Arabic word *al-jabr* meaning restoration.
Algebra has a language of its own. Learners need to understand the terminology in order to answer the questions. A sentence can be translated into a number sentence. For example, Five times z plus three is equal to thirteen. What is the value of z? As a number sentence this is: $5z + 3 = 13$. $5z + 3$ is called an **expression**. An expression is made up of 2 or more terms.
A term may be a **constant**, such as 3 (which has a fixed or constant value), or the product, or quotient, of a number and a variable. For example, $5z$ or $\frac{z}{5}$. $5z$ is a term. 3 is a term. There are 2 terms in the expression $5z + 3$. The variable in this expression is z.
The value of a variable can change according to how it is used in the expression: $y + 2 = 4$; $y - 2 = 4$; $2y = 4$; $\frac{y}{2} = 4$. Remind learners that we don't usually say $5 \times z$. We also do not say $z \div 5$, we say $\frac{z}{5}$.

Generic teaching guidelines for teaching algebra 1

Ensure that learners understand the concept of input and output values as a means of determining variables. Remind learners that variables are unknowns which are represented by letters of the alphabet. Constants have a fixed value like the 5 in $y + 5 = z$. Revise solving equations by inspection and by trial and improvement. Encourage learners to check their solutions by substitution.

Resources

Flashcards with new vocabulary, cardboard and colour pens to create posters, number and variable cards to create number sentences. Each learner should have their own calculator.

Unit 1 Algebraic language

Learner's Book page 237

Unit focus

- learning about algebraic language
- recognising and interpret rules and relationships in symbolic form
- identifying variables in expressions
- identifying constants.

Background information on algebraic language

Word sentences, such as 'seven times *p* divided by two is equal to fourteen' can be written in a number sentence such as: $7p \div 2 = 14$ or $\frac{7p}{2} = 14$. Learners should be familiar with this from their work of equivalent forms in functions and relationships Chapter 11.
$7p \div 2 = 14$ is an **equation**. This means the left hand side ($7p \div 2$) is equal to the right hand side (14).
$7p \div 2$ is an **expression**. 14 is also an expression. They are separated by an = sign. Expressions consist of 1 or more terms. **Terms** are separated by plus or minus signs. This means $\frac{7p}{2}$ is 1 term. 14 is also 1 term. It is not necessary for learners to know this yet, but it helps to be aware as the teacher of the next steps in future grades.
In the expression $3c + 5b - 6$ there are 3 terms. They are $3c$, $5b$ and 6.
Numbers like 6 in the last expression are called **constants**. They have a fixed value, so 6 = 6 = 6... Letters like *c* and *b* are called **variables**. Their value is not fixed. For example in this expression a + 7 the *a* could have any value. If $a = 0$ then $a + 7 = 7$, if $a = 1$ then $a + 7 = 8$ and so on.

Exercise 1

Learner's Book page 238

Guidelines on how to implement this activity

Introduce learners to the concept of an algebraic language. Explain that as a language, algebra has its own symbols and conventions. Show learners what they already know about algebra from working with variables and equations from Chapter 11. Revise the equal to sign and greater than and less than sign. Work through the algebraic conventions as listed in the Learner's Book. Make sure the learners grasp that a constant, such as 2 in $y + 2$, cannot change its value. Allow some learners to work on the board to substitute different values for y, and then to calculate the sum. The *y* in the expression is not a constant. Its value can change. We call it a variable. Variables can be used with all operations. Show examples: $3a$ means 3 times *a*; $\frac{b}{4}$ means *b* divided by 4; $7 - d$ means 7 minus *d* and $8 + f$ means 8 plus *f*.

Allow learners to write multiplication terms with a number and a variable in a short form. For example, $3 \times z = 3z$. Give a number of similar examples. Do the same with division, addition and subtraction. For example $y \div 2 = \frac{y}{2}$ and $x - 5 = x - 5$ and $w + 9 = w + 9$.

Suggested answers

1.1 variables: r; y, constants: –8; 3
1.2 variables: a; c, constants: 4; –6
1.3 variables: b, constants: 65; 8
1.4 variables: a, constants: –15; 7
2 $2 + z = 4x \times 3$ / $2z + 4 = 3 \times x$ / Allow multiple answers
3 $y - 2 = 1 + 3$ / $3y - 1 = 2$ / Allow multiple answers
4.1 $3y$
4.2 $7f$
4.3 $5 + 4c$
4.4 $5a - \frac{1}{2}$
4.5 $\frac{2}{t} - 6b$
5.1 False
5.2 True
5.3 False
5.4 True
5.5 True

Remedial

Write a simple expression on the board. For example, $y + 4$. Ask learners to write expressions on the board. Discuss what they have written. Ask: Is there a number in your expression? Tick it if there is. Is there a letter of the alphabet in your expression? Tick it if there is. Is there an operation sign (+, –, ×, ÷) in your expression? Tick it if there is. If they are correct move on, otherwise repeat until all have understood.
Go back to $y + 4$. Ask: What operation has to be performed? Addition. What do you have to add? 4 and y. Do you all know the value of 4? Can 4 change its value? (no) Can 4 be equal to 5? (no) 4 is a constant. Ask learners to write other constants on the board. Do you all know the value of y? (no) Can y change its value? (yes) How? By adding a sum to the calculation. So:

y	+	4	=	Sum
1	+	4	=	5
2	+	4	=	6
3	+	4	=	7

The 4 remains constant. The y has changed (varied) from 1 to 2 to 3. y is called a variable.

Extension

Encourage learners to write word problems and then translate them into number sentences. They should then identify the expressions, the terms, the constants and the variables in the number sentences.
Ask learners to use time as the basis for some number sentences. This could include minutes, seconds or weeks. For example, they could start: There are p months with 30 days in them and so forth.

Exercise 2

Learner's Book page 240

Guidelines on how to implement this activity

Revise the work the learners did in Chapters 10 and 11, where they created equations to describe patterns and relationships. Show learners that they used algebra. Discuss equations and what being equal means – learners must know that the LHS of an equation must equal the RHS. Work through the table in the Learners' Book that shows common verbal descriptions and their operational equivalent. Ask learners to come up with an example of each. Encourage learners to complete this exercise on their own, but to discuss their working and thinking in groups.

Suggested answers

1.1 False, $99 + 10 = 109$

1.2 False, $5 \times 7 = 35$

1.3 True

1.4 True

1.5 True

1.6 False, $18 - 12 = 6$

1.7 True

1.8 False, $14 + 6 = 20$

1.9 True

1.10 True

2.1

	Marbles	× 3	+ 2	– 1	Total
Muneeb	10	30	32	31	31
Katlego	20	60	62	61	61
Harry	12	36	38	37	37

2.2 Total number of marbles = $(3n + 2) - 1$

3 3b

4.1 $\frac{a}{2} + 7$

4.2 $\frac{2b}{5}$

4.3 $\frac{7c}{2} + 6$

4.4 $7 \times (5 + 9) = 7(5 + 9)$

4.5 $5 \times (8 - 3) = 5(8 - 3)$

5.1

Input — Rules — Output

$7 \longrightarrow \boxed{+\,8} \longrightarrow \boxed{\div\,3} \longrightarrow 5$

5.2

Input — Rules — Output

$14 \longrightarrow \boxed{-\,2} \longrightarrow \boxed{\times\,7} \longrightarrow 84$

5.3

Input — Rules — Output

$21 \longrightarrow \boxed{\times\,10} \longrightarrow \boxed{+\,7} \longrightarrow 217$

5.4.1 $\frac{7 + 8}{3} = 5$

5.4.2 $7(14 - 2) = 84$

5.4.3 $(21 \times 10) + 7 = 217$

6.1

Cakes	1	2	3	4	5	6	7
Cost	R2,50	R5,00	R7,50	R10,00	R12,50	R15,00	R17,50

6.2 Cost = $2{,}50 \times n$

Remedial

Work through some additional examples of working between verbal descriptions and using algebra. Do as many additional examples as needed until learners can manage on their own.

Extension

Ask learners how they might represent a square number or a cube number using algebra.

Unit 2 Number sentences and equations

Learner's Book page 242

Unit focus

- creating number sentences to describe problem situations
- solving number sentences using inspection
- solving number sentences using trial and error
- using substitution to find the value of an equation
- analysing and interpret number sentences in problem situations.

Background information on number sentences and equations

Number sentences can be derived from word sentences. For example, the sum of seven times y and nine is equal to fifty-one. This can be written: $7y + 9 = 51$. Number sentences can also describe word problems. This means algebra can be used to solve word problems. The benefits of writing a number sentence are that it's faster to read and often easier to interpret. This means it can be solved by inspection. Sometimes number sentences have to be solved by trial and improvement. This means you estimate or guess the answer, and then test to see if the guess is correct.

Exercise 1

Learner's Book page 243

Guidelines on how to implement this activity

Revise from the previous unit creating a number sentence. Learners should know that number sentences can also be equations, if they have a LHS and a RHS, which are equivalent. An equation has to have an equal to sign. Explain how to check if equations are valid by checking first the LHS and then the RHS.
Next discuss how to use equations to find missing values. Learners should already know how to use inspection or trial and error to solve for variables. Revise by doing a few examples of solving simple equations together on the board. Show learners how to check their answers for trial and error by substituting in the possible value. Encourage learners to work on their own to complete this exercise.

Suggested answers

1.1 True, $4 + 5 = 9$
1.2 True, $y = 12$; $12 - 2 = 10 > 7$
1.3 False, $5 \times 5 = 25$; $25 > 20$

1.4	True, $\frac{28}{7} = 4$						
1.5	False, $9 - 3 = 6$; $6 > 3$						
2.1	$a = 15$	**2.2**	$b = 38$	**2.3**	$c = 10$	**2.4**	$y = 35$
2.5	$z = 14$	**2.6**	$a = 9$	**2.7**	$b = 4$	**2.8**	$c = 22$
2.9	$r = 9$	**2.10**	$g = 38$	**2.11**	$b = 8$	**2.12**	$a = 32$

3.1	$3x - 6 = 21$; $3x = 27$; $x = 9$	**3.2**	$9y = 18$; $y = 2$
3.3	$2b = 14$; $b = 7$	**3.4**	$3c = 18$; $c = 6$
3.5	$d + 2 = 5$; $d = 3$	**3.6**	$28 + e = 30$; $e = 2$
3.7	$27 - f = 20$; $f = 7$	**3.8**	$29 - g = 9$; $g = 20$
3.9	$65x = 130$; $x = 2$	**3.10**	$\frac{12}{c} = 3$; $c = 4$
3.11	$f = 6$	**3.12**	$a = 28$

4.1 $z = 3{,}53$
4.2 $0{,}09 \times 1\,000 - y = 73$; $90 - y = 73$; $y = 17$
4.3 $7{,}57 - x = 2{,}59$; $x = 4{,}98$
4.4 $\frac{249}{100} = r$; $2{,}49 = r$
4.5 $b + 5 = 18$; $b = 13$
4.6 $7c - 32 = 10$; $7c = 42$; $c = 6$
4.7 $\frac{45}{d} = 9$; $d = 5$
4.8 $\frac{3}{4} \times (y - 7) = 39$; $y - 7 = 52$; $y = 59$
4.9 $h + 5 = 8$; $h = 3$
4.10 $j - 4 = 4$; $j = 8$
4.11 $3 + a = 20$; $a = 17$
4.12 $\frac{10}{a} = 5$; $a = 2$

Remedial

Encourage all learners to try questions 1 – 3. Some learners may struggle with certain aspects in question 4, as they involve working with fractions and decimals. Revise fraction and decimal work from previous chapters and encourage learners to refer back to these notes in order to answer these questions.

Exercise 2

Learner's Book page 245

Guidelines on how to implement this activity

Revise the various verbal descriptions and their translation in algebraic language, and remind learners that the unknown is the variable. Have learners practise writing word problems in algebra. Give learners a word problem to solve. Write the problem on the board. Discuss the problem. Ask learners to try to write a number sentence to describe the problem. Allow some learners to write their sentences on the board. Discuss the number sentences that learners provided. Are they all basically the same? If not, how do they differ? Are they all workable? Are they all correct? If not, why not? Show learners the correct number sentence and solve it using inspection or trial and error. Repeat this with different examples until learners feel confident to manage the exercise on their own.

Suggested answers

1.1 $a - 1$ **1.2** $\frac{a}{2}$ **1.3** $a + 1$; $a + 2$

2.1 $x + y = 45$ **2.2** $p \times q = 24$ / $pq = 24$

2.3 $r - s = 16$ **2.4** $\frac{m}{5} = 7$

3.1

x	y	x	y	x	y	x	y	x	y	x	y	x	y
0	45	7	38	7	38	21	24	28	17	35	10	42	3
1	44	8	37	8	37	22	23	29	16	36	9	43	2
2	43	9	36	9	36	23	22	30	15	37	8	44	1
3	43	10	35	10	35	24	21	31	14	38	7	45	0
4	41	11	34	11	34	25	20	32	13	39	6		
5	40	12	33	12	33	26	19	33	12	40	5		
6	39	13	32	13	32	27	18	34	11	41	4		

3.2

p	1	2	3	4	6	8	12	24
q	24	12	8	6	4	3	2	1

3.3 Has infinitely many solutions. For example, $48 - 32 = 16$; $r = 48$; $s = 52$

3.4 $m = 35$

4.1 Answer is 8.

4.2 Answer is 8.

5 Let b represent the number of bats, m the number of bushbabies and n the number of owls.

$b = 8$m; $n = \frac{b}{2}$

6 $6x = 4\ 494$; $x = 749$

749 thorn trees and $5 \times 749 = 3\ 745$ boababs

7 $10y = 4\ 760$; $y = 476$

Dysentery: 476; bleeding: $4 \times 476 = 1\ 904$; abscesses: $5 \times 476 = 2\ 380$

Remedial

If learners continue to have difficulty, revise by means of a simple problem on the board. For example, Tom has 3 books and his sister Mary has y books. Together they have 7 books. How many books does Mary have? Ask learners to write a number sentence to show the problem. If they have difficulties write the following on the board:

Tom's books + Mary's books = 7 books.

This is: $3 + y = 7$ (number sentence)

To solve subtract the 3 from the LHS and from the RHS.

So: $3\ (-\ 3) + y = 7\ (-\ 3)$

$y = 4$

Extension

In number sentences the LHS and the RHS are equal. This means they are balanced, like an old fashioned scale. When you add or subtract on one side, to keep the balance you must do the same to the other side. Although they are balanced they don't necessarily look even. For example, $3a + 4b + 6c - 7d = 0$. Ask learners to study the work of artist Alexander Calder. He makes three dimensional balanced 'mobiles.' Ask learners to use scrap material to try to make a balanced 3D work where the sides are not identical, but have the same 'weight.'

Exercise 3

Learner's Book page 247

Guidelines on how to implement this activity

Revise how you used substitution to check whether the answers were correct when you used trial and error for solving equations. Discuss how using a formula requires using substitution. Do a few examples using the formula for perimeter and area of shapes. Show learners how we can determine the value of a whole expression when we are given the values of the variables. Do a few examples together as a class on the board. Encourage learners to try this exercise on their own.

Suggested answers

1.1 $y = (2 \times 3) - 5 = 6 - 5 = 1$

1.2 $y = (2 \times 4) - 5 = 8 - 5 = 3$

1.3 $y = (2 \times \frac{5}{2}) - 5 = 5 - 5 = 0$

1.4 $y = (2 \times 6{,}3) - 5 = 12{,}6 - 5 = 7{,}6$

1.5 $y = (2 \times 12) - 5 = 24 - 5 = 19$

1.6 $y = (2 \times 100) - 5 = 200 - 5 = 195$

2.1 $A = \frac{1}{2} \times 4\text{ cm} \times 3\text{ cm} = 6\text{ cm}^2$

2.2 $A = \frac{1}{2} \times 7\text{ m} \times 4\text{ m} = 14\text{ m}^2$

2.3 $A = \frac{1}{2} \times 4{,}3\text{ cm} \times 2{,}1\text{ cm} = 4{,}515\text{ cm}^2$

2.4 $A = \frac{1}{2} \times 3{,}4\text{ m} \times 2{,}67\text{ m} = 4{,}539\text{ m}^2$

3.1 $3 \times 2 \times 0 = 0$

3.2 $3 \times 9 - 6 + 1 = 27 - 6 + 1 = 27 - 5 = 22$

3.3 $14 - \frac{3 \times 4}{2} = 14 - 6 = 8$

3.4 $19 - 5{,}67 + 4{,}12 = 17{,}45$

3.5 $1\frac{5}{6} - \frac{4}{7} = \frac{11}{6} - \frac{4}{7} = \frac{77 - 24}{42} = \frac{53}{42} = 1\frac{11}{42}$

3.6 $\frac{6}{10} - \frac{43}{100} = \frac{60}{100} - \frac{43}{100} = \frac{17}{100}$

Remedial

Learners should not have problems with the concept, as they have worked with substitution and formulae before. If they do experience problems, refer them back to the relevant chapters to revise. Alternatively work through the problem areas with them explaining any gaps in their understanding. Certain questions do include fractions and decimals and learners may need to revise the chapters on decimals and fractions in order to cope.

Extension

Provide additional examples involving complex calculations including fractions and decimals to challenge those learners that require extension.

Consolidation

Learner's book page 249

Before doing this consolidation exercise, encourage learners to review the work covered in this chapter. Advise learners to use the summary and to revise their work. This exercise can be used as an informal assessment task for you to track how learners are coping with the chapter and the concepts covered.

Suggested answers

1 $5y + 8 = 7x$ / $8y + 58 = 7x$ / Accept alternatives (2)

2 $30(14 + 7)$ (2)

3.1

Input | Rules | Output

$5 \longrightarrow \boxed{+6} \longrightarrow \boxed{\times 2} \longrightarrow 22$

3.2

Input | Rules | Output

$10 \longrightarrow \boxed{-4} \longrightarrow \boxed{\div 3} \longrightarrow 2$

3.3

Input | Rules | Output

$5 \longrightarrow \boxed{\times 10} \longrightarrow \boxed{+3} \longrightarrow 53$

(3)

4

Sweets	1	2	3	4	5	6	7
Cost	65c	R1,30	R1,95	R2,60	R3,25	R3,90	R4,55

(4)

5.1 $z = 16$

5.2 $y = 31$

5.3 $x = 5$

5.4 $q = 2$ (8)

6.1 $a > 2$

6.2 $b < 17$

6.3 $c > 4$

6.4 $x < 3$ (4)

7.1 $4z - 30 = 10$; $4z = 40$; $z = 10$

7.2 $3y + 7 = 22$; $3y = 15$; $y = 5$

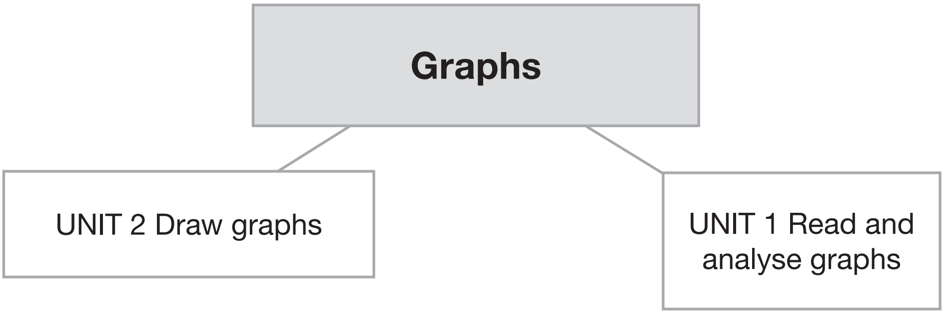
7.3 $9 = 3x$; $x = 3$

7.4 $240 = 24p$; $p = 10$ (7)

[30]

Chapter 13 Graphs

Overview of concepts



	Content	Time allocations	LB page
Unit 1	Read and analyse graphs	3 hours	251
Unit 2	Draw graphs	3 hours	258

Background information on graphs

In the Intermediate Phase, learners encountered graphs in the form of data bar graphs and pie charts. This means they have some experience reading and interpreting graphs. However, in the Senior Phase, they are introduced to line graphs that show functional relationships described in terms of dependent and independent variables.

In Grade 7, the focus is on drawing, analysing and interpreting global graphs only. That is, learners do not have to plot points to draw graphs and they focus on the features of the global relationship shown in the graph.

Examples of contexts for global graphs include:

- the relationship between time and distance travelled
- the relationship between temperature and time over which it is measured
- the relationship between rainfall and time over which it is measured.

Generic teaching guidelines for teaching graphs

The focus in this chapter is analysing and interpreting graphs by asking: is the graph linear or non-linear, and is the graph constant, increasing or decreasing? Learners are also required to draw graphs and identify trends.

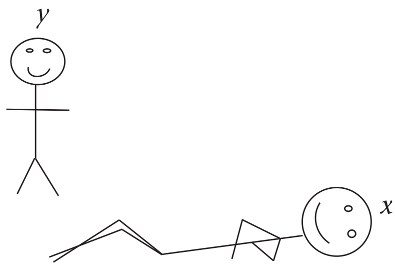
Learners must be reminded that a graph shows a relationship between two variables in a graphic way.

Do some revision of simple number sentences to prepare learners. A number sentence such as $y = x$, describes the relationship between x and y. This relationship between two variables x and y can be shown in a graphic way using a graph's x and y axes.

To explain the dependent and independent variables, you can use examples like the following:

- temperature *depends* upon the time of day. In this case temperature is the dependent variable (y) and time of day is the *independent* variable (x).
- Temperature also *depends* upon the time of year (seasons). Again, temperature is the dependent variable (y) and time of year is the *independent* variable (x).

You can also use the example that sometimes adults or parents are asked how many "dependents" they have. The child is called the "dependent". They can associate the child with y and the parent with x:



The horizontal axis is for the independent variable x. Learners can remember horizontal by associating with the horizon. The vertical axis is perpendicular on the horizontal axis and is for the dependent variable y. The value of y *depends* on the value of x.

Resources

Have pictures of graphs from newspapers, magazines or the internet to show learners. Cardboard and colour pens, blank graph grids and planes for learners to use, examples of increasing, decreasing, and linear graphs to show learners. Each learner should have their own calculator.

Unit 1 Read and analyse graphs

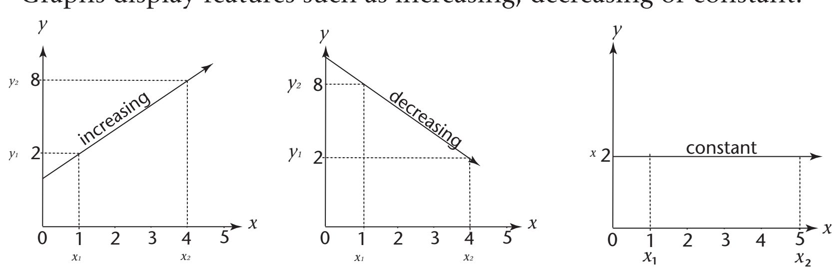
Learner's Book page 251

Unit focus

- learning about line graphs and dependent and independent variables
- analysing and interpreting graphs by looking at trends such as:
  - distinguishing between linear and non-linear in a graph
  - distinguishing between increasing, decreasing or a constant graph.

Background information on reading and analysing graphs

- A line graph shows the relationship between the dependent variable y and the independent variable x
- A graph that forms a straight line is called linear graph
- If the graph does not form a straight line it is called non-linear graph
- Graphs display features such as increasing, decreasing or constant:



Revision exercise
Learner's Book page 251

Guidelines on how to implement this activity

Use this exercise to determine learners' prior knowledge. You can identify any problem areas or knowledge deficits that need to be addressed before continuing with the rest of the chapter.

Suggested answers

1.1 (single) bar graph
1.2 Personal Loans of South African consumers: 2008 - 2011
1.3 Year
1.4 Billions of Rand
1.5 Every consecutive year after 2008, the rand value of personal loans increased.
1.6 It would seem that every year, from 2008 onwards, people are borrowing more money than the year before and increasingly so.

Remedial

Provide the appropriate remediation to learners who were not able to manage this exercise.

Exercise 1
Learner's Book page 254

Guidelines on how to implement this activity

Discuss what a graph is. Ask learners why we draw graphs? Ensure learners know that when reading a graph they must look for the following to get a better understanding of a graph:

- the graph's title - this will tell them more about what the graph is about
- the *x*-axis label - will tell them what is the independent variable
- the *y*-axis label - will tell them what is the dependent variable.

Revise what independent and dependent variables are and be sure to remind learners that the value of the *y*-axis value is dependent on what the *x*-axis value is.
Show learners representations of linear and non-linear graphs. What do learners notice about linear graphs – they form a line. Non-linear graphs can be scattered or curves. Ask learners to identify when a graph is linear or non-linear. Learners can complete this exercise on their own.

Suggested answers

1 Non-linear
2 Linear
3 Linear
4 Non-linear
5 Non-linear

Remedial

Learners might confuse the dependent and independent variables. They need to simply associate the dependent with the *y*-axis and the independent with the *x*-axis. They can look at as many line graphs as possible and read the *x*-axis title which will tell them which is the independent variable.

On the same graph, by reading the y-axis title, they will know what the dependent variable is. Once learners have identified the dependent and independent variables, let learners express in their own words: "The ... (for example temperature) depends on the ... (time of day)".

Extension

Have learners look in newspapers to find other types of graphs, such as pie charts or bar graphs. Ask learners to look for line graphs that are not linear. They should provide an explanation as to why certain line graphs are linear and others non-linear.

Exercise 2

Learner's Book page 256

Guidelines on how to implement this activity

Show learners an example of an increasing graph. Explain to learners that a line graph is increasing if the value of y increases between x_1 and x_2 which means $y_2 > y_1$. Learners can make the association with riding a bicycle "uphill". Show learners a decreasing graph. Explain that a line graph is decreasing if the value of y decreases between x_1 and x_2 which means $y_2 < y_1$. Learners can make the association with riding "downhill". Show learners that a line graph is constant if the value of y stays the same between x_1 and x_2. Learners can make the association with a level road. Explain that analysing graphs in this way is referred to as reading the trends of the graph. Work through the worked example in the Learner's Book together as a class. Ask learners to supply the trends they can see. Do additional examples if necessary. Ensure all learners understand the concept before starting Exercise 2.
If necessary remind learners that a line segment is a piece of straight line and that to analyse and interpret a graph means to read it with understanding.

Suggested answers

1.1 Non-linear
1.2 Dependent variable: time; independent variable: year
1.3 The graph is increasing.
1.4 It takes commuters longer to travel to work if the roads are poorly maintained and traffic lights are broken.
1.5 The Gautrain runs on time and can travel faster than a car; therefore a commuter will get to work faster.
2.1 Between December and January, February and April
2.2 Between January and February, April and May
2.3 No, because the graph is not a horizontal line.
2.4 The independent variable (months) is on the horizontal axis.
2.5 The dependent variable (number of bicycles) is on the vertical axis.
2.6 Over the peak holiday season the number of bicycles rent increased.

Remedial

Learners might still get confused with the terms "increasing" and "decreasing". They just need to learn that and make some kind of the association for example with uphill and downhill. Let learners find as many line graphs as possible in newspapers or magazines to bring to school.

They can look in the business section of many of the newspapers. There should be many line graphs that show for example the oil price or economic data such as inflation. Once they have at least two or three line graphs, let them paste them in their books and identify the following:
- the graph's title, x-axis label and y-axis label
- dependent and independent variables
- increasing trends (if any)
- decreasing trends(if any)
- constant trends(if any).

Learners can write down the names of the titles and the variables. On the graphs themselves they can use different coloured pens to circle the different trends. Let the learners explain what they have learnt from each of the graphs which they analysed.

Extension

Learners are not expected to plot points (x, y); however, they will benefit from a practical data collection exercise listed below. Once learners have collected their data they can use it in Unit 2 to draw line graphs.
Assist learners to find a temperature gauge (thermometer) that can measure ambient temperature. On a daily basis, for at least two weeks, let them read off the thermometer and record the temperature and the time of day.
If a thermometer is not available, let them come up with something else to measure, for example rainfall. Or they can decide to record the percentage of cloud cover. For example, if they look at the sky they can estimate a percentage. On a daily basis they can record the day and time along with the percentage.
You could also allocate either of the three options to small groups of two or three learners per group, so that there will be three different data sets to draw graphs from.

Unit 2 Draw graphs

Learner's Book page 258

Unit focus

- drawing line graphs from a description
- identifying features of a graph such as:
  - an increase
  - a decrease
  - a constant.

Background information on drawing graphs

Learners are not expected to plot points (x, y); however, provide them with graph paper if available. Otherwise they must just draw neatly and plan before they start drawing. They have to consider the following aspects when drawing graphs:
- Identify the independent (x) variable and dependent (y) variable.
- What is the range of the x and y values respectively?
- In their Exercises, most numeric values such as mm rainfall or distance travelled in km or percentages appear on the y-axis.
- Depending on the range of values, what intervals will be suitable for the y axis?

Most importantly, learners must work neatly. Even if the line graph they produce is not precise, the scales must be sensible and the line segments must exhibit linearity – straight lines preferably drawn with a ruler.

Exercise 1

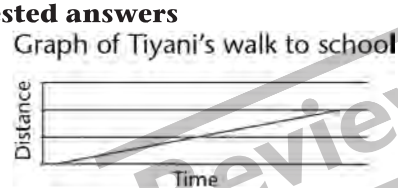
Learner's Book page 260

Guidelines on how to implement this activity

Revise increasing, decreasing and constant trends on graphs and what they look like. Explain to learners that they are now going to draw their own graphs. Show learners that when they draw a line graph, they should remember to do the following:
- add an appropriate title to describe the data being displayed
- identify the independent (*x*) variable and dependent (*y*) variable
- use suitable intervals and a suitable scale for each axis
- label the horizontal (*x*) axis and vertical (*y*) axis.

Work through the example in the Learner's Book on drawing a graph. Draw the graph on the board, and have learners fill in the heading, axes and the points on the graph. Once complete ask the learners questions based on the graph. Use the points of analysis learners learnt in Unit 1. Repeat with as many examples you feel are necessary. Provide grid or graph paper for learners to use as they complete this exercise.

Suggested answers

1


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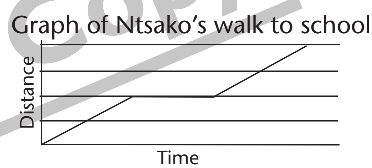

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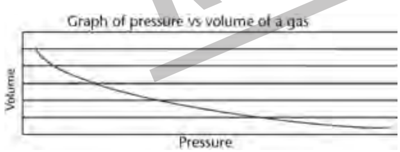

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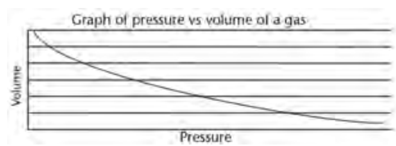


Remedial

Some learners might find it hard to identify the dependent and independent variables. Let learners ask themselves questions. For example, does the season depend on the temperature or does the temperature depend on the season? The one that depends upon the other is on the *y*-axis.
Learners must plan their graphs carefully, otherwise the graph they draw is either not on a suitable scale or they have to keep redrawing it to fit it in.

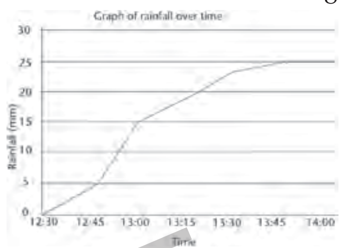
Extension

Learners can draw the line graphs from the data they collected on daily temperature or rain fall or cloud cover. Ask the learners to report on their conclusions by looking at the trends in each graph:
- What do they notice about the rainfall/temperature/cloud cover trends?
- For each conclusion: why do they think that is the case?

Exercise 2 — Learner's Book page 261

Guidelines on how to implement this activity

This exercise leads on from the work covered in the previous exercise. The graphs become more complex, and include more data. It becomes increasingly necessary for learners to plan and prepare their graphs. Work through the example in the Learner's Book. Show learners how to use the graph to make inferences about the data. Do an additional example if required. Worked example no. 3 deals with a broken line graph that is not required for Grade 7, but can be used for extension purposes. Learners can discuss their answers in pairs, but each learner should show all the working and produce a graph.



Suggested answers

1.1 Graph of daily rainfall

1.2 The independent variable is time.

1.3 The dependent variable is rainfall.

1.4 →

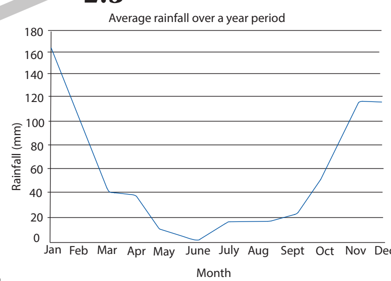
1.5 The graph is increasing from 12:30 till 13:45. From 13:45 till 14:00 the rainfall is constant.

1.6 As the storm approaches, it starts to rain harder and therefore the graph is increasing from 12:30 till 13:45. From 13:45 onwards the storm has arrived and the rainfall stays the same.

2.1 Average monthly rainfall over a year

2.2 The independent variable is the month and the dependent variable is the rainfall.

2.3



2.4.1 The graph is decreasing.

2.4.2 The graph is constant.

2.4.3 The graph is increasing.

2.5 It rains less as it gets colder through the year (January to June) and it starts to rain more as the temperature increases (June to December). Alternatively learners may talk about summer rainfall areas.

Remedial

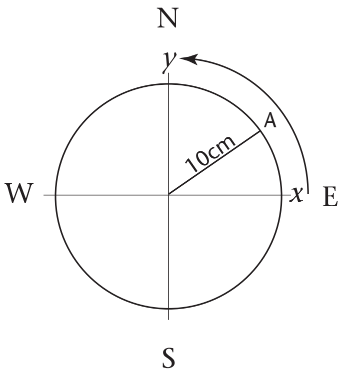
When drawing graphs encourage learners to read each question through at least twice and identify each variable. They can then plan suitable intervals and scales for each axis. Learners often lose unnecessary marks because they do not label each axis or forget the title. Remind learners about these three things to check once they have finished drawing the graph.

Extension

If time allows, learners can complete the following Extension Exercise:

Extension Exercise 1

A circle has a radius of 10 cm as shown.



1 What is the value of y when point A is directly North (N)?

2 What is the value of y when point A is directly East (E)?

3 What is the value of y when point A is directly West (W)?

4 What is the value of y when point A is directly South (S)?

Consolidation

Learner's book page 264

Before doing this consolidation exercise, encourage learners to review the work covered in this chapter. Advise learners to use the summary to revise their work. This exercise can be used as an informal assessment task for you to track how learners are coping with the chapter and the concepts covered.

Suggested answers

1.1 The rate of unemployment increased over the period 2009 – 2012. (1)

1.2 The independent variable is the year and the dependent variable is the unemployment (in percentage). (2)

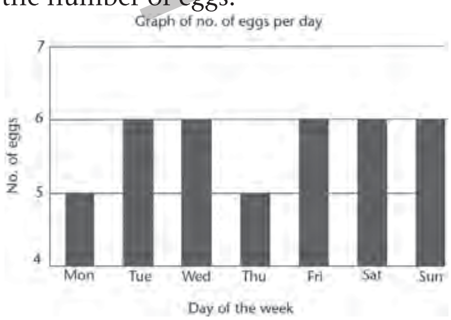
1.3 The graph is non-linear, since the whole graph is not one straight line. (1)

1.4 The graph exhibits an increasing trend. This is portray by the upward slanting line segments. (2)

1.5 Yes, this is a problem. The result is that increasingly more people do not earn an income and therefore will struggle to take care of their families and pay expenses. (2)

2.1 The independent variable is the day of the week and the dependent variable is the number of eggs. (2)

2.2



(5)

2.3 Tuesday and Wednesday, Friday till Sunday (2)

2.4 6. (1)

2.5 Between Monday and Tuesday, Thursday and Friday (1)

2.6 Between Wednesday and Thursday (1)

2.7 $2 \times (5 + 6 + 6 + 5 + 6 + 6 + 6) = 2 \times 40 = 80$ (2)

[22]

Chapter 14 Transformation geometry

Overview of concepts

Transformation geometry

UNIT 1
Transformations

UNIT 2
Symmetry

UNIT 3
Enlargements and reductions

Content		Time allocations	LB page
Unit 1	Transformations	3 hours	266
Unit 2	Symmetry	3 hours	272
Unit 3	Enlargements and reductions	3 hours	276

Background information on transformation geometry

In transformation geometry learners recognise, describe and perform translations, reflections and rotations with geometric figures and shapes. In the Intermediate Phase learners referred to the translations as slides, rotations as turns and reflections as flips. Learners perform transformations on grid paper - they do not work on a Cartesian plane in Grade 7. Learners need to be able to work with symmetry, and to identify and draw lines of symmetry in geometric figures.
In Grade 7 learners work with enlargements and reductions of geometric figures on squared paper and compare them in terms of shape and size.

Generic teaching guidelines for teaching transformation geometry

Introduce transformations with a simple shape like a square or equilateral triangle. Demonstrate on squared paper how the shape can be transformed. This means introduce translations (slides), reflections (mirror images) and rotations (turns). Demonstrate how to locate positions on ordered grids by using horizontal and vertical change. For example, 10 blocks down and 3 blocks to the right. Demonstrate how to draw lines of symmetry as in joining opposite corners of a square. Still using squared paper, demonstrate enlargement and reduction of shapes.

Resources

Grid paper, tracing paper or transparencies, marker pens, cardboard, colour pens and a ruler, scissors and drawing apparatus to construct the figures. If possible use computer software to show transformations in action.

Unit 1 Transformations

Learner's Book page 266

Unit focus

- revising your knowledge of translations, reflections and rotations
- learning how to recognise the types of transformations
- describing transformations of shapes
- practising doing translations, reflections and rotations.

Background information on transformations

Transformations include translations, reflections and rotations. We use letters and numbers to label grids. This grid system is used to find places in atlases. We can use coordinates when performing transformations. In a translation all points in a figure move the same distance in the same direction.
A reflection is a mirror image. For example, a geometrical figure 'flipped' over an axis, which is a mirror line, or axis of symmetry, forms a reflection.
A rotation is where a figure is turned around a centre of rotation in a clockwise or anticlockwise direction.

Exercise 1

Learner's Book page 267

Guidelines on how to implement this activity

Revise transformations that learners completed in the Intermediate Phase. Ensure learners replace their vocabulary from the intermediate phase, namely slides, flips and turns, with the correct terms of translation, reflections and rotations. Do examples on the board, or on a projector, where learners need to recognise the various transformations. Use grids to make this easier.

Suggested answers

1	Translation	**2**	Reflection
3	Rotation and translation	**4**	Translation
5	Reflection		

Remedial

Repeat as many physical manipulations of shapes on a grid as required, to help learners to have a concrete understanding of transformations.

Exercise 2

Learner's Book page 271

Guidelines on how to implement this activity

To best teach performing transformations, it is necessary that learners do as many concrete manipulations of transformations as possible. Use the following exercise to teach learners how to perform translations:

Give each learner two sheets of squared paper. Instruct learners to label the sheets A and B. On sheet A, using the blocks as guides, draw a square 5 blocks by 5 blocks, (call it X) a right angle triangle with vertical 6 blocks and horizontal 3 blocks (call it Y) and a rectangle 3 blocks by 2 blocks (call it Z). Cut these 3 shapes out.

On sheet B, copy the 3 shapes. Call the square L, the triangle M and the rectangle N. Fit the cut out square X on square L on page B. Now slide X to one side. Draw around it and call it L1. Slide it another way, draw around it and call it L2. Sliding a shape like this is called translating.

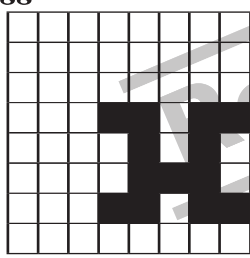
Fit the cut out triangle Y on triangle M on page B. Flip Y so that the two verticals of Y and M stay together. Draw around Y and call the new triangle M1. Take note, it is identical to M, but facing the other way. M1 is a reflection of M.

Fit the cut out rectangle Z on rectangle N on page B. Pin Z down through a centre point. Turn (rotate) Z slowly in a clockwise direction, through 90°, 180°and to 360°.

After completing the above activity learners should be able to manage the following exercise.

Suggested answers

1



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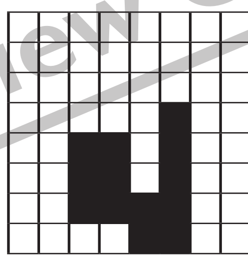

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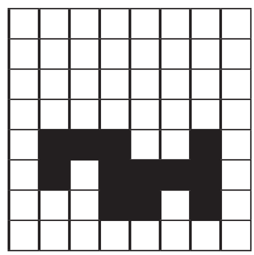

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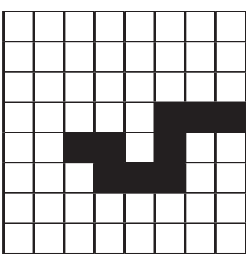

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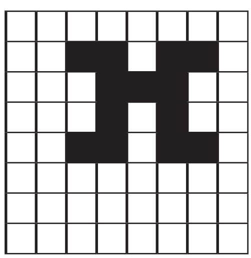

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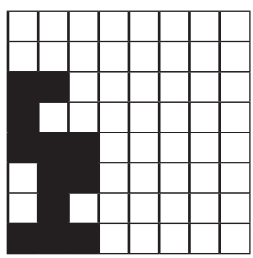


Remedial

Learners who have difficulty understanding transformations should be allowed to practice the concepts with concrete objects. Slide a piece of cardboard around the surface of the desk. This sliding is called translation. Ask them to translate a book to a new position, then a ruler, and an eraser. Next use the piece of cardboard again.

Put it flat on the desk. Then ask learners to stand it on an edge, and then flip it over on its back. This is called reflection. Try it with a text book or a lunchbox. Last, use a ruler. Put the index finger of one hand firmly on the end of the ruler to hold it in a position. Using the other hand turn the ruler in a propeller like movement until you get back to the original position. This is called rotation.

Extension

Transformations allow many opportunities for art. Let learners look at Op art of Vasarely and Kelly . Encourage learners to apply transformations of shapes to create paintings. Cut out geometric shapes from old cereal boxes. Experiment with creating 2D shapes and then using them in transformations on a large 'mural'.

Unit 2 Symmetry

Learner's Book page 272

Unit focus

- identifying lines of symmetry, and
- drawing lines of symmetry on 2D shapes.

Background information on symmetry

Learners have dealt with symmetry in earlier grades. The concept should be familiar, however you may need to revise the terminology with learners. A line that divides a shape into 2 identical halves is called an axis of symmetry. This means if you draw a square, or any geometric shape, and draw a line to cut it into 2 identical parts, that line will be the axis of symmetry. The 2 halves are symmetric about the axis of symmetry. When a shape has rotational symmetry it means it fits exactly over itself when rotated. For example when a square is rotated 360° it will fit exactly over itself at 90°, at 180°, at 270° and at 360°. A square will turn four times in a full rotation.

Exercise 1

Learner's Book page 273

Guidelines on how to implement this activity

Discuss what symmetry is. Encourage learners to provide their ideas and knowledge of symmetry. Give learners squared paper and ask them to draw a square 6 blocks by 6 blocks. They must cut out the square and fold it diagonally. The 2 sides are identical. The fold line is the axis of symmetry. The 2 sides are symmetric about the axis of symmetry. Ask learners to cut out other shapes, and discover if they are symmetrical. Allow learners to do this until you feel confident they all have grasped the concept of symmetry and an axis of symmetry. Learners can complete this exercise on their own.

Suggested answers

1.1 $+ - \times \div$
1.2 They all are.
2 Example: S A R A H
The A and the H in SARAH are symmetrical.
3 Symmetrical shapes: 3.2; 3.3

Remedial

If learners experience problems in drawing lines of symmetry on shapes or figures in their books, encourage them to trace the figures onto scrap paper and to use the folding method to check if their lines are the lines of symmetry.
If learners experience problems with understanding how a shape can have more than 1 line of symmetry, ask learners to draw 2 squares each 6 blocks by 6 blocks on squared paper. Cut them out. Instruct learners to fold one in half diagonally. The 2 halves should be identical triangles. The fold line is called the axis of symmetry. Instruct them to fold the other square in half horizontally. The 2 halves should be identical rectangles. The fold line is also called the axis of symmetry. This is how a shape can have more than 1 axis of symmetry.

Extension

Learners must explore a variety of shapes for symmetry. The shapes should be based on geometrical shapes. They can decorate the shape. For example, a rectangle in a rectangle in a rectangle, each a different colour. These can then be attached to symmetry charts demonstrating axes of symmetry and rotational symmetry.

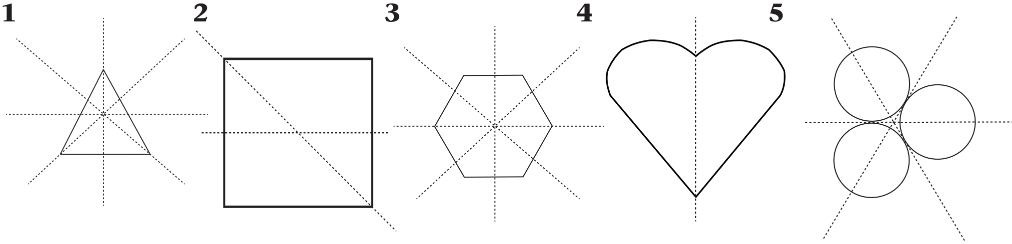
Exercise 2 Learner's Book page 275

Guidelines on how to implement this activity

To explore rotational symmetry provide learners with additional piece of grid paper. Ask learners to draw and cut out a rectangle ABCD 4 blocks by 6 blocks. Fold the rectangle diagonally from A to C and from B to D. The 2 folds cut at the centre of the rectangle at E. Draw a horizontal line on the paper. Use a straight pin to pin the rectangle down through E so that BC is on the line you have ruled. Turn the rectangle clockwise or anti-clockwise till the rectangle comes to rest on the line again. This time AD will be on the line. This is called rotational symmetry. The symmetry happens when the rectangle has turned 180°. A rectangle has a rotational symmetry of 2. Have learners work through this exercise using shape cut outs to help them identify rotational symmetry.

Suggested answers

1 **2** **3** **4** **5**



Extension

Have learners investigate additional shapes, such as octagons, nonagons and a circle. What do learners notice about these shapes and their rotational symmetry?

Unit 3 Enlargements and reductions

Learner's Book page 276

Unit focus

- learning to draw shapes accurately
- increasing the size of shapes in proportion
- reducing the size of shapes in proportion.

Background information on enlargements and reductions

An enlargement or reduction of a geometrical figure is a form of transformation in which all lines in the figure are increased or decreased in proportion. The enlargement, or reduction, of a geometrical figure is similar to the original figure. This means that the corresponding sides of the two figures are in proportion and the interior angles are equal. The angles are not changed by the change in size. The enlargement, or reduction, is worked on a scale, just as maps and architectural drawings are worked on scales.

Revision exercise

Learner's Book page 276

Guidelines on how to implement this activity

Use this revision exercise to revise with learners similar shapes. Learners should be able to recognise when shapes are larger or smaller, and when shapes are the same shape but different sizes. Ensure learners know what an enlargement and a reduction is.

Suggested answers

1 D **2** B **3** B and D

Exercise 1

Learner's Book page 278

Guidelines on how to implement this activity

Give learners squared paper. Ask learners to enlarge the given triangle in the Learner's Book by making BC 3 blocks and AB 6 blocks. Next they must enlarge the square by making its size 8 blocks by 8 blocks. Discuss how they have enlarged the triangle. Explain the concept of a scale factor. Ensure learners understand how to enlarge a shape by a given scale factor. Provide learners with additional grid paper and ask them to draw a rectangle, of 3 squares by 2 squares. Ask learners to increase by a factor of 2. Repeat with additional shapes until learners understand how the scale factor works. Show learners how to identify the scale factor used if you are given two figures. Do a few examples until learners are able to do this themselves. When investigating the effect of the scale factor on area, encourage learners to use their constructions and perform the calculations.

Suggested answers

1.1 - 1.5 Learners' own constructions

2 Area of enlarged shape = (factor of enlargement)^2 × area of original shape

Remedial

If learners are struggling to understand the concept of enlargement prescribe additional practise such as what follows: Give learners squared paper. Tell them to draw a square 4 blocks by 4 blocks. Next draw a square 8 blocks by 8 blocks. Ask: Is the 1st square bigger or smaller? (bigger) Are the 2 squares the same shape? (yes) Are all squares the same shape? (yes) Are all rectangles the same shape? (no). Tell them to draw a rectangle 4 blocks by 1 block. Next draw a rectangle 8 blocks by 2 blocks. Ask: Is the 1st rectangle bigger or smaller? (smaller) Are the 2 rectangles the same shape? (yes) Are all rectangles the same shape? (no) Draw a rectangle 2 blocks by 3 blocks. Discuss.

Extension

Ask learners to do a scale drawing of the classroom. Allow them to measure the length and breadth of the room. The size of desks and tables, and then work out a reasonable scale. For example, 1 metre = 2 cm. Those who complete this task could do a scale drawing of their own bedroom or kitchen at home.
Some learners might like to work out the area of the classroom. For example, 10 m by 9 m = 90m^2 and 20 cm by 18 cm = 360 cm^2. They could also compare perimeters.

Exercise 2

Learner's book page 279

Learners should be adept at enlargement and determining the scale factor of the enlargement. Repeat the activities you did for Exercise 1, but this time have learners construct reduced figures. Explain that reductions are smaller than the original. Again show learners how to use the scale factor and how to determine the scale factor when given two figures. Learners must also understand that reductions, like enlargements, are similar shapes to the original.

Suggested answers

1.1 - 1.4 Learners' own constructions

2 $\frac{4}{2}$ = 2, the factor of reduction is 2

Remedial

Encourage learners to repeat the constructions using simple squares until they understand the concept more fully. When they can work with squares encourage them to work with more complex shapes.

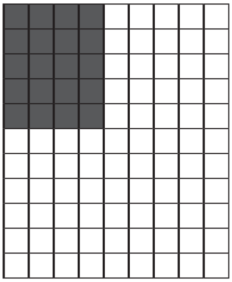
Consolidation

Learner's book page 281

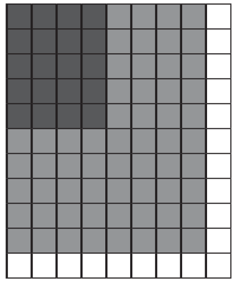
Before doing this consolidation exercise, encourage learners to review the work covered in this chapter. Advise learners to use the summary and to revise their work. This exercise can be used as an informal assessment task for you to track how learners are coping with the chapter and the concepts covered.

Suggested answers

1.1



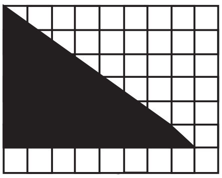
1.2



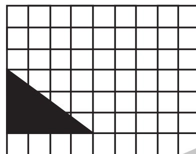
(1)
(1)

1.3 B is bigger
1.4 Twice as big (1)(1)
1.5 20 small squares
1.6 80 small squares (1)(1)
1.7 The area has multiplied by 4 (2)

2.1



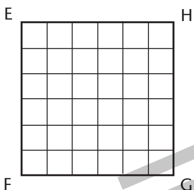
2.2



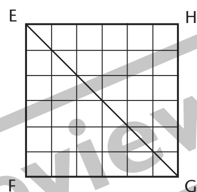
(3)

2.3 C is bigger
2.4 D is twice as small (2)

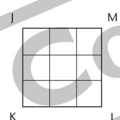
3.1



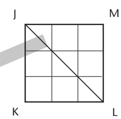
3.2



3.3



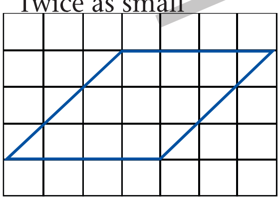
3.4

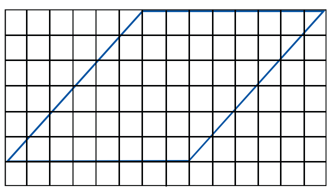


(2)
(1)
(1)
(1)

3.5 Square EFGH
3.6 36 small squares
3.7 9 small squares (3)
3.8 Twice as small
3.9 18 small square
3.10 4,5 small squares (4)
3.11 Twice as small (2)

4.1





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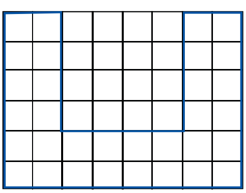
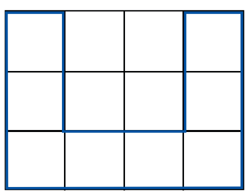
4.2





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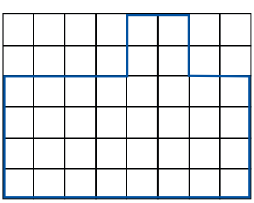
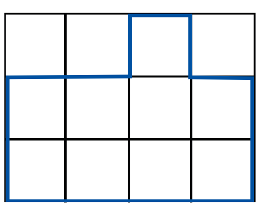
4.3





(1)

[30]

Chapter 15 Geometry of 3D objects

Overview of concepts

Geometry of 3D objects

UNIT 2
Building 3D models

UNIT 1
Classifying 3D objects

Content		Time allocations	LB page
Unit 1	Classifying 3D objects	5 hours	283
Unit 2	Building 3D models	4 hours	288

Background information on geometry of 3D objects

Learners have worked with 3D objects in various forms since the Foundation Phase. They have progressed from informal terms such as boxes and balls to the appropriate terminology of prisms, pyramids and spheres.
In Grade 7 learners learn how to name prisms and pyramids based on the shape of their bases. Furthermore learners are taught about the features of these objects, namely the faces, vertices and edges. Learners must be able to recognise and count these properties.

Generic teaching guidelines for teaching geometry of 3D objects

As an introduction discuss learners' understanding of the terms polyhedra and polygons. Allow them to give examples of each. For example, polygons are 2D shapes like squares, parallelograms, and triangles. Ask learners to point to examples in the classroom. For example, the board, charts, and door. Polyhedra are 3D shapes such as boxes and books.
Have as many examples of different objects available for learners to have a concrete experience of the objects. Learners must physically count the faces, vertices and edges. This helps learners to conceptualise the concepts.

- Show examples of polyhedra such as cereal boxes and dice.
- Let learners cut open small boxes to examine the 'nets'.
- Explain what is meant by 'net'.
- Identify 3D objects in the classroom. For example, pencil boxes, lunch boxes, cupboards and so forth.

Resources

A variety of 3D objects to display in the class – both solid and cardboard creations. Toothpicks and prestick to create skeleton models. Photocopied examples of the nets in the Learner's Book, colour card and pens. Scissors, rulers, glue and tape for construction.

Unit 1 Classifying 3D objects

Learner's Book page 283

Unit focus

- learning about the properties of polyhedra
- describing, sort and compare polyhedra.

Specific background information on classifying 3D objects

Prisms and pyramids are classified according to the shape of their base. 3D objects can be solid, such as a brick, or hollow, such as a box. Buildings and structures are 3D objects. Usually they are cuboids, but the pyramids in Egypt and the famous Louvre museum entrance are square based pyramids.

Exercise 1

Learner's Book page 284

Guidelines on how to implement this activity

Show learners physical objects of pyramids, prisms and cylinders. Have as many different types of each object as possible, and ask learners to suggest ways of classifying the different objects. Guide learners to the conclusion that we classify the objects according to their bases. Have learners sort the objects and provide names for each of the objects.

Suggested answers

1	cube	**2**	triangular pyramid
3	cylinder	**4**	rectangular prism
5	pentagonal pyramid	**6**	rectangular prism
7	square pyramid		

Remedial

Learners may struggle to sort the prisms and pyramids because they don't have sufficient understanding of 2D shapes. Revise the basic geometric shapes, as well as their basic properties. For example, a square always has 4 equal sides, 4 equal angles that are all 90˚ and whose opposite sides are parallel. Make sure basics are grasped. Try to make the lesson as practical as possible by pointing out examples of geometric examples all around. For example, opposite walls are parallel and corners of books are 90˚. Have learners list polyhedra they see in their homes. Some examples could be: stove, refrigerator, or washing machine.

Exercise 2

Learner's Book page 287

Guidelines for implementing this exercise

Introduce learners to the terms: faces, edges and vertices. Use a 3D object to demonstrate to learners what each term means. Use a box as an example to show the faces on a cube or cuboid. Learners should observe that a cube has 6 faces. Count the edges on the cube: 12 edges. How many vertices (corners) are there? 8 vertices. Ideally provide each learner with an object and ask them to point to each of the properties. Separate learners into groups and provide each group with a selection of 3D objects. Ask learners to name each 3D object, and record the number of faces, vertices and edges of each object. It is important that learners have an opportunity to work with physical objects before having to work with the objects abstractly in their books. Learners should complete this exercise on their own, but can discuss their observations in their groups.

Suggested answers

1

Faces	Vertices	Edges
6	8	12

2

Faces	Vertices	Edges
3	0	2

3

Faces	Vertices	Edges
4	4	6

4

Faces	Vertices	Edges
8	12	18

5

Faces	Vertices	Edges
6	6	10

6

Faces	Vertices	Edges
6	8	12

7

Faces	Vertices	Edges
5	5	8

Remedial

If learners are struggling to remember the terms used, have memory competitions with two teams. Have a list of relevant 3D and 2D terms and read them out one at a time. The first member of each team must go to the board and draw or write a description. For example, a 'square' answer could be L or 'a four sided polygon with equal sides. As soon as a learner has sat down call out the next word. Each team may only have 1 member at the board at a time. This will help encourage learners to learn the terms.

Extension

Prescribe the challenge activity to learners that have managed easily with the work so far.

Challenge

Learner's Book page 287

Guidelines for implementing this exercise

Encourage learners to record the relationship between the number of faces, edges and vertices with a table similar to the one below (advise learners not to use a cylinder):

Shape	Cube	triangular prism	tetrahedron
Number of faces	6	5	4
Number of edges	12	9	6
Number of vertices	8	6	4
F + V – E	6 + 8 – 12 = 2	5 + 6 – 9 = 2	4 + 4 – 6 = 2

If you subtract the number of edges from the sum of the number of faces and vertices, then the answer is always two.
F + V – E = 2 (Euler's formula)

Unit 2 Building 3D models

Learner's Book page 288

Unit focus

- revising what nets are,
- using nets to build prisms, and
- using nets to build pyramids.

Background information on building 3D models

Explain to learners the importance of building correctly. When builders build with bricks they ensure that all bricks are the same size and that they are laid evenly. Remind learners that when building a 3D model, learners must always measure carefully and work carefully and accurately. Nets are the 2D representations of 3D objects. They must be able to be bent into the form of a 3D object.

Revision exercise

Learner's Book page 288

Guidelines for implementing this exercise

Revise faces of objects. Learners need to use their knowledge of the number and shapes of faces in order to cope with this exercise. Use this exercise to assess learners' ability to cope with the rest of this unit.

Suggested answers

1 6 faces, 4 rectangles, 2 squares
2 6 faces, 5 triangles, 1 pentagon
3 5 faces, 3 rectangles, 2 triangles
4 6 faces, 6 squares (rectangles)
5 4 faces, 4 triangles

Remedial

Revise the shapes of faces, and how to count the number of faces.

Exercise 1 Learner's Book page 291

Guidelines for implementing this exercise

Discuss what a net is. Ask learners about their previous exposure to nets. Provide learners with boxes to cut open and to observe the nets. Have learners name the 3D object and trace the net of the object into their books or onto larger pieces of card. Show learners how to use a net to count the faces, vertices and edges. Ask learners with different shaped boxes to get together to compare the different nets of their boxes. Learners can do Exercise 1 in a group to discuss which objects they feel the nets will make.

Suggested answers

1 cube
2 rectangular prism
3 triangular pyramid
4 triangular prism
5 square based pyramid

Remedial

If learners are finding it difficult to determine the object formed by the net, encourage them to trace the net onto paper, cut it out and see what object is formed by the net.

Exercise 2 Learner's book page 293

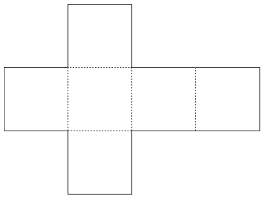
Guidelines for implementing this exercise

Discuss how nets are used to construct 3D models. Explain the dotted lines and that they mean folding lines. Show learners the tabs and explain that they are used to stick the model together. Provide learners with cards and work through the worked example in the Learner's Book for how to construct a cube. Once complete discuss the process with learners. What was easy to do? What was difficult? Remind learners to work as neatly and accurately as possible. Learners must use a sharp pencil and a ruler to accurately trace or draw the net. Learners must then ensure they cut out neatly and correctly. Provide colour cards for learners to construct with. Learners should complete this exercise in groups, with each learner constructing at least 2 of the prescribed objects.

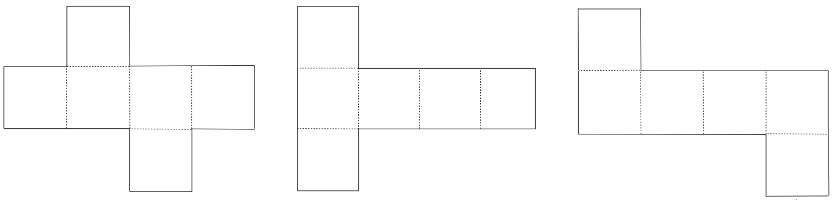
Suggested answers

1 Learners build their own models.
2.1 6 faces
2.2 squares
2.3 6 faces
2.4 6 faces

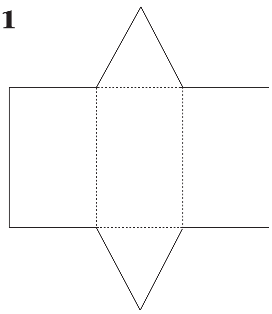
3.1



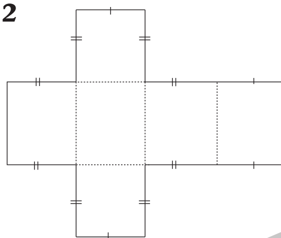
3.2



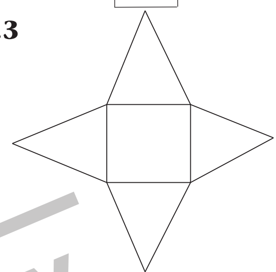
4.1.1



4.1.2



4.1.3



Remedial

If certain learners are experiencing problems with drawing or tracing the net, work carefully with them as they do this. Provide tips on how to hold the ruler and pencil correctly. If learners accidentally cut off the tab or side of a net, encourage them to start again. Remind learners of the importance of neatness and accuracy.

Extension

Do a team project to build a large pyramid. Show as many pictures of the pyramids as possible. Explain to learners that the pyramids of Egypt were built out of cube or cuboid shaped rocks carefully fitted together in a 3D 'tessellation.' Replicate this in the classroom using matchbox size cuboids.

Consolidation

Learner's book page 296

Before doing this consolidation exercise, encourage learners to review the work covered in this chapter. Advise learners to use the summary and to revise their work. This exercise can be used as an informal assessment task for you to track how learners are coping with the chapter and the concepts covered.

Suggested answers

1

polyhedra	polygons
cube	square
cuboid	rectangle
tetrahedron	equilateral triangle
triangular prism	pentagon

(4)

2.1.1 cube
2.1.2 triangular pyramid
2.1.3 rectangular prism
2.1.4 triangular prism (4)
2.2 Cube: six square faces
Rectangular prism: two square faces and four rectangular faces
Triangular prism: two triangular faces and four rectangular faces
Triangular pyramid: four triangular faces (4)
3 A triangular pyramid has four vertices and six edges. (2)
4 A rectangular prism has 12 edges and 8 vertices. (2)
5

Shapes	Cube	Triangular–based pyramid	Rectangular prism	Cylinder	Pentagon–based prism
Number of faces (F)	6	4	6	2	6
Number of vertices (V)	8	4	8	0	6
Number of edges (E)	12	6	12	2	10
F + V – E	2	2	2	0	2

(10)

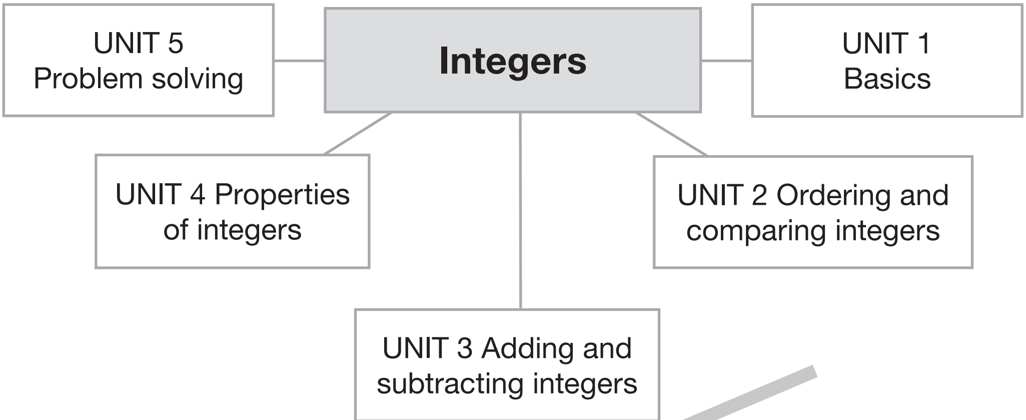
5.1 The answer is 2 in each case.
5.2 The difference between the sum of the number of faces and vertices, and the number of edges equals two except in the case of a cylinder. (3)
6 Learners' own constructions (3)
7.1 pentagonal pyramid
7.2 rectangular prism
7.3 equilateral triangular pyramid
7.4 pentagonal prism
7.5 triangular prism (5)
8 Learners' own work (3)

[43]

CHAPTER 15

Chapter 16 Integers

Overview of concepts



Content		Time allocations	LB page
Unit 1	Basics	1 hour	300
Unit 2	Ordering and comparing integers	2 hours	302
Unit 3	Adding and subtracting integers	2 hours	305
Unit 4	Properties of integers	2 hours	310
Unit 5	Problem solving	2 hours	315

Background information

Negative numbers were used by the Chinese in 200 B.C. They used red rods to represent positive numbers and black rods to represent negative numbers. Financial institutions write in black to represent positive and red to represent negative, just the opposite of what the Chinese did.
This is the learners' first official introduction to negative numbers. Integers are the set of counting numbers (1; 2; 3; 4 …) plus zero (0) and negative numbers (–1; –2; –3 ...) They do not include fractions, they are whole numbers.
Integers can be written: …–5; –4; –3; –2; –1; 0; 1 ; 2; 3; 4; 5; …

Generic teaching guidelines for teaching integers

It is important that learners understand that they have already encountered negative numbers in their everyday life. Remind learners that negative numbers are part of everyday life. For example if you have R7,00 and you have to pay R10,00 for something, you have – R3,00. On very cold days the temperature can be –3 °C and so on.

Ask some learners to write a number sequence on the board showing –5 to +5. Discuss the use of plus in front of positive numbers. It is not generally used. Discuss the use of minus in front of negative numbers. It is always used. Discuss the value of integers. The bigger the number after a minus sign, the smaller its value. For example, –9 999 999 < –1 and –3 > –4 The bigger the number after a plus sign, or the bigger a positive number, the greater its value. For example, 9 999 999 > 1 and 3 < 4

Resources

Number lines, thermometers, examples of weather reports with negative numbers, and other examples from newspapers and magazines. Cardboard and colour pens to create posters and additional number lines. Number cards and comparison cards.

Unit 1 Basics

Learner's Book page 300

Unit focus

- revising knowledge of whole numbers,
- learning about numbers below 0, and
- learning about a group of numbers called integers.

Background information on basics

As this is learners' first exposure to negative numbers and the terminology integers, provide relatable and contextual examples to help learners with the concept. Integers will be a new word, as may be the concept of infinity, and it is important that learners are provided with the necessary time and practice to understand and memorise these concepts.
Learners are expected to be able to count forwards and backwards in whole numbers, natural numbers and integers: ... –3; –2; –1; 0; 1; 2; 3 … and … 3; 2; 1; 0; –1; –2; –3 … They will also be expected to compare given numbers.

Exercise 1

Learner's Book page 301

Guidelines on how to implement this activity

Start with a relevant context such as weather and what a temperature of –3 °C means. Draw a number line on the board from 0 to 10. Ask learners where they think –3 is on the number line. Once learners have accurately identified where –3 will lie (below the 0), extend the number line to –10, but leave out the numerals. Ask learners to come up and place the numerals correctly on the number line. If you wish you can extend the number line to –20 and repeat the exercise. Introduce the word integer to learners. Explain that an integer is a whole number, either positive or negative. As a class count orally from –20 to 0, and then from –30 to 0. Ask learners to count in 2s from –20 to 20. Repeat until learners understand they can count in intervals that include negative numbers. Learners should complete this exercise on their own.

Suggested answers

1 Yes, any number is either a whole number or a negative natural number.

2 An integer is either a whole number or a negative natural number.

3.1	4	**3.2**	–7	**3.3**	1
3.4	–9 is smaller than –8	**3.5**	1	**3.6**	–1

3.7 –8°C, because –8 is a smaller number than 1.

4.1	Yes	**4.2**	Yes	**4.3**	No	**4.4**	No
4.5	No	**4.6**	No	**4.7**	No	**4.8**	No
4.9	Yes	**4.10**	No	**4.11**	Yes	**4.12**	Yes

Remedial

Use number sequences with variables for learners to complete. For example: –10; *a*; *b*; –7; *c*; *d*; –4; *e*.; *f*; *g*; *h*; *j*; 2

Unit 2 Counting, ordering and comparing integers

Learner's Book page 302

Unit focus

- count forwards in integers for any interval
- count backwards in integers for any interval
- order and compare integers.

Background information on counting, ordering and comparing integers

To order and compare integers it is necessary to understand the positional value of the numbers. When using integers learners must remember that negative numbers decrease in value the bigger the number gets. For example –1 000 000 is a much smaller number than 0. If you have 0 cents, then you are richer than if you have –1 000 000 cents. –1 000 000 cents means you owe 1 000 000 cents.
To order integers you have to remember that numbers on a number line become smaller to the left and greater to the right.

Exercise 1

Learner's Book page 303

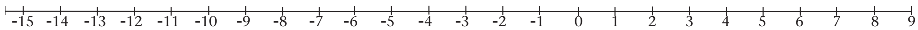
Guidelines on how to implement this activity

Revise with learners: What is a whole number? What is a natural number? What is an integer? Write a whole number sequence and an integer number sequence on the board and allow learners to read them aloud, backwards and forwards. Let learners count in a variety of number intervals, for example: 2; 4; 6; … 20 and –8; –6; –4; … 10 and 5; 10; … 50 and 25; 20; 15; … –30, forwards and backwards to help learners consolidate their understanding of negative numbers. Ensure learners see that counting in negative numbers, replicates the sequence for counting in positive numbers, except in reverse. Do examples together as a class, including numbers up to –100. Learners should complete this exercise on their own.

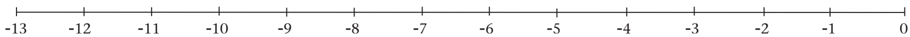
Suggested answers

1.1 –36; –33; –30; –27; –24; –21; –18; –15; –12; –9; –6; –3; 0; 3; 6; 9; 12
1.2 –55; –45; –35; –25; –15; –5; 5; 15; 25; 35; 45
1.3 –29; –26; –23; –20; –17; –14; –11; –8; –5; –2
1.4 –34; –24; –14; –4; 6; 16; 26; 36; 46

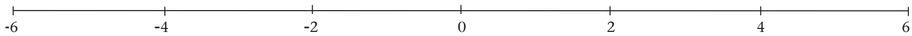
2.1



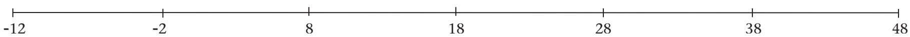
2.2



2.3



2.4



3.1 –99; –96; –93; –90; –87; –84; –81; –78
3.2 –275; –250; –225; –200; –175; –150; –125
3.3 –17; –15; –13; –11; –9; –7; –5; –3; –1
3.4 15; 11; 7; 3; –1; –5; –9; –13; –17
3.5 –1 010; –909; –808; –707; –606; –505; –404; –303
3.6 9; 6; 3; 0; –3; –6; –9; –12; –15

Remedial

Revise counting forwards from negative numbers. For example, –19; –18 … until every learner has had a turn. Now do the reverse. Start with 20. Continue 19; 18; …

Extension

Encourage learners to count in intervals for numbers that are not multiples. Increase the number range up to –1 000.

Exercise 2

Learner's Book page 304

Guidelines on how to implement this activity

Ask learners to draw number lines from –10 to 10. Use the number line to ask learners to fill in <, > or = in statements like: is 3 greater or less than –1 and 5 * 3 and –1 * –2. Ask 2 or 3 learners at a time to work on the board and write answers to questions like: a number 1 greater than 7; a number smaller than –2; 5 less than 3. Remind learners that the larger the negative number, the smaller the number is. For example, –10 000 is smaller than 1. Explain how learners can use the number line to determine the size of numbers. The further to the left, the smaller the number and the further to the right the larger the number. Learners should do this exercise on their own.

Suggested answers

1.1	<	**1.2**	>	**1.3**	>		
1.4	>	**1.5**	<	**1.6**	>		
2	5; 1; 0; –2; –4; –6						
3.1	<	**3.2**	>	**3.3**	<	**3.4**	>
3.5	>	**3.6**	>	**3.7**	<	**3.8**	<
3.9	=	**3.10**	<	**3.11**	=	**3.12**	>

3.13 <
4.1 –56; –49; –28; 0; 63
4.2 –70; –35; –21; 49; 56
4.3 –42; 7; 21; 35; 70
4.4 –63; –14; –7; 14; 28; 42

Remedial

If learners are experiencing problems, provide learners with additional exercises using number lines to write down numbers: 1 less than 10; 2 less than 10; 3 less than 10; 1 less than 9. Learners must work from their number lines to write down numbers: 1 more than –10; 2 more than –10; 3 more than –10; 1 more than –9. Encourage learners to work in pairs to fill in <, > and = for number pairs like: 2 * 3; 3* 4; 4 * 5. Next ask learners to arrange numbers in ascending sequence. For example, 2; 1; 3; 7; 4; 9; 6; 5; 8 and then in descending sequence. Now arrange the following in ascending sequence: 1; –3; 0; 3; 7; –2; 4; 9; –1; 6; 5; 8; 2 and then in descending sequence.

Extension

Have a competition with 2 teams. Ask learners from both teams to write numbers on the board. For example, write negative 17 (–17) and 4 (4). The next learner up must fill in <, > or = signs. The team with the most correct, wins.
Learners who are comfortable with whole numbers, natural numbers and integers can make number charts comparing different types of numbers:

Whole numbers	0; 1; 2; 3; 4; ...
Natural numbers	0; 1; 2; 3; 4; ...
Counting numbers	1; 2; 3; 4; ...
Integers	– 4; –3; –2; –1; 0; 1; 2; 3; 4; ...
Even numbers	2; 4; 6; 8; ...

Unit 3 Adding and subtracting integers

Learner's Book page 305

Unit focus

- revising adding and subtracting on a number line
- adding integers
- subtracting integers.

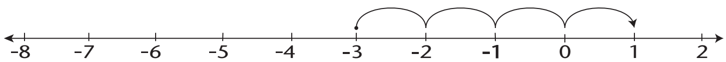
Specific background information on adding and subtracting integers

To add and subtract integers it is necessary to understand what is meant by an additive inverse. An example is 8 + (–8) = 0. To do the calculation we multiply the plus and the minus and the product is minus (+ × – = –). 8 + (–8) = 8 – 8 = 0. The number 8 and –8 are additive inverses of each other.
Other examples are: 24 + (–24) = 0 and –6 + 6 = 0 and (+13) + (–13) = 0
Consider the following: 8 + (–18) = 8 –18 = –10 and 24 + (–4) = 24 – 4 = 20 and –6 + 66 = 60 and (+13) + (–33) = –20

Exercise 1 Learner's Book page 307

Guidelines on how to implement this activity

Show learners how to use a number line to add positive integers. Next show learners how to add starting with negative integers. To add –3 + 4, mark –3 on the line: Start from –3 count 4. Count from –2 (1); –1 (2); 0 (3); 1 (4) The sum of –3 + 4 = 1.



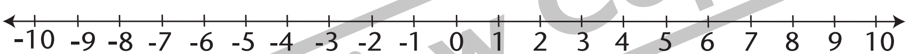
Do additional examples such as 0 + 4; –3 + 5; –1 + 2; –5 + 8
Show learners what happens when we add a negative number. To add we would move right on the number line, but because the number is negative, we move to the left. Essentially we subtract these numbers. Do a few examples, such as 5 + (–3); –4 + (–3). Learners should use a number line as they complete this exercise on their own.

Suggested answers

1	+11	**2**	+3	**3**	–4	**4**	–7
5	+6	**6**	–9	**7**	–9	**8**	–2
9	–5	**10**	+4	**11**	–1	**12**	–2

Remedial

Allow learners who have difficulty with the concept to do simple addition examples like: 1 + 2 and 2 + 3 and 3 + 4. Next ask them to draw a number line like this:

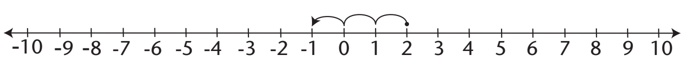


Using the number line, do the following calculations: 1 + 2; 2 + 3; 3 = 4, and so forth. Compare the answers with their original answers. Now, using the number line add 1 + (–1). Start at 1. Count back –1. 1 + (–1) = 0. Do a number of similar examples, like 2 + (–3) = –1 and 2 + (–4) = –2.

Exercise 2 Learner's Book page 309

Guidelines on how to implement this activity

To show subtracting integers ask learners to draw a number line from –10 to 10. To subtract 2 – 3 mark 2 on the line:



Start from 2 count 3 in a negative (minus) direction. Count from 1 (1); 0 (2); –1 (3). The difference between 2 – 3 = –1 . Then look at some of the same calculations: Say all positive numbers are red and all negative numbers are black. Take 7 – 3 and mark it out like this: 1111111 – 111. What is the difference? There are 4 more red (positive) numbers, so the answer is 4. Look at 3 – 6. 111 – 111111 What is the difference? There are 3 more black (negative) numbers, so the answer is –3. Check these answers against the number line results.
Show learners how to calculate 9 – (–1). Explain that the negative would make us work in minus direction, which is left on a number line, but again we change direction and move right. Essentially we are adding.

Explain to learners that a minus sign in front of a number means a change in direction. Do repeated examples until learners know how to subtract negative numbers. Learners should complete this exercise on their own.

Suggested answers

1.1	–2	**1.2**	+10	**1.3**	–7	**1.4**	+1
1.5	–5	**1.6**	+13	**1.7**	+4	**1.8**	+11
1.9	+1	**1.10**	–8	**1.11**	0	**1.12**	+12
2.1	+5	**2.2**	+2	**2.3**	0	**2.4**	+12
2.5	0	**2.6**	–16	**2.7**	–14	**2.8**	–24
2.9	–10	**2.10**	+321	**2.11**	–255	**2.12**	0

Remedial

Provide additional simple examples for learners to practise. Simply practise and checking of their answer should help learners master this concept. Learners should use the number line for addition and subtraction for as long as necessary.

Extension

Learners who find integers easy can be asked to make wall charts demonstrating addition and subtraction of integers. Encourage them to use colours or shapes to clarify the concept. For example, 2 + (–5) where □□ represents the positive 2 and ■■■■■ represents the negative 5. So 2 + (–5) = □□ – ■■■■■ = –■■■ = –3

Unit 4 Properties of integers

Learner's Book page 310

Unit focus

- revising properties of whole numbers,
- applying the commutative property to integers,
- investigating whether inverse operations apply to integers, and
- appling the associative property to integers.

Background information on properties of integers

The commutative property, the distributive property and the associative property can be applied to integers. For example:
The commutative property: 3 + 4 = 7 and 4 + 3 = 7
– 3 + (–2) = –5 and –2 + (–3) = –5
The associative property: 3 + (4 + 5) = (3 + 4) + 5 = 12
– 3 + [(–4) + (–5)] = [(–3) + (–4)] + (–5) = –12
As learners have only worked with addition and subtraction of integers at this point, they cannot investigate the distributive property or the properties of 0 and 1. These will be addressed in grade 8. Learners should investigate the relationship between addition and subtraction, and whether the inverse operation is valid when working with negative integers.

Exercise 1 Learner's Book page 312

Guidelines on how to implement this activity

Revise the commutative properties of whole numbers. Revise with learners that the commutative property means that we can add numbers in any order. Do a few examples with positive whole numbers showing how the commutative property works. Do examples with negative numbers together as a class. Have learners come up to the board and do the calculations. Revise with learners that this property does not apply to subtraction. Ensure learners see that the commutative property works with all integers including negative numbers. Learners should complete this exercise on their own.

Suggested answers

1 $(-15) + (+42) = (+42) + (-15) = 27$

2 $(+27) + (-31) = (-31) + (+27) = -4$

3 $(-45) + (-30) = (-30) + (-45) = -75$

4 $(-7) + (-99) = (-99) + (-7) = -106$

5 $(+67) + (-99) = (-99) + (+67) = -32$

6 $(-101) + (-34) + (-3) = (-101) + (-3) + (-34) = (-34) + (-101) + (-3)$
$= (-3) + (-34) + (-101) = -138$

7 $(+768) + (-459) + (-32) = (+768) + (-32) + (-459) = (-32) + (+768) + (-459)$
$= (-459) + (+768) + (-32) = 277$

8 $(-56) + (+100) - (-99) = (+100) + (-56) - (-99) = 143$

Remedial

-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10

Encourage learners who have difficulties with these concepts to work with a number line.
Revise the commutative property. For example, $3 + 1 = 1 + 3 = 4$. Demonstrate how this is calculated on the number line. Do the same with $-3 + (-1) = -1 + (-3) = -4$. Concentrate on the commutative property if necessary, and go over other properties only when commutative is well grasped.

Extension

Encourage stronger learners to use larger number ranges, including hundreds and thousands to test the commutative property.

Exercise 2 Learner's Book page 313

Guidelines on how to implement this activity

Revise the associative properties of whole numbers. Revise with learners that the associative property means that we can group numbers in any order, in order to add them. Do a few examples with positive whole numbers showing how the associative property works. Do examples with negative numbers together as a class, such as $(1 + 2) + 3 = 1 + (2 + 3)$ and $[(-1) + (-2)] + (-3) = (-1) + [(-2) + (-3)]$.

Have learners come up to the board and do the calculations. Revise with learners that this property does not apply to subtraction. Ensure learners see that the associative property works with all integers including negative numbers. Learners should complete this exercise on their own.

Suggested answers

1 $23 + (51 + 67) = 23 + 118 = 141$
$(23 + 51) + 67 = 74 + 67 = 141$

2 $14 + [-53 + (-62)] = -14 + (-115) = -129$
$[-14 + (-53)] + (-62) = (-67) + (-62) = -129$

3 $54 + [-25 + (-36)] = 54 + (-61) = -7$
$[54 + (-25)] + (-36) = 29 + (-36) = -7$

4 $-440 + (775 + 6) = -440 + 781 = 341$
$(-440 + 775) + 6 = 335 + 6 = 341$

5 $-46 + (-59 + 16) = -46 + (-43) = -89$
$[-46 + (-59)] + 16 = (-105) + 16 = -89$

6 $56 + [-32 + (-21)] = 56 + (-53) = 3$
$[56 + (-32)] + (-21) = 24 + (-21) = 3$

Remedial

Allow learners to use number lines while investigating the properties of integers.

Extension

Encourage stronger learners to use larger number ranges, including hundreds and thousands to test the associative property.

Exercise 3

Learner's Book page 314

Guidelines on how to implement this activity

Revise with learners the understanding that addition and subtraction are opposite operations. Subtraction undoes addition and vice versa. Do a few examples with positive whole numbers showing how the inverse operations undo each other. Next try examples with negative numbers. Have learners come up to the board and do the calculations. Ensure learners see that the inverse operations of addition and subtraction work with all integers including negative numbers. Learners should complete this exercise on their own.

Suggested answers

1 $(-5) + (+5) = 0$; $0 - (+5) = (-5)$
2 $(+6) - (-9) = (+15)$; $(+15) + (-9) = (+6)$
3 $(+16) - (+8) = (+8)$; $(+8) + (+8) = (+16)$
4 $(-27) - (-56) = (+29)$; $(+29) + (-56) = (-27)$
5 $(-13) + (-7) = (-20)$; $(-20) - (-7) = (-13)$
6 $(-99) - (-101) = (+2)$; $(+2) + (-101) = (-99)$
7 $(+88) - (-90) = (+178)$; $(+178) + (-90) = (+88)$
8 $(-54) + (-34) = (-88)$; $(-88) - (-34) = (-54)$

Remedial

Allow learners to use number lines while investigating this property.

Extension

Encourage stronger learners to use larger number ranges, including hundreds and thousands to test the inverse operations.

Unit 5 Problem solving

Learner's Book page 315

Unit focus

- using what you know about integers to solve problems.

Background information on problem solving

Learners have learnt about the steps for problem solving since the Foundation Phase, however learners often continue to struggle to interpret the question and formulate the correct equation to help them conceptualise the problem.
The steps learners should know are:

- to read the question carefully
- identify the information they have been given
- identify what has been asked
- organising the information into a number sentence to solve the problem
- perform the calculation
- judge the reasonableness of the answer.

Learners can still use placeholders if necessary for their number sentences, but should be encouraged to use variables instead.

Exercise 1

Learner's Book page 316

Guidelines on how to implement this activity

Revise the work covered in the previous 4 units of this chapter. Ensure learners can add and subtract with negative integers, and can use the properties of integers effectively. Introduce problem solving. It is important to ensure that learners can read the question with understanding – ask learners to interpret some examples you give them. Learners' inability here may require you to check their reading skills, their maths skills and their logical thinking skills.
Discuss problem solving and ask learners to supply the various strategies they employ when problem solving. Revise the steps for problem solving as set out in the Learner's Book. Work through the example in the Learner's Book, and be sure to identify each step you perform in the worked example. Remind learners to supply their answers with the correct units. Discuss with learners how they will deal with negative numbers in the examples. Allow learners to discuss the problems and formulation of the number sentence for each problem together in small groups, but that each learner must perform their own working.

Suggested answers

1 Total pocket money: 5 × 15,00 = 75,00
Total expenses: 67,50 + 36,00 = 103,50
75,00 – 103,50 = –28,50

2 27 – 12 = 15 °C

3 345 × 24 = 8 280; –277 × 30 = –8310
–8310 is the furthest from 0.

4 245 + (–366) = –121; –27 + (–42) = –69
–69 is closer to 0.

5.1 36 °C

5.2 16 °C

5.3 22 °C

5.4 3 °C; 11 °C; 27 °C; 33 °C

5.5 33 °C; 27 °C; 11 °C; 3 °C

6.1 –45 + (–38) – 90 = –173

6.2 788 – (–12) + 34 = 834

6.3 12 × (–12) + 144 = 0

6.4 –42 × (–21) –30 = 852

6.5 –173; 0; 834; 852 0 is the closest to 0.

7 Mary: 2 × 15 = 30; 30 – 50 = –20 (Mary has R20 too little)
Nandi: 4 × 15 = 60; 60 – 50 = 10 (Nandi has R10 extra)
Jasmin: 3 × 15 = 45; 45 – 50 = –5 (Jasmin has R5 too little)

8.1 62 – 58 = 4

8.2 65 – 60 = 5

9.1 2 × 7 + 18 = 14 + 18 = 32

9.2 18 – (2 × 10) = 18 – 20 = –2

10.1 –7 – (–9) = 2

10.2 –5 – (–3) = –2

10.3 –9 – (–3) = –6

Remedial

Ask learners to work in pairs. One learner must read a simple story problem to the other. They must then discuss the problem. Both must make notes while they are having the discussion.
Try to give 3 or 4 very simple problems to be solved. When they have solved the first one you should check it to make sure they are on the right track. As long as they are, they can start the second and so on.

Extension opportunities

Encourage learners to create their own word problems using negative integers. Learners can then swop their word problems with one another.

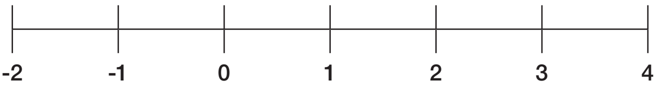
Consolidation

Learner's book page 319

Before doing this consolidation exercise, encourage learners to review the work covered in this chapter. Advise learners to use the summary and to revise their work. This exercise can be used as an informal assessment task for you to track how learners are coping with the chapter and the concepts covered.

Suggested answers

1



2.1	39 + 47 = 86	**2.2**	(39) + (–47) = –8	
2.3	(–28) + (+54) = +26	**2.4**	(–23) + (–54) = –77	(4)
3.1	(+79) – (+234) = –155	**3.2**	(+99) – (+14) = +85	
3.3	(+72) – (+73) = –1			(4)
4.1	4 + (–18) = –14	**4.2**	(–83) + 73 = –10	
4.3	(–33) + (–65) = –88	**4.4**	40 × (–98) = –3 920	(4)
5.1	422 × (3 × 60) = 422 × 180 = 75 960			
5.2	–72 × (20 × 26) = –72 × 520 = –37 440			
5.3	132 × (–13 × 50) = 132 × (–650) = –85 800			
5.4	–42 × (–34 × –43) = –42 × (+1 462) = –61 404			(4)
6.1	30 000 + 2 100 + 1 700 + 700 = R34 500			(1)
6.2	R2 000			(2)
6.3	Yes			(1)

[20]

Chapter 17 Patterns, functions and relationships

Overview of concepts

Patterns, functions and relationships

UNIT 1
Patterns

UNIT 2
General rule

UNIT 3
Input and output rules

UNIT 4
Equivalent forms

Content		Time allocations	LB page
Unit 1	Patterns	1,5 hours	321
Unit 2	General rule	1,5 hours	323
Unit 3	Input and output rules	1,5 hours	326
Unit 4	Equivalent forms	1,5 hours	329

Background information on Patterns, functions and relationships

Learners have worked with functions and relationships in Chapter 7, and with patterns in Chapter 10. They should be familiar with the concepts covered in these previous chapters. This chapter focuses on extending learners' knowledge by including integers in patterns, functions and relationships.

Input		Rule		Output
4	$\longrightarrow$	$\boxed{+\,2}$	$\longrightarrow$	a

Generic teaching guidelines for teaching patterns, functions and relationships

Revise with learners how to use flow diagrams to show the relationship between numbers, and how to translate information from flow diagrams to tables.

Resources

Objects to create patterns, number cards, number lines, blank flow diagrams, blank tables. Each learner should have their own calculator.

Unit 1 Patterns

Learner's Book page 321

Unit focus
- revising numeric patterns
- using integers in patterns
- extending patterns using a rule involving integers.

Background information on patterns
Patterns are all around us. Encourage learners to look for them everywhere. Time and measurement each follow and are structured according to patterns. In mathematics patterns are found everywhere. For example, geometric shapes with equal sides, equal angles, and reduced sizes. In algebra number patterns lay the foundation for much of the future algebraic thinking and manipulation.
Identify number patterns similar to those in each row below:

1st term	2nd term	3rd term	4th term	5th term	6th term	Answer Pattern
4	8	12	16	20	24	$n + 4 = x$
1	2	4	8	16	32	$2n = x$

Revision exercise
Learner's Book page 321

Guidelines on how to implement this activity
Use this exercise to assess learners' knowledge of patterns. Use it to identify any problem areas and provide the appropriate remediation to learners experiencing any problems.

Suggested answers

1.1 29; 34; 39; 44; 49 **1.2** 47; 54; 61; 68; 75

2.1 Square of the term number: 25; 36; 49

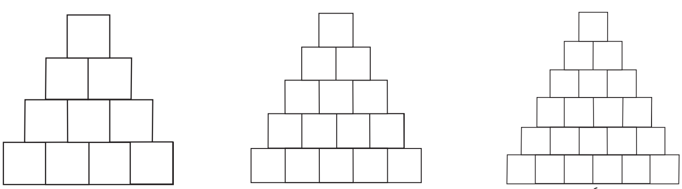
2.2 Add 13 to each term: 65; 78; 91

2.3 Subtract 4 from each term: 12; 8; 4

3

Series	3; 6; 9; …	1; 2; 3; …	10; 20; 30; …
1st term	3	1	10
2nd term	6	2	20
3rd term	9	4	40
4th term	12	5	50

4.1
i



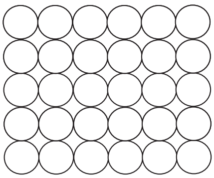
4 5 6

ii

Input	1	2	3	4	5	6	7	8	9	10
Output	1	3	6	10	15	21	28	36	45	55

iii The difference between the output's values increases by +1 each time.
(Another solution: $\frac{1}{2}$ × Input of term *n* × input of term *n* + 1 = Output of term *n*)

4.2
i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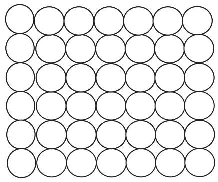
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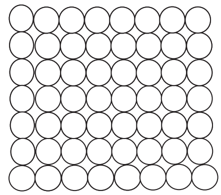

7

ii

Input	1	2	3	4	5	6	7	8	9	10
Output	2	6	12	20	30	42	56	72	90	110

iii The difference between the output's values increases by +2 each time.
(Another solution: Input of term *n* × input of term *n* + 1 = output of term *n*)

5.1 15; 22; 29; 36; 43 **5.2** 0; 7; 14; 21; 28

Exercise 1

Learner's Book page 322

Guidelines on how to implement this activity

Discuss with learners what they think number patterns that contain negative integers will look like. Start with simple examples of patterns that grow by adding a constant amount each time. Instead of limiting this to starting in the positive numbers, start with a negative number. Do an example on the board as a class. Learners should be able to relate this to counting with integers from the previous unit. Be careful when working with formulae that involve negative numbers, as learners have yet to learn the facts about multiplying with integers. Learners should complete this exercise on their own.

Suggested answers

1.1 –54; –48; –42; –36; –30; –24; –18; –12
1.2 90; 81; 72; 63; 54; 45; 36; 27; 18; 9
1.3 –1 001; –901; –801; –701; –601; –501; –401; –301
1.4 –3; 6; –12; 24; –48; 96; –192; 384; –768; 1 536
2.1 Add 6 to the previous term value to get the next term value.
2.2 Subtract 9 from the previous term value to get the next term value.
2.3 Add 100 to the previous term value to get the next term value.
2.4 Multiply previous term value by 2 to get the next term value.
3.1 $1 \times (-5) - 6 = 1$; $2 \times (-5) - 6 = -4$; $3 \times (-5) - 6 = -9$;
$4 \times (-5) - (-6) = -14$; $5 \times (-5) - (-6) = -19$
The sequence is: 1; –4; –9; –14; –19
3.2 $\frac{1}{4} + (-2) = -1\frac{3}{4}$; $\frac{2}{4} + (-2) = -1\frac{1}{4}$; $\frac{3}{4} + (-2) = -1\frac{1}{4}$; $\frac{4}{4} + (-2) = -1\frac{5}{4} + (-2) = -\frac{3}{4}$
The sequence is: $-1\frac{3}{4}$; $-1\frac{1}{2}$; $-1\frac{1}{4}$; -1; $\frac{3}{4}$

3.3 $(-2) + (-4) \times (2 + 1) = -14$; $(-2) + (-4) \times (2 + 2) = -18$; $(-2) + (-4) \times (2 + 3) = -22$;
$(-2) + (-4) \times (2 + 4) = -26$; $(-2) + (-4) \times (2 + 5) = -30$
The sequence is: –14; –18; –22; –26; –30

3.4 $\frac{4}{7} \times 1 - (-1) = \frac{11}{7}$; $\frac{4}{7} \times 2 - (-1) = \frac{15}{7}$; $\frac{4}{7} \times 3 - (-1) = \frac{19}{7}$;
$\frac{4}{7} \times 4 - (-1) = \frac{23}{7}$; $\frac{4}{7} \times 5 - (-1) = \frac{27}{7}$
$\frac{11}{7}$; $\frac{15}{7}$; $\frac{19}{7}$; $\frac{23}{7}$; $\frac{27}{7}$

3.5 $(-6) + (+3) \times (-4) + 1 = -18 + 1 = -17$; $-18 + 2 = -16$; $-18 + 3 = -15$;
$-18 + 4 = -14$; $-18 + 5 = -13$
The sequence is: –17; –16; –15; –14; –13

Remedial

Often learners who are struggling have problems with the basics like numbers, addition, subtraction, multiplication, or division. Encourage learners to work with numbers using concrete help, such as using an abacus, counting pebbles or beans. Allow learners to make simple number cards to play card games. Help learners to be successful by giving them easy examples so that they can feel in charge. For example:
Copy and complete these number patterns:
10; 12; 14; ...; ...; 20; ...; ...; ...; 28
5; 10; ...; ...; ...; ...; 35
And then extend to include the negative integers.

Extension

Ask learners to write their names in block capitals. For example, JACOB SMUTS. Now they must rearrange the letters to create a pattern. In the above name there are 10 letters. Alphabetically they are A; B; C; J; M; O; S; S; T; U. Look for possibilities. Which are symmetric figures? A; B; C; M; O; T; U. That leaves J and S. Try to create something out of that. Learners could alternatively build a picture out of the letters or put them in a pattern by repeating, making mirror images or using other transformations.

Unit 2 General rule

Learner's Book page 323

Unit focus

- revising how to find the rule that governs a sequence,
- finding the *n*th term, and
- working with integers in the general rule of patterns.

Background information on the general rule

Learners should be familiar with finding the general rule from earlier chapters.
Learners need to be able to find the general rule for patterns that involve negative integers, or work with rules that involve integers.
Flow diagrams and tables help learners to identify the relationship between the input and output values. Learners should be able to translate effectively between these two equivalent representations.

Exercise 1

Learner's Book page 324

Guidelines on how to implement this activity

Discuss with learners the strategies they have worked with in previous chapters to find the general rule. Do a few examples as a class on the board to revise methods for finding the general rule. Work through the example in the Learner's Book. Have learners come up to the board to write their working out for finding the rule on the board. Discuss the various strategies and encourage learners to use what makes sense for them. Discuss the impact of negative integers on patterns. Ask learners how they think negative integers may impact in the general rule. Do an example of a negative integer in the rule, and of substituting in term numbers. Allow learners to work in pairs or small groups to finish the exercise.

Suggested answers

1.1 169
1.2 $(n + 9)^2$
1.3 $(5 + 9)^2 = 14^2 = 196$
1.4 $(20 + 9)^2 = 29^2 = 841$
1.5 $(25 + 9)^2 = 34^2 = 1\ 156$
2.1 $(-11 \times n) - 23$
2.2 $-11 \times 10 - 23 = -133$
2.3 $-11 \times 25 - 23 = -298$
2.4 $-11 \times 100 - 23 = -1\ 123$
2.5 $(n \times -11) - 23$ (commutative property of multiplication)
3.1 $a = \sqrt{100} + (-3) = 7$; $b = \sqrt{132} + (-3) = 8{,}49$; $c = \sqrt{144} + (-3) = 9$
3.2 $p = (3 \times 3) + (-12) = -3$; $q = (7 \times 3) + (-12) = 9$; $r = (11 \times 3) + (-12) = 21$; $s = (15 \times 3) + (-12) = 33$

4

	Term 1	Term 2	Term 3	Term 4
Terms	5	12	19	26
Difference	7 (5 + 7 = 12)	7 (12 + 7 = 19)	7 (19 + 7 = 26)	7 (26 + 7 = 33)
7 × number ($7n$)	7	14	21	28
2nd difference	7 – 5 = 2	14 – 12 = 2	21 – 19 = 2	28 – 26 = 2
Wrong by:	2	2	2	2

Remedial

If learners are struggling with the concept of negative numbers and rules, encourage them to make cards with plus signs on them and cards with minus signs on them. They can then use the cards to represent number expressions. For example if the expression is $6 + (-4)$ then they put down 6 plus cards. Next put 4 minus cards down. Pair the plus and minus cards. They should be left with 2 plus cards.
The solution is $6 + (-4) = 2$.

Unit 3 Input and output values

Learner's Book page 326

Unit focus

- revise working with rules and relationships
- revise finding input and output values
- include integers in input, output and rules of functions and relationships.

Specific background info on input and output values

Learners have worked with input and output values in detail in previous chapters. The concept, strategies and methods should be familiar to them. Learners should also know that in order to find inputs, they have to work backwards with the given function and the output values.

Exercise 1

Learner's Book page 327

Guidelines on how to implement this activity

In order for learners to manage this unit successfully, you should ensure that learners can perform the maths operations adequately; that learners understand that the input number is the given number; that learners know that the rules are applied one at a time to the given number, and the output is the result of applying the rules to the input. Work through examples with positive numbers of finding the input and output values. Focus on how we find input values when given output values. We need to use the function in reverse. Introduce examples with negative integers. Remember to ensure that learners only work with adding and subtracting integers to find inputs and outputs, as learners have yet to cover multiplication and division of integers. Do a few examples as a class and then encourage learners to complete the exercise on their own.

Suggested answers

1.1

Input	Rule	Rule	Output
3	+ 8	x 2	22
8	+ 8	x 2	32
13	+ 8	x 2	42

1.2

Input	Rule	Rule	Output
10	Input squared	x 2	200
20	Input squared	x 2	800
30	Input squared	x 2	1800

2

	Term 1	Term 2	Term 3	Term 4
Terms	3	8	13	18
Difference	+ 5	+ 5	+ 5	+ 5
5 × number (5)n	5	10	15	20
	5 – 3 = 2	10 – 8 = 2	15 – 13 = 2	20 – 18 = 2
Wrong by:	2	2	2	2

3.1 Any term value is twice the term value before it.
3.2 Any term value is half of the one before it.
3.3 Any term value is the term number cubed.
3.4 Any term value is equal to the previous term value plus four.
3.5 Any term value is equal to the previous term value minus six.
4.1 96; 192; 384 **4.2** 8; 4; 2 **4.3** 125; 216; 343
4.4 –64; –60; –56 **4.5** –29; –35; –41

5 $a = (14 + 5) \times 2 - 12 = 19 \times 2 - 12 = 38 - 12 = 26$
$b = (21 + 5) \times 2 - 12 = 26 \times 2 - 12 = 52 - 12 = 40$
$c = (28 + 5) \times 2 - 12 = 33 \times 2 - 12 = 66 - 12 = 54$
$d = (35 + 5) \times 2 - 12 = 40 \times 2 - 12 = 80 - 12 = 68$

6 $a = (3 - 10) \times 3 + 2 = -7 \times 3 + 2 = -21 + 2 = -19$
$b = (6 - 10) \times 3 + 2 = -4 \times 3 + 2 = -12 + 2 = -10$
$x = (-1 - 2) \div 3 + 10 = -3 \div 3 + 10 = -1 + 10 = 9$
$y = (8 - 2) \div 3 + 10 = 6 \div 3 + 10 = 2 + 10 = 12$
$z = (17 - 2) \div 3 + 10 = 15 \div 3 + 10 = 5 + 10 = 15$

7

Given	Input	Rules	Output
15; 45; ×3	15	×3	45
95; 19; ×5	19	×5	95
+36; –14; 18; 40	18	–14; +36	40
55; 105; –10; –40	105	–10; –40	55
32; ÷2; ÷2; 128	128	÷2; ÷2	32

8.1 $a = (-2) + (-3) = -5$; $b = (-3) - (-2) = -1$; $d = (-1) - (-2) = 1$; $e = (-2) + (+3) = 1$;
$f = (+3) - (-2) = 5$;
$g = (-2) + (+15) = 13$

Input	–5	–3	–1	1	3	5	15
Output (–2) + (n)	–7	–5	–3	–1	1	3	13

8.2 $a = 3 + (-10) - (-6) = -1$; $b = 4 - 3 + (-6) = -5$; $c = 3 + (-3) - (-6) = 6$;
$d = 3 + (+3) - (-6) = 12$;
$e = 13 - 3 + (-6) = 4$; $g = 3 + (+15) - (-6) = 24$

Input	–10	–8	–5	–3	3	4	15
Output 3 + n – (–6)	–1	1	4	6	12	13	24

Remedial

Learners who have difficulties must be given encouragement and the chance to improve. Provide additional simple examples for learners to practise and gain a sense of mastery over the work. Pair weaker learners with stronger learners for them to observe how the work should be tackled and addressed.

Extension

Learners should be encouraged to identify and learn more about other forms of machines that form a function. For example, computers receive an instruction from the keyboard or mouse (an input), the computer performs the required operation, and generates the output onto the screen. Learners could do a project of the necessary inputs required and the outputs that are generated.

Unit 4 Equivalent forms

Learner's Book page 329

Unit focus

- revising how flow diagrams, tables, formulae and number sentences are equivalent forms of representing relationships
- including integers, namely negative numbers, in these equivalent forms.

Background information on equivalent forms

Learners should know and be able to represent patterns and relationships in each of the following forms: verbally; in flow diagrams; tables; equations and formulae. Learners should be able to identify the form and represent it in another form that is equivalent. Learners should be familiar and adept at portraying positive whole numbers in this way. This unit asks learners to represent relationships and patterns that contain negative integers in each of these forms.

Exercise 1

Learner's Book page 330

Guidelines on how to implement this activity

Revise with learners the different forms we can use to represent patterns and relationships. Do an example representing one function in each of the different forms. Have learners come up to the board and supply these different representations. Do additional examples as a class including negative numbers. Encourage learners to work in pairs to complete this exercise.

Suggested answers

1.1

Input	Rule 1	Rule 2	Rule 3	Output
5	×2	+10	+(–50)	c
10	×2	+10	+(–50)	d
15	×2	+10	+(–50)	e
x	×2	+10	+(–50)	48
40	×2	+10	+(–50)	f

1.2 $x = [(48 - (-50)) + 10] \times 2 = (98 - 10) \times 2 = 88 \div 2 = 44$

$c = 5 \times 2 + 10 + (-50) = -30$

$d = 10 \times 2 + 10 + (-50) = -20$

$e = 15 \times 2 + 10 + (-50) = -10$

$f = 40 \times 2 + 10 + (-50) = 40$

2.1

If the value of x is:	What is: $x + 5 = y$	What is: $3x = y$	What is: $\frac{x}{2} = y$	What is: $x - 1 = y$
4	9	12	2	3
8	13	24	4	7
16	21	48	8	15
100	105	300	50	99

2.2 Output value equals input value plus five.
Output value equals input value times three.
Output value equals input value divided by two.
Output value equals input value minus one.

2.3

Input | Rule | Output

$x \longrightarrow \boxed{+\ 5} \longrightarrow y$

Input | Rule | Output

$x \longrightarrow \boxed{\times\ 3} \longrightarrow y$

Input | Rule | Output

$x \longrightarrow \boxed{\div\ 2} \longrightarrow y$

Input | Rule | Output

$x \longrightarrow \boxed{-\ 1} \longrightarrow y$

3

p	–4	–3	–2	–1	0	1	2	3	4
q	10	9	8	7	6	5	4	3	2

4.1 i) 7; 10; 13; 16; 19; 22 → +3

ii)

Input | Rule | Output

$x \longrightarrow \boxed{+3} \longrightarrow y$

iii) The difference between two consecutive terms is 3.

4.2 i) 7; 9; 11; 13; 15; 17; 19; 21; 23; 25; 27; 29; 31; 33; 35; 37; 39 → +2

ii)

Input | Rule | Output

$x \longrightarrow \boxed{+2} \longrightarrow y$

iii) The difference between two consecutive terms is 2.

4.3 i) 17; 23; 29; 35; 41; 47 → +6

ii)

Input | Rule | Output

$x \longrightarrow \boxed{+6} \longrightarrow y$

iii) The difference between two consecutive terms is 6.

5.1 Output = Input – 2

5.2 Output = 2 × Input + 8

5.3 Output = 5 × Input + 100

5.4 Output = 3 × Input + 24

Remedial

Learners should be encouraged to use number lines to help them with adding and subtracting any integers. When working with patterns that involve multiplication of a negative integer, learners must use the pattern to determine the value, as they have yet to cover multiplication of negative numbers.

Consolidation

Learner's Book page 333

Before doing this consolidation exercise, encourage learners to review the work covered in this chapter. Advise learners to use the summary and to revise their work. This exercise can be used as an informal assessment task for you to track how learners are coping with the chapter and the concepts covered.

Suggested answers

1

Series	5; 10; 15; …	4; 11; 15; 22; 26; …	3; 10; 13; 20; …
5th term	25	26	23
6th term	30	33	30
7th term	35	37	33
8th term	40	44	40

(8)

In series A, the difference between two consecutive terms is 5.
In series B, the difference between the first two terms is 7 and the difference between the 2nd and 3rd term is 4. This pair of term differences, such as 7 and 4, repeat onwards in the sequence in that order.
In series C, the difference between the first two terms is 7 and the difference between the 2nd and 3rd term is 3. This pair of term differences, such as 7 and 3, repeat onwards in the sequence in that order.

2.1 27; 35

12th term $= \frac{1}{2} \times (12 \times 13) + 12 = 90$

nth term $= \frac{1}{2} \times [n \times (n + 1)] + n$ (4)

2.2 25; 31

12th term $= (6 \times 12) - 5 = 67$

nth term $= 6n - 5$ (4)

3 $p = (6 \times 2) - 19 = -7$; $q = (10 \times 2) - 19 = 1$; $r = (14 \times 2) - 19 = 9$;
$s = (18 \times 2) - 19 = 17$ (4)

4

Given	Input	Rules	Output
×5; –4; –5; –25	–4	×5; –5	–25
5; ×12; 63; +60	5	×12; +3	63
37; +8; 6; +23	6	+8; +23	37
+1; –10; 7; –2	7	–10; +1	–2
36; ÷2; 8; ×9;	8	×9; ÷2	36

(15)

5

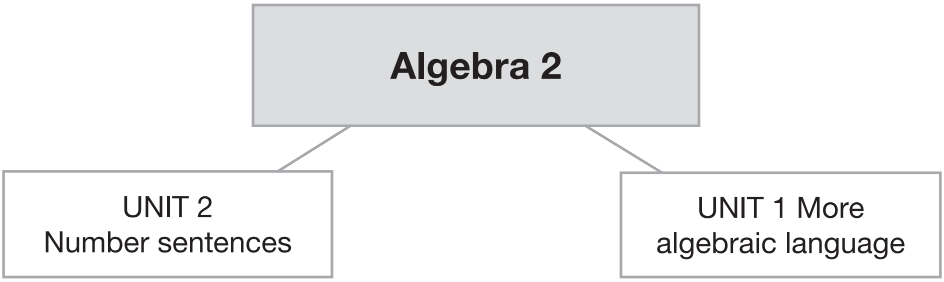
a	0	1	2	3	4	5	6	7
b	7	6	5	4	3	2	1	0

(5)

[40]

Chapter 18 Algebra 2

Overview of concepts



	Content	Time allocations	LB page
Unit 1	More algebraic language	3 hours	335
Unit 2	Number sentences	4 hours	337

Background information on Algebra 2

Learners should know and understand the concept of variables and constants. In algebraic language a variable is a symbol, usually a letter of the alphabet, representing a number. For example, $y + 2 = 5$. In this case the y is equal to 3. Variables and constants must be recognised in given formulae and equations. They must recognise and interpret rules or relationships represented in symbolic form in flow diagrams and tables. In this chapter learners are exposed to the use of integers in algebraic language and number sentences.

Generic teaching guidelines for teaching Algebra 2

Demonstrate the use of variables in number sentences, and demonstrate to learners that a variable may have different values in different positions. For example:

$1 + a = 5$	or	$5 - a = 3$	or	$5a = 15$	or	$\frac{5}{a} = 1$
In the above $a = 6$;		2;		3;		5

Ensure that learners know that although a variable's value changes, a constant has the same value always. For example:

$8 + a = 10$ so $a = 2$ or $8 - a = 3$ so $a = 5$

In the example the constant is 8, and its value does not change.

When teaching number sentences, encourage learners to use variables as unknowns in order to prepare them for future grades.

Resources

Vocabulary cards created in Chapter 12, number and variable cards to create number sentences, cardboard and colour pens to create posters. Each learner should have their own calculator

Unit 1 More algebraic language

Learner's Book page 335

Unit focus

- revising the algebraic language
- identifying variables and constants in algebraic expressions.

Background info on more algebraic language

Learners know that written language can be translated to algebraic language. For example, 'three times *z* less five' can be written as $3z - 5$. $3z - 5$ is an algebraic expression. It has 2 terms. Terms are separated by plus (+) or minus (–) signs, never by multiplication or division signs. 6 times *z*, or 6 multiplied by *z*, is written $6z$ or $6.z$ in algebra. The multiplication sign (×) is not usually used. 7 divided by *z* is written $\frac{7}{z}$. The division sign (÷) is not usually used.

Exercise 1

Learner's Book page 335

Guidelines on how to implement this activity

Revise the work learners did in Chapter 12 on algebraic language. Revise terms, constants and variables. Discuss how to identify variables and constants in algebraic expressions. Remind learners that the sign in front of the term must be included, as it is very important. We don't need to say +5 when it is positive as the positive is implied, but we do have to include the sign, when it is negative.
Revise how to translate from everyday language into algebra, and include examples that contain negative integers. Also revise how to work with negative numbers in algebraic expressions by substituting. Do examples together as a class, encouraging learners to come up to the board and write up their answers. Learners should be able to manage the exercise on their own.

Suggested answers

1.1 $-6x + 5$ **1.2** $\frac{y - (-5)}{2}$

1.3 $2x + 23$ **1.4** $\frac{2a}{4} + (-9)$

1.5 $s + 95 = 185$

2 $a = [(11 - 5) + 30] \div 3 = [6 + 30] \div 3 = 36 \div 3 = 12$
$b = [(47 - 5) + 30] \div 3 = [42 + 30] \div 3 = 45 \div 3 = 15$
$c = (6 \times 3) - 30 + 5 = 18 - 30 + 5 = -7$
$d = (18 \times 3) - 30 + 5 = 54 - 30 + 5 = 29$
$e = (30 \times 3) - 30 + 5 = 90 - 30 + 5 = 65$

3

Value of x	$1 + x =$	$2x =$	$x - 1 =$	$\frac{x}{2} =$
1	2	2	0	$\frac{1}{2}$
2	3	4	1	1
–1	0	–2	–2	$-\frac{1}{2}$
–2	–1	–4	–3	–1
–3	–2	–6	–4	$-\frac{3}{2}$

4.1 i) variable: y

ii) constants: + 5; + (80 ÷ 4)

iii) $3y + 5 = 20$; $3y = 15$; $y = 5$

4.2 i) variable: z

ii) constants:–42; + 3^2;–3

iii) $4z - 42 = 9 - 3$; $4z - 42 = 6$; $4z = 48$; $z = 12$

4.3 i) variable: x

ii) constants: – (–4); + 96; + (–10)

iii) $2x - (-4) = 86$; $2x = 82$; $x = 41$

4.4 i) variable: b

ii) constants: (–6); – (–6); – (–7)

iii) $0 = b - (-7)$; $b = -7$

4.5 i) variable: a

ii) constants: + (45 ÷ 9); 0

iii) $5 = 0 - a$; $a = -5$

5

Number sentences	Variables	constants
$a + 7 = 13$	a	+7; +13
$23 - b = 5$	b	+23; +5

6.1 $x = (42 \div 2) + (-8) = 21 + (-8) = 13$

$y = [26 - (-8)] \times 2 = 34 \times 2 = 68$

$z = (94 \div 2) + (-8) = 47 + (-8) = 39$

6.2

Input	13	26	39
Rules	– (–8); ×2	– (–8); ×2	– (–8); ×2
Output	42	68	94

Remedial

Allow learners who understand the concept to work with those who are struggling, either in pairs or in groups. Most learners can calculate $\Delta - 10 = 1$ so to transfer that concept to using letters of the alphabet as variables is not too difficult. If $\Delta - 10 = 1$ is the same as $y - 10 = 1$, then it should be as easy. Work in pairs to write 'matching' equations, one learner using letters of the alphabet and the other using squares or triangles as place holders. For example: $b + 4 = 6$ and $\Delta + 4 = 6$. In both cases the unknown or variable is 2.

Extension

Encourage learners to find out more about Ancient Egypt and Babylon where mathematicians were able to solve linear equations like $ax = b$.
Extend learners to find pairs of variables that will satisfy equations with more than 1 variable. For example, $2x + y = 21$. Provide a domain for x values, such as (–10; 10)

Unit 2 Number sentences

Learner's Book page 337

Unit focus

- revising solving algebraic number sentences
- including integers in number sentences.

Background info on number sentences

Number sentences are simply word sentences that have been translated into an algebraic form. For example, "five is bigger than four" is written in English. "Vyf is groter as vier" is written in Afrikaans. "$5 > 4$" is written in Algebraic form. Why do you think mathematicians have devised this special language? It is much quicker and shorter to write.
Although any letter of the alphabet can be used for a variable, those most commonly used are x, y and z. We also use a, b, c and n quite often.
Learners have worked with solving number sentences in Chapter 12, and this unit revises this and extends learners to include negative numbers in their working.

Exercise 1

Learner's Book page 338

Guidelines for implementing this exercise

Spend some time translating word sentences into number sentences. For example, twenty minus three times x is equal to two. $20 - 3x = 2$. Let learners work in pairs. Each should write 3 number sentences. Exchange work and translate number sentences into word sentences. For example, $3a - 2 = 1$ is 'three times a minus two equals one.' Discuss ways of solving number sentences. Inspection is the simplest, but needs the equation to be simple to work. Do a few examples of this, and be sure to include examples with negative numbers. Next, discuss using trial and improvement to equations. Learners should say how they determine which number to try. Ensure all learners know how to substitute correctly. Again be sure to include negative numbers in the examples you do as a class. Encourage learners to work in pairs, but do all their own working.

Suggested answers

1

Values of y	$9y$	$7 + 7y$	$3y - 9$	$5y + y$	$10 + 2y$	$8y - 4$
0	0	7	–9	0	10	–4
3	27	28	0	18	16	20
–5	–45	–28	–24	–30	0	–44
7	63	56	12	42	24	52
15	135	112	36	90	40	116

2.1 $x = 6$
2.2 $y = -23$
2.3 $z = 5$
2.4 $p = 72$
2.5 $a = 5$
2.6 $2b^2 = 18$; $b^2 = 9$; $b = \pm 3$
3.1 $4(3) - (3) + (-7) = 12 - 3 + (-7) = 2$; $y = 3$
3.2 $(6) + 4(6) - 5 = 6 + 24 - 5 = 25$; $z = 6$
3.3 $3(3)^2 - 6(3) - 9 = 3(9) - 18 - 9 = 27 - 18 - 9 = 0$; $a = 3$
3.4 $3(10)^2 + 4(10) - 100 = 3(100) + 40 - 100 = 240$; $b = 10$
3.5 $(-4) + 4(-4) + 50 = -4 + (-16) + 50 = 30$; $y = -4$
4.1 $3x = 15$
4.2 $2y + y = 45$
4.3 $7z + 6 = 27$
4.4 $p = 10 \times 15$
5.1 $x = 5$
5.2 $3y = 45$; $y = 15$
5.3 $7z = 21$; $z = 3$
5.4 $p = 150$

6

Given	Find the value of the variable	Identify the constants
$5 - a + 8 = 20$	–7	+5; +8; +20
$b - 70 = 9$	79	–70; +9
$3 + 7c = 45$	6	=3; +45
$\frac{88}{d} - 2 = 6$	11	–2; +6

7.1 $5 \times 90y = 4\,500$
7.2 $450y = 4\,500$; $y = 10$
8.1 $5(2)^2 + 3(2) - 10 = 5(4) + 6 - 10 = 16$; $y = 2$
8.2 $3(5)^2 - 4(5) - 15 = 3(25) - 20 - 15 = 75 - 35 = 40$; $z = 5$
8.3 $2(0) - 6(0) - 9 = 0 - 0 - 9 = -9$; a = 0
8.4 $2(7) + (-7) - 100 = 14 + (-7) - 100 = -93$; $b = 7$

Remedial

Provide drill exercises for solving simple number sentences. Using a number of simple examples is helpful. For example, copy and solve for x:

$6 + 0 = x$	and	$0 + x = 6$	and	$x + 0 = 6$
$5 + 1 = x$		$1 + x = 6$		$x + 1 = 6$
$4 + 2 = x$		$2 + x = 6$		$x + 2 = 6$

If learners can't add, subtract whole numbers including integers, multiply or divide whole numbers, they might have problems. Ensure that they have plenty of practice at using numbers.

Make sure you check each learner's work regularly. Don't allow mistakes to become ingrained.

Extension

From c. 750 C.E to c. 1258 C.E much scientific learning was pursued in Arabic. The word for 'thing' in Arabic is 'shei'. This was translated into 'xei'. In algebra, this was shortened to x. Allow learners to investigate this claim and see what else they can find out about the use of certain letters of the alphabet as variables.

Consolidation

Learner's book page 341

Before doing this consolidation exercise, encourage learners to review the work covered in this chapter. Advise learners to use the summary and to revise their work. This exercise can be used as an informal assessment task for you to track how learners are coping with the chapter and the concepts covered.

Suggested answers

1 $a = [(8 - 2) + 34] \div 5 = [6 + 34] \div 5 = 40 \div 5 = 8$
$b = [(48 - 2) + 34] \div 5 = [46 + 34] \div 5 = 80 \div 5 = 16$
$c = [(4 \times 5) - 34] + 2 = [20 - 34] + 2 = -14 + 2 = -12$
$d = [(12 \times 5) - 34] + 2 = [60 - 34] + 2 = 26 + 2 = 28$
$e = [(20 \times 5) - 34] + 2 = [100 - 34] + 2 = 66 + 2 = 68$ (4)

2

Value of x	$7 + x =$	$9x =$	$x - 24 =$	$\frac{x}{5} =$
5	12	45	–19	1
10	17	90	–14	2
15	22	135	–9	3
20	27	180	–4	4
25	32	225	1	5

(18)

3

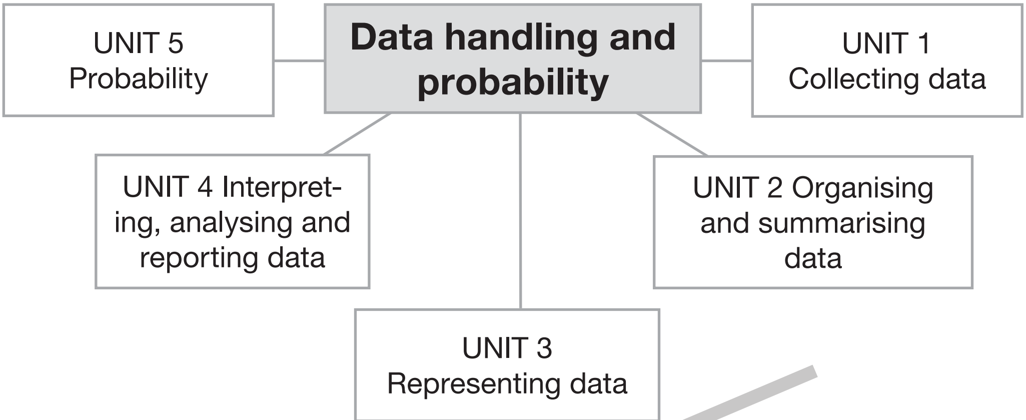
Algebraic expression	Variable/s	Constant	Number of terms	Name of expression
$7z + 15$	z	15	two	binomial
$8y$	y	No constant	one	monomial
$3x - 6w + 9$	$x; w$	9	three	trinomial
$1 - 57v$	v	1	two	binomial

(16)

4.1 $3(3)^2 + 4(3) - 20 = 3(9) + 12 - 20 = 19; y = 3$
4.2 $2(6)^2 + 3(6) - 1 = 2(36) + 18 - 1 = 89; z = 6$
4.3 $(5)^2 - 5(5) - 1 = 25 - 25 - 1 = -1; a = 5$
4.4 $2(9)^2 + 10(9) - 200 = 2(81) + 90 - 200 = 52; b = 9$ (8)
5.1 $5 \times 100y = 25\,000$
5.2 $500y = 25\,000; y = 50$ (4)
[20]

Chapter 19 Data handling and probability

Overview of concepts



Content		Time allocations	LB page
Unit 1	Collecting data	1 hour	343
Unit 2	Organising and summarising data	3 hours	348
Unit 3	Representing data	3 hours	355
Unit 4	Interpreting, analysing and reporting data	3,5 hours	360
Unit 5	Probability	4,5 hours	366

Background information on data handling and probability

Learners have worked on data handling in earlier grades. The following are new in Grade 7:

- samples and populations
- multiple choice questionnaires
- stem-and-leaf displays
- grouping data in intervals
- mean
- range
- histograms
- scales on graphs.

The National Population Census of 2011 will be a good example to explain the concept of a population.

In this chapter learners will do revision on data handling. They will also complete exercises to:

- collect data using appropriate sources;
- organise and record data using tallies, tables and other displays (such as stem-and-leaf);
- group, sort and summarise numerical data (thereby grouping into intervals);
- draw graphs to display data;
- read and interpret data in order to draw conclusions and make predictions; and
- work out the probability of outcomes and relative frequency.

Generic teaching guidelines for teaching data handling and probability

The topic of data handling lends itself to many practical examples. Data handling is also an excellent way to bring environmental and social topics into the mathematics classroom.

To introduce data handling, do some revision by referring to the following:

- Remind learners that our world is filled with information. Let them think of examples of how we get and exchange information. For example, watching TV, reading a magazine article, or sending an sms.
- Teach how data is the building block of information.
- Ask learners to identify as many data sources as possible.
- Ask learners to collect newspapers or appropriate magazines to find as many examples of data handling as possible. It could be a graph or statistics that were published, or an opinion poll. The business section in a newspaper will often have many good examples.
- Let learners work in groups of about 4 or 5. Each group can make a poster to illustrate data handling by displaying their examples which they got from the newspapers. These posters can be put up in class.
- Revise the creation of questionnaires and tally tables.
- Encourage learners to reflect on their experiences with questionnaires. How many of the learners have completed or answered questions for a questionnaire before? Did the questions require yes/no answers or were they in the form of multiple choice questions?
- Ask learners to find out what the Census 2011 was about. They can speak to their parents, family members or people they share a household with. They can also look at this website: http://www.statssa.gov.za/

Resources

Examples of graphs and data from newspapers and magazines. Blank tables and graph axes, blank circles to use for pie charts, examples of issues happening in the community that learners could investigate. For probability have dominoes, dice and coins available. Have cardboard and colour pens for learners to use for both data and probability.

Unit 1 Collecting data

Learner's Book page 343

Unit focus

- posing different types of questions to collect data
- designing a questionnaire
- selecting appropriate sources
- differentiating between samples and populations.

Specific background info on collecting data

As learners learn more about collecting data, they should be exposed to a variety of contexts that deal with social and environmental issues.
They should work with given data sets represented in a variety of ways that include big number ranges, percentages and decimal fractions.
It will be useful to use as examples in class two or three data sets. For example, one that deals with social issues, one that deals with environmental issues and one that deals with economic issues. You can use and refer to these throughout the units of this chapter. It will create a sense of continuity and help the learners to get familiar with the data sets, so that they can rather focus on applying the new concepts.

Exercise 1

Learner's Book page 344

Guidelines on how to implement this activity

This exercise focuses on planning a questionnaire. Work through the 4 steps for collecting data. Discuss each step as a class. At this stage it is only necessary for learners to focus on the planning. Learners are encouraged to identify possible problems/ questions they would want to investigate, then to discuss appropriate data sources; and then to think about the types of questions they would ask, and whether the questions would provide the desired outcome. Let learners do this exercise in groups.

Suggested answers

1 There are many possible issues learners may choose to investigate. Allow learners to select options that interest them. Issues to investigate could include: litter, crime, lack of safe recreation spaces, leisure choices, fashion choices etc.

2 Questionnaires can be administered to fellow students, staff, family or community members. Questionnaires must be answered by someone rather than done by desk research.

3 Learners responses will differ depending on their chosen topic. Encourage them to use the suggestions in the learner book to refine their questions. Learners can exchange questions and get feedback from their peers about the clarity of the questions.

Remedial

Learners must first set aside enough time to plan their questionnaires well. They must imagine and practise what type of responses they may expect based on the questions they formulate.

Extension

Learners can identify another issue to investigate – social, environmental or economic. They can identify appropriate sources.

Exercise 2

Learner's Book page 345

Guidelines on how to implement this activity

Discuss ways to collect data. Ask learners to identify where they might be able to find certain information. Provide examples, such as the number of white cars in a parking lot? Or the number of births in March 2012? Ask learners where they can find this information? What information can they find by asking questions of people around them? It is important that learners are able to differentiate between statistical data that can be looked up in government records or online, and data that needs to be collected from people by means of a survey. Have a discussion and record the different types of data you can collect from a physical sample of people, and the types of data you can look up online. Let learners complete this exercise as a class.

Suggested answers

1 (1) and (vi)
(2) and (v)
(3) and (vii)
(4) and (ii)
(5) and (iii)
(6) and (iv)
(7) and (i)

Extension opportunities

If the learners have access to the Internet or if the school has Internet facilities, encourage learners to use SuperWEB of Statistics South Africa (StatsSA), where they can access a database of their choice (such as Census@School) and "design" their own results sets. First experiment with the site in your own time so that you can adequately assist and guide them: http://interactive.statssa.gov.za/superweb/login.do No log in is required.
If not Census@School they can select, for example 2 Household Surveys->General Household Survey->2011->General Household Survey 2011. Learners can use this site to get data on for example, the number of people with cell phones. The website provides the option to save each result set as a Microsoft Excel file and either the actual number of people or percentages.
Learners should report their findings. Even if they don't save the Excel files they can write down the results. For example, the number of South Africans who have cellphones, or access to running water.

Exercise 3 Learner's Book page 346

Guidelines on how to implement this activity

Discuss the different types of questionnaires. What type of answers can you get? Discuss Yes or No examples – are these too limited. What information do you get. To teach the concept of drawing up a multiple choice questionnaire as opposed to only yes/no answers, you could refer to another example from Census 2011 about the type of cooking fuel used. The results are shown in the table below. Ask learners how the questionnaire could have been designed to collect data like this, since some households may use a combination of fuel types:

Percentage distribution cooking fuel type: General Household Survey 2011	
Electricity from mains	70,90%
Electricity from generator	0,00%
Gas	2,00%
Paraffin	4,70%
Wood	17,80%
Coal	1,10%
Candles	0,00%
Animal dung	0,30%
Solar energy	0,00%
Other (specify)	1,20%
None	0,10%
Unspecified	1,90%
Total	100,00%

Have learners complete Exercise 3 in pairs.

Suggested answers

1 Yes or no responses
2 Short sentence
3 Not without assistance. Someone could ask the questions to the blind person and complete the questionnaire on their behalf.

Exercise 4 Learner's Book page 346

Guidelines on how to implement this activity

This exercise follows on from Exercise 3. Encourage learners to design their own questionnaire based on the problems they identified to investigate in Exercise 1. When learners have to collect data as part of the questionnaire exercise concerning an issue in their own environment, remind them to be focussed on their own safety. It is not a requirement they should go out alone in the neighbourhood from door to door asking questions. They can also conduct the survey at school.

Suggested answers

1

Littering Questionnaire		
1) Is there a lot of rubbish lying around on the school premises? Circle either Yes or No.	Yes	No
2) Do you think there are enough rubbish bins on the school premises? Circle either Yes or No.	Yes	No
3) If you said 'No', do you think adding more rubbish bins would help? Circle either Yes or No.	Yes	No

Remedial

Some learners may feel intimidated to approach peers or people with the questionnaire which they designed. Encourage them to identify people that will be willing to participate – if not peers, perhaps household members, neighbours, staff at a community library or learners who travel to school together with them on the same bus.

Let them work in groups of two, but encourage participation by both so that each learner does his/her share of the work.

Exercise 5

Learner's Book page 347

Guidelines on how to implement this activity

Use a data set in your class examples from Census 2011 about internet access. Use this data set to explain the concept of a data population and also how the questionnaires were designed to obtain yes/no answers:

Internet connection in the household?	
Yes	8,70%
No	89,30%

Internet for students at a school/university/college?	
Yes	5,80%
No	92,20%

Internet in a library or community hall/Thusong centre?	
Yes	3,30%
No	94,80%

Internet Café 2 km or less from the household?	
Yes	4,10%
No	93,90%

Internet at place of work?	
Yes	15,40%
No	82,60%

Suggested answers

1 Names of 1 000 Grade six learners.

2 Data sample because it is smaller collection of names representing the larger group.

3 $x = 100$; $y = 1\ 000$

Remedial options

Some learners may have difficulty to understand the difference between a sample and population. Refer to the explanation provided in the Learner's Book. A sample is a portion of the whole. A population is the whole group.
Give learners more practice to identify samples and populations by providing more examples. For example, the entire population of whale sharks is x. A marine biologist studies a school of y whale sharks in a particular part of the ocean. Which is the population and which is the sample?

Unit 2 Organising and summarising data

Learner's Book page 348

Unit focus

- revising tally tables
- grouping data into intervals
- organising data in stem-and-leaf displays
- determining the mean, median, mode and range to summarise data.

Specific background info on organising and summarising data

The content can be defined as two topics: organising and summarising.
Organising involves using:
- tally marks
- tables
- stem-and-leaf displays.

Organising also involves grouping data into intervals. An interval has a bottom and top limit. Grouping is often preceded by sorting. For example learners, must first sort a data set before grouping it into intervals. Learners can practise sorting data sets in ascending and descending order. Remind that ascending order is from small to large and descending order is from large to small.
Summarising data involves:
- working with a given numerical data set that is ungrouped – not yet sorted or grouped into intervals;
- determining the mean value;
- determining the median;
- determining the mode.
- identifying the largest and smallest scores in a data set, and
- determining the difference between them (range).

Revision exercise Learner's Book page 348

Guidelines on how to implement this activity

Use this revision exercise to establish the prior knowledge and skill level that your learners bring to this unit. Can learners read a table correctly? Can learners use tallies and frequency correctly.

Suggested answers

1

Refreshment	Tally marks	Frequency (f)
Fruit juice	II	2
Fizzy drink	I	1
Ice lolly	III	3
Water	𝍸	5

2 Most athletes prefer water, since it has the highest frequency, 5.

3 Fizzy drink is least popular, since it has the lowest frequency, 1.

Remedial

If learners struggle with this exercise, revise the concepts of tallies and frequency before continuing with the rest of the unit. Learners should be familiar with tally marks. Tally marks are used to keep count. A frequency table can be drawn from a tally table to give the total number of tally marks. The f symbol indicates 'frequency'. Remind learners that every tally mark | counts one. To show a group of 5, draw a line through 4 tally marks: 𝍸

Exercise 1

Learner's Book page 350

Guidelines on how to implement this activity

Do a simple example as a class for organising data. Start off with an example of sorting a list of names or surnames in alphabetical order. Use intervals of letters: A-F, G-K, L-P, Q-U, V-Z.

Learners should practise using large numbers and decimal fractions. Organising and summarising data provides an excellent opportunity to revise common fractions, mixed numbers and decimal fractions. A good way to revise this is by letting learners sort the numbers in a given data set in ascending or descending order. That will reveal possible problem areas with decimal fractions. Use interesting numbers such as 0,008; 0,800; 1,001 etc. Introduce a range of numbers for learners to sort for example: 2,3; $\frac{23}{10}$; 0,008,1 $\frac{12}{6}$. Learners should be able to notice equivalence: 2,3 = 2 $\frac{3}{10}$. If they are required to find the mode of this data set, it will be 2,3 or 2 $\frac{3}{10}$.

Discuss how intervals should be written in the correct bracket notation with closed and open brackets: an open bracket means the number is not included and a closed bracket means the number is included. For example, for [0-100); [100-200), 100 is excluded in the first interval but included in the second. In this way 100 is not counted twice.

When learners sort a data set they should get into the habit of lightly crossing through each value that has been counted with pencil, to avoid skipping some values or counting one value more than once.

Suggested answers

1

English	isiZulu	isiNdebele	siSwati	Sesotho
English	isiZulu	isiNdebele	siSwati	Sesotho
English	isiZulu	isiNdebele		Sesotho
English	isiZulu	isiNdebele		Sesotho
English	isiZulu	isiNdebele		Sesotho
	isiZulu			Sesotho
				Sesotho

2

Language	Tally	Frequency
English	𝍸	5
isiZulu	𝍸 I	6
isiNdebele	𝍸	5
siSwati	II	2
Sesotho	𝍸 II	7

3 The most common language is Sesotho, and the least common is siSwati.

Value that occurs more than once

Make the difference clear: when grouping data into intervals for example, for [0-100); [100-200), 100 is excluded in the first interval but included in the second. If the number 100 appears once in a given data set, it must only be counted once. So, 100 must only appear in one of the two intervals : [100-200). Learners might want to put the 100 in both intervals, but then it will be counted twice whereas it appears only once.

Exercise 2 Learner's Book page

Guidelines on how to implement this activity

Discuss ways that we have organised data – namely in a table, and recording the frequency. Another way of organising data is with a stem and leaf display. Explain what they stem and leaf display involves. Discuss the intervals and how they become the stem, while each unit of data becomes a leaf. Work through the worked example in the Learner's Book, explaining carefully how to set them out. Do an additional example if you feel learners need an additional explanation. Let learners complete this exercise on their own.

Suggested answers

Stem (Tens)	Leaf (Units)	Frequency(f) of leaf units
1	8 9	2
2	1 7 9	3
3	1 3 6 7 8	5
4	0 1	2
5	1	1
6	1 4 6	3
		16

2 The age group [30,39), with a frequency of 5.
3 The age group [50,59), with a frequency of 1.

Exercise 3

Learner's Book page

Guidelines on how to implement this activity

Mean, mode and median are measures of central tendency and these are used to summarise a set of data. Make a poster to put up in class with the "three m's: mmm". Explain each on the poster:

- The **mean** value gives you the average. You can find the mean from ungrouped data.
- The **mode** is the value that occurs most often in the set (highest f). Teach them that if they first sort the data set it is easier to notice which value is the mode.
- The **median** is the middle value in a sorted data set. They must first sort the data set in ascending or descending order.

For the median: if the data set contains an even number of data elements the mode will be the value which is halfway between the two middle values. That will have to be calculated. A very simple example is a data set that contains two numbers: 1 and 2. The median is $1\frac{1}{2}$ or 1,5.
On the other hand, if the data set contains an uneven number of data elements the median will be the value which is in the middle. It can just be identified and read off – it does not have to be calculated. For example, a data set: 2; 4; 6 has median = 4. Work through the worked example in the Learner's Book and have learners complete this exercise on their own.

Suggested answers

1 Sum of all vaccinations = 200
The mean is: $\frac{200}{18}$ = 11,11 (11 injections per day)
2 Yes, because you can only physically have a whole number of injections
3 4; 6; 6; 7; 8; 8; 8; 9; 10; 10; 12; 12; 13; 14; 16; 18; 19; 20
4 8
5 $\frac{10+10}{2} = 10$

Remedial

Learners often make mistakes when they have to calculate values such as mean, mode or median if they have made a mistake while sorting. Specifically with mode – they may conclude that there is no mode in a particular data set just because they forgot one of the values. Learners may confuse mode, median and mean. Let them come up with some association to remember each one and the difference among them.

Extension

To give learners an opportunity to work with big number ranges, they can look at an example of data from Census 2011 that uses millions and smaller:

Household size (number of members):		1	2	3	4	5
Number of children aged 17 and younger:	0	2 271 462	3 643 128	2 007 730	1 257 275	591 771
	1	24 824	762 482	3 320 704	2,250,268	1 284 646
	2	-	22 733	992 438	4 025 032	2 475 594
	3	-	-	24 439	782247	2 540 494
	4	-	-	-	7 564	394 964
	5	-	-	-	-	2 194
TOTAL (let the learners calculate these)						

In addition to calculating the totals, learners can answer questions like the following: Which number of children per household has the highest frequency? (Answer: 2 children in a household of 4 has the highest frequency.)

Exercise 4

Learner's Book page 354

Guidelines on how to implement this activity

Discuss the range of a data set. Learners have just revised the measures of central tendency, and they will now be introduced to a measure of dispersion. Explain the concept of range. Discuss as a class what the range tells us. Why is this information useful? Ask learners to supply examples of data where the range would be important. Learners can complete this exercise on their own.

Suggested answers

1 Smallest value is 4 and largest value is 20.
2 20 – 4 = 16
3.1 March, 1 400 visitors
3.2 400
3.3 1 400 – 400 = 1 000
3.4 The number of visitors ranged from 400 to 1 400.

Remedial options

Provide learners with additional practice by letting them complete exercises such as the following:

Remedial exercise 1

Group the following data set: 201 kg; 200,2 kg; 200 kg; 200 $\frac{2}{10}$ kg; 99,99 kg; 100,001 kg into the following intervals: [0-100 kg); [100-200 kg), [200-300 kg).

Remedial exercise 2

Find the mode of the above data set. (Learners should notice that 200,2 kg occurs twice and is therefore the mode.)

When learners sort a data set they should get into the habit of lightly crossing through each value that has been counted with pencil, to avoid skipping some values or counting one value more than once.

Extension opportunities

Extension exercise 1

Learners can collect data about the approximate distance each of the learners in their class travel to school each day.

- Determine intervals. For example, 0-5 km; 5-10 km; 10-15 km; 15-20 km; 20 km or more.
- Use a multiple choice questionnaire where each response must fit only into 1 of the possible intervals.
- Collect and record the data.
- Summarise the data set by using a stem-and-leaf display.

Extension exercise 2

Provide learners with the following data set from Census@School 2009:

Type of dwelling	
House or brick structure on a separate stand or yard	4 456 960
Traditional dwelling/hut/structure made of traditional materials	1 288 066
Flat in block of flats	201 324
Town/cluster/semi-detached house (simplex)	254 887
House/flat/room in back yard	434 549
Informal dwelling/shack in back yard	258 818
Informal dwelling/shack NOT in back yard	263 732
Room/flatlet not in back yard but on a shared property	137 041
Caravan or tent	8 279
Private ship/boat	4 094
Workers' hostel (bed/room)	23 312
Other (specify)	100 312
Unspecified	745 508
Total	8 176 881

- Sort the data set in ascending order and group the data into sensible intervals.
- Determine the range of the data set.

Unit 3 Representing data

Learner's Book page 355

Unit focus

- displaying or represent data using bar graphs, double bar graphs, histograms with given intervals and pie charts.

Background information on representing data

Graphs and charts are used to represent data. Learners will draw a variety of graphs by hand/technology to display and interpret data (grouped and ungrouped), including:
- bar graphs and double bar graphs
- histograms with given intervals
- pie charts.

Remind them of these important features of a graph:
- an appropriate title (heading) to describe the data display
- except for the pie chart, there is a horizontal and a vertical axis
- each axis must have a label or title
- a suitable scale for each axis according to the data values.

The concept of a scale is important. To display values from 200 – 1000 on a vertical axis, learners should not use a scale of units or tens, since the axis will be too long making the graph too big. Rather use intervals of a 100 or 200, depending on the values to plot.

Exercise 1

Learner's Book page 356

Guidelines on how to implement this activity

Revise graphs with learners. Discuss the properties of graphs. Learners should be able to tell you that graphs have to have a heading, have vertical and horizontal axes, and have an appropriate scale. Do an example of reading a bar graph, a double bar graph, and an example of drawing a bar graph. Use simple information so that learners can revise the concept of drawing the graph. Learners can complete Exercise 1 on their own.

Suggested answers

1.1 The Mmutlana family did not plant any trees in 2010.

1.2 The bar on the left in 2008 and the bar in 2010. The bars are the highest because both families planted the most number of trees over the four years, 2008 – 2011.

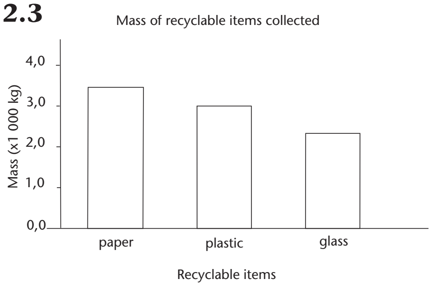
1.3 $10 \times (2 + 2 + 5 + 3) = 10 \times 12 = 120$

1.4 $10 \times (5 + 1 + 0 + 2) = 10 \times 8 = 80$

2.1 Recyclable items: paper, plastic and glass on the horizontal axis and kilograms on the vertical axis.

2.2 In thousands of kilograms

2.3



2.4 Mass of recyclable items collected

2.5 The first (paper) bar. It means that the mass of the paper is greatest of all the recyclable items.
2.6 The third (glass) bar. It means that the glass has the smallest mass of all the recyclable items.

Exercise 2

Learner's Book page 358

Guidelines on how to implement this activity

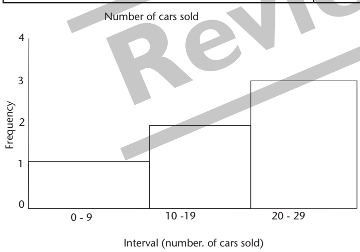
Histograms are used to represent grouped data shown in intervals on the horizontal axis of the graph. Point out the differences between histograms and bar graphs. The bars in a histogram touch each other. Histograms show data categories in consecutive, non-overlapping intervals. Revise how we worked with intervals in unit 2. Show learners by means of a worked example how those intervals work on a histogram. Work through the worked example as a class, and then encourage learners to try the exercise on their own.

Suggested answers

1 discrete data
2 8 falls into [0,10); 15 and 16 fall into [10,20); 20, 23 and 25 fall into [20,30).
3

Number of cars per month		
Interval	Elements per interval	*f*
[0,10)	8	1
[10,20)	15; 16	2
[20,30)	20; 23; 25	3

4



Remedial

Histograms might be difficult for some learners. Make sure they understand the concept of intervals and frequency. If needed they can first draw a table with the intervals and a frequency column from the data set. The horizontal axis of the histogram represents the intervals. The vertical axis represents the frequency.

Extension

Learners can research how many gold medals were won at the 2011 Olympic Games by at least 5 countries, including South Africa. As part of their research, learners should write down for which sport each country won each of their gold medals.

Extension exercise 1

Use the Olympic gold medal data and sort the data set per country according to intervals such as 1 to 5 medals; 6 to 10 medals; 10 to 15 medals and more than 15 medals.

- Draw up a frequency table to show the number of medals per country.
- From the frequency table, convert to percentages and draw a pie chart.
- From the frequency table, draw a histogram of the data.
- Learners should draw at least two conclusions about the data. For example, the country who won the least and the most medals respectively.

Exercise 3

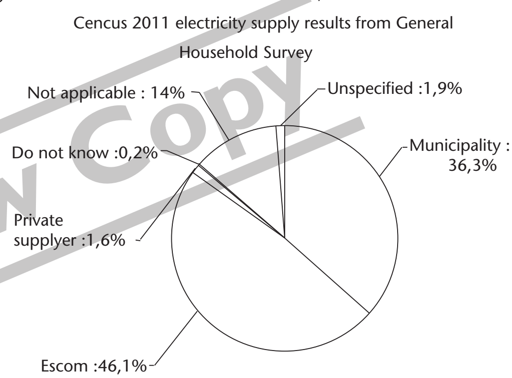
Learner's Book page 359

Guidelines on how to implement this activity

Drawing pie charts to represent data do not have to be accurately drawn with a compass and protractor etc. Learners can use any round object to draw a circle, then divide the circle into halves and quarters and eighths if needed, as a guide to estimate the proportions of the circle that need to be shown to represent the data. What is important is that the values or percentages associated with the data, are shown proportionally on the pie chart.

Drawing, reading and interpreting pie charts is a useful context to revisit equivalence between fractions and percentages. For example, 25% is represented by a $\frac{1}{4}$ sector of the circle or pie chart.

An example of a pie chart drawn with technology (Microsoft Excel):



Work through the examples in the Learner's Book. Ensure learners have the correct tools to construct the pie chart. Allow learners to work in pairs, however each learner must construct their own pie chart in their exercise book.

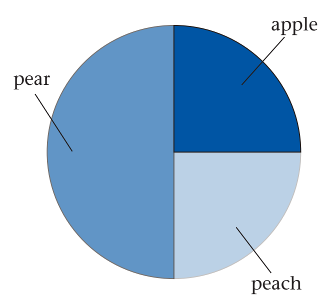
Suggested answers

1 Apple Trees: $\frac{50}{100} \times \frac{100}{1} = 25\%$

Peach Trees: $\frac{50}{100} \times \frac{100}{1} = 25\%$

Pear Trees: $\frac{100}{200} \times \frac{100}{1} = 50\%$

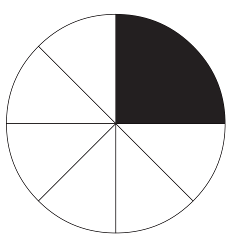
2 Learners own work should look like this



3 Pie chart of orchard trees

4 Learners own work

5 Learners own work should look like this.



Remedial

Learners experiencing problems with percentages and fractions can benefit from exercises to recognize equivalence between fractions and percentages and to find percentages from whole numbers and vice versa.

Remedial exercise

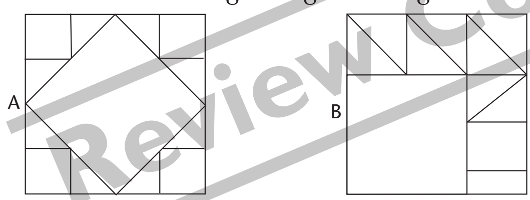
Soccer is the favourite sport of 25% percentage of 300 learners.

- How many learners is that (find the whole number of learners)? (Answer: 25% = $\frac{25}{100}$ × 300 = 25 × 3 = 75 learners).
- Draw a pie chart and colour in the given percentage.
- Of the rest of the learners, 20% prefer tennis, 50% prefer rugby. Represent these also on the same pie chart.
- If 15 learners enjoy swimming most, what percentage is that? Show that also on the same pie chart.

Learners might have difficulty to get the proportions right when representing data on a pie chart. Encourage them to divide the circle into halves and quarters and eighths if needed, as a guide to estimate the proportions of the circle that need to be shown to represent the data.

Remedial exercise

- Look at each of the following shapes A and B. Identify in each shape the number of squares and the number of right-angled triangles.



- In a tally and frequency table, record the number of squares and number of right-angled triangles in each shape.
- Draw a pie chart of the data set for each shape. Show on each chart the number of squares and the number of right-angled triangles.
- Looking at the pie charts, which of the two shapes has the most squares and the most triangles?

Extension opportunities

Learners can complete a data cycle about the type of transport that 20 learners in their class or in their grade use to travel to school each day.

- Design and use a multiple choice questionnaire where each response must fit only into 1 category. For example, 1) bus; 2) taxi; 3) walk; 4) car.
- Collect and record the data.
- Summarise the data set by using a stem-and-leaf display.
- Draw a pie chart to represent the responses. Remember a title. For example, "Transport type of Grade 7s for travelling to school"

Unit 4 Interpreting, analysing and reporting data

Learner's Book page 360

Unit focus

- critically reading and interpreting data represented in words, bar graphs, double bar graphs, pie charts and histograms
- analyse data by answering questions about data categories including data intervals, data sources and contexts, scales used on a graph, central tendencies (mean, mode, median)
- report (summarise) data in short paragraphs that include drawing conclusions about the data, making predictions based on the data, identifying sources of error and bias in the data, choosing appropriate summary statistics for the data (mean, mode, median).

Background info on interpreting, analysing and reporting data

There are three components: interpreting data, analysing data and reporting data.
As part of interpreting data, learners must critically read and interpret data represented in:

- words
- bar graphs
- double bar graphs
- pie chart
- histograms.

As part of analysing data, learners must answer questions related to:

- data categories, including data intervals
- data sources and contexts
- measures of central tendencies (mean, mode and median)
- scales used on graphs.

As part of reporting data, learners must summarize data in short paragraphs that include:

- drawing conclusions about the data
- making predictions based on the data
- identifying sources of error and bias in the data
- choosing appropriate summary statistics for the data (mean, mode, median, range).

Exercise 1

Learner's Book page 362

Guidelines on how to implement this activity

This unit demands quite a lot from learners. However, by using one or two examples as set out in the Learner's Book, learners should be able to grasp the key outcomes of interpreting, analysing and reporting.
If possible, source some examples from newspapers or magazines of data represented in a variety of formats or summarised in certain ways. The main goal is for learners to develop their critical analysis skills. Create a discussion opportunity in class to point out bias and how data can be manipulated to support a viewpoint.

Interpreting
Learners should compare the same data set represented in different ways. For example, in a pie chart or a bar graph or a table. They should discuss what information is shown and what is hidden. Learners should evaluate what form of representation works best for the given data.
Learners should compare graphs on the same topic but where data has been collected from different groups of people, at different times, in different places or in different ways.
Learners should discuss differences between the data with an awareness of bias related to the impact of the data sources and methods of data collection on the interpretation of the data.

Analysing
Learners should compare different ways of summarising the same data sets, developing an awareness of how data reporting can be manipulated and evaluating which summary statistics best represent the data.
Learners should compare graphs of the same data, where the scales of the graphs are different. Here learners should discuss differences with an awareness of how representation of data can be manipulated. They should evaluate which form of representation works best for the given data.

Suggested answers

1.1 No, a house must first be put up for sale before it can be bought, and hence SOLD by the real estate company.
1.2 The only information that is given is the statement that the house(s) were sold by the real estate company.
1.3 Yes, there is information missing. For example, When the houses were put up for sale, at what price were the houses put up for sale?
1.4 No, they are not.
2.1 No context is given.
2.2 No. It doesn’t tell us what is being measured (employment per industrial sector).
2.3 Agriculture
2.4 Yes, 18% is high compared to other countries like USA and Britain OR No, 18% is at least 6% below Shouth Africa's average unemployment.
2.5 Yes, it does, but not enough. Now it is clear why only 2% of the sample worked in mining.
3.1 0 – 100 interval
3.2 The fish caught below the mass of a 100 tons appeared 4 times in the data sample.
3.3 The fish caught with mass of between 200 (including 300) and 300 tons as well as the fish caught with mass of between 300 (including 300) and 400 tons. Both of the bars are equal in height, but shorter than all the other bars.
3.4 The value (number) that occurs most often in a data sample (set).
3.5 0 – 100 interval
3.6 No data context is given.
3.7 No, we do not.
3.8 Yes, more information would make it easier to interpret and provide a more accurate analysis.

Remedial

Learners might find these topics too abstract. There could be some overlap between interpretation and analysis, but the main goal is for the learners to develop their critical analysis skills.

Exercise 2

Learner's Book page 364

Guidelines on how to implement this activity

Encourage learners to write reports in short paragraphs on the data. Discuss as a class what should be included in these reports. Reporting on the measures of central tendencies and dispersion are useful, as are any clear predictions, and any possible bias. Discuss why reporting is useful and how the end user of the report must be kept in mind in order to deliver a relevant result. Encourage learners to do this exercise on their own, but have each learner present their findings to a group.

Suggested answers

1.1 The second graph, due to its smaller scale we can predict with greater accuracy what the number of Aids orphans will be by 2015.

1.2 Due to the first graph's larger scale, the number of Aids orphans almost seems constant over the period 2006 – 2012. If we had to predict into the future based on this graph, our prediction would be inaccurate.

1.3.1 Those people who did not turn to subsistence farming or any other form of work, would then contribute to the percentage of unemployment.

1.3.2 No conclusion.

1.3.3 This explains the high percentage of the Agriculture industry.

1.4 The unemployment rate would most likely rise due the negative effect the drought will have on the farmers (Agriculture).

1.5 Yes, due to the prospect of more and various work opportunities in cities, the unemployed will likely move into cities.

1.6 Due to their young age, their career prospects are good.

2.1 The sum of all the houses built over the 12 month period, divide by the number of months (12).

2.2 No, there were months where fewer than 15 houses were built.

2.3 Using the average as a guarantee of the number of houses that will be built each month is misleading.

2.4

2.4.1

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
No of houses	3	15	16	9	20	14	20	19	20	19	21	10

2.4.2 Total number of houses built = 186

2.4.3 $12 \times 15 = 180$ houses

2.5 January, April, June and December. A reason for the lower number of houses built over theses months could be due to public holidays and/or vacations. The lower numbers does affect the average by bringing the average value down.

2.6 Median: $\frac{14 + 20}{2} = 17$; mode: 20

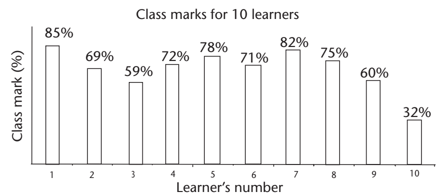
2.7 Due to the large range, the median would be a better summary of central tendency.

2.8 No, the manager should not have advertised the way he did. A more truthful advertisement would be, "I promise that my teams can build an average of 15 houses per month!"

Remedial options

Use the following exercise as revision and consolidation. At the same time it will reveal problem areas as learners discover for themselves how bias, data manipulation and misinterpretation can occur.

Give the learners one initial data set that has been represented, for example, in a bar graph:



- Let learners work out the mean (average), mode and median for the given data set.
- Learners should discuss and compare the answers for these three measures of central tendency. Which of the three summary statistics best represent the data?
- How does learner no. 10's percentage affect the mean value?
- Let learners group the data into intervals: 0-25%; 25-50%; 50-75%; 75-100%. They must draw up a frequency table from where they can draw a histogram.
- Let them discuss: which graph - the bar graph or the histogram - best represent the data set?
- Let them write a short paragraph to explain their conclusion.

Extension opportunities

- Learners can research and collect data on at least two endangered animal species that also occur in South Africa. For example, the African Wild Dog and the Black Rhino. They can refer to a website such as: http://worldwildlife.org/
- Let learners list the threats (also listed on this website), such as illegal wildlife trade.
- Learners can also read more at: http://www.africanconservancy.org/about/documents/Facts.pdf to get facts like the following:

Plight of Rhinos (Source: International Rhino Foundation)

- Of the dozens of species of rhino that once roamed the earth, only 5 now exist.
- Where there were once over 100,000 black rhinos on the plains of Africa, there are now only 2,707 on the entire continent.
- The staggering decimation of the rhino population is due to poaching, to satisfy the demand for the horn for use in Eastern traditional medicines and as dagger handles.
- Prices up to US$40,000 a kilo have been recorded for the much prized rhino horn - more than 5 times the price of gold.

The African Elephant (Source: CITES)

- 5 — 10 million African elephants existed in 1930. Less than 1% of that number (approximately 600,000) remained when they were added to the international list of the most endangered species in 1989.
- Demand for ivory combined with loss of habitat from human settlement led to these huge declines in population.

African Wild Dog (Source: American Museum of Natural History)

- Listed as one of the worlds most endangered canids, and the most endangered predator in Africa, there are now only between 4,000-5,000 African wild dogs in the wild.
- A century ago, African wild dog packs numbering a hundred or more animals could be seen roaming the Serengeti plains. Today, pack size averages about 10, and the total population on the Serengeti is probably less than 60 dogs.
- Due to their large home ranges, African wild dogs are particularly vulnerable to habitat destruction.
- They are widely regarded as pests, and poisoned, shot, trapped and snared in many areas.
- Their most serious threat, though, is introduced diseases. Burgeoning human populations have brought the African wild dogs into frequent contact with domestic dogs, many of which carry canine distemper and rabies.
- Learners can create an Awareness poster by displaying some of the data on animal numbers they collected in a graph such as a bar graph or pie chart.
- They can also represent at least three threats in the way they think will be best suited.

Unit 5 Probability

Learner's Book page 366

Unit focus

- probability experiments and outcomes,
- trials and events, and
- relative frequency.

Background info on probability

In the Intermediate Phase learners did experiments with coins, dice and spinners. In this grade experiments can be done with other objects like marbles in a bag or choosing different cards from a deck of playing cards.

Learners are required to perform simple experiments where the possible outcomes are equally likely.

Learners are expected to:

- List all the possible outcomes.
- Determine the probability of each possible outcome, using the definition of probability.
- Predict (with reasons) the relative frequency of the possible outcomes for a series of trials based on probability.
- Compare relative frequency with probability and explain possible differences.

Exercise 1

Learner's Book page 368

Guidelines on how to implement this activity

Probability is a topic which is very relevant with many practical examples, like the weather forecast and competitions that involve a chance of winning a prize. To introduce the topic, learners can find examples of a weather forecast in some newspapers or examples of competitions on packaging material or in magazines.

Let them bring these to class to put up as illustrations of probability. Discuss possible outcomes and how we record these as fractions. Work through the worked example and encourage learners to complete the exercise on their own.

Suggested answers

1 Learners work in pairs and conduct their own experiments. Check that learners set out their frequency tables correctly.

2.1 trials

2.2

Frequency of actual outcomes					Total:
Outcome	A	B	C	D	
f	3	2	1	4	10

Remedial

Probability can be expressed in different ways:

- as a percentage: there is a 60% chance of rain.
- as a fraction: the team has a 50/50 chance of winning the game.
- in words: she has a very good chance of being chosen.

This topic provides an opportunity to establish whether learners are able to calculate percentages from fractions and recognize equivalence between percentages and fractions. If there is a problem with either, provide these learners with exercises to practise that.

Extension

Learners can discuss the following and see if they agree or disagree: the possible different sequences in a 52-card pack is a figure 68 numerals long! Why could that be a possibility? Let them use 5 or 10 different playing cards and build and record as many sequences as possible with the five or ten cards.

Exercise 2

Learner's Book page 369

Guidelines on how to implement this activity

When discussing relative frequency ensure learners know the following: The relative frequency is the observed number of actual successful outcomes for a finite (a certain discrete number) sample of trials. Show learners an example: to calculate the relative frequency of heads as an outcome, define a head as a successful outcome. If a coin is tossed 50 times and there are 27 outcomes of heads and 23 outcomes of tails, the relative frequency of heads = $\frac{27}{50}$ = 54%. Similarly, to calculate the relative frequency of tails as an outcome, define tails as a successful outcome. The relative frequency of tails = $\frac{23}{50}$. The probability of each outcome is $\frac{1}{2}$ (1 of 2 equally likely outcomes). That means the probability of the coin landing on heads is 50% . The difference between relative frequency and probability is seen in the two answers: $\frac{27}{50}$ compared to $\frac{1}{2}$, or 54% compared to 50%.

- The difference between the two answers is due to the small sample size (only 50 tosses).

- The more trails in the experiment (for example 500 tosses), the closer the relative frequency gets to probability.
- To illustrate this, let learners work in groups and depending on how many groups, let each group toss the coin a certain number of times. For example, group 1: 10 times; group 2: 50 times; group 3: 100 times; group 4: 200 times. Let the groups record the result of each outcome (heads/tails).
- Once the experiment is completed, learners should calculate the relative frequency of their group's outcomes as heads.
- Make the comparison.
- As resources, bring marbles, coins and if possible playing cards to school for experiments that the learners can perform.

Suggested answers

1.1 This answer is based on the experiment learners performed in Exercise 1.

1.2

	A	B	C	D
Relative frequency	$\frac{3}{10}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{2}{5}$

2.1 two

2.2 two

2.3 five

2.4 $\frac{2}{5}$

2.5 $\frac{0}{5}$

2.6 blue, red, green, orange or yellow ball

2.7 Yes

2.8 Each coloured ball has the same probability: $\frac{1}{5} \times \frac{100}{1} = 20\%$

2.9 After each draw, the possible outcomes would have one coloured ball less. The 5^{th} child would have no choice but to choose the only (one) coloured ball left.

Remedial

Some learners may confuse probability with relative frequency. The above mentioned illustration should help to clarify the difference.
Provide more opportunities for learners to calculate probability and relative frequency, such as the following two exercises:

Remedial exercise 1

Learners can take turns to draw a marble from a bag with a few differently coloured marbles. Let them perform at least 20 trials. They should record the outcome of each trial and do their calculations to find the probability and relative frequency of a chosen outcome (for example drawing a blue marble from the bag).

Remedial exercise 2

Learners can take turns to draw a playing card from a deck of ten different cards which face down. Let them perform at least 20 trials. They should record the outcome of each trial and do their calculations to find the probability and relative frequency of a chosen outcome (for example drawing a Queen of Hearts).

Extension opportunities

Extension exercise 1

Learners can design a competition where a person has a 0,1 chance of winning (which means 1 chance in 10 or $\frac{1}{10}$). How many contestants are needed?

Extension exercise 2

Learners can now design a competition where a person has a 0,001% chance of winning something. Remind them that to find the number of contestants (x), they will have to solve for x in: $\frac{1}{x} \times 100 = 0{,}001\%$.

- If for each entry costs R2,00 per sms, how much money is collected if R1,00 of every R2,00 is profit?
- If the competition pays out R50 000 prize money, how much is the total profit?

Consolidation

Learner's book page 372

Before doing this consolidation exercise, encourage learners to review the work covered in this chapter. Advise learners to use the summary and to revise their work. This exercise can be used as an informal assessment task for you to track how learners are coping with the chapter and the concepts covered.

Suggested answers

1 Learners own work. Make sure that they follow the data cycle correctly. (17)

2.1 She won the prize she wanted on her first attempt. If she used more chances she may not have ended up with the week holiday in Thailand. (1)

2.2 (4)

Frequency table of contestants who entered the lucky draw		
Prize no.:	Names of contestants competing for each prize	*f*
1	Tim, Sue, Bongani, Richard, Josua, Joseph	6
3	Bongani, Fouzia, Portia	3
5	Amy, Sue, Richard, Tsego, Josua	5

A lucky draw determined the winner of all six prizes.

2.3 Prize 1 (6 people) (2)

2.4 $\frac{4}{28} = \frac{1}{7}$ (3)

2.5 True (3)

[30]

Chapter 20 Programme of Assessment

This Chapter provides all the resources you need to ensure your learners meet the requirements for promotion.

- It includes two options for each of the required formal programme of assessment tasks
- Exemplar exams for learners to use to practise for their exams. These exemplars are in the Learner's book with the memoranda supplied in this section of the Teacher's Guide.
- Control tests for you to use as part of the POA, along with the memoranda.
- A June and a December exam paper, with memoranda.

The following table lays out the programme for you. The shaded cells are only in the teacher's guide to ensure the tests and exams are unseen by the learners.

Term	Task		Learner's Book page	Teacher's Guide page
1	Assignment	Option 1 Financial maths	375	245
		Option 2 Constructions	377	246
	Control Test 1			247
2	Investigation	Option 1 Functions and relationships	379	250
		Option 2 The relationship between volume and surface area	381	251
	Control Test 2			253
	Exemplar June exam	Exemplar paper for revision purposes	396	255
	June Exam			258
3	Assignment	Option 1 Patterns	383	265
		Option 2 Algebra 1	385	267
	Project	Option 1 Functions, relationships and graphs	386	268
		Option 2 Transformations	388	269
	Control Test 3			270
4	Assignment	Option 1 Integers	390	272
		Option 2 Algebra 2	391	273
	Investigation	Option 1 Data	392	274
		Option 2 Probability	394	276
	Exemplar December exam	Exemplar paper for revision purposes	399	277
	December exam			279

We suggest that in order to prepare learners adequately for formal assessment you use the allocated Revision time prescribed in the CAPS for revising work.

This Series advises that you use the Consolidation exercises and Summaries at the end of each chapter to revise. The Consolidation exercises have mark allocations to enable informal assessment of how learners are managing the specific content area

Assignment 1 Option 1: Memo

1.1 Decrease
1.2 Increase
1.3. Total debits = R335,40
1.4 Total credits = R303,80
1.5 Closing balance = balance brought forward – total debits + total credits
R320,80 – R335,40 + R303,80 = R289,20
1.6 R320,80 – R289,50 = R31,60
1.7 The opening balance is more than the closing balance, therefore savings decreased.
1.8 The closing balance for March will be the balance brought forward on the April statement, that is R289,20. (8)
2 Jabu's monthly repayments will be: R860,00 ÷ 12 = R71,67 (rounded off) (3)
3 Discount: R139,80 × 10% = R13,98
Cash price: R139,80 – R13,98 = R125, 82
Discount: R89,99 × 10% = R9,00
Cash price: R89,99 – R9,00 = R80,99
Discount: R47,65 × 10% = R4,77
Cash price: R47,65 – R 4,77 = R42,88 (6)
4.1 $(\frac{1}{2} \times R45{,}99 \times 2) + (\frac{1}{2} \times R130{,}99) + (165{,}00 - \frac{R165{,}00}{4}) +$
$(R245{,}00 - (\frac{75}{100} \times R245{,}00)) = R296{,}49$ (rounded off) (4)
4.2 Price: $\frac{1}{2} \times R130{,}99 = R65{,}50$
Monthly cost (excluding interest): $\frac{R65{,}50}{12} = R5{,}46$
Interest per month: R5,46 × 15% = R0,82
Therefore interest per year is R0,82 × 12 = R9,83
Or, interest per year is R5,46 × 12 × 15% = R9,83
Total cost of jeans: R65,50 + R9,83 = R75,33.
(This means that the jeans cost Amanda R9,83 more on credit than if she had paid cash.) (4)

Total marks [25]

CHAPTER 20

Assignment 1 Option 2: Memo

1.1 Learners construct triangle DEF with $D\hat{E}F = 40°$, $E\hat{F}D = 90°$, and side EF = 45mm. Ensure that learners measure accurately and that they use a ruler and protractor. DE should measure 60 mm. (3)

1.2 Learners construct triangle ABC with $B\hat{A}C = 45°$ and sides AB = 3 cm and AC = 7 cm. This is a scalene triangle. $A\hat{B}C$ should measure 110°. (3)

1.3 Learners construct triangle XYZ with $X\hat{Y}Z = 100°$ and sides XY = 4,5 cm and YZ = 6 cm. XZ should measure 8 cm. (3)

2.1 Learners construct rectangle ABCD with horizontal sides AB and CD = 60 mm and vertical sides AD and BC = 45 mm, using a set square or a protractor and a ruler. Measurements of diagonals AC and BD should be 75 mm. (4)

2.2 Learners construct square PQRS with sides 47 mm, using a ruler, and a set square or a protractor. The diagonals PR and QS should be 66 mm. (4)

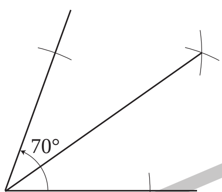
2.3 Learners construct a regular pentagon EDGHI with sides 3 cm and interior angles 108° using a ruler and a protractor. (5)

3.1 Equilateral triangles have three equal sides. (1)

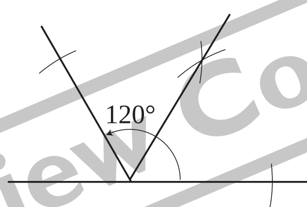
3.2 Isosceles triangles have two equal sides. (1)

3.3 Scalene triangles have no equal sides. (1)

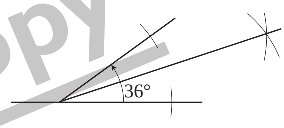
4.1



4.2



4.3



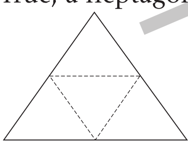
(2)
(2)
(2)

5.1 True, a square is a polygon. (1)

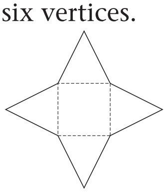
5.2 False, a circle is not a polygon as it has no straight sides. (1)

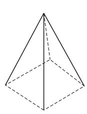
5.3 True, a heptagon is a polygon. (1)

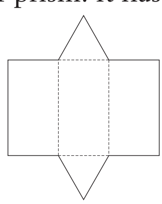
6.1 (3)

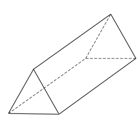


6.2 The net on the left is of a square-based pyramid. It has five faces, five vertices and eight edges. (3)
The net on the right is of a triangular prism. It has five faces, nine edges and six vertices.









7 A triangle and a rectangle are needed to make a triangular prism. (2)

8 Is the net of an octahedron with six rectangular faces and two regular hexagon faces. It has 18 edges and 12 vertices. It could also be named as a hexagonal prism. (3)

Total marks [45]

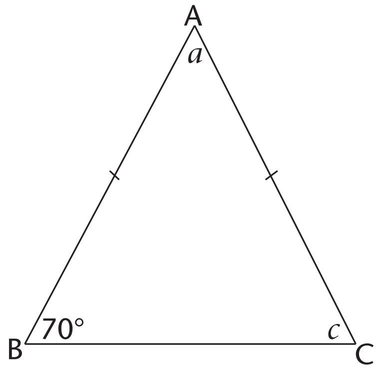
Term 1 Test 1

1.1 Calculate in columns
1.1.1 12 500 – 6 799 (2)
1.1.2 3 450 × 52 (2)
1.2 Determine the HCF of 18; 48 and 72. (1)
1.3 Determine the LCM of 9; 13 and 15. (1)
1.4 Jack, Sipho and Constance spend the day painting a fence for Jack's dad. Jack's dad agrees to pay them R270 for the days' work. If Jack painted 2m of fence, Constance 3m of fence and Sipho 4m of fence, decide on the fairest means of splitting the R270. (2)
1.5 Xolani takes a loan from a bank of R10 000. She agrees to pay it back over 3 years, at 9% interest per year. Calculate the total amount Xolani will have paid back to the bank after 3 years. (2)

2.1 Write the following in exponential form.
2.1.1 3 × 3 × 3 × 3 × 3 (1)
2.1.2 56 × 56 (1)
2.2 Write the following in expanded notation.
2.2.1 5^{10} (1)
2.2.2 9^2 (1)
2.3 Calculate the following:
2.3.1 $5^2 + 3^3 - 2^4$ (2)
2.3.2 $(4 \times 10^3) + (3 \times 10^2) - (2 \times 10^2)$ (2)
2.3.3 $(2^3)^2$ (2)

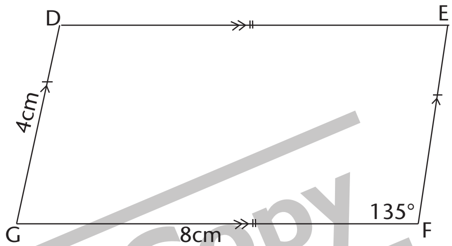
3.1 Construct angle <DEF = 65° (1)
3.2 Construct line AB and its perpendicular bisector XY. (3)
3.3 Construct triangle QSR with QR = 40 mm; SR = 24 mm and <QSR = 70°. (3)
3.4 Construct line AB parallel to line CD. Use a ruler and a compass. (3)

4.1 State whether the following are true or false. If false please provide the correct statement.
4.1.1 A parallelogram has opposite sides and angles equal. (1)
4.1.2 A kite has two pairs of parallel sides. (1)
4.1.3 An isosceles triangle has two sides equal. (1)
4.1.4 The interior angles of a rhombus add up to 270°. (1)
4.1.5 A reflex angle lies between 0° and 270°. (1)

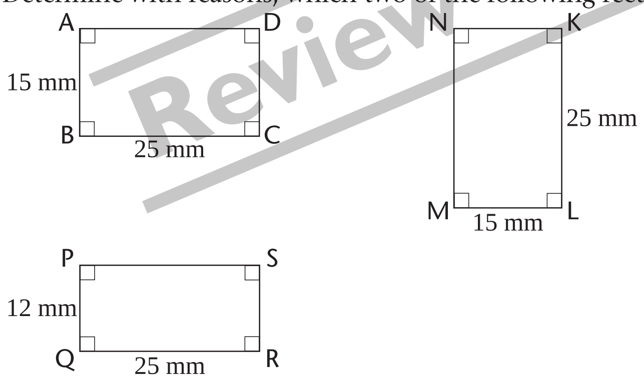


4.2.1 Determine the values of *a* and *c* in triangle ABC. (2)

4.2.2 Determine all the missing sides and angles in the following parallelogram. (2)



4.2.3 Determine with reasons, which two of the following rectangles are congruent. (1)



Total marks [40]

Term 1 Test 1: Memo

1.1.1	12 500 – 6 799 = 5 701	(2)
1.1.2	3 450 × 52 = 179 400	(2)
1.2	The HCF of 18; 48 and 72 is 6.	(1)
1.3	The LCM of 9; 13 and 15 is 585.	(1)
1.4	Jack : Constance : Sipho = 2 : 3 : 4 So we should divide R270 in the ratio 2 : 3 : 4 Jack's share: $\frac{2}{9} \times 270$ = R60 Constance's share: $\frac{3}{9} \times 270$ = R90 Sipho's share: $\frac{4}{9} \times 270$ = R120	(2)
1.5	A = 10 000 (1 + 3 × 0,09) = R12 700	(2)
2.1.1	$3 \times 3 \times 3 \times 3 \times 3 = 3^5$	(1)
2.1.2	$56 \times 56 = 56^2$	(1)
2.2.1	$5^{10} = 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$	(1)
2.2.2	$9^2 = 9 \times 9$	(1)
2.3.1	$5^2 + 3^3 - 2^4 = 25 + 27 - 16 = 36$	(2)
2.3.2	$(4 \times 10^3) + (3 \times 10^2) - (2 \times 10^2) = 4\,000 + 300 - 200 = 4\,100$	(2)
2.3.3	$(2^3)^2 = (8)^2 = 64$	(2)
3.1	Accurate construction of angle $D\hat{E}F = 65°$ using a protractor.	(1)
3.2	Accurate construction of line AB and its perpendicular bisector XY.	(3)
3,3	Accurate construction of triangle QSR with QR = 40 mm; SR = 24 mm and $Q\hat{S}R = 70°$.	(3)
3.4	Accurate construction of line AB parallel to line CD using a ruler and a compass.	(3)
4.1.1	True	(1)
4.1.2	False, a kite has no parallel sides.	(1)
4.1.3	True	(1)
4.1.4	False, the interior angles of a rhombus add up to 360°.	(1)
4.1.5	False, a reflex angle lies between 180° and 270°.	(1)
4.2.1	$c = 70°$ (equal angles in an isosceles triangle) $a = 180° - 70° - 70° = 40°$ (sum of angles in a triangle)	(2)
4.2.2	$\hat{D} = 135°$ (opposite angles in a parallelogram) $\hat{E} = \hat{G} = 45°$ (sum angles in a quadrilateral is 360°) DF = 8 cm and DG = 4 cm (opposite sides in a parallelogram)	(2)
4.2.3	ABCD is congruent to MLKN as they have all their sides and angles equal.	(1)

Total marks [40]

Functions and relationships

Overview
In this investigation, learners work in pairs as they investigate a growth pattern set in the context of the spread of alien vegetation.
Resources
No special resources are required.
Assessment
Use the memorandum below to assess this investigation.
Answers
The area covered by alien vegetation, measured at the end of each year
1

The area covered by alien vegetation, measured at the end of each year									
At the end of	2001	2002	2003	2004	2005	2006	2007	2008	2009
Area (ha)	2	4	8	16	32	64	128	256	512

(7)

2 The area is increasing more and more quickly. (2)

3 No. Learners give their own reasons as to why it is impossible that the area will continue doubling forever. The forest has a limited size; even if the vegetation spreads beyond the forest, there will be geographical boundaries like mountains, rivers, the sea, and so on. Humans will also intervene at some point to combat the spread of the alien vegetation. (2)

4 $8 = 2 \times 2 \times 2$
$16 = 2 \times 2 \times 2 \times 2$
$32 = 2 \times 2 \times 2 \times 2 \times 2$
$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$
$128 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$
$256 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$
$512 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$ (7)

5 $y = x \times 10\ 000$ (2)

6 $\times\ 10\ 000$ (2)

7 $x = y \div 10\ 000$ (2)

8 $\div\ 10\ 000$ (2)

9.1 25 000 m^2

9.2 3 100 m^2 (2)

10.1 3,65 ha

10.2 0,82 ha (2)

Total marks [30]

Investigation 1 Option 2: Memo

Volume and surface area

Rubric

Assessment criteria	1	2	3	4	5	6	7	8	9	10
1 Correct formula for surface area of a rectangle										
2 Correct formula for volume of a rectangular prism										
3 Build model according to instruction set										
4 Explain volume in relation to model										
5 Correct use of volume formula to solve for height										

You can use the following suggested answers to guide your assessment.

1.1 Surface area = length × breadth

1.2 Volume = length × breadth × height

2.2 The volume is the area of the base rectangles multiplied by the height of the five rectangles pushed together. This is almost equivalent to the area of one of the rectangles as it is a very small height.

2.3 The volume has increased – it is now the area of the base rectangle multiplied by the height of the structure, including the pillars, which is much greater than the height of the squashed structure.

3.1 Volume of prism A = 30 × 20 × 10
= 6 000 cm^3
Surface area of prism A = 2 × 30 × 20 + 2 × 30 × 10 + 2 × 20 × 10
= 2 200 cm^2

3.2 Volume of prism B = 60 × 20 × 20
= 24 000 cm^3
Surface area of prism B = 2 × 60 × 20 + 2 × 60 × 20 + 2 × 20 × 20
= 5 600 cm^2

3.3 Volume of prism C = 15 × 5 × 20
=1 500 cm^3
Surface area of prism C = 2 × 15 × 5 + 2 × 15 × 20 + 2 × 5 × 20
= 950 cm^2

3.4 Volume of prism D = 90 × 30 × 20
= 54 000 cm^3
Surface area of prism D = 2 × 90 × 30 + 2 × 90 × 20 + 2 × 30 × 20
= 10 200 cm^2

3.5

Prism	A	B	C	D
Area of base	300 cm^2	1 200 cm^2	75 cm^2	2 700 cm^2
Surface area	2 200 cm^2	5 600 cm^2	950 cm^2	10 200 cm^2
Volume	6 000 cm^3	24 000 cm^3	1 500 cm^3	54 000 cm^3

3.6 All of these prisms have the same height, so the ratio between the area of their bases and their volumes is the same. If the area of the base is increased, the volume increases in the same ratio (e.g. from prism A to B, the area of the base is four times bigger and the volume is four times bigger). But there is no uniform relationship between the total surface area of the prisms and their volumes.

Rating Code	Description of competence	Percentage
7	Outstanding achievement	80 - 100
6	Meritorious achievement	70 - 79
5	Substantial achievement	60 - 69
4	Adequate achievement	50 - 59
3	Moderate achievement	40 – 49
2	Elementary achievement	30 – 39
1	Not achieved	0 - 29

Total marks [30]

Term 2 Test 2

1.1 Copy and complete:

1.1.1 $\frac{2}{3} = \frac{\square}{6} = \frac{6}{\square} = \frac{\square}{12} = \frac{\square}{66}$ (2)

1.1.2 $\frac{4}{5} = \frac{\square}{15} = \frac{20}{\square} = \frac{\square}{35} = \frac{\square}{125}$ (2)

1.2 Arrange the following fractions in ascending order:

1.2.1 $\frac{2}{7}$; $\frac{3}{5}$; $\frac{1}{2}$; $1\frac{1}{9}$ (2)

1.2.2 $\frac{6}{7}$; $\frac{12}{16}$; $\frac{2}{3}$; $\frac{9}{35}$ (2)

1.3 Calculate:

1.3.1 $1\frac{1}{3} + 2\frac{2}{5}$ (2)

1.3.2 $4\frac{2}{3} - 2\frac{3}{5}$ (3)

1.3.3 $\frac{3}{7} \times \frac{1}{3} \times \frac{2}{6}$ (2)

2.1 Complete the following number sequences.

2.1.1 1, 32; 1, 27; 1,22; _____; _____; _____; _____; _____ (2)

2.1.2 0,001; 0,007; _____; _____; 0,025; ______; ______; _____ (2)

2.2 Calculate (without a calculator)

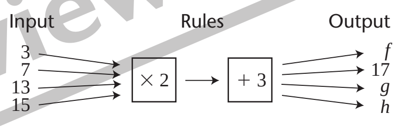
2.2.1 565,78 + 9,765 + 19, 567 (3)

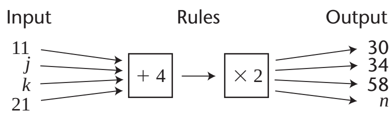
2.2.2 45 – 0,876 (2)

2.2.3 7,65 × 0,56 (3)

2.2.4 569,84 ÷ 34 (3)

3.1 Copy and complete the flow diagrams below and calculate the values of the placeholders (letters). (6)





3.2 Copy and complete the tables below and then draw flow diagrams for the tables.

3.2.1 (3)

Input	2	4	6	8
Output	4	8		

3.2.2 (4)

Input	1	2	3	4	5	6	7	8	9
Output	5	7	9						

Total marks [43]

Term 2 Test 2 Memo

1.1.1 $\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12} = \frac{44}{66}$ (2)

1.1.2 $\frac{4}{5} = \frac{12}{15} = \frac{20}{25} = \frac{28}{35} = \frac{100}{125}$ (2)

1.2.1 $\frac{2}{7}$; $\frac{1}{2}$; $\frac{3}{5}$; $1\frac{1}{9}$ (2)

1.2.2 $\frac{2}{3}$; $\frac{12}{16}$; $\frac{9}{35}$; $\frac{6}{7}$ (2)

1.3.1 $1\frac{1}{3} + 2\frac{2}{5} = \frac{4}{3} + \frac{12}{5}$

$= \frac{20 + 36}{15}$

$= \frac{56}{15}$ 56/15

$=3\frac{11}{15}$ (2)

1.3.2 $4\frac{2}{3} - 2\frac{3}{5} = \frac{14}{3} - \frac{13}{5}$

$= \frac{70 - 39}{15}$

$= \frac{31}{15}$

$=2\frac{1}{15}$ (3)

1.3.3 $\frac{3}{7} \times \frac{1}{3} \times \frac{2}{6} = \frac{1}{21}$ (2)

2.1.1 1, 32; 1, 27; 1,22; 1,17; 1,12; 1,07; 1,02; 0,97 (2)

2.1.2 0,001; 0,007; 0,013; 0,019; 0,025; 0,031; 0,037; 0,043 (2)

2.2.1 565,78 + 9,765 + 19,567 = 595,112 (3)

2.2.2 45 – 0,876 = 44,124 (2)

2.2.3 7,65 × 0,56 = 4,284 (3)

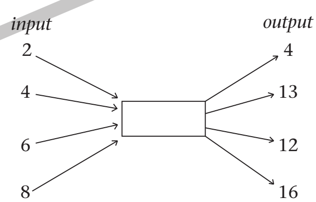
2.2.4 569,84 ÷ 34 = 16,76 (3)

3.1 $f = 9$; $g = 29$; $h = 33$ (3)

$j = 13$; $k = 25$; $n = 50$ (3)

3.2.1 (2)

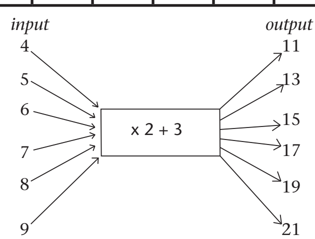
Input	2	4	6	8
Output	4	8	12	16



(1)

3.2.2 (3)

Input	1	2	3	4	5	6	7	8	9
Output	5	7	9	11	13	15	17	19	21



(1)

Total marks [43]

June exemplar: Memo

1.1	45 786 + 23 887 = 69 673	
	69 673 – 23 887 = 45 786 or 69 673 – 45 786 = 23 887	(3)
1.2	397 × 21 = 8 337	
	8 337 ÷ 21 = 397 or 8 337 ÷ 397 = 21	(3)
2.1	The associative property	(1)
2.2	The distributive property	(1)
3.1	The factors of 154 are 1; 2; 7; 11; 14; 22; 77 and 154	(2)
3.2	2; 7 and 11 are prime factors	(1)
4.	150 minutes = 2,5 hours	
	Speed = distance/time	
	$= \frac{180}{2,5}$	
	= 72 km/h	(2)
5.1	$8^3 = 8 \times 8 \times 8$	(1)
5.2	$4^5 = 4 \times 4 \times 4 \times 4 \times 4$	(1)
5.3	$a^{12} = a \times a \times a \times a \times a \times a \times a \times a \times a \times a \times a \times a$	(1)
6	10^1; 7^2; 3^4; 6^3; 4^4	(2)
7.1	$\sqrt{25} + 3^2 = 5 + 9 = 14$	(2)
7.2	$7^2 + 3^1 = 49 + 3 = 52$	(2)
7.3	$3^3 + \sqrt{64} = 27 + 8 = 35$	(2)
7.4	$(5 \times 10^3) - (2 \times 10^2) + 5^3 = 5\,000 - 200 + 125 = 4\,925$	(2)
8	A: iv acute	
	B: v right	
	C: ii reflex	
	D: i obtuse	
	E: iii straight	(5)
9	Accurate construction of the following angles using a protractor:	
9.1	$M\hat{N}O$ of 57°	
9.2	$D\hat{E}F$ of 114°	
9.3	$Q\hat{R}S$ of 90°	(3)
10	Accurate construction of the following 2D shapes:	
10.1	A rectangle with length 4cm and breadth 2,5 cm.	(2)
10.2	A circle with radius of 3,8 cm	(2)
10.3	An equilateral triangle with each side measuring 3,5 cm.	(3)
11	Accurate construction of line RT perpendicular to QS.	(2)
12	ΔABC is a **right-angled** triangle.	
	ΔDEF is an **equilateral** triangle.	
	ΔJKL is an **isosceles** triangle.	
	ΔPQR is a **scalene** triangle.	(5)
13	In ABCD: AB = AD = AC = 14 cm	
	$\hat{A} = 45°$ and $\hat{D} = 135°$	(2)
	In KLMN: MN = 12 cm and KN = 5 cm	
	$\hat{L} = \hat{M} = \hat{N} = 90°$	(2)
	In PQRS: PS = 5 cm and RS = 9cm	
	$\hat{S} = \hat{Q} = 135°$	(2)

14.1 $2\frac{3}{4} + 5\frac{1}{2} = \frac{11}{4} + \frac{11}{2}$

$= \frac{11}{4} + \frac{22}{4}$

$= \frac{33}{4} = 8\frac{1}{4}$ (2)

14.2 $1\frac{2}{3} + 3\frac{1}{7} = \frac{5}{3} + \frac{22}{7}$

$= \frac{35}{21} + \frac{66}{21}$

$= \frac{101}{21}$

$= 4\frac{17}{21}$ (3)

14.3 $4\frac{1}{4} - 2\frac{4}{9} = \frac{17}{4} - \frac{22}{9}$

$= \frac{153}{36} - \frac{88}{36}$

$= \frac{65}{36}$

$= 1\frac{29}{36}$ (3)

14.4 $2\frac{5}{6} \times 4\frac{3}{8} = \frac{17}{5} \times \frac{35}{8}$

$= \frac{595}{48}$ (3)

15 (4)

Common fraction	Decimal fraction	Percentage
	0,375	37,5%
	0,3	30%
	0,75	75%
	0,6	60%
	0,16	16%

16.1 $1{,}897 \times 0{,}34 = 0{,}64498$ (3)

16.2 $2{,}34 \times 12{,}56 = 29{,}3904$ (3)

17.1 output value = 18 (1)

17.2 input value = 72 (2)

18 (2)

Input	5	9	10			32
Output	12	20		26	42	

19.1 Area $= 9 \times 3 + (\frac{1}{2} \times 3 \times 3)$

$= 31{,}5\text{ cm}^2$ (2)

Perimeter $= 9 + 4 + 12 + 3 = 28$ cm (1)

19.2 Area $= 3 \times 3 + (\frac{1}{2} \times 3 \times 2{,}6)$

$= 12{,}9\text{ cm}^2$ (2)

Perimeter $5 \times 3 = 15$ cm (1)

20 Area of triangle PQR $= \frac{1}{2} \times 5 \times 4{,}3$

$= 10{,}75\text{ cm}^2$

Area of rectangle $= 2{,}7 \times 1{,}8$

$= 4{,}86\text{ cm}^2$

Shaded area $= 10{,}75\text{ cm}^2 - 4{,}86\text{ cm}^2$

$= 5{,}89\text{ cm}^2$ (4)

21.1.1 Volume = $4 \times 3 \times 2$
$= 24 \text{ cm}^3$ (1)
Surface area = $2 \times 4 \times 3 + 2 \times 2 \times 3 + 2 \times 2 \times 4$
$= 52 \text{ cm}^2$ (2)

21.1.2 Volume = $8 \times 8 \times 74$
$= 4\ 736 \text{ cm}^3$ (1)
Surface area = $2 \times 8 \times 8 + 2 \times 74 \times 8 + 2 \times 74 \times 8$
$= 2\ 496 \text{ cm}^2$ (2)

22.1 Total volume = $50 \times 30 \times 25$
$= 37\ 500 \text{ cm}^3$ (2)

22.2 Amount of water = $50 \times 30 \times 20$
$= 30\ 000 \text{ cm}^3$
= 30 litres (2)

Total marks [100]

June exam paper

Time: 2 hours

Total: 100 marks

1.1 Copy and complete the table below. (5)

a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*
2	3	5							29

1.2 What are the numbers in Question 1.1 called? (1)

2 Calculate and check by doing the inverse operation:

2.1 967 + 935 = ____ Inverse: ____ = ____

2.2 324 × 628 = ____ ____ = ____ (4)

3 Calculate the following by applying the distributive property:

3.1 32(15 + 45)

3.2 53(98 – 67) (2)

4 Calculate and complete by filling in the table: (5)

Cube or square root	Value	Expanded notation	Exponential form
$\sqrt{25}$	5		
		9 x 9	
			13^2
$\sqrt[3]{8}$			
			73

5 Tom has 86 marbles. He loses 5^2 marbles in a game against Ernie. With the marbles he has won Ernie has 12^3 marbles.

5.1 How many marbles does Tom have left?

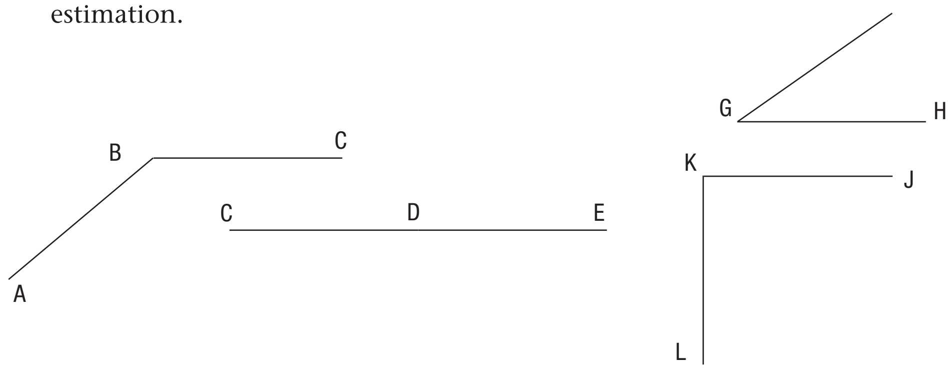
5.2 How many marbles does Ernie have?

5.3 How many marbles do they have together? (3)

6 Calculate and complete the pattern by replacing the variables with their values in the sequence.

1;........8;........27;........64;........*x*;........*y*;........*z*;........*a*;........729 (2)

7 Look at the angles below then complete the table. Say what kind of angles they are (acute, obtuse, and so on) and estimate what size you think they are. Then use your protractor to measure them and compare the actual size and the estimation. (6)



Angle	Type	Estimated size	Measured size
AB̂C			
CD̂E			
FĜH			
JK̂L			

8 Using a pencil, ruler and protractor draw the following angles and say what kind of angles they are:

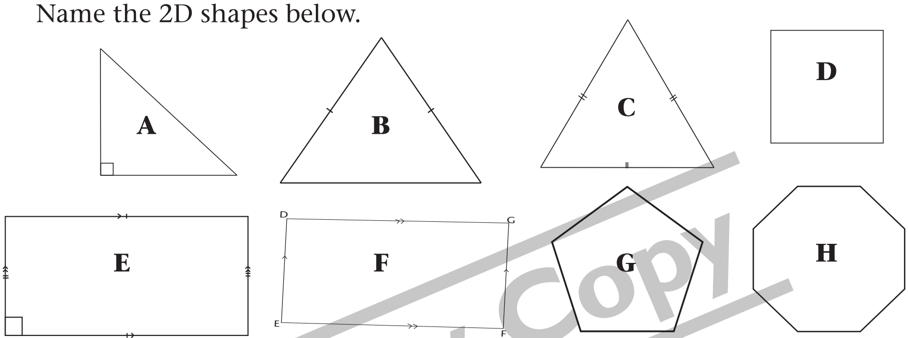
8.1 AB̂C = 90°

8.2 CD̂E = 25°

8.3 DÊF = 170°

8.4 What is an angle bigger than 180° called? (7)

9 Name the 2D shapes below. (4)



10 Draw angle AB̂C = 80° with vertex at B. Bisect the angle. Measure the two angles formed. (3)

11 Draw a square ABCD with horizontal base BC = 5 cm long.

11.1 What is the perimeter of the square you have constructed?

11.2 Measure the four angles in the square. What is their sum?

11.3 Are the four sides of a square always equal? (5)

12 Calculate and complete by filling in the answers on the table: (4)

Fraction in simplest form	Decimal fraction	Percentage	Ratio in simplest form
$\frac{1}{2}$			
	0,36		
		75%	
			9:10

13 Calculate and write your answers in simplest form:

13.1 $\frac{5}{6} + \frac{1}{3}$

13.2 $3\frac{1}{2} - 1\frac{4}{5}$

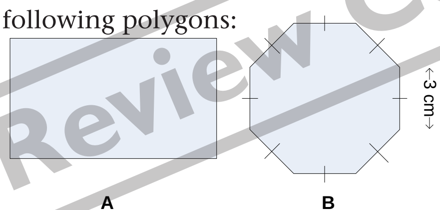
13.3 $3\frac{2}{3} + 5\frac{3}{4}$

13.4 $\frac{2}{3} \times \frac{9}{10}$ (8)

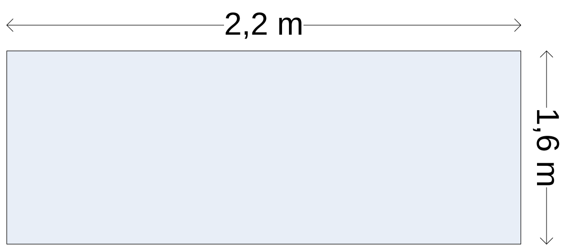
14 Calculate
14.1 3,24 × 4,3
14.2 65,58 × 8,2
14.3 451,64 × 3,1
14.4 4,354 x 9,67 (4)
15 Calculate the values of the variables.
15.1 3,24 + 4,3 + 12,007 = x
15.2 345,58 + 23,2 = y
15.3 4 761,689 + 3,006 = z
15.4 4,834 + 229,67 = a (4)
16 Round off to 2 decimal places:
16.1 23,897
16.2 23,009
16.3 23,093
16.4 23,991 (2)
17 Complete the table below by sorting the numbers into the correct columns. (4)

Given	Input	Rules	Output
+ 34; 87; 121			
999; x 3; 2; 335			
391; ÷ 2; 782			
1579; + 564; 1015			

18 Look at the following polygons:



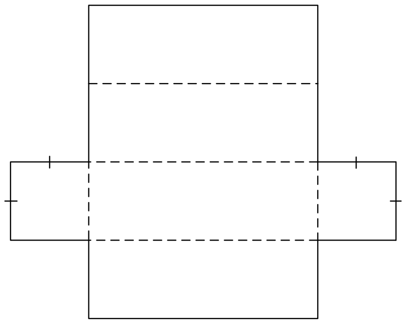
Write down the letter in each case to identify:
18.1 the regular polygon and the irregular polygon. (1)
18.2 Give reasons for your answers in Question 1. (1)
(2)
19 A polygon has length = 6 cm and breadth = half of the length.
19.1 Write down the formula to calculate its perimeter. (1)
19.2 Use the formula to calculate the perimeter and convert your answer to mm. (1)
(2]
20 Thembi makes a table cloth with the following dimensions:



20.1 Write down the formula to calculate the area of the table cloth and calculate the area in m^2. Round off to 1 decimal. (2)

20.2 If she wants to make another table cloth of 6 m^2 and the fabric is 4 m long, how wide must it be? (3)

21 A net is given of a prism with $l = 0,5$ m and $b = h = 25$ cm:



21.1 Name the 3D object. (1)

21.2 Use appropriate formulas to calculate the areas of each of the object's faces in m^2. Show your calculations. (3)

21.3 Calculate the total surface area of the object. (1)

22 Complete the sentences by writing down the missing words:

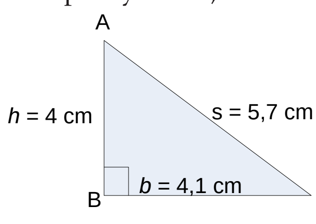
22.1 The amount of space occupied by a prism is called its ______. (1)

22.2 The amount of space inside a prism is called its ________. (1)

23.1 Write down and use the appropriate formula to calculate the volume of the following prism in cm^3: (2)
[AW: OSM7_POA8_AW15; rectangular prism]

23.2 If a large container has a capacity of 1 L, how many of these prisms will fit into the container? (2)

24 Triangle ABC is given:



24.1 Write down the formula to calculate the area of ΔABC. Calculate its area in cm^2. (2)

24.2 4 of these triangles are placed together to form a rectangle. Make a drawing for yourself of this new polygon. What would the total area of this rectangle? (1)

24.3 Use the formula and your answer in **24.2** to find the values of l and b to substitute into the formula. (2)

24.4 Use your formula to test the answer you got in **24.2**. (1)

Total marks [100]

June exam paper: Memo

1.1 (5)

a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*
2	3	5	7	11	13	17	19	23	29

1.2 Prime numbers (1)

2.1 967 + 935 = 1902
Inverse: 1902 – 935 = 967 (1902 – 967 = 935) (2)

2.2 324 × 628 = 203 472
Inverse: 203 472 ÷ 628 = 324 (203 472 ÷ 324 = 628) (2)

3.1 32(15 + 45) = 32 × 15 + 32 × 45 = 480 + 1 440 = 1 920 (1)

3.2 53(98 – 67) = 53 × 98 – 53 × 67 = 5 194 – 3 551 = 1 643 (1)

4 (5)

Cube or square root	Value	Expanded notation	Exponential form
$\sqrt{25}$	5	5×5	5^2
$\sqrt{81}$	9	9×9	9^2
$\sqrt{169}$	13	13×13	13^2
$\sqrt[3]{8}$	2	2×2×2	2^3
$\sqrt[3]{343}$	7	7×7×7	7^3

5.1 $86 - 5^2 = 86 - 25 = 61$ (1)

5.2 $12^3 = 1\ 728$ (1)

5.3 61 + 1 728 = 1 789 (1)

6 (2)

x	*y*	*z*	*a*
125	216	343	512

7 (6)

Angle	Type	Estimated size	Measured size
$A\hat{B}C$	reflex angle		220°
$C\hat{D}E$	straight angle		180°
$F\hat{G}H$	acute angle		35°
$J\hat{K}L$	reflex angle		290°

8.1 Accurate construction; right angle

8.2 Accurate construction; acute angle

8.3 Accurate construction; obtuse angle

8.4 Reflex angle (7)

(4)

9

A right-angled triangle

B isosceles triangle

C equilateral triangle

D square

E rectangle
F parallelogram
G pentagon
H octagon (4)
10 Accurate construction with the two angles having the same size, 40°. (3)
11 Accurate construction of square (2)
11.1 perimeter = 4 × 5 cm = 20 cm (1)
11.2 sum of the 4 angles = 4 × 90° = 360° (1)
11.3 Yes, otherwise it is not a square. (1)
(4)

12

Fraction in simplest form	Decimal fraction	Percentage	Ratio in simplest form
$\frac{1}{2}$	0,5	50%	1:2
$\frac{9}{25}$	0,36	36%	9:25
$\frac{1}{4}$	0,75	75%	3:4
$\frac{9}{10}$	0,9	90%	9:10

13.1 $\frac{5}{6} + \frac{1}{3} = \frac{5}{6} + \frac{2}{6} = \frac{7}{6} = 1\frac{1}{6}$ (2)
13.2 $3\frac{1}{2} - 1\frac{4}{5} = \frac{7}{2} - \frac{9}{5} = \frac{35}{10} - \frac{18}{10} = \frac{17}{10} = 1\frac{7}{10}$ (2)
13.3 $3\frac{2}{3} + 5\frac{3}{4} = \frac{11}{3} + \frac{23}{4} = \frac{44}{12} + \frac{69}{12} = \frac{113}{12} = 9\frac{5}{12}$ (2)
13.4 $\frac{2}{3} \times \frac{9}{10} = \frac{18}{30} = \frac{3}{5}$ (2)
14.1 3,24 × 4,3 = 13,932
14.2 65,58 × 8,2 = 537,756
14.3 451,64 × 3,1 = 1 400,084
14.4 4,354 × 9,67 = 42,103 18 (4)
15.1 x = 19,547
15.2 y = 368,78
15.3 z = 4 764,695
15.4 a = 234,504 (4)
16.1 23,90
16.2 23,01
16.3 23,09
16.4 23,99 (2)
17 (4)

Given	Input	Rules	Output
+ 34; 87; 121	87	+ 34	121
999; x 3; 2; 335	335	-2; ×3	999
391; ÷ 2; 782	782	÷2	391
1579; + 564; 1015	1 015	+ 564	1 579

18.1 A is an irregular polygon and B is a regular polygon. (1)

18.2 A is irregular, because not all the side are equal in length. B is regular, because all angles are equal in size and all sides are equal in length. (1)

19.1 Perimeter = $2l + 2b$ (1)

19.2 Perimeter = 2×6 cm + 2×3 cm = 12 + 6 = 18 cm
18×10 mm = 180 mm (1)

20.1 Area = $l \times b$ = 2,2 m × 1,6 m = 3,52 m^2 (1)
3,5 m^2 (rounded to one decimal place) (1)

20.2 $6 = 4 \times b$; $b = \frac{6}{4} = \frac{3}{2} = 1{,}5$ m (3)

21.1 Rectangular prism (1)

21.2 25 cm = 0,25 m
Area = $l \times b$ = 0,5 m × 0,25 m = 0,125 m^2 (4 faces have the same area)
Area = $b \times h$ = 0,25 m × 0,25 m = 0,0625 m^2 (2 faces have the same area) (3)

21.3 Total surface area = 4 × 0,125 m^2 + 2 × 0,0625 m^2 = 0,625 m^2 (1)

22.1 volume (1)

22.2 capacity (1)

23.1 50 mm = 5cm; 55 mm = 5,5 cm; 48 mm = 4,8 cm
Volume = $l \times b \times h$ = 5,5 cm × 4,8 cm × 5 cm = 132 cm^3 (2)

23.2 1ℓ = 1 000 cm^3; $\frac{1\,000}{132} = 7{,}58$ (7 prisms would fit into the container.) (2)

24.1 Area = $\frac{1}{2} \times b \times h = \frac{1}{2} \times 4{,}1 \times 4 = 8{,}2$ cm^2 (2)

24.2 Total area = 8,2 cm^2 × 4 = 32,8 cm^2 (1)

24.4 length = 8,2 cm and breadth = 4 cm (2)

24.5 8,2 × 4 = 32,8 cm^2 (1)

Total marks [100]

Assignment 2 Option 1: Memo

Patterns

1.1 ◊□□△◊□□△◊□□△◊ (2)
1.2 Three symbols make up the pattern (one of which is repeated twice) (1)
1.3 Repeat pattern (1)
1.4 Regular pattern (1)
2.1 An example of a regular repeat pattern using three or four geometric shapes:
◊⌂□◊⌂□◊⌂□◊⌂□ (2)
2.2 An example of a mirror image pattern using geometric shapes:
◀□△△□▶ (2)
3 Some examples are: the national flag, sports logos, road signs, advertising logos, art and beadwork, symbols for the police, for the army and for religions. (2)
4.1 (2)

4.2 (4)

No. of bunches	1	2	3	4	5	6	7	8	9
No. of cherries	3	6	9	12	15	18	21	24	27

5.1 (4)

2	4	6	8	10	12	14	16	18	20	22	24
8	16	24	32	40	48	56	64	72	80	88	96

5.2 (4)

1	3	5	7	9	11	13	15	17	19
3	9	15	21	27	33	39	45	51	57

6 Learners draw a hundred board like the one on page 47 of the Learners Book. (1)
6.1 They list any three horizontal rows, for example, 1 to 10 or 11 to 20. (3)
6.2 They list any three vertical rows, for example, 1; 11; 21; 32. (3)
6.3 They list any three diagonal rows, for example, 6; 17; 28; 39; 50. (3)
7.1 2; 4; 6; 8; 10; 12; 14; 16; 18; 20 (1)
7.2 7; 14; 21; 28; 35; 42; 49; 56; 63; 70 (1)
7.3 11; 22; 33; 44; 55; 66; 77; 88; 99; 110; 121 (1)
7.4 69; 72; 75; 78; 81; 84; 87; 90; 93; 96 (1)
7.5 1 000; 990; 980; 970; 960; 950; 940; 930; 920; 910; 900; 890 (1)

8

8.1 is multiples of 2 from 2 to 20; or 7a is even numbers from 2 to 20. (1)

8.2 is multiples of 7 from 7 to 70. (1)

8.3 is multiples of 11 from 11 to 121. (1)

8.4 is multiples of 3 from 69 to 96. (1)

8.5 is multiples of 10 in descending order from 1 000 to 890. (1)

9.1 $4 \times 3 + 1$; $5 \times 3 + 1$; $6 \times 3 + 1$ (3)

9.2 4;7;10;13;16;19 (2)

9.3 Each number is 3 more than the number before it. (2)

9.4 Answers will vary as learners create a sequence of similar number sentences. (2)

Total marks [55]

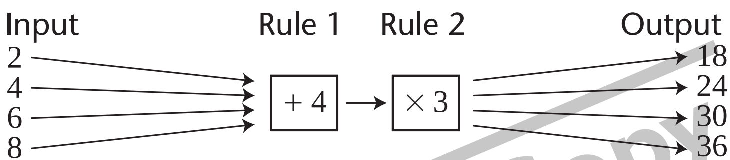
Assignment 2 Option 2: Memo

Algebra 1

1.1	$\frac{2}{3} = \frac{4}{6} = \frac{8}{12} = \frac{30}{45} = \frac{50}{75}$	(4)
1.2	$\frac{4}{7} = \frac{12}{21} = \frac{24}{42} = \frac{20}{35}$	(4)
1.3	$x = 16$	(2)
1.4	$y = 12$	(2)
1.5	$z = 27$	(3)
1.6	$a = 40$	(3)
2		(5)

a	0	1	2	3	4	5	6	7	8
b	8	7	6	5	4	3	2	1	0

3 (5)



4.1	$p = 5$	(2)
4.2	$g = 7$	(2)
4.3	$x = 8$	(2)
4.4	$y = 10$	(2)
4.5	$z = 18$	(2)
4.6	$a = 4$	(2)
4.7	$a = 3$	(2)
4.8	$x = 4$	(3)

Total marks [45]

Project 1 Option 1: Memo

Functions, relationships and graphs

Part 1

1.1 Number of dresses = $\frac{x}{2,5}$ (2)
Number of sides = $6x$ (2)

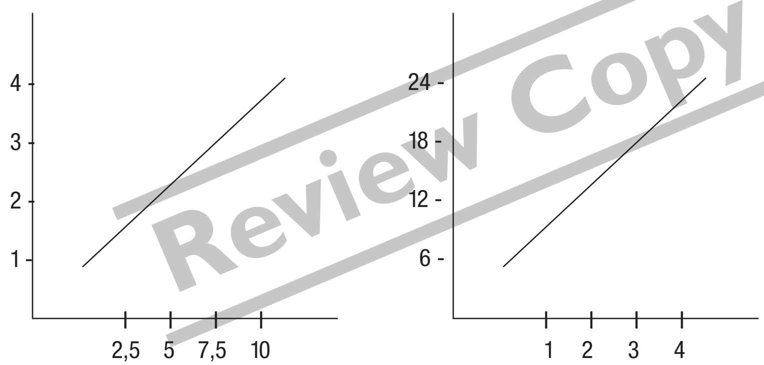
1.2 (2)

Amount of fabric and number of dresses				
x: Amount of fabric, m	2,5	5	7,5	10
y: Number of dresses	1	2	3	4

(2)

Number of hexagons and sides				
x: Number of hexagons	1	2	3	4
y: Total number of sides	6	12	18	24

1.3 (2)
(2)



1.4 The relationships are linear because the graphs are straight lines. (2)

Part 2

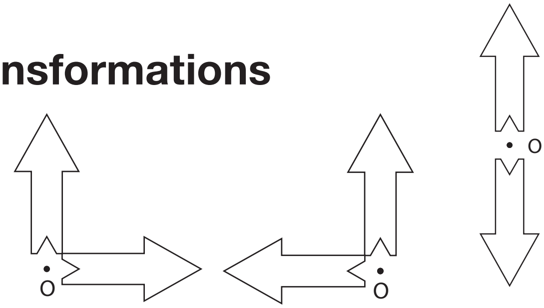
2.1 Temperature for 5 days recorded in tables. (3)
2.2 Line graphs drawn matching data in tables. (3)
2.3 Valid observation made. (2)
2.4 Valid observation made. (2)
2.5 Valid observation about the relationship between the forecast temperature and measured temperatures at morning and midday made. (2)
2.6 Graph/table selected with appropriate reason. (2)

Total marks [30]

Project 1 Option 2: Memo

Transformations

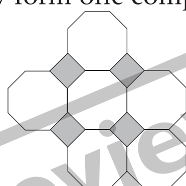
1.1 (1)
1.2 (1)
1.3 (1)



2 (4)

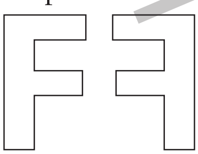
Polyhedron	No. of vertices	No. of faces	No. of edges
Tetrahedron	4	4	6
Pentahedron	5	5	8
Cube	8	6	12
Octahedron	6	8	12

3.1 Rectangles, square, triangles (2)
3.2 The sum of the angle measurements for each point where the polygons join is 360° since they form one complete revolution round each such a point. (1)
4.1 Squares (1)
4.2 (2)

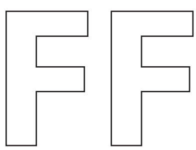


5.1 Slide – translation (2)
5.2 Turn – rotation (2)
5.3 Flip – reflection (2)

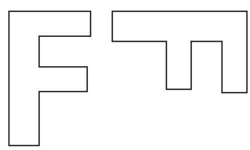
6.1



6.2

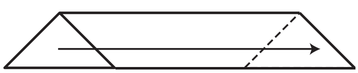


6.3 (3)

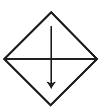


7.1 Trapeziums, triangles, squares, hexagons (1)
7.2 (1)

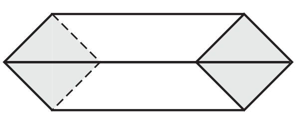
7.3 (2)



7.4 (2)



7.5 (1)



7.6 This is a rectangular prism. (1)

Total marks [30]

Term 3 Test 3

1.1 Complete the following number patterns

1.1.1 9; 18; 27; ___; ____; ____; ____; ____ (2)

1.1.2 1; 4; 5; 9; 14; 23; ___; ____; ____; ___ (2)

1.2.1 For 1.1.1 and 1.1.2 explain how you calculated the next term. (2)

1.2.2 For 1.1.1 write the general term, and determine the 40^{th} term in the sequence. (2)

1.3 Given the formula: $C = (2x + 7)^2$.

1.3.1 Represent the formula as a flow diagram. (2)

1.3.2 Determine C if $x = 2$. (1)

1.3.3 Determine x if the output is 81. (1)

2.1 Write in algebraic language.

2.1.1 Any number multiplied by 7, divided by 2, and then added to 1. (1)

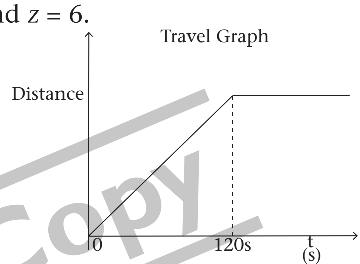
2.1.2 When a certain number is multiplied by 3 and added to 5 the answer is 21. (1)

2.2 Determine the value of x by trial and error.

2.2.1 $3x - 6 = 24$ (1)

2.2.2 $x - 5 = \frac{3}{2}$ (1)

2.3 Determine the value of $\frac{3xy}{z} + 1$, if $x = 5$, $y = 2$ and $z = 6$. (2)

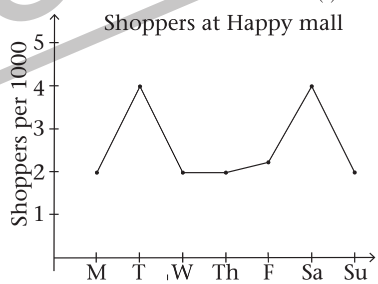


3.1 Use the graph alongside to answer the following questions.

3.1.1 What do you think the graph represents? (1)

3.1.2 What happens at a 120s? (1)

3.1.3 Is this graph linear? (1)



3.2 Use the graph alongside to answer the following questions.

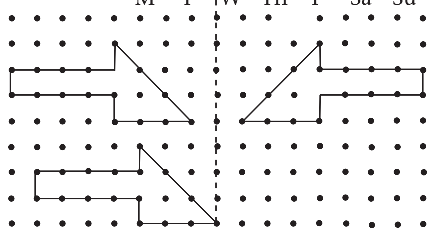
3.2.1 What in the independent variable in this graph? (1)

3.2.2 Describe the trend from the graph. (1)

3.2.3 Give possible reasons for these trends. (1)

4.1 How many lines of symmetry does a square have? (1)

4.2 Look at the following diagram.



4.2.1 Describe the transformation from A to B. (1)

4.2.2 Describe the transformation from A to C. (1)

4.3 If a rectangle is enlarged by a factor of 2, by what factor does the area of the rectangle increase? Use an example to prove your answer. (3)

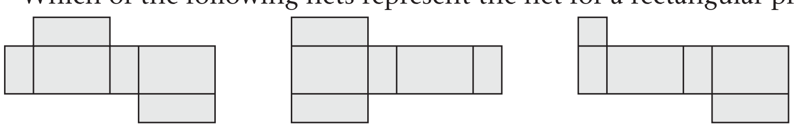
5.1 How many faces does a cube have? (1)

5.2 How many vertices does a triangular prism have? (1)

5.3 Explain the difference between a prism and a pyramid. (2)

5.4 Draw the net of a triangular pyramid. (2)

5.5 Which of the following nets represent the net for a rectangular prism? (2)



Total marks [38]

Term 3 Test 3: Memo

1.1.1 9; 18; 27; 36; 45; 54; 63; 72 (2)

1.1.2 1; 4; 5; 9; 14; 23; 37; 60; 97; 157 (2)

1.2.1 For 1.1.1 – add nine to the previous term
For 1.1.2 – add the two previous terms to get the next term (2)

1.2.2 General term is $9n$ and the 40th term is $9 \times 40 = 360$. (2)

1.3.1 $x \rightarrow \boxed{\times 2} \rightarrow \boxed{+7} \rightarrow \boxed{\text{answer}^2} \rightarrow C$ (2)

1.3.2 $C = (2(2) + 7)^2 = 121$ (1)

1.3.3 $x = 1$ (1)

2.1.1 $\frac{7x}{2} + 1$ (1)

2.1.2 $3x+5 = 21$ (1)

2.2.1 $x = 10$ (1)

2.2.2 $x = 6{,}5$ (1)

2.3 $\frac{3xy}{z} + 1 = \frac{3 \times 5 \times 2}{6} + 1 = 6$ (2)

3.1.1 The graph represents the motion of something (car, bike, person walking etc) which travels at a constant speed for 120s and then stops. (1)

3.1.2 The car/bike/person stops moving. (1)

3.1.3 Yes. (1)

3.2.1 The independent variable in this graph is the day of the week. (1)

3.2.2 The busiest shopping days are Tuesday and Saturday and the number of shoppers drops in between those days. (1)

3.2.3 On Saturday people do not have to go to work so they can shop. By Tuesday they may have run out of things and need to shop again. (1)

4.1 4 lines of symmetry (1)

4.2.1 The transformation from A to B is a reflection across the dotted line. (1)

4.2.2 The transformation from A to C is a shift or translation by 4 units downwards and 1 unit to the right. (1)

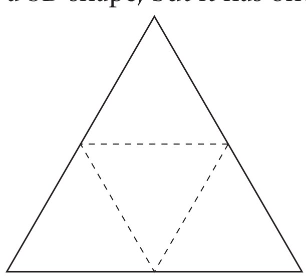
4.3 The area of the rectangle increases by a factor of 4. For example, if a rectangle with length 4 cm and width 2 cm is enlarged by a factor of 2, its new dimensions are 8 cm and 4 cm, so its new area (32 cm^2) is 4 times the old area (8 cm^2) (3)

5.1 A cube has 6 faces. (1)

5.2 A triangular prism has 9 vertices. (1)

5.3 A prism is a 3D shape with two identical polygon bases, joined by rectangular faces. A pyramid is also a 3D shape, but it has one polygon base with at least 3 triangular faces. (2)

5.4 (2)



5.5 (i) and (ii) represent the net for a rectangular prism (2)

Total marks [38]

Integers

1.1 $0 < 5$; $0 > -7$; $3 > -3$; $-1 > -7$; $-11 < 7$ (5)

1.2 Differences: $1 - (-1) = 2$

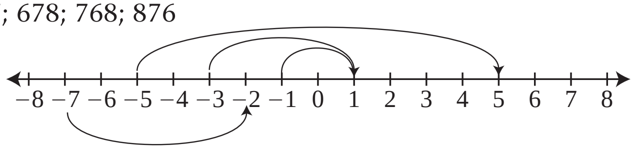
$1 - (-3) = 4$

$5 - (-5) = 10$

$-2 - (-7) = 5$ (4)

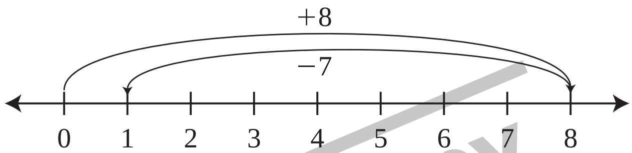
2 –867; –786; –687; 678; 768; 876 (1)

3.1 $(+8) + (-7) = +1$



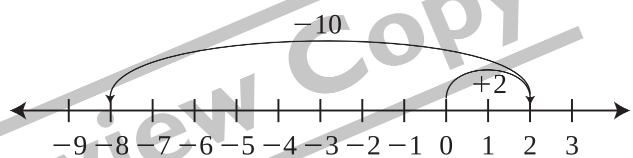
(2)

3.2 $(+2) + (-10) = -8$



(2)

3.3 $(-5) + (-4) = -9$



(2)

3.4 $(-3) + (-9) = -12$ (2)

3.5 $(-1) + (-3) - (-4) = 0$ (2)

Encourage learners to check the above using calculators.

Total marks [20]

Assignment 3 Option 2: Memo

Algebra 2

1.1 a^2

1.2 $3b + 4c$

1.3 $d + 5f$

1.4 $g/2$

1.5 $h - 7$

1.6 $2j + 3k - 4m$ (6)

2 There is 1 term in 1.1
There are 2 terms in 1.2
There are 2 terms in 1.3
There is 1 term in 1.4
There are 2 terms in 1.5
There are 3 terms in 1.6 (6)

3 y and z are the variables. (2)

4 3 and 7 are the constants. (2)

5.1 $7 + 5 + x = 20$
$x =$ R8 (2)

5.2 $7y = 63$
$y = 9$ km (2)

6.1 $4 + a = 3(8 - 2)$
$4 + a = 30$
$a = 26$ (2)

6.2 $\frac{27}{b} - 5 = 1 + 1 + 1 + 1$
$\frac{27}{b} = 9$
$b = 3$ (2)

6.3 $21(7 + c) = 168$
$7 + c = 8$
$c = 1$ (2)

6.4 $15d + 36 = 6 \times 11$
$15d = 30$
$d = 2$ (2)

6.5 $3f + 4f + 5f = 240$
$12f = 240$
$f = 20$ (2)

Total marks [30]

Investigation 2 Option 1: Memo

Data investigation

Marking guidelines

1 Some examples of a linear process are doing a school project, cooking a meal, building a model, etc.

2 The data cycle is a cycle as the process is continuous – conclusions always relate back to the original question and sometimes result in the question being adapted and the process may start again.

3 Questions should be suitable and not too broad.

4 Appropriate sources for the collection of data include peers, family, newspapers, books, magazines and to some extent, the internet (learners should not rely exclusively on the internet).

5 Learners could have used a questionnaire or interviews to collect data – check that their questions are appropriate. If they relied on secondary sources, check that the data collected is suitable and fits the question they are asking.

6 A frequency table shows the number of data points in each category or group. Learners may have to choose their own groups – check that their intervals fit the data (not too large or small) and are the intervals the same size.

7 See above for guidelines about intervals. If appropriate, check whether data is arranged in ascending/descending order.

8 Check whether learners have chosen the appropriate graph for their data. Their graph should have a title and the axes should be clearly labelled. Check the scale on the vertical axis. For discrete data, learners should choose a bar graph and the bars should not touch. For continuous data learners should choose a histogram and the bars should touch. For a pie chart, check whether the segments are the correct sizes and are accurately labelled.

9 Check whether the correct mean, median and mode have been calculated. For a stem and leaf diagram, check that the data values are in the correct order and that learners have used a key.

10 Learners' conclusions should relate back to their original question. They should also mention any features they had noticed or wondered about and investigated. Check whether learners have given reasons based on what they have found out in their investigation. Learners should have used some statistical language in their conclusion.

Rubric

Assessment criteria	1	2	3	4	5	6	7	8	9	10
1 Pose relevant questions about an issue										
2 Select appropriate data sources										
3 Collect data										
4 Organise data in a frequency table										
5 Group the data (in ascending/descending order or into intervals)										
6 Represent the data in a bar graph, pie chart or histogram										
7 Analyse and summarise the data (calculate either mean, mode or modus or summarise in a stem-and-leaf display)										
8 Interpret the data and report conclusions										

Total marks [20]

Investigation 2 Option 2: Memo

Probability

Marking guidelines

1 P(heads) = $\frac{5}{10} = \frac{1}{2}$ = 50%

2 P(tails) = $\frac{5}{10} = \frac{1}{2}$ = 50%

3 Outcomes can be recorded in a table.

4 Relative frequency = $\frac{\text{no of heads}}{10}$

5 Relative frequency = $\frac{\text{no of tails}}{10}$

6 The answers may be different, as the answers in (a) and (b) are theoretical probabilities and the coin was only tossed 10 times in (d) and (e).

7 P(heads) = $\frac{25}{50} = \frac{1}{2}$ = 50%

8 P(tails) = $\frac{25}{50} = \frac{1}{2}$ = 50%

9 Outcomes can be recorded in a table.

10 Relative frequency = $\frac{\text{no of heads}}{10}$

11 Relative frequency = $\frac{\text{no of tails}}{10}$

12 The answers may be closer together as the coin was tossed more times (but there may still be a difference). Learners should note that the theoretical probability has not changed.

13 The relative frequency may be closer to the theoretical probability of 50%.

14 The relative frequency may be closer to the theoretical probability of 50%.

Rubric

Assessment criteria	10	9	8	7	6	5	4	3	2	1
1 Calculate probability and express it as a fraction										
2 Calculate probability and express it as a percentage										
3 Calculate relative frequency										
4 Explain the difference between probability and relative frequency										

Total marks [30]

December exemplar: Memo

1.1 True (1)

1.2 False (1)

2 Number of learners taking part in soccer = $\frac{3}{18} \times 900 = 150$

Number of learners taking part in cricket = $\frac{7}{18} \times 900 = 350$

Number of learners taking part in rugby = $\frac{8}{18} \times 900 = 450$ (2)

3 15% of 158 799 = $\frac{15}{100} \times 158\ 799$ = R23 819,85

Amount he sold it for = R158 799 – R23 819,85 = R134 979,15 (2)

4.1 $\sqrt[3]{64} - 2^3 = 4 - 8 = -4$ (2)

4.2 $122 - 43 + 19 = 144 - 64 + 19 = 99$ (2)

4.3 $\sqrt{100}(\sqrt{4} + \sqrt{25}) = 10(2 + 5) = 70$ (2)

4.4 $72(43 - 25) = 49(64 - 32) = 1\ 568$ (2)

5 Accurate construction of EF parallel to HG (3)

6 Accurate construction of isosceles triangle JKL, with JK = KL = 3 cm and $K\hat{J}L = K\hat{L}J = 55°$. (3)

7.1 True

7.2 False, an equilateral triangle has all three sides equal.

7.3 False, a kite has two pairs of adjacent sides equal.

7.4 True

7.5 True

7.6 True (6)

8.1 $5\frac{6}{7} - 2\frac{3}{5} = \frac{41}{7} - \frac{13}{5}$

$= \frac{205 - 91}{35t}$

$= \frac{114}{35}$

$= 3\frac{9}{35}$ (2)

8.2 $4\frac{5}{9}$ of 27 = $\frac{41}{9} \times 27$

$= 41 \times 3$

$= 123$ (2)

8.3 $7\frac{1}{3} \times 3\frac{7}{12} = \frac{22}{3} \times \frac{43}{12}$

$= \frac{473}{18}$

$= 26\frac{5}{18}$ (2)

9.1 8,976 × 0,67 = 6,01392 (3)

9.2 90,009 × 4,65 = 418,54185 (3)

10.1 Leaners draw the next 3 terms - check for accuracy (3)

10.2 (3)

Term number	1	2	3	4	5	6	10	15
Number of dots	1	4	7	10	13	16	28	43

10.3 You add three dots to each term to get to the next term. (1)

10.4 term number × 3 – 2 term (2)

10.5 There are 298 dots in the 100th term. (1)

11.1 –16; –13; –10; –7 ; –4 ; –1 ; 2 ; 5 (1)

11.2 5; 3; 1; –1 ; –3; –5; –7; –9 (1)

11.3 –10; –150; –190; –230; –270; –310; –350 (1)

12.1 Perimeter = 9 + 2 + 4 + 3 + 3,7 + 1 + 1,3 + 4 = 28 m (1)
Area = 4 × 1,3 + 3,7 × 5 + 4 × 2 = 31,7 m² (2)

12.2 Let the unknown side be x:
$x = \sqrt{3^2 + 5^2} = 5{,}83$ (Pythagoras)
Perimeter = 13 + 3 + 8 + 5,83 = 29,83 cm (1)
Area = $8 \times 3 + \frac{1}{2} \times 5 \times 3 = 31{,}5$ cm² (1)

13.1 Volume of the pool = 8 × 4 × 15 = 480 m³ (2)

13.2 Amount of water = 8 × 4 × 14,7 = 470,4 m³ = 470,4 kl (3)

14.1 $2x - 5 = 17$
$x = 11$ (2)

14.2 $\frac{\sqrt{x}+12}{2} = 10$
$\sqrt{x} = 8$
$x = 64$ (2)

15.1 $2g + 1 = 7$
$g = 3$ (1)

15.2 $3f + 3 = 19$
$f = \frac{16}{3}$ (1)

15.3 $15 - 4a = -1$
$a = 4$ (2)

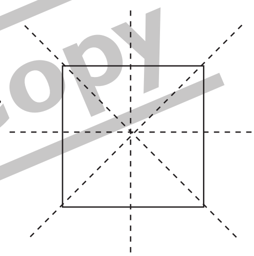
16.1 Days of the week (1)

16.2 13 learners were absent on Monday. (1)

16.3 The graph is increasing from Tuesday to Friday. (1)

16.4 Valid reason given. (1)

17.1 4 lines of symmetry: (2)



17.2 7 lines of symmetry (2)

18 Each of its sides is half of the original size, so the whole shape gets smaller. (2)

19

Object	No. of faces	No. of vertices	No. of edges
Cube	6	8	12
Square pyramid	5	5	8
Cylinder	3	None	2

20.1 (–3) + (–2) – 5 = –10 (2)

20.2 (–111) + (–12)– (–8) = –115 (2)

21 For example, (–3 + 4) – 4 = –3 + (4 – 4) = –3 (any suitable example) (2)

22.1 38 learners (1)

22.2 Mean = $\frac{3 \times 1 + 8 \times 2 + 13 \times 3 + 8 \times 4 + 5 \times 2 + 4 \times 6}{38} = \frac{124}{38} = 3{,}26$ (2)

22.3 $\frac{13}{38} \times 100 = 34{,}21\%$ (1)

22.4 Her sample is her class and her population is the school. (2)

22.5 Yes/no and appropriate reason given. (1)

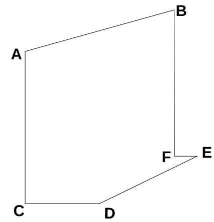
23.1 No – the answer should be $\frac{3}{10}$ (1)

23.2 Relative frequency of choosing the yellow ball = $\frac{1}{10}$ (1)

23.3 They only carried out the experiment 10 times – if they had carried it out more times the answers may have been closer together. (1)

December exam paper

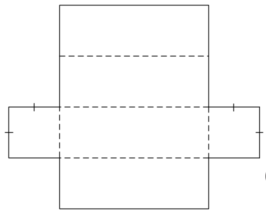
1 Polygon ABCDEF has BF = AC = 2 m; AB = 2,1 m; CD = 0,9 m; DE = 1,2 m and EF = 0,2 m.



1.1 Is it a regular or irregular polygon? Give a reason for your answer. (1)

1.2 Calculate its perimeter and write down the answer in metres. (2)

2 A net is given of a prism with l = 0,5 m and b = h= 25 cm:



2.1 Name the 3D object. (1)

2.2 Use appropriate formulae to calculate the areas of each of the object's faces in m². Show your calculations. (3)

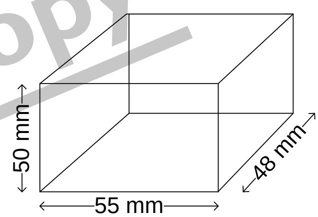
2.3 Calculate the total surface area of the object. (1)

3 Complete the sentences by writing down the missing words:

3.1 The amount of space occupied by a prism is called its ______. (1)

3.2 The amount of space inside a prism is called its ________. (1)

4.1 Write down and use the appropriate formula to calculate the volume of the following prism in cm³: (2)



4.2 If a large container has a capacity of 1 L, how many of these prisms will fit into the container? (2)

5 Complete the number sequences:

5.1 1; 4; 9; …; …; …

5.2 1; …; 6; 10; …

5.3 26; 39; …; …; 78

5.4 Name each type of number sequence shown above. (5)

6 Complete the flow diagram:

Input		Rules				Output
43	→	6	→	7	→	*a*
47	→	16	→	3	→	*b*
51	→	20	→	4	→	*c*

(3)

7 Describe the relationships in each sequence of numbers by describing the operation that has been performed:

7.1 3; 6; 12; 24; 48…

7.2 7; 10; 13; 16; 19; 22…

7.3 160; 80; 40; 20; 10; 5

7.4 45; 40; 35; 30; 25; 20 (4)

8 Calculate the missing variables:

8.1 $4a + 7 = 19$

8.2 $7b - 9 = 12$

8.3 $(c + 8) \times 3 = 42 - 9$

8.4 $678 \div 102 = d$ (4)

9 Determine the following in algebraic language:

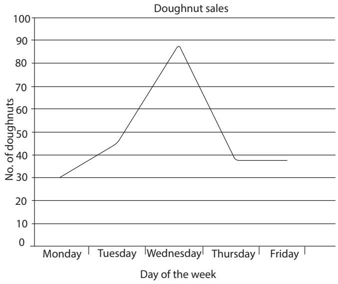
9.1 If a is a whole number, write the number that comes before a.

9.2 If b is a whole number, write the number that comes after b.

9.3 If c is a whole number, write product of 5 and c.

9.4 If d is a whole number, write half of d. (4)

10



A school tuck shop sells doughnuts. Sales were recorded for one week. Read and analyse the graph to answer the questions.

10.1 Between which days are the line segments increasing? (1)

10.2 Does the graph show a constant trend? Motivate your answer. (1)

10.3 Identify the x- and y-variables. Which variable is dependent and which is independent? (2)

11 Rainfall was measured for a period of 4 months.

Month	Rain measured in mm:
January	18
February	12
March	28
April	22

11.1 Use the data to draw a line graph on the graph paper provided. Provide clear labels for each axis and a suitable title. (3)

11.2 Identify at least one decreasing trend by writing down between which months this trend can be observed. (1)

12 Write down which of the following letters are symmetrical: M, N, O, P and Q? Draw in the line of symmetry. (4)

13 A post card is 90 mm by 60 mm.

13.1 Draw an enlargement of the postcard 12 cm by 9 cm (B).

13.2 Draw in a vertical line to divide B into two equal halves.

13.3 What is the length and breadth of each of the smaller rectangles on B?

13.4 Use the vertical line on the enlarged drawing as a line of reflective symmetry, and do a reflected design using triangles about the vertical. (4)

14 Complete the table below to show the properties of each of the following polyhedra: cube; triangular prism.

	Shapes	Cube	Triangular prism
F	Number of faces		
V	Number of vertices		
E	Number of edges		
	F + V – E		

(4)

15 Draw a net for a rectangular prism and label: (2)
15.1 equal faces x (1)
15.2 equal edges y (1)

16 Draw a number line to show the difference between –4 and 3. (1)
17 Calculate the following:
17.1 (99) + (–76)
17.2 (–68) + (+84)
17.3 (–45) + (–54) (3)
18 Calculate the following:
18.1 (+79) – (+234)
18.2 (+99) – (+14)
18.3 (+72) – (+73) (3)
19 Calculate and use the commutative property:
19.1 (–79) × 60
19.2 67 × (–90)
19.3 (–23) × (–19) (3)
20. Calculate using the associative property:
20.1 (312 × –42) × 30
20.2 (64 × –25) × –62
20.3 (24 × –44) × –24 (3)
21 Calculate the value of the variables:
$x^2 \times 2 = 18$
$y^2 \times 2 = 50$
$z^3 \times 2 = 16$
$z^3 \times 2 = 2$

(4)

22 Copy and complete the table:

Algebraic expression	Variable(s)	Constant(s)	Number of terms	Monomial, binomial or trinomial
$3z$				
$2x - 5w + 3$				
$7a + 10$				
$9b$				
$4c - 8 + 1$				

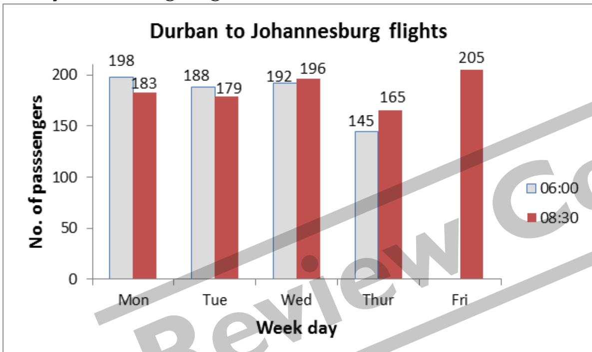
(5)

23 A wildlife conservationist keeps count of some of the animal species in a certain area of a game reserve. On a specific day he counts the following animals:

Animal	Kudu	Giraffe	Zebra	Wildebeest	Impala	Leopard
No. counted	2	2	12	6	25	1

23.1 Do these numbers of animals represent a data sample or a data population of the wildlife in the game reserve? Give a reason. (1)
23.2 Sort the data set in ascending order. (1)
23.3 Group the data into 3 intervals: [0-10); [10-20); [20-30). (3)
23.4 Summarise the data using a stem-and-leaf display. (3)
23.5 Does the data set have a mode? If it does, what is the value? (1)
23.6 What is the range of the data set? (1)

24 An airline company collects data about the number of passengers that take early morning flights from Durban to Johannesburg.



24.1 What type of graph is this? (1)
24.2 Is the graph a many-to-one graph or not? Give a reason for your answer. (2)
24.3 Which of the two flights had the most passengers on Wednesday morning? (1)
24.4 Give a possible reason why there were no passengers plying at 06:00 on Friday. (1)
24.5 What was the total number of passengers that took the 06:00 flight for the week? (2)

25 Michelle performs a probability experiment by rolling a die. She rolls the die 8 times and records the actual outcomes after each roll:

Roll no.	1	2	3	4	5	6	7	8
Outcome	6	4	6	1	6	3	2	1

25.1 Which outcome occurred most often? (1)
25.2 Is there a possible outcome that never occurred? (1)
25.3 Calculate the relative frequency for the outcome of getting a 3. (1)

Total marks [100]

December exam paper: Memo

1.1 Irregular polygon, since all the sides do not have the same length (or all angles are not equal in size). (1)

1.2 Perimeter = 2,1 m + 2 m + 0,2 m + 1,2 m + 0,9 m + 2 m = 8,4 m (2)

2.1 Rectangular prism (1)

2.2 25 cm = 0,25 m
Area = $l \times b$ = 0,5 m × 0,25 m = 0,125 m^2 (4 faces have the same area)
Area = $b \times h$ = 0,25 m × 0,25 m = 0,0625 m^2 (2 faces have the same area) (3)

2.3 Total surface area = 4 × 0,125 m^2 + 2 × 0,0625 m^2 = 0,625 m^2 (1)

3.1 volume (1)

3.2 capacity (1)

4.1 50 mm = 5 cm; 55 mm = 5,5 cm; 48 mm = 4,8 cm
Volume = $l \times b \times h$ = 5,5 cm × 4,8 cm × 5 cm = 132 cm^3 (2)

4.2 1ℓ = 1 000 cm^3; $\frac{1\,000}{132}$ = 7,58 (7 prisms would fit into the container.) (2)

5.1 1; 4; 9; 16; 25; 36

5.2 1; 3; 6; 10

5.3 26; 39; 52; 65; 78

5.4 square numbers; triangular numbers; multiples of 13 (5)

6.1 $a = (43 + 6) \div 7 = 49 \div 7 = 7$

6.2 $b = (47 - 16) \times 3 = 31 \times 3 = 93$

6.3 $c = (51 \times 20) + 4 = 1\,020 + 4 = 1\,024$ (3)

7.1 Multiply a sequence number by 2 to get the next number in the sequence.

7.2 Add 3 to a sequence number to get the next number in the sequence.

7.3 Divide a sequence number by 2 to get the next number in the sequence.

7.4 Subtract 5 from a sequence number to get the next number in the sequence. (4)

8.1 $a = (19 - 7) \div 4 = 12 \div 4 = 3$

8.2 $b = (12 + 9) \div 7 = 21 \div 7 = 3$

8.3 $c = ((42 - 9) \div 3) - 8 = (33 \div 3) - 8 = 11 - 8 = 3$

8.4 $d = 678 \div 100 = 6{,}78$ (4)

9.1 $a - 1$

9.2 $b + 1$

9.3 $5c$

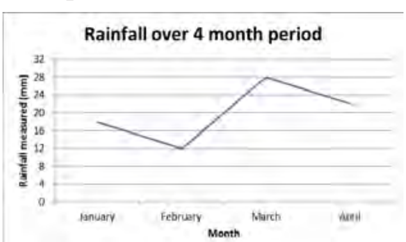
9.4 $\frac{1}{2}d$ or $\frac{d}{2}$ (4)

10.1 Monday to Wednesday (1)

10.2 No, since all four line segments together do not form a horizontal line. (1)

10.3 The “day of the week” denotes the x-variable and “no. of doughnuts” denotes the y-variable. The x-variable is the independent variable and the y-variable is the dependent variable. (2)

11.1

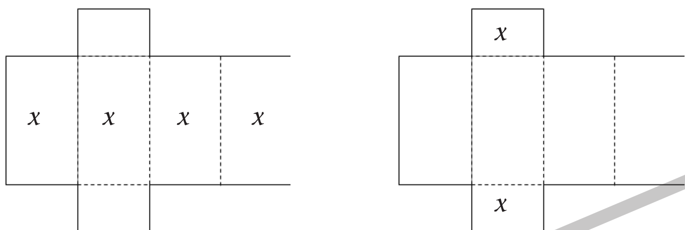


(3)

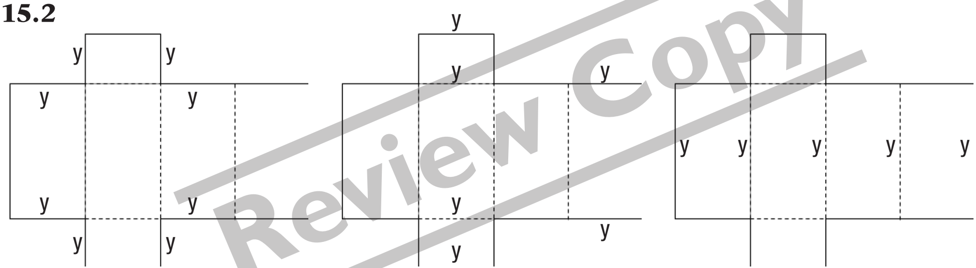
11.2 Between January – February and March – April. (1)
12 M and O are symmetrical. (4)
13.1 Check learners drawings for accuracy
13.2 Check learners drawings for accuracy
13.3 The length is 6 cm and the breadth is 9 cm. (4)
14 (4)

	Shapes	Cube	Triangular Prism
F	Number of faces	6	5
V	Number of vertices	8	6
E	Number of edges	12	9
	F + V – E	2	2

15.1

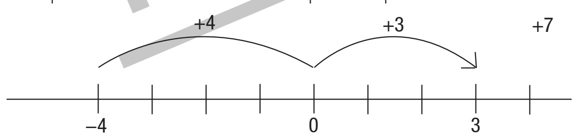


15.2



(4)

16



(1)

17.1 99 – 76 = 23(1)
17.2 – 68 + 84 = 16
17.3 – 45 – 54 = –99 (3)
18.1 79 – 234 = –155
18.2 99 – 14 = 85
18.3 72 – 73 = –1 (3)
19.1 –79 × 60 = 60 × (–79) = –4 740
19.2 67 × (– 90) = – 90 × 67 = –6 030
19.3 –23 × (–19) = –19 × (–23) = –437 (3)
20.1 (312 × –42) × 30 = 312 × (–42 × 30) = 312 × –1 260 = –393 120
20.2 (– 64 × –25) × –62 = –64 × (–25 × –62) = –64 × 1 550 = –99 200
20.3 (– 24 × –44) × –24 = –24 × (–44 × –24) = –24 × 1 056 = –25 344 (3)

21.1 $x = \sqrt{18 \div 2} = \sqrt{9} = 3$

21.2 $y = \sqrt{50 \div 2} = \sqrt{25} = 5$

21.3 $z = \sqrt[3]{16 \div 2} = \sqrt[3]{8} = 2$

21.4 $a = \sqrt[3]{2 \div 2} = \sqrt[3]{1} = 1$ (4)

22 (5)

Algebraic expression	Variable/s	Constant/s	Number of terms	Name of expression
$3z$	z		1	monomial
$2x - 5w + 3$	x, w	3	3	trinomial
$7a + 10$	a	10	2	binomial
$9b$	b		1	monomial
$4c - 8 + 1$	c	- 8 + 1	3	trinomial

23.1 Data sample, because it is a smaller set that represents the whole population. (1)

23.2 1; 2; 2; 6; 12; 25 (3)

23.3

Number of animals		
Interval	Elements per interval	f
	1; 2; 2; 6	4
	12	1
	25	1

23.4 Learners' own work

23.5 Yes, the mode is 2. (1)

23.6 The range is 25 – 1=24. (1)

24.1 Double bar graph (1)

24.2 No, since there is no unit defined representing a collection of passengers. (2)

24.3 The 08:30 flight had the most passengers. (1)

24.4 There was no 06:00 flight, the next earliest time was 08:30. (1)

24.5 198 + 188 + 192 + 145 = 723 (2)

25.1 6 (1)

25.2 Yes, 5. (1)

25.3 Relative frequency = $\frac{1}{3}$ (1)