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Wiskunde

ONDERWYSERSGIDS

GRAAD 9

MET EKSAMENWENKE EN VOORBEELDVRAESTELLE

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Erkennings

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Contents

Term 3

Term 4

How this course works

This series meets the requirements of the National Curriculum and Assessment Policy Statement (CAPS) for the Senior Phase.
In Grade 9, this series consists of two core components: a Learner's Book and a Teacher's Guide.

The Learner's Book

The Learner's Book provides content and subject knowledge as well as activities for learners to develop, practise and consolidate their mathematical knowledge and skills. Written texts are supported by diagrams and illustrations that help to explain content. Examples, exercises and illustrations are representative of South Africa's diverse population.
Exercises steadily become more challenging so that learners progressively develop their understanding of concepts.

The Teacher's Guide

The Teacher's Guide provides you, the teacher, with planning, teaching and assessment tools. Teachers receive guidance on how to teach important concepts and advice on how to teach each exercise.

How to use this Teacher's Guide

The Teacher's Guide covers all Mathematics content and provides rich resources to ensure complete curriculum coverage and the successful development of mathematical concepts and skills in Grade 9.

The Teacher's Guide supports you by providing support and information on how to teach the subject. Some of the features you will find in the Teacher's Guide include the following:

Instructional time allocation

The instructional time in the Senior Phase is as follows:

Subject	Teaching hours per week	Total hours per term
Home Language	5	50
First Additional Language	4	40
Mathematics	4,5	45
Natural Science	3	30
Social Sciences	3	30
Technology	2	20
Economic Management Sciences	2	20
Life Orientation	2	20
Creative arts	2	20
Total	27,5	275

The CAPS for Mathematics

Each CAPS document provides:
- an overview of topics and content areas for its subject (see below)
- the weighting prescribed for each content area (see below)
- a teaching plan for the subject (see Section C – Planning and assessment).

The following content areas comprise the Senior Phase Mathematics curriculum:
- Numbers, Operations and Relationships
- Patterns, Functions and Algebra
- Space and Shape (Geometry)
- Measurement
- Data Handling.

Each content area has a prescribed weighting to ensure complete curriculum coverage.

Content Area	Grade 7	Grade 8	Grade 9
Numbers, Operations and Relationships	30%	25%	15%
Patterns, Functions and Algebra	25%	30%	35%
Space and Shape (Geometry)	25%	25%	30%
Measurement	10%	10%	10%
Data Handling (Statistics)	10%	10%	10%
Total	100%	100%	100%

- Defining the subject, CAPS and teaching terminology for the teacher.

- Providing Formal Assessment Tasks as required by the CAPS.

Chapter 17 Programme of Assessment

This chapter provides all the resources you need to ensure your learners meet the requirements for promotion.
- Two options for each of the required formal programme of assessment tasks.
- Exemplar exams for learners to use to practise for their exams. These exemplars are in the Learner's book with the memoranda supplied in this section of the Teacher's Guide.
- Control tests for you to use as part of the POA, along with the memoranda.
- A June and a December exam paper, with memoranda.

The following table lays out the programme for you. The shaded cells are only in the Teacher's Guide to ensure the tests and exams are unseen by the learners.

Term	Task		Learner's Book page	Teacher's Guide page
1	Assignment	Option 1 Numbers and fractions	427	313
		Option 2 Algebra	328	314
	Control Test 1			315
2	Investigation	Option 1 Quadrilaterals	429	318
		Option 2 Congruency	431	320
	Control Test 2			321
	Exemplar June exam	Exemplar paper for revision purposes	433	324
	June Exam			327
3	Assignment	Option 1 Algebra	438	336
		Option 2 Graphs	439	336
	Project	Option 1 Project	441	338
		Option 2 Volume and surface area	444	340
	Control Test 3			341
4	Assignment	Option 1 3D objects	446	345
		Option 2 Probability	447	346
	Investigation	Option 1 Transformation	448	346
		Option 2 Data	450	348
	Exemplar December exam	Exemplar paper for revision purposes	452	349
	December exam			355

We suggest that in order to prepare learners adequately for formal assessment you use the allocated Revision time prescribed in the CAPS for revising work.

This Series advises that you use the Consolidation exercises and Summaries at the end of each chapter to revise. The Consolidation exercises have mark allocations to enable informal assessment of how learners are managing the specific content area.

Teaching plan for Mathematics Grade 9

This teaching plan shows:
- the pacing of the topics for the course by term
- where to find the relevant content and activities in the Learner's Book
- when Formal Assessment takes place, cross-referenced to suitable activities in the Learner's Book.

This teaching plan follows the time allocations as set out in the CAPS for Mathematics. It assumes six hours of teaching per week.

Term	Content/topics (as per CAPS)	Learner's Book	LB pp.	TG pp.	Time allocation	Assessment
1	Whole numbers	Chapter 1 Units 1–5	11–34	30–42	4,5 hrs	Informal in class assessment Assignment option for POA LB pp.
	Integers	Chapter 1 Units 6–9	35–61	43–58	4,5 hrs	
	Common fractions	Chapter 2 Units 1–2, 5	64–77	59–67	4,5 hrs	
	Decimal fractions	Chapter 2 Units 3–5	78–91	68–76	4,5 hrs	
	Exponents	Chapter 3	92–115	77–98	5 hrs	Informal in class assessment
	Numeric and geometric patterns	Chapter 4 Units 1–2	118–127	99–109	4,5 hrs	Informal in class assessment
	Functions and relationships	Chapter 4 Units 3–4	128–138	109–115	4 hrs	Informal in class assessment
	Algebraic expressions	Chapter 5 Units 1–3	141–155	116–127	4,5 hrs	Informal in class assessment Assignment option for POA
	Algebraic equations	Chapter 5	156–165	127–134	5 hrs	

- Structuring the course into units, with advice on pacing content according to the CAPS.

The National Curriculum and Assessment Policy Statements

This series is based on the National Curriculum Statement Grades R–12 (NCS, January 2012), which is the policy document for learning and teaching in South Africa. The NCS consists of three documents, namely:
- Curriculum and Assessment Policy Statements (CAPS) for all approved subjects for Grades R–12,
- National Policy pertaining to the Programme and Promotion Requirements of the National Curriculum Statement Grades R–12, and
- National Protocol for Assessment Grades R–12 (January 2012).

Together these documents are the basis for determining minimum outcomes, processes and procedures for the assessment of learner achievement in public and independent schools.

Each CAPS document has four sections:
- Section 1: Introduction to the Curriculum and Assessment Policy Statements for the specific subject
- Section 2: The specific subject's aims, time allocations and requirements to offer it as a subject
- Section 3: Overview of topics and teaching plan for the specific subject
- Section 4: Assessment in the specific subject.

Sections 2, 3 and 4 of the CAPS documents, together with the National Policy pertaining to the Programme and Promotion Requirements of the NCS, represent the norms and standards of the National Curriculum Statement Grades R–12.

Instructional time allocation

The instructional time in the Senior Phase is as follows:

Subject	Teaching hours per week	Total hours per term
Home Language	5	50
First Additional Language	4	40
Mathematics	4,5	45
Natural Science	3	30
Social Sciences	3	30
Technology	2	20
Economic Management Sciences	2	20
Life Orientation	2	20
Creative arts	2	20
Total	27,5	275

The CAPS for Mathematics

Each CAPS document provides:
- an overview of topics and content areas for its subject (see below),
- the weighting prescribed for each content area (see below), and
- a teaching plan for the subject (see Section C–Planning and assessment).

The following content areas comprise the Senior Phase Mathematics curriculum:
- Numbers, Operations and Relationships;
- Patterns, Functions and Algebra;
- Space and Shape (Geometry);
- Measurement; and
- Data Handling.

Each content area has a prescribed weighting to ensure complete curriculum coverage.

Inhoud Area	Grade 7	Grade 8	Grade 9
Numbers, Operations and Relationships	30%	25%	15%
Patterns, Functions and Algebra	25%	30%	35%
Space and Shape (Geometry)	25%	25%	30%
Measurement	10%	10%	10%
Data Handling (Statistics)	10%	10%	10%
Total	100%	100%	100%

Topic overview

	Grade 7	Grade 8	Grade 9
Term 1	• Mental calculations • Order and compare whole numbers (9 digits) • Properties of whole numbers • Calculations with whole numbers • Addition and subtraction (6 digits) • Multiplication and division (4-digit by 2-digit) • Multiples and factors (of 2- and 3-digit whole numbers) • Prime factors • LCM and HCF (3-digit whole numbers) • Solve problems (ratio and rate; percentages, decimal fractions; financial context) • Exponents • Measure angles • Construct geometric figures • Classify 2D shapes • Similar and congruent 2D shapes • Solve problems	• Order and compare whole numbers (prime numbers to 100) • Properties of whole numbers • Calculations with whole numbers • Multiples and factors • Solve problems (ratio and rate; percentages, decimal fractions; financial context) • Count, order and compare integers • Calculations with integers • Properties of integers • Solve problems • Represent numbers in exponential form • Calculations in exponential form • Laws of exponents • Numeric and geometric patterns • Input and output values or rules for patterns and relationships • Equivalent forms • Algebraic language • Expand and simplify algebraic expressions • Set up equations and solve by inspection	• Properties of whole numbers and integers • Calculations with whole numbers and integers • Multiples and factors • Solving problems including ratio and rate; direct and indirect proportion; percentages, decimal fractions and financial context • Calculations with integers • Properties of integers • Solving problems • Common fractions • Decimal fractions • Exponents • Calculations in exponential form • Solving problems • Numeric and geometric patterns • Input and output values or rules for patterns and relationships • Equivalent forms • Algebraic language • Expand and simplify algebraic expressions • Equations (use factorisation; of the form where a product of factors = 0)

Term 2	• Common fractions • Percentages • Decimal fractions • Equivalent forms • Solve problems • Input and output values for patterns and relationships • Equivalent forms (verbal, flow diagrams, tables, formulae, number sentences) • Area and perimeter of 2D shapes • Conversion of SI units • Surface area and volume of 3D objects	• Algebraic language • Expand and simplify algebraic expressions • Set up equations and solve by using additive and multiplicative inverses • Construct and investigate geometric figures • Classify 2D shapes • Similar and congruent triangles • Angle relationships • Solve problems	• Investigate properties of geometric figures by construction • Classify 2D shapes • Similar and congruent triangles • Solving problems • Angle relationships • Using the Theorem of Pythagoras • Area and perimeter of 2D shapes (polygons and circles)
Term 3	• Numeric and geometric patterns • Input and output values for patterns and relationships • Equivalent forms • Algebraic language • Number sentences • Interpret and draw graphs • Transformations • Classify 3D objects • Build 3D models	• Common fractions • Percentages • Decimal fractions • The Theorem of Pythagoras • Area and perimeter of 2D shapes • Surface area and volume of 3D objects • Solve problems • Data handling	• Input and output values or rules for patterns and relationships • Algebraic language • Expand and simplify algebraic expressions • Factorise algebraic expressions • Equations • Draw and interpret graphs • Draw linear graphs from given equations • Surface area and volume of 3D objects (include cylinders)
Term 4	• Integers • Numeric and geometric patterns • Input and output values for patterns and relationships • Algebraic language • Number sentences • Data handling • Probability	• Input and output values or rules for patterns and relationships • Equivalent forms • Solve algebraic equations • Interpret and draw graphs • Transformations • Enlargements and reductions • Classify 3D objects • Build 3D models • Probability	• Transformations • Enlargements and reductions • Classify 3D objects • Build 3D models • Data handling • Probability

Planning

Types of planning tools

The following planning tools are provided:
- A teaching plan
- A sample lesson plan

Teaching plan for Mathematics Grade 9

This teaching plan shows:
- the pacing of the topics for the course by term;
- where to find the relevant content and activities in the Learner's Book; and
- when Formal Assessment takes place, cross–referenced to suitable activities in the Learner's Book.

This teaching plan follows the time allocations as set out in the CAPS for Mathematics. It assumes six hours of teaching per week.

Term	Inhoud/topics (as per CAPS)	Learner's Book	LB bladsy	TG bladsy	Time allocation	Assessment
1	Whole numbers	Chapter 1 Units 1–5	11–34	28–43	4,5 hrs	Informal in class assessment Assignment option for POA LB p. 427
	Integers	Chapter 1 Units 6–9	35–62	43–58	4,5 hrs	
	Common fractions	Chapter 2 Units 1–2, 6	63–77	59–67	4,5 hrs	
	Decimal fractions	Chapter 2 Units 3–5	78–91	68–76	4,5 hrs	
	Exponents	Chapter 3	92–116	77–98	5 hrs	Informal in class assessment
	Numeric and geometric patterns	Chapter 4 Units 1–2	117–127	99–109	4,5 hrs	Informal in class assessment
	Functions and relationships	Chapter 4 Units 3–4	128–138	109–115	4 hrs	Informal in class assessment
	Algebraic expressions	Chapter 5 Units 1–3	140–155	116–127	4,5 hrs	Informal in class assessment Assignment option for POA LB p. 428
	Algebraic equations	Chapter 5 Unit 4	156–165	127–134	5 hrs	
	Revision				4 hrs	Informal in class assessment
	End of term test				1 hr	Formal assessment

Term	Inhoud/topics (as per CAPS)	Learner's Book	LB bladsy	TG bladsy	Time allocation	Assessment
2	Constructions	Chapter 6	166–193	135–150	9 hrs	Informal in class assessment Investigation option for POA LB p. 429
	Geometry of 2D shapes	Chapter 7 Units 1–3	195–208	151–158	9 hrs	Informal in class assessment Investigation option for POA LB p. 431
	Geometry of straight lines	Chapter 7 Units 4–6	209–226	158–167	9 hrs	Informal in class assessment
	Theorem of Pythagoras	Chapter 8 Unit 1	227–234	169–175	5 hrs	Informal in class assessment
	Test				1 hr	Formal assessment
	Area and perimeter	Chapter 8 Units 2–4	235–252	175–191	5 hrs	Informal in class assessment
	Revision				5 hrs	Informal in class assessment
	Mid Year Exam				2 hrs	Formal assessment

Term	Inhoud/topics (as per CAPS)	Learner's Book	LB bladsy	TG bladsy	Time Allocation	Assessment
3	Functions and relationships	Chapter 9	252–264	192–200	5 hrs	Informal in class assessment
	Algebraic expressions	Chapter 10 Units 1–2	265–282	201–213	9 hrs	Informal in class assessment Assignment option for POA LB p. 336
	Algebraic equations	Chapter 10 Units 3–5	283 –294	214–222	9 hrs	
	Graphs	Chapter 11	295–321	223–238	12 hrs	Informal in class assessment Assignment and Project option for POA LB p. 336, 338
	Volume and surface area	Chapter 12	322–343	239–255	5 hrs	Informal in class assessment Project option for POA LB p. 340
	Revision				4 hrs	Informal in class assessment
	End of term test				1 hr	Formal assessment

Term	Inhoud/topics (as per CAPS)	Learner's Book	LB bladsy	TG bladsy	Time Allocation	Assessment
4	Transformation Geometry	Chapter 13	344–372	256–272	9 hrs	Informal in class assessment Investigation option for POA LB p. 448
	Geometry of 3D objects	Chapter 14	373–389	273–279	9 hrs	Informal in class assessment Assignment option for POA LB p. 446
	Collect, organise and summarise data	Chapter 15 Units 1–2	390–397	281–288	4 hrs	Informal in class assessment Investigation option for POA LB p. 450
	Represent data	Chapter 15 Unit 3	398–402	288–293	3 hrs	
	Analyse, interpret and report data	Chapter 15 Unit 4	403–409	293–297	3,5 hrs	
	Probability	Chapter 16	410–425	298–311	4,5 hrs	Informal in class assessment Assignment option for POA LB p. 447
	Revision				10 hrs	Informal in class assessment
	End of year exam				2 hrs	Formal assessment

Example of a lesson plan

Some teachers may find daily lesson plans useful, although these are not a formal policy requirement. An example of how to complete a lesson plan is below.

Date:	**Grade:** 9	**Term:** 1
Chapter: 1	**Unit:** 1	**Contact time:** 5 hours
Inhoud/concept: Whole numbers	**Exercise:** 1–5	**Resources required:**
Exercise 1		
Links with previous knowledge or exercise: Learners should already be able to: • do mental calculations with whole numbers including multiplication facts up to 12×12 • do calculations with whole numbers, integers, common and decimal fractions in written form as well as using a calculator • identify prime numbers up to 100 • list the properties of whole numbers, integers, common and decimal fractions • understand the equivalent forms of common fraction and decimal fraction forms of the same number • order and compare whole numbers, integers, common and decimal fractions.		
Links with next activity: This exercise helps learners revise the necessary concepts in preparation for comparing and operating with whole numbers and integers.		
Teaching plan • Spend time assessing learners' knowledge of the basic skills, as listed above, before doing the exercises in this unit. Learners may struggle if this revision is not done. • Revise the various terms used in this unit, such as prime, composite, whole numbers, counting numbers, substitution and integers. • Ensure learners can distinguish between the different kinds of numbers. • Do at least one example together as a class. Use either the worked example in the Learner's Book or supply an additional example. • Learners complete the exercise on their own. • Mark the work in class, by writing the answers on the board, and learners mark their own work. • Walk round the class and observe how learners coped with the exercise. • Identify learners who experienced difficulty and assign similar practice examples for homework.		
Assessment: Informal self-assessment with teacher supervision		
Teacher reflection:		

Assessment

Assessment is the planned process of identifying, gathering and interpreting information about learners' performance on an on going basis. Assessment should be both informal and formal, and a variety of assessment tasks should be used. Learners should timeously receive feedback on both informal and formal assessment.

The four steps of assessment

1 Generate and collect evidence of achievement.
2 Evaluate the evidence.
3 Record the findings.
4 Use the findings to guide future learning and teaching.

Types of assessment

Type of assessment	Description
Baseline assessment	Establishes whether learners meet basic skills and knowledge level required. Helps teacher plan for the year and for each learner. Is administered at the beginning of the year and before a particular topic. Results are used as a guide for teaching and not for promotion purposes.
Diagnostic assessment	Informs the teacher about certain specific problem areas that may hinder performance. May help determine whether a learner's problems are content or psycho-social based. Appropriate interventions should follow on from diagnostic assessment. Results should inform interventions and not be used for promotion purposes.
Formative assessment	Used to aid the learning process and not for promotion purposes. Usually informal, to provide the teacher and learner with a more frequent account of where the learner is at. Teachers can use this form of assessment to modify and adapt their own teaching.
Summative evaluation	Carried out after completion of a topic or cluster of topics. Is an assessment of learning that has taken place. Recorded and used for promotion. This is usually formal assessment, making up the Formal Programme of Assessment.

Informal assessment

Informal assessment is the monitoring of learners' progress through observations, discussions, practical demonstrations, learner–teacher conferences, and informal classroom interactions. All daily exercises completed in the mathematics classroom can be used for informal assessment.
CAPS tells us that informal assessment should be used to provide feedback to the learners and to inform planning for teaching, but need not be recorded or taken into account for promotion. It should not be seen as separate from learning activities taking place in the classroom. Learners or teachers can mark these assessment tasks.

Formal assessment

Certain tasks make up the formal programme of assessment for the year. Formal assessment tasks are marked and formally recorded by the teacher for progression and certification purposes. All formal assessment tasks are subject to moderation for the purpose of quality assurance and to ensure that appropriate standards are maintained. Formal assessment provides teachers with a systematic way of evaluating how well learners are progressing in a grade and in a particular subject. Examples of formal assessments include tests, examinations, projects, assignments and investigations. Formal assessment tasks form part of a year–long formal programme of assessment in each grade and subject.

Formal Programme of Assessment

Formal assessment requirements of Mathematics

The forms of assessment used should be appropriate for the learners' ages and developmental levels. Learners must complete formal assessments each term. Formal assessments include formally assessed tasks, along with projects and examinations.
The Formal Programme of Assessment as prescribed by the CAPS is shown below. This Programme of Assessment is generic across the three grades in the Senior Phase, and lists the types of formal assessments required each term.

Minimum requirements for formal assessment

	Forms of assessment	Minimum requirements per term				Number of tasks per year	Weighting
		Term 1	Term 2	Term 3	Term 4		
SBA (School-based Assessment)	Tests	1	1	1		3	40%
	Examination		1			1	
	Assignment	1		1	1	3	
	Investigation		1		1	2	
	Project			1		1	
	Total	2	3	3	2	10*	
Final examination		End of year				1	60%

* To be completed before the final examination at the end of the year

Types of formal assessment for Mathematics

Tests and examinations

These are individual assessment tasks. Tests and examinations for formal assessment should cover a substantial amount of content. Tests and examinations must be completed under strictly controlled conditions.

Each test and examination must cater for a range of cognitive levels in the correct allocation (see the table below).

Cognitive level	Description of skill to be demonstrated
Knowledge ≈ 25%	• Estimation and appropriate rounding of numbers • Straight recall • Identification and direct use of correct formula • Use of mathematical facts • Appropriate use of mathematical vocabulary
Routine procedures ≈ 45%	• Performing of well-known procedures • Simple applications and calculations which might involve many steps • Derivation from given information may be involved • Identification and use (after changing the subject) of correct formulae generally similar to those encountered in class
Complex procedures ≈ 20%	• Problems involving complex calculations and/or higher-order reasoning • Investigate elementary axioms to generalise them into proofs for straight line geometry, congruence and similarity • No obvious route to the solution • Problems not necessarily based on real-world contexts • Making significant connections between different representations • Require conceptual understanding
Problem-solving ≈ 10%	• Unseen non-routine problems (which are not necessarily difficult) • Higher-order understanding and processes are often involved • Might require the ability to break the problem down into its constituent parts

Projects

Learners complete *one* project in Mathematics in each grade. Projects can be used to test a range of skills and competencies. It is prescribed that learners complete a project in Term 3 of each grade. Projects must provide learners with the ability to demonstrate their understanding of a mathematical concept and apply it to a real–life situation. Be wary of prescribing projects that are beyond the cognitive level of the learners, or that will simply involve replicating or "cutting and pasting" facts and data from reference material.

Assignments

An assignment is also an individual task, similar to tests and examinations. However, assignments should be an extended piece of work with a focus on more demanding work than that covered in class. Three assignments per year are required by the CAPS. Assignments can include past questions, but should also include more challenging aspects encouraging the learner to use additional material to help them. Assignments can be completed at home.

Investigations

An investigation should be used to discover rules or concepts. It is recommended that learners should conduct investigations in class as much as possible, and that the final written task definitely be done in class. Rubrics are used to assess investigations. *Two* investigations per year are required by the CAPS.
The skills involved in investigations include:
- organising and recording ideas and discoveries in tables and diagrams,
- explaining ideas in appropriate forms,
- showing clear understanding of concepts and procedures through calculations, and
- generalising and drawing conclusions.

Guidelines for Assessment Tasks

Tasks should be designed to cover the content and concepts of the subject and include a variety of activities selected to assess learner's knowledge and skills.
Before handing out an assessment task to learners, teachers should ensure that they are able to answer all the questions themselves. When teachers set an assessment task, they should draw up a memorandum of answers and/or a rubric for the assessment. Refer to the seven–point rating code or scale of achievement when constructing a rubric.
Feedback should acknowledge strengths and clearly identify areas of weakness for development. Action plans on how learners will be supported should accompany this feedback. It is important that the feedback provided to learners encourages them to do better, and builds their self–confidence.

Planning for assessment

We have provided a full assessment plan for you to use.

Programme of Assessment

Term	Task	Topics	Page ref	
			LB	TG
1	Assignment	Option 1: Numbers and fractions	427	313
		Option 2: Algebra	428	314
	Test	Chapters 1–5		315
2	Investigation	Option 1: Quadrilaterals	429	319
		Option 2: Congruency	431	320
	Test	Chapters 6 and 7		321
	June Examination	Chapters 1–8		327
3	Assignment	Option 1: Algebra	438	336
		Option 2: Graphs	439	336
	Project	Option 1: Graphs	441	338
		Option 2: 3D objects	444	340
	Test	Chapters 9–12		341
4	Assignment	Option 1: 3D objects	446	345
		Option 2: Probability	447	346
	Investigation	Option 1: Transformation	448	347
		Option 2: Data	450	349
	December Examination	Chapters 1–17		355

Inclusive assessment

Teachers need to develop adaptive and alternative methods to assess learners with barriers to learning, so that learners are given opportunities to demonstrate competence in ways that suit their abilities. Here are some examples of how to assess these learners while still maintaining the validity of the assessment.

- Some learners may need concrete apparatus for a longer time than their peers.
- Assessments tasks, especially written tasks, may have to be broken up into smaller sections for learners who cannot concentrate or work for a long time, or they may be given short breaks during the tasks. Learners can also be given extra time to complete tasks.
- Some learners may need to do their assessment tasks in a separate venue to limit distractions.
- A variety of assessment instruments should be used, as a learner may find that a particular assessment instrument does not allow him/her to show what they can do.

- Learners who cannot read can have tasks read to them and they can orally dictate answers. Assessment can also include a practical component in which learners can demonstrate their competence without having to use language.
- A sign language interpreter can be used.
- Assessment tasks could be available in Braille or enlarged with bolded text.
- Assessment can include the use of Dictaphones or computers with voice synthesisers.
- The forms of assessment used should be age and developmental level appropriate. The design of these tasks should cover the content of the subject and include a variety of tasks designed to achieve the objectives of the subject.

Recording and reporting assessment

- Recording: Recording documents the level of a learner's performance in a specific assessment task. It indicates learner progress towards the achievement of the knowledge as prescribed in the curriculum. Records of learner performance should be used to verify the progress made by teachers and learners in the teaching and learning process.
- Reporting: Learners' performance can be reported in a number of ways. These include report cards, parents' meetings, school visitation days, parent–teacher conferences, phone calls, letters, class or school newsletters etc. Teachers in all grades report in percentages against the subject. The various achievement levels and their corresponding percentage bands are as shown in the table below.

Rating code	Description of competence	Marks
7	Outstanding achievement	80–100
6	Meritorious achievement	70–79
5	Substantial achievement	60–69
4	Adequate achievement	50–59
3	Moderate achievement	40–49
2	Elementary achievement	30–39
1	Not achieved	20–29

Metacognitive Strategies

What are metacognitive strategies and how can I use them?

Metacognition is the process of thinking about how you think. Adults often do this automatically. Before taking on something new, we may ask ourselves: What do I already know about this? What will help me understand it better? How is it structured? As we engage with a text or action, we may ask ourselves: Did I understand that? Why do I think that? How does this connect with what I already know? How could I apply this in my life? Then we evaluate what we have learnt or done by asking questions like: Did I understand that well? What strategies helped and what strategies didn't help? What should I do the next time I take on a task like this? Learners, however, are often unaware of how they think and engage with learning material. You help learners to learn independently by explicitly guiding them to plan, monitor, and evaluate their reading and learning strategies. This is particularly effective for those learning in English as a second language and for learners who are struggling. It can dramatically improve their performance.

You teach metacognitive skills by asking learners to explain what they are thinking and what strategies they are using to understand material. This is best done in small groups. You can also use "think aloud" strategies when engaging with texts and images. "Think–alouds" are often effective when reading texts to learners, and during small–group and pair reading exercises. Here is an example of how to teach metacognitive strategies using a 'think aloud':

1. Choose a short piece of text and note where you will stop during reading to model your thought processes.
2. Things to include in this planning stage could be:
   - reading the text title and the table of contents,
   - looking at the images and predicting what the text may be about, and
   - skim–reading the text looking for headings, words in bold, and summaries. As you skim read, think about what you already know about the subject and what more you would like to know.
3. In class, explain what you will be doing to the learners. Start by explaining how you planned before reading the text.
4. To monitor understanding during reading, you can explain where you stopped to ask yourself whether you understood the content. If the text has a long or complex sentence, describe how you divided it up to understand it. Find places where you could ask questions such as:
   - Why would this ....?
   - Is this similar to ....?
   - How can I figure out what this new word means?
   - What does the writer want me to know?
   - What do I think will happen next? Why do I think that?
   - Do I need to re–read this for detailed information?
5. Now show learners how to evaluate their metacognitive strategies by asking and answering questions such as:
   - Did I read and understand this well?
   - What helped me to understand? What didn't help?
   - What should I do next time I read about this topic?
   - What will help me remember what I read?

"Think–alouds" can be used very effectively in Maths classes to model logical and analytical thought. This technique can be used to make clear to learners hidden thought processes such as where to begin with a problem, what instructions mean or how to pick out the key information in problem solving examples.

By engaging with how learners think, you can better prepare them for their lives and learning in the future.

What is Mathematics?

Mathematics is a language. It uses symbols and notations to describe relationships. Mathematics is a human activity that involves observing, representing and investigating patterns and relationships in both the physical and social dimensions. Mathematics helps develop key mental processes such as logical and critical thinking, accuracy and problem–solving. All of these processes contribute to a learner's decision–making ability.

The specific aims of Mathematics

The aims of Mathematics are to develop:
- a critical awareness of mathematical relationships,
- confidence and competence in Mathematics without fear of the subject,
- curiosity and a love of Mathematics,
- appreciation for the beauty and elegance of Mathematics,
- recognition of the subject as a creative art,
- the deep conceptual understanding required to understand Mathematics,
- the acquisition of specific skills in order to apply Mathematics,
- study related subject matter, and
- to encourage further studies in Mathematics.

Specific skills for Mathematics

In order to develop the essential skills, the learner should:
- develop correct mathematical language use;
- develop number vocabulary, number concept and application skills;
- learn to listen, communicate, reason logically and apply the knowledge gained;
- learn to investigate, analyse, represent and interpret information;
- learn to pose and solve problems; and
- be aware that Mathematics plays a key role in real–life situations.

Inhoud area focus in the Senior Phase

Inhoud area	General focus	Specific focus
Numbers, Operations and Relationships	Meaning of different kinds of numbers Relationships between different kinds of numbers Relative sizes of different numbers Representations of numbers in various ways Operations with numbers Estimation and checking solutions	Represent numbers in a variety of ways and move flexibly between representations Recognise and use properties of operations with different number systems Solve a variety of problems Use an increased range of numbers and are able to perform multiple operations correctly and fluently
Patterns, Functions and Algebra	Achieve efficient manipulation skills which will carry over into other domains of the subject Describe patterns and relationships through the use of symbolic expressions, graphs and tables Identify and analyse regularities and change in patterns Make predictions and solve problems	Investigate numerical and geometric patterns to establish the relationships between the variables Express rules governing patterns in algebraic language or symbols Develop algebraic manipulative skills to recognise the equivalence between different representations of the same relationship Analyse situations in a variety of contexts Use different and equivalent representations: algebraic language, formulae, expressions, equations and graphs
Space and Shape (Geometry)	Properties of shapes and objects Relationships between these properties Orientations, positions and transformations of two-dimensional shapes and three-dimensional objects	Draw and construct a wide range of geometric figures and solids using appropriate instruments Appreciate the use of constructions to investigate the properties of geometric figures and solids Develop clear and precise descriptions and classification categories of geometric figures and solids Solve geometric problems drawing on known properties of geometric figures and solids
Measurement	Select and use appropriate units, instruments and formulae Make sensible estimates Be aware of sensibleness and reasonableness of measurement and results	Use formulae for measuring area, perimeter, surface area and volume of geometric figures and solids Select and convert between units of measurement Use the Theorem of Pythagoras to solve problems involving right-angled triangles
Data Handling	Ask questions and find answers in order to describe events and the social, technological and economic environment Collect, organise, represent, analyse, interpret and report data Enabled to make informed predictions by the study of probability Describe randomness and uncertainty	Pose questions for investigation Collecting, summarise and represent and critically analyse data Interpret, report and make predictions about situations Better understand probability – include single and compound events and their relative frequency in simple experiments

Teaching Mathematics in the Senior Phase

The Senior Phase can be a difficult phase to teach. Each grade brings with it an added challenge. In Grade 7 learners are in the final year of Primary School and preparing for the transition to High school. In Grade 8 learners are adjusting to a new school, new peers and usually a new way of teaching and learning. In Grade 9 learners need to select their subjects for matric, and ultimately start preparing for their career and a life outside of school. Each of these periods add additional pressure and stress to learners in this phase.

Senior Phase learners do Mathematics every day. In this phase learners move from Mathematics with a primarily arithmetic focus to that of a more formal and abstract approach. Essentially learners start doing 'real' mathematics. In order to cope with this shift, learners must be challenged to think abstractly and critically, and not to merely copy formulae and do substitutions.

The writing of formal tests and exams becomes even more important. The Mathematics teacher must spend time developing exam techniques which include unpacking terminology used in exams, such as determine, identify, deduce, predict, present, summarise, expand, suggest, illustrate, and so on. The Learner's Book provides many built–in opportunities for learners to engage with these. Section D of the Teacher's Guide also provides a list of important terminology. Presentation answers, time management, and stress management are all important areas in which learners should receive on–going coaching. Mathematics teachers should work closely with Life Orientation teachers in order to support learners.

The volume and depth of material that learners are expected to engage with in this phase are higher. Learners are expected to start marking their own work (from the board) and this is new to many Grade 8 learners. They will need support and instruction as they learn how to manage this form of responsibility. Learners will need help to understand that marking their own work is a form of self–assessment. This self–assessment should inform learners' awareness of their own strengths and weaknesses.

Grade 9 is a crucial year in the teaching of Mathematics: learners are required to make a choice between Mathematics and Mathematical Literacy in Grade 10. This choice will be based on their experience and level of success achieved in Grade 9. For learners who have some idea of their future career, the choice between these two might be somewhat more straightforward. Both Mathematics and Life Orientation teachers are a valuable source of information and guidance for learners making curriculum and career choices. Take care to inform learners as early as possible of the consequences of choosing Mathematics or Mathematical Literacy for future study and career prospects. It is of upmost importance that teachers lay a good foundation for basic algebra and geometry in Grades 8 and 9 in order to facilitate learners who wish to pursue further Mathematics in Grade 10.

Inclusive teaching

What is inclusive teaching?

In the Senior Phase, it is crucial that learners find themselves in an environment where they can develop an interest in learning and the belief that they can learn. Inclusive Education is defined as the creation of a learning environment that promotes the full personal, academic and professional development of all learners irrespective of race, class, gender, disability, religion, culture, sexual preference, learning styles and language.
Inclusion is about acknowledging and respecting:

- that all children have the right to learn;
- that all children can learn;
- that all learners need support;
- that all learners are unique and have different, but equally valued, learning needs;
- that all learners need the opportunity to build on their own unique strengths;
- that the learner is the centre of the teaching and learning process; and
- that there are differences in learners, such as age, gender, language, culture, learning styles, disabilities, HIV status and so on.

Inclusion is also about:

- enabling educational structures, systems and learning methodologies to meet the needs of all learners;
- acknowledging both formal and informal learning–it embraces learning that occurs in the home, community and so on;
- changing attitudes, behaviour, methodologies and environments to meet the needs of all learners;
- ensuring maximum participation of all learners in the culture and curriculum of all educational institutions; and
- identifying and minimising barriers to learning that can occur at any level of the system.

Some of the learners in your class may already suffer from exclusion or think negatively about education. There is no reason for their exclusion from class activities. It is the responsibility of the teacher to ensure the inclusion of these learners. This means adapting activities to suit their needs and capabilities. It is equally important that the class is not divided because of this. Rather, learners with these challenges should be accepted and helped where possible by their peers. Learners should at all times be discouraged from teasing, bullying or ignoring learners with special needs. When these attitudes are directed towards a learner they create in that learner a barrier to learning.

Practical guidelines for inclusive teaching

- Have a detailed understanding of each learner's background, strengths, unique abilities, needs and barriers. Then use this information to inform your planning and give a clearer focus.
- Remember that the teacher is a facilitator of learning.
- Keep the content and material as relevant as possible.
- Break down learning into small, manageable and logical steps. Keep instructions clear and short (plan beforehand).
- Grade activities according to the different levels and abilities of learners. Try to ensure that learners remain challenged enough without undue stress.
- Develop a balance between individual, peer tutoring, cooperative learning and whole class teaching.
- Use learners to help one another in the form of group types, peer assisted learning, buddy systems and so on. Ensure that learners feel included and supported in the classroom by both the teacher and their peers.
- Set up pairs and groups of learners where members can have different tasks according to strengths and abilities. Promote self−management skills and responsibility through group roles and the types of tasks you set. However, ensure learners do not become 'pigeon−holed' in to a particular role in a group. Encourage learners to take on tasks that build on their strngths and develop their weaker areas.
- Motivate learners and affirm their efforts and individual progress. Build confidence. Encourage questioning, reasoning, experimentation with ideas and risking opinions.
- Determine the learner's Zone of Proximal Development (ZPD) and use it for effective teaching and learning. Vygotsky described the ZPD as the distance between what the learner already knows and understands and what he/she can understand with 'expert' support. Learning is thus a social interaction as the teacher or more skilled peer who mediates and supports the learner as he/she understands a new concept.
- Spend time on consolidating new learning. Use different ways to do this until all learners understand the concept. Make time to go back to tasks so that learners can learn from their own and others' experiences and methods.
- Use and develop effective language skills (expressive and receptive, verbal and non−verbal).
- Experiment with a variety of teaching methods and strategies to keep learners interested and to cater for and develop different learning styles. Use games, co−operative group work, brainstorming, problem−solving, debates, presentations, and so on.

Learners with barriers to learning

A barrier to learning is anything that prevents a learner from participating fully and learning effectively. This includes learners who were formerly disadvantaged and excluded from education because of the historical, political, cultural and health challenges facing South Africans. Some other examples of barriers to learning may be learners who are visually or hearing impaired; learners who are left handed or learners who are intellectually challenged. Barriers to learning cover a wide range of possibilities and learners may experience more than one barrier. Some barriers, therefore, require more than one adaptation in the classroom and varying types and levels of support.

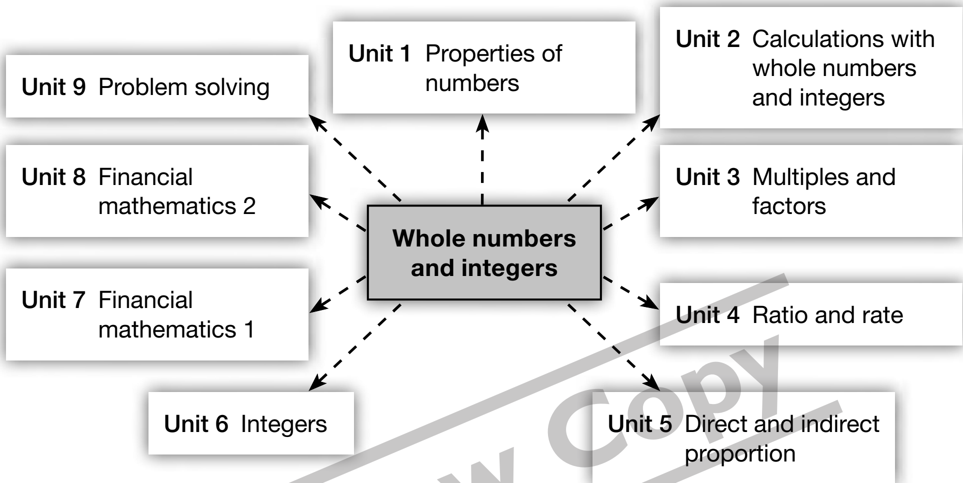
These learners may require and should be granted more time for:

- completing tasks,
- acquiring thinking skills (own strategies), and
- assessment activities.

Teachers need to adapt the number of activities to be completed without interfering with the learners gaining the required skills.

Chapter 1 Numbers and integers

Chapter overview



Content		Time allocations	LB page
Unit 1	Properties of whole numbers	1 hour	12
Unit 2	Calculations with whole numbers	1 hour	19
Unit 3	Multiples and factors	1 hour	22
Unit 4	Ratio and rate	1 hour	26
Unit 5	Direct and indirect proportion	1 hour	31
Unit 6	Integers	1 hour	35
Unit 7	Financial mathematics 1	1 hour	41
Unit 8	Financial mathematics 2	1 hour	48
Unit 9	Problem solving	1 hour	54

Background information

In previous grades, the learners learnt:
- about whole numbers, integers, common and decimal fractions; how to do calculations (mental, on paper and using a calculator); and some of the properties of operations;
- how to calculate squares, square roots, cubes and cube roots of integers
- about ratio and rate including speed as distance per time unit
- topics of financial mathematics such as profit and loss, budgets, accounts, loans, simple interest and hire purchase
- to solve word problems involving the above topics.

Interesting information about some of the topics covered in this chapter can be found on web sites of the internet, for example:
- a value of π up to 1 000 places of decimal. Just Google "value of Pi" to access the relevant web sites.
- Also on financial topics such as interest rates; e.g. the prime interest rate.

Teaching guidelines

- This chapter covers a wide variety of topics and implied skills, so you need to monitor learners' mastering of the learning material closely.
- When demonstrating Worked examples on the board, encourage learners to participate by answering questions like "By what prime number is this whole number divisible?" (when doing HCF and LCM).
- Exercises must at least partially be done in the classroom under supervision. Monitor the learners' progress and provide help and remediation where required.
- In order to draw graphs of direct and indirect proportion relationships, the learners must be provided with suitable grid paper, like cm^2-dotted paper.
- Information about current interest rates of banks and other financial institutions, like money lenders, should be obtained in order to make the learning of simple and compound interest practical and relevant for the learners.

Resouces

- Number cards, comparison card (cards with <, > and =), number lines, HTU grids up to millions, grid paper, cardboard, colour pens, poster boards, counters, scissors, rulers. Each learner should have their own calculator. Provide current financial information from newspaper and banks regarding budgets and interest rates. Have store flyers advertising sales, utility bills and store accounts to show learners the use of financial mathematics every day.

Unit 1 Properties of whole numbers and integers

Learner's Book page 12

Unit focus

This unit focusses on the following:
- the properties of natural numbers;
- whole numbers, integers, rational numbers, irrational numbers; and
- the sets of numbers that constitute thereal number system.

Background information

Learners should already know how to:
- do mental calculations with whole numbers including multiplication facts up to 12×12
- do calculations with whole numbers, integers, common and decimal fractions in written form as well as using a calculator
- identify prime numbers up to 100
- identify the properties of whole numbers, integers, common and decimal fractions
- work with the equivalent forms of common fraction and decimal fraction forms of the same number
- order and compare whole numbers, integers, common and decimal fractions.

If necessary spend time assessing learners' knowledge of these skills before doing the exercises in this unit.

Exercise 1

Learner's Book page 13

Guidelines on how to implement this activity

Ensure learners understand the various terms used in this unit and are able to distinguish between the different kinds of numbers. For this exercise revise prime numbers, composite numbers and substituting values to complete the table.

Suggested answers

1.1 5, 19, 47 and 53 **1.2** No prime numbers **1.3** 21, 39 and 95
2 $39 = 3 \times 13$ $95 = 5 \times 19$

Remedial

Some learners may have difficulty distinguishing between prime numbers and composite numbers (Exercise 1). Revise the definition of a prime number and how to apply the definition.

Extension

Challenge learners to investigate whether there is a largest prime number. There are web sites about this topic, just Google "prime number".

Exercise 2

Learner's Book page 15

Guidelines on how to implement this activity

Do revision of the types of numbers the learners are supposed to know, especially the names and characteristics of each. Number lines drawn on the board or posters are very useful aids for this purpose. Motivate the extension of $\mathbb{N}_0$ to $\mathbb{Z}$ to make subtraction always possible and from $\mathbb{Z}$ to $\mathbb{Q}$ to make division (except division by 0) always possible.

Suggested answers

1.1	$4 > 0$	**1.2**	$-1 < 1$	**1.3**	$-2 < 0$
1.4	$0 > -10$	**1.5**	$-3 > -4$	**1.6**	$-10 < -7$
1.7	$-25 < 24$	**1.8**	$-100 < -99$		
2.1	$5 - 9 = -4$	**2.2**	$-1 + 110 = 109$		
2.3	$2 - (-10) = 12$	**2.4**	$0 - (-111) = 111$		
2.5	$12 \times (-12) = -144$	**2.6**	$(-7) \times (-6) = 42$		
2.7	$(-39) \div (-3) = 13$	**2.8**	$10 \div (-3) = -\frac{10}{3} = -3{,}333$		

Remedial

Some learners may have difficulty with ordering and operations with integers. Revise these skills with them by starting with positive numbers and extending this to include negative numbers. Have number lines available for learners to refer to.

Exercise 3

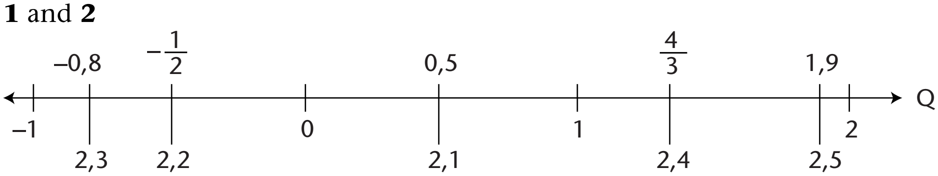
Learner's Book page 16

Guidelines on how to implement this activity

Discuss the concept of rational numbers. Make sure that the learners can recognise all the various forms of rational numbers, including recurring decimals. Do examples together as a class using number lines on the board.

Suggested answers

1 and **2**



Remedial

Some learners may have difficulty fully understanding the definition of $\mathbb{Q}$ (the set of rational numbers) and how to indicate the position of a rational number (that is not an integer) on a number line. Explain this to them with examples.

Exercise 4

Learner's Book page 17

Guidelines on how to implement this activity

The concept of irrational numbers may be strange to the learners although they should know that π is an irrational number. Use various calculator displays (8-digit and 10-digit) to bring over the idea that an irrational number has an infinite non-recurring decimal form.

Suggested answers

1 $\sqrt{3} = 1{,}73205$ correct to 5 decimal places

2 $\sqrt{10} = 3{,}162278$ correct to 6 decimal places

3 $\sqrt{0{,}5} = 0{,}71$ correct to 2 decimal places

4 $\sqrt[3]{2} = 1{,}2599$ correct to 4 decimal places

5 $\sqrt[3]{0{,}5} = 0{,}7937$ correct to 4 decimal places

6 $\sqrt[3]{-10} = -2{,}154435$ correct to 6 decimal places

Remedial

Some learners may find it difficult to conceptualise the set of irrational numbers, $\mathbb{Q}'$. Help them to understand that there are numbers whereby the decimal form does not terminate and does not display a recurring pattern. Some learners may have difficulty in writing rational approximations, correct to a required number of decimal places, of irrational numbers.Work through a few examples with them.

Extension

Challenge learners to investigate the value of π in decimal form: the number of decimal places that experts have calculated using computers. Google "value of Pi".

Exercise 5

Learner's Book page 18

Guidelines on how to implement this activity

The purpose of this unit is to get the learners to form a concept of the Real number system. Before starting the exercise, revise the Real number system using the summary in the Learner's Book. We can use Exercise 5 as an informal assessment. Learners who manage Exercise 5 have acquired the concept well.

Suggested answers

Number	$\mathbb{N}_0$	$\mathbb{Z}$	$\mathbb{Q}$	$\mathbb{Q}'$	$\mathbb{R}$
0	yes	yes	yes	no	yes
–15	no	yes	yes	no	yes
$0{,}\overline{142857}$	no	no	yes	no	yes
$\sqrt{3}$	no	no	no	yes	yes
$\sqrt{\frac{1}{4}}$	no	no	yes	no	yes
$\frac{1}{6}$	no	no	yes	no	yes
$\sqrt{19}$	no	no	no	yes	yes
$-\sqrt{144}$	no	yes	yes	no	yes
$\frac{3}{11}$	no	no	yes	no	yes
$\sqrt{0{,}9}$	no	no	no	yes	yes
$\sqrt[3]{64}$	yes	yes	yes	no	yes
$\sqrt[3]{-1}$	no	yes	yes	no	yes
$\sqrt{-1}$	no	no	no	no	no

Remedial

Some learners may have difficulty in understanding the inclusiveness of the set of real numbers of all the other kinds of numbers dealt with in this unit. Use diagrams to help learners understand the interrelationships between the sets of numbers.

Unit 2 Calculations with numbers and integers

Learner's Book page 19

Unit focus

This unit focusses on the following:
- Revision of calculation techniques for working with whole numbers such as
  - estimating your answer;
  - adding, subtracting and multiplying with columns;
  - long division;
  - rounding off and compensating; and
  - using a calculator.

Background information

Learners should already be familiar with all the following concepts. This should be revision, however it is important that you ensure learners can do:
- mental calculations with whole numbers including multiplication up to 12 × 12;
- calculations with whole numbers mentally as well as using a calculator;
- identify prime numbers up to 100; and
- list the properties of whole numbers.

Exercise 1 Learner's Book page 21

Guidelines on how to implement this activity

- Establish whether the learners can answer multiplication facts orally and quickly. Remediate if necessary.
- Revise the columns-method for doing addition and subtraction with whole numbers by doing relevant examples on the board with the participation of the learners. Also do long multiplication.
- Revise the long division method for whole numbers by doing a couple of graded examples, from single-digit-divisors to double-digit divisors. Some learners are likely to have difficulty with this. Let the learners start Exercise 1 in the class and finish it, if necessary, for homework.
- Revise the rounding off and compensating of big whole numbers and the estimation of calculations with whole numbers. Let the learners start Exercise 2 in the class and finish it, if necessary, for homework.
- Revise the properties of whole numbers such as the commutative, associative and distributive properties. Let the learners start Exercise 3 in the class and finish it, if necessary, for homework.

Suggested answers

1.1 109 786 + 45 379 = 155 165

1.2 239 876 + 345 789 + 654 897 = 1 240 562

1.3 324 987 + 90 000 + 87 654 = 502 641

1.4 65 900 – 56 599 = 9 301

1.5 108 778 – 99 987 = 8 791

1.6 459 653 – 107 996 = 351 657

1.7 4 589 × 545 = 2 501 005

1.8 8 945 × 489 = 4 374 105

1.9 15 852 × 745 = 11 809 740

1.10 89 450 × 852 = 76 211 400

2.1 50 232 ÷ 56 = 897

2.2 34 650 ÷ 22 = 1 575

2.3 78 950 ÷ 25 = 3 158

2.4 52 275 ÷ 123 = 425

2.5 97 976 ÷ 296 = 331

3.1 345 889 + 231 123
Estimate:
3500 000 + 230 000 ≈ 580 000
350 000 − 345 889 = 4 111
230 000 − 231 123 = −1 123
4 111 + (−1 123) = 2 988
345 889 + 231 123
= 580 000 – 2 988
= 577 012

3.2 654 098 + 34 876
Estimate:
650 000 + 35 000 ≈ 685 000
650 000 − 654 098 = − 4 098
35 000 − 34 876 = 124
−4 098 + 124 = −3 974
654 098 + 34 876
= 685 000 – (−3 974)
= 688 974

3.3 549 076 + 23 110
Estimate:
550 000 + 23 000 ≈ 573 000
550 000 − 549 076 = 924
23 000 − 23 110 = −110
924 + (−110) = 814
549 076 + 23 110
= 573 000 − 814
= 572 186

3.4 998 432 − 654 008
Estimate:
1 000 000 − 650 000 ≈ 350 000
1 000 000 − 998 432 = 1 568
650 000 − 654 008 = −4 008
1 568 − (−4 008) = 5 576
998 432 − 654 008
= 350 000 − 5 576
= 344 424

3.5 556 984 − 443 056
Estimate:
560 000 − 440 000 ≈ 120 000
560 000 − 556 984 = 3 016
440 000 − 443 056 = −3 056
3 016 − (−3 056) = 6 072
556 984 − 443 056
= 120 000 − 6 072
= 113 928

3.6 198 008 − 88 999
Estimate:
200 000 − 90 000 ≈ 110 000
200 000 − 198 008 = 1 992
90 000 − 88 999 = 1 001
1 992 − 1 001 = 991
198 008 − 88 999
= 110 000 − 991
= 109 009

Remedial

- Since the basic multiplication facts are so important when doing long multiplication and long division as well as factorising whole numbers and estimation of answers, remediation of this must be done with those learners who have not mastered it yet.
- Some learners may have difficulty with long multiplication and long division. Let them attempt easy examples first and then progress to examples involving bigger whole numbers.
- Some learners may have difficulty with rounding off big whole numbers. Help them with this. They will probably also struggle with estimating the answer of some calculations and the skill of compensating. Let them attempt easy examples first and then progress to examples involving bigger whole numbers.
- Some learners may have difficulty with understanding the commutative, associative and distributive properties. Work through some more examples to help them with this.

Extension

- Challenge learners to investigate prime numbers beyond 100 and to find the biggest prime number they can determine.
- Encourage learners to do research on the history of the various notations of whole numbers: how the ancient Egyptians, the Babylonians and the Romans wrote whole numbers and how the Indians wrote whole numbers and were the first to use a symbol for 0.

Unit 3 Multiples and factors

Learner's Book page 22

Unit focus

This unit focusses on the following:
- prime factors
- Lowest Common Multiple (LCM) and Highest Common Factor (HCF)
- finding square roots and cube roots using prime factors.

Background information

In order to manage the material on multiples and factors, it is imperative that learners are able to:
- do mental calculations with whole numbers including multiplication facts up to 12×12
- identify prime numbers up to 100
- determine the LCM and the HCF of numbers to at least 3-digit whole numbers, by inspection or factorisation.

Exercise 1

Learner's Book page 24

Guidelines on how to implement this activity

- Revise the prime factorisation of composite numbers within the range of the learner's knowledge of the multiplication facts (i.e. up to 12×12).
- Revise the concepts Highest Common Factor and Lowest Common Multiple by using composite numbers not bigger than 144.
- The Worked examples on page xx are relevant.
- Let the learners start Exercise 1 in the class and finish it, if necessary, for homework.

Suggested answers

1 $9 = 3 \times 3$ $16 = 2 \times 2 \times 2 \times 2$ $18 = 2 \times 3 \times 3$
$20 = 2 \times 2 \times 5$ $21 = 3 \times 7$ $165 = 3 \times 5 \times 11$

2.1 $42 = 2 \times 3 \times 7$ and $70 = 2 \times 5 \times 7$
HCF (42; 70) $= 2 \times 7 = 14$

2.2 $42 = 2 \times 3 \times 7$ and $70 = 2 \times 5 \times 7$
LCM (42; 70) $= 2 \times 3 \times 5 \times 7 = 210$

2.3 $132 = 2 \times 2 \times 3 \times 11$ and $286 = 2 \times 11 \times 13$
HCF (132; 286) $= 2 \times 11 = 22$

3.1 $4 = 2 \times 2$, 5 is prime, and $9 = 3 \times 3$
LCM (4; 5; 9) $= 2 \times 2 \times 5 \times 3 \times 3 = 180$

3.2 11 is prime, $55 = 5 \times 11$ and $65 = 5 \times 13$
LCM (11; 55; 65) $= 5 \times 11 \times 13 = 715$

3.3 $48 = 2 \times 2 \times 2 \times 2 \times 3$, $56 = 2 \times 2 \times 2 \times 7$ and $72 = 2 \times 2 \times 2 \times 3 \times 3$
HCF (48; 56; 72) $= 2 \times 2 \times 2 = 8$

Remedial

Some learners may have difficulty in understanding the concepts HCF and LCM. First demonstrate it to them with relatively small whole numbers and then let them try similar examples and gradually attempt bigger numbers.

Extension

Challenge learners to create pairs of composite numbers with bigger prime factors like 13, 17, 19 etc. and then to determine the HCF and LCM of these pairs.

Exercise 2

Learner's Book page 25

Guidelines on how to implement this activity

Discuss the divisibility of composite numbers:

- When is a number divisible by 2?
  Answer: When the units digit is 0 or an even number.
- When is a number divisible by 5?
  Answer: when the units digit is 0 or 5.
- When is a number divisible by 3?
  Answer: when the sum of its digits is divisible by 3.

Demonstrate by means of the Worked examples how to prime factorise larger numbers, and how to use prime factorization to find the square root and cube root of larger numbers. Let the learners start Exercise 2 in the class and finish it, if necessary, for homework.

Suggested answers

1.1 $60 = 2^2 \times 3 \times 5$
$135 = 3^3 \times 5$
$\text{LCM} = 2^2 \times 3^3 \times 5 = 540$
$\text{HCF} = 3 \times 5 = 15$

1.2 572; 598
$572 = 2^2 \times 11 \times 13$
$598 = 2 \times 13 \times 23$
$\text{LCM} = 2^2 \times 11 \times 13 \times 23 = 13\ 156$
$\text{HCF} = 2 \times 13 = 26$

1.3 10 469; 4 123
$10\ 469 = 19^2 \times 29$
$4\ 123 = 7 \times 19 \times 31$
$\text{LCM} = 7 \times 1$
$9^2 \times 29 \times 31 = 2\ 271\ 773$
$\text{HCF} = 19$

1.4 529; 1 369
$529 = 23^2$
$1\ 369 = 37^2$
$\text{LCM} = 23^2 \times 37^2 = 724\ 201$
$\text{HCF} = 1$

2.1 $216 = 2^3 \times 3^3$
$= 2 \times 3 \times 2 \times 3 \times 2 \times 3$
$\sqrt[3]{216} = 2 \times 3 = 6$

2.2 $1\ 521 = 3^2 \times 13^2$
$= 3 \times 13 \times 3 \times 13$
$\sqrt{1\ 521} = 3 \times 13 = 39$

2.3 $1\ 764 = 2^2 \times 3^2 \times 7^2$
$= 2 \times 3 \times 7 \times 2 \times 3 \times 7$
$\sqrt{1\ 764} = 2 \times 3 \times 7 = 42$

2.4 $2\ 744 = 2^3 \times 2^7$
$= 2 \times 7 \times 2 \times 7 \times 2 \times 7$
$\sqrt[3]{2\ 744} = 2 \times 7 = 14$

2.5 $42\ 875 = 5^3 \times 7^3$
$= 5 \times 7 \times 5 \times 7 \times 5 \times 7$
$\sqrt[3]{42\ 875} = 5 \times 7 = 35$

Remedial

Some learners may struggle to prime factorise composite numbers. Revise the rules for divisibility by 2, 3 and 5 and also revise the multiples of prime numbers like 7 and 11.

Extension

Challenge learners to create perfect square and perfect cube numbers with prime factors bigger that 19.

Unit 4 Ratio and rate

Learner's Book page 26

Unit focus

This unit focusses on the following:
- solving problems about ratio and rate; and
- solving problems involving speed, distance and time.

Background information

Learners should already know how to:
- compare two or more quantities of the same kind (ratio)
- compare two quantities of different kinds (rate).

Learners must know how to share according to a given ratio where the whole is given, and to increase or decrease a number according to a given ratio. In Grade 8 learners also learnt how to use ratio and rate, including speed, distance and time, to solve poroblems in context.

Exercise 1

Learner's Book page 27

Guidelines on how to implement this activity

Revise the concepts *ratio* and *rate* by demonstrating the Worked examples. Let the learners do the exercises in the class and finish it, if necessary, for homework. Monitor their progress and provide help where required.

Suggested answers

1.1 Number of boys = 4 × 5 = 20
1.2 Number of girls = 6 × 5 = 30
1.3 Number of boys : number of girls = 20 : 30 = 2 : 3
1.4 Number of boys : number of learners = 20 : 50 = 2 : 5
1.5 Number of girls : number of learners = 30 : 50 = 3 : 5
1.6 2 : 5 is 40%
1.7 3 : 5 is 60%
2.1 108 : 144 = 3 : 4
2.2 12,5 : 7,5 = 5 : 3
2.3 0, 39 : 1,3 : 0,26 = 3 : 10 : 2

3 The sum of the ratio parts:
$9 + 5 + 3 = 17$
$27 \text{ cm} = \frac{9}{17}$
So $\frac{1}{17} = 27 \div 9 = 3$
Breadth $= 5 \times 3 = 15$ cm
Height $= 3 \times 3 = 9$ cm

4.1 Capital investment of Andiswa : Benjani : Chris
$= 25\ 000 : 50\ 000 : 40\ 000$
$= 5 : 10 : 8$

4.2 The sum of the ratio parts = 23
R9 200 ÷ 23 = R400
Andiswa will get 5 × 400 = R2 000
Benjani will get 10 × 400 = R4 000
Chris will get 8 × 400 = R3 200

5.1 Cost during peak hours = 2,25 × 3 = R6,75
Cost during off-peak hours = 1,18 × 3 = R3,54
Difference = R6,75 − R3,54 = R3,21

5.2 45 seconds = 0,75 minutes
Cost during peak hours = 2,65 × 0,75 = R1,9875
Cost during off-peak hours 1,30 × 0,75 = R0,975
Difference = R1,9875 − R0,975 = R1,0125 = R1,01 to two decimal places

Remedial

- Some learners may have difficulty expressing a ratio as a percentage. Help them with easy examples (such as 2 : 5 = 4 : 10 = 40 : 100 = 40%) at first, followed by more challenging ones.
- Some learners may have difficulty simplifying ratios. Help them with easy examples (such as R120 : R80 = 12 : 8 (by dividing by R10) = 3 : 2 (by dividing by 4) at first, followed by more challenging ones.

Exercise 2

Learner's Book page 30

Guidelines on how to implement this activity

Revise the concepts speed, distance and time including the various units commonly used for this. Demonstrate by means of Worked examples on the board. Let the learners start Exercise 3 in the class and finish it, if necessary, for homework.

Suggested answers

1.1 $S = \frac{D}{T}$

$= \frac{1\ 500}{18}$

$= 83,\dot{3}$km/h

1.2 $T = \frac{D}{S}$

$= \frac{1\ 500}{72}$

$= 20,8\dot{3}$ hours
$= 20$ hours $+ (0,8\dot{3} \times 60$ minutes$)$
$= 20$ hours 50 minutes

2.1 42,2 km = 42 200 m
2 hours 33 minutes and 29 seconds
$= (2 \times 60 \times 60) + (33 \times 60) + 29 = 9\ 209$ seconds
$S = \frac{D}{T}$
$= \frac{42\ 200 \text{ m}}{9\ 209 \text{ s}}$
$= 4,58$ m/s (to two decimal places)

2.2 $T = (2 \times 60 \times 60) + (35 \times 60) + 11 = 9\ 311$ seconds
Time difference $= 9\ 311 - 9\ 209$
$= 102$ seconds
To find the distance covered by the second athlete, we multiply his speed in the last minutes by the time difference between the two athletes:
$D = S \times T$
$= 4,5 \times 102$
$= 459$ metres behind

3.1 $S = \frac{D}{T}$
$= \frac{4\ 200 \text{ m}}{75,6 \text{ s}}$
$= 55,\dot{5}$m/s
$S = \frac{D}{T}$
$= \frac{4,2 \text{ km}}{(75,6 \div 60 \div 60)\text{h}}$
$= 200$ km/h

3.2 $T = \frac{D}{S}$
$= \frac{4,2}{240}$
$= 0,0175$ hours
$= 0,175 \times 60 \times 60$seconds
$= 63$ seconds

4.1 $R = S \times T$
$= 2,4 \times 6,5$
$= 15$ revolutions

4.2 $T = \frac{R}{S}$ $\quad R = T.S$
Top loader: $R = 600 \times 45$
$= 24\ 000$ r.p.m
Front loader: $R = 1\ 400 \times 45$
$= 6\ 300$ r.p.m

4.3 $S = \frac{R}{T}$
$= \frac{10}{4 \div 60}$
$= 150$ rotations per minute (r.p.m.)

Remedial

Some learners may have difficulty understanding and applying the relationship between speed, distance and time. Help them with easy examples followed by more challenging ones. Some learners may have difficulty converting units of speed like km/h to m/s. Revise these units with them and demonstrate examples of conversions.

Unit 5 Direct and indirect proportion

Learner's Book page 31

Unit focus

This unit focusses on the following:
- direct and indirect proportion
- solve problems using direct and indirect proportion.

Background information

Learners should already know about the concepts of ratio and rate. They should also know about relationships between two variable quantities that can be represented in various equivalent ways such as in words, by flow diagrams, tables or formulae. Learners should also know how to interpret and draw global graphs and how to use tables or ordered pairs to plot points and draw graphs on the Cartesian plane.

Exercise 1

Learner's Book page 32

Guidelines on how to implement this activity

Since direct and indirect proportion is new to the learners it is important to demonstrate various examples on the board. Total payments for the purchase of a number of units (e.g. for tins of cool drink) in relation to the price per unit, offers many examples of direct proportion. Let the learners do Exercise 1 in the class and discuss the answers with them once they have finished the exercise.

Suggested answers

1 Yes, the table shows direct proportion as $y = 3x$
2 No, the table does not show direct proportion as the rate is not constant.
3 Yes, the table shows direct proportion as $y = \frac{x}{2}$
4 Yes, the table shows direction proportion as $y = 5x$
5 No, the table does not show direct proportion as the rate is not constant.
6 Yes, the table shows direct proportion as $y = -\frac{x}{11}$

Exercise 2

Learner's Book page 33

Guidelines on how to implement this activity

Work through an example of a graph of a direct proportion such as the example in the Learner's Book. The verbal description of a direct proportion as worded in the Learner's Book is very important. So analyse and discuss it with the learners and link it to the equation $y = cx$ and the straight line graph. Demonstrate the Worked examples to the learners. Stress the characteristic of an indirect proportion as a relationship of the form xy = constant. Let the learners do Exercise 2 in the class and discuss the answers with them once they have finished this exercise.

Suggested answers

1 The table shows indirect proportion as $xy = 30$

2 The table shows neither direct nor indirect proportion.

3 The table shows direct proportion as $y = -\frac{x}{5}$

4 The table shows indirect proportion as $xy = 60$

5 The table shows direct proportion as $y = 5x$

6 The table shows neither direct nor indirect proportion.

Exercise 3

Learner's Book page 34

Guidelines on how to implement this activity

Compare the characteristics of *direct* and *indirect* proportion. Let the learners start with Exercise 3 in the class and discuss the answers with them once they have finished Questions 1 and 2. Work through Question 3 of Exercise 3 with participation of the learners since it requires some skill to draw the curve in the first quadrant. Compare the characteristics of direct and indirect proportion with reference to their respective graphs.

Suggested answers

1.1

Length of each piece (x)	1 m	1,5 m	2 m	3 m	5 m	6 m	7,5 m	10 m	15 m
Number of pieces (y)	30	20	15	10	6	5	4	3	2

1.2 $xy = 30$

1.3 Indirectly proportional as the product xy is constant.

2.1

Number of workers (n)	8	12	20	24	30
Time taken in hours (t)	30	20	12	10	8
Man-hours for the task	240	240	240	240	240

2.2 $nt = 240$

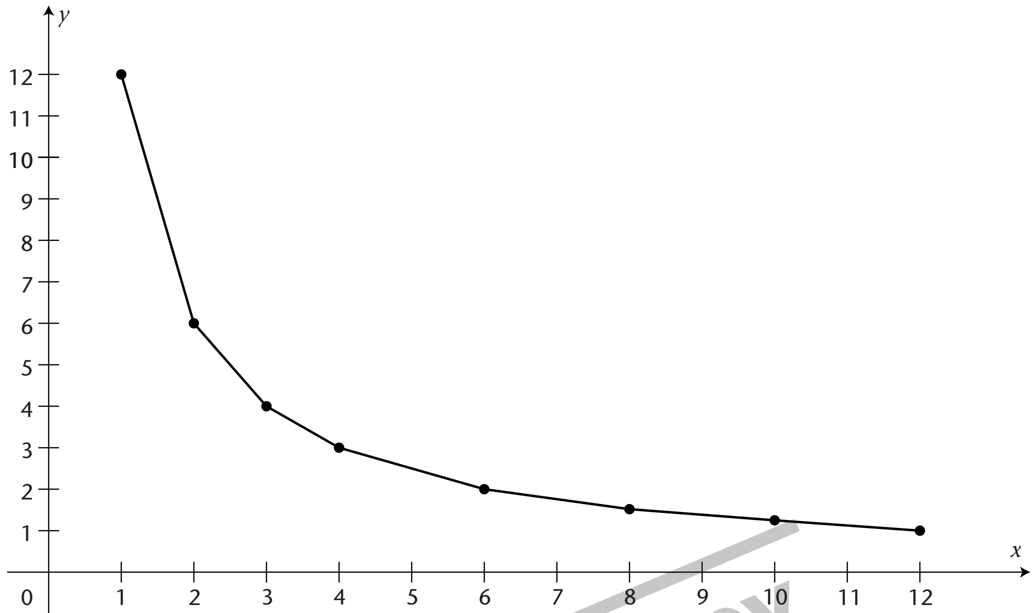
2.3 Indirectly proportional as the product nt is constant.

3.1 Construction.

3.2

Length of rectangle: l	12 cm	10 cm	8 cm	6 cm	4 cm	3cm	2cm	1 cm
Breadth of rectangle: b	1 cm	1,2 cm	1,5 cm	2 cm	3 cm	4 cm	6 cm	12 cm
Area of rectangle	12 cm²	12 cm²	12 cm²	12 cm²	12 cm²	12 cm²	12 cm²	12 cm²

3.3 and **3.4**



Unit 6 Integers

Learner's Book page 35

Unit focus

This unit focusses on the following:
- revise calculations with integers;
- revise the use of the commutative, associative and distributive properties of addition and multiplication for integers; and
- revise the additive and multiplicative inverses of integers.

Background information

In previous grades, the learners learnt to:
- do calculations involving all four operations with integers;
- do calculations involving all four operations with numbers that involve the squares, cubes, square roots and cube roots of integers;
- recognise and use commutative, associative and distributive properties of addition and multiplication of integers; and
- recognise and use additive and multiplicative inverses for integers.

Exercise 1

Learner's Book page 35

Guidelines on how to implement this activity

Revise the number line for integers by drawing such a number line on the board and ask questions to find out what the learners still remember about integers and the properties of integers.

Suggested answers

1.1 $7 + (-4) = 3$
1.2 $-7 + 4 = -3$
1.3 $-7 + (-4) = -11$
1.4 $12 + (-12) = 0$
1.5 $7 + (-10) = -3$
1.6 $0 + (-17) = -17$
1.7 $29 + (-29) = 0$
1.8 $-108 + (-12) = -120$
1.9 $19 + (-6) + (-13) = 0$
2.1 $10 - (-4) = 14$
2.2 $(-6) - (+9) = -15$
2.3 $23 - (-11) = 34$
2.4 $(-37) - (+10) = -47$
2.5 $(-185) - (-5) = -180$
3.1 $2\ 137 - (-1\ 563) = 3\ 700$
3.2 $-3\ 789 - 4\ 211 = -8\ 000$
3.3 $-14\ 392 - (-21\ 500) = 7\ 108$

Exercise 2

Learner's Book page 36

Suggested answers

1.1 $6 + (-6) = 0$
1.2 $10 + (-10) = 0$
1.3 $-13 + 13 = 0$
1.4 $0 = -25 + 25$
2.1 They are the same distance from 0, just in opposite directions.

Exercise 3

Learner's Book page 36

Guidelines on how to implement this activity

Continue using number lines to help learners. Remind learners that when subtracting negative integers, we essentially add them. Monitor learners as they complete Exercise 3, and assist if any learners are experiencing any difficulty.

Suggested answers

1.1 $10 - (-4) = 14$
1.2 $-6 - (+9) = -15$
1.3 $32 - (-13) = 45$
1.4 $-37 - (+10) = -47$
1.5 $-185 - (-15) = -170$
1.6 $2\ 047 - 2\ 470 = -423$
2.1 $2\ 137 - (-1\ 563) = 3\ 700$
2.2 $-9\ 873 - 4\ 227 = -14\ 100$
2.3 $-5\ 678 - (-943) = -4\ 735$
2.4 $-24\ 300 - (-25\ 500) = 1\ 200$
2.5 $257\ 984 - 258\ 631 = -647$

Exercise 4

Learner's Book page 37

Guidelines on how to implement this activity

Revise the multiplication and division of integers. Revise what happens with the signs when we multiply and divide. Let the learners do Exercise 4 in the classroom and monitor their progress.

Suggested answers

1.1 $(-10) \times 75 = -750$
1.2 $12 \times (-8) = -96$
1.3 $(-125) \times (-100) = 12\,500$
1.4 $100 \times (-125) = -12\,500$
1.5 $60 \div (-20) = -3$
1.6 $(-75) \div 15 = -5$
1.7 $(-54) \div (-9) = 6$
1.8 $0 \div (-37) = 0$
1.9 $(-235) \times 0 = 0$
1.10 $(-1) \div 0 =$ undefined
1.11 $(-2) \times (-3) \times (-4) = -24$
1.12 $(-5) \times 3 \times (-4) \times (-1) = -60$
1.13 $[(-24) \div (-8)] \div (-3) = -1$

Remedial

Some learners may have difficulty with doing correct calculations with integers, especially where negative integers or large integers are involved. Revise the basic "sign rules" with them and show them how to use brackets and how to remove brackets, especially where negative integers are involved.

Exercise 5

Learner's Book page 39

Guidelines on how to implement this activity

Demonstrate the commutative, associative and distributive properties for the operations with integers on the board. Also, the fact that 1 is the identity for multiplication and that the only integers that have multiplicative inverses are 1 and –1. Motivate the latter statement by showing that, for example: although $(-2) \times \left(-\frac{1}{2}\right) = 1$, the multiplicative inverse of –2, it is not an integer but the rational number $-\frac{1}{2}$. Let the learners do Exercise 5 in the classroom and discuss the answers once they completed the exercise.

Suggested answers

1.1 Yes, $7 \times (5 + 3) = 7 \times 8 = 56$ and $(7 \times 5) + (7 \times 3) = 35 + 21 + 56$
1.2 Yes, $(10 + 5 + 3) \times 2 = 18 \times 2 = 36$
and $(10 \times 2) + (5 \times 2) + (3 \times 2) = 20 + 10 + 6 = 36$
1.3 Yes, $5 \times (11 - 4) = 5 \times 7 = 35$ and $(5 \times 11) - (5 \times 4) = 55 - 20 = 35$
1.4 Yes, $(9 - 4) \times 3 = 5 \times 3 = 15$ and $(9 \times 3) - (4 \times 3) = 27 - 12 = 15$
1.5 $a \times (b - c) = (a \times b) - (a \times c)$
Multiplication of integers is distributive over subtraction.
2.1 No, $14 - 9 = 5$ but $9 - 14 = -5$
Subtraction of integers is not commutative.
2.2 No, $2 - (5 - 6) = 2 - (-1) = 3$ but $(2 - 5) - 6 = -3 - 6 = -9$
Subtraction of integers is not associative.

2.3 No, $12 \div (6 \div 2) = 12 \div 3 = 4$ but $(12 \div 6) \div 2 = 2 \div 2 = 1$
Division of integers is not associative.

2.4 No, $-8 \div 4 = -2$ but $4 \div (-8) = -0{,}5$
Division of integers is not commutative.

3.1 $83 \times 57 + 83 \times 43 = 83 \times (57 + 43) = 83 \times 100 = 8\ 300$

3.2 $15 \times (-8) + 25 \times (-8) = -8 \times (15 + 25) = -8 \times 40 = -320$

3.3 $8 \times (-73) - 8 \times (-23) = 8 \times (-73 - (-23)) = 8 \times -50 = -400$

Remedial

Some learners may have difficulty with interpreting and applying the associative and distributive properties for operations with integers. Demonstrate these with easy examples and progress to more involved cases like applying the distributive property "in reverse".

Exercise 6

Learner's Book page 40

Guidelines on how to implement this activity

Demonstrate examples of calculating the squares, square roots, cubes and cube roots of integers. Let the learners do Exercise 6 in pairs.

Suggested answers

1 $[(-13)^3] + [52 - (-4)^3] = 2\ 318$

2 $\sqrt{81} - \sqrt[3]{-125} = 14$

3 $[(-6)^3 \div (-3)^2] = -24$

4 $235 - [67 - (-7)(-4)] = 196$

5 $[(15)^2 \times (-2)^3] \div [(-10)^2] = -18$

6 $\sqrt{16} + \sqrt[3]{-64} = 0$

Remedial

Some learners may have difficulty with calculating square roots and cube roots of integers. Let them first attempt easy cases like 3^2 and $(-3)^3$ and the related square and cube roots.

Unit 7 Financial mathematics 1

Learner's Book page 41

Unit focus

This unit focusses on the following:

- profit and loss
- discount
- vat
- budgets
- accounts
- loans
- simple interest
- hire purchase.

Background information

In Grade 8 the learners learnt to solve problems that involve whole numbers, percentages and decimal fractions in financial contexts.

Exercise 1

Learner's Book page 43

Guidelines on how to implement this activity

- Establish what the learners remember of concepts they learnt in Grades 7 and 8 by asking questions that require the learners to give simple examples of profit, loss, discount etc.
- Demonstrate the Worked examples (about profit and loss) on the board.
- Let the learners do Questions 1 to 3 of Exercise 1 and monitor their progress. Demonstrate the Worked example (about budgeting) and then let the learners do Questions 4 and 5 of Exercise 1.

Suggested answers

1 Profit $= 20 - 12{,}5$
$= \text{R}7{,}50$
Percentage profit $= \frac{7{,}50}{12{,}50} \times \frac{100}{1}$
$= 60\%$

2.1 He made a loss as the selling price is R250 less than the cost price.

2.2 Percentage loss $= \frac{250}{750} \times 100$
$= 33{,}33\%$

3 Income on sold packs $= 280 \times 29{,}99$
$= \text{R}8\ 397{,}20$
Profit $= 8\ 397{,}20 - 7\ 500$
$= \text{R}897{,}20$

4.1

Budget for a concert			
Income		**Expenses**	
Item	**Total**	**Item**	**Total**
120 adult tickets @ R20 each	R2 400,00	Material for costumes (140 m @ R8,20/m)	R1 148,00
250 learner tickets @ R10 each	R2 500,00	370 programmes @ 80c each	R296,00
80 pre-school tickets @ R5 each	R400,00	Flowers for the music and art teachers	R50,00
		Paint for scenery	R75,50
		Other	R200,00
Total income	**R5 300,00**	**Total expenses**	**R1 769,50**

4.2 Profit = total income – total expenses

4.3 To cover any other expenses not listed in the previous categories, as well as any unexpected expenses.

4.4 The number of programmes allows each person who can read to have a programme. Pre-schoolers who cannot read are excluded from the programme count.

4.5 Profit = R5 300 – R1 769,50 = R3 530,50

4.6 $37 \times 5 + 23 \times 20 = \text{R}645$

Amount needed from learners tickets = R1 769,50 − R645 = R1 124,50

No of learner tickets to sell = R1 124,50 ÷ 10 = 112,45

You would need to sell 113 learner tickets.

4.7 Total income = R645 + 10x

Total expenses = R1 769,50 + R37 + R23 + x

= x + R1 829,50

$645 + 10x = x + 1\,829{,}50$ (to break even, income and expenses are equal)

$9x = 1\,184{,}50$

$x = 131{,}6$

You would need to sell 132 learner tickets

5 Total income = $45 \times 20 + 98 \times 10 + 17 \times 5$

= R 1 965

Total expenses = R1 769,59 + R296 + R45 + R98 + R17

= R2 225,50

Loss = R2 225,50 − R1 965 = R260,50

Remedial

Some learners may find it difficult to understand the English wording of a problem. Help them to comprehend such a problem and understand the meanings of terms like profit, loss etc. Some learners may have difficulty doing calculations involving percentages. Demonstrate relevant examples to them and see whether they have mastered it. Some learners may have difficulty interpreting and drawing up a budget. Help them with simple examples related to their own management of personal finances.

Exercise 2

Learner's Book page 46

Guidelines on how to implement this activity

Revise the concept of simple interest and demonstrate Worked examples. Discuss the formulas $SI = \frac{Pnr}{100}$ and $SI = Pni$ and demonstrate by means of examples. Let the learners do Exercise 2 and monitor their progress.

Suggested answers

1.1 $S = Pni$

$= \text{R}2\,450 \times 2{,}5 \times 0{,}1$

$= \text{R}612{,}50$

1.2 R2 450 + R612,50 = R3 062,50

2 $S = Pni$

$400 = 1\,200 \times 2 \times i$

$2i = 0{,}33$

$i = 0{,}1666$

$i = 16{,}67\%$

3 $S = Pni$

$2\,160 = 6\,000 \times n \times 0{,}09$

$2\,160 = 540n$

$n = 4$ years

4.1 Balance = R4 999 – R999
= R4 000
S = 4 000 × 0,5 × 0,15
= R300

4.2 Total amount = R300 + R4 999
= R5 299

4.3 Discount = $\frac{5}{100}$ × 4 999 = R249,95
Cash price =
4 999 – 249,95 = R4 749,05

5 $S = Pni$
= 500 × 4 × 0,03
= R60

Remedial

Some learners may find it difficult to understand and calculate simple interest and /or hirepurchase. Demonstrate relevant examples to them and check whether they have mastered it.

Exercise 3

Learner's Book page 47

Guidelines on how to implement this activity

Discuss the concept of hirepurchase and demonstrate by means of the Worked examples. Let the learners do Exercise 3 and monitor their progress.

Suggested answers

1.1 Deposit = $\frac{20}{100}$ × R65 000
= R13 000
Yes, Sam has enough money for the deposit.

1.2 72 months

1.3 Total cost = R13 000 + (R800 × 72) = R70 600

2.1 Balance = R65 000 – R15 000 = R50 000
Monthly payment = R50 000 ÷ 100 = R500

2.2 Cost =R15 000 + (R500 × 9 × 12) = R69 000

2.3 Yes, because it will cost him less than the shop agreement.

Extension

- Ask learners to obtain copies of financial statements of businesses like budgets, year-end financial reports of companies (usually published in newspapers) and balance sheets. Let them write a short report to show how they interpret these statements or do an oral presentation about it.
- Let some learners investigate what the current interest rates of banks, investment companies and other businesses like money lenders are, then to report back to the class.

Unit 8 Financial mathematics 2

Learner's Book page 48

Unit focus

This unit focusses on the following:
- compound interest,
- exchange rates, and
- commission and rentals.

Background information

In Grade 8 the learners learnt to solve problems that involve simple interest and exchange rates. Simple interest was also dealt with in the preceding Unit 7.

Exercise 1

Learner's Book page 49

Guidelines on how to implement this activity

Since compound interest is a new topic for the learners, it is important to demonstrate the concept with the Worked example on page 40 and /or further examples of your own choice on the board. Also compare through calculation and an example such an investment with a simple interest investment of the same principal value, at the same rate and for the same period. Make sure that the learners can distinguish between interest compounded annually, compounded every half year or every quarter. Let the learners start Exercise 1 in the class where you can monitor their progress. The remainder of the exercise (or a selection of it) can be given as homework.

Suggested answers

1.1 Year 1: Principal value = R500

Interest earned in 1st year = $\frac{Pnr}{100} = \frac{\text{R}500 \times 9 \times 1}{100}$ = R45

Value of investment = R500 + R45 = R545

Year 2: Principal value = R545

Interest earned in 2nd year = $\frac{Pnr}{100} = \frac{\text{R}545 \times 9 \times 1}{100}$ = R49,05

Value of investment = R545 + R49,05 = R594,05

Year 3: Principal value = R594,05

Interest earned in 3rd year = $\frac{Pnr}{100} = \frac{\text{R}594{,}05 \times 9 \times 1}{100}$ = R53,46

Final value of investment = R594,05 + R53,45 = R647,50

1.2 Year 1: Principal value = R2 000

Interest earned in 1st year = $\frac{Pnr}{100} = \frac{\text{R}2\,000 \times 8 \times 1}{100}$ = R160

Value of investment = R2 000 + R160 = R2 160

Year 2: Principal value = R2 160

Interest earned in 2nd year = $\frac{Pnr}{100} = \frac{\text{R}2\,160 \times 8 \times 1}{100}$ = R172,80

Value of investment = R2 160 + R172,80 = R2 332,80

1.3 Year 1: Principal value = R750

Interest earned in 1st year = $\frac{Pnr}{100} = \frac{\text{R}750 \times 7{,}5 \times 1}{100}$ = R56,25

Value of investment = R750 + R56,25 = R806,25

Year 2: Principal value = R806,25

Interest earned in 2nd year = $\frac{Pnr}{100} = \frac{\text{R}806{,}25 \times 7{,}5 \times 1}{100}$ = R60,47

Value of investment = R806,25 + R60,47 = R866,72

Year 3: Principal value = R866,72

Interest earned in 3rd year = $\frac{Pnr}{100} = \frac{\text{R}866{,}72 \times 7{,}5 \times 1}{100}$ = R65

Final value of investment = R866,72 + R65 = R931,72

2.1 Year 1: Principal value = R90

Interest earned in 1st year = $\frac{Pnr}{100} = \frac{\text{R}90 \times 10 \times 1}{100}$ = R9

Value of loan = R90+ R9 = R99

Year 2: Principal value = R99

Interest earned in 2nd year = $\frac{Pnr}{100} = \frac{\text{R}99 \times 10 \times 1}{100}$ = R9,90

Value of loan= R99 + R9,90 = R108,90

Interest paid = R108,90 – R90 = R18,90

2.2 Year 1: Principal value = R5 500

Interest earned in 1st year = $\frac{Pnr}{100} = \frac{\text{R}5\ 500 \times 9{,}1 \times 1}{100}$ = R500,50

Value of loan = R5 500 + R500,50 = R6 000,50

Year 2: Principal value = R6 000,50

Interest earned in 2nd year = $\frac{Pnr}{100} = \frac{\text{R}6\ 000{,}50 \times 9{,}1 \times 1}{100}$ = R546,05

Value of loan= R6 000,50 + R546,05 = R6 546,55

Interest paid = R6 546,55 – R5 500 = R1 046,55

2.3 Year 1: Principal value = R100 000

Interest earned in 1st year = $\frac{Pnr}{100} = \frac{\text{R}100\ 000 \times 11{,}2 \times 1}{100}$ = R11 200

Value of loan = R100 000+ R11 200 = R111 200

Year 2: Principal value = R111 200

Interest earned in 2nd year = $\frac{Pnr}{100} = \frac{\text{R}111\ 200 \times 11{,}2 \times 1}{100}$ = R12 454,40

Value of loan= R111 200 + R12 454,40 = R123 654,40

Year 3: Principal value = R123 654,40

Interest earned in 3rd year = $\frac{Pnr}{100} = \frac{\text{R}123\ 654{,}40 \times 11{,}2 \times 1}{100}$ = R13 849,29

Final value of loan = R123 654,40 + R13 849,29 = R137 503,69

Interest paid = R137 503,69 – R100 000 = R137 503,69

3.1 Interest rate per half year = 7,5% ÷ 2 = 3,75%

Half-year 1: Principal value = R150 000

Interest earned in 1st half-year = $\frac{Pnr}{100} = \frac{\text{R150 000} \times 3{,}75 \times 1}{100}$ = R5 625

Value of loan = R150 000+ R5 625 = R155 625

Half-year 2: Principal value = R155 625

Interest earned in 2nd half-year = $\frac{Pnr}{100} = \frac{\text{R155 625} \times 3{,}75 \times 1}{100}$ = R5 835,94

Value of loan= R155 625 + R5 835, 94 = R161 460,94

Half-year 3: Principal value = R161 460,94

Interest earned in 3rd half-year = $\frac{Pnr}{100} = \frac{\text{R161 460,94} \times 3{,}75 \times 1}{100}$ = R6 054,79

Final value of loan = R161 460,94 + R6 054,79 = R167 515,73

3.2 Interest rate per quarter = 8% ÷ 4 = 2%

Quarter 1: Principal value = R19 000

Interest earned in 1st quarter = $\frac{Pnr}{100} = \frac{\text{R19 000} \times 2 \times 1}{100}$ = R380

Value of investment = R19 000+ R380 = R19 380

Quarter 2: Principal value = R19 380

Interest earned in 2nd quarter = $\frac{Pnr}{100} = \frac{\text{R19 380} \times 2 \times 1}{100}$ = R386,60

Value of investment = R19 380+ R386,60= R19 767,60

Quarter 3: Principal value = R19 767,60

Interest earned in 3rd quarter = $\frac{Pnr}{100} = \frac{\text{R19 767,60} \times 2 \times 1}{100}$ = R395,35

Value of investment = R19 767,60 + R395,35 = R20 162,95

Quarter 4: Principal value = R20 162,95

Interest earned in 4th quarter = $\frac{Pnr}{100} = \frac{\text{R20 162,95} \times 2 \times 1}{100}$ = R403,26

Final value of investment = R20 162,95 + R403,26 = R20 566,21

4 Interest per month = 1%

Month	1	2	3	4	5	6
Opening balance	R0,00	R101,00	R203,01	R306,04	R410,10	R515,20
Money deposited	R100,00	R100,00	R100,00	R100,00	R100,00	R100,00
Interest calculated at month end	R1,00	R2,01	R3,03	R4,06	R5,10	R6,15
Final balance	R101,00	R203,01	R306,04	R410,10	R515,20	R621,35

Remedial

Some learners may find it difficult to write down the steps to calculate the initial value, interest earned and value of investment per term as in the Worked example. Explain each step to them and monitor their progress.

Further remediation may be offered by giving learners examples with simpler numbers to work with. For example, ask learners to:

Calculate the final amount received if R200 is invested at 10% interest compounded annually for 3 years.

Exercise 2

Learner's Book page 51

Guidelines on how to implement this activity

- Demonstrate how the formula for the total amount (or future value), A, of an investment at compound interest is derived, starting with the formula for simple interest.
- Demonstrate the Worked example and/or similar examples of your own choice. Let the learners start Exercise 2 in the class where you can monitor their progress. The remainder of the exercise (or a selection of it) can be given as homework.

Suggested answers

1 $A = P(1 + i)^n$
$= 4\,500(1{,}09)^5$
$= R6\,923{,}81$

2.1 $A = P(1 + i)^n$
$= 4\,350\left(1 + \frac{0{,}02}{12}\right)^{12}$
$=$ R4 437,80

2.2 Total purchase price = R2 000 + R4 437,80 = R6 437,80

3

Years	0	5	10	15	20
$A = P(1 + ni)$	R10 000,00	R15 500,00	R21 000,00	R26 500,00	R32 000,00
$A = P(1 + i)^n$	R10 000,00	R16 850,58	R28 394,21	R47 845,89	R80 623,12

Remedial

Some learners may find it difficult to apply the formula for compound interest, $A = P(1 + i)^n$. Demonstrate further examples to them from easy to more challenging.

Extension

Ask learners to investigate what the current rates of banks and other financial institutions are for investments or loans at compound interest. Challenge some learners to draw graphs of investments (of the same initial amount) at simple and at compound interest at the same interest rates and for varying times of investment such as in the case of Question3 of Exercise 2.

Exercise 3

Learner's Book page 52

Guidelines on how to implement this activity

Discuss exchange rates and supply information about recent exchange rates between the rand and other currencies. Demonstrate the Worked example and / or similar examples of your own choice. Let the learners start Exercise 3 in the class where you can monitor their progress. The remainder of the exercise (or a selection of it) can be given as homework.

Suggested answers

1.1 R1 000 = €95 **1.2** £50 = R661 **1.3** \$250 = R2 130
1.4 R1 000 = £76 **1.5** ¥1 000 = R1 340 **1.6** 250 Pula = R270
2 Revenue = \$1 560 × 15 300 = \$23 868 000 × R8,23 = R196 433 640
3.1 20% of 3 000 = 600
Total cost = ¥3 600 × 1,32 =R4 752
3.2 15% of R4 752 = R712,80
Selling price = R4 752 + R712,80 =R5 464,80

Remedial

Some learners may find it difficult to apply exchange rates correctly. Demonstrate a variety of examples to them, from easy to more challenging.

Exercise 4

Learner's Book page 53

Guidelines on how to implement this activity

Discuss commission and rentals and let the learners do Exercise 4 in the class where you can monitor their progress. The remainder of the exercise can be given as homework.

Suggested answers

1.1 Commission = 2 × 20 + 1,50 × 20 = R70
1.2 Commission = 2 × (72 ÷ 3) + 1,50 × 15 = R70,50
1.3 Let the number of rotis be x:
$1{,}5x = 55{,}50$
$x = 37$ rotis
2.1 Total hiring cost = (135 + 25 + 2 × 18) × 3 = R588
2.2 One day extra = 135 + 25 + 2 × 18 = R196

Unit 9 Problem solving

Learner's Book page 54

Unit focus

This unit focusses on the following:
- solving problems relating to topics covered in this chapter.

Background information

In previous grades and in the previous units of this chapter, learners learnt to solve a variety of problems involving number systems, financial topics, ratio, rate and speed.

Exercises 1 – 8

Guidelines on how to implement this activity

The exercises and Worked examples of this unit are similar to those of Units 1 to 8 and are designed to extend and consolidate the exercises of those units. The learners can be immediately put to work to do the exercises while the teacher monitors their progress and provides help where required. If necessary the teacher can demonstrate similar and further examples.

Exercise 1

Learner's Book page 54

Suggested answers

1

Number	$\mathbb{N}_0$ Whole number	$\mathbb{Z}$ Integer	$\mathbb{Q}$ Rational number	$\mathbb{Q}'$ Irrational number	$\mathbb{R}$ Real number
2^9	yes	yes	yes	no	yes
$0{,}1\dot{6}$	no	no	yes	no	yes
$\sqrt{\frac{4}{9}}$	no	no	yes	no	yes
$-\sqrt{144}$	no	yes	yes	no	yes
$\sqrt[3]{-10}$	no	no	no	yes	yes

Exercise 2

Learner's Book page 54

Suggested answers

1.1 $60 = 2^2 \times 3 \times 5$
$135 = 3^3 \times 5$
HCF $(60; 135) = 3 \times 5 = 15$

1.2 $72 = 2^3 \times 3^2$
$120 = 2^3 \times 3 \times 5$
LCM $(72; 120) = 2^3 \times 3^2 \times 5 = 360$

1.3 $589 = 19 \times 31$
$310 = 2 \times 5 \times 31$
HCF $(589; 310) = 31$

1.4 $245 = 5 \times 7^2$
$35 = 5 \times 7$
LCM $(245; 35)$ $5 \times 7^2 = 245$

Exercise 3 Learner's Book page 55

Suggested answers

1 Circumference of L : Circumference of G = 8 : 5
2 Circumference of S : Circumference of G = 3 : 5
3 Area of L : Area of G = 64 : 25
4 Area of S : Area of G = 9 : 25
5 Circumference of L : Circumference of S = 8 : 3

Exercise 4 Learner's Book page 56

Suggested answers

1.1 $S = \frac{V}{T}$
$= \frac{1\,875}{10 \times 60}$
$= 3{,}125$ kl/s

1.2 $V = ST$
$= 12 \times 600$
$= 7\,200$ kl

2.1 $S = \frac{V}{T}$
$= \frac{500 - 125}{15}$
$= 25$ ml/min

2.2 $T = \frac{V}{S}$
$= \frac{250 \text{ ml}}{17 \text{ ml/min}}$
$= 14{,}7$ minutes

Exercise 5 Learner's Book page 57

Suggested answers

1.1 The values are in direct proportion as $y = \frac{x}{2}$
1.2 The values are in neither direct nor indirect proportion.
1.3 The values are in indirect proportion as $xy = 60$
2.1 Number of litres = 200 ÷ 8,33 = 24 litres
2.2 Number of litres = 200 ÷ 10 = 20 litres
2.3 It is indirectly proportional as the product of the amount of petrol and the price per litre is always R200 in this situation. As the price per litre increases, the amount of petrol bought decreases.

Exercise 6 Learner's Book page 58

Suggested answers

1.1 $A = P(1 + in)$
$= 500(1 + 0{,}09 \times 3)$
$=$ R635

1.2 $A = P(1 + in)$
$= 2\,000(1 + 0{,}08 \times 20)$
$=$ R5 200

1.3 $A = P(1 + in)$
$= 750(1 + 0{,}075 \times 7{,}5)$
$=$ R1 171,88

2.1 $S = Pni$
$= 90 \times 10 \times 0{,}1$
$=$ R90

2.2 $S = Pni$
$= 5\,500 \times 5\frac{3}{60} \times 0{,}091$
$=$ R2 527,53

3.1 $A = P(1 + i)^n$
$= 150\,000\left(1 + \frac{0{,}075}{2}\right)^{1{,}5 \times 2}$
$= R167\,515{,}72$

3.2 $A = P(1 + i)^n$
$= 19\,000\left(1 + \frac{0{,}08}{4}\right)^{1 \times 4}$
$= R20\,566{,}21$

4 $A = P(1 + i)^n$
$10\,000 = P(1 + 0{,}09)^5$
$10\,000 = P(1{,}09)^5$
$P = \frac{10\,000}{1{,}09^5}$
$P = R6\,499{,}31$

Exercise 7

Learner's Book page 58

Suggested answers

1.1 R1 000 = 1 000 × ¥21 = ¥21 000
1.2 ¥1 000 = R(1 000 ÷ 21) = R47,62
1.3 US$1 000 = R12 200 = ¥256 200
1.4 ¥1 000 = US$3,90
1.5.1 US$40 = R12,20 × 40 = R488
1.5.2 ¥10 000 = R(10 000 ÷ 21) = R476,19
1.5.3 R500
In descending order: R500; US$40; ¥10 000

Remedial

In monitoring the work of the learners, the teacher must give special attention to those learners who struggle to do some of the problems.

Extension

- Challenge learners to solve problems found in old exam papers or other sources.
- Challenge learners to create and solve their own problems based on the concepts covered in this chapter.

Consolidation

Learner's Book page 61

Before doing this consolidation exercise, encourage learners to review the work covered in this chapter. Advise learners to use the summary and to revise their work. This exercise can be used as an informal assessment task for you to track how learners are coping with the chapter and the concepts covered. The mark allocation provides guidelines on how to assess learners.

Suggested answers

1.1 Any number that can be written as a fraction is a rational number.

1.2 False **1.3** True **1.4** False

1.5 True **1.6** True

1.7 The set of real numbers, $\mathbb{R}$, is the union of rational and irrational numbers. (7)

2.1 $9 \times 2\,075 = (9 \times 2\,000) + (9 \times 75) = 18\,000 + 675 = 18\,675$ (1)

2.2 $8 \times 3\,000 + 8 \times 900 + 8 \times 50 + 8 \times 7 = 8 \times 3\,957$ (1)

2.3 $84 = 2^2.3.7$
$462 = 2.3.7.11$
$\text{HCF} = 2.3.7 = 42$
$\text{LCM} = 2^2.3.7.11 = 924$ (2)

3.1 X:Y:Z = 7 000 : 5 000 : 4 000 = 7 : 5 : 4 (3)

3.2 Xolani should get $\frac{7}{16} \times 100 = 43{,}75\%$

Yvonne should get $\frac{5}{16} \times 100 = 31{,}25\%$

Zanele should get $\frac{4}{16} \times 100 = 25\%$ (3)

3.3 Xolani: 43,75% of 4 800 = R2 100
Yvonne: 31,25% of 4 800 = R1 500
Zanele: 25% of 4 800 = R1 200 (3)

3.4 $A = P(1 + i.n)$
$= 7\,500(1 + 0{,}12 \times 2)$
$= \text{R}9\,300$ (3)

3.5 R1,30 = ¥1 (Yen)
¥5750 = R(1,3 × 5 750) = R7 475 (3)

4 $S = \frac{D}{T}$
$= \frac{150}{8{,}5}$
= 17,65 cm/min (to two decimal places) (2)

5 $T = \frac{D}{S}$
$= \frac{0{,}5}{70}$
= 0,007 hours
= 25,7 seconds (2)

6.1 $D = S \times T$
$= 114\,000 \times 24$
$= 2\,736\,000$ km (2)

6.2 $D = S \times T$
$= 114\,000 \times (365{,}25 \times 24)$
$= 999\,324\,000$ km (2)

7.1 $S = \frac{V}{T}$
$V = S \times T$
$= 85 \times 30$
= 2 550 litres (2)

7.2 $T = \frac{V}{S}$
$= \frac{2\,500\text{ l}}{85}$
= 29,41 minutes (3)

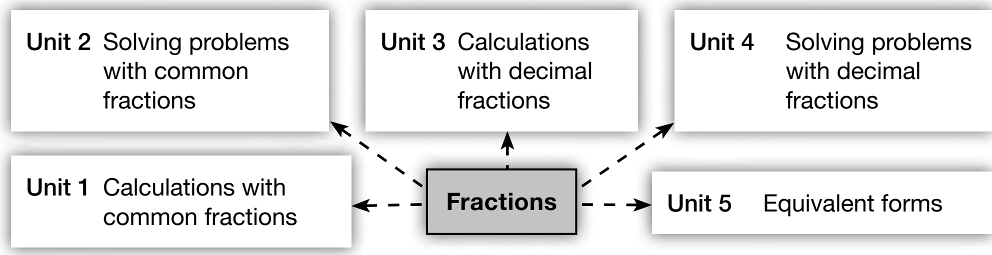
8.1 15% of 2 200 $= \frac{15}{100} \times 2\,200$
= R330
Cash price = R2 200 − R330
= R1 870 (3)

8.2 Deposit = 20% of 4 310 = R862
Monthly payment
= 5% of 4 310 = R215,50
Total cost = R862 + R215,50 × 20
= R5 172 (3)

[45]

Chapter 2 Fractions

Chapter overview



Content		Time allocations	LB page
Unit 1	Calculations with common fractions	2 hours	64
Unit 2	Solving problems with common fractions	2 hours	74
Unit 3	Calculations with decimal fractions	2 hours	78
Unit 4	Solving problems with decimal fractions	2 hours	85
Unit 5	Equivalent forms	1 hour	87

Background information

Many learners experience problems when working with fractions, both common fractions and decimals. For common fractions the notion of two numbers, one over the other, operating as one number can be confusing. Although learners have been working with fractions since the foundation phase, many continue to struggle with the concepts and operations involved. It is vital that learners master basic fractions as the shift from Grade 9 onwards is more towards complex algebraic fractions. If learners are still struggling with basic fractions they will not manage the necessary concepts required to master the algebra component of FET mathematics.

Teaching guidelines

Spend time revising all the core concepts involved with fractions. Go as far back as revising numerators and denominators, and work through simplification and basic adding. Have picture aids, number lines and place value charts for learners to use as they complete the chapter.

Resources

Picture aids showing common fractions, number cards, comparison cards, number lines, HTU charts including decimal fractions up to 10 000ths. Cardboard, and colour pens to create posters and vocabulary cards. Each learner should have their own calculator.

Unit 1 Calculations with common fractions

Learner's Book page 64

Unit focus

This unit focusses on the following:

- revising the conversion of mixed numbers to common fractions in order to perform calculations with them;
- relooking at how to simplify fractions;
- working with equivalent fractions; and
- performing basic and complex operations.

Background information on calculations with common fractions

Learners need to remember that in order to operate with mixed numbers, it is necessary to convert from mixed numbers to improper fractions. Learners must revise simplification and always provide their answer in the simplest form.

Exercise 1

Learner's Book page 67

Guidelines on how to implement this activity

Revise the concept of common fractions. Discuss numerators, denominators and mixed numbers. Encourage the class to supply as much of the information as possible. This should be revision. Identify learners who may be struggling and provide additional basic fraction problems to these learners. Revise how to simplify common fractions. Work through the examples in the Learner's Book together as a class, or encourage learners to work through the examples in small groups.
Integrate the newly revised fraction knowledge with algebraic fractions. Discuss how to simplify fractions with variables. Do examples together as a class, and then supply additional examples for learners to work through in pairs.

Suggested answers

1.1 $2 \times 11 + 9 = 31$

$2\frac{9}{11} = \frac{31}{11}$

1.2 $9 \times 12 + 5 = 113$

$9\frac{5}{12} = \frac{113}{12}$

1.3 $5 \times 75 + 73 = 448$

$5\frac{73}{75}x = \frac{448}{75}x$

1.4 $10 \times 13 + 8 = 138$

$10\frac{8}{13}xy = \frac{138}{13}xy$

2.1 $\frac{120}{360} = \frac{1}{3}$

2.2 $\frac{4x}{14x} = \frac{2}{7}$

2.3 $\frac{48xy}{12y} = 4x$

2.4 $\frac{13ab}{69xy} = \frac{13ab}{69xy}$

2.5 $\frac{105ay}{85\,xy} = \frac{21a}{17x}$

2.6 $\frac{77xyz}{11zyx} = 7$

Remedial

It is very effective to introduce fractions to learners by making use of equal sharing situations with a remainder that can be shared out. This approach is also valuable in remedial intervention with regard to the conceptual understanding of fractions. When these tasks are done with learners they should not express their answers as decimals. Learners must be encouraged to *draw* their solutions. One reason for this is that the teacher can easily diagnose which learners are experiencing conceptual problems and which are simply not able to name or write the fraction.

1 Three sisters share four chocolate slabs equally. How much of a chocolate slab will each sister get? Use a diagram to explain your answer.

2 Three friends share five fizzer sweets equally. How many fizzer sweets does each friend get? Use a diagram to explain your answer.

3 Five brothers share twenty-one finger biscuits equally. How many finger biscuits will each brother get? Use a diagram to explain your answer.

4 Four girls share seven sandwiches equally. How many sandwiches will each girl get? Use a diagram to explain your answer.

5 Ten boys share twelve hotdogs equally. How many hotdogs will each boy get? Draw a diagram to explain your answer.

Also use a fraction wall – put one up in the classroom to provide a visual aid for learners – even though they are in Grade 9 already.

ASK: Which piece is bigger?

a $\frac{1}{4}$ or $\frac{2}{8}$

b $\frac{1}{5}$ or $\frac{2}{10}$?

Exercise 2

Learner's Book page 68

Guidelines on how to implement this activity

Revise the concept of equivalence and fractions. Remind learners that in order to create equivalence we need to multiply the numerator and denominator by the same amount. Do some examples together as a class. Encourage learners to work in pairs and find as many equivalent fractions of a given fraction as possible in a time limit. Run this as a competition, and provide incentives to the winners.

Suggested answers

1 $\frac{4}{5} = \frac{80}{100}$

2 $\frac{3}{2} = \frac{21}{14}$

3 $\frac{15}{100} = \frac{3}{20}$

4 $\frac{12}{144} = \frac{1}{12}$

5 $\frac{132}{100} = \frac{33}{25}$

6 $\frac{13}{39} = \frac{26}{78}$

Exercise 3

Learner's Book page 71

Guidelines on how to implement this activity

Revise the basic operations with fractions. Do simple examples at a Grade 6 and 7 level to start with, and increase in complexity until learners can manage at the desired level of the activity. Remind learners that when we add and subtract fractions we have to have a lowest common denominator (LCD). Revise how to find the LCD. Always start with simple calculations, so learners revise the core concept and do not get confused by the complex calculations. Gradually include algebraic examples into the worked examples you do together as a class, until learners can manage the activity on their own.
Remember to explain to learners carefully why we inverse multiply when we divide. This is sometimes a confusing aspect of multiplication, yet it can be explained rather simply. Use halving to show how this works.

Suggested answers

1
$$\frac{25}{2} + \frac{46}{5} = \frac{125}{10} + \frac{92}{10} = \frac{217}{10} = 21\frac{7}{10}$$

2
$$\frac{x}{3} + \frac{x}{5} = \frac{5x}{15} + \frac{3x}{15} = \frac{8x}{15}$$

3
$$\frac{4}{ab} + \frac{9}{2ab} = \frac{8}{2ab} + \frac{9}{2ab} = \frac{17}{2ab}$$

4
$$\frac{10}{9} - \frac{11}{18} = \frac{20}{18} - \frac{11}{18} = \frac{9}{18} = \frac{1}{2}$$

5
$$2\frac{1}{3} - 1\frac{5}{6} = \frac{7}{3} - \frac{11}{6} = \frac{14}{6} - \frac{11}{6} = \frac{3}{6} = \frac{1}{2}$$

6
$$\frac{7x}{25} - \frac{x}{5} = \frac{7x}{25} - \frac{5x}{25} = \frac{2x}{25}$$

7
$$\frac{3x}{4} - \frac{2x}{9} = \frac{27x}{36} - \frac{8x}{36} = \frac{19x}{36}$$

8
$$\frac{10}{7} \times \frac{2}{3} = \frac{20}{21}$$

9
$$2\frac{3}{4} \times 4\frac{5}{3} = \frac{11}{4} \times \frac{17}{3} = \frac{187}{12} = 15\frac{7}{12}$$

10
$$\frac{4a}{5y} \times \frac{3a}{9} = \frac{12a^2}{45y} = \frac{4a^2}{15y}$$

11
$$\frac{3a^2}{a} \times \frac{b}{2a} = \frac{3a^2b}{2a^2} = \frac{3b}{2}$$

12
$$\frac{1}{2} \div 2 = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

13 $\frac{15}{9} \div \frac{5}{3} = \frac{15}{9} \times \frac{3}{5}$
$= \frac{3}{3} \times \frac{1}{1}$
$= 1$

14 $2\frac{1}{3} \div 1\frac{1}{5} = \frac{7}{3} \times \frac{6}{5}$
$= \frac{7}{1} \times \frac{2}{5}$
$= \frac{14}{5}$
$= 2\frac{4}{5}$

15 $a \div \frac{a}{3} = \frac{a}{1} \times \frac{3}{a}$
$= 3$

16 $\frac{b}{6} \div \frac{2}{b} = \frac{b}{6} \times \frac{b}{2}$
$= \frac{b^2}{12}$

Remedial

Work in a small group with any learners who may struggle with these operations. It is vital that learners grasp this as it is necessary for future algebraic manipulation at FET level.

CHAPTER 2

Exercise 4

Learner's Book page 73

Guidelines on how to implement this activity

In order for learners to be able to work with squares, square roots, cubes and cube roots of fractions, it may be necessary to revise these concepts of whole numbers. Only start this section when you are confident that learners can manage these concepts with whole numbers. Introduce fractions to squares and cubes. Show how learners should look at the numerator and denominator independently, find the square or cube, and then put them back together as a fraction. Work through as many examples together as a class until learners will be able to manage the activity on their own.

Suggested answers

1 $(\frac{11}{19})^2 = \frac{11}{19} \times \frac{11}{19}$
$= \frac{121}{361}$

2 $\left(\frac{9}{4}\right)^3 = \frac{9}{4} \times \frac{9}{4} \times \frac{9}{4}$
$= \frac{729}{64}$

3 $\left(\frac{7}{2}\right)^2 = \frac{7}{2} \times \frac{7}{2}$
$= \frac{49}{4}$

4 $\left(2\frac{1}{3}\right)^3 = \left(\frac{7}{3}\right)^3$
$= \frac{7}{3} \times \frac{7}{3} \times \frac{7}{3}$
$= \frac{343}{27}$

5 $\sqrt{\frac{10 + 6}{1}} = \sqrt{16}$
$= 4$

6 $\sqrt[3]{\frac{60 - 6}{2}} = \sqrt[3]{27}$
$= 3$

7 $\sqrt[3]{\frac{216}{343}} = \frac{\sqrt[3]{216}}{\sqrt[3]{343}}$
$= \frac{6}{7}$

8 $\sqrt{\frac{121}{144}} = \frac{\sqrt{121}}{\sqrt{144}}$
$= \frac{11}{12}$

9 $\left(\frac{1}{8}\right)^2 + \sqrt[3]{\frac{81}{3}} = \frac{1}{64} + \sqrt[3]{27}$
$= \frac{1}{64} + 3$
$= 3\frac{1}{64}$

10 $\sqrt{\frac{25}{9}} - \left(\frac{1}{3}\right)^3 = \frac{\sqrt{25}}{\sqrt{9}} - \frac{1^3}{3^3}$
$= \frac{5}{3} - \frac{1}{27}$
$= \frac{45}{27} - \frac{1}{27}$
$= \frac{44}{27}$

11 $\sqrt[3]{\frac{64}{125}} - \left(\frac{4}{3}\right)^2 = \frac{\sqrt[3]{64}}{\sqrt[3]{125}} - \frac{4^2}{3^2}$
$= \frac{4}{5} - \frac{16}{9}$
$= \frac{36}{45} - \frac{80}{45}$
$= \frac{-44}{45}$

12 $\sqrt{9a^6b^{12}} \times \left(\frac{1}{3}\right)^3 = 3a^3b^6 \times \frac{1}{27}$
$= \frac{a^3b^6}{9}$

13 $\sqrt[3]{\frac{216x^3}{8y^6}} + \left(\frac{x}{y}\right)^2 = \frac{6x}{2y^2} + \frac{x^2}{y^2}$

14 $(\frac{2abc^2}{4})^2 + \sqrt[3]{\frac{343}{8}} = \frac{4a^2b^2c^4}{16} + \frac{7}{2}$

15 $\sqrt{\frac{a^6}{b^{10}}} \times \left(\frac{x^2}{y^3}\right)^3 = \frac{a^3}{b^5} \times \frac{x^6}{y^9}$
$= \frac{a^3x^6}{b^5y^9}$

16 $\sqrt{\frac{(2a+b)^2}{(a+b)^2}} + (\frac{a}{b})^3 = \frac{2a+b}{a+b} + \frac{a^3}{b^3}$

Unit 2 Solving problems with common fractions

Learner's Book page 74

Unit focus

This unit focusses on the following:
- solving problems in contexts involving common fractions, mixed numbers and percentages.

Background information on solving problems with common fractions

When solving problems, always revise with learners the steps they should follow. These involve:
- always carefully read the question and determine what you are being asked to solve
- identify the information you are given
- create a number sentence to solve the problem
- substitute in your values and solve
- write the final answer as a solution to the original problem.

Exercise 1

Learner's Book page 76

Guidelines on how to implement this activity

Work through the examples in the Learner's Book with the learners together as a class. Encourage learners to draw pictures or diagrams of the problem to help them to visualise what they are being asked to find.

Suggested answers

1 $\frac{1}{4}$ of 10 kg $= \frac{1}{4} \times 10$

$= \frac{5}{2} = 2\frac{1}{2}$ kg

$\frac{1}{5}$ of 10 kg $= \frac{1}{5} \times 10$

$= 2$ kg

Number of kilograms left $= 10 - 2\frac{1}{2} - 2$

$= 5\frac{1}{2}$ kg

2 $\frac{214}{8} = \frac{107}{4} = 26\frac{3}{4}$

The next whole number is 27

3 $\frac{1}{4}$ of 48 $= \frac{1}{4} \times 48$

= 12 learners get 80% or more

$\frac{1}{3}$ of 12 $= \frac{1}{3} \times 12$

= 4 learners get full marks

4 Perimeter of rectangle $= 2l + 2b$

$= 2\left(2\frac{1}{4}\right)+2\left(1\frac{1}{2}\right)$

$= 2\left(2\frac{1}{4}\right) + 2\left(1\frac{1}{2}\right)$

$= 2\left(\frac{9}{4}\right) + 2\left(\frac{3}{2}\right)$

$= \frac{9}{2} + 3$

$= 7\frac{1}{2}$ metres

5 Number of doses $= 3 \times 9 = 27$

Medicine per dose $= 810 \div 27 = 30$ ml

6 $\frac{3}{4}$ of R29 $= \frac{3}{4} \times 29$

$= \frac{87}{4}$

$= 21\frac{3}{4}$

$=$ R21,75

$\frac{2}{5}$ of R69 $= \frac{2}{5} \times 69$

$=$ R46

$2\frac{3}{8}$ of R45 $= 2\frac{3}{8} \times 45$

$= \frac{19}{8} \times 45$

$= \frac{855}{8}$

$= 106\frac{7}{8}$

$=$ R106,88

Total cost = R21,75 + R46 + R106,88 = R174,63

7 $A = \frac{1}{2}bh$

$= \frac{1}{2} \times 50\frac{1}{2} \times 24\frac{3}{5}$

$= \frac{1}{2} \times \frac{101}{2} \times \frac{123}{5}$

$= \frac{12\ 423}{20}$

$= 621\frac{3}{20}\text{ cm}^2$

8 Total fraction painted $= \frac{1}{7} + \frac{1}{6} + \frac{1}{4}$

$= \frac{12}{84} + \frac{14}{84} + \frac{21}{84}$

$= \frac{47}{84}$

Fraction still to be painted $= 1 - \frac{47}{84} = \frac{37}{84}$

9.1 Time spent travelling, having lunch and tea $= 1\frac{1}{4} + \frac{45}{60} + 2\left(\frac{15}{60}\right) + 1$

$= \frac{75}{60} + \frac{45}{60} + \frac{30}{60} + \frac{60}{60}$

$= \frac{210}{60}$

= 3 hours 30 minutes

Fraction of Zola's day (using 12 hours) $= \frac{3\frac{1}{2}}{12}$

$= \frac{7}{24}$

9.2 $\frac{7}{24} \times 100 = 29{,}17\%$

10.1 Fraction owned by Siseko, Sivuyile and Nomsa $= \frac{3}{8} + \frac{1}{4} + \frac{1}{16}$

$= \frac{6}{16} + \frac{4}{16} + \frac{1}{16}$

$= \frac{11}{16}$

Fraction owned by Margaret $= 1 - \frac{11}{16}$

$= \frac{16}{16} - \frac{11}{16} = \frac{5}{16}$

10.2 $\frac{5}{16}$ of R2 400 000 $= \frac{5}{16} \times 2\ 400\ 000$

$= 5 \times 150\ 000$

$=$ R750 000

11 $2\frac{1}{3} \times 55 = \frac{7}{3} \times 55$

$= 128{,}33$

Profit per jacket $= 128{,}33 - 55$

$=$ R73,33

Amount earned $= 73 \times 73{,}33 - 2(55)$

$= 5\ 353{,}33 - 110$

$=$ R5 243,33

12 Total deposited $= 3\ 210 + 2\ 890 + 1\ 876 + 4\ 234 + 1\ 010$

$=$ R13 220

5% of 13 220 $= \frac{5}{100} \times 13\ 220$

$=$ R661

Amount Jackson has $=$ R13 220 $-$ R661

$=$ R12 559

13 Profit in December $= \frac{1}{2}$ of 128 000

$=$ R64 000

$\frac{5}{8}$ of 64 000 $= \frac{5}{8} \times 64\ 000$

$=$ R40 000

Mandy gets R40 000

14 Ntombizodwa's overtime earning $= 1\frac{1}{2} \times 12$

$= \frac{3}{2} \times 12$

$=$ R18 per hour

Monwabiso's hourly pay $= 1\frac{1}{4} \times 18$

$= \frac{5}{4} \times 18$

$=$ R22,50

Amount Monwabiso earns per day $=$ R22,50 $\times 8$

$=$ R180

15 Shaded portion $= \frac{2}{8} = \frac{1}{4}$ of the rectangle

Remedial

If learners experience problems with this activity, pair them with a stronger learner. Ask the stronger learners to explain their thought processes and outline how they tackled the problems to the learners experiencing difficulties.

Unit 3 Calculations with decimal fractions

Learner's Book page 78

Unit focus

This unit focusses on the following:
- performing multiple calculations with decimal fractions, using a calculator;
- performing multiple operations with numbers that involve squares, cubes, square roots and cube roots of decimal fractions;
- using your knowledge of place value to estimate the number of decimal places in the result before performing calculations; and
- using rounding off and a calculator to check results.

Background information on decimal fractions

Learners seem to be able to grasp decimal fractions up to 100 rather easily, because of their exposure to them in money and measurement. Extending this knowledge to thousandths and beyond is sometimes more challenging. Learners need to be able to seamlessly convert between decimals and common fractions. Use a chart like the one below to help learners revise commonly used decimal fraction conversions. Make the table as comprehensive as you feel is necessary. Encourage learners to learn these off by heart.

Common fraction	Decimal
$\frac{1}{2}$	0,5
$\frac{1}{4}$	0,25
$\frac{1}{8}$	0,125
$\frac{1}{10}$	0,1
$\frac{1}{5}$	0,2
$\frac{1}{3}$	$0,\dot{3}$
$\frac{2}{3}$	$0,\dot{6}$

Exercise 1

Learner's Book page 79

Guidelines on how to implement this activity

Revise the notion of decimals and the HTU chart. Be sure to include decimals beyond thousandths. Revise converting between common and decimal fractions. Have learners create their own chart like the one above for commonly used decimal conversions. Include as many fractions as you feel is necessary. Do some examples together as a class – having different members of the class supply the answers. Include examples with mixed numbers and how these are included in decimal fractions.

Suggested answers

1	$0{,}37 = \frac{37}{100}$	**2**	$0{,}850 = \frac{850}{1000} = \frac{17}{20}$
3	$3{,}003 = 3\frac{3}{1000}$	**4**	$0{,}0825 = \frac{825}{10\,000} = \frac{22}{400}$
5	$2{,}25 = 2\frac{25}{100} = 2\frac{1}{4}$	**6**	$0{,}0001 = \frac{1}{10\,000}$
7	$10{,}020 = 10\frac{20}{1000} = 10\frac{1}{50}$	**8**	$100{,}10 = 100\frac{10}{100} = 100\frac{1}{10}$

Remedial

Test learners on the commonly used conversions. Have them memorise the table and have mini tests on this. This will help improve learners' ability when working with decimal fractions.

Exercise 2

Learner's Book page 80

Guidelines on how to implement this activity

Discuss what a reciprocal value is. Show learners how a reciprocal value works, and that it is the inverse of the number. Discuss that reciprocals, when multiplied have to provide an answer of 1. Show by means of multiplication of common fractions that this is so. Do examples together as a class, showing learners how to use their calculators to find reciprocals.

Suggested answers

1 Reciprocal of 100 = $\frac{1}{100} = 0{,}01$

2 Reciprocal of 150 = $\frac{1}{150} = 0{,}00\dot{6}$

3 Reciprocal of 10 = $\frac{1}{10} = 0{,}1$

4 Reciprocal of 40 = $\frac{1}{40} = 0{,}025$

5 Reciprocal of 85 = $\frac{1}{85} = 0{,}01176\ldots$

6 Reciprocal of 625 = $\frac{1}{625} = 0{,}0016$

7 Reciprocal of 1 250 = $\frac{1}{1\,250} = 0{,}0008$

8 Reciprocal of 8 625 = $\frac{1}{8\,625} = 0{,}0001159\ldots$

9 Reciprocal of 930 = $\frac{1}{930} = 0{,}001075\ldots$

10 Reciprocal of 1 357 = $\frac{1}{1\,357} = 0{,}0007369\ldots$

Exercise 3

Learner's Book page 81

Guidelines on how to implement this activity

Discuss finite and infinite decimals. Provide examples of each, and have learners practise finding finite and infinite decimals on their calculators. Show learners how to write an infinite recurring decimal using the dot method. Allow learners to use their calculators to complete this activity.

Suggested answers

1.1	$\frac{45}{100} = 0{,}45$	**1.2**	$\frac{6}{100} = 0{,}06$	**1.3**	$\frac{9}{1\,000} = 0{,}009$
1.4	$\frac{801}{100} = 8{,}01$	**1.5**	$\frac{24}{21} = 1{,}\dot{1}4285\dot{7}$		
1.6	$\frac{55}{19} = 2{,}8947368421053$ OR $\frac{55}{9} = 6{,}\dot{1}$				
1.7	$4\frac{7}{9} = 4{,}\dot{7}$	**1.8**	$\frac{99}{100} = 0{,}99$	**1.9**	$\frac{5\,001}{200} = 25{,}005$
1.10	$\frac{71}{14} = 5{,}0\dot{7}1428\dot{5}$				
2.1	$\frac{1}{4} = 0{,}25$ finite	**2.2**	$\frac{2}{5} = 0{,}4$ finite	**2.3**	$\frac{9}{18} = 0{,}5$ finite
2.4	$\frac{11}{12} = 0{,}91\dot{6}$ infinite	**2.5**	$\frac{8}{9} = 0{,}\dot{8}$ infinite	**2.6**	$\frac{12}{20} = 0{,}6$ finite
2.7	$\frac{93}{100} = 0{,}93$ finite	**2.8**	$5\frac{3}{7} = 5{,}\dot{4}2857\dot{1}$ infinite		
2.9	$9\frac{1}{8} = 9{,}125$ finite	**2.10**	$100\frac{75}{100} = 100{,}75$ finite		

Exercise 4

Learner's Book page 82

Guidelines on how to implement this activity

Calculation techniques with decimals have been learnt since Grade 7. Revise the addition and subtraction method, in each case using a worked example on the board. Emphasise the need for neatness and laying out the sum correctly – especially with regard to place value. Remind learners to use the comma as the guide for laying out the sum correctly. Remind learners that decimals can also be negative numbers and can be coefficients for variables. Include variables into a worked example. Remind learners that we can only add or subtract like terms, and that this applies to decimals that contain variables as well. Encourage learners to try this exercise on their own, but allow them to work in pairs if necessary. Learners should not use a calculator in this exercise as learners are revising the physical calculations involved.

Suggested answers

1	$0{,}25 + 0{,}75 = 1$	**2**	$0{,}3 + 0{,}\ 7 = 1$
3	$0{,}5 + 0{,}055 = 0{,}555$	**4**	$1{,}45ax + 2{,}5ax = 3{,}95ax$
5	$12{,}3yb^2 + 0{,}5b^2 = 12{,}3yb^2 + 0{,}5b^2$	**6**	$11{,}1 - 0{,}1 = 11$
7	$0{,}5 - 0{,}24 = 0{,}26$	**8**	$1{,}652 - 3{,}2 = -1{,}548$

9 $0{,}4x^2 - 0{,}3x^2 - 2{,}12x^2 = -2{,}02x^2$

10 $7{,}777a^3 - (22{,}45a^3 - 14{,}6a^3) = 7{,}777a^3 - 7{,}85a^3$
$= -0{,}073a^3$

11 $0{,}345x^3y^2 - 12{,}4x^2y^3 = 0{,}345x^3y^2 - 12{,}4x^2y^3$

Exercise 5

Learner's Book page 84

Guidelines on how to implement this activity

Revise the methods for multiplying and dividing decimals. Learners have done these operations since Grade 7, however they will require revision as they can be tricky and complicated. In order to multiply and divide decimals it becomes necessary to convert the decimal to a whole number, by multiplying by the necessary power of 10. Revise these key concepts with learners. Work through the examples in the Learner's Book and do as many additional examples as you feel is necessary for the learners to confidently tackle the activity. Be sure to include worked examples that use variables with the decimals. Remind learners that all the properties of multiplying and dividing variables still apply.

Suggested answers

1.1 3,12 cm × 100 = 312 cm

1.2 0, 124 m × 10 = 1,24 m

1.3 3,567 km × 1 000 = 3 567 km

1.4 0,245 g × 10 = 2,45 g

1.5 $\frac{12{,}891\text{ kg}}{100} = 0{,}12891\text{ kg}$

1.6 $\frac{98{,}256\text{ g}}{10} = 9{,}8256\text{ g}$

1.7 $\frac{24{,}986\text{ g}}{1} = 24{,}986\text{ g}$

1.8 $50{,}89a^2b \times 30b^2c = 1\,526{,}7a^2b^3c$

1.9 $\frac{500{,}002}{5} = 100{,}0004$

1.10 $\frac{300{,}015}{3} = 100{,}005$

1.11 $\frac{1\,002}{20} = 50{,}1$

1.12 $\frac{5\,000{,}500}{50} = 100{,}01$

2.1 $\frac{13{,}46}{0{,}45} = 29{,}9\dot{1}$

2.2 456,781 × 2,678 = 1 223,259518

2.3 0,0045 × 123,765 = 0,5569425

2.4 45 005,435 × 0,001 = 45,005435

2.5 $\frac{1\,001{,}001}{24{,}6} = 40{,}691097560976$

2.6 $\frac{0{,}002}{0{,}006} = 0{,}\dot{3}$

2.7 0,45 m × 5,67 m = 2,5515 m

2.8 701 km × 34,9 km = 24 464,9 km

2.9 2,5 kg × 4,8 = 12 kg

2.10 $\frac{45\text{ km}}{56\text{ min}} = 0{,}803\text{ km.min}^{-1}$

2.11 $\frac{1\,012\text{ km}}{12{,}5\text{ hr}} = 80{,}96\text{ km.hr}^{-1}$

2.12 $\frac{140{,}9\text{ km}}{2\text{hr } 13\text{ min}}$

$2\text{ hr } 13\text{ min} = 2 + \frac{13}{60} = 0{,}217 + 2 = 2{,}217\text{ hr}$

$\frac{140{,}9\text{ km}}{2{,}217\text{ hr}} = 63{,}55\text{ km.hr}^{-1}$

2.13 $\frac{712\text{ km}}{7\text{ hr } 10\text{ min}}$

$7\text{ hr } 10\text{ min} = 7 + \frac{10}{60} = 7 + 0{,}167 = 7{,}167\text{ hr}$

$\frac{712\text{ km}}{7{,}167\text{ hr}} = 99{,}34\text{ km.hr}^{-1}$

2.14 $\frac{605{,}34\text{ km}}{9\text{ hr } 14\text{ min}}$

$9\text{ hr } 14\text{ min} = 9 + \frac{14}{60} = 9 + 0{,}234 = 9{,}234\text{ hr}$

$\frac{605{,}34\text{ km}}{9{,}234\text{ hr}} = 65{,}56\text{ km.hr}^{-1}$

2.15 $\frac{45{,}675\text{ km}}{3\text{ hr } 25\text{ min}}$

$3\text{ hr } 25\text{ min} = 3 + \frac{25}{60} = 3 + 0{,}417 = 3{,}417\text{ hr}$

$\frac{45{,}675\text{ km}}{3{,}417\text{ hr}} = 13{,}37\text{km.hr}^{-1}$

2.16 €3,5 × 10,16 = €10,16

2.17 £3 × 12,82 = £35,56

2.18 $\$25 \times 8{,}339 = \$208{,}475 = \$208{,}48$ to 2 decimal places

2.19 $\frac{R100}{10{,}164} = R9{,}8386462922826 = R9{,}84$ to 2 decimal places

2.20 $\frac{R240}{12{,}825} = R18{,}713450292398 = R18{,}71$ to 2 decimal places

2.21 $\frac{R1\ 200}{8{,}34} = R143{,}88489208633 = R143{,}88$ to 2 decimal places

Remedial

Learners may require additional revision of the methods of multiplying and dividing. Make yourself available to provide more detailed explanations and material for these learners. Restrict examples to pure decimals until learners can confidently manage, before introducing variables, negative numbers and powers.

Unit 4 Solving problems with decimal fractions

Learner's Book page 85

Unit focus

This unit focusses on the following:
- solving problems in context involving decimal fractions.

Background information on solving problems with decimal fractions

When solving problems, always revise with learners the steps they should follow:
- Always carefully read the question and determine what you are being asked to solve.
- Identify the information you are given.
- Create a number sentence to solve the problem.
- Substitute in your values and solve.
- Write the final answer as a solution to the original problem.

Exercise 1

Learner's Book page 86

Guidelines on how to implement this activity

Revise the steps for working with word problems. Encourage learners to draw a picture or diagram to help them visualise the word problems. Work through the example in the Learner's Book and provide additional examples if you feel learners need additional practice before starting the exercise.

Suggested answers

1 Perimeter $= 4s$
$= 4 \times 9{,}34$
$= 37{,}36$ cm

2 Area $= l \times b$
$= 9{,}68 \times 0{,}99$
$= 9{,}5832\ \text{m}^2$

3 Area $= \frac{1}{2}bh$
$= 0{,}5 \times 0{,}526 \times 24{,}5$
$= 6{,}4435\ \text{cm}^2$

4 $1\ 200 \div 0{,}1025 = R11\ 707{,}32$ (to two decimal places)

5 Total amount received = 674,75 + 1 345,98 + 987,45
= R3 008,18
Amount each waitron receives = R3 008,18 ÷ 6
= R501,36 (to two decimal places)

6 Height = 2,567 × 50
= 128,35 cm

7 23,45 cm = 234,5 mm
Actual distance = 234,5 × 10
= 2 345 km

8 7,5 ÷ 25 = 0,3

9 Length = area ÷ width
= 456,58 ÷ 76,987
= 5,93 cm (to two decimal places)

10.1 Convert hours and minutes to hours: 12 hours 3 minutes = $\frac{123}{60}$ = 12,05 hours
Use the speed equation: $S = \frac{D}{T} = \frac{56,412}{12,05}$ = 4,68 km/hr (correct to two decimal places)

10.2 D = 4,68 × 6
= 28,08 km

11.1 Total weight = 56,25 + 101,2 + 115,58 + 67,99 + 99,55 + 78,8 + 86,5
+ 120,78 + 135,7 + 45,1 + 50
= 957,45kg
No, they are not safe in the lift as their total weight exceeds 800 kg.

11.2 Average weight = $\frac{957,45}{11}$
= 87,04 kg

12 22,5% of 9 700
= $\frac{22,5}{100}$ × 9 700
= R2 182,50 saved per month
Cost of the two bulls = R 10 000
Number of months to save R10 000 = 10 000 ÷ 2182,5
= 4,5819…
It will take him 5 months to save enough money.

13 Number of tablets needed = 25 ÷ 7,5
= 3,333…
She should put 4 tablets in the bucket.

Remedial

Allow learners to work in pairs or small groups to help tackle the problems. Encourage learners to discuss their thinking, methods and operations with one another.

Unit 5 Equivalent forms

Learner's Book page 87

Unit focus

This unit focusses on the following:

- revising common fractions where one denominator is a multiple of another; and
- revising common fraction, decimal fraction and percentage fraction forms of the same number.

Background information on equivalent forms

Learners have worked with equivalent forms of fractions, decimals and percentages for the past few grades. They should be proficient in converting between them. Most importantly learners need to remember that: $0{,}1 = \frac{10}{100} = 10\%$

Exercise 1

Learner's Book page 88

Guidelines on how to implement this activity

This exercise focusses on equivalent fractions. Learners need to revise how to make fractions equivalent to one another by multiplying both the numerator and the denominator by the same value. Show learners number lines and fraction walls to revise the concept of equivalence.

Suggested answers

1 Yes, $\frac{1}{2} = \frac{3}{6}$

2 No, $\frac{1}{3} = \frac{3}{9}$ not $\frac{6}{9}$

3 Yes, $\frac{1}{4} = \frac{6}{24}$

4 No, $\frac{1}{3} = \frac{3}{9}$ not $\frac{4}{9}$

5 No, $25\frac{3}{4} = \frac{103}{4}$ and $250\frac{3}{4} = \frac{1\,003}{4}$

6 No, $10\frac{1}{4} = \frac{41}{4}$ and $100\frac{1}{2} = \frac{201}{2} = \frac{402}{4}$

7 No, $\frac{125}{5} = \frac{250}{10}$ not $\frac{50}{10}$

8 Yes, $\frac{9}{10} = \frac{90}{100}$

9 No, $\frac{7}{10} = \frac{210}{300}$ not $\frac{21}{300}$

10 No, $\frac{27}{810} = \frac{1}{30}$ and $\frac{2}{6} = \frac{10}{30}$

Exercise 2

Learner's Book page 89

Guidelines on how to implement this activity

This exercise focusses on the equivalence of the different forms of fractions. Have learners revise by using a table like the one below showing some of the common different forms.

Common fraction	Decimal	Percentage
$\frac{1}{2}$	0,5	50%
$\frac{1}{4}$	0,25	25%
$\frac{2}{5}$	0,1	10%

Suggested answers

1 Yes, $\frac{25}{100} = 0,25$

2 No, $\frac{1}{4} = 0,25$ not 7,5

3 No, $0,9 = \frac{9}{10}$ and not $\frac{1}{9}$

4 No, $\frac{34}{10} = 3,4$ not 0,34

5 Yes, $45\% = \frac{45}{100} = 0,45$

6 No, $1\frac{1}{2} = 1,5$ not 1,25

7 Yes, $33,3\% = \frac{1}{3}$

8 No, $1,1 = 110\%$ and $100\% = 1$

Remedial

Have learners create a comprehensive table of common equivalent forms, and encourage learners to learn this by rote. This will help learners to be able to convert more easily between these different forms.

Consolidation

Learner's Book page 91

Before doing this consolidation exercise, encourage learners to review the work covered in this chapter. Advise learners to use the summary and to revise their work. This exercise can be used as an informal assessment task for you to track how learners are coping with the chapter and the concepts covered. The mark allocation provides guidelines on how to assess learners.

1.1 $5\frac{2}{11} = \frac{57}{11}$

1.2 $11\frac{5}{21} = \frac{236}{21}$

1.3 $5\frac{73}{74}y = \frac{443}{74}y$

1.4 $10\frac{8}{15}pq = \frac{158}{15}pq$ (4)

2.1 $\frac{130}{390} = \frac{1}{3}$

2.2 $\frac{8x}{24x} = \frac{1}{3}$

2.3 $\frac{144an}{12an} = 12$

2.4 $\frac{18\,ab}{164xy} = \frac{9ab}{82xy}$

2.5 $\frac{105ay}{85\,xy} = \frac{21a}{17x}$

2.6 $\frac{121xyz}{11zyx} = 11$ (6)

3.1 $\frac{3}{5} = \frac{60}{100}$

3.2 $\frac{3}{2} = \frac{150}{100}$

3.3 $\frac{25}{100} = \frac{5}{20}$

3.4 $\frac{3}{8} = \frac{375}{1000}$

3.5 $\frac{3}{2} = \frac{3000}{2000}$ (5)

4.1 Yes, $\frac{25}{100} = 0,25$

4.2 No, $\frac{1}{4} = 0,25$ not 2,5

4.3 No, $0,9 = 90\%$ not 95%

4.4 No, 39 is not equal to 1,39

4.5 No, $35\% = \frac{35}{100} = 0,35$ not 3,5

4.6 Yes, $1\frac{1}{4} = 1\frac{25}{100} = 1,25$

4.7 Yes, $66,\dot{6}\% = \frac{2}{3}$ (8)

4.8 No, $1,1 = 110\%$ not 101% ($101\% = 1,01$)

5.5 Area to be paved = $19{,}6 \times 4{,}7$
$= 92{,}12 \text{ m}^2$
Area of one brick = $8{,}5 \times 25{,}3$
$= 215{,}05 \text{cm}^2$
$92{,}12 \text{ m}^2 = 92{,}12 \times 100 \times 100 \text{ cm}^2$
$= 921\ 200 \text{ cm}^2$
Number of bricks needed = $921\ 200 \text{ cm}^2 \div 215{,}05 \text{ cm}^2$
$= 4\ 283{,}65$
$= 4\ 284$ bricks are needed (to the nearest whole number) (2)

5.6 $13{,}75$ cm = $137{,}5$ mm
If 4 mm = 10 km, then 1 mm = $10 \div 4 = 2{,}5$ km
Actual distance = $137{,}5 \times 2{,}5$
$= 343{,}75$ km (2)

[35]

Chapter 3 Exponents

Chapter overview

Exponents

- **Unit 1** Comparing and representing numbers in exponential form
- **Unit 2** Laws of exponents
- **Unit 3** Calculations with exponents
- **Unit 4** Scientific notation and solving problems with exponents

Content		Time allocations	LB page
Unit 1	Comparing and representing numbers in exponential form	1 hour	93
Unit 2	Laws of exponents	2 hours	96
Unit 3	Calculations with exponents	$1\frac{1}{2}$ hours	104
Unit 4	Scientific notation and solving problems with exponents	$1\frac{1}{2}$ hours	108

Background information on exponents

In this chapter, exponents as done in Grades 7 and 8 are consolidated and extended to include integer exponents and scientific notation involving negative exponents. There is additional focus on the laws of exponents as well as square and cube roots of exponents.
Practical examples of the application of scientific notation can be found in science text books, reports about population statistics etc.

Teaching guidelines

- Revision and consolidation of the learning material done in previous grades is important.
- Care must be taken that learners apply the laws of exponents in the correct way.
- When doing calculations involving numbers in exponential form, learners must be shown how to use the relevant keys of a calculator.
- Simple exponential equations are introduced in Unit 3. Care must be taken that learners do not confuse this kind of equation with other kinds of equations they learnt about in previous grades.

Resources

Posters showing the laws of exponents, or posterboard and colour pens to create posters, vocabulary cards, number lines and HTU tables include decimal fractions. Examples of scientific notation used in chemistry and astronomy to show learners the use of such notation in context. Each learner should have their own calculator.

Unit 1 Comparing and representing numbers in exponential form

Learner's Book page 93

Unit focus

This unit focusses on the following:
- revising the writing and comparing of numbers in exponential form;
- relooking at the square roots of perfect square numbers; and
- the cube roots of perfect cube numbers.

Background information on comparing and representing numbers in exponential form

Learners should be familiar with representing numbers in exponential form. This unit should serve as revision, laying the groundwork for the rest of the chapter.

Exercise 1

Learner's Book page 93

Guidelines on how to implement this activity

Revise writing numbers in expanded form, with the index representing how many times the number must be multiplied by itself. Do a few examples together on the board. Start with numbers, and be sure to include variables. Discuss how to write numbers from expanded form into exponential form. Introduce expanding of fractions and writing fractions in exponential form. Introduce variables as well. Show learners how to calculate when numbers are in expanded or exponential form. Focus in particular on fractions. Learners may need to revise how to multiply fractions in order to manage this part of the exercise.

Suggested answers

1.1 3^7 **1.2** $(-10)^6$ **1.3** x^5
1.4 $\left(\frac{1}{2}\right)^3$ **1.5** $(1{,}\dot{3})^2$

2.1 $2 \times 2 \times 2 \times 2 \times 2 \times 2$
2.2 $(-4) \times (-4) \times (-4)$
2.3 $\left(\frac{2}{3}\right) \times \left(\frac{2}{3}\right) \times \left(\frac{2}{3}\right) \times \left(\frac{2}{3}\right)$
2.4 $(0{,}1) \times (0{,}1)$
2.5 $(-1) \times (-1) \times (-1) \times (-1) \times (-1) \times (-1) \times (-1)$

2.6	$\frac{x}{n} \times \frac{x}{n} \times \frac{x}{n}$				
3.1	64	**3.2**	-64	**3.3**	$\frac{16}{81}$
3.4	0,01	**3.5**	-1	**3.6**	$\frac{x^3}{n^3}$

Remedial

If learners are struggling with this exercise, it is important to address the problem areas now. Learners who are not able to manage this unit will not be able to cope with the rest of the chapter. Particular problem areas may be multiplying fractions and decimals, and working with variables. Be sure to remediate any problems before allowing learners to progress to the remainder of the chapter.

Exercise 2

Learner's Book page 94

Guidelines on how to implement this activity

When working with squares and cubes, ask learners to prepare a squares and cubes chart. Create a table with 3 columns. In the first column learners write the numbers 1 – 12, in the second column learners find the square of the number (this means the learners multiply the number by itself and write the answer). In the third column learners find the cube (they multiply the answer by itself twice). When finding cubes, learners only need to go up to six cubed. Encourage learners to memorise the chart. Show learners that this chart also works for square roots and cube roots.
Once learners have had a chance to memorise the chart, play games calling out numbers and asking learners to supply the square or cube.
Show learners how using our existing knowledge of squares and cubes, we can find the squares cubes, square roots and cube roots of fractions and decimals. Do a few examples together as a class.

Suggested answers

1.1	6	**1.2**	11	**1.3**	$\frac{1}{3}$
1.4	$\sqrt{\frac{9}{4}} = \frac{3}{2}$	**1.5**	0,1	**1.6**	x^2
2.1	$6^2 = 36$	**2.2**	$11^2 = 121$	**2.3**	$\left(\frac{1}{3}\right)^2 = \frac{1}{9}$
2.4	$\left(\frac{3}{2}\right)^2 = \frac{9}{4} = 2\frac{1}{4}$	**2.5**	$(0,1)^2 = 0,01$	**2.6**	$(x^2)^2 = x^4$
3.1	3	**3.2**	-1	**3.3**	4
3.4	-3	**3.5**	5	**3.6**	0,5
4.1	$3^3 = 27$	**4.2**	$(-1)^3 = -1$	**4.3**	$4^3 = 64$
4.4	$(-3)^3 = -27$	**4.5**	$(5)^3 = 125$	**4.6**	$(0,5)^3 = 0,125$

Remedial

Have regular quizzes of the squares and cubes chart to ensure learners learn these important concepts.

Extension

Encourage stronger learners to learn the cubes of numbers up to 12^3.

Exercise 3

Learner's Book page 95

Guidelines on how to implement this activity

Discuss comparing whole numbers and how we do this. Introduce square and cube numbers into comparisons. Show learners that sometimes it is necessary to calculate the square and cube number in order to compare them. However if either the bases or the index is the same, for both numbers, we can compare them without calculating. Do a few examples together before learners do this exercise on their own.

Suggested answers

1 $8^2 = 64$ and $7^2 = 49$
So $8^2 < 7^2$

2 $6^4 = 1\ 296$ and $6^6 = 46\ 656$
So $6^4 < 6^6$

3 $3^{21} = 1{,}046 \times 10^{10}$ and $3^{22} = 3{,}138 \times 10^{10}$
So $3^{21} < 3^{22}$

4 $2^4 = 16$ and $3^4 = 81$
So $2^4 < 3^4$

5 $5^5 = 3\ 125$ and $4^6 = 4\ 096$
So $5^5 < 4^6$

6 $2^5 = 32$ and $3^5 = 243$
So $2^5 < 3^5$

7 $x^5 < x^4$

8 $5^a < 3^a$

9 $b^2 < b^3$

10 $6^x < 7^x$

Remedial

Spend more time revising comparing whole numbers with learners who struggle to grasp comparing exponential numbers.

Unit 2 Laws of exponents

Learner's Book page 96

Unit focus

This unit focusses on the following:
- revising some of the laws of exponents you learnt in Grade 8,
- revising multiplying and dividing using the laws,
- learning about negative exponents, and
- learning how to apply these laws to multiplying brackets in algebra.

Background information on laws of exponents

Learners know the following laws of exponents from Grade 8:
- $a^m \times a^n = a^{m+n}$
- $a^m \div a^n = a^{m-n}$
- $(am)^n = a^{m \times n}$
- $(a \times t)^n = a^n t^n$
- $a^0 = 1, a \neq 0$

These laws need to be extended to include integer exponents. Learners also need to learn the law that states: $a^{-m} = \frac{1}{a^m}$

Exercise 1

Learner's Book page 97

Guidelines on how to implement this activity

Revise the laws for multiplying and dividing powers. Remind learners that we can only apply the exponent laws when the bases are the same. Work through the examples in the Learner's Book. Be sure to include examples with variables, decimals and fractions. When working with powers in brackets, revise how to use the appropriate law to multiply out the bracket. Learners should complete this exercise on their own.

Suggested answers

1.1 $7^9 \times 7^4 = 7^{9+4} = 7^{13}$

1.2 $(-5)^7 \times (-5)^3 = (-5)^{7+3} = (-5)^{10}$

1.3 $(0,6)^5 \times (0,6)^1 = (0,6)^6$

1.4 $3^6 \times 3^8 \times 3^{10} = 3^{24}$

1.5 $7^9 \div 7^4 = 7^{9-4} = 7^5$

1.6 $10^8 \div 10^7 = 10^{8-7} = 10^1 = 10$

1.7 $(-12)^{11} \div (-12)^{11} = (-12)^{11-11} = (-12)^0 = 1$

1.8 $\left(\frac{5}{8}\right)^6 \times \left(\frac{5}{8}\right)^4 = \left(\frac{5}{8}\right)^{6+4} = \left(\frac{5}{8}\right)^{10}$

1.9 $(-1,1)^9 \div (-1,1)^6 = (-1,1)^{9-6} = (1,1)^5$

2.1 $5^3 \times 5^3 = 5^6$

2.2 $3^2 \times 3^2 = 3^4$

2.3 $7^6 \div 7^3 = 7^{6-3} = 7^3$

2.4 $x^3 \div x^3 = x^{3-3} = x^0 = 1$

2.5 $(10 \times 10 \times 10) \div (5 \times 5) = 40$

2.6 $y^3 + y^3 = 2y^3$

2.7 $x^3 - x^3 = 0$

3.1 $7^{2+4} \div 7^{3\times 2}$

$= 7^6 \div 7^6$

$= 7^{6-6}$

$= 7^0$

$= 1$

3.2 $\left(\frac{3}{4}\right)^5 \div \left(\frac{3}{4}\right)^2$

$= \left(\frac{3}{4}\right)^{5-2}$

$= \left(\frac{3}{4}\right)^3$

$= \frac{27}{64}$

3.3 $9^2(3^2 + 9^0 - 9)$

$= 9^2(9 + 1 - 9)$

$= 9^2(1)$

$= 81$

3.4 $ab^2 \times a^3b^4 \div a^4b^6$

$= a^4b^6 \div a^4b^6$

$= a^{4-4}.b^{6-6}$

$= a^0.b^0$

$= 1 \times 1$

$= 1$

3.5 $2(2x^2y^3)(-2x^2y)$

$= 2(-4x^4y^4)$

$= -8x^4y^4$

3.6 $\frac{15p^{14r^5}}{25p^{9r^6}}$

$= \frac{3p^5}{5r}$

4.1 $3^4 \times 3^7$

$= 3^{4+7}$

$= 3^{11}$

4.2 $3^4 + 3^7$

$= 81 + 2\ 187$

$= 2\ \ 268$

4.3 $(-a)^2$

$= a^2$

4.4 $-7^2a \times ab$

$= -49\ a^2b$

4.5 $6x^3 \times 4x^2$

$= 24x^5$

4.6 $6x^3 + 4x^2$

cannot be simplified

4.7 $-3ab^2 \times 5ab^2$

$= -15a^2b^4$

4.8 $6p^2q \times 2pq$

$= 12p^3q^2$

4.9 $(2xy)(-3x^{2y})(-4xy^3)$

$(2xy)(12x^3y^4)$

$= 24x^4y^5$

4.10 $2xy - 3x^2y - 4xy^3$

Cannot be simplified.

Remedial

Use this exercise to assess learners' capability with working with exponents. Learners have covered all this work in Grade 8, and if learners are struggling some remediation is required. Provide additional Grade 8 work on exponents to provide as homework until learners can manage with prescribed material. If necessary work in a small group with these learners and re-explain the concept of exponents and exponent laws.

Exercise 2

Learner's Book page 102

Guidelines on how to implement this activity

Introduce negative exponents to learners. Show learners the new law that converts negative exponents to positive, $a^{-m} = \frac{1}{a^m}$. Do examples of this rule together on the board. Next, revise raising a power to another power. Show learners how we multiply the exponents when we do this. Show learners the reasoning behind this by writing the examples in expanded notation. Do a few examples together as a class. Lastly show learners the powers of products and quotients. Work through the examplesin the Learner's Book and do additional examples if required. Pay attention to working with exponents in algebraic fractions. Tell learners that when working with exponents, it is mathematically incorrect to have an answer with negative exponents. We have to convert all negative exponents to positive, by implementing the new law we have learnt.
Learners should try this exercise on their own, but can work in pairs if necessary.

Suggested answers

1.1 a^5

1.2 a^6

1.3 $\frac{1}{7^3}$

1.4 $3x^4$

1.5 $\left(\frac{m}{n}\right)^3$

1.6 $x^0 = 1$

1.7 $\frac{7}{p^3}$

1.8 $16xy$

1.9 $\frac{xy - x^2y}{x^2}$

$= \frac{y}{x} - y$

2.1 $2^{-3}.6^2.3$

$= 2^{-3}.(2.3)^2.3^1$

$= 2^{-3+2}.3^{2+1}$

$= 2^{-1}.3^3$

$= \frac{3^3}{2}$

$= \frac{27}{2}$

2.2 $\frac{9^{-2} \times (12)^2}{4^2 \times 6}$

$= \frac{3^{-2} \times (2^2 \times 3)^2}{(2^2)^2 \times (2 \times 3)}$

$= \frac{3^{-2} \times 2^4 \times 3^2}{2^4 \times 2 \times 3}$

$= \frac{3^{-2+2} \times 2^4}{2^{4+1} \times 3}$

$= \frac{3^0 \times 2^4}{2^5 \times 3}$

$= \frac{1}{2 \times 3}$

$= \frac{1}{6}$

2.3 $\frac{5^n.\ 5^{n+1}}{25^n}$

$= \frac{5^{2n+1}}{(5^2)n}$

$= 5^{2n+1-2n}$

$= 5^1$

$= 5$

2.4 $(3x^{-1})^3$

$= 3^3.(x^{-1})^3$

$= 27 \times x^{-3}$

$= \frac{27}{x^3}$

2.5 $\left(\frac{3}{x^{-2}}\right)^2$

$= (3x^2)^2$

$= 9x^4$

2.6 $2(a^2b^3)^2(-2a^2b)$

$= 2a^4b^6 \times -2a^2b$

$= -4a^{4+2}b^{6+1}$

$= -4a^6b^7$

2.7 $\frac{7^{2n-1}}{7^{n+2}}$

$= 7^{2n-1-(n+2)}$

$= 7^{2n-1-n-2}$

$= 7^{n-3}$

3.1 $\frac{5^6}{5^2}$

$= 5^{6-2}$

$= 5^4$

$= 625$

3.2 $\frac{3^5}{3^8}$

$= 3^{5-8}$

$= 3^{-3}$

$= \frac{1}{3^3}$

$= \frac{1}{27}$

3.3 $\frac{33x^4y^2}{11xy}$

$= 3x^{3y}$

3.4 $\frac{8x^8y^3}{2x^2y}$

$= 4x^6y$

3.5 $\frac{16a^2b^3}{10a^5b^2}$

$= \frac{8b}{5a^3}$

3.6 $\frac{8x^7y^3}{32x^6y^2}$

$= \frac{xy}{4}$

3.7 $\frac{24a^4b^2c^3}{12b^2c^4}$

$= \frac{2a^4}{c}$

3.8 $\frac{7x^7y^8}{28x^5y^7}$

$= \frac{x^2y}{4}$

4.1 $\frac{9^{n-1} \times 12^n}{4^{n+1} \times 27^n}$

$= \frac{(3^2)^{n-1} \times (3.2^2)^n}{(2^2)^{n+1} \times (3^3)^n}$

$= \frac{3^{2n-2} \times 3^n \times 2^{2n}}{2^{2n+2} \times 3^{3n}}$

$= \frac{2^{2n} \times 3^{3n-2}}{2^{2n+2} \times 3^{3n}}$

$= 2^{2n-(2n+2)} \times 3$
$(3n-2)-3n$

$= 2^{2n-2n-2} \times$
$3^{3n-2-3n}$

$= 2^{-2} \times 3^{-2}$

$= (2 \times 3)^{-2}$

$= 6^{-2}$

$= \frac{1}{6^2}$

$= \frac{1}{36}$

4.2 $\frac{x^{-m} \times (xy)^{2m} \times x^{-2}}{xy}$

$= \frac{x^{-m} \times x^{2m}y^{2m} \times x^{-2}}{xy}$

$= \frac{x^{-m+2m-2}.y^{2m}}{xy}$

$= \frac{x^{m-2}.y^{2m}}{xy}$

$= x^{m-2-1}.y^{2m-1}$

$= x^{m-3}.y^{2m-1}$

4.3 $\frac{2^{-3}.x.y^{-2}}{4^2.x^{-1}.y^2}$

$= \frac{x.x}{2^3 4^2 y^2.y^2}$

$= \frac{x^2}{8 \times 16y^4}$

$= \frac{x^2}{128y^4}$

4.4 $\frac{-15a^9b^2}{5a^3b^6}$

$= \frac{-3a^{9-3}}{b^{6-2}}$

$= \frac{-3a^6}{b^4}$

4.5 $(4p^{-2}q^0r)^3$

$= \left(\frac{4r}{p^2}\right)^3$

$= \frac{4^3r^3}{p^6}$

$= \frac{64r^3}{p^6}$

Remedial

Allow learners to work in pairs and discuss their thinking and answers. Have the exponent laws easily available for learners to refer to, including the new laws they have learnt.

Exercise 3

Learner's Book page 103

Guidelines on how to implement this activity

This section extends learners' use of exponents in algebra. Learners can get confused when integrating two concepts, so be sure to explain carefully and allow plenty of time for learners to integrate their thinking of the two concepts. Work through the examples in the Learner's Book, but provide additional examples until you are confident learners will be able to manage the exercise. This exercise focusses on the distributive law with exponents and dividing of polynomials. Learners should try this exercise on their own, but again if necessary they can work in pairs.

Suggested answers

1.1 $2p(5 - 6q^2) = 10p - 12q^2p$

1.2 $3p(1 - p + 2p^2)$
$= 3p - 3p^2 + 6p^3$

1.3 $6a(7 - 2a + 3a^2) = 42a - 12a^2 + 18a^3$

1.4 $-3 + 2(x - 6)$
$= -3 + 2x - 12$
$= 2x - 15$

1.5 $2x(x^2 + 3x) - 6x^2$
$= 2x^3 + 6x^2 - 6x^2$
$= 2x^3$

1.6 $7x - (x^2 - 2x - 1)$
$= 7x - x^2 + 2x + 1$
$= x^2 + 9x + 1$

1.7 $2b(5b^2 - 7b) - 3b^2$
$= 10b^3 - 14b^2 - 3b^2$
$= 10b^3 - 17b^2$

1.8 $-2a(4a^2 + a - 3)$
$= -8a^3 - 2a^2 + 6a$

1.9 $-4x^2y(xy - y + x)$
$= -4x^3y^2 + 4x^2y^2 - 4x^3y$

2.1 $\frac{2x^2 - 4x}{2}$
$= x^2 - 2x$

2.2 $\frac{y^3 - y}{y}$
$= y^2 - 1$

2.3 $\frac{3x^2y - xy^2}{xy}$
$= 3x - y$

2.4 $\frac{5b^3 - 10b}{5b}$
$= b^2 - 2$

2.5 $\frac{3a^4 + a^2 + 6a}{a}$
$= 3a^3 + a + 6$

2.6 $\frac{a^2b - 5ab^3}{ab}$
$= a - 5b^2$

Remedial

Learners who struggle with algebra may require some additional time and remediation in order to manage this exercise. Revise some basic algebra concepts such as the distributive law, multiplying out brackets and dividing polynomials by monomials. Be on hand to assist learners and try to focus them on the exponential aspects rather than being caught up in the algebra manipulations.

Unit 3 Calculations with exponents

Learner's Book page 104

Unit focus

This unit focusses on the following:
- revising and learning more about calculations involving numbers in exponential form; and
- learning how to solve exponential equations.

Background information on calculations with exponents

This unit shows learners how to solve exponents with and without a calculator. It is important that learners are able to use a calculator, but it should not detract from their ability to reason whether the answer they achieve with a calculator is correct or not. Learners are introduced to exponential equations in this unit. Learners only need to be able to solve the basics here, and the concept of exponential equations is explored in more detail in Grade 10. Basic manipulations are all that are required in Grade 9.

Exercise 1

Learner's Book page 104

Guidelines on how to implement this activity

To solve exponents without a calculator means using squares and cubes learners should be familiar with and be able to work out easily. This means squares up to 12^2 and cubes up to 6^3. This is why the squares and cubes chart is so important. Revise the exponent laws again, especially the newer laws and how to apply them. Work through some examples for solving without a calculator and encourage learners to complete this exercise on their own.

Suggested answers

1 $\frac{2^3}{2^{-2}}$

$= 2^3 2^2$

$= 2^{3+2}$

$= 2^5$

$= 32$

2 $\frac{3^5}{3^7}$

$= \frac{1}{3^{7-5}}$

$= \frac{1}{3^2}$

$= \frac{1}{9}$

3 $(-5)^{-2} \times 10^2$

$= \frac{1}{(-5)^2} \times (2x5)^2$

$= \frac{2^2x5^2}{5^2}$

$= 2^2$

$= 4$

4 $\frac{1}{4^{-2}} \div \left(\frac{1}{2}\right)^{-3}$

$= 4^2 \div (2)^3$

$= \frac{(2^2)^2}{2^3}$

$= \frac{2^4}{2^3}$

$= 2$

5 $\frac{7^5}{(-7)^3} + (49)^0$

$= \frac{7^5}{-(7)^3} + 1$

$= -7^{5-3} + 1$

$= -7^2 + 1$

$= -49 + 1$

$= -48$

6 $5^{-3} \times 5^5 - \frac{(-4)^3}{2^{-2}}$

$= 5^{-3+2} - (4^3)(2^2)$

$= 5^{-1} + (2^2)^3(2^2)$

$= \frac{1}{5} + 2^{6+2}$

$= \frac{1}{5} + 2^8$

$= 256\frac{1}{5}$

7 $\frac{3^4 + (-3)^3}{9^2 - (-3)^3}$

$= \frac{81 - 27}{81 - (-27)}$

$= \frac{54}{108}$

$= \frac{1}{2}$

8 $\frac{(4^3(-5)^3)^2}{2^5(-5)^4}$

$= \frac{2^{3 \times 2 \times 2}(-5)^{3 \times 2}}{2^5(-5)^4}$

$= \frac{2^{12}(-5)^6}{2^5(-5)^4}$

$= 2^7(-5)^2$

$= 128 \times 25$

$= 3\ 200$

9 $\left(\frac{3}{5}\right)^2 \div \left(\frac{4}{3}\right)^{-3}$

$= \left(\frac{3}{4}\right)^5 \div \left(\frac{3}{4}\right)^3$

$= \frac{3^5}{4^5} \times \frac{4^3}{3^3}$

$= \frac{3^2}{4^2} = \frac{9}{16}$

10 $\frac{10^{2n-3} \times 10^{6-n}}{10^{n+3}}$

$= \frac{10^{2n-3+6-n}}{10^{n+3}}$

$= \frac{10^{n+3}}{10^{n+3}}$

$= 1$

Remedial

Revise the squares and cubes chart to help learners quickly identify and break down numbers into base primes. Have the laws of exponents easily available for learners to refer to at all times. Encourage learners to check the rules and laws whenever they are unsure or stuck on a problem.

Exercise 2

Learner's Book page 105

Guidelines on how to implement this activity

Discuss substitution in algebra. Demonstrate to learners that in this case it involves substituting a value in order to find the value of the whole expression. Work through the examples in the Learner's Book. Keep examples simple at first, in order for learners to focus on the concept, rather than the intricacies of the calculation. Gradually increase the complexity until learners are able to cope with the exercise on their own.

Suggested answers

1.1
$$\begin{aligned} & x^2 - 3x + 4 \\ &= (-2)^2 - 3(-2) + 4 \\ &= 4 + 6 + 4 \\ &= 14 \end{aligned}$$

1.2
$$\begin{aligned} & y^3 + 3y^2 - y + 5 \\ &= (-1)^3 + 3(-1)^2 - (-1) + 5 \\ &= -1 + 3 + 1 + 5 \\ &= 8 \end{aligned}$$

1.3
$$\begin{aligned} & \left(\frac{3b}{a}\right)^3 \\ &= \left(\frac{3(-1)}{2}\right)^3 \\ &= \left(\frac{-3}{2}\right)^3 \\ &= -\frac{3^3}{2^3} \\ &= -\frac{27}{8} \end{aligned}$$

1.4
$$\begin{aligned} & \tfrac{1}{3}\left(3x^2y^{-3}\right)^2 \\ &= \tfrac{1}{3}\left(9x^4y^{-6}\right) \\ &= \frac{3x^4}{y^6} \\ &= \frac{3(2)^4}{(-1)^6} \\ &= \frac{3 \times 16}{1} \\ &= 48 \end{aligned}$$

2.1
$$\begin{aligned} & a^5 - 3a^2b^2 + 3{,}5^c \\ &= (-2)^5 - 3(-2)^2\left(\tfrac{1}{2}\right)^2 + (3{,}5)^c \\ &= -32 - 3 \times 4 \times \tfrac{1}{4} + 1 \\ &= -31 - 3 \\ &= -34 \end{aligned}$$

2.2
$$\begin{aligned} & (a + b)^c + (a + b + c)(a - b) \\ &= \left(-2 + \tfrac{1}{2}\right)^0 + \left(-2 + \tfrac{1}{2} + 0\right)\left(-2 - \tfrac{1}{2}\right) \\ &= 1 + \left(-1\tfrac{1}{2}\right)\left(-2\tfrac{1}{2}\right) \\ &= 1 + \left(\tfrac{-3}{2} \times \tfrac{-5}{2}\right) \\ &= 1 + \tfrac{15}{4} \\ &= 1 + 3\tfrac{3}{4} \\ &= 4\tfrac{3}{4} \end{aligned}$$

3.1 $\frac{(4x^2y^3)^3}{x^4y}$

$= \frac{64x^6y^9}{x^4y}$

$= 64x^2y^8$

$= 64(-2)^2(-1)^8$

$= 64 \times (+4) \times 1$

$= 256$

3.2 $\frac{p^{m+2}(q^3 + q^0)}{p^2(q^3 + 1)^2}$

$= \frac{5^{1+2}[(-2)^3 + (-2)^0]}{5^2[(-2)^3 + 1]^2}$

$= \frac{5^3(-8+1)}{5^2(-8+1)^2}$

$= \frac{5^3.(-7)}{5^2(-7)^2}$

$= \frac{5}{-7}$

Exercise 3

Learner's Book page 106

Guidelines on how to implement this activity

It is important that learners are able to use their calculators correctly. This exercise is important in helping them to learn the correct input keys when working with exponents. Work through the examples in the Learner's Book, and supply additional examples to ensure learners have managed to master the skill. Ensure each learner is able to use their own calculator. The order of the input keys may not be the same for all calculators and it is important that you ensure each learner knows the correct input sequence for their calculator.

Suggested answers

1.1 $(1,5)^2 = 2,25$

1.2 $2^{10} = 1\,024$

1.3 $\left(\frac{1}{2}\right)^5 = \frac{1}{2^5}$
$= \frac{1}{32}$
$= 0,03$

1.4 $(-3)^7 = -2\,187$

1.5 $(-0,9)^4 = -0,6561$
$= -0,66$

1.6 $\sqrt{2} = 1,4142136$
$= 1,41$

1.7 $\sqrt{169} = 13$

1.8 $\sqrt{0,0625} = 0,25$

1.9 $\sqrt[3]{216} = 6$

1.10 $\sqrt[3]{-1\,000} = -10$

2.1 $4\pi r^2$
$= 4\pi(2,5\text{cm})^2$
$= 78,54\text{cm}$

2.2 $\sqrt{5^2 + 12^2}$
$= \sqrt{25 + 144}$
$= \sqrt{169}$
$= 13$

2.3 $\frac{4}{3}\pi r^3$
$= \frac{4}{3} \times \pi \times 9^3$
$= 3\,053,63$

2.4 $\sqrt[3]{6^3 + 4^3}$
$= \sqrt[3]{216 + 64}$
$= \sqrt[3]{280}$
$= 6,54$

Remedial

Ensure each learner knows how their own calculator works. Provide additional calculators as some learners may have forgotten or may not have calculators.

Extension

Have calculator speed tests where learners have to find the answer on their calculator. Reward with class points.

Exercise 4

Learner's Book page 107

Guidelines on how to implement this activity

Learners must be told that an exponential equation has the variable, for example x, in the exponent. In order to solve for the variable, the bases have to be the same. This is the cornerstone of solving exponential equations. Work through some examples together as a class. Work slowly as this is a new concept and learners may require additional time to grasp the intricacies involved. Learners can work through this exercise in pairs.

Suggested answers

1
$3^x = 81$
$3^x = 3^4$
$x = 4$

2
$2^{-x} = 0{,}125$
$2^{-x} = (0{,}5)^3$
$2^{-x} = \left(\frac{1}{2}\right)^3$
$= 2^{-3}$
$x = 3$

3
$2.5^x = 50$
$5^x = 25$
$5^x = 5^2$
$x = 2$

4
$16^x = 8$
$(2^4)^x = 2^3$
$2^{4x} = 2^3$
$4x = 3$
$x = \frac{3}{4}$

5
$5^{x+1} = 1 = 5^0$
$x + 1 = 0$
$x = -1$

6
$5^y - 1^5 = 0$
$5^y = 1$
$5^y = 5^0$
$y = 0$

7
$\left(-\frac{3}{4}\right)^k = 1 = \left(-\frac{3}{4}\right)^0$
$k = 0$

8
$10^{1-x} = 0{,}01$
$10^{1-x} = 10^{-2}$
$1 - x = -2$
$-x = -2 - 1$
$-x = -3$
$x = 3$

9
$\left(\frac{1}{3}\right)^x = \frac{1}{9}$
$\left(\frac{1}{3}\right)^x = \frac{1}{3^2}$
$3^{-x} = 3^{-2}$
$-x = -2$
$x = 2$

10
$5^{-2} = \frac{1}{25^p}$
$5^{-2} = \frac{1}{(5^2)^p}$
$5^{-2} = 5^{-2p}$
$-2p = -2$
$p = 1$

11
$2\left(\frac{1}{3}\right)^x = 54$
$\left(\frac{1}{3}\right)^x = 27$
$3^{-x} = 3^3$
$-x = 3$
$x = -3$

12
$5^y - 0{,}2 = 0$
$5^y = 0{,}2$
$5^y = \frac{1}{5}$
$5^y = 5^{-1}$
$y = -1$

Remedial

If learners are experiencing problems, provide additional examples for them to work through. Encourage learners to break the problem down and to always start with trying to make the bases the same. Work through some additional examples and explain the thinking process all the way through.

Extension

If learners have managed well, try including more challenging examples from Grade 10 for learners to work with.

Unit 4 Scientific notation and solving problems with exponents

Learner's Book page 108

Unit focus

This unit focusses on the following:
- revising scientific notation,
- learning about scientific notation for numbers smaller than 1, and
- solving problems with exponents.

Background information on scientific notation

Scientific notation is a good context for learners to explore working with exponents. Learners should be informed of the contexts where we find scientific notation. It is used to describe very large numbers, such as in space and geo-physics, and it is used to describe very small numbers, such as in biology and chemistry. Learners have covered scientific notation before in Grade 8. In Grade 9, the work extends to include negative exponents – the negative exponents are used to denote very small numbers.

Exercise 1

Learner's Book page 109

Guidelines on how to implement this activity

Revise place value notation. Have an HTU (including decimals up to 1000ths) chart and illustrate how each place value is a power of 10. Next, illustrate how we use multiplying by powers of ten to denote place value. Do a few examples together as a class before the exercise. Learners should manage this exercise on their own.

Suggested answers

1.1 $5\ 029{,}33 = 5,\ 02933 \times 10^{+3}$

1.2 $9,\ 807 = 9,\ 807 \times 10^{0}$

1.3 $125\ 078 = 1,\ 25078 \times 10^{5}$

1.4 $0{,}0145 = 1{,}45 \times 10^{-2}$

1.5 $123\ 456\ 789 = 1{,}23456789 \times 10^{8}$

2.1 $= 7\ 098{,}157$

2.2 $= 0,\ 64051$

Exercise 2

Learner's Book page 110

Guidelines on how to implement this activity

Learners covered scientific notation of larger numbers in Grade 8. This section should be revision. Discuss with learners where we might find examples of scientific notation in everyday life. Revise how we set out scientific notation. Ensure learners remember to include the multiplication by 10. Do a few examples together as a class, but then learners should manage this exercise on their own.

Suggested answers

1.1	$275{,}09 = 2{,}7509 \times 10^{+2}$	**1.2**	$9\ 853\ 301 = 9{,}853301 \times 10^{6}$
1.3	$4\ 070\ 000 = 4{,}07 \times 10^{6}$	**1.4**	$1\ 430\ 000\ 000$ km $= 1{,}43 \times 10^{9}$ km
2.1	$1{,}04605 \times 10^{5} = 104\ 605$	**2.2**	$7{,}6078 \times 10^{5} = 104\ 605$
2.3	$9{,}75 \times 10^{8} = 975\ 000\ 000$	**2.4**	$4{,}0350816 \times 10^{5} = 403\ 508{,}16$

Remedial

Have additional Grade 8 material on scientific notation available for additional practice for learners experiencing any problems with this exercise.

Exercise 3

Learner's Book page 111

Guidelines on how to implement this activity

Introduce representing numbers smaller than 1 in scientific notation. Show learners examples of very small numbers in biology and science. Demonstrate to learners how we would represent these numbers using scientific notation. Do a few examples together as a class. Be sure to show learners how to work from scientific notation to place value notation. It is also important that learners are able to round off correctly. If necessary spend some time revising rounding off to 2 and 3 decimal places. Learners should manage this activity on their own.

Suggested answers

1.1	$1{,}04605 \times 10^{6} = 1\ 046\ 050$	**1.2**	$9{,}75 \times 10^{8} = 975\ 000\ 000$
1.3	$4{,}00350816 \times 10^{5} = 40\ 035{,}0816$	**1.4**	$1{,}25 \times 10^{-2} = 0{,}0125$
1.5	$9{,}025 \times 10^{-4} = 0{,}0009025$	**1.6**	$4{,}2015 \times 10^{-7} = 0{,}00000042015$
2.1	$6\ 540\ 700 = 6{,}5407 \times 10^{6}$	**2.2**	$123\ 456\ 789 = 1{,}23 \times 10^{8}$
2.3	$531\ 005\ 000\ 000\ 000 = 5{,}31005 \times 10^{14}$		
2.4	$0{,}00258 = 2{,}58 \times 10^{-3}$	**2.5**	$0{,}0000060801 = 6{,}0801 \times 10^{-6}$
2.6	$0{,}000000000074 = 7{,}4 \times 10^{-11}$		

3 You divide by 10^{6} and then multiply by 10^{6}.

4 You multiply by 10^{5} and then divide by 10^{5}.

5 You divide by 1 000.

Extension

Learners can do a mini-project on scientific notation and its uses in mathematics and science. Learners can explore the history of this notation, as well as how it is used today.

Exercise 4

Learner's Book page 112

Guidelines on how to implement this activity

In this section learners need to integrate the laws of exponents to work with scientific notation. Learners apply the laws of multiplication and division with exponents to solve the examples in this exercise. Work through the examples in the Learner's Book and provide additional examples if you feel learners need more practice before doing the exercise. Learners should try to complete this exercise on their own.

Suggested answers

1.1 $4 \times 10^5 \times 7 \times 10^{-3}$
$= 28 \times 10^2$
$= 2,8 \times 10^3$

1.2 $2,2 \times 10^4 \times 5 \times 10^{-4}$
$= 2,2 \times 5 \times 10^0$
$= 11 \times 10^0$
$= 1,1 \times 10^1$

1.3 $9,01 \times 10^{-2} \times 3 \times 10^{-4}$
$= 9,01 \times 3 \times 10^{-6}$
$= 27,03 \times 10^{-6}$
$= 2,703 \times 10^{-5}$

2.1 $(5,4 \times 10^6) \div (9 \times 10^4)$
$= (5,4 \div 9) \times (10^6 \div 10^4)$
$= 0,6 \times 10^2$
$= 6 \times 10^1$

2.2 $(1,8 \times 10^2 \div (6 \times 10^{-2})$
$= (1,8 \div 6) \times (10^{-2} \div 10^{-2})$
$= 0,3 \times 10^4$
$= 3 \times 10^3$

3.1 $9,25 \times 10^{-4} + 1,05 \times 10^{-4}$
$= (9,25 + 1,05) \times 10^{-4}$
$= 10,3 \times 10^{-4}$
$= 1,03 \times 10^{-3}$

3.2 $6,015 \times 10^6 + 2,3 \times 10^5 - 7 \times 10^3$
$= 6\ 015 \times 10^3 + 230 \times 10^3 - 7 \times 10^3$
$= (6\ 015 + 230 - 7) \times 10^3$
$= 6\ 238 \times 10^3$
$= 6\ 238 \times 10^6$
Alternative
$60,15 \times 10^5 + 2,3 \times 10^5 - 0,07 \times 10^5$
$= (60,15 + 2,3 - 0,07) \times 10^5$
$= 62,38 \times 10^5$
$= 6,238 \times 10^6$
Alternative
$6,015 \times 10^6 + 0,23 \times 1$
$0^6 - 0,007 \times 10^6$
$= (6,015 + 0,23 - 0,007) \times 10^6$
$= 6,238 \times 10^6$

4.1 $7{,}205 \times 10^4 - 7{,}0205 \times 10^5$
$= 7{,}205 \times 10^4 - 70{,}205 \times 10^4$
$= (7{,}205 - 70{,}205) \times 10^4$
$= -63 \times 10^4$
$= -6{,}3 \times 10^5$
Alternative
$0{,}7205 \times 10^5 - 7{,}0205 \times 10^5$
$= (0{,}7205 - 7{,}0205) \times 10^5$
$= -6{,}3 \times 10^5$

4.2 $1{,}125 \times 10^{-2} - 2{,}5 \times 10^{-4}$
$= 1{,}125 \times 10^{-2} - 0{,}025 \times 10^{-2}$
$= (1{,}125 - 0{,}025) \times 10^{-2}$
$= 1{,}1 \times 10^{-2}$
Alternative
$112{,}5 \times 10^{-4} - 2{,}5 \times 10^{-4}$
$= (112{,}5 - 2{,}5) \times 10^{-4}$
$= 110 \times 10^{-4}$
$= 1{,}1 \times 10^{-2}$

Remedial

Revise the laws of multiplying and dividing with exponents in more detail if learners are struggling. Learners may have trouble integrating the two concepts. Provide additional practice exercises with simple calculations to allow learners to focus on the new concept, rather than the complex operations.

Exercise 5

Learner's Book page 113

Guidelines on how to implement this activity

Learners are exposed to real life examples of solving problems with scientific notation. Work through the examples in the Learner's Book, and supply additional context if learners ask questions about the background of the questions.
When working with word problems, learners must:
- they must read the question carefully,
- identify what they are being asked,
- identify the information they are given,
- write a number sentence,
- substitute in the numbers, and
- solve and check the reasonableness of the answer.

Allow learners to work in groups to discuss the problems, but encourage them to do the working out on their own.

Suggested answers

1.1 $1{,}86 \times 10^5$ miles per second
$= 1{,}86 \times 1{,}609344 \times 10^5$ km/sec
$= 2{,}9933798 \times 10^5$ km/sec

1.2 $2{,}9933798 \times 10^5$ km/sec
$= 2{,}9933798 \times 3\,600$
$\times 10^5$ km/hour
$= 10\,776{,}16742 \times 10^5$ km/hour
$= 1{,}077616742 \times 10^9$ km/hour

2.1 V of a grain of quartz sand
$\frac{4}{3}\pi r^3$
$= \frac{4}{3}\pi\left(\frac{1}{2} \times 0{,}4 \text{ mm}\right)^3$
$= \frac{4}{3}\pi(0{,}2 \text{ mm})^3$
$= 0{,}0335103 \text{ mm}^3$
$= 3{,}35103 \times 10^{-2} \text{ mm}^3$

CHAPTER 3

2.2 1 mm^3 of sand has a mass of 2,6mg

$3{,}35103 \times 10^{-2}$ mm^3 of sand has a mass of $3{,}35103 \times 10^{2} \times 2{,}6$ mg

$= 8{,}712678 \times 10^{-2}$ mg

3.1 $7 \times 10^9 + 5 \times 365 \times 86\,400$ (1 day = 24hrs = 1 440 min = 86 400 sec)

$= 7 \times 10^9 + 1{,}5768 \times 10^8$

$= 7 \times 10^9 + 0{,}15768 \times 10^9$

$= (7 + 0{,}15768) \times 10^9$

$= 7{,}15768 \times 10^9$ people

3.2 $\frac{1{,}34735 \times 10^9}{7 \times 10^9} \times 100\%$

$= (1{,}34735 \div 7) \times (10^9 \div 10^9) \times 100\%$

$= 0{,}1924786 \times 10^0 \times 100\%$

$= 0{,}1924786 \times 1 \times 100\%$

$= 19{,}24786\%$

$= 1{,}924786 \times 10^1\%$

Remedial

Many learners have problems with word problems. Work with learners who have problems in small groups or individually to interpret the question and explain the questions to them. Ensure learners understand what they are being asked. Help learners develop the number sentence and identify the relevant numbers to substitute. Let learners manage the working out themselves. Provide additional practice examples for learners to work on at home.

Consolidation

Learner's Book page 115

Before doing this consolidation exercise, encourage learners to review the work covered in this chapter. Advise learners to use the summary and to revise their work. This exercise can be used as an informal assessment task for you to track how learners are coping with the chapter and the concepts covered. The mark allocation provides guidelines on how to assess learners.

Suggested answers

1.1 $8^5 \times 8^{-2}$

$= 8^{5-2}$

$= 8^3$

1.2 $12^6 \div 12^3$

$= 12^{6-3}$

$= 12^3$

1.3 $(1{,}5)^4 \times (1{,}5)^2 \times (1{,}5)$

$= 1{,}5^{4+2+1}$

$= 1{,}5^7$

1.4 $\left(-\frac{2}{3}\right)^4 \div \left(-\frac{2}{3}\right)^4$

$= \left(-\frac{2}{3}\right)^{4-4}$

$= \left(\frac{2}{3}\right)^0$

$= 1$

2.1 3^{-2}

$= \frac{1}{3^2}$

$= \frac{1}{9}$

2.2 10^{-3}

$= \frac{1}{10^3}$

$= \frac{1}{1000}$

$= 0{,}001$

2.3 $(-5)^{-1}$

$= \frac{1}{-5^1}$

$= -\frac{1}{5}$

2.4 $\left(\frac{2}{3}\right)^{-1}$
$= \left(\frac{3}{2}\right)^{1}$
$= \frac{3}{2}$
$= 1{,}5$

2.5 $(0{,}5)^{-3}$
$= \left(\frac{1}{2}\right)^{-3}$
$= 2^3$
$= 8$

2.6 $\left(\frac{1}{x}\right)^{-5}$
$= x^5$

(6)

3.1 $\sqrt{49}$
$= \sqrt{7^2}$
7

3.2 $\sqrt{\frac{1}{16}}$
$= \sqrt{\frac{1}{4^2}}$
$= \frac{1}{4}$

3.3 $\sqrt{121}$
$= \sqrt{11^2}$
$= 11$

3.4 $\sqrt[3]{1}$
$= \sqrt[3]{1 \times 1 \times 1}$
$= 1$

3.5 $\sqrt[3]{-64}$
$= \sqrt[3]{-4 \times -4 \times -4}$
$= -4$

(5)

4.1 $ab^2c^3 \times a^3bc^{-2} \times a^0b^{-3}c^{-1}$
$= a^{1+3+0} \times b^{2+1-3} \times c^{3-2-1}$
$= a^4b^0c^0$
$= a^4(1)(1)$
$= a^4$

4.2 $2x^2 \times 3x^3 \times 4x^4$
$= 24x^{2+3+4}$
$= 24x^9$

4.3 $a^3b^2c \div ab^2c^3$
$= a^{3-1}b^{2-2}c^{1-3}$
$= a^2b^0c^{-2}$
$= \frac{a^2}{c^2}$

4.4 $\frac{2^0 \times 2^{-1} \times 2^5}{4^{-1} \times 2^4}$
$= \frac{2^{0-1+5}}{2^{-2+4}}$
$= \frac{2^4}{2^2}$
$= 2^2$
$= 4$

4.5 $\frac{x^2y \times x^3y^2}{xy \times x^4y}$
$= \frac{x^5y^3}{x^5y^2}$
$= x^{5-5}.y^{3-2}$
$= x^0.y$
$= y^1$
$= y$

4.6 $\frac{8b^4 \times 2b \times c^{-2}}{c^2 \times b^2 \times a^0}$
$= \frac{16b^{4+1} \times c^{-2}}{1 \times b^2 \times c^2}$
$= 16b^{5-2} \times c^{-2-2}$
$= 16b^3.c^{-4}$
$= \frac{16b^3}{c^4}$

(6)

5.1 $\frac{6x^2y}{2y}$
$= 3x^2$

5.2 $\frac{9a^3y}{27a^2y^2}$
$= \frac{a}{3y}$

5.3 $\frac{x^4 - 4x^3 + 6x^2}{x^2}$
$= x^2 - 4x + 6$

5.4 $\frac{2a^3 - 6a + 12ab}{2a}$
$= a^2 - 3 + 6b$

5.5 $\frac{ab^3 \times (-2a^3b) \times 3ab}{ab}$

$= -6a^2b^2$

5.6 $\frac{(-9x^2y^4) \times 6xy^{-1}}{3xy \times (-3x^2)}$

$= 6y^2$ (6)

6.1 $6x^7$

6.2 $2x^3$

6.3 $\frac{9}{y^2}$

6.4 $\left(\frac{a}{b}\right)^4$

$= \frac{a^4}{b^4}$

6.5 3^6

6.6 $a^2 - 6b$ (6)

7.1 $3^{2-n} \times 3^n$

$= 3^{2-n+n}$

$= 3^2$

$= 9$

7.2 $2^{x-3} \times 4$

$= 2^{x-3} \times 2^2$

$= 2^{x-3+2}$

$= 2^{x-1}$

7.3 $\frac{3^{x+y}}{3^{x-y}}$

$= 3^{x+y-(x-y)}$

$= 3^{x+y-x+y}$

$= 3^{2y}$

$= 9y$

7.4 $(3^{x+1})^2$

$= 3^{2x+2}$

$= 3^{2x}.3^2$

$= 9^x.9$

$= 9^{x+1}$

7.5 $\frac{9^x}{3^x}$

$= \frac{(3^2)^x}{3^x}$

$= 3^{2x-x}$

$= 3^x$

7.6 $(a^5)^2 \times (a^{-2})^5$

$= a^{10} \times a^{-10}$

$= a^{10-10}$

$= a^0$

$= 1$

7.7 $[3(a^2b^3)]^3$

$= 3^3 \times (a^2b^3)^3$

$= 27 \times a^6b^9$

7.8 $[2(x^3y^{-1})^2]^2$

$= [2 \times x^6 \times y^{-2}]^2$

$= 2^2 \times x^{12} \times y^{-4}$

$= \frac{4x^{12}}{y^4}$

7.9 $(-x^2y^3)^4 \times (-2x^3y)$

$= x^4y^{12} \times -2x^3y$

$= -2x^7y^{13}$

(9)

8.1 $\frac{(ab^4)^5 \times a^3b^2}{(a^2b^3)^3}$

$= \frac{a^5b^{20} \times a^3b^2}{a^6b^9}$

$= \frac{a^{5+3} \times b^{20+2}}{a^6b^9}$

$= \frac{a^8 \times b^{22}}{a^6b^9}$

$= a^{8-6} \times b^{22-9}$

$= a^2b^{13}$

8.2 $\frac{2xy^3 \times 4x^2y}{(2xy^2)^3}$

$= \frac{8x^3y^4}{8x^3y^6}$

$= \frac{1}{y^2}$

8.3 $\frac{(3ab^2)^4 \times (2a^3b)^{-2}}{(2a^2b^3)^{-3} \times ab}$

$= \frac{81a^4b^8 \times (2a^2b^3)^3}{(2a^3b)^2 \times ab}$

$= \frac{81a^4b^8 \times 2a^6b^9}{4a^6b^2 \times ab}$

$= \frac{162a^{10}b^{17}}{4a^7b^3}$

$= \frac{81a^3b^{14}}{2}$

(3)

9.1 $8(x + y)^2$
$= 8\left(\frac{1}{2} + \left(-\frac{1}{4}\right)\right)^2$
$= 8\left(\frac{1}{4}\right)^2$
$= 8 \times \frac{1}{16}$
$= \frac{8}{16}$
$= \frac{1}{2}$

9.2 $\frac{[6(a^5b^3)^2]}{a^8b^7}$
$= \frac{6a^{10}b^6}{a^8b^7}$
$= \frac{6a^2}{b}$
$= \frac{6(-2)^2}{-3}$
$= \frac{6 \times 4}{-3}$
$= \frac{24}{-3}$
$= -8$ (4)

10.1 $(-0{,}15)^3$
$= -0{,}003375$

10.2 5^{10}
$= 9\ 765\ 625$

10.3 $\sqrt{289}$
$= 17$

10.4 $\sqrt[3]{100}$
$= 4,\ 6415888$

10.5 $\sqrt[3]{-0{,}125}$
$= -0{,}5$

10.6 $v = \frac{4}{3}\pi r^3$
$= \frac{4}{3}\pi(12{,}5 \text{ cm})^3$
$= 8\ 181{,}230868 \text{ cm}^3$

10.7 $\sqrt[3]{a^3 + b^2}$
$= \sqrt[3]{(-7)^3 + 13^2}$
$= \sqrt[3]{-343 + 169}$
$= \sqrt[3]{-174}$
$= -5{,}5827702$ (7)

11.1 $2^x = 64$
$2^x = 2^6$
$x = 6$

11.2 $2^{1-x} = \frac{1}{2}$
$2^{1-x} = 2^{-1}$
$1 - x = -1$
$-x = -2$
$x = 2$

11.3 $5^y = 1$
$5^y = 5^0$
$y = 0$

11.4 $4^y = 32$
$(2^2)^y = 2^5$
$2y = 5$
$y = 2{,}5$

11.5 $3^{-x} = 81$
$3^{-x} = 3^4$
$-x = 4$
$x = -4$

11.6 $7^{y+2} = \frac{1}{49}$
$7^{y+2} = \frac{1}{7^2}$
$7^{y+2} = 7^{-2}$
$y + 2 = -2$
$y = -4$

11.7 $\left(\frac{1}{2}\right)^{x-1} - 2^{-6} = 0$
$\left(\frac{1}{2}\right)^{x-1} = 2^{-6}$
$(2^{-1})^{x-1} = 2^{-6}$
$2^{-x+1} = 2^{-6}$
$-x + 1 = -6$
$-x = -7$
$x = 7$

11.8 $(25)^{\frac{1}{x}} = 125$
$(5^2)^{\frac{1}{x}} = 5^3$
$5^{\frac{2}{x}} = 5^3$
$\frac{2}{x} = \frac{3}{1}$
$3x = 2$
$x = \frac{2}{3}$

11.9 $2^{x+1} = (0{,}5)^{x+1}$
$2^{x+1} = \left(\frac{1}{2}\right)^{x+1}$
$2^{x+1} = (2^{-1})^{x+1}$
$2^{x+1} = 2^{-x-1}$
$x + 1 = -x - 1$
$2x = -2$
$x = -1$ (9)

CHAPTER 3

12.1 $2{,}5065 \times 10^5$
250 650

12.2 $1{,}04709 \times 10^7$
$= 10\ 470\ 900$

12.3 $6{,}402 \times 10^{-3}$
$= 0{,}006402$

12.4 $3{,}09 \times 10^{-5}$
$= 0{,}0000309$ (4)

13.1 465 000
$= 4{,}65 \times 10^5$

13.2 35 million
$= 35 \times 10^6$
$= 3{,}5 \times 10^7$

13.3 2.3 billion
$= 2{,}3 \times 10^9$

13.4 0,000723
$= 7{,}23 \times 10^{-4}$

13.5 0,000010203
$1{,}0203 \times 10^5$ (5)

13.6 You multiply by 10^4
And then divide by 10^4 (2)

14.1 $(1{,}5 \times 10^5) \times (1{,}2 \times 10^3)$
$= (1{,}5 \times 1{,}2) \times (10^5 \times 10^3)$
$= 1{,}8 \times 10^8$

14.2 $(2{,}5 \times 10^5) \times (9 \times 10^{-2})$
$= (2{,}5 \times 9) \times 10^5 \times 10^{-2}$
$= 22{,}5 \times 10^3$
$= 2{,}25 \times 10^4$

14.3 $(6{,}5 \times 10^5) \div (2 \times 10^3)$
$(6{,}5 \div 2) \times (10^5 \div 10^3)$
$3{,}25 \times 10^2$

14.4 $(2{,}43 \times 10^{-4}) + (8{,}65 \times 10^{-4})$
$= (2{,}43 + 8{,}65) \times 10^{-4}$
$= 11{,}08 \times 10^{-4}$
$= 1{,}108 \times 10^{-3}$

14.5 $(9{,}5 \times 10^5) - (1{,}05 \times 10^6)$
$= (9{,}5 \times 10^5) - (10{,}5 \times 10^5)$
$= (9{,}5 - 10{,}5) \times 10^5$
$= -1 \times 10^5$
Alternative
$(0{,}95 \times 10^6) - (1{,}05 \times 10^6)$
$= (0{,}95 - 1{,}05) \times 10^6$
$= -0{,}1 \times 10^6$
$= -1 \times 10^5$

14.6 $(1{,}75 \times 10^{-5}) - (2{,}05 \times 10^{-6})$
$= (1{,}75 \times 10^{-5}) - (0{,}205 \times 10^{-5})$
$= (1{,}75 - 0{,}0205) \times 10^{-5}$
$= 1{,}545 \times 10^{-5}$
Alternative
$(17{,}5 \times 10^{-6}) - (2{,}05 \times 10^{-6})$
$= (17{,}5 - 2{,}05) \times 10^{-6}$
$= 15{,}45 \times 10^{-6}$
$= 1{,}545 \times 10^{-5}$ (6)

15.1 $\dfrac{1{,}43637 \times 10^8 \text{ km}}{2{,}9933798 \times 10^5 \text{ km/sec}}$
$= 0{,}479848898 \times 10^3$ sec
$= 479{,}848898$ sec

15.2 $\dfrac{479{,}848898}{60}$ min
$= 7{,}997481643$ min (4)

16.1 d = 7,5 micrometers
$= 7{,}5 \times 10^{-6}$ m
$r = \frac{d}{2} = 7{,}5 \times 10^{-6} \div 2$ m
$= 3{,}75 \times 10^{-6}$ m

16.2 $C = \pi D$
$= \pi \times 7{,}5 \times 10^{-6}$ m
$= 23{,}5619449 \times 10^{-6}$ m
$= 2{,}35619449 \times 10^{-5}$ m

16.3 $V = \frac{4}{3}\pi r^3$
$= \frac{4}{3}\pi\left(\frac{7{,}5}{2} \text{ micrometers}\right)^3$
$= 220{,}8932335$ micrometers^3
$= 2{,}208932335 \times 10^2$ micrometers^3 (9)

[95]

Chapter 4 Patterns, functions and relationships

Chapter overview

Pattern, functions and relationships

Unit 1 Investigate and extend patterns

Unit 2 The general rule

Unit 3 Input and output values

Unit 4 Equivalent forms

Content		Time allocations	LB page
Unit 1	Investigate and extend patterns	2 hours	118
Unit 2	The general rule	$2\frac{1}{2}$ hours	123
Unit 3	Input and output values	2 hours	128
Unit 4	Equivalent forms	2 hours	132

Background information on patterns, functions and relationships

Learners have already worked with mathematical patterns in previous grades, so the work in this chapter serves mainly to consolidate their existing knowledge of numeric and geometric patterns. No new content is introduced in this chapter.
In the study of geometric patterns, the focus is on the underlying number patterns that are embedded within the geometric patterns. The focus is not, for example, on discovering the properties of geometric figures. This is done elsewhere in this course, for example in Chapter 7.
The scope of number patterns to be covered in this chapter includes the following:
- patterns with a constant difference (for example, where the same number is added or subtracted each time);
- patterns with a constant ratio (for example, where the same number is used to multiply or divide each time); and
- quadratic sequences (with a second constant difference).

Note that your learners are not expected to know or use these actual terms which will be introduced and defined formally in later grades.

Teaching guidelines

Learners often enjoy working with patterns, functions and relationships, as they perceive this as being different from mainstream mathematics. If you emphasise the fun aspect of this work, you will make it accessible to all your learners.

- When working with number patterns, be aware that the general principles are best illustrated by simple examples. Do not over-complicate this work with examples that involve sign changes and fractions, until you feel confident that your learners are comfortable with the basics.
- When working with geometric patterns, encourage your learners to understand how each pattern is constructed. In particular, they should be very clear on what is required to move from one stage to the next (for example by adding three matches each time to form another square).
- There are many beautiful pictures of objects in nature that show patterns. Collect your own pictures and display them on your classroom walls. These pictures will not only enhance the appearance of your classroom, but will also stimulate your learners to associate mathematics with physical beauty as well as with numeracy. As you will see in Unit 1, some objects in nature show patterns that can be linked to the Fibonacci sequence. However, do not try to link every object in nature to the Fibonacci sequence.
- When finding the general rule for a number pattern, your learners are required to describe these rules in their own words, as well as in algebraic language. Different people have different strengths, so some learners will find the verbal descriptions easier to do than the algebraic formulae, or vice versa.
- Your learners will already be familiar with flow diagrams from previous grades and should find this aspect of the work accessible and pleasant to do.

Resources

Matches, counters, or straws to use for constructing geometric patterns. Pictures of patterns in nature, pine cones, sunflowers and see shells can all be used to demonstrate pattern formation. Number grids and blank tables and flow diagrams for learners to use to translate patterns into equivalent forms.

Unit 1 Investigate and extend patterns

Learner's Book page 118

Unit focus

This unit focusses on the following:

- investigating and extending numeric and geometric patterns
- looking for relationships between numbers, including patterns represented in physical or diagram form, in tables and algebraically
- creating your own numeric and geometric patterns.

Background information on investigating and extending patterns

In this unit, your learners will investigate and extend numeric and geometric patterns, looking for relationships between numbers. These patterns include patterns represented in physical or diagram form, in tables and algebraically. Learners will also create their own numeric and geometric patterns.

Exercise 1

Learner's Book page 118

Guidelines on how to implement this activity

This unit starts off with a revision exercise in which your learners will practise the skills that they have already learnt, as they investigate and extend numeric and geometric patterns. Encourage learners to work on their own, but if necessary learners can work in pairs.

Suggested answers

1.1 1; 3; 5; 7; 9; 11; 13

1.2 $\frac{1}{3}$; $\frac{1}{4}$; $\frac{1}{5}$; $\frac{1}{6}$; $\frac{1}{7}$; $\frac{1}{8}$; $\frac{1}{9}$

1.3 2; 6; 10; 14; 18; 22; 26

1.4 2; 6; 18; 54; 162; 486; 1 458

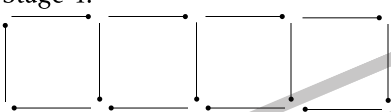
1.5 1; 4; 9; 16; 25; 36; 49

1.6 2; −3; −8; −13; −18; −23; −28

1.7 AC; ACE; ACEG; ACEGI; ACEGIK; ACEGIKM

1.8 1; /1/; /1/1/; /1/1/1/; /1/1/1/1/; /1/1/1/1/1/

2.1 Stage 4:



Stage 5:

2.2

Stage	1	2	3	4	5
Number of matches	4	7	10	13	16

2.3 3 matches.

2.4 She starts with 1 match and adds 3 each time, so the formula to find the number of matches is:
$3n + 1$. If she uses up 50 matches:
$3n + 1 = 50$
$3n = 49$
$n = 16,3$
$3 \times 16 = 48$ so she has 1 match left over.

Remedial

- The ability to extend a number pattern relies on your learners' ability to see the pattern in the given terms. If your learners struggle with this, help them to find the pattern by a process of elimination. They should ask themselves:
  - Is the same number being added or subtracted each time?
  - Is the same number being used to multiply or divide each time?

  If neither of the above applies, they need to look for other possibilities.

- Quadratic patterns (for example, patterns with a constant second difference) are also not dealt with formally in Grade 9. However, your learners should be able to recognise the squares of numbers: 1; 4; 9; 16; 25 and so on. They should then also be able to recognise these squares after they have been manipulated slightly, for example 2; 8; 18; 32; 50 and so on. (In this case, the square numbers have simply been doubled.)
- For learners who struggle, give them simple patterns to practice with, for example:

  1; 2; 3; 4; 5; …
  1; 2; 4; 8; 16; …
  20; 18; 16; 14; 12; …
  32; 16; 8; 4; 2; …

  Then you can move on to patterns where negative numbers appear as well, for example:

  $-1; -2; -3; -4; -5; \ldots$
  $-1; -2; -4; -8; -16; \ldots$
  $-20; -18; -16; -14; -12; \ldots$
  $32; -16; 8; -4; 2; \ldots$
- If learners struggle with patterns that involve fractions, it is often helpful to view the numerators and the denominators separately. For example, in the pattern:

  $\frac{1}{5}; \frac{2}{7}; \frac{3}{9}; \frac{4}{11}; \ldots$

  the numerators are: 1; 2; 3; 4; … and the denominators are 5; 7; ,9; 11; …

Exercise 2

Learner's Book page 121

Guidelines on how to implement this activity

The unit moves on to a treatment of patterns in nature, with specific reference to the Fibonacci sequence. We spend some time on this, because this is the last opportunity that your learners will really have of engaging with patterns in nature in the context of school Mathematics. From Grade 10 onwards, the treatment of patterns becomes more academic as the focus moves on to a more formal treatment of sequences that include arithmetic, geometric and quadratic sequences.
Learners are often referred to examples of natural objects like pineapples, sunflowers and pine cones and are told that they can see number patterns like the Fibonacci sequence in these objects, yet they are seldom guided to see these patterns for themselves. When told that a particular object shows the Fibonacci sequence, learners may expect to see the entire Fibonacci sequence on display. Guide them to understand that in the case of the pine cone that we have included, two sets of spirals are clearly visible. One set of 13 spirals runs anti-clockwise from the centre of the cone, while the other set of 8 spirals runs clockwise. These numbers, 8 and 13, are consecutive numbers in the Fibonacci sequence. So, it is often the case that an object from nature will show two consecutive numbers of the Fibonacci sequence. Other objects that we mention are sunflowers. In some varieties, the spirals are arranged in groups of 55 and 34 spirals respectively. In others, the spirals are arranged in groups of 89 and 55 spirals respectively. Point out that 34, 55 and 89 are consecutive Fibonacci numbers. Point out all these consecutive pairs of numbers in the list of the first 16 numbers in the Fibonacci sequence. This list appears on page 119 of the Learner's Book.

Point out to your learners that the Fibonacci sequence is remarkable because:

- it can be simply derived, starting with the numbers 0 and 1 (as explained in the Learner's Book) and
- objects in nature frequently show sets of things that occur in pairs of numbers that happen to be consecutive pairs in this sequence.

The discussion about the Fibonacci sequence leads naturally into a definition of a recursive pattern, which learners will find in the *Did you know?* feature.

Work through the Worked examples in the Learner's Book with your class. These examples have been carefully chosen to illustrate different kinds of patterns that your learners should be able to investigate and extend. Make sure that all your learners understand each example before asking them to do Exercise 2.

Walk through your class as they do this exercise and keep an eye open for learners who struggle to interpret Question 4 correctly. Make sure that each learner creates two number patterns and two geometric patterns. When it comes to number patterns, it is easiest for them to work with a constant difference – for example, adding or subtracting the same number each time. If they want to create number patterns with a constant ratio, it is easiest if they multiply by the same number each time. If they try to divide by the same number each time, they may well end up with uncomfortable fractions quickly. One way around this problem is to multiply first, then write down the terms of the number pattern in reverse, thus creating a pattern that shows repeated division.

Suggested answers

1.1

1^2	=	1	=	1
2^2	=	4	=	1 + 3
3^2	=	9	=	1 + 3 + 5
4^2	=	16	=	1 + 3 + 5 + 7
5^2	=	25	=	1 + 3 + 5 + 7 + 9
6^2	=	36	=	1 + 3 + 5 + 7 + 9 + 11
7^2	=	49	=	1 + 3 + 5 + 7 + 9 + 11 + 13
8^2	=	64	=	1 + 3 + 5 + 7 + 9 + 11 + 13 + 15
9^2	=	81	=	1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17
10^2	=	100	=	1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19

1.2 Each successive term in the pattern is a square number (1; 4; 9; 16 etc) and each square number can be formed by adding consecutive odd numbers.

2.1 64; 49; 36; 25; 16; 9; 4

2.2 $49; 7; 1; \frac{1}{7}; \frac{1}{49}; \frac{1}{343}; \frac{1}{2\,401}$

2.3 $-2; -4; -8; -16; -32; -64; -128$

2.4 $\frac{1}{2}; \frac{2}{3}; \frac{3}{4}; \frac{4}{5}; \frac{5}{6}; \frac{6}{7}; \frac{7}{8}$

2.5 10; 12; 17; 25; 36; 50; 67

2.6 $2^1; 2^3; 2^9; 2^{27}; 2^{81}; 2^{243}; 2^{729}$

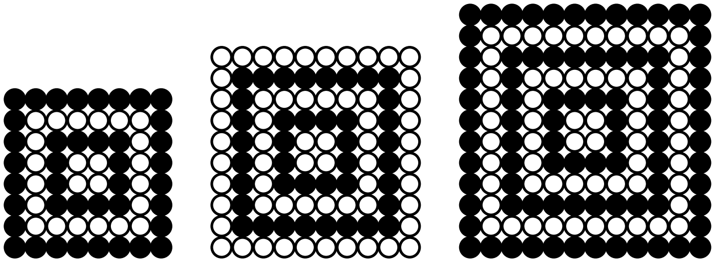
2.7 $x; 2x; 3x; 4x; 5x; 6x$

2.8 $x; x^2; x^3; x^4; x^5; x^6$

2.9 $xy; x^2y^3; x^3y^5; x^4y^7; x^5y^9; x^6y^{11}$

2.10 $13p - q; 12p - 2q; 11p - 3q; 10p - 4q; 9p - 5q; 8p - 6q$

3.1



3.2

Stage	1	2	3	4	5	6
Number of chocolate buttons added	4	12	20	28	36	44
Total number of chocolate buttons	4	16	36	64	100	144

3.3 These are consecutive natural numbers.
3.4 Eight buttons are added each time, starting from four.
3.5 The numbers are all square numbers, starting from four.
4 Learners supply own patterns

Remedial

Algebraic patterns can also be confusing. However, at this level, the algebraic manipulations will always be simple. Learners should be able to cope with patterns like: a; $2a$; $3a$; $4a$; … and a; a^2; a^3; a^4; …

If they have to work with algebraic patterns that consist of more than one term, it is again useful to view these terms separately. For example, the algebraic pattern:

$$a + 9b;\ 2a + 8b;\ 3a + 7b;\ \ldots$$

can be viewed as a combination of two simpler patterns: a; $2a$; $3a$; … and $9b$; $8b$; $7b$; …

Extension

Challenge your learners to look back at Question 4 and to analyse the patterns that they created. Ask them to create more patterns so that they have included at least one of each of the following:

- a pattern that involves negative numbers only
- a pattern that involves a mixture of positive and negative numbers
- a pattern that involves fractions
- a pattern that involves decimal numbers
- a recursive pattern
- an algebraic pattern.

Ask your learners to write down the next three terms in the following pattern:

$1;\ \frac{1}{5};\ 0{,}03;\ \frac{1}{250};\ \frac{1}{2\,000};\ \ldots$

(The pattern can be written as 1; 0,2; 0,03; 0,004; 0,0005; … .
So, the answer is 0,00006; 0,000007; 0,0000008)

Unit 2 The general rule

Learner's Book page 123

Unit focus

This unit focusses on the following:
- describing and justifying the general rules for patterns, and
- describing patterns in your own words or in algebraic language.

Background information on the general rule

In this unit, your learners will describe and justify the general rules for patterns in their own words and in algebraic language. This is revision of what they have done before, but we do introduce the notation T_n for the n^{th} term (or the general term). This notation provides your learners with a useful short-hand way of referring to the general term.

Exercise 1

Learner's Book page 124

Guidelines on how to implement this activity

This unit starts off by explaining why we need to give more than three terms in order to fully describe a particular number pattern. We show your learners how there are different ways of continuing a number pattern if only the first three terms are given. For example:

If the first three terms of a number pattern are 1, 2 and 4, the pattern could be:
- 1; 2; 4; 8; 16; 32; … (multiply each term by 2 to get the next term).

Alternatively, the pattern could be:
- 1; 2; 4; 7; 11; 16; … (add 1, then add 2 then add 3, and so on).

So, we need to give at least four terms in order to uniquely describe a number pattern.

A much better way of describing a number pattern is to give the general rule for the pattern. To this end, we introduce the notation T_n for the n^{th} term (or the general term). Your learners must see T_n as referring to a single concept. It does not stand for T multiplied by n. So it is important that they write this notation correctly, with a capital T and a subscript n.
Using this powerful notation, T_1 is the first term, T_2 is the second term, and so on.
Work through the Worked examples on page of 124 of the Learner's Book with your class, which show how to write the general rule for a number pattern in words.
The next set of Worked examples shows how to write the general rule for a number pattern in algebraic language, using the notation T_n. This is done by writing the pattern in terms of the number of each term, and then generalising the result.
Show learners how to use the general rule to find any term in the pattern by means of substituting in the term number. Work through the examples in the Learner's Book and ensure learners are comfortable using the formula.

Suggested answers

1.1 $T_n = 5n + 3$
$T_1 = 5(1) + 3 = 8$
$T_2 = 5(2) + 3 = 13$
$T_3 = 5(3) + 3 = 18$
$T_{15} = 5(15) + 3 = 78$

1.2 $T_n = 1{,}5n - 0{,}5$
$T_1 = 1{,}5(1) - 0{,}5 = 1$
$T_2 = 1{,}5(2) - 0{,}5 = 2{,}5$
$T_3 = 1{,}5(3) - 0{,}5 = 4$
$T_{15} = 1{,}5(15) - 0{,}5 = 2$

1.3 $T_n = n^2 - 1$
$T_1 = (1)^2 - 1 = 0$
$T_2 = (2)^2 - 1 = 3$
$T_3 = (3)^2 - 1 = 8$
$T_{15} = (15)^2 - 1 = 224$

1.4 $T_n = 3n^2 - n$
$T_1 = 3(1)^2 - 1 = 2$
$T_2 = 3(2)^2 - 2 = 10$
$T_3 = 3(3)^2 - 3 = 24$
$T_{15} = 3(15)^2 - 15 = 660$

1.5 $T_n = \frac{1}{2}n$
$T_1 = \frac{1}{2(1)} = \frac{1}{2}$
$T_2 = \frac{1}{2(2)} = \frac{1}{4}$
$T_3 = \frac{1}{2(3)} = \frac{1}{6}$
$T_{15} = \frac{1}{2(15)} = \frac{1}{30}$

1.6 $T_n = \frac{1}{2^n}$
$T_1 = \frac{1}{2^1} = \frac{1}{2}$
$T_2 = \frac{1}{2^2} = \frac{1}{4}$
$T_3 = \frac{1}{2^3} = \frac{1}{8}$
$T_{15} = \frac{1}{2^{15}} = \frac{1}{32\ 768}$

1.7 $T_n = 3^{n-1} + 1$
$T_1 = 3^{1-1} + 1 = 2$
$T_2 = 3^{2-1} + 1 = 4$
$T_3 = 3^{3-1} + 1 = 10$
$T_{15} = 3^{15-1} + 1 = 4\ 782\ 970$

1.8 $T_n = (-1)^n.4n$
$T_1 = (-1)^1.4(1) = -4$
$T_2 = (-1)^2.4(2) = 8$
$T_3 = (-1)^3.4(3) = -12$
$T_{15} = (-1)^{15}.4(15) = -60$

2.1 Divide by ten to get the next term – each term is ten times smaller than the previous term.
2.2 Add five to get the next term – each term is five more than the previous term.
2.3 Divide by seven to get the next term.
2.4 Divide by four to get the next term – each term is four times smaller than the previous term.
2.5 Square root to get the next term – each term is the square root of the previous term.
2.6 Multiply by one more each time (by two, then by three, then by four etc).

Remedial

- You may expect a number of your learners to struggle with parts of this exercise. This is essentially a mixed exercise that requires that each question must be tackled on its own merits. This means that your learners will have to read each question and interpret what it says and what it requires them to do.
- If your learners are unsure how to tackle each question, allow them to discuss the questions in pairs or in small groups.

Exercise 2

Learner's Book page 126

Guidelines on how to implement this activity

Learners often struggle to find the general term when there is a constant difference between successive terms in a number pattern. We explain this process in the Learner's Book. Work through this text carefully with your class, making sure that they all understand the idea that the general term will be of the form $T_n = an + c$, where a is the constant difference and c is a constant.

The Worked examples that follow show how one can use the general term of a pattern to find any term in the pattern by substitution.

Make sure that all your learners understand all the work that has been covered thus far in this unit before asking them to do Exercise 2. This exercise will give your learners plenty of practice in the concepts that have been covered thus far. It is a long exercise, so do not rush your class. Ask those learners who have not been able to complete this exercise in class to do so for homework.

In Question 5, your learners should realise that not all numbers behave like this! For example, in the case of the number 17: $1^1 + 7^2 = 50$, which is not equal to 17.

In Question 6, if your learners have forgotten what a recursive pattern is, refer them back to the *Did you know?* in the Learner's Book.

Suggested answers

1.1 3; 9; 27; 81

1.2 2; 6; 18; 54

1.3 81; 27; 9; 3; 1

2.1 $T_n = n^2 + 1$
$T_{10} = 10^2 + 1 = 101$

2.2 $T_n = 50n - 250$
$T_{10} = 50(10) - 250 = 250$

2.3 $T_n = -2n + 101$
$T_{10} = -2(10) + 101 = 81$

2.4 $T_n = \frac{1}{n}$
$T_{10} = \frac{1}{10}$

2.5 $T_n = 3n - 103$
$T_{10} = 3(10) - 103 = -73$

2.6 $T_n = an + b^n$
$T_{10} = 10n + b^{10}$

3.1 $8^1 + 9^2 = 8 + 81 = 89$
$1^1 + 3^2 + 5^3 = 1 + 9 + 125 = 135$
$5^1 + 9^2 + 8^3 = 5 + 81 + 512 = 598$
$2^1 + 4^2 + 2^3 + 7^4 = 2 + 16 + 8 + 2\,401 = 2\,427$
$2^1 + 6^2 + 4^3 + 6^4 + 7^5 + 9^6 + 8^7$
$= 2 + 36 + 64 + 1\,296 + 16\,807 + 531\,441 + 2\,097\,152 = 2\,646\,798$

3.2 Each number is equal to the sum of its digits raised to successive powers – first digit to the power of one, second digit to the power of two, third digit to the power of three etc.

3.3 Each number is equal to the sum of its digits raised to successive powers – first digit to the power of one, second digit to the power of two, third digit to the power of three etc.

3.3.1 Yes, $1^1 + 7^2 + 5^3 = 175$ **3.3.2** No, $2^1 + 5^2 + 3^3 = 54$

3.3.3 Yes, $5^1 + 1^2 + 8^3 = 518$ **3.3.4** Yes, $1^1 + 3^2 + 0^3 + 6^4 = 1\ 306$

4.1 Calculators will display the answer to $1\ 001^4$ in scientific notation.

4.2 The numbers are symmetrical (also called palindromic) – they are the same whether you read them from left to right or right to left.

4.3

Number	Square	Cube	Fourth power
11	$11^2 = 121$	$11^3 = 1\ 331$	$11^4 = 14\ 641$
101	10 201	1 030 301	104 060 401
1 001	1 002 001	1 003 003 001	1 004 006 004 001
10 001	100 020 001	1 000 300 030 001	10 004 000 600 040 001
100 001	10 000 200 001	1 000 030 000 300 001	100 004 000 060 000 400 001
1 000 001	1 000 002 000 001	1 000 003 000 003 000 001	1 000 004 000 006 000 004 000 001

5.1

Hexagon number	1	2	3	4	5
Number of matches	6	12	18	24	30

5.2 $T_n = 6n$

5.3 $T_{20} = 6(20) = 120$

6.1 A recursive pattern is one in which any term in the pattern (usually from the 2nd or 3rd terms onwards) can be described in terms of the previous term (or terms).

6.2 Many possible answers.

Remedial

- When substituting different values of n into the general term in Question 1, watch out for the following common errors:
  - Learners apply the bodmas rules incorrectly.
  - Learners apply the laws of exponents incorrectly.
  - Learners make sign mistakes.
  - Learners struggle to calculate with decimal fractions.
- If your learners are given the first few terms of a number pattern and are asked to work out the rule for the n^{th} term of the pattern, advise them to always check their answer by substituting in 1, 2, 3 and so on in the place of n. This should then give them the 1st, 2nd and 3rd terms of the pattern respectively. This will help them to decide for themselves whether or not their answer is correct. Checking their own answers gives learners a sense of independence and control that will help them on many occasions in the future.
- Both Questions 2 and 4 contain some number patterns that learners may struggle with. If they do not manage to answer a particular question, advise them to carry on with the exercise and to answer only those questions that they are able to solve. Then make time for a class discussion afterwards, in which you can discuss those patterns that your learners found to be problematic.

Extension

- For those of your learners who have completed the whole exercise, ask them to create more recursive patterns for their partners to solve, modelled on Question 6.
- Learners could also try to find some new numbers of their own that obey the pattern in Question 5. They will have to do this by trial-and-error.

Challenge

Learner's Book page 127

Guidelines on how to implement this activity

The Challenge exercise at the end of this unit has been included for those learners who have coped well with this work covered so far in this unit.

Suggested answers

1 $T_n = 2^n$ **2** $T_n = 3 \times 2^n$ **3** $T_n = \frac{32}{2^n}$

Unit 3 Input and output values

Learner's Book pages 128

Unit focus

This unit focusses on the following:

- determining input values and output values,
- determining rules for patterns and relationships, and
- using flow diagrams, tables, formulae and equations.

Background information on input and output values

In this unit, your learners will find input values, output values or rules for patterns using flow diagrams, tables, formulae and equations. This work should be familiar to them from earlier grades.

Exercise 1

Learner's Book page 129

Guidelines on how to implement this activity

Work through the Worked example on page of 129 of the Learner's Book with your class. Make sure that all your learners understand how to find the rule for the flow diagram, as well as how to find the output number for a given input number, and vice versa.

Now ask your class to do Exercise 4. This is another long exercise. Ask those learners who have not been able to complete this exercise in class to do so for homework.

Suggested answers

1.1.1 $T_6 = 3(6) - 13 = 5$ **1.1.2** $T_{13} = 3(13) - 13 = 26$

1.1.3 $T_{29} = 3(29) - 13 = 74$

1.2.1 $3n - 13 = -1$
$3n = 12$
$n = 4$

1.2.2 $3n - 13 = 77$
$3n = 90$
$n = 30$

1.2.3 $3n - 13 = 131$
$3n = 144$
$n = 48$

2.1 $T_n = \frac{n}{5} + 2$

2.2.1 $T_5 = \frac{5}{5} + 2 = 3$

2.2.2 $T_{13} = \frac{13}{5} + 2 = 4{,}6$

2.2.3 $T_{31} = \frac{31}{5} + 2 = 8{,}2$

2.3.1 $\frac{n}{5} + 2 = 4$
$\frac{n}{5} = 2$
$n = 10$

2.3.2 $\frac{n}{5} + 2 = 6{,}6$
$\frac{n}{5} = 4{,}6$
$n = 23$

2.3.3 $\frac{n}{5} + 2 = 11{,}4$
$\frac{n}{5} = 9{,}4$
$n = 47$

3.1

Shape	1	2	3	4	5	6	7	8
Number of rectangles	1	2	3	4	5	6	7	8
Perimeter of the shape (cm)	18	24	30	36	42	48	54	60

3.2.2 $6r + 12$

3.3.1 P = 6(10) + 12 = 72

3.3.2 P = 6(55) + 12 = 342

3.4 number of rectangles → $\boxed{\times 6}$ → $\boxed{+ 12}$ → perimeter

3.5 perimeter → $\boxed{- 12}$ → $\boxed{\div 6}$ →no of rectangles

4.1

Shape	1	2	3	4	5	6	7	8
Number of rectangles	1	3	5	7	9	11	13	15
Perimeter of the shape (cm)	18	42	66	90	114	138	162	186

4.2.1 $24n - 6$

4.3.1 P = 24(10) − 6 = 234

4.3.2 P = 24(150) − 6 = 3 594

4.4 shape number → $\boxed{\times 2}$ → $\boxed{- 1}$ → no of rectangles

4.5 perimeter → $\boxed{+ 6}$ → $\boxed{\div 24}$ → shape number → ×2 → − 1 → no of rectangles

4.6.1 $24n - 6 = 402$
$n = 17$
$r = 2(17) - 1$
$= 33$

4.6.2 $24n - 6 = 1914$
$n = 80$
$r = 2(80) - 1$
$= 159$

5.1 20 × 25 = R500

5.2

Day	1	2	3	4	5	6	7	8
Lungi's wage	1c	2c	4c	8c	16c	32c	64c	128c

5.3 $w = 2^{n-1}$ or $w = \frac{2^n}{2}$

5.4

Day	5	10	15	20	25
Lungi's wage	16c	512c	16 384c	524 288c	16 777 216c

5.5 R167 772,16

5.6 R167 772,16 – R500 = R167 272,16

Remedial

The ability to read any given question carefully and to interpret it correctly is a skill that your learners will require throughout the rest of their school Mathematics. This skill applies as much to Mathematical Literacy as it does to Mathematics in later grades. (In Mathematical Literacy, much of the learning is context-based and therefore relies on reading and interpretative skills.) In Question 1 of Exercise 4, for example, there is an important but subtle difference between the wording in Questions 1.1 and 1.2. In Question 1.1 they are asked to find the value of the term if the term number is given. In Question 1.2, they are asked to do the opposite; in other words, to find the term number if the value of the term is given.

- One way to address this problem is to allow your learners to discuss any questions that they do not understand in pairs or in small groups.
- Another way is to set a few minutes aside for your learners to read through the questions in an exercise, before starting to answer the questions. Then you can have a brief class discussion about any problem areas before your class starts to work on the exercise.

In Questions 3.2 and 4.2, your learners should be able to identify the correct expression by inspection. If they struggle to do this, they can also test each expression by substituting in values of 1, 2, 3 and so on for the variable in each expression to see which expression matches the given situation.

Unit 4 Equivalent forms

Learner's Book page 132

Unit focus

This unit focusses on the following:

- determining, interpreting and justifying equivalence of different forms of the same relationship or rule.

Background information on equivalent forms

In the final unit of this chapter, learners will work with equivalent descriptions of the same rule presented in words, in flow diagrams, in tables, by formulae and by equations. The focus here is on equivalent descriptions of the same rule presented in different forms.

Exercise 1
Learner's Book page 134

Guidelines on how to implement this activity

This unit starts off by explaining that there are often equivalent forms of describing a number pattern. The following example of a pattern is given: 1; 4; 9; 16; … . Then we show that there are two different ways of describing this pattern in words. We extend this idea by explaining that there are also different devices that we can use to describe the same pattern. These include verbal descriptions, flow diagrams, tables, formulae and equations.

Make sure that all your learners understand that equivalent descriptions of the same rule are descriptions that look different, but that have the same meaning.

Work through the Worked example in the Learner's Book with your class. This example gives your class a foretaste of what is to come in Chapter 11, where they will be working with graphs of straight lines in the Cartesian plane. (In order to graph linear relationships, they will also be working with tables of *x*- and *y*-values.)

In this Worked example, a table of *x*- and *y*-values is given. The example shows your learners how to find the rule for the relationship between the *x*- and the *y*-values. Missing input and output values are calculated and an *x*-value is found for which the corresponding *y*-value is an even number.

Read through the text just above Exercise 1 with your class. It is important that your class make the following distinction:

- when we talk about input values and output values in general, then the input values can have any values that we like. They can be negative as well as positive, and they can be fractions as well as integers.
- However, when we talk about number patterns, then the term number (usually *n*) is always a positive integer. This makes intuitive sense, because we talk about the first second and third terms of a number pattern, and so on. So the term numbers can only be 1, 2, 3 and so on.

Once you feel confident that your learners understand what has been discussed thus far in this unit, ask them to do the exercise.

Suggested answers

1.1

X	1	2	3	5	8	16	50	100
Y	$\frac{3}{2}$	1	$\frac{1}{2}$	$-\frac{1}{2}$	–2	–6	–23	–48

1.2.1 $y = -\left(\frac{1}{2}x - 2\right)$ **1.2.4** $y = -\frac{1}{2}(x - 4)$

1.3.2 $x \rightarrow \boxed{\times -\frac{1}{2}} \rightarrow \boxed{+\ 2} \rightarrow y$

1.4.1 (0; 2)

1.4.4 (−1; 2,5) **1.4.6** (−10; 7)

2.1

Shape number (s)	1	2	3	4	5	6
Number of plastic triangles (t)	1	4	9	16	25	36
Perimeter of shape (p) (cm)	7,5	15	22,5	30	37,5	45

2.2 They are all square numbers.

2.3.3 $t = s^2$

2.3.5 $s = \sqrt{t}$

2.4.2 $s = \frac{p}{7,5}$

2.4.3 $p = 7,5s$

2.5.1 $s = \sqrt{100} = 10$
$p = 7,5(10) = 75$ cm

2.5.2 $s = \sqrt{144} = 12$
$p = 7,5(12) = 90$ cm

2.5.3 $s = \sqrt{256} = 16$
$p = 7,5(16) = 120$ cm

2.6.1 $s = \frac{75}{7,5} = 10$
$t = 10^2 = 100$

2.6.2 $s = \frac{750}{7,5} = 100$
$t = 100^2 = 10\ 000$

2.6.3 75 m = 7 500 cm
$s = \frac{7\ 500}{7,5} = 1\ 000$
$t = 1\ 000^2 = 1\ 000\ 000$

3.1

Number of hexagons (h)	1	2	3	4	5
Number of triangles (t)	6	12	18	24	30
Perimeter of shape (p) (cm)	15	25	35	45	55

3.2 $t = 6h$

3.3 $p = 10h + 5$

3.4.1 $p = 10(250) + 5 = 2\ 505$ cm
$p = 10(100) + 5 = 1\ 005$ cm

3.4.2 $600 = 6h$
$h = 100$

3.4.3 $155 = 10h + 5$
$10h = 150$
$h = 15$

3.4.4 $495 = 10h + 5$
$10h = 490$
$h = 49$
$t = 6(49) = 294$

Remedial

- In Question 1.2, to evaluate equations 1.2.1–1.2.4, your learners should simplify the RHS by eliminating the brackets.To evaluate equations 1.2.5 and 1.2.6, they will need to make y the subject of the formula in each equation. Only then can they compare the equations to the given rule to decide whether or not they are equivalent.
- In Question 1.3, your learners should be able to compare the flow diagrams to the given rule by inspection. If they struggle to do this, they should test each flow diagram by substituting in different values of x to see whether or not the flow diagram gives the same corresponding output values.
- Your learners should be aware that it is possible to have more than one correct answer in the following questions: 1.2, 1.3, 1.4, 2.3 and 2.4. They should therefore test every given option and not stop testing once they have found a match.
- In Question 2.6.3, your learners should notice that a different unit is used (metres instead of centimetres). At this level they should be very comfortable with converting metres to centimetres however, keep a look out for learners who do not do this conversion correctly or who fail to realise that a conversion is necessary.
- Question 3.4 is another example of a question that requires careful reading and interpretation on the part of your learners. For each sub-question in Question 3.4, they should be very clear on what is being asked. Be prepared to remediate if they do not interpret each sub-question correctly.

Extension

The Challenge exercise is designed as an extension opportunity for those learners who have coped well with this work.
Learners who have completed the Challenge could answer the following additional questions:

- Find four more ordered pairs of your own choosing that will satisfy the rule in Question 1.
- Use your equation in Question 3.3 to work out:
  - the perimeter of a shape that contains 300 hexagons
  - the perimeter of a shape that contains 300 triangles
  - the number of hexagons in a shape with a perimeter of 305 cm
  - the number of triangles in a shape with a perimeter of 635 cm.

Challenge

Learner's Book page 136

Guidelines on how to implement this activity

This Challenge exercise is designed to be attempted by those learners who have coped well with this unit so far.

Suggested answer

$p = 10\left(\frac{t}{6}\right) + 5$

Consolidation

Learner's Book page 138

Before doing this consolidation exercise, encourage learners to review the work covered in this chapter. Advise learners to use the summary and to revise their work. This exercise can be used as an informal assessment task for you to track how learners are coping with the chapter and the concepts covered. The mark allocation provides guidelines on how to assess learners.

Suggested answers

1.1	$\frac{1}{2}$; 1; 2; 4; 8; 16; 32	(1)
1.2	−5; 5; −6; 6; −7; 7; −8; 8	(1)
1.3	–2,5; –2; –1,5; –1; –0,5; 0; 0,5	(1)
1.4	0; 1; 8; 27; 64; 125; 216	(1)
1.5	6; 6,9; 6,96; 6,969; 6,9696; 6,96969; 6,969696	(1)
1.6	61; 60; 58; 55; 51; 46; 40	(2)
1.7	x; $5x^2$; $125x^3$; $625x^4$; $3\ 125x^5$; $15\ 626x^6$	(2)
1.8	$a + 10b$; $a^2 + 7b$; $a^3 + 4b$; $a^4 + b$; $a^5 - 2b$; $a^6 - 5b$	(2)
2.1	Subtract four to get each new term, or each term is four less than the previous term.	(1)
2.2	Divide by five to get each new term, or each term is five time smaller than the previous term.	(1)
2.3	Double the numerator and subtract two from the denominator of a term to get the next term.	(2)

3.1 $T_1 = 1 + 6 = 7$
$T_2 = 2 + 6 = 8$
$T_3 = 3 + 6 = 9$
$T_{15} = 15 + 6 = 21$ (2)

3.2 $T_1 = 2{,}5 - 1{,}5(1) = 1$
$T_2 = 2{,}5 - 1{,}5(2) = -0{,}5$
$T_3 = 2{,}5 - 1{,}5(3) = -2$ (2)

3.3 $T_1 = \frac{5(1)}{2} = 2{,}5$
$T_2 = \frac{5(2)}{2} = 5$
$T_3 = \frac{5(3)}{2} = 7{,}5$
$T_{15} = \frac{5(15)}{2} = 37{,}5$ (2)

3.4 $T_1 = (1)^2 + 2(1) = 3$
$T_2 = (2)^2 + 2(2) = 8$
$T_3 = (3)^2 + 2(3) = 15$
$T_{15} = (15)^2 + 2(15) = 255$ (2)

3.5 $T_1 = 4.3(1) = 12$
$T_2 = 4.3(2) = 24$
$T_3 = 4.3(3) = 36$
$T_{15} = 4.3(5) = 60$ (2)

3.6 $T_1 = 4.3^1 = 12$
$T_2 = 4.3^2 = 36$
$T_3 = 4.3^3 = 108$
$T_{15} = 4.3^{15} = 57\ 395\ 628$ (3)

4 Many possible patterns. (7)

5.2 The number is equal to the sum of each digit raised to the power of itself. (3)

6.1

x	1	2	3	4	9	16	25	80
y	1	$\frac{5}{2}$	4	$\frac{11}{2}$	13	$\frac{47}{2}$	37	$\frac{239}{2}$

(4)

6.2.2 $y = \frac{1}{2}(3x - 1)$

6.2.4 $2y = 3x - 1$

6.2.5 $3x = 2y + 1$ (3)

6.3.2 $x \longrightarrow \boxed{\times \frac{3}{2}} \longrightarrow \boxed{-\frac{1}{2}} \longrightarrow y$

6.3.4 $x \longrightarrow \boxed{\times 3} \longrightarrow \boxed{-1} \longrightarrow \boxed{\div 2} \longrightarrow y$ (2)

6.4.2 $\left(0; -\frac{1}{2}\right)$

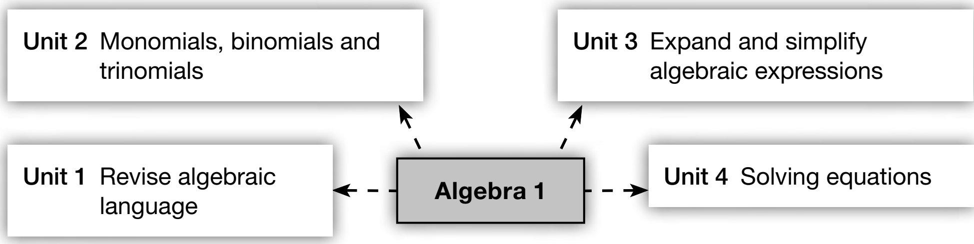
6.4.3 $(-1; -2)$

6.4.5 $(-3; -5)$ (3)

[50]

Chapter 5 Algebra 1

Overview of concepts



Content		Time allocations	LB page
Unit 1	Revise algebraic language	1 hour	141
Unit 2	Monomials, binomials and trinomials	1 hour	144
Unit 3	Expand and simplify algebraic expressions	$2\frac{1}{2}$ hours	146
Unit 4	Solving equations	4 hours	156

Background information on algebra

From earlier grades, learners should already be familiar with basic algebraic concepts. They should already know that:

- an algebraic expression is any meaningful collection of numbers, letters or mathematical symbols,
- the letters in an algebraic expression are called variables,
- the coefficient of a term is the number (together with its sign) that appears in front of the term,
- when no constant is shown with a variable term, the coefficient of the term is 1,
- a term without a sign in front of it is always positive,
- the exponent of a number or a variable is the small number that is written to the right and above the number or variable,
- a number or a variable with no exponent shown has an exponent of 1,
- when you multiply (or divide) two terms with the same sign, the result is positive, and
- when you multiply (or divide) two terms with different signs, the result is negative.

Learners often tend to think that algebra is a whole different branch of mathematics with different rules from "normal arithmetic". It is important that they understand that algebra is simply an extension of normal arithmetic. The introduction of variables allows us to generalise our answers and to make our findings applicable to more than one specific value of the variable(s) concerned.

Generic teaching guidelines for teaching algebra

- The importance of a clear understanding of the basic principles of algebra cannot be over-emphasised. A lack of understanding of these principles can negatively impact learners in future grades. An analysis of mistakes made throughout Grades 9–12 has shown that sign mistakes are extremely common, as is the incorrect application of the basic rules of algebra. Be prepared to repeat these ideas over and over again, and to remediate any misconceptions as and when they occur.
- Learners should always write as clearly as possible. This is of course important throughout their schoolwork, but never more so than in algebra. They should be careful to distinguish clearly between the letter "z" and the number "2", as well as between the letter "x" and the multiplication symbol "$\times$".
- Advise learners to always work through any given rule. For example, if a learner feels that they can simplify $a^2 + a^3 = a^5$, show them how to test this theory for at least one numerical value. For example, if they choose $2^2 + 3^2 = 4 + 8 = 12$, which is not the same as $2^5 = 32$.
  However, there is a pitfall to this idea: If they choose a = 2 to test whether or not $a + a = a^2$, they will find that:
  $a + a = 2 + 2 = 4$, $2a = 2(2) = 4$, $a \times a = 2 \times 2 = 4$ and $a^2 = 2^2 = 4$.
  This result ***might*** seem to suggest that $a + a = a^2$, or that $a \times a = 2a$.
  This is why you should encourage learners to always test the given rule with at least two different numbers. They need to understand that one example does not prove a theory, but one counter-example does disprove it!
- It is vitally important that your learners understand the value of checking their solutions. The value of this is two-fold:
  - If a learner checks a solution and finds it to be correct, this reinforces their work and gives them confidence in their abilities.
  - If a learner checks a solution and finds it to be incorrect, this serves as a pointer to the fact that they need to look for their mistake(s). This allows them to become more independent as they work and to discover their own mistakes, instead of waiting for their work to be marked. It also empowers them in test and examination situations.
- It is equally important that learners check that they have copied a problem down correctly before even starting to solve it. A common mistake for learners to make is to copy down a problem incorrectly. Also advise your learners to use a pencil when cancelling factors – this makes their work much easier for them to check afterwards.

Resources

A balance scale to show the importance of balancing equations, vocabulary cards, cardboard, colour pens, ruler, and each learner should have their own calculator.

Unit 1 Revise algebraic language

Learner's Book page 141

Unit focus

This unit focusses on the following:
- revise what learners already know about algebraic language,
- recognise and identify conventions for writing algebraic expressions,
- identify and classify like and unlike terms in algebraic expressions, and
- recognise and identify coefficients and exponents in algebraic expressions.

Background information on algebraic language

In this unit, your learners will revise what they already know about algebraic language. This is a pure revision of Grade 8 work.

Revision exercise

Learner's Book page 141

Guidelines on how to implement this activity

This exercise revises the skills and concepts learners should have learnt in Grade 8. Use this exercise as a diagnostic tool to help you to establish their prior knowledge and level of competence with this work.You can use this exercise to informally assess learners.

Suggested answers

1	A, C and D						
2	A, B, C						
3	A, B, D						
4.1	10			**4.2**	6		
5.1	$k = 3$	**5.2**	$k = 7$	**5.3**	$k = 5$	**5.4**	$k = 7$
5.5	$k = 7$	**5.6**	$k = 8$	**5.7**	$k = 4$	**5.8**	$k = 3$
5.9	$k = 8$	**5.10**	$k = 3$				

Remedial

If learners experience difficulty with this exercise, revise Grade 8 work before proceeding with the rest of this unit.

Exercise 1

Learner's Book page 143

Guidelines on how to implement this activity

Revise all the concepts that are explained in the Learner's Book. Discuss the sign rules that form the backbone of algebraic operations. Work through some examples of your own on the board, before asking your class to do Exercise 1. Walk through your class as they do this exercise and keep an eye open for problems that need general remediation.

Suggested answers

1.1	5 terms	**1.2**	1	**1.3**	5

1.4	3	**1.5**	1	**1.6**	5		
2.1	$6x$	**2.2**	$6p^2q^3$	**2.3**	$18a^3$	**2.4**	$\frac{3m}{2n}$

1.7 No, because all the terms are unlike (have different powers).
1.8 They have been written in descending order of powers or exponents.

3.1	$x + x = 2x$	**3.2**	$x \times x = x^2$
3.3	$x - x = 0$	**3.4**	$x \div x = 1$
3.5	$x \times 1 = x$	**3.6**	$x \times 0 = 0$
3.7	$1 \div x = \frac{1}{x}$	**3.8**	$0 \div x = 0$
3.9	$x \div 1 = x$	**3.10**	$x \div 0$ is undefined.
4.1	$y = x + 5$	**4.2**	$y = 5x$
4.3	$y = x - 5$	**4.4**	$y = \frac{1}{5}x$ or $y = \frac{x}{5}$
4.5	$y = \sqrt{x}$	**4.6**	$y = x^2$

Remedial

- If any of your learners are struggling with the concepts in this unit, they will not be able to master the rest of the work in this chapter. Set them additional exercises from the Grade 8 textbook for extra practice.
- Write the sign rules on the board and keep them there throughout this chapter. Your learners must learn these by rote:
  $+ \times + = +$ and $+ \div + = +$
  $- \times - = +$ and $- \div - = +$
  $+ \times - = -$ and $+ \div - = -$
  $- \times + = -$ and $- \div + = -$
- Another rule that your learners must know by rote is that we can only collect like terms. To "prove" this to your class, write a simple expression on the board, for example $2x + y + 3x + 4y$. By substituting in different values for x and y, they will see that the correct answer is consistently obtained only when like terms are collected.
- Another idea that learners struggle with is to understand that a term without an explicit coefficient has a coefficient of 1. Learners may find it helpful to write in the "invisible" 1 when collecting like terms.
- When collecting like terms, learners must realise that the sign in front of each term is a part of the term. A term without an sign in front of it is positive. Learners may find it helpful to write in the "invisible" +.

Extension

Learners who have coped well with this unit could spend some time thinking about how they would explain these abstract concepts to younger learners.
You can also give stronger learners some problems from Grade 10 to practise.

Unit 2 Monomials, binomials and trinomials

Learner's Book page 144

Unit focus

This unit focusses on the following:
- recognising and differentiate between monomials, binomials and trinomials.

Background information on monomials, binomials and trinomials

In this unit, learners will recognise and differentiate between monomials, binomials and trinomials. It is important that they understand the definition of a polynomial, which defines the sub-divisions of monomials, binomials and trinomials.

Exercise 1

Learner's Book page 145

Guidelines on how to implement this activity

Work through the definitions in the Learner's Book with your class. Make sure that they all understand that a polynomial is a specific kind of algebraic expression and that not all algebraic expressions are polynomials. The important distinction is that a polynomial has variables with exponents that are positive whole numbers. Once you have established this definition, it is a simple leap to the idea of a monomial (a polynomial with one term), a binominal (a polynomial with two terms), and a trinomial (a polynomial with three terms).
Work through the Worked examples in the Learner's Book with your class, making sure that they all understand which expressions are polynomials, which are not, and the reasons for each. Give them some more examples of your own to consolidate these ideas, if you feel this is required. Discuss ordering the terms in polynomials, as well as the degree of polynomials. Learners should complete Exercise 1 on their own.

Suggested answers

1.1 It is not a polynomial as the third term has a variable in the denominator.
1.2 It is a polynomial with six terms.
1.3 It is not a polynomial as the second term has an exponent that is not a whole number.
1.4 It is a polynomial with four terms.
1.5 It is a polynomial and is a binomial as it has two terms.
1.6 It is not a polynomial as the exponent of the second term is negative.
2.1 $6 + 2x + x^2$ (2^{nd} degree)
2.2 $2x^2 + 7x^3 + 5x^4 + x^5$ (5^{th} degree)
2.3 $7 + 4x + 3x^2 + 5x^4 + x^6$ (6^{th} degree)
2.4 $-4 - 2x + 6x^2 + 15x^4 + 14x^7$($7^{\text{th}}$ degree)
2.5 $1 + 8xy - 5x^2y^3 + 2x^3y + 3x^4y^2 - x^6y^5$ (11^{th} degree polynomial)

3.1 $x^2 + 2x + 6$
3.2 $x^5 + 5x^4 + 7x^3 + 2x^2$
3.3 $x^6 + 5x^4 + 3x^2 + 4x + 7$
3.4 $14x^7 + 15x^4 + 6x^2 - 2x - 4$
3.5 $-x^6y^5 + 3x^4y^2 + 2x^3y - 5x^2y^3 + 8xy + 1$

Remedial

If learners struggle with Question 1, refer them back to the Worked examples about how to recognise polynomials, monomials, binomials and trinomials.
If learners struggle with Questions 2 and 3, refer them back to the definitions where these ideas are explained.
Give your learners plenty of practice by making up some questions of your own for them to do.

Extension

The Challenge is designed as an extension opportunity for those learners who have coped well with this work. Learners who need additional challenges can make up their own questions along the lines of the challenge for one another to solve.

Challenge

Learner's Book page 145

Guidelines on how to implement this activity

The Challenge question has been included for those learners who have coped well with this work.

Suggested answers
This is a model answer. There are many other possible expressions, but all should be 5^{th} degree.
$x^3y^2 + x^2y + xy^2 + xy + 4$ (5 degree)

Unit 3 Expand and simplify algebraic expressions

Learner's Book page 146

Unit focus

This unit focusses on the following:
- addition and subtraction like terms in algebraic expressions,
- multiplication and division of monomials, binomials and trinomials by integers or monomials,
- simplification of algebraic expressions,
- determining the squares, cubes, square roots and cube roots of single algebraic terms or like algebraic terms,
- determining the numerical value of algebraic expressions by substitution,
- multiplication and division of polynomials by integers or monomials,
- finding the product of two binomials, and
- finding the square of a binomial.

Background information on expanding and simplifying algebraic expressions

The focus of this unit is on expanding and simplifying algebraic expressions. We begin simplifying expressions by collecting like terms, then move on to multiplying and dividing expressions by monomials. Your learners will also find the squares and cubes of expressions, as well as the square roots and cube roots of expressions. Finally, we focus on the rules for multiplying two binomials, which includes finding the squares of binomials.

In Grade 8, learners covered most of this work, including multiplying and dividing binomials and trinomials by monomials. In Grade 9, these ideas are expanded to include multiplying and dividing polynomials by monomials. The skill of multiplying two binomials is new in Grade 9.

Exercise 1

Learner's Book page 148

Guidelines on how to implement this activity

Discuss adding and subtracting like terms, and ensure learners know what like terms are and how to add and subtract them. Discuss multiplying and dividing terms. Ensure learners know how to perform these operations on algebraic terms.

Work through the Worked examples with your class. The first two examples deal with simplifying expressions by collecting like terms. The next two deal with multiplying trinomials by monomials and the last two deal with dividing trinomials by monomials. Point out how every term in the trinomial is multiplied (or divided) by the monomial.

Once you feel satisfied that your learners understand these examples, ask them to do Exercise 1. The questions in this exercise are structured in such a way that your learners are clearly led to see what they should be doing. Once your class has completed this exercise, go through some of the solutions with them, to make sure that they have coped with this work before moving on to the next section.

Suggested answers

1.1 $2x - 4 + 6x + 10 = 8x + 6$

1.2 $7b - \mathrm{a} + 3a - 6b = \mathrm{b} + 21$

1.3 $-2xy + 5x - 2y - 3y + 9xy = 7xy + 5x - 5y$

1.4 $2a^2 + \mathrm{a} - 3a^2 + 4a = -a^2 + 5a$

1.5 $3ac - 5ab - 6ac - 3ab - 10 = -3ac - 8ab - 10$

1.6 $6xy^2 - 7x^2y + 9x^2y - 3xy^2 = 3xy^2 + 2x^2y$

1.7 $t^3 - 4t - 1 + 2t^2 - 3t + 3 - 3t^2 + 9t - 2 = t^3 - t^2 + 2t$

1.8 $x^3 - 3x^2 - 3x - 5 - 2x^3 - 3x^2 + x + x^3 - 2x^2 + 3x + 3 = -8x^2 + x - 2$

2.1 $x^2 \times x^3 = x^5$ **2.2** $p \times p^4 = p^5$

2.3 $x^2 \times x^4 = x^6$ **2.4** $y^4 \times y^5 = y^9$

3.1 $2(6p^2 + 5) = 12p^2 + 10$

3.2 $p(2p^2 - p + 1) = 2p^3 - p^2 + p$

3.3 $6a(3a^2 - 2a + 7) = 18a^3 - 12a^2 + 42a$

3.4 $2y(x + 5y - 7) = 2xy + 10y^2 - 14y$

3.5 $-(x^2 - 2x - 1) = -x^2 + 2x + 1$

3.6 $2b(5b^2 - 7b - 3) = 10b^3 - 14b^2 - 6b$

3.7 $-2a(4a^2 + a - 3) = -8a^3 - 2a^2 + 6a$

3.8 $-4x^2y(xy - y + x) = -4x^3y^2 + 4x^2y^2 - 4x^3y$

4.1 $\frac{x^8}{x^2} = x^6$ **4.2** $\frac{p^6}{p^3} = p^3$ **4.3** $\frac{b^5}{b} = b^4$ **4.4** $\frac{y^{11}}{y^4} = y^7$

5.1 $\frac{2a + 4b}{2} = a + 2b$ **5.2** $\frac{9a - 27b + 30c}{3} = 3a - 9b + 10c$

5.3 $\frac{16r + 24s}{8} = 2r + 3s$ **5.4** $\frac{36ab + 12ab^2}{12ab} = 3 + b$

5.5 $\frac{12x^3 + 28x^2}{4x} = 3x^2 + 7x$ **5.6** $\frac{25a^3 - 15a^2}{5a} = 5a^2 - 3a$

5.7 $\frac{4a^6 + 6a^3 - 10a}{-2a} = -2a^5 - 3a^2 + 5$ **5.8** $\frac{21x^7y^3 + 14x^5y^2}{-7xy^2} = -3x^6y - 2x^4$

Remedial

- Learners who persist in collecting unlike terms will need a lot of reminding that they may only collect like terms. Be prepared to remediate this problem on an on-going basis. This problem tends to persist throughout the grades.
- Another persistent problem is that learners make sign mistakes. Even when doing a simple addition or subtraction, for example –4 – 3, some learners will say that the answer is 7, "because a minus times a minus is a plus". There is no easy remedy for this problem, other than to keep reminding them that they are not multiplying in this case, but rather they are subtracting 3 (or adding –3). Illustrate this concept on a number line. Alternatively, this is one occasion where checking their answers on a calculator can be very useful, but then they need to know how to enter the negative numbers correctly on their own model of calculator.
- Another example of a sign mistake is when learners evaluate $(-1)^2$ and get –1. Alternatively, they evaluate $-(1)^2$ and get 1. In both of these cases, it is helpful to write out the brackets. So, $(-1)^2$ becomes $(-1)(-1)$ and $-(1)^2$ becomes $-(1)(1)$.

Exercise 2

Learner's Book page 151

Guidelines on how to implement this activity

Discuss with learners how they can simplify expressions that contain brackets, squares, cubes, roots and substitution. Although this section covers a number of different concepts, this work is still a revision of Grade 8 work. Full Worked examples are once again provided. Work through these with your class.
Once you feel satisfied that your learners understand these examples, ask them to do Exercise 2. In Questions 2.1–2.4, your learners will probably find it helpful to write out the terms. For example, $(4xyz)^2$ can be written out as $(4xyz)(4xyz)$, and $(4xyz)^3$ can be written out as $(4xyz)(4xyz)(4xyz)$. This makes it easier for your learners to tackle these questions in a systematic way.
Once your class has completed this exercise, go through some of the solutions with them to make sure that they have coped with this work before moving on to the next section.

Suggested answers

1.1 $\begin{aligned} 7a + 4 - 3(a - 6) &= 7a + 4 - 3a + 18 \\ &= 4a + 22 \end{aligned}$

1.2 $\begin{aligned} 3(5x - 2y) + 2(4x - 3y) &= 15x - 6y + 8x - 6y \\ &= 23x - 12y \end{aligned}$

1.3 $\begin{aligned} 5(5x - 2y) - 4(4x - 3y) &= 25x - 10y - 16x + 12y \\ &= 9x + 2y \end{aligned}$

1.4 $\begin{aligned} 3(2x - 9)(- 5x + 1) &= 6x - 27 + 5x - 5 \\ &= 11x - 32 \end{aligned}$

1.5 $\begin{aligned} 2(a^2 - 3a - 4) - (2a^2 - 3a - 4) &= 2a^2 - 6a - 8 - 2a^2 + 3a - 4 \\ &= -3a - 12 \end{aligned}$

1.6 $\begin{aligned} 2x(y^2 - 2y) - y(2xy - 5x) &= 2xy^2 - 4xy - 2xy^2 + 5xy \\ &= xy \end{aligned}$

1.7 $\begin{aligned} -2(4x - 3) - 6 + 5(3x - x + 1) &= -8x + 6 - 6 + 15x - 5x + 5 \\ &= 2x + 5 \end{aligned}$

1.8 $\begin{aligned} a^2(a^3 - 2a^2 + a) + 10a^4 &= a^5 - 2a^4 + a^3 + 10a^4 \\ &= a^5 + 8a^4 + a^3 \end{aligned}$

2.1 $(4xyz)^2 = 16x^2y^2z^2$

2.2 $(-2ab^2c)^2 = 4a^2b^4c^2$

2.3 $(4xyz)^3 = 64x^3y^3z^3$

2.4 $(-2ab^2c)^3 = -8a^3b^6c^3$

2.5 $\sqrt{144z^4} = 12z^2$

2.6 $\sqrt{100p^2 - 36p^2} = \sqrt{64p^2} = 8p$

2.7 $\sqrt[3]{-27a^3b^6} = -3ab^2$

2.8 $\sqrt[3]{99k^3 - 100k^3} = \sqrt[3]{-k^3} = -k$

3.1 $-(9 - a) = -(9 - (-1)) = -10$

3.2 $x^2 + 7 = 2^2 + 7 = 11$

3.3 $y^2 - (x - y)^2 = (-4)^2 - (2 - (-4))^2 = 16 - 36 = -20$

3.4 $4ab^2 - 2xy + \frac{0}{x} = 4(-1)(3)^2 - 2(2)(-4) + 0 = -36 + 16 = -20$

3.5 $(3x + 2y)^2 = (3(2) + 2(-4))^2 = (6 - 8)^2 = 4$

3.6 $(y - x)(y - x) = (-4 - 2)(-4 - 2) = 36$

3.7 $(-y)^2 - 5(-y) + 13 = (-(-4))^2 - 5(-(-4)) + 13 = 16 - 20 + 13 = 9$

3.8 $6\left(\frac{x}{y}\right) - 2\left(xy + \frac{y}{x}\right) = 6\left(\frac{2}{-4}\right) - 2\left((2)(-4) + \frac{-4}{2}\right) = -3 - 2(-10) = 17$

Remedial

When learners calculate the square root of an expression, remind them to always simplify the expression under the root first. So $\sqrt{25y^4 - 16y^4}$ becomes $\sqrt{9y^4}$ and NOT $\sqrt{25y^4} - \sqrt{16y^4}$! This idea should already be familiar to them from previous grades.

Exercise 3

Learner's Book page 152

Guidelines on how to implement this activity

This section expands the idea of multiplying and dividing expressions by monomials to expressions with four or more terms. This work is easy to grasp if your learners have coped with the previous work.
Work through the two Worked examples in class, then ask learners to do Exercise 3. Learners should be encouraged to do this exercise on their own.

Suggested answers

1 $x(2x^4 + x^3 - 3x^2 + x - 13) = 2x^5 + x^4 - 3x^3 + x^2 - 13x$

2 $\frac{24a + 18b - 6c + 10d - 14e}{2} = 12a + 9b - 3c + 5d - 7e$

3 $6xy(5x^3y + 4x^2y^2 - 3xy^3 - 2xy) = 30x^4y^2 + 24x^3y^3 - 18x^2y^4 - 12x^2y^2$

4 $\frac{8a - 4a^2 + 2a^3 - 6a^4}{2a} = 4 - 2a + a^2 - 3a^3$

5 $-p^2(6p^4 - p^3 + 2p^2 - 3p + 9) = -6p^6 + p^5 - 2p^4 + 3p^3 - 9p^2$

6 $\frac{27m^5 + 9m^4 - 12m^3 + 15m^2 - 3m}{-3m} = -9m^4 - 3m^3 + 4m^2 - 5m + 3$

Remedial

If learners are experiencing difficulty, have them revise basic fraction work before retrying these examples. Show learners how to use arrows when multiplying to help them, and how to break up each term over the denominator.

Exercise 4

Learner's Book page 155

Guidelines on how to implement this activity

The following section of multiplying binomials is new in Grade 9. This is very important work and it is a skill that your learners will rely on heavily in the future. Make sure that you spend enough time on explaining the concept to them. The FOIL method is widely used and has proven to be effective over the years. Make sure that your learners understand the basic concept, which is that every term in the first set of brackets is multiplied by every term in the second set of brackets. Finally, like terms are collected when possible.

Exercise 4 gives your learners plenty of practice in multiplying binomials. Walk through your classroom as they work. Remediate any problems as they occur. If many learners are not managing to put this new concept into practice, ask the class to stop working and work through some of the solutions with them, before asking them to continue with the exercise.
In Question 3, remind your class that to "find the product" means to multiply.

Suggested answers

1.1 $(x + y)(y + z) = xy + xz + y^2 + yz$ **1.2** $(x - y)(y + z) = xy + xy - y^2 - yz$

1.3 $(p + q)(r - t) = pr - pt + qr - qt$ **1.4** $(x + y)(x - y) = x^2 - y^2$

1.5 $(2a + b)(c + d) = 2ac + 2ab + bc + bd$

1.6 $(a + 2b)(3a - d) = 3a^2 - ad + 6ab - 2bd$

1.7 $(4x + y)(5 - z) = 20x - 4xz + 5y - yz$

1.8 $(x + 7)(x - 6) = x^2 + x - 42$

1.9 $(x + y)(x - 2y) = x^2 - xy - 2y^2$

1.10 $(4a + 3b)(2c + d) = 8ac + 4ad + 6bc + 3bd$

1.11 $(4m + n)(3m - 5n) = 12m^2 - 17mn - 5n^2$

1.12 $(m + 3n)(m - 2n) = m^2 + nm - 6n^2$

2.1 $(A + B)(A - B) = A^2 - AB + AB - B^2$
$= A^2 - B^2$

2.2 $(A + B)^2 = (A + B)(A + B)$
$= A^2 + AB + AB + B^2$
$= A^2 + 2AB + B^2$

2.3 $(A - B)^2 = (A - B)(A - B)$
$= A^2 - AB - AB + B^2$
$= A^2 - 2AB + B^2$

3.1 $(x + 5)(x - 5) = x^2 - 25$ **3.2** $(x + 5)^2 = x^2 + 10x + 25$

3.3 $(x - 5)^2 = x^2 - 10x + 25$ **3.4** $(a + 4)(a - 4) = a^2 - 16$

3.5 $(a + 4)^2 = a^2 + 8a + 16$ **3.6** $(a - 4)^2 = a^2 - 8a + 16$

3.7 $(1 + 3a)(1 - 3a) = 1 - 9a^2$ **3.8** $(1 + 3a)^2 = 1 + 6a + 9a^2$

3.9 $(1 - 3a)^2 = 1 - 6a + 9a^2$ **3.10** $(2x + 7)(2x - 7) = 4x^2 - 49$

3.11 $(2x + 7)^2 = 4x^2 + 28x + 49$ **3.12** $(2x - 7)^2 = 4x^2 - 28x + 49$

3.13 $(5a + 3)(5a - 3) = 25a^2 - 9$ **3.14** $(5a + 3)^2 = 25a^2 + 30a + 9$

3.15 $(5a - 3)^2 = 25a^2 - 30a + 9$

Remedial

If your learners struggle to find the products by inspection in Question 3, encourage them to rather write out the brackets twice and then to use the FOIL method of multiplication. It is more important that they are able to expand the product correctly than to do so more quickly by inspection.

Extension

Learners who need additional challenges can make up their own questions along the lines of the challenge for one another to solve.en.

Challenge
Learner's Book page 155

Guidelines on how to implement this activity

The Challenge exercise has been included for those learners who have coped well with this work. Set them to work on this while you spend time with those learners who are still busy with Exercise 4.

Suggested answers

1 $(6x + 7y)(6x - 7y) = 36x^2 - 49y^2$
2 $(6x + 7y)^2 = 36x^2 + 84xy + 49y^2$
3 $(6x - 7y)^2 = 36x^2 - 84xy + 49y^2$
4 $(10a + 3y)(10a - 3y) = 100a^2 - 9y^2$
5 $(10a + 3y)^2 = 100a^2 + 60ay + 9y^2$
6 $(10a - 3y)^2 = 100a^2 - 60ay + 9y^2$

Unit 4 Solving equations

Learner's Book page 156

Unit focus

This unit focusses on the following:
- set up equations to describe problem situations,
- analyse and interpret equations that describe a given situation,
- solve equations by inspection,
- solve equations using additive and multiplicative inverses,
- solve equations using the laws of exponents, and
- determine the numerical value of an expression by substitution.

Background information on solving equations

The focus of this unit is on setting up and solving equations. This is revision of Grade 8 work. It forms the basis of new work on equations that your learners will do in Chapter 10.

Exercise 1
Learner's Book page 157

Guidelines on how to implement this activity

Read through the Did you know? in the Learner's Book with your class. Although they are not expected to memorise any of the facts in this feature, they should be made aware that algebra is a discipline that has been with us for many centuries.
Work through the Worked examples with your class about translating situations into mathematical equations. These are fairly simple examples, so make sure that all your learners understand them. Move on to the next two Worked examples about solving equations by inspection. These examples revise concepts that should already be familiar to your class from earlier grades. Learners should manage this exercise on their own.

Suggested answers

1.1 $5x - 10 = 60$ **1.2** $\frac{x}{7} + 4 = 11$

2.1.1 Neo's age = $x - 7$

2.1.2 Ludo's age in 2 years' time = $x + 2$

2.1.3 Neo's age in 2 years' time = $x - 7 + 2 = x - 5$

2.2 $x + 2 = 2(x - 5)$

3.1 B $x + 11 = 20$ **3.2** C $x - 8 = 14$

3.3 A $2x + 23 = 31$

4.1 $2x + 6 = 18$
$12 + 6 = 18$ so $2x = 12$
$2 \times 6 = 12$ so $x = 6$

4.2 $3x - 2 = 7$
$9 - 2 = 7$ so $3x = 9$
$3 \times 3 = 9$ so $x = 3$

4.3 $17 - 2x = 11$
$17 - 6 = 11$ so $2x = 6$
$2 \times 3 = 6$ so $x = 3$

4.4 $\frac{x}{2} = 2{,}5$
$\frac{5}{2} = 2{,}5$ so $x = 5$

Remedial

If your learners have trouble translating verbal descriptions of problem situations into equations, consider compiling a list of these words and their meanings with your class. For example, "sum" means addition, "difference" means subtraction, "product" means multiplication, "quotient" means division, and so on. You could keep a poster of these words on the wall of your classroom and add to it as new words arise.

Extension

Ask learners to think up problems of their own for one another to solve. This is particularly challenging as they will have to think up problems to which there are sensible solutions. This can be trickier than it seems!

Challenge

Learner's Book page 158

Guidelines on how to implement this activity

The Challenge is suitable for all your learners to attempt. Set it as an optional exercise for learners to try.

Suggested answers

$x(x + 1) = x + 1 + 35$
$x(x + 1) = x + 36$

Exercise 2

Learner's Book page 160

Guidelines on how to implement this activity

Introduce the more formal methods of solving equations to learners. Read through the text with your class in which an equation is likened to a balanced scale. This is a useful mental picture for your learners to bear in mind as they work. The key to keeping an equation in balance is this: "What you do to the LHS, you must also do to the RHS". Furthermore, your learners must realise that all our efforts are concentrated on getting the unknown variable on its own on the LHS of the equation, with a constant on the RHS. Once we have achieved this, we will have solved the equation. Work through the Worked examples with your class. Spend enough time on each example so that all your learners understand the various steps involved. Explain the concept of additive and multiplicative inverse and how we use these concepts to solve equations. Once the equation has been solved, show your learners how to check the solution by substituting it into the original equation. This act of checking the solution to an equation is extremely important, as it will empower your learners to feel more confident and independent.
Ask your class to do Exercise 2. In Question 2 they are required to check their solutions to Question 1. Do not allow them to view this question as an optional extra – they must check each and every answer to Question 1. In future, they will not always be reminded to check their solutions, but they should make a habit of doing so.

Suggested answers

1.1
$3x - 8 = 7$
$3x - 8 + 8 = 7 + 8$
$3x = 15$
$x = 5$

1.2
$9 - 2x = 19$
$9 - 9 - 2x = 19 - 9$
$-2x = 10$
$x = -5$

1.3
$2x - 9 = 15$
$2x - 9 + 9 = 15 + 9$
$2x = 24$
$x = 12$

1.4
$11 - x = 21$
$11 - 11 - x = 21 - 11$
$-x = 10$
$x = -10$

1.5
$4x - 1 = -9$
$4x - 1 + 1 = -9 + 1$
$4x = -8$
$x = -2$

1.6
$3x + 7 = x + 8$
$3x + 7 - 7 = x + 8 - 7$
$3x = x + 1$
$3x - x = x - x + 1$
$2x = 1$
$x = 0{,}5$

1.7
$4x + 3 = 31$
$4x + 3 - 3 = 31 - 3$
$4x = 28$
$x = 7$

1.8
$x + 3(x + 1) = 2x + 8$
$x + 3x + 3 = 2x + 8$
$4x + 3 = 2x + 8$
$4x + 3 - 3 = 2x + 8 - 3$
$4x = 2x + 5$
$4x - 2x = 2x - 2x + 5$
$2x = 5$
$x = 2{,}5$

1.9 $\frac{2x}{5} - 6 = 4$

$\frac{2x}{5} - 6 + 6 = 4 + 6$

$\frac{2x}{5} = 10$

$2x = 50$

$x = 25$

1.10 $4(x + 1) = 14$

$4x + 4 = 14$

$4x + 4 - 4 = 14 - 4$

$4x = 10$

$x = \frac{10}{4}$

$x = 2{,}5$

1.11 $x - 7 = 3(x + 3)$

$x - 7 = 3x + 9$

$x - 7 + 7 = 3x + 9 + 7$

$x = 3x + 16$

$x - 3x = 3x - 3x + 16$

$-2x = 16$

$x = -8$

1.12 $10(x - 2) = 2(4x + 9)$

$10x - 20 = 8x + 18$

$10x - 20 + 20 = 8x + 18 + 20$

$10x = 8x + 38$

$10x - 8x = 8x - 8x + 38$

$2x = 38$

$x = 19$

1.13 $\frac{x}{2} - \frac{1}{4} = \frac{7}{12}$

$6x - 3 = 7$

$6x - 3 + 3 = 7 + 3$

$6x = 10$

$x = \frac{10}{6}$

$x = 1\frac{2}{3}$

1.14 $\frac{4x}{5} + 16 = 32$

$\frac{4x}{5} + 16 - 16 = 32 - 16$

$\frac{4x}{5} = 16$

$4x = 80$

$x = 20$

2.1 $3(5) - 8 = 15 - 8 = 7$

2.2 $9 - 2(-5) = 9 + 10 = 19$

2.3 $2(12) - 9 = 24 - 9 = 15$

2.4 $11 - (-10) = 11 + 10 = 21$

2.5 $4(-2) - 1 = -8 - 1 = -9$

2.6 $3(0{,}5) + 7 = 8{,}5$

$0{,}5 + 8 = 8{,}5$

2.7 $4(7) + 3 = 28 + 3 = 31$

2.8 $2{,}5 + 3(2{,}5 + 1) = 2{,}5 + 10{,}5 = 13$

$2(2{,}5) + 8 = 13$

2.9 $\frac{2(25)}{5} - 6 = 10 - 6 = 4$

2.10 $4(2{,}5 + 1) = 4(3{,}5) = 14$

2.11 $-8 - 7 = -15$

$3(-8 + 3) = 3(-5) = -15$

2.12 $10(19 - 2) = 170$

$2(4(19) + 9) = 2(85) = 170$

2.13 $\frac{\frac{5}{3}}{2} - \frac{1}{4} = \frac{5}{3} \times \frac{1}{2} - \frac{1}{4}$

$= \frac{5}{6} - \frac{1}{4}$

$= \frac{10 - 3}{12}$

$= \frac{7}{12}$

2.14 $\frac{4(20)}{5} + 16 = 16 + 16 = 32$

Remedial

For learners who struggle with the idea of keeping an equation in balance, write the following on the board:

What you do to the LHS, you must also do to the RHS.

If $x + a = y$, then $x = y - a$ (Subtract a from both sides of the equation.)
If $x - a = y$, then $x = y + a$ (Add a to both sides of the equation.)
If $ax = y$, then $x = \frac{y}{a}$ (Divide both sides of the equation by a.)
If $\frac{x}{a} = y$, then $x = ay$ (Multiply both sides of the equation by a.)

Ask your learners to copy these guidelines down in the back of their exercise books and to refer to them as they check their answers.

Extension

Ask learners to think up problems of their own for one another to solve. This is particularly challenging as they will have to think up problems to which there are sensible solutions. This can be trickier than it seems!

Exercise 3

Learner's Book page 162

Guidelines on how to implement this activity

Revise the work learners have covered in Exercise 2 by going through some of the solutions as a class. It is important that learners feel confident that they have mastered these skills before continuing with new work.
Learners need to be able to integrate their knowledge of equations with the laws of exponents to solve equations. Read through the notes from the Learner's Book with your class and work through the Worked examples with them. Once you feel confident that your learners understand these examples, ask them to do Exercise 3.

Suggested answers

1.1 $x^2 = 9$; $x = \sqrt{9}$; $x = \pm 3$

1.2 $x^3 = 27$; $x = \sqrt[3]{27}$; $x = 3$

1.3 $x^3 = 8$; $x = \sqrt[3]{8}$; $x = 2$

1.4 $2x^2 = 50$; $x^2 = 25$; $x = \sqrt{25}$; $x = \pm 5$

1.5 $\frac{x^4}{2} = 8$; $x^4 = 16$; $x = \sqrt[4]{16}$; $x = \pm 2$

1.6 $x^5 = -32$; $x = \sqrt[5]{-32}$; $x = 2$

2.1 $2x = 16$; $2^x = 2^4$; $x = 4$

2.2 $16x = 256$; $16^x = 16^2$; $x = 2$

2.3 $4.3^x = 108$; $3^x = 27$; $x = 3$

2.4 $x^2 = \frac{9}{4}$
$x = \sqrt{\frac{9}{4}}$
$x = \pm\frac{3}{2}$

2.5 $3^x + 30 = 111$
$3^x = 81$
$3^x = 3^4$
$x = 4$

2.6 $10.2^x = 80$
$2^x = 8$
$2^x = 2^3$
$x = 3$

Remedial

Refer those learners who struggle with Exercise 3 back to the Worked examples that precede the exercise. The principles that are illustrated in the examples will serve as useful guidelines for them

Extension

Give learners additional problems to solve, similar to the problems in Exercise 2 and 3.

Exercise 4

Learner's Book page 162

Guidelines on how to implement this activity

Learners need to be able to determine the value of an expression when given a value for a variable. Discuss substitution and what it means. Revise how to substitute by including brackets around the number being substituted to prevent problems with signs. Work through the Worked examples as a class. Ensure learners understand how to find the value for the equation for a given variable. Learners should complete this exercise on their own.

Suggested answers

1
$$y = \frac{5t}{-6} - 7t$$
$$y = \frac{5(-1)}{-6} - 7(-1)$$
$$= \frac{5}{6} + 7$$
$$= 7\frac{5}{6}$$

2
$$m = \frac{3x^2 - 4}{2}$$
$$m = \frac{3(3)^2 - 4}{2}$$
$$= \frac{27 - 4}{2}$$
$$= \frac{23}{2}$$
$$= 11\frac{1}{2}$$

3
$$y = \frac{7(-x) + 7}{-2x}$$
$$y = \frac{7(-7) + 7}{-2(7)}$$
$$= \frac{-42}{-14}$$
$$= 3$$

4
$$t = \frac{\sqrt{16x^2} + 1}{-3}$$
$$t = \frac{\sqrt{16(-2)^2} + 1}{-3}$$
$$= \frac{\sqrt{64} + 1}{-3}$$
$$= \frac{9}{-3}$$
$$= -3$$

Remedial

Learners need to be able to determine the value of an expression when given a value for a variable. Discuss substitution and what it means. Revise how to substitute by including brackets around the number being substituted to prevent problems with signs. Work through the Worked examples as a class. Ensure learners understand how to find the value for the equation for a given variable. Learners should complete this exercise on their own.

Consolidation

Learner's Book page 164

Before doing this consolidation exercise, encourage learners to review the work covered in this chapter. Advise learners to use the summary and to revise their work. This exercise can be used as an informal assessment task for you to track how learners are coping with the chapter and the concepts covered. The mark allocation provides guidelines on how to assess learners.

Suggested answers

1.1 3 terms (1)

1.2 1 term (1)

2 $5x^2 - 3x + 1$ (1)

3 $-3x + x^2 - 5 + 15x + 4x^2 - 4 - 2x^2 - 7x = 3x^2 + 5x - 9$ (1)

4.1 Yes, because all the exponents are positive whole numbers and there are no variable denominators. (2)

4.2 $x^6y - x^4y + 5x^3y^2 - 2x^2y^3 - 6xy + 31$ (1)

4.3 7th degree (1)

5.1 $\sqrt{25k^2 + 144k^2} = \sqrt{169k^2} = 13k$

5.2 $\sqrt[3]{-216x^6y^6} = -6x^2y^2$ (2)

6.1 $2xy$

6.2 $4x^2y^2$ (2)

7.1 $-3(x - y) = -3(-2 - 4)$
$= 18$

7.2 $y = 5x = 5(-1) = -5$
$-3(x - y) = -3(-1 - (-5))$
$= -3(4)$
$= -12$ (2)

8.1 $3x + 21y - 6x + 5y - 7 = -3x + 26y - 7$ (1)

8.2 $6b - 4(3a - 2b) + 3a = 6b - 12a + 8b + 3a = 14b - 9a$ (1)

8.3 $-3k(2k + 2) + k(k - 1) = -6k^2 - 6k + k^2 - k = -5k^2 - 7k$ (2)

8.4 $3 + 4a(a - 2b) - 2 + 5b(a - 3b) = 3 + 4a^2 - 8ab - 2 + 5ab - 15b^2$
$= 4a^2 - 3ab - 15b^2 + 1$ (2)

8.5 $\frac{6x^2y}{3y} = 2x^2$ (1)

8.6 $\frac{x^4 + 4x^3 - 6x^2 + 13x}{x} = x^3 + 4x^2 - 6x + 13$ (1)

8.7 $\frac{2a^3 - 6a - 12ab + 38ab^2}{2a} = a^2 - 3 - 6b + 19b^2$ (1)

8.8 $\frac{ab^3 - 2a^3b + 3ab - 5a^2b^2}{ab} = b^2 - 2a^2 + 3 - 5ab$ (1)

8.9 $(a + 5b)(3a - 4b) = 3a^2 - 4ab + 15ab - 20b^2$
$= 3a^2 - 11ab - 20b^2$ (1)

8.10 $(m - n)(2m - 7n) = 2m^2 - 7mn - 2mn + 7n^2$
$= 2m^2 - 9mn + 7n^2$ (1)

9.1 $(p + 4)(p - 4) = p^2 - 16$

9.2 $(p + 4)^2 = p^2 + 8p + 16$

9.3 $(p - 4)^2 = p^2 - 8p + 16$

9.4 $(5 + 6a)(5 - 6a) = 25 - 36a^2$

9.5 $(5 + 6a)^2 = 25 + 60a + 36a^2$

9.6 $(5 - 6a)^2 = 25 - 60a + 36a^2$ (6)

10.1 $6x - 7 = 35$

10.2 $x^2 + \sqrt{x} = 18$ (2)

11.1 $x + x - 7 = 23$

11.2 $x + x - 7 = 23$
$2x - 7 + 7 = 23 + 7$
$2x = 30$
$x = 15$

11.3 Sam's brother is 8 years old.
$15 + 8 = 23$ (3)

12.1 $3x + 15 = x + 29$
$3x - x = 29 - 15$
$2x = 14$
$x = 7$ (1)

12.2 $7(x + 2) = 3(2x + 5)$
$7x + 14 = 6x + 15$
$7x - 6x = 15 - 14$
$x = 1$ (2)

12.3 $\frac{2x}{3} = 2{,}5 - \frac{x}{6}$
$4x = 15 - x$
$5x = 15$
$x = 3$ (2)

12.4 $4x^2 = 196$
$x^2 = 49$
$x = \pm 7$ (2)

12.5 $x^3 - 2 = -10$
$x^3 = -10 + 2$
$x^3 = -8$
$x = \sqrt[3]{-8}$
$x = -2$ (2)

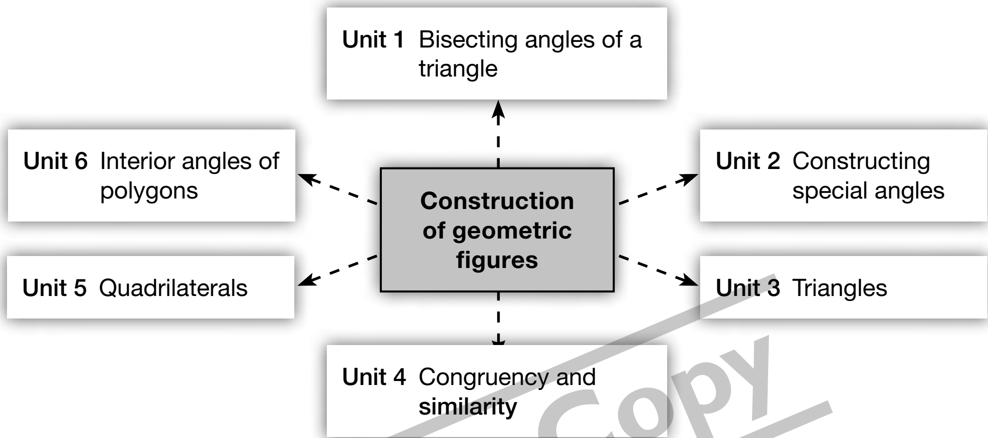
12.6 $3^x = 81$
$3^x = 3^4$
$x = 4$ (1)

12.7 $6^x - 40 = 176$
$6^x = 176 + 40$
$6^x = 216$
$6^x = 6^3$
$x = 3$ (2)

[50]

Chapter 6 Construction of geometric figures

Chapter overview



Content		Time allocations	LB page
Unit 1	Bisecting angles of a triangle	1 hour	167
Unit 2	Constructing special angles	1 hour	172
Unit 3	Triangles	2 hours	175
Unit 4	Congruency and similarity	2 hours	179
Unit 5	Quadrilaterals	2 hours	185
Unit 6	Interior angles of a polygon	1 hour	188

Background information

In previous grades, the learners learnt:

- basic constructions of triangles and quadrilaterals, and
- about some properties of these geometric figures.

Constructions are a useful tool for investigating the properties of geometric figures. In this chapter learners will investigate many more properties of 2-dimensional geometric figures through construction and measurement, particularly the minimum conditions for the congruency of two triangles.

Teaching guidelines

- Make sure learners are competent and comfortable using a compass and know how to measure and read angles on a protractor.

- Revise constructions of angles before proceeding with new constructions.
- Begin with constructions of lines so that learners can explore angle relationships on straight lines.
- Revise drawing of circles and the fact that all radii of a circle are equal.
- Construction of special angles are done by:
  - bisecting a right angle to get an angle of 45°
  - drawing an equilateral triangle to get an angle of 60°
  - bisecting a 60° angle to get an angle of 30°

Resources

Each learner should have a complete geometry set. This should include a compass, a protractor, a set square and a ruler. Grid paper, graph paper, cardboard, colour pens, scissors and glue should also be available. Each learner should have their own calculator.

Unit 1 Bisecting angles of a triangle

Learner's Book page 166

Unit focus

This unit focusses on the following:
- learning how to accurately construct lines and triangles using geometry tools,
- revising the properties and definitions of triangles, and
- learning how to accurately measure and bisect angles within triangles.

Background information on bisecting angles

Bisecting an angle uses an arc drawn to cut both arms of the angle. With centre at the points where the arcs cut the lines, and a slightly bigger radius, two more arcs are drawn. A line from the vertex of the angle to the point where these two arcs meet is the bisector of the angle. The angle bisectors of a triangle are concurrent. The point where they meet is the "incentre", that is the centre of the inscribed circle of the triangle.

Exercise 1

Learner's Book page 168

Guidelines on how to implement this activity

Work through the example showing how to draw specific line lengths. Ensure learners are able to hold and manipulate a pair of compasses correctly. If necessary, do additional examples together as a class before learners do the exercise on their own.

Suggested answers

1 Use a ruler to check that the learners have drawn a line of 3.5 cm and that it is labelled AB.

2 Use a ruler to check that the learners have drawn a line of 8 cm and that it is labelled PQ.

3 Use a ruler to check that the learners have drawn a line of 6,7 cm and that it is labelled TR.

Remedial

If language difficulties result in some learners not understanding the verbal instructions, practical demonstration may help. Rotating the compass incorrectly will result in inaccuracy. Make sure the learners hold the compass at the top and not by the arms.

Exercise 2

Learner's Book page 169

Guidelines on how to implement this activity

It is best to demonstrate a construction step by step and let the class perform each step after the teacher, so make use of board instruments or a data projector or Smart Board and software such as "Geogebra" or "Geometer's Sketchpad" if available. If necessary, on completion of the activity, revise the meaning of congruency.

Suggested answers

1 Check construction using a ruler.
2 Check construction using a ruler.
3 In triangle ABC: $\hat{A} = 53°$, $\hat{B} = 37°$, $\hat{C} = 90°$
  In triangle PQR: $\hat{P} = 82°$, $\hat{Q} = 57°$, $\hat{R} = 41°$
  In comparing, learners' triangles should be identical. Learners may need to 'turn' their diagrams to see triangles are identical.
4 It is safe to say that if two triangles have all their sides equal, then their angles will also be equal. We say that these two triangles are congruent (identical).

Remedial

Some learners may be confused by the fact that there is 0° and 180° at both sides of the protractor. A simple demonstration of which way the arms have rotated to form the angle is useful. Learners may create inaccurate constructions. This could be because of using a blunt pencil. Advise learners to sharpen their pencils, and to always work carefully and neatly with constructions. Clumsiness and carelessness can be addressed by repetition and practice.

Exercise 3

Learner's Book page 171

Guidelines on how to implement this activity

First make sure each member of the class understands the meaning of the word "bisect". Let each learner draw a triangle and measure the angles, then demonstrate each step to bisecting the angles, one at a time. It may be necessary to give individual assistance to some learners at first. Once each angle has been bisected they must measure the two new angles to make sure that each is half of the original angle.

Suggested answers

1.1 Check construction with a ruler and a protractor.
1.2 Check construction with a ruler and a protractor.
1.3 Angle C = 60°. The sum of the interior angles of the triangle is180°.
1.4 Construction.
1.5 In △ABD:
$\hat{A} = 35°$
$\hat{B} = 25°$
$\hat{D} = 120°$
The sum of the interior angles of the triangle is180°.
1.6 Construction.
1.7 The sum of the interior angles in both triangles is the same = 180°. Also, the angle bisectors meet at a common point.
2.1 Construction.
2.2 Construction.

Extension

Draw an inscribed circle in a triangle.Learners who are proficient in the constructions can:

- draw any triangle
- bisect its angles
- construct a perpendicular from the incentre, where the angle bisectors meet, to any one of the sides of the triangle
- with this perpendicular distance as radius and the incentre as centre, they can draw the circle which should be tangential to all three sides of the triangle.

Unit 2 Constructing special angles

Learner's Book page 172

Unit focus

This unit focusses on the following:

- construct a 90° angle without using a protractor,
- construct a 45° angle without using a protractor,
- construct a 60° angle without using a protractor, and
- construct a 30° angle without using a protractor.

Background information

Learners must be able to construct a right angle at a point on a line by drawing two arcs of equal radius on either side of the point. With centres at the points where these arcs cut the line and a larger radius, they draw two more arcs. The line from the original point to where these two arcs meet is perpendicular to the original line. The construction is demonstrated by means of a worked example in the text book.

- Bisect a 90° angle to obtain 45°.

Using equal arcs to construct 60° is also demonstrated in the text book. Make sure that the learners see that this construction can result in an equilateral triangle.

- Bisect a 60° angle to obtain 30°.

Although these angles are constructed without the use of a protractor, it is informative for learners to measure the finished angles with a protractor to check that they are correct.

Exercise 1

Learner's Book page 173

Guidelines on how to implement this activity

As in Unit 1, step by step demonstration, with the class performing each step of the construction after the teacher demonstrates it, is the best way to start. It is advisable to check on what the learners are drawing during the process, in case some are struggling and inhibited from asking for help.
Once the class has understood the steps of each construction they can be set another construction to do without the teacher leading them through it. This is a good opportunity for the teacher to check on the progress of each individual.

Suggested answers
Check construction with a ruler and a protractor.

Exercise 2

Learner's Book page 174

Guidelines on how to implement this activity

Ensure learners have drawn their constructions from Exercise 1 neatly and correctly before continuing with this exercise. Learners must continue to work alone in order to master the necessary skills in construction.

Suggested answers

1 Triangle ABC is an equilateral triangle as all three of its angles are equal to 60°.
2 Draw an angle of 60° and bisect it to produce two angles of 30° each.
3 Construction.
3.1 The size of the third angle is 30°.
3.2 A right-angled triangle.
3.3 No it is not possible to have more than one right angle in a triangle, because if there were more than one right-angle the sum of the interior angles would exceed 180° and the shape would not be a triangle.

Remedial

Learners may require additional assistance to master working with the geometry tools. Provide additional simple constructions for them to practice with. Ensure you provide as many practical demonstrations as necessary to help learners understand what they have to do. Encourage neatness and accuracy for all constructions.

Extension

- Accurately construct squares and rectangles using rulers and compasses but no protractors.

- Bisect their angles and compare the angle bisectors of the square with those of the rectangle, taking note of whether or not the angle bisectors are also diagonals.
- Construct two equilateral triangles, on top of one another so that they have a common side, again using only ruler and compass. What kind of quadrilateral results?

Unit 3 Triangles

Learner's Book page 175

Unit focus

This unit focusses on the following:
- learning more about angles of triangles and how to measure them, and
- constructing triangles with three known measurements using a calculator.

Background information

A triangle is a basic geometric shape, a 2-dimensional polygon with 3 sides and 3 vertices. The sum of the 3 interior angles is always equal to 180° and an exterior angle is equal to the sum of the 2 interior opposite angles. Furthermore the exterior angle and its adjacent interior angle are supplementary. The triangle inequality states that the sum of any two sides of a triangle must be greater than the third side. This too could be investigated by construction if time permits.

Exercise 1

Learner's Book page 176

Guidelines on how to implement this activity

Learners will investigate and consolidate their knowledge of the interior and exterior angles of triangles by construction and measurement. Accuracy is important here, so it may be advisable to construct triangles with special angles, 90°, 60°, 30° for example, without a protractor, to demonstrate the sum of the interior angles. Then go on to construct triangles with other angles, measured with a protractor.

Suggested answers

1.1 $\hat{A} = 53°$
$\hat{B} = 63°$
$A\hat{C}D = 116°$
$\hat{A} + \hat{B} = 53° + 63° = 116° = A\hat{C}D$

1.2 $\hat{A} = 28°$
$\hat{B} = 123°$
$A\hat{C}D = 151°$
$\hat{A} + \hat{B} = 28° + 123° = 151° = A\hat{C}D$

1.3 Own constructions

Remedial

- The spatial positions "adjacent" and "opposite" are often confusing. It may help to make learners stand beside (adjacent to) one classmate but facing (opposite) another. Moving physically into these positions often helps to get the idea across.
- Terminology such as "the sum of" may cause confusion. Stress that it means the answer you get when you add the interior angles.
- An exterior angle is formed by making one side longer, making an angle on the outside of the triangle.

Exercise 2

Learner's Book page 178

CHAPTER 6

Guidelines on how to implement this activity

Learners extend their knowledge of constructing triangles. This exercise serves to show learners that they can draw accurate triangles given only three pieces of information. It is important which three pieces of information they are given. This exercise leads into the next unit on congruency.

Suggested answers

1 AB = 4,5 cm
$\hat{A} = 43°$
$\hat{C} = 47°$
Learners' triangles should be identical.

2.1 Construction **2.2** Construction **2.3** Construction

2.4 Construction

2.5 BC = 5,8 cm, $\hat{B} = 81°$ and $\hat{C} = 44°$
Learners' triangles should be identical.

3 No, you cannot construct an accurate triangle. The triangle could have side lengths of any set of values. You could draw many triangles with the same three angles, but of different sizes.

Extension

- Investigate the sum of the exterior angles of a triangle. Extend each of the threesides in one direction only. Measure the three exterior angles and add them. Compare your answer with a friend. Try another, different triangle. What do you notice?
- Try to construct a triangle with the following sides: 6 cm; 4 cm; 10 cm. What do you notice?
- Try to construct a triangle with sides 12 cm; 3 cm; 7 cm. Again, what do you notice?
- Write a rule about the lengths of the sides of a triangle, based on your observations. A rule like this is called a "conjecture".

Unit 4 Congruency and similarity

Learner's Book page 179

Unit focus

This unit focusses on the following:
- learning about congruency,
- learning how to prove that two triangles are congruent,
- learning about similarity, and
- learning how to prove that two triangles are similar.

Background information on congruency

Congruent figures are identical in all their dimensions. In the case of triangles, only three dimensions need to be known in order to conclude that they are congruent, or not. These are:
- all three corresponding sides equal (SSS),
- two sides and the angle between them equal (SAS),
- two angles and a corresponding side equal (ASA), and
- a right angle, the hypotenuse and one other side equal (RHS).

Similar figures have corresponding angles equal and sides in constant proportion. Thus they are the same shape but might not be the same size. They are scale models of each other.

Congruent figures are also similar, but similar figures are not necessarily congruent.

Exercise 1

Learner's Book page 181

Guidelines on how to implement this activity

Construction is a useful way to investigate congruency. Review the previous exercises in this chapter using the table in the Learner's Book to show the different triangles we have constructed.
In addition, have each member of the class construct accurately, a triangle with the lengths of all three sides given and then measure each angle. They then compare their triangles with the constructions drawn by others and it should soon be apparent that, although they only measured the three sides, all three corresponding angles have the same measurements and the triangles are identical. In the same way, each can construct a triangle in which two sides and the angle between them are given. Once again, compare the outcome. Repeat this to illustrate each of the four cases for congruency. Consolidate the four cases of congruency and ensure learners can recognise these before embarking on this exercise. Learners must complete this exercise on their own.

Suggested answers

1 Yes, $\triangle PQR \equiv \triangle TSU$ (SAS)
$PR = TU$
$\hat{P} = \hat{T}$
$\hat{R} = \hat{U}$

2 Yes, $\triangle ABC \equiv \triangle MTP$ (RHS)
$BC = TP$
$\hat{A} = \hat{M}$
$\hat{C} = \hat{P}$

3 Yes, $\triangle XYZ \equiv \triangle HKD$ (ASA)
$XY = HK$
$YZ = KD$
$\hat{Y} = K$

4 No, these two triangles are not congruent as the equal angles are not in the corresponding positions.

5 Yes, $\triangle ABC \equiv \triangle QRP$ (SSS)
$\hat{A} = \hat{Q}$
$\hat{B} = \hat{R}$
$\hat{C} = \hat{P}$

6 No, these two triangles are not congruent as the equal angles and side are not in the corresponding positions.

Remedial

Some learners have difficulty visualising the fact that two triangles are identical in all respects. In these cases it is useful to cut out the pairs of congruent triangles with scissors and fit them on top of each other. If confusion arises over what is meant by "corresponding sides and angles" colour coding may help, so that, for example, the green lines are equal and the angles opposite them have a green dot and they are equal.

Challenge

Learner's Book page 182

Guidelines on how to implement this activity

Learners must draw as many different triangles as they can with the same angles. Learners should see by means of constructing that their answer is that two triangles with corresponding angles equal (AAA) will not be congruent as their sides could be different lengths. It is possible to construct two triangles with all corresponding angles equal, but with different side lengths (i.e. one triangle will be an enlargement of the other).

Exercise 2

Learner's Book page 184

Guidelines on how to implement this activity

Discuss similar triangles using the results from the challenge activity. Show learners that the triangles they constructed with equal angles are similar triangles. Discuss the concept of similarity and the proportion of sides.

Suggested answers

1 Check with a protractor and a ruler that all angles are equal and all sides are different lengths.

2 The sides should all be in proportion.

3 Yes, two triangles with two pairs of equal angles are similar because the third pair of angles will also be equal as the sum of the interior angles of any triangle is the same (180°).

Remedial

Some learners may confuse congruency with similarity. Again if they construct two triangles with equal corresponding angles but sides of different lengths it is easy to demonstrate the difference between congruency and similarity by cutting out the triangles and comparing the physical shapes.

Extension

Congruency can be used to investigate and later to prove many known properties of geometric figures, for example:

- Show the class how to draw a right-angled triangle on squared paper.
- Let them draw the reflection of this triangle, about its own vertical side.
- Investigation of the large triangle that results from drawing these two triangles should show very clearly that the altitude of an isosceles triangle bisects the triangle.
- Let the class now draw two congruent right-angled triangles, cut them out and fit them together to form a rectangle. Then discuss what properties of a rectangle this illustrates.

Unit 5 Quadrilaterals

Learner's Book page 185

Unit focus

This unit focusses on the following:

- investigating the sides, angles and diagonals of quadrilaterals; and
- revising the properties of squares, rectangles, parallelograms, trapeziums, rhombi and kites.

Background information on quadrilaterals

The word "quadrilateral" simply means "four sided" so any flat geometric figure with four sides and four interior angles is a quadrilateral. There are many quadrilaterals which have special properties.

The class will already know some of them but they can be revised and investigated by means of construction. It is important that learners are familiar with the definitions of special types of quadrilaterals:

- A parallelogram is a quadrilateral with both pairs of opposite sides parallel.
- A rhombus is a special parallelogram with all four sides equal.
- A rectangle is a special parallelogram with 90° angles at its vertices.
- A square is a special rectangle with all four sides equal.

- A trapezium is any quadrilateral with one pair of opposite sides parallel.
- A kite is a quadrilateral with two pairs of adjacent sides equal.

Investigation 1

Learner's Book page 185

Guidelines on how to implement this activity

Revise the properties of a square. Do this initially without a reference to see what learners can remember. List the properties on the board. Then refer learners back to their construction of a square in Unit 2. Learners use this to complete the exercise. Once learners have completed the exercise, revise all the properties and ensure learners have listed them all correctly.

Suggested answers

1.1 All sides are equal in length.
1.2 All angles are equal to 90°.
1.3 Yes the opposite sides are parallel.
1.4 The diagonals are equal in length.
1.5 The diagonals are perpendicular.
1.6 Yes the diagonals bisect each other.

Investigation 2

Learner's Book page 186

Guidelines on how to implement this activity

First ensure that all learners have a clear understanding of the definitions of each of the above quadrilaterals. They should then construct them accurately, in order to investigate all their properties.
Since construction and measurement are very time consuming, divide the class into groups of five. One group member then constructs a parallelogram, one a rhombus, one a rectangle, one a kite and one a trapezium. After construction, they must measure and record all interior angles, draw and measure the diagonals etc. Using their constructions they must fill in the table of properties given in the text book.

Suggested answers

1 Constructions
2 Measurements
2.1 Rectangle – opposite sides are equal in length.
Parallelogram – opposite sides are equal in length.
Rhombus – all four sides are equal in length.
Trapezium – no sides are equal in length.
Kite – adjacent side are equal in length.
2.2 Rectangle – all angles are right angles.
Parallelogram – opposite angles are equal.
Rhombus – opposite angles are equal.
Trapezium – no angles are equal.
Kite – one pair of opposite angles are equal.

2.3 Rectangle – opposite sides are parallel.
Parallelogram – opposite sides are parallel.
Rhombus – opposite sides are parallel.
Trapezium – one pair of opposite sides are parallel.
Kite – no parallel sides.

3 Construction and measurements

3.1 Rectangle – diagonals are equal and bisect each other.
Parallelogram – diagonals are not equal but do bisect each other.
Rhombus – diagonals are not equal but do bisect each other.
Trapezium – diagonals are not equal and do not bisect each other.
Kite – diagonals are not equal and one diagonal is bisected.

3.2 Rectangle – diagonals are not perpendicular.
Parallelogram – diagonals are not perpendicular.
Rhombus – diagonals are perpendicular.
Trapezium – diagonals are not perpendicular.
Kite – diagonals are not perpendicular.

4

Quadrilateral	**Parallelogram**	**Rhombus**	**Rectangle**	**Square**	**Trapezium**	**Kite**
Opposite sides equal	✓	✓	✓	✓		
Adjacent sides equal		✓		✓		✓
Interior angles all 90^0			✓	✓		
Opposite angles equal	✓	✓	✓	✓		
Only one pair of opposite angles equal						✓
Opposite sides parallel	✓	✓	✓	✓		
Only one pair of opposite sides parallel					✓	
Diagonals bisect	✓	✓	✓	✓		One pair
Diagonals equal			✓	✓		
Diagonals perpendicular		✓		✓		✓

5.1 True **5.2** True **5.3** True

6 The sum of the interior angles of each quadrilateral should be 360°.

Remedial

Some learners may struggle with construction using instruments such as the compass and protractor. If this is the case, allow those learners to construct their quadrilaterals on squared paper, since the purpose is to investigate their interior angles and diagonals etc. Measurement may be difficult if the constructed figures are too small. Encourage learners to construct quite big figures, so that they can easily measure sides, diagonals and angles. Learners must mark sides or angles which are equal with the same identifying mark or colour, so that they can see at a glance, the properties of the quadrilateral.

Extension

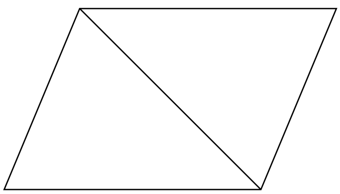
As a class project, each group can make a poster out of their five constructed quadrilaterals. They should use colour and any other imaginative ways to illustrate the properties clearly. As an incentive, the posters which best illustrate the properties of quadrilaterals should be put up on the classroom wall. If resources permit, the "winners" who create the best posters could get a chocolate or some small reward.

Challenge

Learner's Book page 187

Guidelines on how to implement this activity

Encourage learners to work in pairs for this activity. Learners should have the necessary construction equipment. Allow learners to work with limited assistance from the teacher.



The diagonal divides the quadrilateral above into two triangles. The sum of the interior angles in each triangle is 180°, thus the sum of the interior angles in the quadrilateral is 360°.

Extension

Learners can cut out some pairs of triangles which are congruent and investigate which special types of quadrilaterals can be made by putting them together.Compare what types of quadrilaterals can be formed by putting together two triangles which are not congruent.

Unit 6 Interior angles of a polygon

Learner's Book page 188

Unit focus

This unit focusses on the following:

- how to calculate the sum of the interior angles of different polygons.

Background information on the interior angles of polygons

"Poly" means "many" and a polygon is any 2-dimensional, geometric figure with many sides or edges. The polygons with the fewest sides are triangles and next are quadrilaterals, both of which have been studied. Moving on to investigate the sum of the interior angles of other polygons by construction need not be too difficult or time consuming.

Exercise 1

Learner's Book page 189

Guidelines on how to implement this activity

Each class member can draw any irregular pentagon (5-sided figure) just using a ruler and without necessarily measuring the sides or angles. All the pentagons will be different but when they measure the interior angles and add them, they should all get a total of 540°. In the same way, if they draw a hexagon (6-sided figure) the sum of the angles should be 720°. The reasons why they obtain these sums can then be illustrated as described in the text book. From one vertex only, diagonals are drawn to divide the polygon into the minimum possible number of triangles. These can be used to calculate the angle sum, because the sum of the angles in each triangle is 180^0. Investigation of polygons by this means leads to the formula for the sum of the interior angles of a polygon of n sides: $S = 180°(n - 2)$

Suggested answers

1 Construction. **2** Construction.

3

Polygon	Number of triangles	Sum of interior angles
Triangle (3 sides)	1	1 × 180 = 180°
Quadrilateral (4 sides)	2	1 × 180 = 360°
Pentagon (5 sides)	3	3 × 180 = 540°
Hexagon (6 sides)	4	4 × 180 = 720°
Heptagon (7 sides)	5	5 × 180 = 900°
Octagon (8 sides)	6	6 × 180 = 1 080°
Nonagon (9 sides)	7	7 × 180 = 1 260°
Decagon (10 sides)	8	8 × 180 = 1 440°
Any polygon (n sides)	$n - 2$	$(n - 2) \times 180$

Remedial

Learners may struggle with inaccurate measurement of angles, so check that learners are using their protractors correctly. Confusion may arise about drawing diagonals from one vertex only. Learners often start drawing diagonals randomly from all vertices, resulting in numerous triangles. Demonstrate the procedure to anybody making this mistake.

Exercise 2

Learner's Book page 189

Guidelines on how to implement this activity

Using the formula learners have discovered in the previous exercise, show learners how to use the formula effectively to find the size of one angle in a polygon. Work through the example in the Learner's Book and ensure learners are able to adapt and work with the formula.

Suggested answers

1 $S = 180°(n - 2)$
$= 180°(5 - 2)$
$= 180° \times 3$
$= 540°$
Interior angle $= \frac{540°}{5}$
$= 108°$

2 $S = 180°(n - 2)$
$= 180°(8 - 2)$
$= 180° \times 6$
$= 1\ 080°$
Interior angle $= \frac{1\ 080°}{8}$
$= 135°$

3 $S = 180°(n - 2)$
$= 180°(12 - 2)$
$= 180° \times 10$
$= 1\ 800°$
Interior angle $= \frac{1\ 800°}{12}$
$= 150°$

Remedial

Learners may experience difficulty with the formula. This can be resolved by revising basic algebraic formulae and the meaning of variables.

Extension

- Calculate the size of each interior angle of a regular octagon, decagon and dodecagon.
- What happens to the size of each angle as the number of sides increases?
- Construct each of these regular polygons, using a ruler and protractor.
- What would happen to the shape of the polygon as the number of its sides gets bigger and bigger?
- How many sides do you think it would need before it becomes approximately circular?

Consolidation

Learner's Book page 192

Before doing this consolidation exercise, encourage learners to review the work covered in this chapter. Advise learners to use the summary and to revise their work. This exercise can be used as an informal assessment task for you to track how learners are coping with the chapter and the concepts covered. The mark allocation provides guidelines on how to assess learners.

1.1	Construction	(1)
1.2	Construction	(2)
1.3	Construction	(2)
1.4	Construction	(2)
1.5	Yes, the line from D passes through B. ABD is an isosceles triangle.	(1)
2.1	$\triangle ABC \equiv \triangle ADC$ (RHS)	(2)
2.2	$\triangle KIT \equiv \triangle KET$ (SSS)	(2)
2.3	$\triangle DGH \equiv \triangle MKN$ (SAS)	(2)
2.4	The triangles are not congruent.	(2)

3.1 A square because the diagonals bisect each other and are equal in length. (2)

3.2 A parallelogram because the opposite side are equal in length. (2)

3.3 A rectangle because the opposite sides are parallel and the interior angles are all equal to 90°. (2)

3.4 A kite because the diagonals intersect at 90° and the longer diagonal bisects the shorter one.

4.1 Yes, because $\widehat{A} = 72°$ and $\widehat{F} = 63°$ (sum of angles in a triangle is 180°) thus all three pairs of corresponding angles are equal (AAA).

4.2 $\frac{DE}{AB} = \frac{DF}{AC}$

$\frac{DF}{2} = \frac{6}{3}$

$\frac{DF}{2} = 2$ (2)

DF = 4 units (2)

5.1 $S = 180°(n - 2)$

$= 180°(20 - 2)$

$= 180° \times 18$

$= 3\ 240°$ (2)

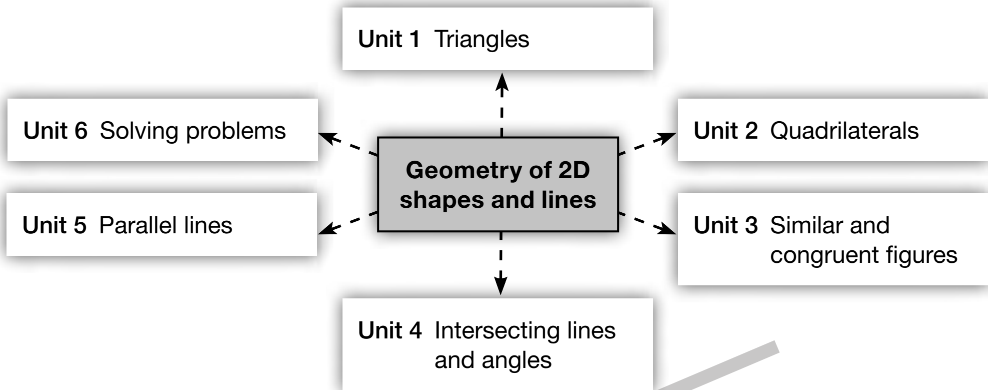
5.2 Size of interior angle $= \frac{3\ 240°}{20}$

$= 162°$ (2)

[31]

Chapter 7 Geometry of 2D shapes and lines

Overview of concepts



Chapter 6		Tyd toewysings	LB bladsy
Unit 1	Triangles	3 hours	195
Unit 2	Quadrilaterals	3 hours	200
Unit 3	Similar and congruent figures	3 hours	204
Unit 4	Intersecting lines and angles	3 hours	209
Unit 5	Parallel lines	3 hours	213
Unit 6	Solving problems	3 hours	220

Background information on geometry of 2D shapes and lines

Geometry is an important part of mathematics as it demonstrates to learners how to display logic and reasoning in a way that algebra and arithmetic does not. Learners learn how to formulate an argument or proof and then construct their answer in such a way as to validate their claims. This skill is not only important for further mathematics at FET, but also for other subjects and learning areas.

Geometry also provides an opportunity for more visual learners to excel. Spatial and visual acuity becomes key, and the concepts explored in geometry take learners beyond the arithmetic realm of mathematics.

The following should already be known to the learners from their Grade 8 work:

- the sum of the interior angles of a triangle;
- what an exterior angle is;
- the properties of isosceles, equilateral and right-angled triangles;
- the Theorem of Pythagoras; and
- the properties of quadrilaterals.

Generic teaching guidelines for teaching geometry of 2D shapes and lines

When introducing geometry in Grade 9, revise the above listed prior knowledge as learners may have forgotten them. It may also be necessary to recap the important facts that were investigated by construction in Chapter 6. Pay special attention to

- interior and exterior angles of triangles,
- interior angles of quadrilaterals,
- conditions for congruency, and
- conditions for similarity.

In general when teaching geometry, be sure to remind learners to always draw the diagram in their exercise books. This allows learners to work with the diagram and to fill in all the information they know about the figure before starting the example. Encourage learners to brainstorm their ideas and to discuss their thinking.

Pay special attention to how learners set their work out. Learners must learn how to set out geometry problems correctly, providing reasons and support for their arguments.

Resources

Learners will still require a protractor and a compass. Have cardboard and colour pens to make posters of the properties of the 2D shapes and lines, as well as vocabulary cards. Have polygon cut-outs for learners to investigate with. Each learner should have their own calculator.

Unit 1 Triangles

Learner's Book bladsy 195

Unit focus

This unit focusses on the following:

- revising the properties of triangles, and
- finding unknown angles in triangles.

Background information on triangles

The word "triangle" means "three angles". The study of triangles is fundamental to geometry as their properties have so many applications. It is important that learners understand the basic properties of triangles and can apply the Theorem of Pythagoras in Grade 9 because this forms the foundation in Grade 10 for the study of trigonometry.

Exercise 1

Learner's Book bladsy 196

Guidelines on how to implement this activity

Revise the following triangles and their properties with learners: equilateral, isosceles and right-angled triangles. Draw each triangle on the board and have learners come up to the board to fill in the properties. Discuss different angle sizes and how we can use the size of an angle to name a triangle, for example: an acute-angled triangle or an obtuse-angled triangle. Learners should complete this exercise on their own.

Suggested answers

1	equal; 60°	**2**	two; equal	**3**	90°
4	scalene; acute-angled	**5**	obtuse		

Remedial

If learners struggle to remember the properties of these triangles, ask the learners to create a chart that lists the properties of the triangles studied in this unit. Learners can then refer to this chart while they work through problems until they have memorised the properties.

Extension

Give learners various polygons and have them divide each one into triangles. Then have learners name each triangle formed.

Exercise 2

Learner's Book bladsy 197

Guidelines on how to implement this activity

Revise how to use the properties of the triangles to solve for missing angles. Do a few examples together as a class. Learners should manage this activity on their own.

Suggested answers

1 $a + 60° + 40° = 180°$
$a = 180° - 100$
$= 80°$

2 $b = 73° + 67°$
$= 142°$

3 $2c + 76° = 180°$
$2c = 180° - 76°$
$2c = 104°$
$c = 52°$

4 $d = 2(60°)$
$= 120°$

5 $e + 2(73°) = 180°$
$e = 180° - 146°$
$= 34°$

6 $f + 90° = 144°$
$f = 144° - 90°$
$= 54°$

Remedial

Allow learners to refer to the charts they made for the previous exercise on the properties of the triangles. This will help them to see how to solve for the missing angles.

CHAPTER 7

Exercise 3

Learner's Book bladsy 199

Guidelines on how to implement this activity

This section introduces learners to writing equations and providing reasons for solving triangles. This is an important concept that learners have to master. Learners must understand that each step in their reasoning process must have a reason. Learners also need to familiarise themselves with the correct symbols and notation for writing their reasons. Explore some examples of the correct notation and test learners on these before working through any of the examples in the Learner's Book.
Do a few examples using equations to solve triangles. Be sure to provide a reason for each step. Allow learners to do Questions 1 and 2 in pairs, but the remainder of the exercise should be done individually.

Suggested answers

1 $3x + 2x + x = 180°$ (∠'s in △)
$6x = 180°$
$x = \frac{180°}{6}$
$= 30°$

2 $x + x + 3x + 20° = 180°$ (Isosceles △, base angles =; ∠'s in △)
$5x + 20° = 180°$
$5x = 180° - 20°$
$5x = 160°$
$x = 32°$

3 $2x + 23° + 37° + x = 180°$ (∠'s in △)
$3x + 60° = 180°$
$3x = 180° - 60°$
$3x = 120°$
$x = 40°$

4 $2x - 20° + x + 5° + 90° - x = 180°$ (∠'s in △)
$2x + 75° = 180°$
$2x = 180° - 75°$
$= 105°$
$x = 52{,}5°$

5 $2x + 3x = 4x + 32°$ (Ext. ∠ = 2 opp. int. ∠s)
$5x = 4x + 32°$
$5x - 4x = 32°$
$x = 32°$

Remedial

Learners who have trouble with algebra and understanding equations will experience difficulty here. Provide some basic equation revision for these learners before they start the exercise.

Unit 2 Quadrilaterals

Learner's Book bladsy 200

Unit focus

This unit focusses on the following:
- revising the properties of quadrilaterals, and
- revising solving quadrilaterals with known properties

Background information on quadrilaterals

The properties of quadrilaterals have been investigated by construction in Chapter 6. Learners should be familiar with them and the natural next step is to apply this knowledge to finding unknown sides and angles in quadrilaterals. Diagonals are new in Grade 9 and may require additional focus over the other properties.

Exercise 1

Learner's Book bladsy 201

Guidelines on how to implement this activity

Ask learners what a quadrilateral is. Ask them to identify any properties that are common to all quadrilaterals. Revise the specific properties of each quadrilateral. Work with each quadrilateral individually. Draw the quadrilateral on the board, and have learners come up and fill in the properties. Have learners create a table in their exercise books, like the one in the Learner's Book, that summarises all the properties of each quadrilateral. Encourage learners to colour code the properties that are common to two or more quadrilaterals. Allow learners to refer to the chart as they complete this exercise on quadrilaterals.

Suggested answers

1.1	parallelogram	**1.2**	kite		
1.3	rectangle (or a square if the sides are also equal)				
1.4	square	**1.5**	rhombus		
1.6	rectangle	**1.7**	rhombus		
1.8	square	**1.9**	square		
2.1	true	**2.2**	true	**2.3**	true
2.4	false	**2.5**	true	**2.6**	true
2.7	false	**2.8**	false		

Remedial

Learners may find all the properties for the various quadrilaterals confusing, which will cause them difficulty when they need to solve problems. Have learners memorise the properties and test them on these.

Extension

Learners can create colourful posters on the different quadrilaterals and their properties to put up around the classroom.

Exercise 2

Learner's Book bladsy 202

Guidelines on how to implement this activity

The natural progression in learning the properties of quadrilateral is to use these properties to solve missing lines and angles in quadrilaterals. Remind learners how to develop equations and set out solving problems in geometry, as they did with solving triangles in Unit 1. Learners must use the correct symbols and notation and provide a reason for each step in their reasoning process. Do a few examples with learners, paying attention to setting out the solution properly and providing reasons.

Suggested answers

1.1 $B\hat{C}D = 2(B\hat{C}E)$
$= 2 \times 36°$
$= 72°$

1.2 $E\hat{B}C + B\hat{C}E + B\hat{E}C = 180°$
$E\hat{B}C + 36° + 90° = 180°$
$E\hat{B}C = 180°\ 36°\ 90°$
$= 54°$

2 $O\hat{P}Q = O\hat{Q}P = x$ (Isosceles △; diagonals of a rectangle bisect; diagonals =)
$\therefore 2x + 128° = 180°$ (∠'s of a △)
$\therefore 2x = 180° - 128°$
$2x = 52°$
$x = 26°$
$P\hat{O}Q + Q\hat{O}E = 180°$ (straight line)
$\therefore Q\hat{O}R = 180°\quad 128° = 52°$
$52° + 2y = 180°$ (∠'s of a △; diagonals of rectangle)
$2y = 180° - 52°$
$= 128°$
$y = 64°$

3.1 $\hat{C} = \hat{A}$ (opp. ∠'s parallelogram =)
$= 105°$

3.2 $\hat{A} + \hat{B} + \hat{C} + \hat{D} = 360°$ (sum int. ∠'s parallelogram =)
$2(105°) + \hat{B} + \hat{D} = 360°$
$\hat{B} + \hat{D} = 360° - 210° = 150°$
but $\hat{B} = \hat{D}$ (opp. ∠'s parallelogram =)
$= \frac{150°}{2}$
$= 75°$

4.1 Square. Diagonals are equal and bisect opposite angles.

4.2 $x = 45°$

Remedial

Learners experiencing problems with the exercise may be struggling with equations or the properties of quadrilaterals. Identify the problem area and provide simple examples that only require one property of the quadrilateral for the solution.

Unit 3 Similar and congruent figures

Learner's Book bladsy 204

Unit focus

This unit focusses on the following:
- revising congruent triangles,
- revising similar triangles, and
- solving similar and congruent triangles.

Background information on similar and congruent figures

Any geometric figure can be similar or congruent. Learners need to know the conditions for similarity and congruency. We study triangles in the most detail, so the focus should be on triangles.
The concept of similarity is linked to the concepts of enlargement and reduction, which are also studied in transformation geometry. Learners will need to be able to identify the scale factor of the enlargement or reduction, as well as its effect on the perimeter and area of the figure.

Exercise 1

Learner's Book bladsy 205

Guidelines on how to implement this activity

Revise the conditions for congruency and similarity in triangles. Discuss the conditions for congruency and show examples of each case on the board.
Explain the special case for triangles and similarity: if all the angles are equal, the shape is similar and the sides are automatically in proportion.
Revise how to label congruent and similar figures and the correct notation for congruency and similarity. Move on to how to use congruency to find missing sides of angles. Do a few examples on the board, and draw learners' attention to how the work is set out, the notation used and that you always provide a reason for each step.
Learners should do this activity on their own.

Suggested answers

1.1 Yes (SSS)

1.2 $A\hat{B}D = 180° \; 2(58°)$ (Isosceles $\triangle$)
$= 64°$
$D\hat{B}C = A\hat{B}D$ ($\triangle ABD \equiv \triangle CBD$)
$= 64°$

2.1 $\triangle PQR = \triangle UST$ (AAS)

2.2 PR

2.3 $\hat{P} = 180° \; 73° \; 41°$ ($\angle$'s in $\triangle$)
$= 66°$
but $\hat{U} = \hat{P}$ (corresponding $\angle$'s)
$= 66°$

Remedial

Learners may need additional revision for the conditions of congruency and accurately identifying them. Provide problems from Grade 8 where learners have to identify congruency.

Exercise 2

Learner's Book bladsy 207

Guidelines on how to implement this activity

Discuss how to use similarity to find missing sides and angles. Finding missing angles is easy because the angles are the same in similar triangles. However, solving missing sides can be more problematic. Learners need to remember that the sides of similar shapes are in proportion. This means working with ratios and fractions. Do a few examples together as a class, keeping the numbers as simple as possible so that learners can focus on the concept and not the complex calculation. Increase the complexity of the numbers until learners can cope with the prescribed exercise. Learners should do this exercise on their own.

Suggested answers

1 Yes, figures are similar (AAA).

$\frac{25}{10} = \frac{5}{2}$

$\frac{a}{7} = \frac{5}{2}$

$a = 7 \times \frac{5}{2}$

$= 17{,}5$ units

$\frac{12}{b} = \frac{5}{2}$

$\therefore 5b = 24$

$b = \frac{24}{5}$

$= 4{,}8$ units

or $b = 12 \div \frac{5}{2} = 4{,}8$ units

2 Yes, figures are similar (AAA).

$\triangle DGE \mathrel{|||} \triangle HFE$

$\frac{DG}{FH} = \frac{4}{16} = \frac{1}{4}$

$\therefore \frac{b}{3} = \frac{4}{1}$

$\therefore b = 3 \times 4 = 12$ units

$\frac{a}{14} = \frac{1}{4}$

$a = \frac{14}{4}$

$= 3{,}5$ units

Remedial

Learners with problems working with fractions may struggle. Be on hand to provide assistance and advise learners to revise working with fractions from earlier chapter.

Unit 4 Intersecting lines and angles

Learner's Book bladsy 209

Unit focus

This unit focusses on the following:

- revising the relationships between angles formed by intersecting lines,
- solving problems using these relationships, and
- practising using equations to find unknown angles.

Background information on perpendicular and intersecting lines

Whenever straight lines meet, angles are formed at a vertex. Revise the following concepts with your learners:

- If two intersecting lines form an angle of 90°, then the lines are perpendicular.
- When two straight lines cross, the vertically opposite angles are equal.
- Angles that add up to 180° are called supplementary angles. The angles on a straight line are supplementary.
- Angles that add up to 90° are called complementary angles.

Exercise 1

Learner's Book bladsy 210

Guidelines on how to implement this activity

Discuss intersecting lines and the angles they form. Revise the seven key points of intersecting lines as laid out in the Learner's Book. Ensure learners understand and can recognise and apply each of the points. Discuss how we can use these points to solve for missing angles.
Do a few examplesof using these points to solve problems as a class. Keep the numbers as simple as possible to encourage learners to understand the concept. Remind learners to always supply reasons for their answers.

Suggested answers

1 $a + 47° = 180°$ (adj. suppl. ∠'s)
$a = 180° - 47°$
$= 133°$

2 $3x = 180°$ (adj. suppl. ∠'s)
$x = 60°$

3 $q = 119°$ (vert. opp. ∠'s)
$p + q = 180°$ (adj. suppl. ∠'s)
$p + 190° = 180°$
$p = 180° - 119°$
$= 61°$

4 $2x + 84° = 180°$ (adj. suppl. ∠'s)
$2x = 180° - 84°$
$2x = 96°$
$x = 48°$

5 $y + 33° = 90°$ (adj. compl. ∠'s)
$y = 90° - 33°$
$= 57°$

6 $m + 58° + 2(90°) = 360°$
(adj. suppl. ∠'s; revolution)
$m = 306° - 180° + 58°$
$= 122°$

Remedial

Learners who struggle with working with equations will find this exercise challenging. Revise solving equations by using additive and multiplicative inverses with these learners. Remind them that what we do to the one side we must do to the other side too.

Extension

Learners can create additional problems for one another to solve. Alternatively, stronger learners can work with groups of struggling learners to help them master working with intersecting lines.

Exercise 2

Learner's Book bladsy 212

Guidelines on how to implement this activity

This exercise requires learners to develop equations, sometimes of quite a complex nature. Learners may require additional support determining the initial equation. Do as many examples as necessary until learners are able to develop their own equations. Remind learners to always supply a reason for their answers. Learners should do this activity on their own.

Suggested answers

1 $3x - 24° + 2x + x = 180°$ (straight line)
$6x - 24° = 180°$
$6x = 180° + 24°$
$6x = 204°$
$x = 35°$

2 $180° - 95° = 85°$ ($\angle$'s on a str. line)
$180° = 85° + (2x - 20°) + (x + 10°)$ ($\angle$'s on a str. line)
$180° - 85° + 20° - 10° = 2x + x$
$3x = 105°$
$x = 35°$

3 $x + 50° + 90° + x + 40° = 360°$ (revolution)
$2x + 180° = 360°$
$2x = 360° - 180°$
$2x = 180°$
$x = 90°$

4 $2x = 138°$ (vert. opp. $\angle$'s)
$x = 69°$

Remedial

Provide additional help and examples for learners struggling to develop the number sentences. Learners who have difficulty with algebra may also find this exercise problematic. Revise gathering like terms and solving equations with additive and multiplicative inverses.

Unit 5 Parallel lines

Learner's Book bladsy 213

Unit focus

This unit focusses on the following:
- revising rules around parallel and transversal lines,
- relooking at the angles created by these lines,
- solving unknown angles using parallel lines and state your reasons, and
- learning how to confirm whether lines are parallel.

Background information on parallel lines

Parallel lines never meet and the perpendicular distance between them is constant. Revise the following concepts with learners. When parallel lines are cut by a transversal:

- alternate angles are equal,
- corresponding angles are equal, and
- co-interior angles are supplementary.

Also revise the converse:

- If alternate angles are equal, the lines are parallel.
- If corresponding angles are equal, the lines are parallel.
- If co-interior angles are supplementary, the lines are parallel.

Exercise 1

Learner's Book bladsy 214

Guidelines on how to implement this activity

Ask learners to define parallel lines and to provide examples of parallel lines. Discuss what a transversal is. Ask learners what difference there is to a transversal cutting any lines as opposed to cutting parallel lines.
Draw the three special forms of angles formed by a transversal cutting parallel lines: corresponding, alternate and co-interior. Work through each of these special forms of angles and discuss the angles formed. Learners need to be able to identify the angles from the shapes formed. Learners should manage this exercise on their own.

Suggested answers

1 *a* and *b*, *p* and *c*, *d* and *n*, *I* and *f*, *j* and *k*
2 *p* and *n*, *h* and *f*, *e* and *g*, *j* and *m*
3 *p* and *b* *e* and *f*, *h* and *g*, *j* and *l*

Exercise 2

Learner's Book bladsy 216

Guidelines on how to implement this activity

Demonstrate to learners how to use parallel lines to solve for missing angles. Remind learners that when working with geometry we have to set our work out correctly and provide reasons in the correct notation. Revise the notation with learners. Do a few examples of finding the missing angles on the board as a class. Allow learners to provide as much of the reasoning and calculations as possible. Learners can do Questions 1 and 2 in pairs, but should do the remainder of the exercise on their own.

Suggested answers

1	$a = 81°$	(corr. $\angle$'s $\parallel$ lines)	**2**	$d = 131°$	(alt. $\angle$'s $\parallel$ lines)
	$b = 81°$	(corr. $\angle$'s $\parallel$ lines)			

3	$b = 110°$	(corr. ∠'s ‖ lines)	**4**	$a = 71°$	(corr. ∠'s ‖ lines)
	$a + b = 180°$	(straight line)		$b + 71° = 180°$	(co-int. ∠'s ‖ lines)
	$a = 180° - 110°$			$b = 180° - 71°$	
	$= 70°$			$= 109°$	
5.1	$b = 60°$	(corr. ∠'s ‖ lines)	**5.2**	△PQR ||| △PST	(AAA)
	$c = 40°$	(corr. ∠'s ‖ lines)			
	$a + b + c = 180°$				
	$a + 40° + 60° = 180°$				
	$a = 180° - 100°$				
	$= 80°$				

Remedial

Learners need to integrate some of their knowledge of the previous units into this exercise, especially Question 5. Ensure learners are able to work with similar triangles before starting this exercise.

Exercise 3

Learner's Book bladsy 217

Guidelines on how to implement this activity

Until now, learners have used the concept of parallel lines to find angles. In this section, learners use the reverse concept to prove that certain lines may be parallel based on the angles that the lines form with a transversal. Discuss with learners the reverse understanding of the concept.
Do a few simple examples where the lines need to be proven as parallel. Extend this working to include working with figures and not simply lines. Learners need to use their knowledge of triangles and quadrilateral to be able to answer all the examples in this exercise. Do an example involving parallel lines and quadrilaterals to better prepare learners for the exercise.

Suggested answers

1	$C\hat{E}F = 180° - 45° = 135°$	(straight line)
	∴ AB‖CD	(corr. ∠'s =)
2	AB‖CD	(corr. ∠ =)
3.1	AD‖BD	(alt. ∠'s =)
3.2	Trapezium: one pair of opposite sides are equal.	
4.1	PQ‖SR	(alt. ∠'s =)
	QR‖PS	(alt. ∠'s =)
4.2	Parallelogram: opposite sides are parallel.	

Remedial

Learners may require additional revision of quadrilaterals in order to manage this exercise. Allow learners to take out their charts on the properties of quadrilaterals and to keep this available as they complete this exercise.

Exercise 4

Learner's Book bladsy 219

Guidelines on how to implement this activity

As the geometry becomes more complex, learners need to be able to integrate all the knowledge they have learnt in this chapter in order to manage solve problems. Learners need to be able to:

- create equations to help them solve for missing lines and angles;
- recognise and apply the properties of lines, angles, triangles and quadrilaterals;
- set their work out correctly;
- always provide a reason; and
- write their reasons in the correct notation.

Suggested answers

1 $2x + 15° = 90° - x$ (alt. $\angle$'s || lines)
$3x = 75°$
$x = 25°$

2 $3x - 50° = 2x$ (corr. $\angle$s || lines)
$3x - 2x = 50°$
$x = 50°$

3 $3x + 51° + 102° - x = 180°$
(co-int. $\angle$'s || lines)
$2x + 153° = 180°$
$2x = 180° - 153°$
$2x = 27°$
$x = 13{,}5°$

4 $x + 17° + 2x - 5° = 180°$
(corr. $\angle$s || lines)
$3x + 12° = 180°$
$3x = 168°$
$x = 56°$

Remedial

Learners may struggle to integrate their knowledge from previous units. Show learners how to approach a problem in a systematic and step by step way. Help learners to break down the problems into smaller questions. Learners should only focus on one question at a time and answer only what is being asked.

Extension

Provide complex problems for your stronger learners. Choose problems that involve finding more than one variable or require using congruency together with properties of parallel lines to solve.

Unit 6 Solving problems

Learner's Book bladsy 220

Unit focus

This unit focusses on the following:

- applying your knowledge of the geometry of 2D shapes and lines to solve problems.

Background information on solving problems

Learners need to be able to apply all they have learnt in thischapter to solving problems. Learners must have a clear understanding of all the theory and an ability to interpret the question in order to solve various problems.

Exercise 1

Learner's Book bladsy 221

Guidelines on how to implement this activity

Spend time revising the properties of triangles, as well as similarity and congruency. Discuss with learners the general approach to problem solving:

1 Identify what you are being asked to find.
2 Identify the information you are given.
3 Create an equation to solve the problem, with reasons.
4 Substitute values into the equation.
5 Solve the equation and judge the reasonableness of your answer.

Do an example together as a class. Encourage learners to attempt the problem on their own, but allow learners to work in pairs if necessary. Then work through the solution on the board as a class. Ask different learners to come up and write each step of the solution.

Suggested answers

1
$x + 6°$
Let LC = x
$x + 6° + x + 90° = 180°$
$2x + 96° = 180°$
$2x = 180° - 96°$
$= 84°$
$x = 42°$
LA = $x + 6° = 42° + 6° = 48°$
LC = $x = 42°$

2
LP : LQ: LR = 1 : 2 : 3
LP = $\frac{1}{6} \times 180° = 30°$
LQ = $\frac{2}{6} \times 180° = 60°$
LR = $\frac{3}{6} \times 180° = 90°$

3.1
$\frac{1}{2}x + \frac{1}{2}x + x = 180°$ (∠'s of △)
$2x = 180°$
$x = 90°$
3 ∠s are: 90°; 45° and 45°

3.2 Isosceles, right

Remedial

If learners experience difficulty in problem solving, assist learners in using the steps highlighted in the guidelines for teaching this exercise above. Try not to simply provide learners with the equation, but guide them towards the correct variables. Learners should perform the calculations themselves, but may require reminders on how to work with equations, especially those containing fractions.

Exercise 2

Learner's Book bladsy 222

Guidelines on how to implement this activity

Learners integrate their knowledge of solving sides and angles with congruent and similar triangles. Revise the necessary basics and do a few examples together as a class. Learners should manage this exercise on their own.

Suggested answers

1.1 $A\hat{D}E$
alt $\angle$'s, || lines (AAA)

1.2 $\triangle ABC|||\triangle DEA$

$\therefore \frac{BC}{EA} = \frac{2}{6} = \frac{1}{3}$

$\frac{DE}{AB} = \frac{X}{1,6} = \frac{1}{3}$

$\therefore 3x = 1,6$

$x = 3 \times 1,6$

$= 4,8$ units

1.3 $\frac{AC}{DA} = \frac{1}{3}$

$\frac{AC}{5} = \frac{1}{3}$

$3AC = 5$

$AC = \frac{5}{3}$

$= 1,6$

$= 1,67$ units

2.1 $D\hat{B}C$
$A\hat{B}D$
Common $\triangle ABD \equiv \triangle CDB$

2.2 $AD = CB$
$AB = CD$
$\hat{E} = \hat{C}$

2.3 ll^m Opposite sides and opposite $\angle =$

Remedial

Learners may require revision of the cases of congruency and similarity. Have learners make a chart consisting of each case and a diagram. Learners can refer to their chart as they work through the exercise.

Exercise 3

Learner's Book bladsy 223

Guidelines on how to implement this activity

Learners need to integrate their knowledge of lines to solve problems. Revise solving problems with lines by doing a few examples together as class. Learners should manage this exercise on their own.

Suggested answers

1 $3x - 31° = 2x + 22°$ (vert. opp. $\angle$'s =)

$3x - 2x = 22° + 31°$

$x = 53°$

one pair of opp $\angle$'s $= 3x - 31°$

$= 3(53°) - 31°$

$= 128°$ each

Other pair $= 180° - 128° = 52°$ each

2 $a = 32°$ (vert. opp. $\angle$'s =)

$b = 58°$ (vert. opp. $\angle$'s =)

3	$\hat{BEF} = 2x + 15°$	(vert. opp. ∠'s)
	$2x + 15°x + 21° = 180°$	(co-int ∠'s = 180°)
	$3x = 180° - 15° - 21°$	
	$3x = 144°$	
	$x = 48°$	
	$\hat{EFD} = x + 21° = 48° + 21° = 69°$	
4	$x + 180 - x = 180°$	
	$\therefore$ Yes PQ∥TR	(co-int. ∠'s suppl.)

Extension

Provide exam-type questions that integrate all the concepts covered in this unit. Learners need to be able to solve various problems when they occur in no particular order (so that it is not clear from which unit the questions come from).

Consolidation

Learner's book bladsy 225

Before doing this consolidation exercise, encourage learners to review the work covered in this chapter. Advise learners to use the summary and to revise their work. This exercise can be used as an informal assessment task for you to track how learners are coping with the chapter and the concepts covered. The mark allocation provides guidelines on how to assess learners.

Suggested answers

1.1	True		
1.2	True, only if diagonals are equal.		
	If diagonals are not equal → rhombus		
1.3	True		
1.4	True		
1.5	True, all the sides are not equal.		
	If sides are equal as well → square		(5)
2.1	$a = 27°$	(alt ∠'s ∥ lines)	
	$b = a$		
	$= 27°$	(alt ∠'s ∥ lines)	(2)
2.2	$c = 62°$	(corresp ∠'s ∥ lines)	
	$a + 111° = 180°$	(co-int ∠'s ∥ lines)	
	$= 69°$		
	$a + b + 62° = 180°$	(∠'s of △)	
	$69° + b + 62° = 180°$		
	$b = 180° - 69° - 62°$		
	$= 49°$		(2)

3.1 $5x - 50° = 90° - 5x$

$5x + 5x = 90° + 50°$ (opp $\angle$ of a parallelogram)

$10x = 140°$

$x = 14°$ (3)

3.2 $x + x + 17° = 67°$ (ext. $\angle$ = 2 opp in $\angle$'s)

$2x = 67° - 17°$

$2x = 50°$

$x = \frac{50°}{2}$

$= 25°$ (3)

3.3 $5x + 44° + 3x = 180°$ (co-int. $\angle$'s $\parallel$ lines)

$8x = 180° - 44°$

$= 136°$

$x = 17°$ (3)

3.4 $5x = 3(180°)$

$5x = 540°$

$x = 108°$ (3)

4.1 $h^2 = 3^2 + 3^2$ (Isosceles $\triangle$, Pythag.)

$= 18$

$h = \sqrt{18}$

$= 4,2$ cm (3)

4.2 $x^2 + 12^2 = 13^2$ (Pythag.)

$x^2 + 144 = 169$

$x^2 = 169 - 144$

$= 25$

$x = \sqrt{25}$

$= 5$ cm

$\frac{12 + y}{12} = \frac{26}{13} = \frac{2}{1}$ ($|||$ $\triangle$s)

$12 + y = 2 \times 12$

$y = 24 - 12$

$= 12$ cm (5)

5 $LW^2 = 8^2 + 7,5^2$

$= 120,25$

$LW = \sqrt{120,25}$

$= 10,965\ldots$ m

His ladder is 10 m < 10,965 m so she will land in the bushes. (4)

[35]

CHAPTER 7

Chapter 8 Area and perimeter

Overview of concepts

Area and perimeter

- **Unit 1** Theorem of Pythagoras
- **Unit 2** Polygons
- **Unit 3** Circles
- **Unit 4** Doubling of dimensions of 2D shapes

Content		Time allocations	LB page
Unit 1	The Theorem of Pythagoras	5 hours	228
Unit 2	Polygons	3 hours	235
Unit 3	Circles	1 hour	241
Unit 4	Doubling of dimensions of 2D shapes	1 hour	246

Background information onarea and perimeter

Learners have worked with area and perimeter in earlier grades. This chapter focusses on:

- using the Theorem of Pythagoras to find sides needed to calculate area and perimeter,
- using properties of triangles and quadrilaterals to find missing sides and angles, and
- working with pi and circles to calculate area and perimeter.

The Great Pyramid of Giza in Egypt has some amazing properties linked to area and perimeter. When using the Egyptian unit of measurement, the cubit, the perimeter of the Great Pyramid is 365,24 cubits, which is equal to the number of days in one year. When you double the perimeter, you get a number that represents one minute of one degree at the equator. The height of the pyramid $\times 10^9$ gives the approximate distance from the earth to the sun. The perimeter divided by 2 and then multiplied by the height of the pyramid is equal to pi (3,1416). For more amazing Egyptian applications of area and perimeter see the following website: www.touregypt.net.

Generic teaching guidelines for teachingarea and perimeter

- When demonstrating Worked examples to the class, it is important to draw neat drawings of plane figures, including given dimensions, on the board.
- Learners should be encouraged to participate in class demonstrations by answering questions such as "Which side of this triangle should be taken as the base and what is the related height?"
- Exercises must be at least partially completed in the class room under supervision of the teacher. The teacher should monitor the learners' progress and provide help and remediation where required.
- Learners' written solutions to problems must be set out correctly in the style demonstrated in the Worked examples. It is important to assist learners in developing the appropriate way of writing answers to calculations and solutions to problems.

Resources

Colour paper, cardboard and colour pens to create posters and polygonal shapes. Grid paper and pencils. Each learner should have their own calculator. Conversion charts for the SI units and their conversions.

Unit 1 The Theorem of Pythagoras

Learner's Book page 228

Unit focus

This unit focusses on the following:
- revising the Theorem of Pythagoras,
- proving that a given triangle is right-angled,
- using the theorem to calculate the length of unknown sides of right-angled triangles, and
- applying Pythagoras' Theorem to solve practical problems involving right-angled triangles.

Background information on triangles

In previous grades, learners got to know the geometric properties of triangles and quadrilaterals. Learners were introduced to the formulas and units for calculating the perimeters and areas of squares, rectangles and triangles.

In Grade 8, learners were introduced to the Theorem of Pythagoras for the first time. They worked with proving that a given triangle is right-angled and how to calculate the length of an unknown side of a right-angled triangle. Learners may require additional revision of the Theorem of Pythagoras.

Exercise 1

Learner's Book page 230

Guidelines on how to implement this activity

Explain the Theorem of Pythagoras to learners. Spend some time revising the theorem and doing examples together as a class. Ensure you cover:

- how to find the length of the missing sides of a triangle,
- how to prove whether a triangle is right angled or not.

Revise Pythagorean triples and have learners memorise at least three of these to help them when working with right-angled triangles. Learners should do this exercise on their own.

Suggested answers

1.1 AC is the longest side, so check whether $AC^2 = AB^2 + BC^2$

$AC^2 = (9\text{ m})^2 = 81\text{ m}^2$

$AB^2 + BC^2 = (5\text{ m})^2 + (7\text{ m})^2$

$= (25 + 49)\text{ m}^2$

$= 74\text{ m}^2$

So $AC^2 \neq AB^2 + BC^2$, this $\triangle ABC$ is not right-angled.

1.2 $DF = \sqrt[2]{61}\text{ mm} = 15{,}62\text{ mm}$

$\therefore$ DF is the longest side

so check whether $DF^2 = FE^2 + DE^2$

$DF^2 = \left(\sqrt[2]{61}\text{ mm}\right)^2 = 244\text{ mm}^2$

$DE^2 + FE^2 = (10)^2 + (12\text{ mm})^2$

$= (100 + 144)\text{ mm}^2$

$= 244\text{ mm}^2$

So $DF^2 = DE^2 + FE^2$

This $\triangle DEF$ is right-angled with DE its hypotenuse and $D\hat{F}E$ its right angle.

1.3 LK is the longest side, so check where $KL^2 = KM^2 + ML^2$

$KL^2 = (50\text{ cm})^2 = 2\,500\text{ cm}^2$

$KL^2 + ML^2 = (14\text{ cm})^2$

$= (196 + 2\,304)\text{ cm}^2$

$= 2\,500\text{ cm}^2$

So $KL^2 = KM^2 + ML^2$, thus $\triangle KLM$ is right-angled with KL its hypotenuse and $\hat{M}$ its right angle.

2.1 $6^2 = 3^2 + x^2$

$x^2 = 6^2 - 3^2$

$= 36 - 9$

$= 27$

$\therefore x = \sqrt{27}\text{ mm}$

2.2 $y^2 = 15^2 + 12^2$

$= 225 + 144$

$= 369$

$\therefore y = \sqrt{369}\text{ m}$

2.3 $\left(\sqrt{20}\right)^2 = 2^2 + z^2$

$z^2 = \left(\sqrt{20}\right)^2 - 2^2$

$= 20 - 4$

$= 16$

$\therefore z = 4$

Remedial

Learners may struggle with applying the Theorem of Pythagoras and will require additional material to practise with. Provide Grade 8 material on Pythagoras for learners to take home and work through.

Extension

Ask learners to draw their own triangles. They must then use the Theorem of Pythagoras to determine if their triangle is right-angled or not, without measuring the angle.

Exercise 2

Learner's Book page 231

Guidelines on how to implement this activity

Learners need to be able to recognise right-angled triangles when they form part of other shapes and not just when they are presented on their own. Show learners examples of complex shapes made up of different shapes. Ask learners to come up and outline possible right angles. Learners then need to use their knowledge of the Theorem of Pythagoras to find sides and angles within these complex shapes. Do a few examples together with the learners. Learners can work in pairs to complete this exercise.

Suggested answers

1 In $\triangle$PQR: $QP^2 = QR^2 + PR^2$

$(9 \text{ cm})^2 = (6 \text{ cm})^2 + h^2$

$h^2 = (9 \text{ cm})^2 - (6 \text{ cm})^2$

$= (81 - 36) \text{ cm}^2$

$= 45 \text{ cm}^2$

In $\triangle$PRS: $PS^2 = PR^2 + RS^2$

$(15 \text{ cm})^2 = h^2 + x^2$

$x^2 = (15 \text{ cm})^2 - (\sqrt{45} \text{ cm})^2$

$= (225 - 45) \text{ cm}^2$

$= 180 \text{ cm}^2$

Thus $x = \sqrt{180}$ cm

$= 13{,}4$ cm correct to one decimal place

2.1 In $\triangle$BCD: $BD^2 = BC^2 + CD^2$
$(10\text{ m}) = x^2 + (6{,}8\text{ m})^2$
$x^2 = (10\text{ m})^2 - (6{,}8\text{ m})^2$
$= (100 - 46{,}24)\text{ m}^2$
$= 53{,}76\text{ m}^2$
Thus $x = \sqrt{53{,}76}$ m
BC = 7,33 m correct to two decimal places

2.2 In $\triangle$ACD: $AD^2 = AC^2 + CD^2$
$(12\text{ m})^2 = AC^2 + (6{,}8\text{ m})^2$
$AC^2 = (12\text{ m})^2 - (6{,}8\text{ m})^2$
$= (144 - 46{,}24)\text{ m}^2$
$= 97{,}76\text{ m}^2$
$AC = \sqrt{97{,}76}$ m
$= 9{,}89$ m correct to two decimal places
But AC = AB + BC
$= y + x$
and $x = 7{,}33$ m from 2.1
so $9{,}89\text{ m} = y + 7{,}33$ m
AB = 2,56 m correct to two decimal places

3.1 In $\triangle$ABC: $AB^2 = BC^2 + AC^2$
$(20\text{ m})^2 = (12\text{ m})^2 + x^2$
$x^2 = (20\text{ m})^2 - (12\text{ m})^2$
$= (400 - 144)\text{ m}^2$
$= 256\text{ m}^2$
$x = \sqrt{256}$ m
$= 16$ m

3.2 BD = BC + CD
= 912 + 16 m
= 28 m

3.3 ED = AB (opposite sides of parm. are equal)
= 20 m
and AE = BD (opposite sides of parm. are equal)
= 28 m
thus perimeter of
ABDE = 2(20 + 28) m = 96 m

3.4 Area of parallelogram ABDE
= base × height
= BD × x
$= (28 \times 16)\text{ m}^2$
$= 448\text{ m}^2$

4.1 In $\triangle$ABC: $AC^2 = AB^2 + BC^2$
$= (24\text{ cm})^2 + (10\text{ cm})^2$
$= (576 + 100)\text{ cm}^2$
$= 676\text{ cm}^2$
$AC = \sqrt{676}$ cm
$= 26$ cm

4.2 In $\triangle$ACD: $AD^2 = AC^2 + CD^2$
$= (26\text{ cm})^2 + (8{,}5\text{ cm})^2$
$= (676 + 72{,}25)\text{ cm}^2$
$= 748{,}25\text{ cm}^2$
$AD = \sqrt{748{,}25}$ cm
$= 27{,}35$ cm correct to two decimal places

Remedial

If learners are struggling to identify the right angle, tell them to use colour markers to outline the triangle shapes. The colour coding helps learners focus on the task at hand.
Learners may also have trouble with the square roots in this exercise. Revise working with square roots and leaving answers in square root form.

Exercise 3

Learner's Book page 233

Guidelines on how to implement this activity

This exercise shows the Theorem of Pythagoras used in real life situations. Learners need to learn how to draw a rough sketch and fill in the right-angled triangle they are working with. Work through the steps of problem solving, namely:

1 Read the question carefully. Draw a sketch to represent the problem.
2 Identify what they are being asked to find.
3 Identify what information they have been given.
4 Develop a number sentence to find what is being asked.
5 Substitute in the correct numbers into the number sentence.
6 Solve the number sentence.
7 Check the reasonableness of the solution.

Do a few examples with learners together as a class. Learners can work in pairs, but each learner must record all their own working.

Suggested answers

1.1 Let h be the height of the gate.

$(2{,}5\text{ m})^2 = h^2 + (1{,}5\text{ m})^2$
$h^2 = (2{,}5\text{ m})^2 - (1{,}5\text{ m})^2$
$= (6{,}25 - 2{,}25)\text{ m}^2$
$= 4\text{ m}^2$
$h = \sqrt{4}\text{ m}$
$= 2\text{ m}$

1.2 Let w be the width of the roof. Since $\triangle$ABC is an equilateral triangle M is the midpoint of BC.

In $\triangle$ABM: $AB^2 = BM^2 + AM^2$
$(4\text{ m})^2 = BM^2 + (1{,}5\text{ m})^2$
$BM^2 = (4\text{ m})^2 - (1{,}5\text{ m})^2$
$= (16 - 2{,}25)\text{ m}^2$
$= 13{,}75\text{ m}^2$
$BM = \sqrt{13{,}75}\text{ m}$
$= 3{,}71$ m correct to two decimal places
thus the width of the roof $= 2 \times BM$
so $w = 2 \times 3{,}71$ m
$= 7{,}42$ m

2 Let AB be the height from ground to window, AC be the distance from the base of the house to the foot of the ladder, and BC the necessary length of the ladder:

In triangle ABC: $BC = \sqrt{AB^2 + AC^2}$
$= \sqrt{(4{,}5)^2 + (2{,}5)^2}$
$= \sqrt{20{,}25 + 6{,}25}$
$= \sqrt{26{,}5}$
$= 5{,}14$m

The distance along AC is 5,14 m which means that a 6 m ladder will be long enough.

3 ED = AB (opp. sides of a rectangle are equal)

$= 4$ m

In $\triangle$CED: $CD^2 = CE^2 + ED^2$

$= (9 \text{ m})^2 + (4 \text{ m})^2$

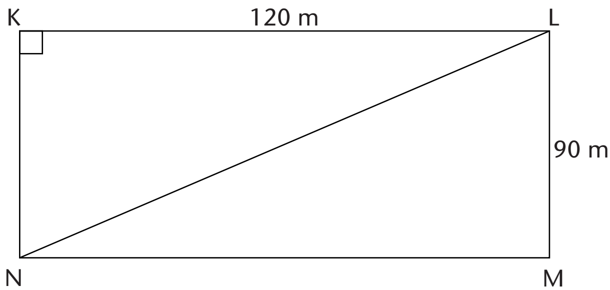
$= (81 + 16) \text{ m}^2$

$= 97 \text{ m}^2$

$CD = \sqrt{97}$ m

$= 9{,}85$ m correct to two decimal places.

4 Let rectangle KLMN represent the soccer field.



In rectangle KLMN:

KN = LM (opp. sides of rectangle are equal)

in $\triangle$KNL: $NL^2 = NK^2 + LK^2$

$= (90 \text{ m})^2 + (120 \text{ m})^2$

$= (8\,100 + 14\,400) \text{ m}^2$

$= 22\,500 \text{ m}^2$

$NL = \sqrt{22\,500}$ m

$\therefore$ diagonal of soccer field is 150 m

5.1 Since $\triangle$ABC is isosceles, M is the midpoint of BC

so BM = 8 m ÷ 2 = 4 m

In $\triangle$ABM: $AB^2 = BM^2 + AM^2$

$(5{,}3 \text{ m})^2 = (4 \text{ m})^2 - h^2$

$h^2 = (5{,}3 \text{ m})^2 - (4 \text{ m})^2$

$= (28{,}09 - 19) \text{ m}^2$

$= 12{,}09 \text{ m}^2$

$h = \sqrt{12{,}09}$ m

$= 3{,}48$ correct to two decimal places

5.2 Area of 2 triangular gables $= 2 \times \left(\frac{1}{2}\text{base} \times \text{height}\right)$

$= 2 \times \frac{1}{2} \times 8 \times 3{,}48 \text{ m}^2$

$= 27{,}84 \text{ m}^2$

Area of 2 rectangular pieces of roof $= 2 \times l \times b$

$= 2 \times 12 \text{ m} \times 5{,}3 \text{ m}$

$= 127{,}2 \text{ m}^2$

Thus Area of whole roof $= (27{,}84 + 127{,}20) \text{ m}^2$

$=155{,}04$ correct to two decimal places

Remedial

Provide additional support for learners as they construct the equations for their problems.
Remind learners to include reasons for their answers and to work systematically.
Provide additional real-life examples for learners to practise with.

Extension

Encourage learners to develop their own real-life problems from their immediate environment. For example, "What is the length of a diagonal of the classroom?" Learners can measure the height and width of a wall and then by Pythagoras determine the length of the diagonal.

Unit 2 Polygons

Learner's Book page 235

Unit focus

This unit focusses on the following:

- revising what was learnt in Grades 7 and 8 about the area and perimeter of squares, rectangles and triangles and the perimeter of certain polygons; and
- learning to calculate the areas of parallelograms, rhombi, trapeziums and kites.

Background information on area and perimeter of polygons

Learners should know how to calculate the perimeter of regular and irregular polygons. They should also be able to calculate the perimeter and area of squares, rectangles and triangles using the appropriate formulae and correct to at least one decimal place. Complex polygons should be broken up into rectangles and triangles to make the calculation more manageable. Learners should know and use the appropriate SI units. Learners also need to be able to convert fluently between the different SI units: $mm^2 \leftrightarrow cm^2 \leftrightarrow m^2 \leftrightarrow km^2$.

Exercise 1

Learner's Book page 238

Guidelines on how to implement this activity

Revise the formulae for calculating the area and perimeter of triangles, rectangles and squares. Ensure learners know these formulae and can use them appropriately. Do a few examples using the formulae to find the area and perimeter of these shapes. Focus on triangles and the base and perpendicular height. Ensure learners can identify the base and height of triangles. Revise units of measurement and area. Work through the examples in the Learner's Book and ensure learners can easily convert between the units. Work through an example of complex shape, such as the pentagon in the Learner's Book. Show learners how the shape can be broken down to make the calculation possible. Learners should attempt this exercise on their own.

Suggested answers

1.1 0,123 m = 123 mm

1.2 17,58 cm = 175,8 mm

1.3 5 250 m = 5,250 km

1.4 6 125 mm = 6,125 m

1.5 1 m^2 = 10 000 cm^2

1.6 1 cm^2 = 100 mm^2

1.7 45 500 cm^2 = 4,55 m^2

1.8 7 400 mm^2 = 74 cm^2 = 0,0074 m^2

2.1 Perimeter = 12×3 m
$= 36$ m
Area = $5 \times (3 \times 3)$ m^2
$= 45$ m^2

2.2 Perimeter = (13 + 13 + 5 + 12)cm
= 43 cm
In $\triangle ABC$: $AC^2 = AB^2 + BC^2$
$= (12 \text{ cm})^2 + (5 \text{ cm})^2$
$= (144 + 25) \text{ cm}^2$
$= 169 \text{ cm}^2$
$AC = \sqrt{169}$ cm
$= 13$ cm
Therefore $\triangle ADC$ is equilateral and M is the midpoint of AC.
$\therefore$ AM = 13 cm ÷ 2 = 6,5 cm
In $\triangle ADM$: $AD^2 = AM^2 + DM^2$
$(13 \text{ cm})^2 = (6{,}5 \text{ cm})^2 + DM^2$
$DM^2 = (13 \text{ cm})^2 - (6{,}5 \text{ cm})^2$
$= (169 - 42{,}25) \text{ cm}^2$
$= 126{,}75 \text{ cm}^2$
$DM = h = \sqrt{126{,}75}$ cm
Area of ABCD = area of $\triangle ABC$ + area of $\triangle ADM$ + area of $\triangle DMC$
$= \frac{1}{2}(5)(12) + \frac{1}{2}(6{,}5)(\sqrt{126{,}75}) + \frac{1}{2}(6{,}5)(\sqrt{126{,}75})$
$= 30 + 36{,}589 + 36{,}589$
$= 103{,}18 \text{ cm}^2$

2.3 Perimeter = $12 \times 4 = 48$ mm
EF = EH = HG = FG = 12 mm (Given)
EFGH is a Rhombus (All sides are equal)
$\therefore$ FH bisects EG at 90°
$\therefore$ FO = OH = 10 mm
In $\triangle OHG$:
$HG^2 - HO^2 = OG^2$
$12^2 - 10^2 = OG^2$
$144 - 100 = OG^2$
$44 = OG^2$
$\sqrt{44} = OG$
$\therefore$ Area $\triangle EGH = \frac{1}{2} \times EG \times OH$
$= OG \times OH$ ($\frac{1}{2}$EG = OGas EG bisects FH prop. Of rhombus)
$= \sqrt{44} \times 10$
$= 66{,}33 \text{ mm}^2$
$\triangle EGH \equiv \triangle EFG$(SSS; EF = EH, FG = GH and side ED is shared)
$\therefore$ Area EGH = Area EFG = 66,33 m^2
$\therefore$ Area EFGH = Area EGH + Area EFG = $66{,}33 \times 2 = 132{,}66$ mm^2

Remedial

Learners experiencing difficulty with this exercise may not remember the basic formulae for calculating area and perimeter. Provide additional practice in the form of Grade 8 material for learners to work on. Keep examples simple so learners can consolidate the basic formulae.

Extension

Encourage learners to find the area and perimeter of advanced shapes such as octagons and dodecahedrons.

Exercise 2

Learner's Book page 239

Guidelines on how to implement this activity

Learners extend their knowledge of area and perimeter of squares and rectangles to include the formulae for other quadrilaterals. Learners learn proofs for these formulae. Learners do not need to memorise these proofs, but they are important for learners to understand why the given formulae work. Do examples on the board using the formulae for finding areas of parallelograms, rhombi and trapeziums. Ensure learners know which values to substitute into the formulae. Learners should do this exercise on their own.

Suggested answers

1 Area of parallelogram PQRS = base × height

$= bh$

$= 13\text{ cm} \times 15\text{ cm}$

$= 195\text{ cm}^2$

2 LM = KN = 28 cm (opp. sides of parm. are equal)

in $\triangle KPL$: $KL^2 = KP^2 + PL^2$

$(26\text{ cm})^2 = h^2 + (10\text{ cm})^2$

$= (676 - 100)\text{ cm}^2$

$= 576\text{ cm}^2$

$h = \sqrt{576}\text{ cm} = 24\text{ cm}$

therefore the area of parallelogram KLMN = base × height

$= bh$

$= 28\text{ cm} \times 24\text{ cm}$

$= 672\text{ cm}^2$

3 Area of trapezium PQRS $= \frac{1}{2}(\text{sum of parallel sides}) \times h$

$= \frac{1}{2}(PS + QR) \times h$

$= \frac{1}{2}(17 + 9)\text{mm} \times 6\text{ mm}$

$= 78\text{ mm}^2$

CHAPTER 8

4 Area of trapezium ABCD $= \frac{1}{2}$(sum of parallel sides) $\times h$

$= \frac{1}{2}$ (AB + DC) $\times h$

$= \frac{1}{2}$ (AB + DC) $\times h$

$= \frac{1}{2}$(8,6 + 11,2) km $\times$ 3,5 km

$=$ 34,65 km^2

5 Area of trapezium EFGH $= \frac{1}{2}$ (sum of parallel sides) $\times h$

$= \frac{1}{2}$ (HE + GF) $\times$ HG

$= \frac{1}{2}$(11 + 9)m $\times$ HG

$=$ 56 m^2

6 Area of rhombus TUVW = side length $\times$ height

$= 5h$

$=$ 12 mm $\times$ 10 mm

$=$ 120 mm^2

Remedial

Learners may struggle to substitute correctly. Identify learners experiencing problems and check their substitution before they begin the calculation. Ensure learners can distinguish between the polygons and apply the correct formula to the correct quadrilateral.

Exercise 3

Learner's Book page 240

Guidelines on how to implement this activity

Work with learners to develop the formula for calculating the area of a kite. Discuss the steps in the process and ensure learners know the properties of a kite. Learners also need to work with irregular shapes and identify the quadrilaterals in a complex shape, and then apply the appropriate formulae to find the missing sides and to calculate the total area. Do a few examples together as a class. Allow learners to work in pairs or small groups to complete this activity.

Suggested answers

1 Area of kite ABCD $= \frac{1}{2}$ ($diagonal_1 \times diagonal_2$)

$= \frac{1}{2}$(AB $\times$ CD)

$= \frac{1}{2}$ (90 cm $\times$ 50 cm)

$=$ 2 250 cm^2

2 Area of parallelogram PQRS $= b \times h$

$=$ 2,1 m $\times$ 0,8 m

$=$ 1,68 m^2

3 Area of trapezium KLMN $= \frac{1}{2}(\text{sum of parallel sides}) \times h$

$= \frac{1}{2}(25 \text{ mm} + 19 \text{ mm}) \times 15 \text{ mm}$

$= 330 \text{ mm}^2$

4 $\triangle$ABE is an isosceles triangle with N the midpoint of BE

BN $= \frac{1}{2}$BE

$= \frac{1}{2}(2{,}4)$m

$= 1{,}2$ m

AN = AM − NM

$= (5 - 1{,}8)$ m

$= 3{,}2$ m

Therefore in $\triangle$ABC: $AB^2 = BN^2 + AN^2$

$= (1{,}2 \text{ m})^2 + (3{,}2 \text{ m})^2$

$= (1{,}44 + 10{,}24) \text{ m}^2$

$= 11{,}68 \text{ m}^2$

Thus AB $= \sqrt{11{,}68}$ m $= 3{,}42$ m correct to two decimal places

FG = BE (see construction)

$= 2{,}4$

let CF $= x$

CD = CF + FG + GD

$4 \text{ m} = x + 2{,}4 \text{ m} + x$ ($\triangle$BCF $\equiv$ $\triangle$ EDC…RHS)

$2x = 4 \text{ m} - 2{,}4 \text{ m}$

$2x = 1{,}6$ m

CF $= x = 0{,}8$ m

BF = EG = NM = 1,8M (Construction)

In $\triangle$BCF: $BC^2 = CF^2 + BF^2$.

$= x^2 + (1{,}8 \text{ m})^2$

$= (0{,}8 \text{ m})^2 + (1{,}8 \text{ m})^2$

$= (0{,}64 + 3{,}24) \text{ m}^2$

$\therefore$ BC $= \sqrt{3{,}88}$ m $= 1{,}97$ m correct to two decimal places

5 Area of figure ABCD = Area of $\triangle$ABE + Area of trapezium BCDE

$= \frac{1}{2}bh + \frac{1}{2}(\text{sum of 11 sides}) \times h$

$= \frac{1}{2} \times BE \times AN + \frac{1}{2}(BE + CD) \times NM$

$= \frac{1}{2} \times 2{,}4 \text{ m} \times 3{,}2 \text{ m} + \frac{1}{2}(2{,}4 \text{ m} + 4 \text{ m}) \times 1{,}8 \text{ m}$

$= 3{,}84 \text{ m}^2 + 5{,}76 \text{ m}^2$

$= 9{,}6 \text{ m}^2$

6 AE = AB (given)

$= 3{,}42$ m

ED = BC (given)

$= 1{,}97$ m

Perimeter of pentagon ABCDE = AB + BC + CD + ED + AE

$= (3{,}42 + 1{,}97 + 4 + 1{,}97 + 3{,}42)$ m

$= 14{,}78$ m

Remedial

This exercise involves working with fractions and decimals. Learners may need assistance with these areas. Provide support by checking learners as they complete each step and helping them to identify where they might be going wrong.

Extension

If learners managed this exercise well, provide more difficult irregular shapes consisting of triangles and quadrilaterals for learners to calculate area and perimeter.

Unit 3 Circles

Learner's Book page 241

Unit focus

This unit focusses on the following:

- revising what was learnt in Grade 8 about the radius, diameter, circumference and area of a circle;
- learning to solve problems about circumference and area of figures that involve both polygons and circles or parts of circles; and
- learning how to solve equations based on the formulae for perimeter, circumference and area.

Background information on circles

Learners worked with circles in Grade 8, but the use of pi can be confusing to learners and may require additional revision. Pi is the relationship between the circumference and the diameter. It is a constant relationship true of any circle. It was known by the Ancient Babylonians in 2 000 BC, and was further explored by the Greeks Hippias and Archimedes in the fifth and third century BC respectively. The mysticism of circles and their believed powers is also ancient. The Celtic tribes of Europe and Britain built temples and monuments in the shapes of concentric circles. Stonehenge in England is an example of such a temple. The idea of a shape with no angles was an intriguing one to ancient tribes and civilizations.

Exercise 1

Learner's Book page 243

Guidelines on how to implement this activity

Discuss circles with learners and why circles are different to other polygons. Revise the parts of a circle with learners. Discuss pi and the relationship it represents. Emphasise that this relationship is constant for any circle. Revise the formulae for circumference and area. For calculation purposes, learners are to use pi as $\frac{22}{7}$. Ensure learners use the fraction and not the rounded off decimal value (3,14), otherwise their answers will differ from the prescribed answers. Work through a few examples of finding the area and circumference of a circle. Learners should do this exercise on their own.

Suggested answers

1.1 C = πd

$= \frac{22}{7} \times 56$ cm

$= (2 \times 8)$ cm

$= 176$ cm

1.2 A = πr^2 where $r = 56$ cm $\div 2 = 28$ cm

$= \frac{22}{7} \times \frac{(28)^2}{1}$ cm^2

$= 22 \times 4 \times 28$ cm^2

$= 2\,464$ cm^2

2.1 A = πr^2

$= 3{,}1415927 \times (25)^2$ mm^2

$= 1\,963{,}50$ mm^2

$= 19{,}64$ cm^2

2.2 Area of semi-circle = $\frac{1}{2}\pi r^2$ where $r = \frac{1}{2}$ of 1,2 cm = 0,6 cm

$= \frac{1}{2} \times 3{,}1415927 \times (0{,}6)^2$cm^2

$= 3{,}39292$ cm^2

$= 339{,}29$ mm^2

3.1 In $\triangle$ABC: $AC^2 = AB^2 + BC^2$

$(3 \text{ cm})^2 = 5^2 + 5^2$

$25^2 = 9^2$

$5^2 = 4{,}5$ cm^2

$5 = \sqrt{4{,}5}$ cm = 2,12 cm

= 2,12 cm

3.2 Area of the square = 5^2

$= (2{,}12 \text{ cm})^2$

$= 4{,}49$ cm^2

3.3 Area of the circle = πr^2 where $r = \frac{1}{2}$ of 3 cm = 1,5 cm

$= 3{,}1415927 \times (1{,}5 \text{ cm})^2$

$= 7{,}07$ cm^2

3.4 Area of shaded part = Area of circle − Area of square

$= (7{,}07 - 4{,}49)$ cm^2

$= 2{,}58$ cm^2

$= 258$ mm^2

4.1 Diameter of circle = 14 m

Thus $r = \frac{1}{2}$ of 14 cm = 7 m

so perimeter of $\frac{1}{2}$ circle = $\frac{1}{2}(2\pi r)$

$= \pi r$

$= 3{,}1415927 \times 7$ m

$= 21{,}99$ m = 22 m

perimeter of the shape = 22 m + 8 m + 14 m + 8 m

= 52 m

4.2 Total area of the shape = Area of $\frac{1}{2}$ circle + Area of rectangle

$= \frac{1}{2}(\pi r^2) + l \times b$

$= \frac{1}{2}(3{,}1415927)(7 \text{ m})^2 + 14 \text{ m} \times 8 \text{ m}$

$= 188{,}969$ m^2

$= 189$ m^2

5.1 $\triangle$KLM is equilateral therefore C is midpoint of LM

In $\triangle$KLM: $KL^2 = LC^2 + KC^2$

$(12 \text{ cm})^2 = (6 \text{ cm})^2 + h^2$

$h^2 = (12 \text{ cm})^2 - (6 \text{ cm})^2$

$= (144 - 36) \text{ cm}^2$

$= 108 \text{ cm}^2$

$h = \sqrt{108} \text{ cm}$

$= 10{,}4 \text{ cm}$

5.2 Area of $\triangle$KLM $= \frac{1}{2}bh$

$= \frac{1}{2} \times 12 \text{ cm} \times 10{,}4 \text{ cm}$

$= 62{,}4 \text{ cm}^2$

5.3 Area of inscribed circle $= \pi r^2$

$= 3{,}1415927 \times (3 \text{ cm})^2$

$= 28{,}274$

$= 28{,}3 \text{ cm}^2$

5.4 Area of shaded parts = Area of $\triangle$KLM – Area of inscribed circle

$= 62{,}4 - 28{,}3$

$= 34{,}1 \text{ cm}^2$

6.1 Area of ring = Area of outer circle Area of inner circle

$= \pi r_2^2 - \pi r_1^2$

$= \pi(r_2^2 - \pi r_1^2)$

$= 3{,}1415927[(14 \text{ mm})^2 - (9 \text{ mm})^2]$

$= 3{,}1415927(196 - 81) \text{ mm}^2$

$= 361{,}283 \text{ mm}^2$

$= 3{,}61 \text{ cm}^2$

6.2 Inner circumference of the ring $= 2\pi r_1$

$= 2 \times 3{,}1415927 \times 9 \text{ mm}$

$= 56{,}548 \text{ mm}$

$= 5{,}65 \text{ cm}$

6.3 Outer circumference of the ring $= 2\pi r_2$

$= 2 \times 3{,}141527 \times 14 \text{ mm}$

$= 87{,}967 \text{ mm}$

$= 8{,}80 \text{ cm}$

7.1 Area of shaded part

= Area of $\triangle$ABC + Area of $\triangle$ADC − Area of circle

$= \frac{1}{2}BC \times AB + \frac{1}{2}AD \times DC - \pi r^2$

$= \frac{1}{2}(8 \text{ cm}) \times (6 \text{ cm}) + \frac{1}{2}(5 \text{ cm})(8{,}7 \text{ cm}) - \pi(3 \text{ cm})^2$

$= 24 \text{ cm}^2 + 21{,}75 \text{ cm}^2 - 12{,}57 \text{ cm}^2$

$= 33{,}18 \text{ cm}^2$

7.2 Area RUTS = $3 \times 8 = 24 \text{ m}^2$

Shaded area RUTS = $24 \text{ m}^2 - 2 \text{ m}^2 = 22 \text{ m}^2$

Area QRM = $\frac{1}{2} \times 2 \times 2 = 2 \text{ m}^2$

Area VNU = $\frac{1}{2} \times 2 \times 2 = 2 \text{ m}^2$

Area PQV = $\frac{1}{2} \times 4 \times 4 = 8 \text{ m}^2$

Area QVNM = $4 \times 2 = 8 \text{ m}^2$

Shaded area QVNM = 8 – circle area

= $8 - (\pi \times 1 \times 1)$

= $8 - 3{,}14$

= $4{,}\ 86$

Total area = shaded area RUTS + Area QRM + Area VNU + Area PQV + shaded area QVNM = 22 + 2 + 2 + 8 + 4,86

= $38{,}86 \text{ m}^2$

Remedial

Learners may require assistance as to how to input fractions on their calculator. Ensure each learner can input the data correctly into their calculators. Revise rounding off and instruct learners only to round off their answers at the very end. Explain how rounding off too early will give an incorrect final answer. Do a simple example of rounding off too soon on the board.

Extension

Learners can explore circles as a unique polygon and research their appeal to different civilizations. Learners can present their findings in a poster for the classroom.

Exercise 2

Learner's Book page 245

Guidelines on how to implement this activity

This exercise revises learners' knowledge of finding area and perimeter of other polygons as well as circles. Revise the formulae for finding the perimeter and area of a square, rectangle, parallelogram and a circle. Remind learners to draw a sketch in their books to help them solve problems.

Suggested answers

1 $4s = \text{P}$

$4s = 42 \text{ m}$

$s = 42 \div 4$

$= 10{,}5 \text{ m}$

2 $\text{A} = s^2 = 36$

$s = \sqrt{36}$

$s = 6$

3 Area of rectangle = $l \times b$

$l \times 0{,}9$ m = $1{,}9$ m^2

$l = \frac{1{,}9}{0{,}95}$ m

= 2 m

4.1 Area of parallegram = $b \times h$

8 cm $\times h$ = 52 cm^2

$h = \frac{52}{8}$ cm

= 6,5 cm

4.2 Perimeter of parallelogram = 2 × (short side + longside)

2(PQ + 8 cm) = 30 cm

PQ + 8 cm = (30 ÷ 2) cm

PQ = 15 cm − 8 cm

= 7 cm

RS = PQ = 7 cm (opp. sides of parm. are equal)

5.1 Area of semi-circle = $\frac{1}{2}\pi r^2$

$\frac{1}{2}(3{,}141527)r^2 = 77$ m^2

$r^2 = 77 \div \frac{1}{2}(3{,}1415927)$ m^2

= 49,019 m^2

= 49 m^2

$\therefore r = \sqrt{49}$ m = 7 m

5.2 Circumference of the whole shape = Circumference of the semi-circle + diameter

$= \frac{1}{2}(2\pi r)$ + AB

$= \pi r + 2r$

$= 3{,}1415927 \times 7$ m + 2(7) m

= 35,99 m

= 36 m

Remedial

If learners struggle with this exercise, remind them about the steps to use for problem-solving. This helps break the problem down into manageable pieces and makes it easier to cope with.

Unit 4 Doubling of dimensions of 2D shapes

Learner's Book page 246

Unit focus

This unit focusses on the following:

- investigating the relationship between the dimensions (length, breadth and radius) and the area and perimeter of 2D figures.

Background information on the relationship between dimensions and area and perimeter

In Grade 8 (as well as in Units 2 and 3 of this chapter), learners have learnt to:

- calculate the perimeter and area of squares, rectangles and triangles using appropriate formulae and correct to at least one decimal place;
- calculate the areas of polygons by deconstructing them into rectangles and/or triangles;
- use and describe the relationship between the radius, diameter and circumference of a circle in calculations; and
- use and describe the relationship between the radius and area of a circle in calculations.

Learners will now use this information to investigate the relationship between the dimension of a shape and its area and perimeter. It is an important skill that integrates learners' knowledge of patterns and relationships with geometry.

Exercise 1

Learner's Book page 248

Guidelines on how to implement this activity

Work through Question 1 of the Worked example from the Learner's Book on the board or use an example of your own choice. Get the learners to participate by asking questions like "What is the perimeter of square A?", "How can this ratio be simplified?" Then you write the correct answers on the board. Let the learners do Question 1 of Exercise 1 in the class and monitor their progress. The results for circles are similar to that of the squares of the Worked example.
Demonstrate Question 2 of the Worked example from the Learner's Book on the board or use an example of your own choice. Let the learners do Questions 2 and 3 of Exercise 1 in the class and monitor their progress. The remainder of the exercise can be given as homework. Learners should do this exercise on their own.

Suggested answers

1.1 Radius of circle B : Radius of Circle B = 2 : 1

1.2 Circumference of circle B = $2\pi r_B = 2 \times (3{,}141527)2$ m
$= 12{,}5663708$ m
Circumference of circle A = $2\pi r_A = 2 \times (3{,}1415927)1$ m
$= 6{,}2831854$ m
Circumference B : Circumference A = 12,5663708 : 6,2831854
= 2 : 1
When the radius of a circle doubles, then its perimeter also doubles.

1.3 Area of circle B = $\pi r_B^2 = 3{,}1415927 \times 1^2$ m^2
$= 12{,}5663708$ m^2
Area of circle A = $\pi r_A^2 = 3{,}1415927 \times 1^2$ m^2
$= 3{,}1415927$ m^2
Area B: Area A = 12,5663708 : 3,1415927
= 4 : 1
When the radius of a circle doubles, then its area increases four fold.

2.1 $b_D : b_C = 8 : 4 = 2 : 1$

$h_D : h_C = 6 : 3 = 2 : 1$

2.2 Perimeter D : Perimeter C = (10 + 6 + 8) : (5 + 3 + 4)

$= 24 : 12 = 2 : 1$

When the lengths of the sides double, the perimeter also doubles.

2.3 Area D : Area C = $\frac{1}{2}(8)(6) : \frac{1}{2}(4)(3)$

$= 24 : 6$

When the lengths of the sides double, the area increases four fold.

3.1 $h_F : h_E = 12 : 6 = 2 : 1$

$s_F : s_E = 16 : 8 = 2 : 1$

3.2 Perimeter F = 4s = 4 × 16 mm = 64 mm

Perimeter E = 4s = 4 × 8 mm = 32 mm

Perimeter F: Perimeter E = 64 : 32 = 3 : 1

When the side of a rhombus doubles, then its perimeter also doubles.

3.3 Area F = $s_F \times h_F$ = 16 mm × 12 mm = 192 mm^2

Area E = $s_E \times h_E$ = 8 mm × 6 mm = 48 mm^2

Area F: Area E = 192 : 48 = 4 : 1

When the side of a rhombus doubles, then its area increases fourfold.

4.1 $s_h : s_g = 6 : 2 = 3 : 1$

4.2 Perimeter H = 4 × s_H = 4 × 6 m = 24 m

Perimeter G = 4 × s_G = 4 × 2 m = 8 m

Perimeter H: Perimeter G = 24 : 8 = 3 : 1

When the side of a square triples, then its perimeter also triples (increases threefold).

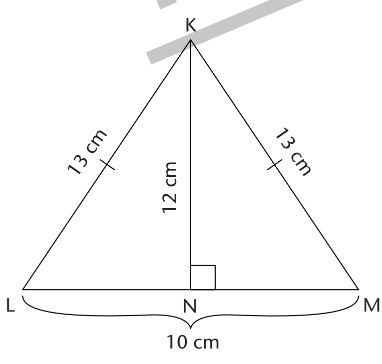
4.3 Area H = $s_H^2 = (b\text{ m})^2 = 36\text{ m}^2$

Area G = $s_G^2 = (2\text{ m})^2 = 4\text{ m}^2$

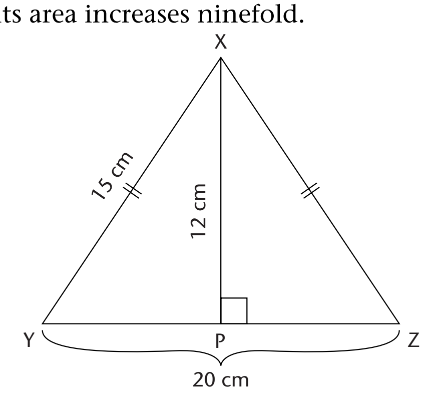
Area H: Area G = 36 : 4 = 9 : 1

When the side of a square triples, then its area increases ninefold.

5.1



5.2



5.3 Area $\triangle$KLM = $\frac{1}{2}bh = \frac{1}{2} \times 10\text{ cm} \times 12\text{ cm}$

$= 60\text{ cm}^2$

Area $\triangle$XYZ = $\frac{1}{2}bh = \frac{1}{2} \times 20\text{ cm} \times 12\text{ cm}$

$= 120\text{ cm}^2$

5.4 $YP = \frac{1}{2}(YZ) = \frac{1}{2}(20 \text{ cm})$

$= 10 \text{ cm}$

In $\triangle \times YP : XY^2 = YP^2 + XP^2$

$= (10 \text{ cm})^2 + (12 \text{ cm})^2$

$= (100 + 144) \text{ cm}^2$

$= 244 \text{ cm}^2$

$XY = \sqrt{244} \text{ cm} = 15{,}6 \text{ cm}$

and $XZ = XY = 15{,}6$ cm (△XYZ given isosceles)

5.5.1 Base YZ : Base LM = 20 : 10 = 2 : 1

5.5.2 $\triangle XYZ = 2 \times 15{,}6 \text{ cm} + 20 \text{ cm}$

$= 51{,}2 \text{ cm}$

Perimeter $\triangle KLM = 2 \times 13 \text{ cm} + 10 \text{ cm}$

$= 63 \text{ cm}$

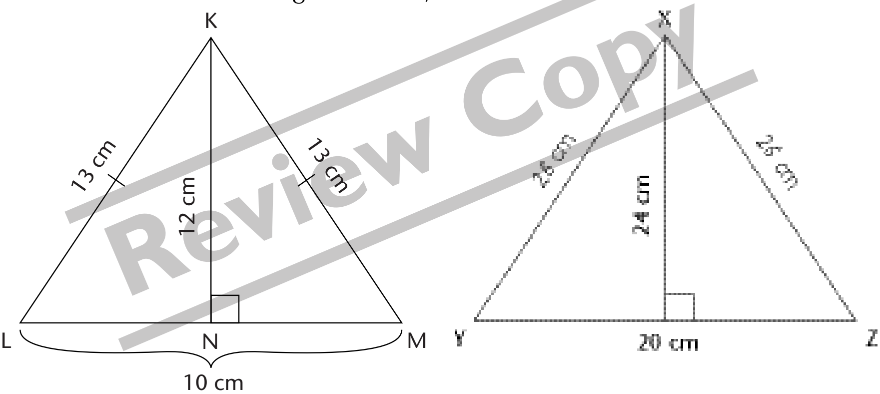
5.5.3 Area $\triangle XYZ = bh = \frac{1}{2} \times 20 \times 12 \text{ cm}^2 = 120 \text{ cm}^2$

Area $\triangle KLM = \frac{1}{2}bh = \frac{1}{2} \times 10 \times 12 \text{ cm}^2 = 60 \text{ cm}^2$

Area △XYZ : △KLM = 120 : 60 = 2 : 1

When the base of a triangle doubles, then its area doubles.

5.6



In $\triangle XYZ$: $XY^2 = (10 \text{ cm})^2 + (24 \text{ cm})^2$

$= (100 + 576) \text{ cm}^2$

$= 676 \text{ cm}^2$

XY = 26 cm (= XZ)

Perimeter $\triangle XYZ = 2 \times 26 \text{ cm} + 20 \text{ cm} = 72 \text{ cm}$

Perimeter △KLM = 36 cm (see 5.5.2)

Perimeter △XYZ : Perimeter △KLM = 72 : 36 = 2 : 1

When the height and the base of a triangle doubles, then its perimeter doubles.

5.7 Area $\triangle XYZ = \frac{1}{2}bh = \frac{1}{2} \times 20 \text{ cm} \times 24 \text{ cm}$

$= 60 \text{ cm}^2$

Area △XYZ : Area △KLM = 240 : 60 = 4 : 1

When the base and the height of a triangle double, then its area increases fourfold.

Remedial

Some learners may struggle to calculate areas or simplify ratios. Help them with this by asking questions such as:

- "By what number can 72 as well as 36 be divided?"
- "How do you calculate the area of a circle?"

For those learners who struggle with the more challenging problem Question 5 of Exercise 1, help them to see that the Theorem of Pythagoras needs to be applied in some cases.

Extension

Challenge learners to tessellate with 2D figures of the same shape but with respective dimensions forming simple ratios such as 2 : 1, for example rectangles C, D and E of Worked example Question 2.

Consolidation

Learner's book page 250

Before doing this consolidation exercise, encourage learners to review the work covered in this chapter. Advise learners to use the summary and to revise their work. This exercise can be used as an informal assessment task for you to track how learners are coping with the chapter and the concepts covered. The mark allocation provides guidelines on how to assess learners.

Suggested answers

1.1 $23\ 450\ \text{cm}^2 = 2{,}345\ \text{m}^2$ **1.2** $7\ 852\ \text{mm}^2 = 7{,}85\ \text{cm}^2$

1.3 $0{,}65\ \text{m}^2 = 6\ 500\ \text{cm}^2$ **1.4** $34\ 560\ \text{cm}^2 = 3{,}456\ \text{m}^2$

1.5 $6\ 543\ 200\ \text{m}^2 = 4{,}5432\ \text{km}^2$ **1.6** $0{,}091\ \text{km}^2 = 91\ 000\ \text{m}^2$ (6)

2 $\text{BD}^2 = \text{BA}^2 + \text{AD}^2$

$= (4\ \text{m})^2 + (9\ \text{m})^2$

$= (16 + 81)\text{m}^2$

$= 97\ \text{m}^2$ (3)

$\text{BD} = \sqrt{97}\,\text{m} = 9{,}85\ \text{m}$

3.1 In $\triangle\text{PQS}: \text{QP}^2 = \text{QS}^2 + \text{PS}^2$

$(39\ \text{mm})^2 = \text{QS}^2 + (22\ \text{mm})^2$

$\text{QR}^2 = (39\ \text{mm})^2 - (22\ \text{mm})^2$

$= (1\ 521 - 484)\text{mm}^2$

$= 1\ 037\ \text{mm}^2$

$\text{QS} = \sqrt{1\ 037}\,\text{mm}^2$

$= 32{,}2\ \text{mm}$ (2)

3.2 In $\triangle\text{PSR}$: $\text{SR} = \text{QS} = 50\ \text{mm} - 32{,}2\ \text{mm} = 17{,}8\ \text{mm}$

$\text{PR}^2 = \text{PS}^2 + \text{SR}^2$

$= (22\ \text{mm})^2 + (17{,}8\ \text{mm})^2$

$= (484 - 316{,}84)\ \text{mm}^2$

$= 167{,}16\ \text{mm}^2$

$\text{PR} = \sqrt{167{,}16}\ \text{mm}$

$= 12{,}9\ \text{mm}$ (2)

3.3 Area $\triangle\text{PQR} = \frac{1}{2}b \times h$

$= \frac{1}{2}(50\ \text{mm}) \times 22\ \text{mm}$

$= 550\ \text{mm}^2$ (2)

4.1 Perimeter of $\triangle ABC$ = sum of 3 sides = $(7 + 24 + 25)$ cm
= 56 cm
Area of $\triangle ABC = \frac{1}{2}bh$
$= \frac{1}{2} \times 7 \text{ cm} \times 24 \text{ cm}$
$= 84$ cm (2)

4.2 Perimeter of rhombus DEFG = $4 \times s$
$= 4 \times 1{,}4$ m
$= 5{,}6$ m
Area of rhombus DEFG = $b \times h$
$= 1{,}4 \text{ m} \times 1{,}1 \text{ m}$ (2)

4.3 Perimeter parm. HJKL = 2(short side + long side)
= 2(34 mm + 51 mm)
= 170 mm
Area of parm. HJKL = $b \times h$
$= 34 \text{ mm} \times 24 \text{ mm}$
$= 816 \text{ mm}^2$
$= 8{,}16 \text{ cm}^2$ (2)

4.4 Perimeter of trapezium MNOP = $(25 + 13 + 20 + 12)$ cm
= 70 cm
Area of trapezium MNOP = $\frac{1}{2}$(sum of parallel sides) $\times h$
$= \frac{1}{2}(25 + 20) \times 12 \text{ cm}^2$
$= 270 \text{ cm}^2$ (2)

4.5 QS = 20 cm
Perimeter of kite QRST = $(2 \times 10 + 2 \times 13{,}4)$ cm
= 46,8 cm
Area of kite QRST = $\frac{1}{2}$(*diagonal* QS)(*diagonal* RT)
$= \frac{1}{2} \times 20 \text{ cm} \times 12 \text{ cm}$
$= 120 \text{ cm}^2$ (2)

5.1 Circumference of circle = πd
$= 3{,}1415927 \times 2{,}4$ m
$= 7{,}53982248$ m
$= 7{,}54$ m
Area of circle = πr^2 where = $\frac{1}{2}$ of 2,4 m = 1,2
$= 3{,}1415927(1{,}2 \text{ m})^2$
$= 4{,}52 \text{ m}^2$ (2)

5.2 Perimeter of semi-circle = $\frac{1}{2}(\pi d) + d$
$= \frac{1}{2} \times 3{,}1415927 \times 84 \text{ cm} + 84 \text{ cm}$
$= 215{,}95$ cm
Area of semi-circle = $\frac{1}{2}(\pi r^2)$ where $r = \frac{1}{2}$ of 84 cm = 42 cm
$= 2\ 770{,}88 \text{ cm}^2$ (2)

5.3 Perimeter of quarter circle $= 2 \times r + \frac{1}{4}(2\pi r)$
$= 2 \times 14 \text{ mm} + \frac{1}{4} \times 2 \times 3{,}1415972 \times 14 \text{ mm}$
$= 49{,}99 \text{ mm}$

Area of quarter circle $= \frac{1}{4}\pi r^2$
$= \frac{1}{4} \times 3{,}1415972 \times (14 \text{ mm})^2$
$= 153{,}94 \text{ mm}^2$ (2)

6.1 Perimeter $= 7 \times 54 \text{ mm} = 378 \text{ mm}$ (2)

6.2 Circumference of big semi-circle $= \frac{1}{2}(2\pi r)$
$= 3{,}1415927 \times 20 \text{ mm}$
$= 62{,}83 \text{ mm}$

Circumference of 2 small semi-circles = Circumerence of 1 small circle
$= 2\pi r$
$= 2 \times 3{,}1415927 \times 10 \text{ mm}$
$= 62{,}83 \text{ mm}$

Thus circumference of shape $= 2 \times 62{,}83 \text{ mm}$
$= 125{,}66 \text{ mm}$ (2)

6.3 Circumference of semi-circle $= \frac{1}{2}(\pi d)$
$= \frac{1}{2} \times 3{,}1415927 \times 5 \text{ m}$
$= 7{,}85 \text{ m}$

Circumference of the shape $= 7{,}85 + 5 + 5$
$= 17{,}85 \text{ m}$ (2)

7.1 Area of shaded part = Area of square − Area of triangle
$= 5^2 - \frac{1}{2}bh$
$= (4 \text{ cm})^2 - \frac{1}{2}(4 \text{ cm})(3 \text{ cm})$
$= 16 \text{ cm}^2 - 6 \text{ cm}^2$
$= 10 \text{ cm}^2$ (2)

7.2 Area of shaded part = Area of triangle − Area of circle
$= \frac{1}{2}bh - \pi r^2$ where $r = \frac{1}{2}$ of 3 cm
$= 1{,}5 \text{ cm}$
$= \frac{1}{2} \times 6 \times 4 \text{ cm}^2 - 3{,}1415927 \times (1{,}5 \text{ cm})^2$
$= 12 \text{ cm}^2 - 7{,}07 \text{ cm}^2$
$= 4{,}93 \text{ cm}^2$ (2)

7.3 Area of shaded part = Area of circle − Area of triangle
$= \pi r^2 - \frac{1}{2}bh$
$= 3{,}15927 \times (5 \text{ m})^2 - \frac{1}{2} \times 10 \text{ m} \times 5 \text{ m}$
$= 53{,}54 \text{ m}^2$ (2)

7.4 Area of shaded part = Area of parm. − Area of circle
$= b \times h - \pi r^2$ where $r = \frac{1}{2}$ of 2,5 m
$= 1{,}25 \text{ m}$
$= 6{,}5 \text{ m} \times 2{,}5 \text{ m} - 3{,}1415927 \times (1{,}25 \text{ m})^2$
$= (16{,}25 - 4{,}91)\text{m}^2$
$= 11{,}34 \text{ m}^2$ (2)

8.1 Circumference = $2\pi r$

$= 2 \times 3{,}1415927 \times 63$ m

$= 11{,}34\ m^2$ (2)

8.2 In 1 minute 500 ℓ of water is sprayed

In 120 minutes 120 × 500 ℓ of water is sprayed

= 60 000 ℓ (2)

8.3 Area of circular field = πr^2

$= 3{,}1415927 \times (63\ m)^2$

$= 12\ 498{,}98\ m^2$ (3)

8.4 10 000 m^2 = 1 ha

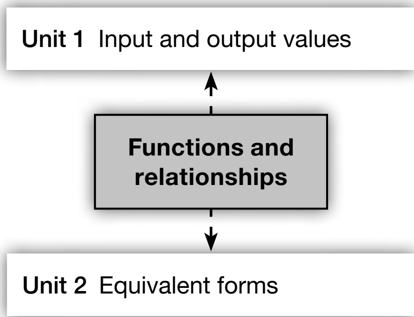
Thus 12 468,98 m^2 = 1,246898 ha

= 1,25 ha (3)

[55]

Chapter 9 Functions and relationships

Overview of concepts



Content		Time allocations	LB page
Unit 1	Input and output values	$2\frac{1}{2}$ hours	253
Unit 2	Equivalent forms	$2\frac{1}{2}$ hours	257

Background information on functions and relationships

Learners have worked with functions and relationships throughout their schooling in mathematics. The focus in Grade 9 is to look at functions as graphs. Learners need to revise the other forms, including flow diagrams, verbal descriptions and tables, however the main objective is to get learners to see graphs as equivalent forms and visual representations of equations. In Grade 10, functions become synonymous with graphs and this section in Grade 9 helps create the foundation for this way of thinking.

Generic teaching guidelines for teaching functions and relationships

Ensure learners can identify and determine the input and output values for each of the required forms, namely verbal descriptions, flow diagrams, tables, formulae and equations. In Grade 9, learners need to be aware of the domain of the input values. They need to know if the function is valid for all input values, or if it is only valid for a limited set of values. Spend some time explaining the domain of values as this is a new concept for learners. Encourage learners to discuss their thinking process when working with relationships. This helps learners to understand their own thinking and the thinking processes of their classmates. Comparing strategies is important for learners to develop and improve metacognitive thinking.

Resources

Blank number lines, blank flow diagrams, and blank tables for learners to use when working with patterns. Colour cardboard and pens to create posters, number and vocabulary cards. Each learner should have their own calculator.

Unit 1 Input and output values

Learner's Book page 253

Unit focus

This unit focusses on the following:
- revising determining input values, output values and rules for relationships
- focussing on finding output values for given equations.

Background information on input and output values

Finding input and output values is the first step in working with patterns and relationships. It helps learners to better understand the pattern or relationship that is presented to them by working with the necessary function. The focus in Grade 9 is on equations and changing the subject of the formula. Using equations to solve for unknown variables becomes increasingly important in FET, and changing the subject of the formula is an important skill for learners to master for further mathematical study.

Exercise 1

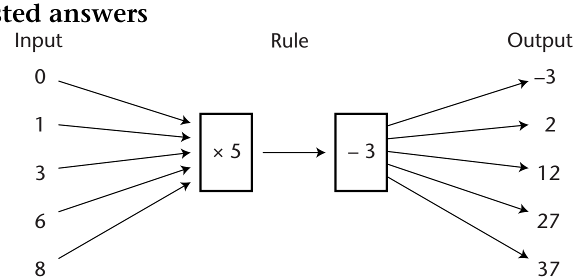
Learner's Book page 255

Guidelines on how to implement this activity

Revise finding input and output values in the different forms. Focus on finding the input values, as this requires learners to work in reverse. This is particularly challenging when working with equations. Learners need to know how to change the subject of the formula in order find the input value. Spend time doing as many examples as required until learners can cope with the prescribed exercise.

Suggested answers

1.1



1.2

Input (x)	0	1	3	6	8
Output (y)	–3	2	12	27	37

1.3 $y = 5x - 3$

2.1 $y = 4(7) + 12 = 40$

2.2 $a = \frac{14(56)}{2} = 784$

2.3 $p = 9 - 14(13) = 9 - 182 = -173$

2.4 $y = \frac{1}{2} + \frac{9}{4} = \frac{2}{4} + \frac{9}{4} = \frac{11}{4} = 2\frac{3}{4}$

2.5 $f = \frac{1}{3(-1)} - (-3) = -\frac{1}{3} + 3 = 2\frac{2}{3}$

2.6 $b = 3(6 - 2(34)) = 3(-62) = -186$

2.7 $c = 15(-4) + 6 = -54$

2.8 $y = 7 - (-7(-1)) = 0$

3.1
$$\begin{aligned} y &= 7x - 9 \\ x &= \frac{y+9}{7} \\ &= \frac{6+9}{7} \\ &= \frac{15}{7} \\ &= 2\tfrac{1}{7} \end{aligned}$$

3.2
$$\begin{aligned} t &= \tfrac{1}{2}s - 15 \\ s &= 2(t + 15) \\ &= 2(60 + 15) \\ &= 150 \end{aligned}$$

3.3
$$\begin{aligned} h &= 7j + 19 \\ j &= \frac{h-19}{7} \\ &= \frac{-3-19}{7} \\ &= \frac{-22}{7} \\ &= -3\tfrac{1}{7} \end{aligned}$$

3.4
$$\begin{aligned} g &= 2d - 5 + d = 3d - 5 \\ d &= \frac{g+5}{3} \\ &= \frac{-1+5}{3} \\ &= \frac{4}{3} \\ &= 1\tfrac{1}{3} \end{aligned}$$

3.5
$$\begin{aligned} f &= \tfrac{4}{5}g + 6 \\ g &= \tfrac{5}{4}(f - 6) \\ &= \tfrac{5}{4}(0 - 6) \\ &= -\frac{15}{2} \\ &= -7\tfrac{1}{2} \end{aligned}$$

4.1
$$\begin{aligned} e &= \frac{4f-7}{2} \\ 2e &= 4f - 7 \\ 2e + 7 &= 4f \\ f &= \frac{2e+7}{4} \end{aligned}$$

4.2
$$\begin{aligned} x &= \frac{2y-1}{4} \\ 4x &= 2y - 1 \\ 4x + 1 &= 2y \\ y &= \frac{4x+1}{2} \end{aligned}$$

4.3
$$\begin{aligned} m &= \frac{2a-4}{2} \\ 2m &= 2a - 4 \\ 2m + 4 &= 2a \\ a &= \frac{2m+4}{2} \\ a &= m + 2 \end{aligned}$$

Remedial

In order to change the subject of the formula, learners need to be able to solve and manipulate equations, including those with fractions. If learners experience difficulty with this exercise, provide material that revises working with complex equations.

Exercise 2

Learner's Book page 256

Guidelines on how to implement this activity

Introduce learners to the concept of a domain for input values. Discuss the concept that some functions are continuous and will apply for all values, while others are limited. Discuss what forms the domain could take, for example, natural numbers, rational numbers or integers. Discuss limited domains and the effect this has on output values. Work through the Worked example as a class, and do some additional examples where you supply a specific domain for the input values.

Suggested answers

1.1

d	7 m	14 m	21 m	28 m	35 m
C	22 m	44 m	66 m	88 m	110 m

1.2 49 m

2.1 $x = \{-2; -1; 1; 2; 4; 5; 10\}$

2.2 $y = \frac{1}{x}$

2.3 No, because we cannot divide by zero.

3.1 $y = \frac{x}{2} + 4$

3.2 output = $\{12; 10; 9; 8\frac{1}{2}; 8\ \frac{1}{4}\}$

3.3

$$\longrightarrow \boxed{\div 2} \longrightarrow \boxed{+4} \longrightarrow$$

Remedial

Learners need an understanding of the number system and how the system is integrated. Spend some time revising this if learners show difficulty in understanding the given domain of the variables.

Extension

Encourage learners to create their own functions with limited domains.

Unit 2 Equivalent forms

Learner's Book page 257

Unit focus

This unit focusses on the following:
- determining, interpreting and justifying the equivalence of different descriptions of the same relationship or rule.

Background information on equivalent forms

Learners need to be able to translate between the different forms of representing relationships. They should be reasonably skilled at working with tables, verbal descriptions and flow diagrams. The focus in Grade 9 is on working with formulae, equations and graphs. Learners need to interpret the graph as a visual representation of the equation. This becomes increasingly important as learners move into the FET phase of their education.

Exercise 1

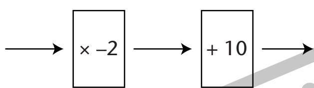
Learner's Book page 259

Guidelines on how to implement this activity

Learners should know by now that the various forms of representing relationships are all equivalent. Ensure learners understand this concept. Show learners an example of a relationship represented verbally, in a flow diagram, in a table, as an equation and as a graph. Ask for learners' input as you display the relationship. Do a few examples together as a class. Show a relationship in one of the forms and ask learners to come up to the board and represent the relationship in another way. Repeat until you feel learners will manage the exercise on their own.

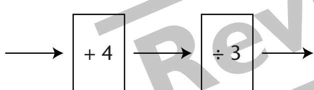
Suggested answers

1.1



1.2 $y = -2x + 10$

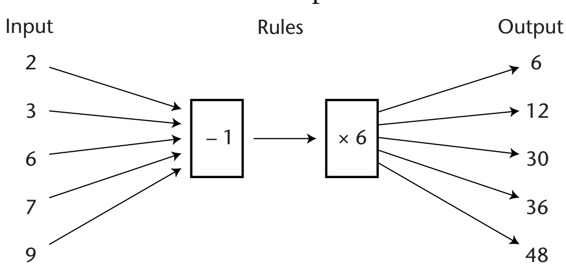
2.1



2.2 $y = \frac{x + 4}{3}$

3.1 Subtract one from each input value and then multiply by six.

3.2



3.3 $y = 6(x - 1)$ $x = \{2; 3; 6; 7; 9\}$

4.1 $y = 1 - x$ or $y = -x + 1$ $x = \{-3; -2; -1; 0; 1; 2; 3\}$

4.2 Subtract the x values from 1 to get the y-values.

5.1 Multiply x by four and then add five to get y.

5.2 Multiply d by π to get C.

5.3 Subtract three from x and then divide by two to get y.

6.1

x	–2	–1	0	1	2	$3\frac{1}{3}$
y	16	13	10	7	4	0

6.2

x	$-\frac{1}{8}$	$-\frac{1}{4}$	$\frac{1}{2}$	2	4	10
y	–8	–4	2	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{10}$

6.3

r	$\sqrt{7}$	$\sqrt{14}$	7	10
A	22	44	154	$\frac{2\,200}{7}$

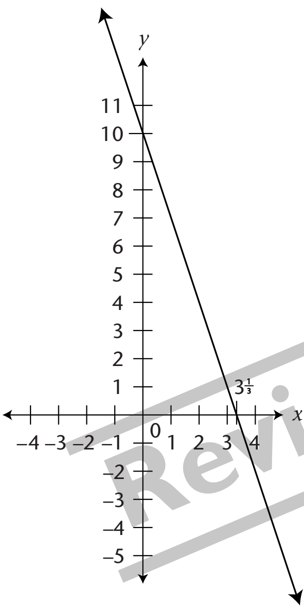
6.4

b	1 m	2,5 m	4 m	10 m
A	9 m^2	22,5 m^2	36 m^2	90 m^2

6.5

b	2 m	3 m	4 m	8 m
l	12 m	8 m	6 m	3 m

7



Remedial

Learners are required to integrate their knowledge of equations and graphs here. Some learners may require additional revision of these concepts. Give learners a variety of simple linear equations and have them draw up a table as shown below:

x	2	1	0	1	2
y					

Get learners to complete the table and then plot the points on the Cartesian plane. Learners must then join the points to draw the graph.
If learners still struggle, limit the domain to only positive values and provide equations that are plotted in only the first quadrant.

Extension

Learners can create a poster showing a relationship portrayed in each of the different forms. The posters can go up in the class to act as a learning aid for learners when they work with functions and relationships.

Exercise 2

Learner's Book page 261

Guidelines on how to implement this activity

Besides being able to translate from one form to another, learners also need to be able to identify whether two different forms represent the same relationship. Work through the examples in the Learner's Book and provide a few additional examples to discuss and work through as a class. Discuss possible key aspects to look for when trying to determine if different forms are equivalent.

Suggested answers

1.1 (ii) and (iiii) are equivalent.

1.2 (i) can be changed to: $y = 5(x + 3) = 5x + 15$

2.1 (ii) and (iii) are equivalent.

2.2 (i) can be changed to:

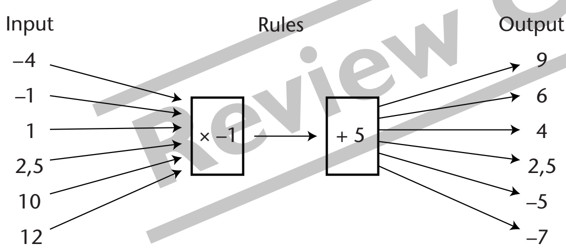
s	10 mm	15 mm	20 mm
A	$100mm^2$	$225\ mm^2$	$400\ mm^2$

3.1 (i), (ii) and (iv) are equivalent.

3.2 (iii) can be changed to:

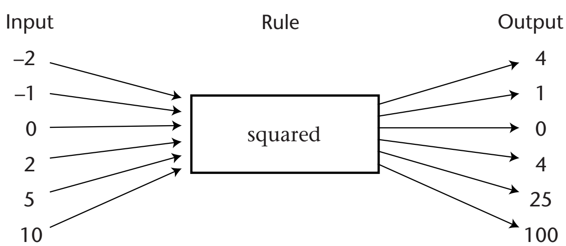
x	10	20	30	40
y	5	3	1	–1

4.1



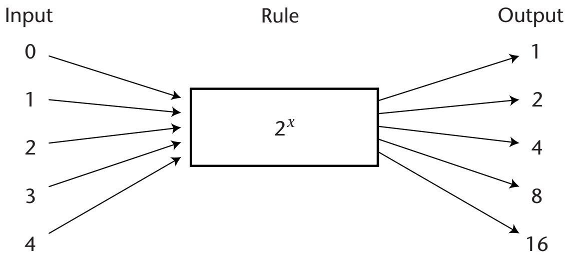
4.2 $y = -x + 5$ or $y = 5 - x$

5.1



5.2 $y = x^2$

6.1



6.2 $y = 2^x$

7

x	0	1	2	3	4
y	1	3	9	27	81

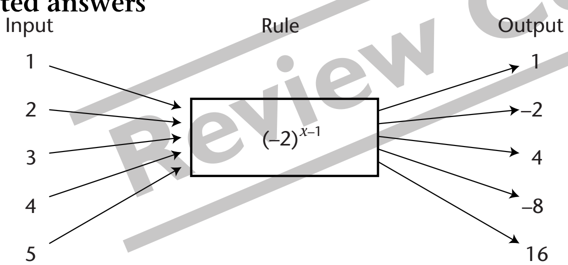
Consolidation

Learner's Book page 264

Before doing this consolidation exercise, encourage learners to review the work covered in this chapter. Advise learners to use the summary and to revise their work. This exercise can be used as an informal assessment task for you to track how learners are coping with the chapter and the concepts covered. The mark allocation provides guidelines on how to assess learners.

Suggested answers

1.1 (1)



1.2 (1)

x	1	2	3	4	5	6
y	1	–2	4	–8	16	–32

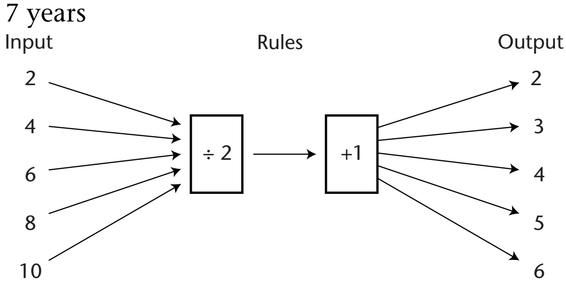
1.3 $y = (-2)^{x-1}$ with domain {1; 2; 3; 4; 5; 6} (2)

2.1 (2)

n	1	2	3	4	5
I	120	240	360	480	600

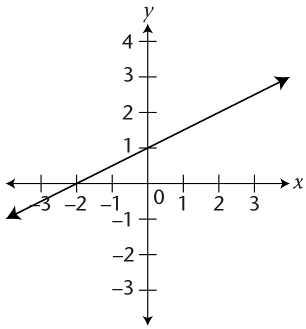
2.2 7 years (2)

3.1 (2)



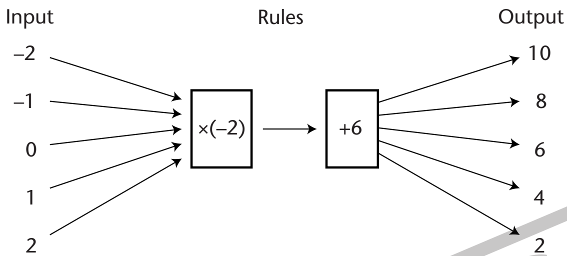
3.2 $y = \frac{x}{2} + 1$ (2)

3.3



(2)

4.1



(5)

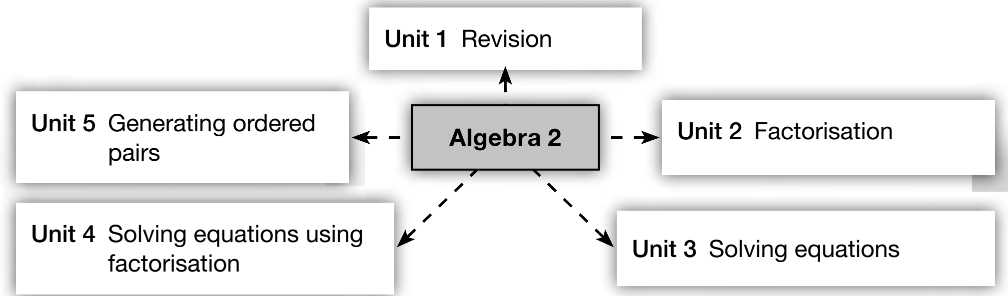
4.2 (i) and (ii) are equivalent.
(iii) can be changed to be "multiply a number by negative two and then add six". (3)

5 (i), (ii) and (iv) are equivalent. (1)
(iii) can be changed to "square the radius of a circle and then multiply by π to calculate the areas of circles with radii 7 cm, 14 cm and 21 cm". (2)

[25]

Chapter 10 Algebra 2

Overview of concepts



Content		Time allocations	LB page
Unit 1	Revision	2 hours	266
Unit 2	Factorisation	7 hours	268
Unit 3	Solving equations	4 hours	283
Unit 4	Solving equations using factorisation	4 hours	287
Unit 5	Generating ordered pairs	2 hours	290

Background information on algebra

This chapter builds directly on the work that your learners did in Chapter 5. It revises important skills from Chapter 5, as well as equipping your learners with new skills, including the very important skill of factorisation. This chapter also lays the foundation for generating tables of ordered pairs, a skill which will be built upon in Chapter 11 (Equations and graphs).

Generic teaching guidelines for teaching algebra

- All of the guidelines that we gave for Chapter 5 apply to this chapter as well. Refer to Chapter 5 to refresh your memory about these.
- The main focus of this chapter is on factorisation. This is such an important concept that you must be prepared to spend time on every aspect of this work. Trinomials, in particular, can be very tricky for most learners and you will need to provide much encouragement and as many additional practice questions as you feel are necessary.
- The value of checking solutions cannot be over-emphasised. When it comes to factorising, consider enforcing that learners check their solutions by refusing to mark their work unless they show clearly that they have checked each and every solution. Although this might seem to be an extreme approach to adopt, your learners will benefit hugely in the long run.

Resources

Cardboard and colour pens, variable and constant cards, and each learner should have their own calculator.

Unit 1 Revision

Learner's Book page 266

Unit focus

This unit focusses on the following:

- revising simplifying algebraic expressions,
- revising determining the numerical value of algebraic expressions by substitution,
- revising multiplying polynomials by integers or monomials,
- revising dividing polynomials by integers or monomials,
- revising finding the product of two binomials, and
- revising finding the square of a binomial.

Background information on revision

In this unit, your learners will revise what they learnt in Chapter 5.

Exercise 1

Learner's Book page 266

Guidelines on how to implement this activity

This unit starts off with a simple revision exercise in which your learners will practise some of the skills that they should already have mastered. Check that your learners are comfortable with this work by going through the solutions with them. Remediate any mistakes before proceeding further.

Suggested Answers

1.1 $x^2 + y^2 + z^2 = (-3)^2 + (2)^2 + (-1)^2$
$= 14$

1.2 $\frac{y + 2z}{xyz} = \frac{1}{xz} + \frac{2}{xy}$

1.3 $\sqrt{x^2y^2z^2} = xyz$

2.1 $(-6ab^2c^3)^2 = 36a^2b^4c^6$

2.2 $\sqrt{2p^2 + 3p^2 + 4p^2} = \sqrt{9p^2} = 3p$

2.3 $\sqrt[3]{9d(3d^2)} = \sqrt[3]{27d^3} = 3d$

3.1 $2(5x - y) - 3(x - y) = 10x - 2y - 3x + 3y = 7x + y$

3.2 $-(2a^2 - 3a - 4) + 2(4a^2 - 3a - 2) = -2a^2 + 3a + 4 + 8a^2 - 6a - 4$
$= 6a^2 - 3a$

3.3 $-3p(5p^3 - 2p^2 - 8p + 1) = -15p^4 + 6p^3 + 24p^2 - 3p$

3.4 $\frac{64m^4 + 32m^3 - 16m^2 + 12m^2 - 4m}{4m} = 16m^3 + 8m^2 - 4m + 3m - 1$
$= 16m^3 + 8m^2 - m - 1$

Remedial

Learners who persist in collecting unlike terms will need a lot of reminding that they may only collect ***LIKE*** terms. Be prepared to remediate this problem on an on-going basis.

If your learners are still making sign problems, write the sign rules on the board and keep them there throughout this chapter. Your learners must learn these by rote:

$+ \times + = +$	and	$+ \div + = +$
$- \times - = +$	and	$- \div - = +$
$+ \times - = -$	and	$+ \div - = -$
$- \times + = -$	and	$- \div + = -$

Exercise 2

Learner's Book page 267

Guidelines on how to implement this activity

Revise the FOIL method of multiplying two binomials that learners learnt in Chapter 5. Work through the Worked examples with your class, making sure that they all understand each example. Then ask them to do Exercise 2 on their own. Walk through your class as they do this exercise and keep an eye open for problems that need general remediation.

Suggested answers

1.1 $(a + b)(a + c) = a^2 + ac + ab + bc$
1.2 $(a - b)(b + c) = ab + ac - b^2 - bc$
1.3 $(m + 2n)(m + 3n) = m^2 + 3mn + 2mn + 6n^2 = m^2 + 5mn + 6n^2$
1.4 $(a + 5b)(5a - b) = 5a^2 - ab + 5ab - 5b^2 = 5a^2 + 4ab - 5b^2$
1.5 $(3x + 2y)(6 - 5y) = 18x - 15xy + 12y - 10y^2$
1.6 $(xy + 8)(xy - 1) = x^2y^2 - xy + 8xy - 8 = x^2y^2 + 7xy - 8$
1.7 $(pq - 3r)(2pq + r) = 2p^2q^2 + pqr - 6pqr - 3r^2 = 2p^2q^2 - 5pqr - 3r^2$
1.8 $(6m + n)(5m - 4n) = 30m^2 - 24mn + 5mn - 4n^2 = 30m^2 - 19mn - 4n^2$
1.9 $(2s + 9t)(2s - 7t) = 4s^2 - 14st + 18st - 63t^2 = 4s^2 + 4st - 63t^2$
1.10 $\left(\frac{1}{2}x + y\right)\left(\frac{1}{2}x - 2y\right) = \frac{1}{4}x^2 - xy + \frac{1}{2}xy - 2y^2 = \frac{1}{4}x^2 - \frac{1}{2}xy - 2y^2$

2.1	$(x + 8)(x - 8) = x^2 - 64$	**2.2**	$(x + 8)^2 = x^2 + 16x + 64$
2.3	$(x - 8)^2 = x^2 - 16x + 64$	**2.4**	$(2a + 3)(2a - 3) = 4a^2 - 9$
2.5	$(2a + 3)^2 = 4a^2 + 12a + 9$	**2.6**	$(2a - 3)^2 = 4a^2 - 12a + 9$
2.7	$(4 + 5a)(4 - 5a) = 16 - 25a^2$	**2.8**	$(4 + 5a)^2 = 16 + 40a + 25a^2$
2.9	$(4 - 5a)^2 = 16 - 40a + 25a^2$	**2.10**	$(xy + 9)(xy - 9) = x^2y^2 - 81$
2.11	$(xy + 9)^2 = x^2y^2 + 18xy + 81$	**2.12**	$(xy - 9)^2 = x^2y^2 - 18xy + 81$
2.13	$(12s + 5t)(12s - 5t) = 144s^2 - 25t^2$	**2.14**	$(12s + 5t)^2 = 144s^2 + 120st + 25t^2$
2.15	$(12s - 5t)^2 = 144s^2 - 120st + 25t^2$		

Remedial

In Question 1.10 of Exercise 2, your learners need to be able to multiply fractions. Although they have been doing this for a number of years by now, you will still find that some learners struggle with this. Show them how $\frac{1}{2} \times \frac{1}{2} = \frac{1 \times 1}{2 \times 2} = \frac{1}{4}$.

In Question 2 of Exercise 2, if your learners struggle to find the products by inspection, encourage them to rather write out the brackets twice and then to use the FOIL method of multiplication. It is more important that they are able to expand the product ***correctly***, than that they are able to do so more ***quickly*** by inspection. Work through the solutions to Exercise 2 with your class, being careful to remediate any problems before moving on to the next unit.

Extension

Have stronger learners attempt the Challenge on their own.

Challenge

Learner's Book page 267

Guidelines on how to implement this activity

The Challenge has been designed for stronger learners. See which of them arrive at the correct solution. Work through the solution with the rest of the learners and show them how elegantly this problem can be solved. Challenge your stronger learners to experiment with the concept behind this challenge and to see if they can come up with similar problems of their own.

Suggested Answers

$$\begin{aligned} 65^2 - 35^2 &= (65 - 35)(65 + 35) \\ &= (30)(100) \\ &= 3\ 000 \end{aligned}$$

Unit 2 Factorisation

Learner's Book page 268

Unit focus

This unit focusses on the following:

- factorising algebraic expressions by taking out a common factor,
- factorising algebraic expressions by recognising the difference of two squares,
- factorising trinomials,
- using factorisation to simplify algebraic expressions, and
- using factorisation to simplify algebraic fractions.

Background information on factorisation

The skill of factorisation is probably one of the most important algebraic skills that your learners will learn at school level. Three kinds of factorising are required in Grade 9: factorising by taking out a common factor, factorising the difference of squares and factorising a trinomial. Of these three, by far the most problematic from most learners' points of view is factorising a trinomial. After learning these three techniques of factorising, your learners will apply their knowledge of factorising to use factorisation to simplifying algebraic expressions and fractions.

Exercise 1

Learner's Book page 271

Guidelines on how to implement this activity

Discuss what it means to factorise a number. Read through the text on factorising a number with your class. Emphasise the key points about factorising in the *Key concepts*, then move on to a discussion about what it means to factorise an algebraic expression. Point out that factorising is the reverse of expanding an expression through multiplying.

Introduce the first technique of factorising, namely taking out a common factor. Work through the worked examples with your class. Emphasise that your learners must always multiply their factorised answers out. They can either do this in a margin on the side of their workbook, or on a piece of scrap paper. Learners must NOT multiply their factorised answers out below each answer. This gives the impression that the multiplication is part of the answer, which it is not. So, it is not correct to say, for example, that:

$3x + 9 = 3(x + 3)$
$\quad\quad\;\; = 3x + 9$

This kind of circular reasoning must be avoided at all costs. Later, when your learners become more proficient, they do not have to write out their checking, but can do it mentally. However, they ***must always*** check their factorising.

Once you feel confident that your learners understand what is involved in taking out a common factor, ask them to do Exercise 1.

Suggested Answers

1.1	$15x - 3 = 3(5x - 1)$	**1.2**	$21x + 28y = 7(3x + 4y)$
1.3	$10x - 25 = 5(2x - 5)$	**1.4**	$p^2 + 6p = p(p + 6)$
1.5	$5t + t^3 = t(5 + t^2)$	**1.6**	$4s^2 - s = s(4s - 1)$
1.7	$16x^4 + 4x^2 = 4x^2(4x^2 + 1)$	**1.8**	$d - 5d^3 = d(1 - 5d^2)$
1.9	$xy + xyz = xy(1 + z)$	**1.10**	$ab - b^2 = b(a - b)$
1.11	$x^3 + x^2 = x^2(x + 1)$	**1.12**	$3b^5 - 18b^2 = 3b^2(b^3 - 6)$
1.13	$5x^3 + 10x + 25 = 5(x^3 + 2x + 5)$	**1.14**	$3xy + 4xz + 5x = x(3y + 4z + 5)$
1.15	$2ab - 8c + 4b = 2(ab - 4c + 2b)$	**1.16**	$4z^3 - 8z = 4z(z^2 - 2)$
1.17	$3xy + 12xz + 6x = 3x(y + 4z + 2)$		
1.18	$10x^2 + 12x^3 + 14x^4 = 2x^2(5 + 6x + 7x^2)$		
1.19	$10ab - 15ab^2 - 5b = 5b(2a - 3ab - 1)$		
1.20	$14y + 7xy - 21y^2 = 7y(2 + x - 3y)$		
2.1	$15a + 16$ cannot be factorised	**2.2**	$15ab - 35b = 5b(3a - 7)$
2.3	$8xy + 16y = 8y(x + 2)$	**2.4**	$8xy + 16y - 24x = 8(xy + 2y - 3x)$
2.5	$15ab - 35b + 12a$ cannot be factorised		
2.6	$7t^2 - 14t + 21 = 7(t^2 - 2t + 3)$	**2.7**	$10p^2 + 11q$ cannot be factorised
2.8	$8xy - 8yz + 24y = 8y(x - z + 3)$		
2.9	$10a^8 - 40a^5 + 20a^4 = 10a^4(a^4 - 4a + 2)$		
2.10	$36x^3y^2z + 18x^2yz^2 - 9x^4z = 9x^2z(4xy^2 + 2yz - x^2)$		

Remedial

In Question 2, some of the expressions cannot be factorised. Some of your learners will find this disconcerting, but reassure them that it is not always possible to factorise an expression. In this case, the expression should simply be left alone.

Extension

Reserve questions 2.9 and 2.10 as extension, so as not to confuse weaker learners.

Exercise 2

Learner's Book page 273

Guidelines on how to implement this activity

The next section of this unit deals with factorising the difference of squares. It is important that your learners understand the phrase "difference of squares". For a difference of squares to be present, there must be two squares separated by a minus sign. Point out that the minus sign is key – this idea does not apply to the sum of squares.

Work through the worked examples with your class. Show them yet again how important it is to always check one's factorising by multiplying the answer out. If the result of this multiplication check is not identical to the original expression, the factorising is wrong and learners must try again. Ask your class to do Exercise 2, which will test their ability to factorise the difference of squares.

Suggested Answers

1.1	$x^2 - 9 = (x - 3)(x + 3)$	**1.2**	$x^2 - 16 = (x - 4)(x + 4)$
1.3	$x^2 - 100 = (x - 10)(x + 10)$	**1.4**	$36 - x^2 = (6 - x)(6 + x)$
1.5	$25 - x^2 = (5 - x)(5 + x)$	**1.6**	$x^2 - 49y^2 = (x - 7y)(x + 7y)$
1.7	$9a^2 - b^2 = (3a - b)(3a + b)$	**1.8**	$4x^2 - 9y^2 = (2x - 3y)(2x + 3y)$
1.9	$81x^2 - 16 = (9x - 4)(9x + 4)$	**1.10**	$25a^2 - 4 = (5a - 2)(5a + 2)$
1.11	$49a^2 - 4 = (7a - 2)(7a + 2)$	**1.12**	$36 - 25x^2 = (6 - 5x)(6 + 5x)$
1.13	$25 - 4x^2 = (5 - 2x)(5 + 2x)$	**1.14**	$144 - 121x^2 = (12 - 11x)(12 + 11x)$

1.15 $1 - x^4 = (1 - x^2)(1 + x^2) = (1 - x)(1 + x)(1 + x^2)$

1.16 $64m^4 - n^4 = (8m^2 - n^2)(8m^2 + n^2)$

2
$$\begin{aligned}(a + b)(a + b) &= a^2 + ab + ab + b^2\\ &= a^2 + 2ab + b^2\\ &\neq a^2 + b^2\end{aligned}$$

3.1	$\frac{1}{9}x^2 - y^2 = \left(\frac{1}{3}x - y\right)\left(\frac{1}{3}x + y\right)$	**3.2**	$x^2 - \frac{1}{16}y^2 = \left(x - \frac{1}{4}y\right)\left(x + \frac{1}{4}y\right)$
3.3	$\frac{16}{25}p^2 - \frac{1}{4}q^2 = \left(\frac{4}{5}p - \frac{1}{2}q\right)\left(\frac{4}{5}p + \frac{1}{2}q\right)$	**3.4**	$\frac{36}{49}a^2 - \frac{81}{64}b^2 = \left(\frac{6}{7}a - \frac{9}{8}b\right)\left(\frac{6}{7}a + \frac{9}{8}b\right)$

Remedial

When taking out a common factor, if learners do not take out the highest common factor, the factorised expression has not been factorised fully. Encourage your learners to always look at their factorised answer critically, asking themselves: "Am I done?" If any factor can be further factorised, they should do so.

Extension

The Challenge question at the end of the exercise has been included for those learners who have coped well with this work and feel ready to be stretched further.

Challenge

Learner's Book page 273

Guidelines on how to implement this activity

This exercise is designed to extend the stronger learners. It should not be prescribed for learners experiencing any difficulty with the material.

Suggested Answers

1 $1\frac{7}{9}x^2 - 1 = \frac{16}{9}x^2 - 1 = \left(\frac{4}{3}x - 1\right)\left(\frac{4}{3}x + 1\right)$

2 $0{,}25s^2 - t^2 = (0{,}5s - t)(0{,}5s + t)$

3 $1{,}21m^2 - 0{,}16n^2 = \frac{121}{100}m^2 - \frac{16}{100}n^2 = \left(\frac{11}{10}m - \frac{4}{10}n\right)\left(\frac{11}{10}m + \frac{4}{10}n\right)$

Exercise 3

Learner's Book page 275

Guidelines on how to implement this activity

Thus far, learners have only been required to do one kind of factorising at a time – either by taking out a common factor, or by factorising the difference of squares. The next section in this unit combines these two concepts. It is absolutely key to all kinds of factorising that learners should first look for a common factor, if one exists. This is the golden rule of factorising that ***always*** applies.

Work through the worked examples in the Learner's Book. Spend some time on each example, because there is much to learn here. Point out that the common factor is always taken out first. In the second example, there are fractions. This example can actually be factorised in two different ways by treating the common factor differently. Point out that this leads to two different solutions. It is important that your learners appreciate that although these solutions are different, they are also equivalent. In other words, when each of these two solutions is multiplied out, they both result in the original expression.

Learners should complete this exercise on their own. Remind learners (and keep on reminding them) that they should ***always*** first take out a common factor, if one exists. Also remind them that some expressions cannot be factorised. They will find a couple of examples of this in Question 2 of this exercise. Finally, remind them that they should always check their factorising by multiplying their answer out. Check your learners' answers before continuing with the next section.

Suggested Answers

1.1 $8x^2 - 50 = 2(4x^2 - 25)$
$= 2(2x - 5)(2x + 5)$

1.2 $18a^2 - 2 = 2(9a^2 - 1)$
$= 2(3a - 1)(3a + 1)$

1.3 $2x^2 - 8 = 2(x^2 - 4)$
$= 2(x - 2)(x + 2)$

1.4 $24 - 6x^2 = 6(4 - x^2)$
$= 6(2 - x)(2 + x)$

1.5 $3x^2 - 48 = 3(x^2 - 16)$
$= 3(x - 4)(x + 4)$

1.6 $144x^4 - 36y^4 = 36(4x^4 - y^4)$
$= 36(2x^2 - y^2)(2x^2 + y^2)$

1.7 $\frac{4}{9}a^2 - 25b^2 = \left(\frac{2}{3}a - 5b\right)\left(\frac{2}{3}a + 5b\right)$

1.8 $\frac{1}{4}a^2 - \frac{1}{9}b^2 = \left(\frac{1}{2}a - \frac{1}{3}b\right)\left(\frac{1}{2}a + \frac{1}{3}b\right)$

1.9 $49x^2 - \frac{1}{25} = \left(7x - \frac{1}{5}\right)\left(7x + \frac{1}{5}\right)$

1.10 $\frac{16}{25}a^2 - b^8 = \left(\frac{4}{5}a - b^4\right)\left(\frac{4}{5}a + b^4\right)$

2.1 $4x^2 + 1$ cannot be factorised
2.2 $3t^2 - 27 = 3(t^2 - 9) = 3(t - 3)(t + 3)$
2.3 $16x^2 - 25x = x(16x - 25)$
2.4 $18 - 8t^2 = 2(9 - 4t^2) = 2(3 - 2t)(3 + 2t)$
2.5 $20x^2y^2 - 45x^2z^2 = 5x^2(4y^2 - 9z^2) = 5x^2(2y - 3z)(2y + 3z)$
2.6 $4t^3 + 4t^2 + 16t = 4t(t^2 + t + 4)$
2.7 $x - 9x^3 = x(1 - 9x^2) = x(1 - 3x)(1 + 3x)$
2.8 $16a^2 - 32 = 16(a^2 - 16) = 16(a - 4)(a + 4)$
2.9 $4a^4 - 36a^2b^2 = 4a^2(a^2 - 9b^2) = 4a^2(a - 3b)(a + 3b)$
2.10 $7ab - 28b^3 = 7b(a - 4b^2) = 7b(a - 2b)(a + 2b)$
2.11 $12z^4 - 3z^2 = 3z^2(4z^2 - 1) = 3z^2(2z - 1)(2z + 1)$
2.12 $24x^3 + 15x = 3x(8x^2 + 5)$
2.13 $9x^4 - 36y^2 = 9(x^4 - 4y^2) = 9(x^2 - 2y)(x^2 + 2y)$
2.14 $32b^2 + 8b^4 = 8b^2(4 + b^2)$
2.15 $32b^2 - 8b^4 = 8b^2(4 - b^2) = 8b^2(2 - b)(2 + b)$
2.16 $8xy + 24xy^2 - 16xy^3 = 8xy(1 + 3y - 2y^2)$
2.17 $\frac{1}{2}a^2 - \frac{1}{8}b^2 = \frac{1}{2}\left(a^2 - \frac{1}{4}b^2\right) = \frac{1}{2}\left(a - \frac{1}{2}b\right)\left(a + \frac{1}{2}b\right)$
2.18 $8x^4 - 50y^2 = 2(4x^4 - 25y^2) = 2(2x^2 - 5y)(2x^2 + 5y)$
2.19 $4x^2y - 49xy^2 = xy(4x - 49y)$
2.20 $\frac{1}{64}x^6 - 25 = \left(\frac{1}{8}x^3 - 5\right)\left(\frac{1}{8}x^3 + 5\right)$

Remedial

If your learners struggle to recognise a perfect square, give them the following guidelines: Any variable or number that has an even exponent is a perfect square. Also, any number that can be written with an even exponent is a perfect square. When factorising the difference of squares, where one of the squares is a constant, learners often halve the constant instead of taking the square root. If they check their answers by multiplying out, they will discover this mistake for themselves.

Extension

Reserve questions 2.17–2.20 for stronger learners.

Exercise 4 Learner's Book page 277

Guidelines on how to implement this activity

The next section deals with factorising expressions that contain brackets. You will find that many learners have a knee-jerk reaction when they see brackets and tend to want to multiply them out before doing anything else. Although this approach is not incorrect, it can save a lot of time if your learners learn to work with the brackets instead of always trying to eliminate them. The first two worked examples illustrate this idea. Work through these with your class and show them how bracketed expressions can themselves sometimes be viewed as common factors or as perfect squares. The second two worked examples show how terms can be grouped by putting in brackets, and how this grouping can lead to useful common factors.
In the case of the second example, the terms could also be grouped as follows:

$$\begin{aligned} ax - bx + 4a - 4b &= ax + 4a - bx - 4b \\ &= a(x + 4) - b(x + 4) \\ &= (x + 4)(a - b). \end{aligned}$$

Show your class how both answers are equivalent, even though the order of the factors is different. (Your learners will explore this idea for themselves in the next Challenge.)
Introduce learners to the concept of a swap-around, which is a useful technique to use when there are common factors present, but the signs are the wrong way round. Work through the worked examples with your class, making sure that your learners understand these new ideas. You may have to pause here and revise a few of these concepts before asking them to do Exercise 4.

Suggested Answers

1.1 $a(x + y) + b(x + y) = (x + y)(a + b)$

1.2 $a(x - y) - b(y - x) = a(x - y) + b(x - y)$
$= (x - y)(a + b)$

1.3 $5(m - n) + p(n - m) = 5(m - n) - p(m - n)$
$= (m - n)(5 - p)$

1.4 $(m + n)^2 + p(m + n) = (m + n)(m + n + p)$

1.5 $7(a + b)^2 - (a + b) = (a + b)(7(a + b) - 1)$

1.6 $(x + y)(a + b) - (x + y)^2 = (x + y)(a + b - (x + y))$
$= (x + y)(a + b - x - y)$

1.7 $(s - t)^2 - 25 = (s - t - 5)(s - t + 5)$

1.8 $(m - n)^2 - p^2 = (m - n - p)(m - n + p)$

1.9 $16 - (m - n)^2 = (4 - (m - n))(4 + (m - n)$
$= (4 - m + n)(4 + m - n)$

1.10 $(a - 3b)^2 - 1 = (a - 3b - 1)(a - 3b + 1)$

1.11 $64 - (2a + b)^2 = (8 - (2a + b))(8 + 2a + b)$
$= (8 - 2a - b)(8 + 2a + b)$

1.12 $49 - (x + y)^2 = (7 - x - y)(7 + x + y)$

1.13 $(a + 3)^2 - (s + 4)^2 = (a + 3 - (s + 4))(a + 3 + s + 4)$
$= (a - s - 1)(a + s + 7)$

1.14 $(a - b)^2 - (b - a) = (a - b)^2 + (a - b)$
$= (a - b)(a - b + 1)$

1.15 $(p + q)^2 - 4(p + q) = (p + q)(p + q - 4)$
1.16 $(a + b)^2 - 9c^2 = (a + b - 3c)(a + b + 3c)$
1.17 $(a + b)^2 - (a + b)c = (a + b)(a + b - c)$
1.18 $x(y - 2) + y(y - 2) - (2 - y) = x(y - 2) + y(y - 2) + (y - 2)$
$= (y - 2)(x + y + 1)$
1.19 $m(n + 2) + 3n + 6 = m(n + 2) + 3(n + 2)$
$= (n + 2)(m + 3)$
1.20 $xy - yz + bx - bz = y(x - z) + b(x - z)$
$= (x - z)(y + b)$

Remedial

When factorising an expression like $x(a + 3) + y(3 + a)$, learners do not recognise that $(a + 3) = (3 + a)$ and instead feel that they need to use some kind of swap-around technique here. Guide them to see that the swap-around is only used when the signs of both terms must be changed.

Extension

Prescribe the challenge activity for learners who have shown skill in mastering factorisation so far.

Challenge

Learner's Book page 277

Guidelines on how to implement this activity

The Challenge question at the end of the exercise has been included for those learners who have coped well with this work.

Suggested answers

1.1 $xy + 3x + yz + 3z = x(y + 3) + z(y + 3)$
$= (y + 3)(x + z)$
1.2 $xy + yz + 3x + 3z = y(x + z) + 3(x + z)$
$= (x + z)(y + 3)$
2 The answers are the same as the questions consist of the same terms arranged in a different order.

Exercise 5

Learner's Book page 280

Guidelines on how to implement this activity

Introduce learners to factorising trinomials. Learners often find factorising a trinomial to be very tricky. The best advice that you can give them is to make an attempt and then to check the validity of their attempt by multiplying the factors out. This process of multiplying the factors out actually teaches them about the various components of a trinomial. No matter how proficient your learners become at factorising – they still should check their answer by multiplying it out. There is simply no exception to this rule. This checking can be done on paper, or even mentally, once your learners become more proficient, but it ***must always*** be done.

Read through the text in the Learner's Book with your class, then work through the worked examples with them. Ask your class to do Exercise 5. Check their progress once they have completed Question 1, in which the signs are all positive. This is the easiest kind of trinomial to factorise as learners do not have to worry about what signs to put where. Make sure that you remediate any problems before asking your class to complete the exercise.

Suggested answers

1.1	$x^2 + 9x + 14 = (x + 2)(x + 7)$	**1.2**	$x^2 + 6x + 9 = (x + 3)(x + 3)$
1.3	$x^2 + 5x + 6 = (x + 2)(x + 3)$	**1.4**	$x^2 + 8x + 16 = (x + 4)(x + 4)$
1.5	$x^2 + 12x + 35 = (x + 5)(x + 7)$	**1.6**	$x^2 + 12x + 36 = (x + 6)(x + 6)$
2.1	$x^2 - 8x + 15 = (x - 5)(x - 3)$	**2.2**	$x^2 - 5x + 6 = (x - 6)(x + 1)$
2.3	$x^2 - 9x + 18 = (x - 6)(x - 3)$	**2.4**	$x^2 - 10x + 16 = (x - 8)(x - 2)$
2.5	$x^2 - 11x + 28 = (x - 7)(x - 4)$	**2.6**	$x^2 - 13x + 36 = (x - 9)(x - 4)$
3.1	$x^2 - 3x - 18 = (x - 6)(x + 3)$	**3.2**	$x^2 + x - 2 = (x + 2)(x - 1)$
3.3	$x^2 - x - 12 = (x - 4)(x + 3)$	**3.4**	$x^2 + x - 6 = (x + 3)(x - 2)$
3.5	$x^2 + 4x - 12 = (x + 6)(x - 2)$	**3.6**	$x^2 - 6x - 16 = (x - 8)(x + 2)$
4.1	$x^2 + 3x - 18 = (x + 6)(x - 3)$	**4.2**	$x^2 + 14x + 49 = (x + 7)(x + 7)$
4.3	$x^2 - 10x + 21 = (x - 7)(x - 3)$	**4.4**	$x^2 - 12x + 27 = (x - 9)(x - 3)$
4.5	$x^2 + 3x - 10 = (x + 5)(x - 2)$	**4.6**	$x^2 - 3x - 28 = (x - 7)(x + 4)$

Remedial

Learners often identify the correct factors when factorising trinomials, but use the wrong signs. Learners will discover this kind of mistake for themselves if they check their factorising by multiplying out. Encourage checking of every answer.

Extension

Reserve question 4 as extension for stronger learners.

Exercise 6

Learner's Book page 281

Guidelines on how to implement this activity

This section extends learners' knowledge of factorisation , as it combines the idea of a common factor and a trinomial. Explain that learners should first take out the common factor, if one exists, before continuing to factorise. Work through the worked examples with your class, then ask them to do Exercise 6.

Suggested answers

1.1 $2x^2 + 20x + 50 = 2(x^2 + 10x + 25)$
$= 2(x + 5)(x + 5)$

1.2 $x^2 - 10x - 9 = -(x^2 + 10x + 9)$
$= -(x + 9)(x + 1)$

1.3 $3x^2 - 18x - 27 = -3(x^2 + 6x + 9)$
$= -3(x + 3)(x + 3)$

1.4 $3x^2 + 27x + 42 = 3(x^2 + 9x + 14)$
$= 3(x + 7)(x + 2)$

1.5 $4x^2 + 20x + 16 = 4(x^2 + 5x + 4)$
$= 4(x + 4)(x + 1)$

1.6 $x^2 - 12x - 11 = -(x^2 + 12x + 11)$
$= -(x + 11)(x + 1)$

1.7 $3x^3 - 6x^2 - 24x = 3x(x^2 - 2x - 8)$
$= 3x(x - 4)(x + 2)$

1.8 $5x^3y - 5x^2y - 10xy = 5xy(x^2 - x - 2)$
$= 5xy(x - 2)(x + 1)$

2.1 $12x + x^2 + 27 = x^2 + 12x + 27$
$= (x + 9)(x + 3)$

2.2 $2x - 24 + 2x^2 = 2x^2 - 2x - 24$
$= 2(x^2 - x - 12)$
$= 2(x - 4)(x + 3)$

2.3 $24 - 10x + x^2 = x^2 - 10x + 24$
$= (x - 6)(x - 4)$

2.4 $30 - 4x + 2x^2 = 2x^2 - 4x - 30$
$= 2(x^2 - 2x - 15)$
$= 2(x - 5)(x + 3)$

2.5 $18 - 16x + 2x^2 = 2x^2 - 16x - 18$
$= 2(x^2 - 8x - 9)$
$= 2(x - 9)(x + 1)$

2.6 $14x - 36 + 2x^2 = 2x^2 - 14x - 36$
$= 2(x^2 - 7x - 18)$
$= 2(x - 9)(x + 2)$

Extension and remediation

Reserve question 2 for stronger learners to do on their own, while you work together with weaker learners to complete the question.

Exercise 7

Learner's Book page 282

Guidelines on how to implement this activity

In this section learners learn how to use factorisation to simplify algebraic fractions. When simplifying a fraction, learners must factorise the numerator and the denominator and then cancel common factors above and below the line. This section tests your learners' knowledge of all the factorisation techniques that they have learnt thus far in this unit. It lays an important foundation for future algebraic work at FET level. Work though the worked examples with your class, then ask them to do this exercise on their own.

Suggested answers

1.1 $\frac{3x + 3y}{xy + y^2} = \frac{3(x + y)}{y(x + y)}$
$= \frac{3}{y}$

1.2 $\frac{6a + 9b - 12c}{6} = \frac{3(2a + 3b - 4c)}{6}$
$= \frac{2a + 3b - 4c}{2}$

1.3 $\frac{4a + 2b}{16a^2 - 4b^2} = \frac{2(2a + b)}{4(2a - b)(2a + b)}$
$= \frac{1}{2(2a - b)}$

1.4 $\frac{81x^4 - 1}{9x^2 - 1} = \frac{(9x^2 - 1)(9x^2 + 1)}{9x^2 - 1}$
$= 9x^2 + 1$

1.5 $\frac{(a - b)^2 - 25}{3a - 3b + 15} = \frac{(a - b - 5)(a - b + 5)}{3(a - b + 5)}$
$= \frac{a - b - 5}{3}$

1.6 $\frac{2(k - 1)}{m(5k - 5)} = \frac{2(k - 1)}{5m(k - 1)}$
$= \frac{2}{5m}$

1.7 $\frac{8a^2 + 4ab}{2a + b} = \frac{4a(2a + b)}{2a + b}$
$= 4a$

1.8 $\frac{b - a}{a^2 - b^2} = \frac{b - a}{(a - b)(a + b)}$

1.9 $\frac{x^2 - y^2}{2x - 2y} = \frac{(x - y)(x + y)}{2(x - y)}$
$= \frac{x + y}{2}$

1.10 $\frac{x^2 - 16}{2x^2 - 8x} = \frac{(x - 4)(x + 4)}{2x(x - 4)}$
$= \frac{x + 4}{2x}$

1.11 $\frac{3x + 9y}{9y^2 - x^2} = \frac{3(x + 3y)}{(3y - x)(3y + x)}$
$= \frac{3}{3y - x}$

1.12 $\frac{49y^2 - 4}{7y + 2} = \frac{(7y - 2)(7y + 2)}{7y + 2}$
$= 7y - 2$

1.13 $\frac{2x^2 - 288}{3x + 36} = \frac{2(x^2 - 144)}{3(x + 12)}$
$= \frac{2(x - 12)(x + 12)}{3(x + 12)}$
$= \frac{2(x - 12)}{3}$

1.14 $\frac{16x^4 - 1}{1 - 2x} = \frac{(4x^2 - 1)(4x^2 + 1)}{1 - 2x}$
$= \frac{(2x - 1)(2x + 1)(4x^2 + 1)}{-(2x - 1)}$
$= -(2x + 1)(4x^2 + 1)$

1.15 $\frac{p^2 - q^2}{4p - 4q} = \frac{(p - q)(p + q)}{4(p - q)}$
$= \frac{p + q}{4}$

1.16 $\frac{c^2 - d^2}{d - c} = \frac{(c - d)(c + d)}{-(c - d)}$
$= -(c + d)$

1.17 $\frac{x^2 - 4x - 12}{x + 2} = \frac{(x - 6)(x + 2)}{x + 2}$
$= x - 6$

1.18 $\frac{x^2 - 9x + 20}{5x^2 - 40x + 75} = \frac{(x - 4)(x - 5)}{5(x^2 - 8x + 15)}$
$= \frac{(x - 4)(x - 5)}{5(x - 5)(x - 3)}$
$= \frac{x - 4}{5(x - 3)}$

1.19 $\frac{x^2 - 64}{x^2 + 15x + 56} = \frac{(x - 8)(x + 8)}{(x + 7)(x + 8)}$
$= \frac{x - 8}{x + 7}$

1.20 $\frac{30a + 25b - 20c}{6a - 4c + 5b} = \frac{5(6a + 5b - 4c)}{6a + 5b - 4c}$
$= 5$

Remedial

Do additional examples with learners who are struggling with this concept. If necessary revise working with fractions and cancelling. Identify where learners are having difficulty and revise the particular problem with these learners.

Extension

The Challenge question at the end of the exercise has been included for those learners who have coped well with this work.

Challenge

Learner's Book page 282

Guidelines on how to implement this activity

Prescribe this activity to learners who have managed admirably in the chapter so far.

Suggested answers

1 $\frac{2x(x^2 - 25)}{(4x^2 - 6x)(x + 5)} = \frac{2x(x - 5)(x + 5)}{2x(2x - 3)(x + 5)}$
$= \frac{x - 5}{2x - 3}$

2 $\frac{(2a + 3c)(a^2 - 25b^2)}{(2a + 10b)(4a^2 - 9c^2)} = \frac{(2a + 3c)(a - 5b)(a + 5b)}{2(a + 5b)(2a - 3c)(2a + 3c)}$
$= \frac{(a - 5b)}{2(2a - 3c)}$

Unit 3 Solve equations

Learner's Book page 283

Unit focus

This unit focusses on the following:
- revise solving equations by inspection
- revise solving equations using additive and multiplicative inverses
- revise solving equations using the laws of exponents.

Background information on solving equations

This unit serves as a revision of the work already done in Chapter 5 on solving equations.

Exercise 1

Learner's Book page 285

Guidelines on how to implement this activity

Read through the text about solving equations with your class, then work through the worked examples with them. This is pure revision of work that they have already done in Chapter 5, so move on directly to Exercise 1. However, do take the time to check their answers to this exercise carefully, as this serves as a baseline assessment that will show you where your learners stand with regard to this work.

Suggested answers

1.1 $x + x + 10 = 7x$

1.2 $x + x + 10 = 7x$
$10 = 7x - 2x$
$5x = 10$
$x = 2$ years old

1.3 Cara is 12 years old.

2.1 $3x - 8 = 10$
$18 - 8 = 10$
$x = 6$

2.2 $\frac{7x}{3} = 21$
$63 \div 3 = 21$
$7 \times 9 = 63$
$x = 9$

2.3 $25 - 3x = 19$
$25 - 6 = 19$
$x = 2$

2.4 $37 = 4x + 5$
$37 = 32 + 5$
$x = 8$

3.1 $4x + 9 = 2x + 25$
$4x - 2x = 25 - 9$
$2x = 16$
$x = 8$

3.2 $8(x - 1) + 6 = 22$
$8x - 8 + 6 = 22$
$8x = 22 + 2$
$8x = 24$
$x = 3$

3.3 $6x - 21 = 5(x - 3)$
$6x - 21 = 5x - 15$
$6x - 5x = -15 + 21$
$x = 6$

3.7 $\frac{x}{5} - \frac{x}{10} = 3$
$2x - x = 30$
$x = 30$

3.8 $\frac{x + 2}{3} = \frac{1 - x}{2}$
$2(x + 2) = 3(1 - x)$
$2x + 4 = 3 - 3x$
$2x + 3x = 3 - 4$
$5x = -1$
$x = -\frac{1}{5}$

3.9 $0{,}3x = 0{,}1x + 4$
$0{,}3x - 0{,}1x = 4$
$0{,}2x = 4$
$x = 20$

3.10 $\frac{x}{50} = \frac{x}{25} - 2$
$x = 2x - 100$
$100 = 2x - x$
$x = 100$

4.1 $x^2 + 9 = 25$
$x^2 = 16$
$x = \pm 4$

4.2 $x^2 = \frac{36}{49}$
$x = \pm \frac{6}{7}$

4.3 $3\,x^3 = 24$
$x^3 = -8$
$x = -2$

4.4 $\frac{-x^3}{2} = \frac{54}{4}$
$x^3 = 27$
$x = 3$

4.5 $x^7 = -128$
$x = \sqrt[7]{-128}$
$x = -2$

4.6 $13^x = 169$
$13^x = 13^2$
$x = 2$

4.7 $2^x + 1 = 17$
$2^x = 16$
$2^x = 2^4$
$x = 4$

4.8 $1 - 5^x = -124$
$1 + 124 = 5^x$
$5^x = 125$
$5^x = 5^3$
$x = 3$

4.9 $2.5^x = 250$
$5^x = 125$
$5^x = 5^3$
$x = 3$

4.10 $\frac{3^x}{5} = 16{,}2$
$3^x = 81$
$3^x = 3^4$
$x = 4$

Remedial
If some of your learners struggle with the basic concepts in Exercise 1, give them as much practice as possible with similar questions of your own.

Extension
Use your stronger learners to help to mentor the weaker learners, or to think up additional practice questions for them.

Unit 4 Solve equations using factorisation

Learner's Book page 287

Unit focus
This unit focusses on the following:
- use the zero-product rule to solve equations
- use factorisation to solve real-life problems.

Background information on solving equations using factorisation

In this unit, your learners will apply what they have learnt about factorising, as they use factorisation to solve equations. They will learn about the all-important zero-product and will apply their knowledge to solving real-life problems as well.

Exercise 1

Learner's Book page 289

Guidelines on how to implement this activity

Read through the text about the zero-product rule with the class and explain this very important concept carefully. The idea that an equation can have more than one solution is not entirely new to your learners, as they have encountered this before, for example in equations of the form $x^2 = k^2$, where the solutions are $x = \pm$ k. For example, there are two solutions to the equation $x^2 = 9$, namely $x = 3$ or $x = -3$.
The zero-product rule is one that they will use frequently from now on, so make sure that your learners understand this concept. Work through the worked examples with your class. Pay special attention to the fact that learners may only divide an equation by a common factor if that common factor is a constant. If the common factor contains a variable and if the equation is divided throughout by that common factor, one solution to the equation may get lost. So, for example, in the second example, it would be incorrect to divide throughout by $(x - 5)$. If they did so, the solution $x = 5$ would get lost. The golden rule here is to factorise the equation fully and then to apply the zero-product rule.
The *Key concepts* explain the difference between solving for an equation in which the highest power is 1 (a linear equation) and an equation in which the highest power is 2 (a quadratic equation). We apply the zero-product in the second case.
Ask your class to do Exercise 1. Walk through your class as they work, and watch them carefully to see how well they are coping with this new work. Remediate any problems as they arise. Make sure that your class has coped with Questions 1 and 2 before you allow them to continue with Questions 3-6.

Suggested answers

1.1 $12x - 6x^2 = 0$
$6x(2 - x) = 0$
$6x = 0$ or $2 - x = 0$
$x = 0$ or $x = 2$

1.2 $6(x - 2) + 8x(x - 2) = 0$
$(x - 2)(6 + 8x) = 0$
$x - 2 = 0$ or $6 + 8x = 0$
$x = 2$ or $x = -0{,}75$

1.3 $0 = x^2 - 50x$
$0 = x(x - 25)$
$x = 0$ or $x = 25$

1.4 $(x + 3) - x(x + 3) = 0$
$(x + 3)(1 - x) = 0$
$x = -3$ or $x = 1$

1.5 $x^2 - 2x - 15 = 0$
$(x - 5)(x + 3) = 0$
$x = 5$ or $x = -3$

1.6 $7(2 - x) + 14x(x - 2) = 0$
$7(2 - x) - 14x(2 - x) = 0$
$(2 - x)(7 - 14x) = 0$
$2 - x = 0$ or $7 - 14x = 0$
$x = 2$ or $x = 0{,}5$

1.7 $5x^4 - 405 = 0$
$5(x^4 - 81) = 0$
$5(x^2 - 9)(x^2 + 9) = 0$
$5(x - 3)(x + 3)(x^2 + 9) = 0$
$x - 3 = 0 \quad \text{or} \quad x + 3 = 0$
or $x^2 + 9 = 0$
$x = 3 \quad \text{or } x = -3$

1.8 $x^2 - 11x + 18 = 0$
$(x - 9)(x - 2) = 0$
$x = 9 \quad \text{or } x = 2$

1.9 $3x^2 - 125 + 2x^2 = 0$
$5x^2 - 125 = 0$
$5(x^2 - 25) = 0$
$5(x - 5)(x + 5) = 0$
$x = 5 \quad \text{or } x = -5$

1.10 $2x^2 + 24x + 70 = 0$
$2(x^2 + 12x + 35) = 0$
$2(x + 7)(x + 5) = 0$
$x = -7 \quad \text{or } x = -5$

2.1 $9x^2 = 18x$
$9x^2 - 18x = 0$
$9x(x - 2) = 0$
$9x = 0 \quad \text{or } x - 2 = 0$
$x = 0 \quad \text{or } x = 2$

2.2 $x^2 - 8x = 6x - x^2$
$2x^2 - 14x = 0$
$2x(x - 7) = 0$
$x = 0 \quad \text{or} \quad x = 7$

2.3 $5(2 - x) = x(2 - x)$
$5(2 - x) - x(2 - x) = 0$
$(2 - x)(5 - x) = 0$
$x = 2 \quad \text{or } x = 5$

2.4 $x - 1 + 2x(1 - x) = 0$
$-(1 - x) + 2x(1 - x) = 0$
$(1 - x)(2x - 1) = 0$
$x = 1 \quad \text{or } x = 0{,}5$

2.5 $x^2 - 8x - 3(8x - x^2) = 0$
$-(8x - x^2) - 3(8x - x^2) = 0$
$(8x - x^2)(-1 - 3) = 0$
$-4x(8 - x) = 0$
$x = 0 \quad \text{or } x = 8$

2.6 $5x - 10 + 3(2 - x) = 0$
$5 - 3(x - 2) = 0$
$2(x - 2) = 0$
$x = 2$

2.7 $x^2 - 14x = 13$
$0 = x^2 + 14x + 13$
$0 = (x + 13)(x + 1)$
$x = -13 \quad \text{or } x = -1$

2.8 $25 - x^2 = x - 5$
$0 = x^2 + x - 30$
$0 = (x - 5)(x + 6)$
$x = 5 \quad \text{or } x = -6$

2.9 $3x^2 - 1 = 2x^2 + 8$
$x^2 - 9 = 0$
$(x - 3)(x + 3) = 0$
$x = 3 \quad \text{or } x = -3$

2.10 $84 - x^2 = 5x$
$0 = x^2 - 5x + 84$
$0 = (x - 12)(x + 7)$
$x = 12 \quad \text{or } x = -7$

3 C $V = 4(x + 2)(x + 3)$ cm^3

4 If $y = 2x^2 - 4x - 3$,

4.1 $y = 2(-3)^2 - 4(-3) - 3 = 27$

4.2 $-3 = 2x^2 - 4x - 3$
$0 = 2x^2 - 4x$
$0 = 2x(x - 2)x = 0 - \; - \text{or } x = 2$

5.1 Square

5.2 Area of base of box $= (16x - 32)(16x - 32) = 256x^2 - 1\,024x + 1\,024 -$ mm^2
$= 256(x^2 - 4x + 4)$
Area of base of one domino $= (2x - 4)(x - 2) = 2x^2 - 8x + 8 -$ mm^2
$= 2(x^2 - 4x + 4)$
Therefore $256 \div 2 = 128$ dominoes will fit into the box.

6.1 $x^2 = 9$

$x = \pm 3$

6.2 $x^2 - 9 = 0$

$(x - 3)(x + 3) = 0$

$x = 3$ or $x = -3$

6.3 The solutions are the same.

Remedial

Learners may attempt to collect unlike terms in an attempt to simplify the equation. This could be an honest mistake, or else it could be a sub-conscious attempt to "make the problem go away". Either way, this kind of thinking needs patient and persistent remediation on your part.

Learners may also divide throughout by a common factor that includes a variable, thus losing one solution. Remind them that they need to factorise fully before applying the zero-product rule. They must guard against discarding possible solutions along the way.

Learners often find the correct roots, but apply the incorrect signs. They need to understand that in order to make a factor equal to zero, x should be the additive inverse of the constant within the brackets. If they check their answers by substituting them into the original equation, they will discover this kind of mistakes for themselves.

In Question 3, learners may need to be reminded that V = length x width x height.

Extension

If you feel that Question 6 will be too confusing for your weaker learners to attempt at this stage, set it as a Challenge question as well.

Challenge

Learner's Book page 289

Guidelines on how to implement this activity

Reserve this exercise only for learners who have coped easily with the work covered in this chapter so far. In Question 3 of the Challenge, your learners will have to expand the concept of the zero-product rule to include three factors, i.e. if A × B x C = 0, then A, B or C must be 0. See if they can make this leap for themselves.

Suggested answers

1

$x^4 - 9 = x^2 + 3$

$x^4 - x^2 - 12 = 0$

$(x^2 - 4)(x^2 + 3) = 0$

$(x - 2)(x + 2)(x^2 + 3) = 0$

$x = 2$ or $x = -2$

2

$x^2 + 8 = x^4 - 64$

$0 = x^4 - x^2 - 72$

$0 = (x^2 - 9)(x^2 + 8)$

$0 = (x - 3)(x + 3)(x^2 + 8)$

$x = 3$ or $x = -3$

3

$x^3 - 7x^2 + 6x = 0$

$x(x^2 - 7x + 6) = 0$

$x(x - 6)(x - 1) = 0$

$x = 0$ or $x = 6$ or $x = 1$

4

$(x + 2)(x - 9) = 9 - x$

$x^2 - 7x - 18 - 9 + x = 0$

$x^2 - 6x - 27 = 0$

$(x - 9)(x + 3) = 0$

$x = 9$ or $x = -3$

Unit 5 Generate ordered pairs

Learner's Book page 290

Unit focus

This unit focusses on the following:
- using substitution in equations to generate tables of ordered pairs
- using equations of the form $y = ax^2 + c$ to generate tables of ordered pairs

Background information on generating ordered pairs

The final unit of this chapter is devoted to generating tables of ordered pairs by using substitution. This work is lays the foundations for the work that they will be doing in the next chapter on graphs.

Exercise 1

Learner's Book page 291

Guidelines on how to implement this activity

Read through the introductory text with your class, then work through the worked example with them. This example requires simple substitution to find the missing values of y, and also the values of x. Learners have to be able to solve second-degree equations by finding the appropriate square root. Ask your class to do Exercise 1, which is a simple application of the work that was covered in the worked example. The Challenge question at the end of the exercise is suitable for all your learners to try, as Exercise 1 is quite short and not particularly demanding. It needs to be made explicit to learners that y's value is dependent on x's value, and that these values only exist together. Ensure learners know the term ordered pair.

Suggested answers

1

x	–3	–1	0	2	3,5
y	–20	–12	–8	0	6

2

x	–3	–1	2,5	–1,5	–11,5
y	$3\frac{3}{4}$	$2\frac{3}{4}$	1	3	8

3

x	–3	–2	–1	0	1	2	3
y	13	3	–3	–5	–3	3	13

4

x	–3	–1	0	1	2
y	10	2	1	2	5

5

x	–2	0	1	2	3
y	–8	4	1	–8	–23

Remedial

- Exercise 1 contains 3 questions which have been carefully graded in increasing order of difficulty. Question 1 is well within the scope of all your learners and requires only that they substitute correctly. Watch out (as usual!) for sign mistakes.
- In Question 2, the y-values are easily found using substitution. To find the x-values, learners will have to do a bit more thinking. Be prepared to remediate any misconceptions as they occur. Provide additional examples of your own for extra practice, if necessary.

- Question 3 builds on Question 2, but is a bit more difficult in that the coefficient of x^2 is negative. However, it is still within the scope of all your learners, provided that they do not lose their heads. Remediate any problems as they arise. Again, these are likely to be sign problems and/or collecting unlike terms in an attempt to simplify the problem.

Extension

Question 4 can easily be solved using substitution. Encourage all your leaners to try their hand at this, although you may have to demonstrate this process on the board for them to see. They should be pleasantly surprised at how simple this method really is, once they have your full worked example on the board on which to model their thinking. Prescribe the challenge activity to all learners.

Challenge

Learner's Book page 291

Guidelines on how to implement this activity

Ask those of your learners who need an extra challenge to devise their own problems along the lines of the Challenge question in this exercise, for a partner to solve. Without helping them at all, see if they grasp the idea that they should start out by making up an equation and then by simple substitution, identify some points that will (or will not) satisfy the equation.

Suggested answers

A (0; 8)
C (−1; 6)
E (−2; 0)
F (3; −10)

Consolidation

Learner's book page 293

Before doing this consolidation exercise, encourage learners to review the work covered in this chapter. Advise learners to use the summary and to revise their work. This exercise can be used as an informal assessment task for you to track how learners are coping with the chapter and the concepts covered. The mark allocation provides guidelines on how to assess learners.

Suggested answers

1.1	$18x - 9y = 9(2x - y)$	(1)
1.2	$36abc + 63cd = 9c(4ab + 7d)$	(1)
1.3	$5p + 6q + 30pq$ cannot be factorised	(1)
1.4	$10a^6 - 20a^5 + 40a^4 = 10a^4(a^2 - 2a + 4)$	(1)
1.5	$k^2 - 100 = (k - 10)(k + 10)$	(1)
1.6	$18p^2 - 2q^2 = 2(9p^2 - q^2)$	
	$= 2(3p - q)(3p + q)$	(2)
1.7	$x^2 + x - 20 = (x - 4)(x + 5)$	(1)
1.8	$12x + x^2 + 20 = (x + 10)(x + 2)$	(1)

1.9 $x^3 - 8x^2 + 7x = x(x^2 - 8x + 7)$
$= x(x - 7)(x - 1)$ (2)

1.10 $(x + y)^3 - 4(x + y) = (x + y)((x + y)^2 - 4)$
$= (x + y)(x + y - 2)(x + y + 2)$ (2)

2.1 $\frac{12a + 30b - 18c}{2} = 6a + 15b - 9c$ (2)

2.2 $2c(c^2 - 2cd + 5d^2 - 3) = 2c^2 - 4c^2d + 10cd^2 - 6c$ (2)

2.3 $\frac{p^2 - q^2}{6p + q} = \frac{(p - q)(p + q)}{6(p + q)}$
$= \frac{p - q}{6}$ (2)

2.4 $\frac{4a + 5b}{16a^2 - 25b^2} = \frac{4a + 5b}{(4a + 5b)(4a - 5b)}$
$= \frac{1}{4a - 5b}$ (2)

2.5 $\frac{2x^2 - 200}{3x - 30} = \frac{2(x - 10)(x + 10)}{3(x - 10)}$
$= \frac{2(x + 10)}{3}$ (2)

2.6 $-3p^2(p^3 + p^2q + pq^2 + pq) = -3p^5 - 3p^4q^2 - 3p^3q^2 - 3p^3q$ (2)

2.7 $\frac{36x^4 - 1}{6x^2 + 1} = \frac{(6x^2 + 1)(6x^2 - 1)}{6x^2 + 1}$
$= 6x^2 - 1$ (2)

3.1.1 $ab + 5a + bc + 5c = a(b + 5) + c(b + 5)$
$= (b + 5)(a + c)$ (1)

3.1.2 $ab + bc + 5a + 5c = b(a + c) + 5(a + c)$
$= (a + c)(b + 5)$ (1)

3.2 The answers are the same as the questions consist of the same terms in a different order. (2)

4.1 $x(x + 1) = 0$
$x = 0$ or $x = -1$ (1)

4.2 $x^2 - 5x + 4 = 0$
$(x - 4)(x - 1) = 0$
$x = 4$ or $x = 1$ (2)

4.3 $x^2 - x - 30 = 0$
$(x - 6)(x + 5) = 30$
$x = 6 -$ or $x = -5$ (2)

4.4 $6x + 12 + 3x(x + 2) = 0$
$6(x + 2) + 3x(x + 2) = 0$
$(x + 2)(6 + 3x) = 0$
$x = -2$ or $x = -2$ (2)

4.5 $8x^2 - 1 = 24 - x^2$
$9x^2 - 25 = 0$
$(3x - 5)(3x + 5) = 0$
$x = \frac{5}{3}$ or $x = -\frac{5}{3}$ (2)

4.6 $x(x - 3) - 4(3 - x) = 0$
$x(x - 3) + 4(x - 3) = 0$
$(x - 3)(x + 4) = 0$
$x = 3$ or $x = -4$ (2)

4.7 $2x^2 - 14x + 12 = 0$
$2(x^2 - 7x + 6) = 0$
$2(x - 6)(x - 1) = 0$
$x = 6$ or $x = 1$ (2)

4.8 $x^2 - 36 = x + 6$
$x^2 - x - 42 = 0$
$(x - 7)(x + 6) = 0$
$x = 7$ or $x = -6$ (2)

4.9 $27 = -3x^2 - 18x$
$3(x^2 + 6x + 9) = 0$
$3(x + 3)(x + 3) = 0$
$x = -3$ (2)

4.10 $5(22 - x) + 10x(x - 22) = 0$
$5(22 - x) - 10x(22 - x) = 0$
$(22 - x)(5 - 10x) = 0$
$x = 22$ or $x = 0,5$ (2)

5 $76^2 - 24^2 = (76 - 24)(76 + 24)$
$= (52)(100)$
$= 5\ 200$ (2)

6.1 $4x^2 - 1 = 0$

$4x^2 = 1$

$x^2 = \frac{1}{4}$ (1)

6.2 $x^2 = \frac{1}{4}$

$x = \sqrt{\frac{1}{4}}$

$x = \pm\frac{1}{2}$ (1)

6.3 $4x^2 - 1 = 0$

$(2x - 1)(2x + 1) = 0$

$x = \frac{1}{2}$ or $x = -\frac{1}{2}$ (1)

6.4 The solutions are the same. (1)

7.1 $x + 2$ (1)

7.2 $x(x + 2) = 48$ (1)

7.3 $x(x + 2) = 48$

$x^2 + 2x - 48 = 0$

$(x - 6)(x + 8) =$

$x = 6$ or $x = -8$ (3)

But age cannot be a negative number, thus Neo's age is 6. Her cousin's age is 8. The product of 6 and 8 is 48.

8.1 $y = (3)^2 - 3 - 9 = -3$ (1)

8.2 $3 = x^2 - x - 9$

$0 = x^2 - x - 12$

$(x - 4)(x + 3) = 0$

$x = 4$ or $x = -3$ (2)

8.3 $y = (-3)^2 - (-3) - 9 = 3$ (1)

8.4 $-3 = x^2 - x - 9$

$0 = x^2 - x - 6$

$(x - 3)(x + 2) = 0$

$x = 3 -$ or $x = -2$ (2)

9.1

x	–4	–3	–1	0	2	3
y	7	0	8	9	5	0

(6)

9.2 None of them satisfy the equation. (2)

[75]

Chapter 11 Graphs

Overview of concepts

Graphs

Unit 1 Analysing and interpreting global graphs

Unit 2 Draw global graphs

Unit 3 Draw algebraic linear graphs

Unit 4 Determine the equations of linear graphs

Content		Time allocations	LB page
Unit 1	Analysing and interpreting global graphs	3 hours	296
Unit 2	Drawing global graphs	3 hours	302
Unit 3	Drawing algebraic linear graphs	3 hours	306
Unit 4	Determining the equations of linear graphs	3 hours	314

Background information on graphs

Learners have worked with global graphs in earlier grades, and were introduced to algebraic graphs in Grade 8. The focus in Grade 9 is graphs as visual representations of a relationship. Increasingly that relationship is an equation. Algebraic graphs are visual representations of an algebraic equation. Often the table is used as an interim step for representing the equation as a graph, however learners are shown new methods of determining algebraic graphs without using a table.
Learners work with global graphs in order to learn how to recognise trends in a graphical format and be exposed to graphs in real life contexts. This improves learners' mathematical literacy. Learners who will pursue Mathematical literacy as a subject will work with global graphs in more depth in their FET work. On the other hand, algebraic graphs prepare learners for work on functions in FET Mathematics.

Generic teaching guidelines for teaching graphs

Learners should be encouraged to draw rough sketches of the global graphs in their exercise books. This allows learners to mark and experiment on the graph. Revise with learners the properties of graphs and ensure learners can identify whether graphs:
- are linear or non linear;
- are constant, increasing or decreasing;
- have maximum or minimum values; and
- represent discrete or continuous data.

When drawing global graphs learners must be neat and create accurate grids and

scales to represent the data. Encourage neatness and accuracy.
Algebraic graphs require additional attention, as these lay the foundation for future work in mathematics. Spend as much time as necessary clarifying plotting points from a table, working with the intercept methods and ultimately determining the equations of the graphs. Learners must be confident with algebraic linear graphs if they are to manage what is required of them at the FET level.

Resources

Examples of graphs from newspapers and magazines. Blank graph grids, excel graphs, cardboard and colour pens to create large poster graphs, scissors and rulers. Each learner should have their own calculator.

Unit 1 Analysing and interpreting global graphs

Learner's Book page 296

Unit focus

This unit focusses on the following:
- revising what you have already learnt about graphs; and
- analysing and interpret global graphs.

Background information on analysing and interpreting global graphs

Analysing and interpreting global graphs is a skill that goes beyond learning mathematics. Being able to accurately interpret data presented in graphical form allows learners to participate in an increasingly numerate world. Learners become able to interpret the information presented to them in newspapers, magazines and online, and make sense of the world around them.

Exercise 1

Learner's Book page 300

Guidelines on how to implement this activity

Discuss the properties of graphs. Show learners examples of graphs that are linear; non-linear; constant; increasing; decreasing; graphs with minimum values; graphs with maximum values; graphs that display continuous data and graphs that display discrete data. Learners need to be able to analyse graphs according to these properties. Work through the worked examples in the Learner's Book. Ensure learners can accurately interpret what each graph is representing, and answer the questions on each graph. Learners can complete this exercise in pairs.

Suggested answers

1.1	May		
1.2	January, February, March and December		
1.3	June	**1.4**	Winter
1.5	July		
2.1	10 cm	**2.2**	1,5 cm
3.1	2,25cm	**3.2**	2,5 cm
4.1	Accept 1,2 – 1,25 cm	**4.2**	Accept 1,2 – 1,25 cm
4.3	Accept 3,1 – 3,2 cm		
4.4	No, because zero multiplied by any number is zero.		
4.5	No		
5.1.1	R10	**5.1.2**	R10
5.1.3	R17,50		
5.2	R15		
5.3.1	No	**5.3.2**	No
5.4	Up to 2 kg	**5.5**	10 kg, for R45

Remedial

Work through additional examples if learners are struggling to interpret graphs correctly. Ensure learners understand and can identify the properties of graphs. Encourage learners to use these properties to report on the graph, before tackling the prescribed questions. Have learners create a chart with the key properties of global graphs on it, and to use this as a reference when analysing and interpreting global graphs.

Extension

Bring in real life examples of global graphs. These can be found in newspapers, magazines and online. Ask learners to interpret what the graphs mean and what information they are relaying. Assign learners into groups and have each group create questions for one of the graphs you have sourced. Swop the graphs around for other groups to answer the questions.

Unit 2 Drawing global graphs

Learner's Book page 302

Unit focus

This unit focusses on the following:
- revising drawing global graphs from given descriptions of a problem situation, and
- revising identifying features of global graphs.

Background information on drawing global graphs

Being able to draw a global graph extends learners ability to translate equivalent forms of relationships. It is an important skill and should be allowed the time required to master it. Learners have to use neatness, accuracy and appropriate scale choice. None of the work in this unit is new to Grade 9, and should be revision of Grade 8 work. It also creates an appropriate introduction to algebraic graphs, as it prepares learners to draw axes and plot points on a grid or plane.

Exercise 1

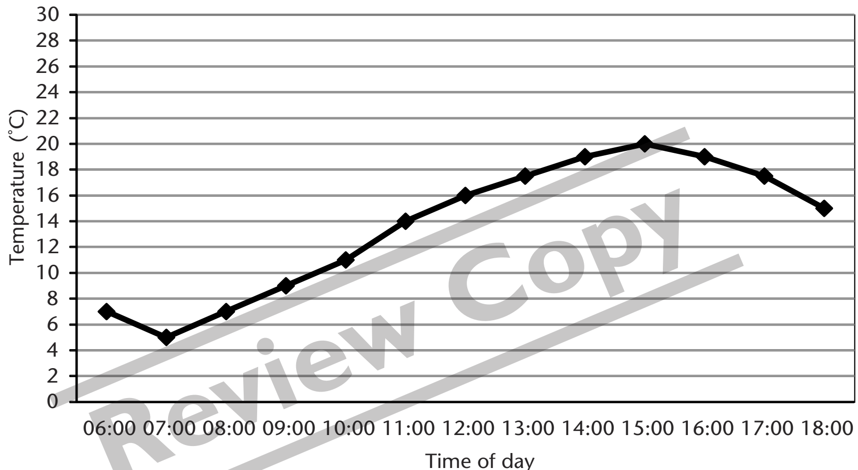
Learner's Book page 304

Guidelines on how to implement this activity

Revise the basics for drawing graphs. This involves setting up the axes, choosing an appropriate scale and labelling correctly. Discuss how to plot points on a grid or plane. Work through the examples in the Learner's Book, and if necessary supply additional examples for learners to practise with. Learners should complete this exercise on their own.

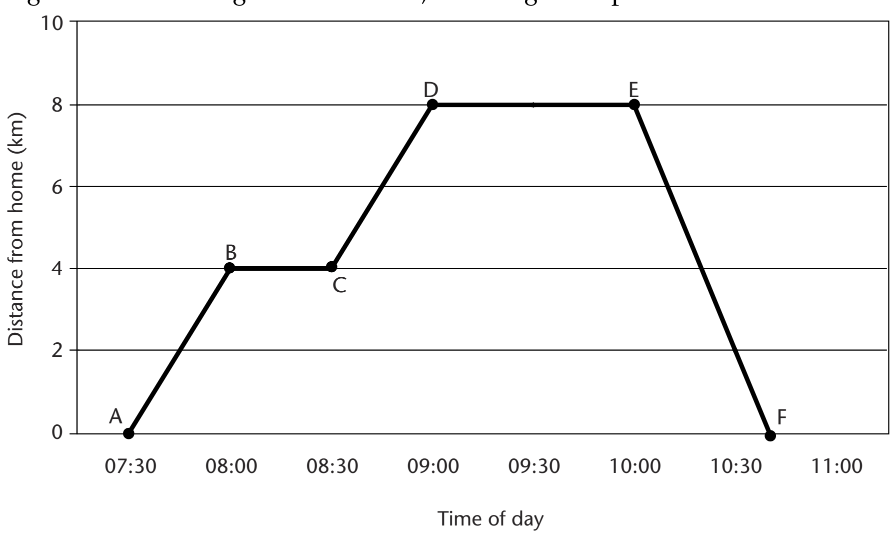
Suggested answers

1.1 to **1.3**



1.4 The temperature decreases between 06:00 and 07:00 to its minimum of 5 °C. It then increases steadily until 15:00, reaching a maximum of 20 °C, when it begins to decrease again until 18:00, reaching a temperature of 15 °C.

2

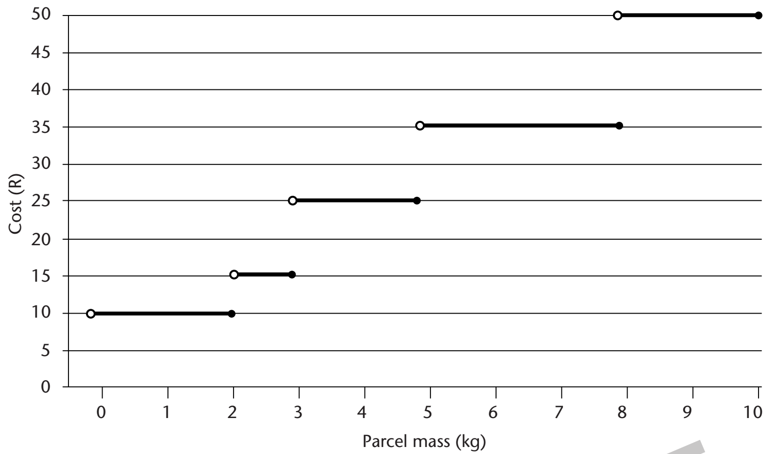


2.1 She cycled 8 km/h.

2.2 She arrived home at 10:40.

2.3 She cycled 16 km in total.

3.1 and **3.2**



3.3.1 R20

3.3.2 R10

Remedial

Learners should master working with global graphs before advancing onto algebraic graphs. Provide the necessary remediation of any problem areas before allowing learners to continue with the next unit.

Extension

Learners can create their own global graphs by conducting experiments and recording the data. Learners can record the temperature every day for 2 weeks to observe any trends. Or learners could keep track of crime rates in the area over a period to investigate if crime is more likely on weekends or weekdays.

Unit 3 Drawing algebraic linear graphs

Learner's Book page 306

Unit focus

This unit focusses on the following:
- plotting points on a Cartesian plane, and
- drawing linear graphs on a Cartesian plane.

Background information on drawing algebraic linear graphs

This is a key area in mathematics. Algebraic linear graphs lay the foundation for much further study in mathematics including analytical geometry and calculus. Learners have to be able to draw algebraic graphs if they are to succeed in FET mathematics. Drawing algebraic graphs is the culmination of working with functions, patterns and relationships. It is providing a graphical representation of the function as an equation.

Plotting points accurately and understanding the quadrant system is vital, and should be taught explicitly. In Grade 8 learners worked with the table method as the interim step in translating between equations and graphs. In Grade 9 learners are taught to translate directly from the equation to the graph by means of the intercept method. The gradient of the line becomes important and from their work on global graphs learners should be able to infer what the level of steepness for the graph means.

Exercise 1

Learner's Book page 307

Guidelines on how to implement this activity

Although learners have worked with the Cartesian plane before it is necessary that learners revise the basics of the plane. Focus on the quadrants and what the value (either positive or negative) of the x and y–values are in each quadrant. Revise how to plot points on a Cartesian plane. Demonstrate, using an example, plotting points on a Cartesian plane. Learners should complete this activity on their own.

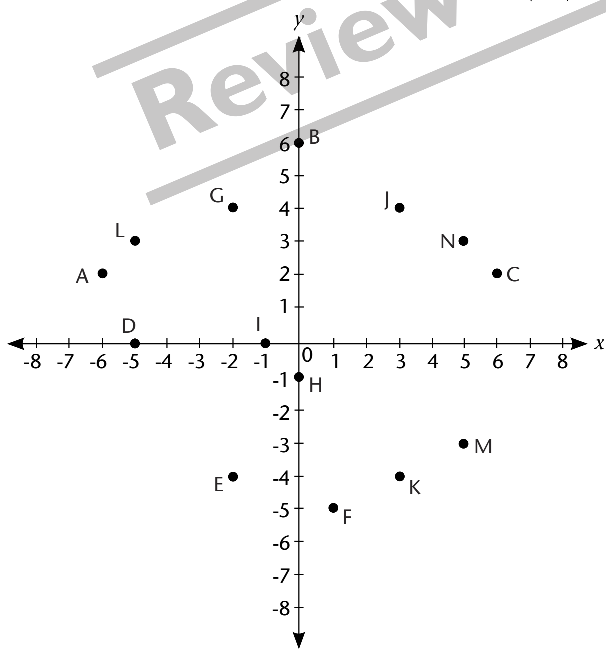
Suggested answers

1.1 A (3; 3) B (−4; 4) C (5; −6) D (0; 5)
E (8; 2) F (7; 6) G (−8; −3) H (−6; −5)
I (−8; 0) J (4; −3)

1.2 Points G and J have the same y-co-ordinate (−3).

1.3 Points G and I have the same x-co-ordinate (−8).

2



Remedial

Provide 1cm by 1cm grid paper for learners when working with algebraic graphs. This helps learners to work neatly and more accurately. It also saves time, as learners don't need to measure out the plane each time they draw a pair of axes.

Exercise 2

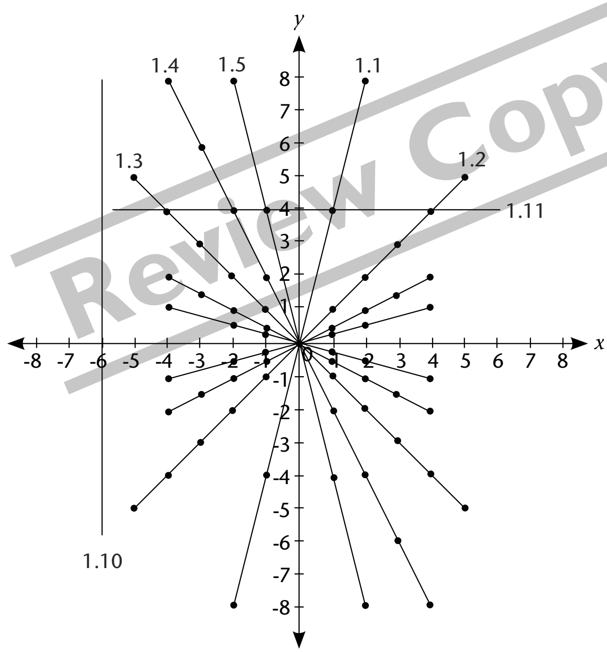
Learner's Book page 308

Guidelines on how to implement this activity

Revise with learners how they used tables to draw graphs in Grade 8. Work through the example in the Learner's Book, using the given x values to determine corresponding y values by substituting the x values into the given function equation $y = 2x$. Revise the meaning of domain of values. Learners covered domain in Chapter 9. Instruct learners that these are the restrictions on the x values. If necessary perform another example together before learners attempt the exercise on their own.

Suggested answers

1.1 to **1.9**



2.1.1 If $m > 0$ then the graph slopes **upwards** from left to right.
2.1.2 If $m < 0$ then the graph slopes **downwards** from left to right.
2.2.1 $y = 4x$
2.2.2 $y = \frac{1}{4}x$
2.3 The point (0; 0). We call this point the origin.

CHAPTER 11

Remedial

Do a fraction example together as a class, as some learners may find working with the fractions complicated and this may impact negatively of their understanding of the concept.

Extension

Ask stronger learners what they notice about lines number 9 and 10. Why do they think these lines have different equations? Ask these learners to discuss their thinking and reasoning in groups.

Exercise 3

Learner's Book page 309

Guidelines on how to implement this activity

Refer learners back to Question 2 in Exercise 2. Ask learners what they noticed about the m value. Learners need to learn that m represents gradient. Learners need to be able to define gradient and understand what it means and its impact on the graph. Show learners how to use the graph, and the grid to calculate the gradient. Remind learners that gradient is always the change in y over the change in x, and not the other way around. Learners should attempt this exercise on their own.
Learners have to understand that the gradient of a straight line is constant. It does not change. This means that anywhere along the linear graph, the gradient will be the same. Learners also need to observe that in examples of horizontal and vertical lines there is no m value. This is because for horizontal lines the gradient is 0, and for vertical lines the gradient is undefined. Show learners using the gradient formula why this is so.

Suggested answers

1.1 $m = 3$ **1.2** $m = \frac{1}{2}$ **1.3** $m = 1$ **1.4** $m = 1$

2 The gradients are the same. The gradient is the same at any point along the linear graph.

Remedial

Ensure learners reach the conclusions outlined above. If learners cannot identify gradient, nor understand that gradient is constant along a line some intervention is required. Work through the exercise with these learners, explaining the analysis. Provide additional examples for learners to practise with at home.

Exercise 4

Learner's Book page 311

Guidelines on how to implement this activity

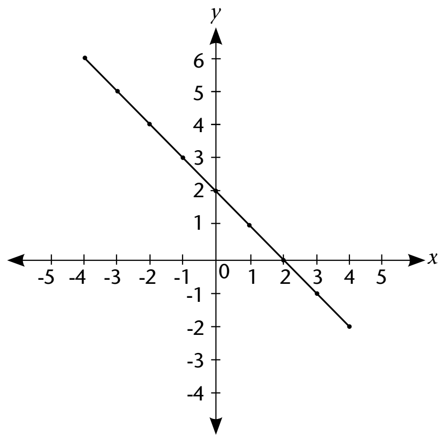
This exercise extends learners to drawing graphs, using a table, with a y intercept. Work through an example with the learners, revising substituting in values and plotting points. Learners must use a rule to draw the straight line. Learners must complete this exercise on their own.

Suggested answers

1.1

x	–4	–3	–2	–1	0	1	2	3	4
y	6	5	4	3	2	1	0	–1	–2

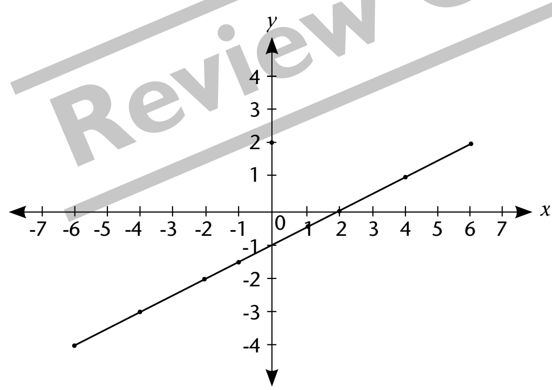
1.2 and **1.3**



2.1 Copy and complete the table of ordered pairs for the equation $y = \frac{1}{2}x - 1$.

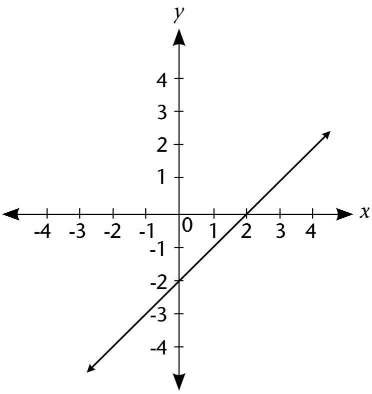
x	–6	–4	–2	–1	0	1	2	4	6
y	–4	–3	–2	–1,5	–1	–0,5	0	1	2

2.2 and **2.3**

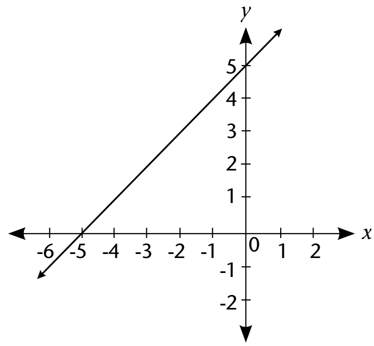


CHAPTER 11

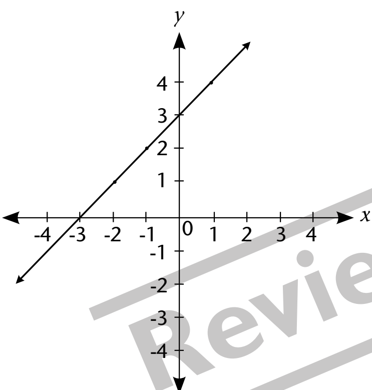
3.1 $y = x - 2$



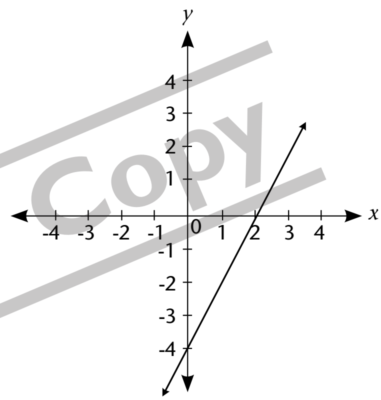
3.2 $y = x + 5$



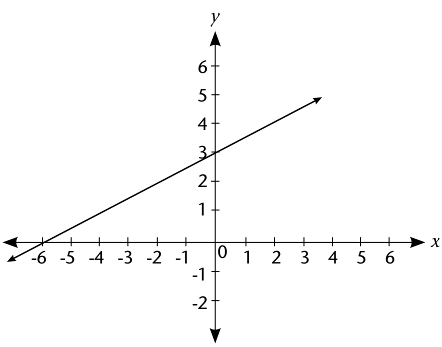
3.3 $y = x + 3$



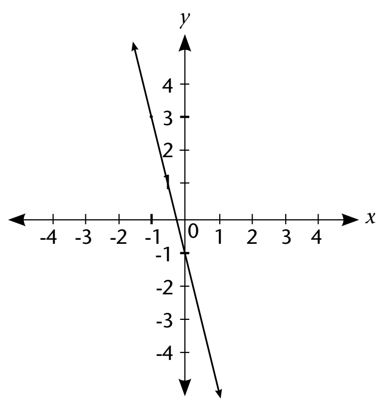
3.4 $y = 2x - 4$



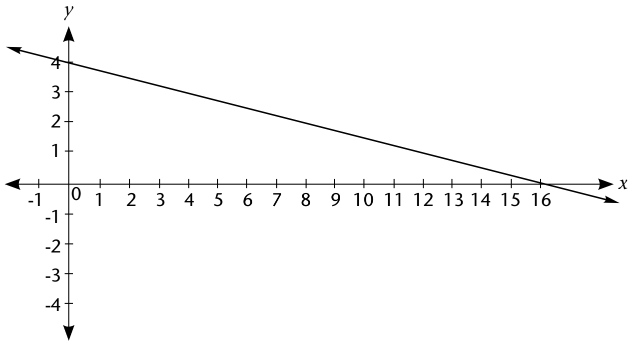
3.5 $y = \frac{1}{2}x + 3$



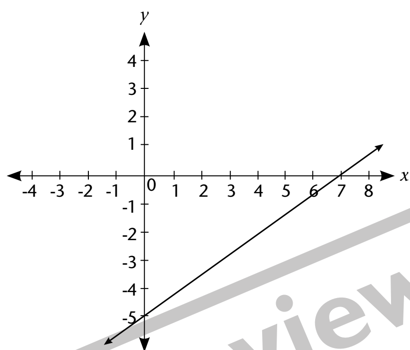
3.6 $y = -4x - 1$



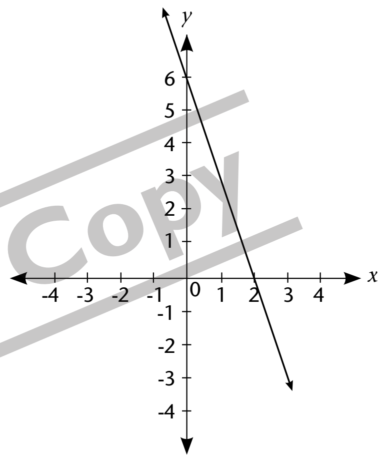
3.7 $y = -\frac{1}{4}x + 4$



3.8 $y = \frac{3}{4}x - 5$



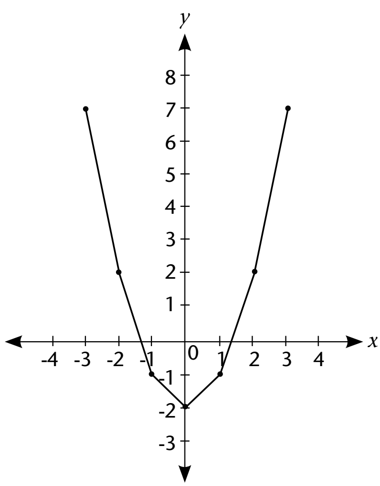
3.9 $y = 6 - 3x$



4

x	–3	–2	–1	0	1	2	3
y	7	2	–1	–2	–1	2	7

4.1 and **4.2**



4.3 This graph is not a straight line. It is a curve which is symmetrical around the y-axis.

Remedial

Some learners may experience problems working with the fraction. Revise multiplying fractions with whole numbers with these learners in order for them to manage this exercise.

Extension

Stronger learners can be encouraged to note that the constant c, as well as representing the y-intercept, also represents a vertical shift of the graph. This is a core concept in FET when learners perform shifts on the axes, and being able to identify this form of transformation now will help learners in their future mathematical endeavours.

Exercise 5

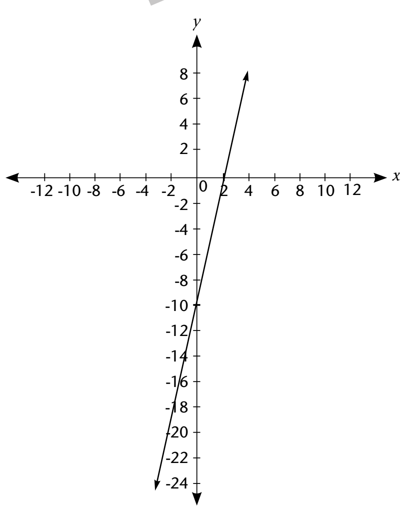
Learner's Book page 313

Guidelines on how to implement this activity

Until now learners have always used the table method for drawing graphs. In this section learners discover how to move directly from the equation to the graph, using the intercept-intercept method. Discuss how you can draw a straight line using only 2 points. Discuss that these can be any two points, however using the x-intercept and y-intercept is easier, as this means we substitute 0 into the equation (because at the y-axis $x = 0$; and at the x-axis $y = 0$). Do a few examples together as a class, using the intercept method. This method is new to learners and may require time for learners to fully grasp the concept and technique. Remind learners to always work neatly and to label their graphs correctly.

Suggested answers

1



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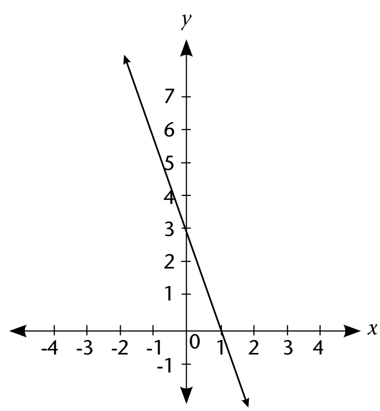

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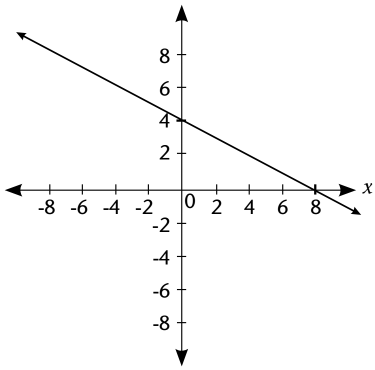

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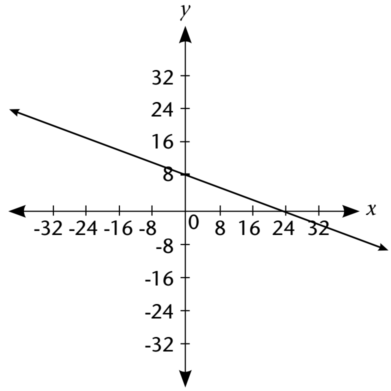

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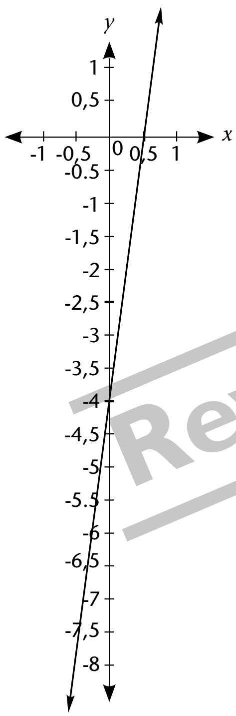

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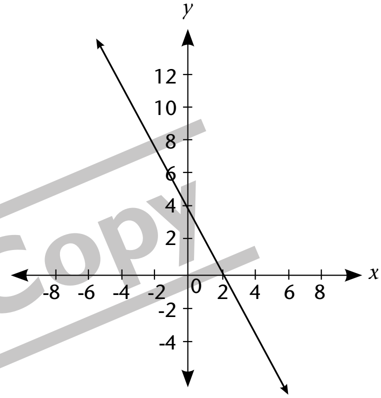


Remedial

Provide additional examples for learners to practise with until they fully grasp the concept of the intercept-intercept method. Learners with problems working with equations may need some revision of solving equations in order to cope with the exercise.

Unit 4 Determining the equations of linear graphs

Learner's Book page 314

Unit focus

This unit focusses on the following:
- learning how to determine an equation from a given graph, and
- learning about special cases of straight line graphs.

Background information on determining the equations of linear graphs

This is a new section for learners. It is a very important section as it lays a lot of the foundation work required for FET mathematics work on graphs. Learners have to be confident in plotting points and drawing these graphs, as those skills underpin their understanding of reading the necessary information, namely the y intercept and gradient from the graph. Ensure learners can draw algebraic linear graphs before starting this unit.

Exercise 1

Learner's Book page 316

Guidelines on how to implement this activity

Discuss the general equation for a straight line graph, $y = mx + c$. Revise what the m and c values represent. Discuss with learners how you could read the value of c off the graph. Learners should be able to identify c as the y intercept, and able to easily read this from the graph. Learners should also remember that m represents the gradient, and that gradient is the change in y over the change in x. Show learners how to determine the gradient of a graph by choosing 2 points on the graph and determining the change in y over the change in x for those 2 points. Do a few examples together as a class and then be available to assist learners as they attempt the first few examples on their own.

Suggested answers

1.1	$y = 2x + 2$	**1.2**	$y = -3x - 3$
1.3	$y = -2x + 5$	**1.4**	$y = x + 3$
1.5	$y = -\frac{5}{4}x + 3\frac{1}{2}$	**1.6**	$y = 2x + 2$

Remedial

Ensure learners have grid paper to work with to aid them to find the gradient.

Extension

Provide additional fraction examples as a challenge for learners who cope easily with this exercise.

Exercise 2

Learner's Book page 317

Guidelines on how to implement this activity

Discuss with learners any special straight line graphs they have worked with in this chapter. What is special about these straight lines? Revise horizontal and vertical lines on the Cartesian plane. Identify the gradients and discuss why the gradients are these values? Discuss the line $y = x$ and its gradient. Again encourage learners to provide reasons as to why the gradient is 1. Learners can complete this exercise in pairs, but make sure each learner shows the full working in their exercise books.

Suggested answers

1 $y = \frac{3}{7}x + 3$

2 $y = \frac{1}{2}x + 5$

3 $y = -2x + 4$

4 $y = -2$

5 $y = \frac{3}{4}x$

6 $y = -\frac{3}{5}x - 3$

Remedial

Learners have to master determining the equations of graphs if they are to succeed in further mathematical work. Provide as much revision and remediation in the form additional material, as needed in order for learners to cope with the prescribed work.

Consolidation

Learner's Book page 321

Before doing this consolidation exercise, encourage learners to review the work covered in this chapter. Advise learners to use the summary and to revise their work. This exercise can be used as an informal assessment task for you to track how learners are coping with the chapter and the concepts covered. The mark allocation provides guidelines on how to assess learners.

Suggested answers

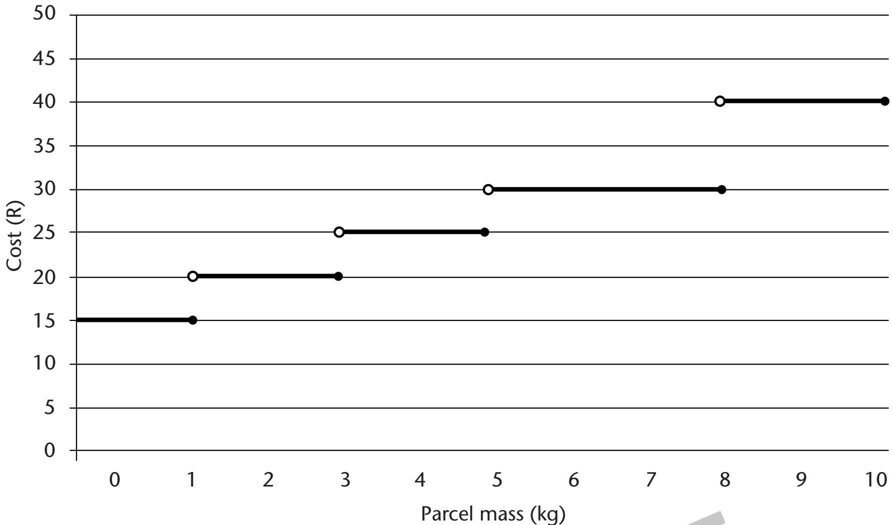
1.1 $\frac{35 \text{ km}}{1{,}5 \text{ hours}} = 23{,}3$ km/hr

1.2 No. For the first hour (A to B) he travels at a speed of 20km/hour. For B to C he travels at a speed of 30km/hr.

1.3 About 1 hour 30 minutes.

1.4 In approximately 30 minutes he travels 20km, this is a speed of 40km/hour. This is the fastest he travels. (5)

2.1 and **2.2** (7)



2.3.1 No difference (2)

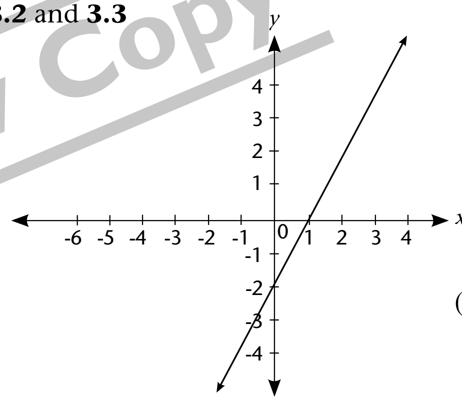
2.3.2 R10 (2)

3.1

x	-3	-2	-1	0	1	2	3
$y = 3x - 2$	-11	-8	-5	-2	1	4	7

(5)

3.2 and **3.3**



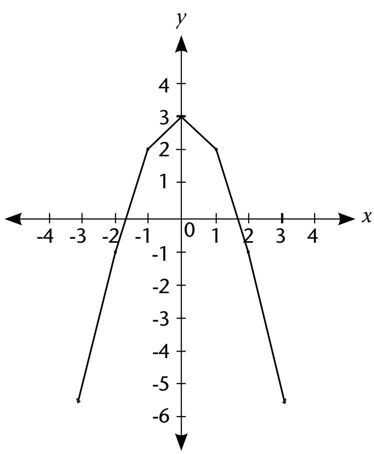
(4)

4

x	-3	-2	-1	0	1	2	3
$y = -x + 3$	-6	-1	2	3	2	-1	-6

(5)

4.1 and **4.2**



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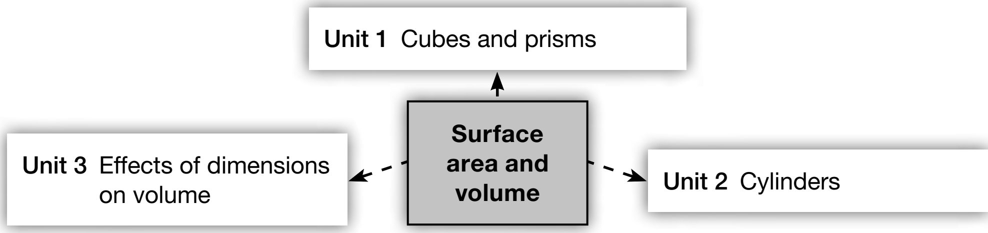
[35]

Chapter 12 Surface area and volume

Overview of concepts



Content		Time allocations	LB page
Unit 1	Cubes and prisms	2 hours	323
Unit 2	Cylinders	2 hours	329
Unit 3	Effects of dimensions on volume	1 hour	336

Background information on surface area and volume

As part of the geometry of 3D objects in Grade 8, learners learnt how to describe, sort and compare polyhedra, as well as how to use nets to create models of cubes and prisms.

As part of measurement in Grade 8, learners learnt how to calculate the surface area, volume and capacity of cubes, rectangular prisms and triangular prisms, using appropriate formulae.

Learners should be able to convert between the appropriate SI units, including:

mm ↔ cm ↔ m and ml (cm) ↔ ℓ ↔ kl

Generic teaching guidelines for teaching surface area and volume

Have as many of the physical objects described in the Learner's Book available in the class for learners to handle. Working with physical objects is an important part of the conceptualisation process for learners. Give learners time to handle the objects and to understand how they are put together.

SI units can be problematic for learners, so be sure to revise these adequately. Create charts that show the conversions between units, and put these up around the class to help learners when completing problems that require conversion.

Resources

3D object such as boxes, cylinders and prisms for learners to observe and experiment with. Conversion charts of the SI units for capacity. Cardboard and colour pens. Scissors and glue to cut open and construct 3D models. Each learner should have their own calculator.

Unit 1 Cubes and prisms

Learner's Book page 323

Unit focus

This unit focusses on the following:
- revising the work done on surface area and volume of cubes, rectangular prisms and triangular prisms;
- using the Theorem of Pythagoras to calculate missing lengths; and
- applying these skills to solve problems of a practical nature.

Background information on cubes and prisms

Learners have been working with cubes and prisms since the Intermediate Phase and are familiar with the forms, the nets and how to construct models of these forms. Learners need to use their understanding of these forms to calculate surface areas and volumes. Learners need to learn the formulae for these objects and be able to understand how the formulae are derived. In order to work with surface area and volume, learners need to understand which units are involved. Learners have worked with the SI units before, but often find them confusing.

Exercise 1

Learner's Book page 324

Guidelines on how to implement this activity

Ask learners what they remember about units for area, volume and capacity,writing learners' responses on the board. Revise the SI units and conversions on the board. Work through the Worked examples or a selection of them on the board. Have learners come up to the board and fill in answers. Let learners start Exercise 1 in the class and monitor their progress. The remainder of the exercise can be given as homework.

Suggested answers

1 1 kl = 1 000 ℓ

2 1 ℓ = 1 000 ℓ

3 1 m^3 = 1 000 000 cm^3

4 1 dm^3 = 1 000 000 mm^3

5 1575 cm^3 = 0,001575 m^3

6 84 555 ml = 84,555 ℓ

7 0,128 kl = 128 ℓ

8 1,0958 m^3 = 1 095,8 dm^3

9 $2{,}5 \times 10^3$ cm^3 = 2 500 cm^3
= 2,5 ℓ

Remedial

Some learners may need help with the conversion between units, especially where they have to decide whether to multiply or divide by a power of ten. Revise the relationship between the various multiples and sub-multiples of units with learners.

Exercise 2

Learner's Book page 326

Guidelines on how to implement this activity

Discuss the formulae for surface area and volume of prisms that learners can remember. Revise the formulas for surface area and volume of cubes and rectangular prisms and work through the Worked examples Questions 1 and 2 (or examples of your own choice) on the board. Let learners do Exercise 2 Questions 1.1, 1.2, 2.1 and 2.2 in the class and monitor their progress.
Revise the formulae for surface area and volume of triangular prisms and demonstrate the Worked example Question 3 (or examples of your own choice) on the board. Show the learners a physical model of a triangular prism and its net so that they can better interpret the drawings in the Learner's Book. Some of the drawings show a triangular prism standing on a triangular face while others show this kind of prism standing on one of its rectangular faces. This should be demonstrated with a physical model.
Discuss Exercise 2 Question 4 by asking the learners how they would calculate the height of the supporting beam of the roof. Let them then do Questions 4, 5 and 6 as homework.

Suggested answers

1.1 Volume of rectangular prism $l \times b \times h$
$= 2{,}25 \times 1{,}5 \text{ m} \times 0{,}5 \text{ m}$
$= 1{,}6875 \text{ m}^3$

1.2 Volume of cube $= 5^3$
$= (12 \text{ cm})^3$
$= 1{,}728 \text{ cm}^3$

1.3 Volume of triangular prism = area of base × height
$= \frac{1}{2}b \times h \times l$
$= \frac{1}{2} \times 4 \text{ m} \times 1{,}75 \text{ m} \times 6{,}4 \text{ m}$
$= 22{,}4 \text{ m}^3$

2.1 Total surface area of rectangular prisms
= 2 × area of base +perimeter of base × height
$= 2lb + 2(l + b)h$
$= 2(2{,}25 \times 1{,}5) + 2(2{,}25 + 1{,}5)0{,}5 \text{ m}^2$
$= 6{,}75 + 3{,}75) \text{ m}^2$
$= 10{,}5 \text{ m}^2$

2.2 Total surface area of cube $= 65^2$
$= 6(12 \text{ cm})^2$
$= 864 \text{ cm}^2$

2.3 Total surface area of triangular prism
$= 2 \times$ area of base + perimeter of base $\times$ height
$= 2 \times bh + (5 + 5 + b) \times l$
$= 2 \times 4 \times 1,75 + (2,66 \times 2 + 4) \times 6,4$ m
$= 14 + 59,648\ \text{m}^2$
$= 73,648\ \text{m}^2$

3.1 Rectangular prism

3.2 Answers will vary

3.3 Volume $= l \times b \times h$ etc.

3.4 Total surface area $= 2lb + 2(l + b)h$

4.1 $\text{h}^2 + (4\ \text{m})^2 = (5,3)^2$ (Isosceles $\triangle$)
$\text{h}^2 = (5,3\ \text{m})^2 - (4\ \text{m})^2$
$= (28,09 - 16)\ \text{m}^2$
$= 12,09\ \text{m}^2$
Thus $h = \sqrt{12,09}$ m
$= 3,48$ m

4.2 Area of 2 triangular gables
$= 2 \times \left(\frac{1}{2}bh\right)$
$= bh$
$= 8\ \text{m} \times 3,48\ \text{m}$
$= 27,84\ \text{m}^2$
Area of 2 rectangular parts
$= 2 \times l \times b$
$= 2 \times 12\ \text{m} \times 5,3\ \text{m}$
$= 127,2\ \text{m}^2$
Area of the whole roof $= (27,84 + 127,2)\ \text{m}^2$
$= 155,04\ \text{m}^2$

4.3 Volume of triangular prism
Area of base $\times\, l$ of prism
$= \frac{1}{2}bh \times l$
$= \frac{1}{2} \times 8\ \text{m} \times 3,48 \times 12\ \text{m}$
$= 167,04\ \text{m}^3$

5.1 $b = \frac{1}{2}$ of breadth of roof at its base
$= \frac{1}{2} \times b$ m
$= 3$ m
$s^2 = b^2 + h^2$
$= (3\ \text{m})^2 + (3\ \text{m})^2$
$= 18\ \text{m}^2$
Thus $5 = \sqrt{18}$ m $= 4,24$ m

5.2 Total area of roof $= 2 \times \frac{1}{2}$ base $\times$ height $+ 2 \times 5l$
$= 6\ \text{m} \times 3\ \text{m} + 2 \times 4,24\ \text{m} \times 9\ \text{m}$
$= 18\ \text{m}^2 + 76,32\ \text{m}^2$
$= 94,32\ \text{m}^2$

6.1 Total area of the four walls = Perimeter of base $\times$ height

$$= 2(l + b) \times h$$
$$= 2(9 \text{ m} + 6 \text{ m}) \times 3 \text{ m}$$
$$= 90 \text{ m}^2$$

6.2 Area of the floor = $l \times b$

$$= 9 \text{ m} \times 6 \text{ m}$$
$$= 54 \text{ m}^2$$

6.3 Volume of air inside the building = $l \times b \times h$

$$= 9 \text{ m} \times 6 \text{ m} \times 3 \text{ m}$$
$$= 162 \text{ m}^3$$

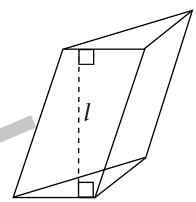
Remedial

Some learners may have difficulty with interpreting 2D drawings of 3D objects. Demonstrate the relationship between the two with physical models and nets of prisms.

Extension

Challenge learners to identify triangular prisms in the classroom or at home. Ask learners to estimate and measure dimensions and then calculate surface areas and volumes.

Show some learners a skew rectangular prism and a skew triangular prism of which the perpendicular distance between parallel, horizontal faces are given as well as the dimensions of all the faces. Challenge them to find formulae to calculate the volume and the surface area of these prisms.



Exercise 3 Learner's Book page 328

Guidelines on how to implement this activity

Discuss with learners that objects are not always easily classified as one particular object. Some objects can be made up of different objects put together. Ideally have a construction of a complex solid made up of a cube and another object. Demonstrate how to dissect the object into two or more easily identifiable objects.

Demonstrate the Worked example on the board with the learners. When working through the Worked example, remind learners to set out the work neatly and correctly. Let learners work through Exercise 3 in pairs.

Suggested answers

1.1 Volume of pool = area of trapezium $\times$ length of prism

$$= \tfrac{1}{2}(1 \text{ m} + 2 \text{ m}) \times 9 \text{ m} \times 5 \text{ m}$$
$$= 13{,}5 \times 5 \text{ m}^3$$
$$= 67{,}5 \text{ m}^3$$

1.2 In $\triangle ABC : AB^2 = AC^2 + BC^2$
$= (9\text{ m})^2 + (1\text{ m})^2$
$= (81 + 1)\text{ m}^2$
$= 82\text{ m}^2$
$AB = \sqrt{82} = 9{,}06\text{ m}$
Therefore the length of the floor of the pool is 9,06 m.
Area of floor $= l \times b$
$= 9{,}06\text{ m} \times 5\text{ m}$
$= 45{,}3\text{ m}^2$

2.1 PD = AD – AP
= 50 – AP But AP = BC = 36 cm
= 50 – 36
= 14 cm
QE = HE – HQ
= 50 – HQ But HQ = GF = 36 cm
= 50 – 36
= 14 cm

2.2 Area ABCP = 38 × 36
= 1 368 cm Area CPD = $\frac{1}{2} \times b \times h$
$= \frac{1}{2} \times PD \times PC$
$= \frac{1}{2} \times 14 \times 38$
= 266 cm

2.3 Volume of washbasin = area of ABCP × width of basin + area CPD + width of basin
= 1 368 × CF + 266 × CF
= 1368 × 90 + 266 × 90
= 123 120 + 23 940
$= 147\ 060\text{ cm}^3$

2.4 1 dm = 1 litre 1 000 cm = 1 dm
147 060 ÷ 1 000 = 147,06 m 1m = 147,06 litres

Remedial

Some learners may have difficulty interpreting combinations of prisms, such as the "swimming bath" problems. A model made from board or stiff paper will be helpful to demonstrate the situation.

Extension

Some crystals found in nature have prismatic forms. Encourage learners to find out more about prismatic forms from newspapers and the Internet.

Unit 2 Cylinders

Learner's Book page 329

Unit focus

This unit focusses on the following:
- calculating the surface area and the volume of a cylinder
- solving problems about cylinders and prisms of a practical nature.

Background information on cylinders

Since cylinders are introduced in Grade 9, it is important to show physical models of cylindrical objects and how their dimensions are measured. Also show the learners a net of a cylinder, showing the curved surface and the two circular ends.

Exercise 1

Learner's Book page 330

Guidelines on how to implement this activity

Work through the properties of a cylinder with the class. Show learners different cylindrical forms. Preferably have the objects available for learners to work with, or have pictures of different cylinders.
Encourage learners to identify cylinders from everyday life. Identify the parts of a cylinder and how you could describe a cylinder in terms of its sides and shapes. Have a model of a cylinder available that you can cut up to show the net of a cylinder.
Show learners that a cylinder is made up of two circles and a rectangle. Using the parts of the cylinder, show learners that the length of the rectangle is the same as the circumference of the circle.
Discuss the formulae for the surface area of a cylinder. Ask learners how they think the formula is derived. Work through an example of calculating the surface area of a cylinder. Encourage learners to come up to the board and assist as you demonstrate.
Let the learners start Exercise 1 in class and monitor their progress. The remainder of the exercise can be given as homework.

Suggested answers

1. Surface area of cylinder $= 2 \times \text{area of base} + \text{circumference of base} \times h$
$= 2\pi r2 + 2\pi rh$ where d = 4 m and $r = \frac{1}{2} \times 14$ cm = 7 cm
$= 2\pi r(r + h)$
$= 2 \times \frac{22}{7} \times 7(7 + 3)$ m^2
$= 44 \times 10$ m^2
$= 4\ 40$ m^2

2.1 Total surface area $= 2\pi r(r + h)$
$= 2 \times 3{,}1415927 \times 3{,}7(3{,}7 + 8)$ cm^2
$= 272$ cm^2

2.2 Total surface area $= 2\pi r(r + h)$, where d = 8,4 cm and $r = \frac{1}{2} \times 8{,}4$ cm = 4,2 cm
$= 2 \times 3{,}1415927 \times 5(5 + 200)$
$= 6\ 440{,}27$ mm^2

2.3 Total surface area $= 2\pi r(r + h)$
$= 2\pi(0,5)(0,5 + 20)$
$= 64,4$ cm 3
Area of curved surface $= 2\pi rh$
Circumference $= 2\pi r = 47,6$ cm
$\therefore$ Area of curved surface $= (47,6)h$
$= (47,6)(80)$
$= 3\ 808$ cm

Remedial

Some learners may have difficulty in writing down the steps of the solution to a problem in a logical and correct way. Let them do suitable examples under your guidance.

Extension

Ask learners to investigate how measurements of cylindrical objects with very small diameters (such as thin wire or fine spaghetti) are measured.

Exercise 2

Learner's Book page 332

Guidelines on how to implement this activity

Discuss the formula for the volume of a cylinder. Demonstrate how to use the formula with an example. Let the learners do Exercise 2 Questions 1 and 2 in the class and monitor their progress. Discuss Exercise 2 Question 3 by asking the learners how they will go about to calculate the volume of the water in the reservoir and how to calculate the full capacity in kilolitres. Let them continue with the exercise and complete it as homework.

Suggested answers

1.1 Volume of cylinder = Area of base × height
$= \pi r^2 h$
$= 3,1415927\ (3,7\text{ cm})^2(8\text{ cm})$
$= 344,07\text{ cm}^3$

1.2 Volume of cylinder $= \pi r^2 h$
$= 3,1415927 \times (4,2\text{ cm})^2 \times 4\text{ cm}$
$= 221,67\text{ cm}^3$

1.3 Volume of cylinder $= \pi r^2 h$
$= 3,1415927 \times (5\text{ mm})^2 \times 200\text{ mm}$
$= 15\ 707,96\text{ mm}^3$

2 Volume of cylinder $= \pi r^2 h$ where d = 35 cm and $r = \frac{1}{2}$of 35 cm = 17,5 cm
$= \frac{22}{7} \times (17,5\text{ cm})^2 \times 40\text{ cm}$
$= 38\ 500\text{ cm}^3$

3.1 $r = \frac{1}{2}$of 5,6 m = 2,8 m

3.2 Volume = Area of base × h
$= \pi r^2 \times h$
$= 3,1415927 \times (2,8\text{ m})^2 \times 2,5\text{ m}$
$= 61,6$ m

3.3 Volume = Area of base × h
$= \pi r^2 h$
$= 3{,}1415927 \times (2{,}8\text{ m})^2 \times 3{,}8\text{ m}$
$= 93{,}6\text{ m}^3$
$= 93{,}6$ kl

4.1 Circumference of coin = 2d
$= 2 \times 18{,}9$ mm
$= 37{,}8$ mm

4.2 Area of one face of coin = πr^2 where $r = \frac{1}{2}$of 18,9 mm = 9,45 mm
$= 3{,}1415927 \times (9{,}45\text{ mm})^2$
$= 280{,}6\text{ mm}^2$

4.3 Volume of coin = Area of base × h
$= \pi r^2 \times h$
$= 3{,}1415927 \times (9{,}45\text{ mm})^2 \times 1{,}8\text{ mm}$
$= 505\text{ mm}^3$

5.1 Volume of tank = $\pi r^2 h$ where d = 1,8 and $r = \frac{1}{2}$of 1,8 m = 0,9 m
$= 3{,}1415927 \times (0{,}9\text{ m})^2 \times 2{,}1\text{ m}$
$= 5{,}3\text{ m}^2$
$= 5{,}3$ kl
$= 5\,300$ ℓ

5.2 Volume = $\pi r^2 h$
$= 3{,}1415927 \times (0{,}9\text{ m})^2 \times 2{,}5\text{ m}$
$= 6{,}3\text{ m}^3$
$= 6{,}3$ kl

5.3 1 kl weighs 1 tonne
thus6,3 kl weighs 6,3 tonnes

6.1 Volume of tank = $\pi r^2 h$ where d = 2 m and r = 1 m
$= 3{,}1415927 \times (1\text{ m})^2 \times 9{,}5\text{ m}$
$= 29{,}85\text{ m}^3$
$= 29{,}85$ kl

6.2 29,85 ÷ 2 kl = 14,925 kl
= 14,93 kl

6.3 Total surface area
= 2 × Area of base + circumference of base × h
$= 2 \times \pi r^2 + 2\pi rh$
$= 2\pi r(r + h)$
$= 2 \times 3{,}1415927 \times 1\text{ m}(1\text{ m} + 9{,}5\text{ m})$
$= 65{,}97\text{ m}^2$

Remedial

Some learners may have difficulty with interpreting the 2D drawings of cylinders. Help them by demonstrating the relationship with physical models and nets of cylinders.

Extension

Challenge learners to identify cylindrical objects in the classroom or around the school. Estimate and measure dimensions (inside as well as outside in the case of cylindrical containers) and calculate surface areas, volumes and capacities.

Exercise 3

Learner's Book page 334

Guidelines on how to implement this activity

Revise working with complex objects as those in Unit 1. Show learners illustrations of objects involving cylinders and prisms. Discuss how you would calculate the volume and surface area of these solids. Demonstrate by working through an example on the board and encourage participation of the learners. Let the learners start Exercise 3 in the class and monitor their progress. The remainder of the exercise can be given as homework.

Suggested answers

1.1 The volume of metal left
= Volume of prism – Volume of cylinder
$= l \times b \times h - \pi r^2 h$ where $r = \frac{1}{2}(14 \text{ mm}) = 7 \text{ mm}$
$50 \times 20 \times 20 \text{ mm}^3 - \frac{22}{7} \times \frac{7^2}{1} \times 50 \text{ mm}^3$
$= 20\,000 \text{ mm}^3 - 7\,700 \text{ mm}^3 = 12\,300 \text{ mm}^3$

1.2 The area of the curved surface of the whole
$= 2\pi r \times h$
$= 2 \times \frac{22}{7} \times \frac{7}{1} \times 50 \text{ mm}$
$= 2\,200 \text{ mm}$

2.1 Volume of whole structure
= Volume of rectangular prism + Volume of half cylinder
$= l \times b \times h + \frac{1}{2}\pi r^2 h$ where $r = \frac{1}{2}(7 \text{ m}) = 3{,}5 \text{ m}$
$= 10 \times 7 \times 3 \text{ m}^3 + \frac{1}{2} \times \frac{22}{7} \times (3{,}5)^2 \times 10 \text{ m}^3$
$= 210 \text{ m}^3 + 192{,}5 \text{ m}^3$
$= 402 \text{ m}^3$

2.2 Total surface area of walls and roof
= Perimeter of rectangle $\times h$ + circumference of half circle $\times h$ + 2 $\times$ area of half circle
$= 2(l + b) \times h + \frac{1}{2}(2\pi r)h + 2(\frac{1}{2}\pi r^2)$
$= 2(10 + 7) \times 3 \text{ m}^2 + \frac{22}{7} \times 3{,}5 \times 10 \text{ m}^2 + \frac{22}{7} \times 3{,}5^2 \text{ m}^2$
$= 102 \text{ m}^2 + 110 \text{ m}^2 + 38{,}5 \text{ m}^2$
$= 250{,}5 \text{ m}^2$

Remedial

Some learners may have difficulty interpreting combinations of prisms and cylinders such as those of Exercise 3. Demonstrate the problems with physical models, showing how the complex shapes are composed of simpler ones.

Extension

Some objects found in nature have cylindrical forms. Ask learners to research five natural cylindrical forms. Have learners put their findings onto a poster to be displayed in the classroom.

Unit 3 Effects of dimensions on volume

Learner's Book page 336

Unit focus

This unit focusses on the following:
- learning how the volume and dimensions of 3D objects can change.

Background information on the effects of dimensions on volume

Learners have worked with the interrelationship between the area of the base of an object and its volume in Grade 8. In this section learners have to extend this knowledge by understanding how changing the dimensions of the object, either on the base, or the height, will impact on the volume. In previous grades it was an observed relationship, for example, the larger the base the larger the volume. In Grade 9 learners have to make critical statements using substitution and values when observing the impact of changing dimensions on volume.

Exercise 1

Learner's Book page 338

Guidelines on how to implement this activity

Discuss how learners think the dimensions of an object affect its volume. Ask learners to hypothesize what they think this effect will be. Work through the worked examples together as a class. Get the learners to participate by asking questions like "What is the ratio of the volume of shapes A and B?", "How can this ratio be simplified?". Then you write the correct answer on the board as one of the steps.
Let learners start with Exercise 1 in the class and monitor their progress. The remainder of the exercise can be given as homework. Once the learners have done their homework, check and / or mark their answers and discuss the answers.

Suggested answers

1.1

△ Prism	b	h	l	Base area = $\frac{1}{2}bh$	Volume = $\frac{1}{2}bh \times l$
D	3cm	4cm	10cm	$\frac{1}{2} \times 3 \times 4 = 6\text{ cm}^2$	$6 \times 10 = 60\text{ cm}^3$
E	3cm	4cm	20cm	$\frac{1}{2} \times 3 \times 4 = 6\text{ cm}^2$	$6 \times 20 = 120\text{ cm}^3$
F	3cm	8cm	10cm	$\frac{1}{2} \times 3 \times 8 = 12\text{ cm}^2$	$12 \times 10 = 120\text{ cm}^2$
G	6cm	8cm	10cm	$\frac{1}{2} \times 6 \times 8 = 24\text{ cm}^2$	$24 \times 10 = 240\text{ cm}^3$
H	6cm	8cm	20cm	$\frac{1}{2} \times 6 \times 8 = 24\text{ cm}^2$	$24 \times 20 = 480\text{ cm}^3$

1.2.1 $V_E : V_D = 120 : 60 = 2 : 1$
$l_E : l_D = 20 : 10 = 2 : 1$
When the length of the triangular prism doubles, the volume doubles.

1.2.2 $V_F : V_D = 120 : 60 = 2 : 1$
$h_F : h_D = 8 : 4 = 2 : 1$
When the height of the triangular prism doubles, the volume doubles.

CHAPTER 12

1.2.3 $V_G : V_D = 240 : 60 = 4 : 1$
$b_G : b_D = 6 : 3 = 2 : 1$
$h_G : h_D = 8 : 4 = 2 : 1$
When the height and the base of the triangular prism doubles, the volume increases fourfold.

1.2.4 $V_H : V_D = 480 : 60 = 8 : 1$
$b_H : b_D = 6 : 3 = 2 : 1$
$h_H : h_D = 8 : 4 = 2 : 1$
$l_H : l_D = 20 : 10 = 2 : 1$
When all three dimensions of a triangular prism doubles, the volume increases eightfold.

2.1 Either the length or the breadth or the height of the rectangular prism can double in order to double the volume of such a prism.

2.2 Either the base length or the base height or the length of the triangular prism can double in order to double the volume of such a prism.

Remedial

Some learners may struggle to calculate volumes or surface areas. Have learners write a list of all the formulae they need to know on a card. Learners can then refer to this card while they work with problems.
Learners may also struggle with simplifying ratios. Give learners simple problems to practice, such as "By what number can 72 as well as 36 be divided?"

Extension

Challenge learners to investigate the formulae for the surface area and volume of a sphere. Ask learners to then calculate the effects of doubling the radius of a sphere on its surface area and volume respectively.

Exercise 2

Learner's Book page 340

Guidelines on how to implement this activity

In the previous exercise, learners investigated the effect of a change in dimension on the volume of a cylinder. In this section learners investigate how to change the dimensions to ensure the volume is doubled. Learners start with a cube and progress to the cylinder. Work through the Worked examples as a class. Be sure to explain each step as this form of working can be confusing to learners. Learners can work on this exercise in pairs. Discuss learners' answers in the class and ensure learners were able to work accurately.

Suggested answers

1.1

Cylinder	r	l	Base area = πr^2	Volume = $\pi r^2 \times l$
A	7cm	10cm	$\frac{22}{7} \times 7^2 = 154$ cm^2	$154 \times 10 = 1\,540$ cm^3
B	14cm	10cm	$\frac{22}{7} \times 14^2 = 616$ cm^2	$616 \times 10 = 6\,160$ cm^3
C	14cm	20cm	$\frac{22}{7} \times 14^2 = 616$ cm^2	$616 \times 20 = 12\,320$ m^3

1.2.1 $V_B : V_A = 6\ 160 : 1\ 540 = 4 : 1$
$r_B : r_A = 14 : 7 = 2 : 1$
When only the radius doubles, the volume of cylinder B is 4 times the volume of cylinder A.

1.2.2 $V_C : V_A = 12\ 300 : 1\ 540 = 8 : 1$
$r_C : r_A = 14 : 7 = 2 : 1$
$l_C : l_A = 20 : 10 = 2 : 1$
When both the radius and the height (l) of the cylinder doubles, the volume of cylinder C is 8 times the volume of cylinder A.

2 Cylinder C $r = 10$ cm, $l = 15$ cm
Cylinder D r = unknown, $l = 15$cm

$$\text{Volume of D} = \tfrac{1}{2}\pi r^2 \text{l}$$
$$= \tfrac{1}{2}\pi r^2 \times 15$$
$$\text{Volume of C} = \tfrac{1}{2}\pi r^2 l$$
$$= \tfrac{1}{2}\pi 10^2 \times 15 \text{ cm}^3$$

Since Volume of D = 2 × Volume of C

$$\tfrac{1}{2}\pi \text{r}^2 \times 15 = 2 \times \tfrac{1}{2}\pi 10^2 \times 15 \text{ cm}^3$$

Divide both sides by $\tfrac{1}{2}\pi \times 15$
Thus $r^2 = 2 \times 10^2$
$r = 10\sqrt{2}$ cm
Thus a cylinder with the same length as a given cylinder will have double the volume of the given cylinder if its base is $\sqrt{2}$ times that of the given cylinder.

3 Cube E: $S_E = 25$ mm
Cube F: $S_F = ?$
Volume of E = $(S_E)^3 = (25 \text{ mm}^3) = 15\ 625 \text{ mm}^2$
Volume of F = $(S_F)^3$
Since volume of F = 2 × Volume of E
$(S_F)^3 \text{ mm}^3 = 2 \times (25 \text{ mm})^3$

$$\therefore S_F = \sqrt[3]{2 \times 25^3}$$
$$= \sqrt[3]{2 \times} \sqrt{25^3}$$
$$= \sqrt[3]{2} \times 25 \text{ mm}$$

Thus a cube with side lengths $\sqrt[3]{2}$ times that of a cube with given sides length will have a volume that is double that of the given cube.

4 Rectangular prism A: $l = 3{,}5$ m, $b_A = 2$ m and $h_A = 1{,}5$ m
Rectangular prism B: $\text{l}_B = 3{,}5$ m, $b_B = 2$ m and $h_B = ?$
Volume of A = $l_A \times b_A \times h_A = 3{,}5 \times 2 \times 1{,}5 \text{ m}^2$
Volume of B = $l_B \times b_B \times h_B = 3{,}5 \times 2 \times h_B \text{ m}^2$
Since Volume of B = 4 × Volume of A
$3{,}5 \times 2 \times h_B \text{ m}^2 = 4(3{,}5 \times 2 \times 1{,}5) \text{ m}^3$
Divide both sides by $3{,}5 \times 2$
Thus $h_B = 4 \times 1{,}5 \text{ m} = 6 \text{ m}$
Thus a rectangular prism with the same length and breadth of a given cylinder will have four times the volume of the given rectangular prism if the height is

four times that of the given rectangular prism.

5 Triangular prism M: $b_M = 6$ mm, $h_M = 1{,}5$ mm $l_M = 25$ mm
Triangular prism N: $b_N = 6$ mm, $h_N = ?$, $l_N = 25$ mm

Volume of triangular prism M = $\frac{1}{2}b_M h_M l_M$

Volume of triangular prism N = $\frac{1}{2} \times 6 \times h_N \times 25$ mm^2

Since volume of N = 2 × Volume of M

$\frac{1}{2} \times 6 \times h_N \times 25 \text{ mm}^2 = 2 \times \left(\frac{1}{2} \times 6 \times 1{,}5 \times 25\right) \text{mm}^2$

Divide both sides by $\frac{1}{2} \times 6 \times 25$
Thus $h_N = 2 \times 1{,}5$ mm = 3 mm
Thus a triangular prism with the same base length and prism length of a given triangular prism will have double the volume of the given triangular prism if the height is double that of the given triangular prism.

6 Cylinder P: $d_P = 7$ m and $l_P = 4{,}25$ m
Cylinder Q: $d_Q = 7$ m, $l_Q = ?$

Volume of cylinder P = $\frac{1}{2}\pi r_P^5 l_P$ where $r_P = \frac{1}{2}$ (7 m)

$= 3{,}5$ m

$= \frac{1}{2}\pi(3{,}5)^2 \times 4{,}25 \text{ m}^2$

Volume of cylinder Q = $\frac{1}{2}\pi r_Q^2\, l_Q$

$= \frac{1}{2}\pi(3{,}5)^2 \times l_Q$

Since Volume of Q = 2 × Volume of P

$\frac{1}{2}\pi(3{,}5)^2 \times l_Q \text{ m}^2 = 2 \times \frac{1}{2}\pi(3{,}5)^2 \times 4{,}25 \text{ m}^2$

Divide both sides by $\frac{1}{2}\pi(3{,}5)^2$
Thus $l_Q = 2 \times 4{,}2$ m
$= 8{,}5$ m
Thus a cylinder with the same base diameter of a given cylinder will double the volume of the given cylinder if the length is double that of the given cylinder.

7 When the length of the cylinder doubles, the volume will double.

Remedial

Learners may struggle to draw and write down conclusions from the ratios they calculated. Work through the examples in the Learner's Book with learners and focus on the change in one dimension at a time. After considering each dimension, write a simple equation on the board to show the effect the altered dimension(s) have on the volume.

Extension

Challenge learners to design prismatic and cylindrical containers where the surface area or volume is more than two times that of a given container.

Consolidation

Learner's Book page 342

Before doing this consolidation exercise, encourage learners to review the work covered in this chapter. Advise learners to use the summary and to revise their work. This exercise can be used as an informal assessment task for you to track how learners are coping with the chapter and the concepts covered. The mark allocation provides guidelines on how to assess learners.

Suggested answers

1.1 $0{,}75 \text{ m}^2 = 7\ 500 \text{ cm}^2$

1.2 $2{,}25 \times 10^4 \text{ mm}^2 = 225 \text{ cm}^2$

1.3 $0{,}025 \text{ km}^2 = 25\ 000 \text{ m}^2$

1.4 $0{,}125 \text{ m}^3 = 125\ 000 \text{ cm}^3$

1.5 $10^5 \text{ mm}^3 = 100 \text{ cm}^3$

1.6 $0{,}125 \text{ kl} = 125\ \ell$

1.7 $1{,}08 \times 10^4 \text{ ml} = 10{,}8\ \ell$ (7)

2.1 Volume of triangular prism $= \frac{1}{2}bhl\left(\text{or } \frac{1}{2}bhH\right)$

$= \frac{1}{2}(14)(12) \times 48 \text{ cm}^3$

$= 4\ 032 \text{ cm}^3$ (2)

2.2 Total surface area of a triangular prism

= Perimeter of triangle × length of prism + 2 × area of △

$= (13 + 15 + 14) \times 48 \text{ cm}^2 + 2 \times \frac{1}{2}(14 \times 12)$

$= 2\ 016 \text{ cm}^2 + 168 \text{ cm}^2$

$= 2\ 184 \text{ cm}^2$ (2)

3.1 Volume of cylinder $= \pi r^2 l\ (\pi r^2 h)$

Where $r = \frac{1}{2}(5{,}6 \text{ cm}) = 28 \text{ cm}$

$\frac{22}{7} \times \left(\frac{28}{1}\right)^2 \times \frac{50}{1} \text{ cm}^3$

$= 123\ 200 \text{ cm}^3$ (3)

3.2 Total surface area of cylinder

= Circumference of circle × length of cylinder + 2 × area of circle

$= 2 \times \frac{22}{7} \times 28 \times 50 \text{ cm}^3 + 2 \times \frac{22}{7} \times (28)^2 \text{ cm}^2$

$= 8\ 800 \text{ cm}^2 + 4\ 928 \text{ cm}^2$

$= 13\ 728 \text{ cm}^2$ (3)

4.1 Circumference of circle $= 2\pi r = 157$ cm

Thus $r = \frac{157}{2\pi}$ cm

$= 24{,}99$ cm

Surface area of cylinder = Circumference of circle × h + 2 × area of circle

$= 157 \times 67 \text{ cm}^2 + 2 \times \pi(24{,}99)^2 \text{ cm}^2$

$= 14\ 442{,}85 \text{ cm}^2$ (2)

4.2 Volume of cylinder

= Area of circle × h

$= \pi(24{,}99)^2 \times 67 \text{ cm}^3$

$= 131\ 448{,}97 \text{ cm}^3$ (2)

5.1 Volume of cylinder
$= \text{Area of circle} \times h$
$= \pi \times 15^2 \times 120 \text{ cm}^3$
$= 84\,823 \text{ cm}^3$
$= 84{,}82\ l$ (2)

5.2 Total surface area of cylinder
$= \text{Circumference of circle} \times h + 2 \times \text{area of circle}$
$= 2\pi(15) \times 120 \text{ cm}^3 + 2 \times \pi(15)^2 \text{ cm}^2$
$= 2\pi 15(120 + 15) \text{ cm}^2$
$= 12\,723{,}45 \text{ cm}^2$
$= 1{,}27 \text{ m}^2$ (3)

6.1 Volume of triangular prism $= \text{Area of triangle} \times l \text{ of prism}$
$= \frac{1}{2}bhl$
$= \frac{1}{2} \times 1{,}2 \times 0{,}5 \text{ m} \times 3 \text{ m}^3$
$= 0{,}9 \text{ m}^3$
Volume of rectangular prism $= \text{Area of rectangle} \times h$
$= l \times b \times h$
$= 1{,}75 \times 0{,}5 \times 3 \text{ m}^3$
$= 2{,}625 \text{ m}^3$
Total volume of structure $= (0{,}9 + 2{,}625) \text{ m}^3$
$= 3{,}525^3$
$= 3{,}5 \text{ m}^3$ (2)

6.2 Length of rectangle = 3 m
Breadth of rectangle $= 0{,}5^2 + 1{,}2^2$
$= 0{,}25 + 1{,}44$
$= 1{,}69$
Breadth of rectangle = 1,3 m
Area of rectangle $= 3 \times 1{,}3$
$= 3{,}9 \text{ m}^2$ (2)

7.1 Volume of rectangular prism $= l \times b \times h$
$= 28 \times 15 \times 32 \text{ cm}^3$
$= 13\,440 \text{ cm}^3$
Volume of $\frac{1}{2}$ cylinder $= \frac{1}{2}(\pi r^2 h)$ $\left(r = \frac{1}{2}(28) = 14 \text{ cm}\right)$
$= \frac{1}{2}(\pi \times (14)^2 \times 32) \text{ cm}^3$
$= 9\,852{,}03 \text{ cm}^3$
Total volume of post box $= 13\,440 + 9852{,}03 \text{ cm}^3$
$= 23\,292{,}03 \text{ cm}^3$ (3)

7.2 Total surface area $= 2(28 + 32) \times 15 \text{ cm}^2 + 28 \times 32 \text{ cm}^2 + \frac{1}{2}(2\pi 14) \times 32 \text{ cm}^2$
$+ 2\left(\frac{1}{2}\pi 14^2\right) \text{ cm}^2$
$= 1\,800 + 896 + 1407{,}43 + 615{,}75 \text{ cm}^2$
$= 4\,719{,}18 \text{ cm}^2$ (3)

8.1 Volume of wood drilled out $= \pi r^2 h$ where $r = 4{,}5$ mm
$= \pi \times (4{,}5)^2 \times 6 \text{ mm}^3$
$= 381{,}7 \text{ mm}^3$ (2)

8.2 Volume of initial rectanglular block $= l \times b \times h$
$= 25 \times 21 \times 6\ \text{mm}^3$
$= 3\,150\ \text{mm}^3$
Volume of wood remaining $= (3\,150 - 381{,}7)\ \text{mm}^3$
$= 2\,768{,}3\ \text{cm}^3$
$= 2{,}7683\ \text{cm}^3$
$= 2{,}8\ \text{cm}^3$ (3)

8.3 Area of curved surface of whole = circumference of base × height
$= 2\pi(4{,}5\ \text{mm}) \times 6\ \text{mm}$
$= 169{,}646\ \text{mm}^2$
$= 1{,}69646\ \text{cm}^2$
$= 1{,}7\ \text{cm}^2$ (3)

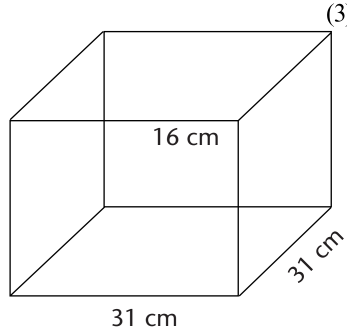
9.1 Area of side view $= 2{,}2 \times 5\ \text{m}^2 + 2(2{,}2 + 1{,}2)25\ \text{m}^2 + 1{,}2\ \text{m}^2 \times 20\ \text{m}^2$
$= (11 + 170 + 24)\ \text{m}^2$
$= 205\ \text{m}^2$ (2)

9.2 Volume of water = Area of base × height
$= 205\ \text{m}^2 \times 24\ \text{m}$
$= 4\,920\ \text{m}^3$
$= 4\,920\ \text{kl}$ (3)

10.1 Capacity of inside = Volume of side circle
πr^2 where $r = \frac{1}{2}(3{,}6)$ m
$= 1{,}8$ m
$= \pi(1{,}8)^2 \times 3$
$= 30{,}536\ \text{m}^3$ ∴ The capacity is 30,536 kilolitres. (3)

10.2 Volume of bigger cirle $= \pi r^2 h$ where $r = \frac{1}{2}(4)$
$= 2$ m
$= \pi(2)^2 \times 3$
$= 37{,}699\ \text{m}^3$
Thus volume of round wall $= (37{,}699 - 30{,}536)\ \text{m}^3$
$= 7{,}163\ \text{m}^3$ (3)

11.1 Number of tiles that will fit in a box
$= 16\ \text{cm} \div 0{,}6\ \text{cm}$
$= 26{,}6$
$= 26$ tiles (2)



11.2 Area of one tile
$= 5^2$
$= (30\ \text{cm})^2$
$= 900\ \text{cm}^2$ (2)

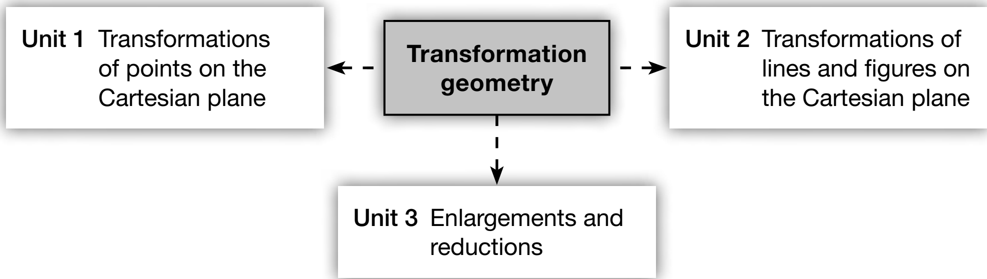
11.3 Area of one box of tiles
$= 900\ \text{cm}^2 \times 26$
$= 23\,400\ \text{cm}^2$
$= 2{,}34\ \text{m}^2$ (3)

11.4 Thus number of boxes needed $= 20{,}8\ \text{m}^2 \div 2{,}34\ \text{m}^2$
$= 8{,}8$
$= 9$ boxes (must round up to a whole number) (3)

[65]

Chapter 13 Transformation geometry

Overview of concepts



Content		Time allocations	LB page
Unit 1	Transformations of points on the Cartesian plane	2 hours	345
Unit 2	Transformations of lines and figures on the Cartesian plane	4 hours	353
Unit 3	Enlargements and reductions	3 hours	362

Background information on transformations

Transformations are a unique way of integrating the Cartesian plane with spatial thinking and reasoning. It lays the groundwork for further work on transforming algebraic graphs in FET. It is necessary that learners know the complexities of the Cartesian plane, and transformations offer an ideal environment for this. Encourage learners to perform as many actual transformations as possible. Only once learners are confident should they move on to the more abstract prediction and algebraic manipulation of co-ordinate points. Keep the work as practical as possible in Grade 9 (as opposed to theoretical).

Generic teaching guidelines for teaching transformations

In order to teach transformations on the Cartesian plane, it is helpful to allow learners to physically make the transformations themselves using transparencies, grid paper and erasable markers. Provide each learner with an erasable marker, grid paper and a square of transparency film. Have learners plot the Cartesian plane, original point, line or figure onto the grid paper. Learners then trace the point, line or figure onto the transparency. Learners shift the transparency according to the desired transformation: flip (reflection), turn (rotation) or slide (translation). Learners see where the new point, line or figure will lie on the plane. Learners mark the point and then draw the shape on the plane to show the transformation.
This method helps learners visualise transformations as they get to see the transformation taking place.

Resources

Grid paper, graph paper, tracing paper, transparencies, marker pens, colour pens, rulers, protractors and scissors. Each learner should have a their own calculator.

Unit 1 Transformations of points on the Cartesian plane

Learner's Book page 345

Unit focus

This unit focusses on the following:
- revising plotting points on the Cartesian plane,
- performing transformations on the Cartesian plane,
- recognising and describe transformations on the Cartesian plane,
- using the rules of transformations on the plane to predict co-ordinates, and
- identifying transformations using co-ordinates and their image.

Background information

This first section focusses on plotting and transforming points. It is necessary that learners understand what the Cartesian plane is, how the four quadrants operate together, and what the signs of the x- and y-values are depending on the quadrants. Transforming points and identifying the effect on the co-ordinates of x and y is very important and lays the groundwork for the rest of the chapter. It is important that learners perform as many actual transformations as possible.

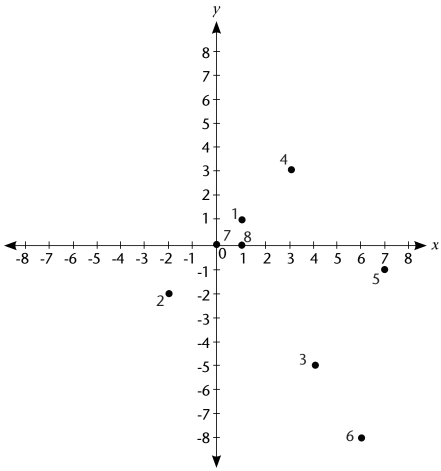
Exercise 1

Learner's Book page 346

Guidelines on how to implement this activity

Discuss the Cartesian plane, and how it is made up of four quadrants. Ask learners what values, positive or negative, y will have in each quadrant. Then ask what values, positive or negative, x will have in each quadrant. Revise how to plot points on the graph and how to work from the x-value to the corresponding y-value to find the point.
Draw a Cartesian plane on the board and call up learners to plot given points. Learners should do Exercise 1 on their own.

Suggested answers



Remedial

Learners can work in small groups if necessary. Playing games that use simple grid reference systems can help learners unpack how to read and work with the Cartesian plane. Always use grid paper to help learners accurately plot points.

Exercise 2

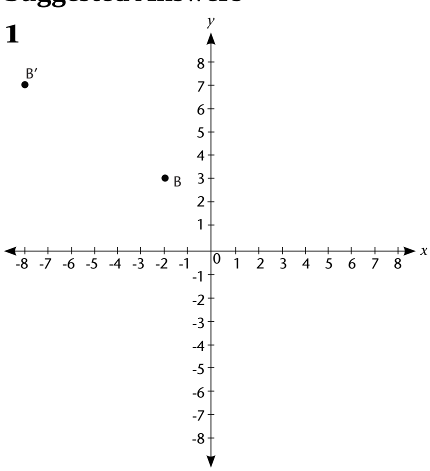
Learner's Book page 350

Guidelines on how to implement this activity

Discuss reflections and translations. Discuss reflection in each of the axes and around the line $y = x$. Do examples plotting reflectionstogether as a class. Have learners perform the reflections using transparencies. Introduce translations of points by having learners perform the translation using grid paper and the transparency if necessary.

Suggested Answers

1



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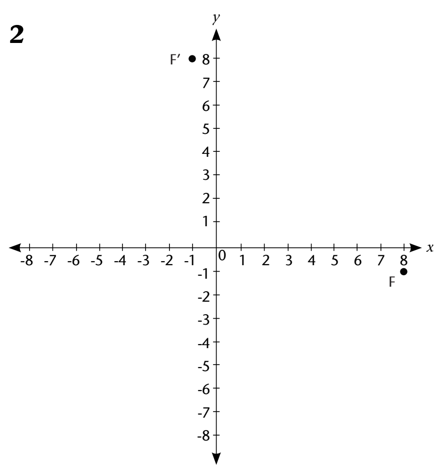

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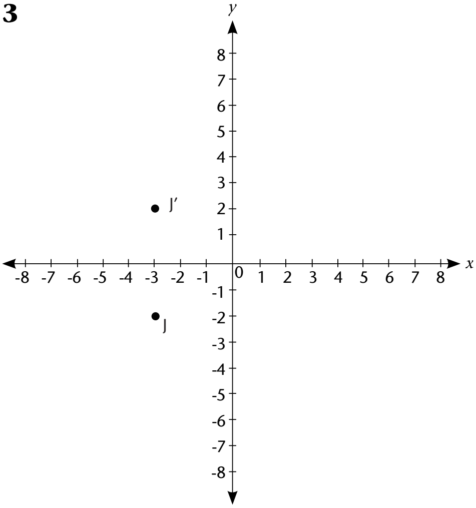

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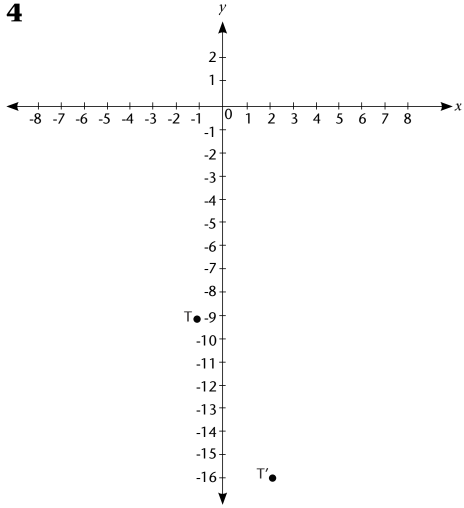

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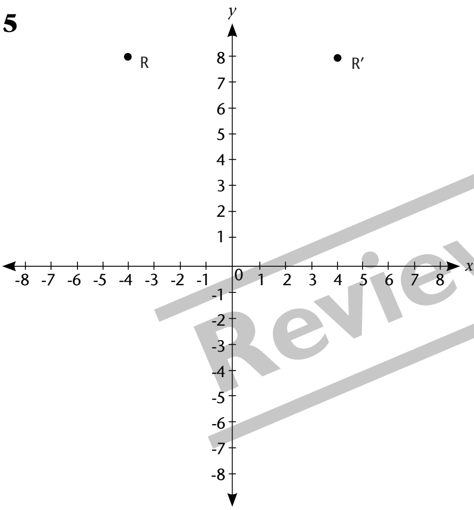

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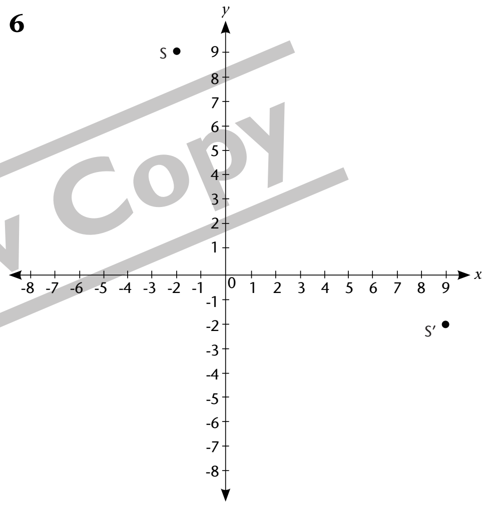


Exercise 3

Learner's Book page 351

Guidelines on how to implement this activity

Review the reflections done in Exercise 2. In each instance, discuss what happened to the co-ordinates of the points reflected. Draw up a set of general rules that can determine what transformation occurred from the change in co-ordinates. Discuss these as a class and have learners test their theories and predictions by performing the transformations.

Review the translation of points. Ask learners to identify any patterns in the co-ordinates after translation. Devise a general rule to use to determine the translation based on the co-ordinates. Discuss these as a class and have learners test their theories and predictions by performing the transformations.
Work through the Worked examples in the Learner's Book for determining the transformation and then encourage learners to work on their own to complete Exercise 3.

Suggested answers

1 A translation by five units to the left and one unit down.
2 A translation by one unit to the left and 11 units down.
3 A reflection in the y-axis or a translation by 24 units to the left.
4 A reflection in the x-axis or a translation by two units down.
5 No transformation has taken place.
6 A reflection in the line $y = x$ or a translation by five units to the left and five units upwards.
7 A reflection in the line $y = x$ or a translation by 12 units to the left and 12 units upwards.
8 A translation by nine units to the right and two units down.
9 A reflection in the line $y = x$ or a translation by 14 units to the left and 14 units upwards.
10 A reflection in the x-axis or a translation by ten units down.
11 A translation by nine units to the right and 33 units down.
12 A reflection in the y-axis or a translation by 20 units to the left.

Exercise 4

Learner's Book page 352

Guidelines on how to implement this activity

Review the examples of reflections done in Exercise 2. Extend the discussion from Exercise 3 on what happened to the co-ordinates of the points reflected. Draw up a set of general rules to predict the co-ordinates of points that are reflected. Discuss these as a class and have learners test their theories and predictions by performing the transformations.
Review the translation of points. Extend the discussion from Exercise 3 to devise a general rule to predict co-ordinates when translating. Discuss these as a class and have learners test their theories and predictions by performing the transformation.
Learners can work in small groups to complete Exercise 4.

Suggested answers

1	A'(−4; 3)	**2**	(11; 1)	**3**	P'(7; 3)
4	(−2; 4)	**5**	Q'(2; −4)	**6**	(−3; −8)
7	(−7; 2)	**8**	(5; −9)	**9**	(−3; −8)
10	(18; 22)	**11**	(8; −3)	**12**	(−4; 5)

Remedial

Provide grid paper and additional transparencies for learners to work out the transformations. Spend as much time as necessary working with point transformations. If learners are not able to manage points, they will struggle with lines and figures. Provide additional examples if extra practice is required.

Extension

Provide line segments and simple figures for learners to experiment with and transform.

Unit 2 Transformations of lines and figures on the Cartesian plane

Learner's Book page 353

Unit focus

This unit focusses on the following:
- revising the rules for transforming points on the Cartesian plane,
- performing transformations of lines and figures on the Cartesian plane,
- recognising and describe transformations on the Cartesian plane,
- using the rules of transformations on the plane to predict co-ordinates, and
- identifying transformations using co-ordinates and their image.

Background information

Transforming lines and figures simply extends the knowledge of transforming points. Instead of transforming one point learners now transform two points and then three or more points, which make up the vertices of the figures. Learners need to understand that they are not doing anything new, simply performing the same transformation on additional points. The size of the lines and figures remains constant during transformation: no change in size or shape occurs.

Exercise 1

Learner's Book page 353

Guidelines on how to implement this activity

Revise the rules for transforming on the Cartesian plane. Discuss the types of transformations and their impact on co-ordinates. Work through each rule, and if time allows, encourage learners to perform the transformation with an example to test if it works. Learners need to be able to recognise the transformation and predict the co-ordinates after a given transformation.

1.1 Translation by eight units to the left and one unit down.
1.2 Reflection across the line $y = x$.
1.3 Reflection across the line $y = x$.
1.4 Reflection across the x-axis.
1.5 Translation by one unit to the right.
1.6 Reflection across the y-axis.

2.1	A'(17; 11)	**2.2**	(−7; −1)	**2.3**	P'(−7; 9)
2.4	(−3; 1)	**2.5**	Q'(9; −8)		

Remedial

Learners should work through the exercises in Unit 1 again, if they have struggled with this activity.

Exercise 2

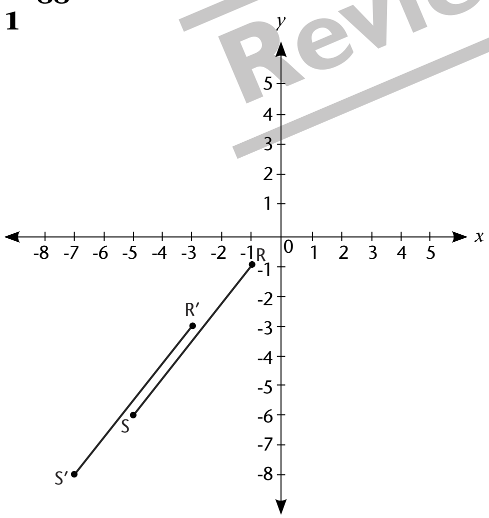
Learner's Book page 356

Guidelines on how to implement this activity

Discuss how moving a line segment is simply moving two points, connected by a line. Learners then need to transform two points instead of one. Do a few examples, showing this concept to learners. Again use transparencies and have learners perform the transformation themselves as much as possible. Discuss translations and reflections of line segments.
Introduce rotations to learners. Ensure learners use the transparency here when performing rotation examples, and that they rotate around the origin of the plane. Revise what is meant by clockwise and anti-clockwise. Have learners perform some rotation examples. From their physical transformations, have learners note any possible general rules for rotation. When predicting co-ordinates, learners only need to know rotations of 90° clockwise or anti-clockwise. However, learners can physically perform rotations around any degree.

Suggested answers

1



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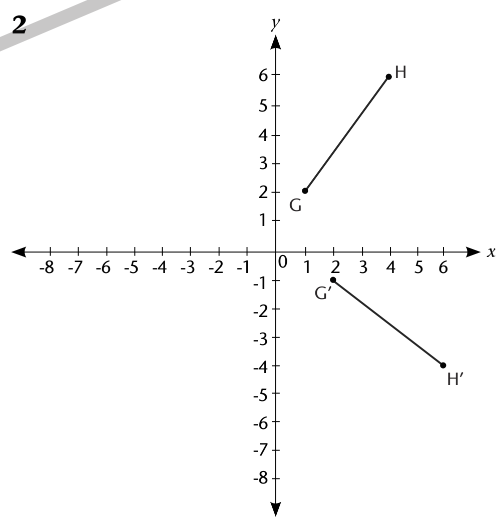

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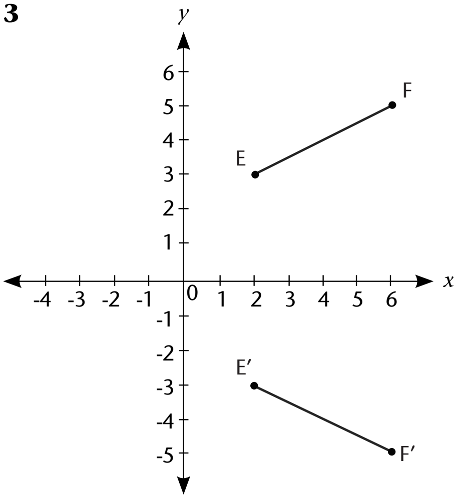

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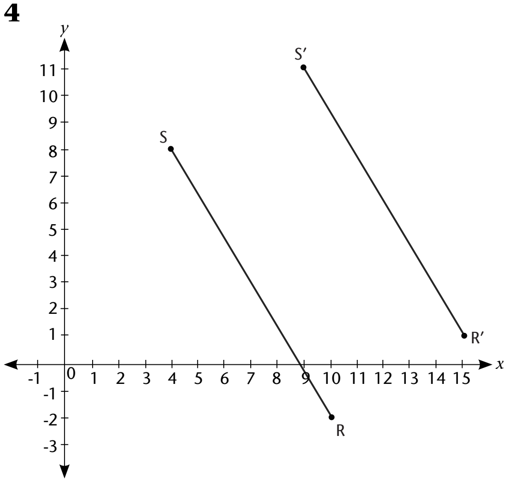

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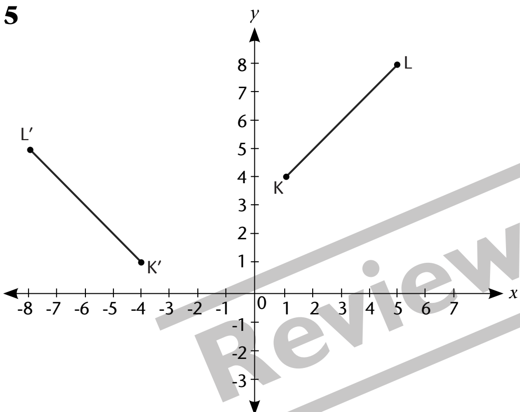

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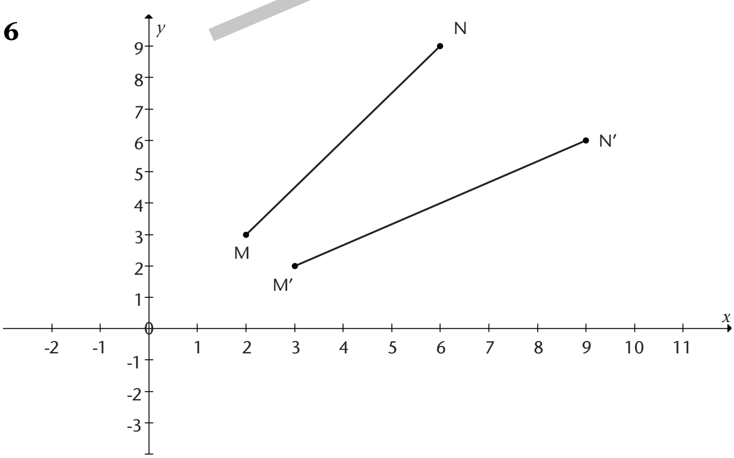


7 Line segment AB has been translated two units to the right and one unit down.
Line segment CD has been translated five units to the right and four units down.

Exercise 3

Learner's Book page 359

Guidelines on how to implement this activity

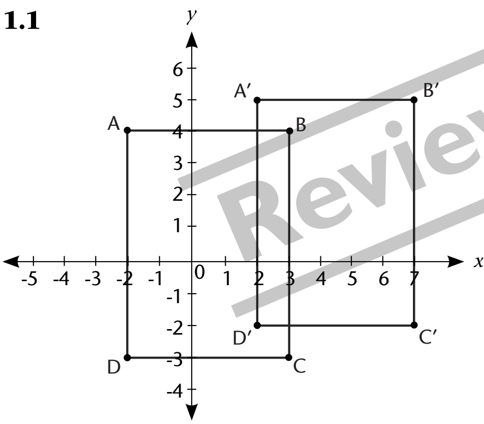
Transforming a figure extends a transformation from two points to three, four or more points. It is important that learners note that the transformed figure is congruent. This means all the sides, angles and proportions remain the same, but that the position of the shape on the Cartesian plane has changed. When translating a shape, the shape remains not only congruent but also parallel and co-linear to the original. Encourage learners to perform transformations using transparencies to help them visualise the transformation, and to accurately plot the points of the transformed figure.

Show learners how they can use the general rules for transformation to predict the co-ordinates of each vertex of the figure. Learners then test their predictions by performing the transformation.

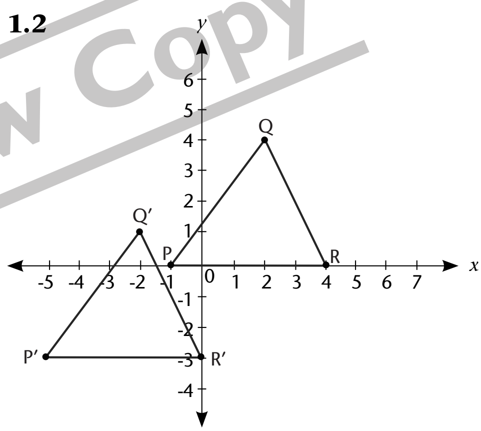
Be careful with rotations as learners need to only know predictions of rotations of 90 degrees, but can be asked to perform the actual rotation, without prediction, beyond this.

Suggested answers

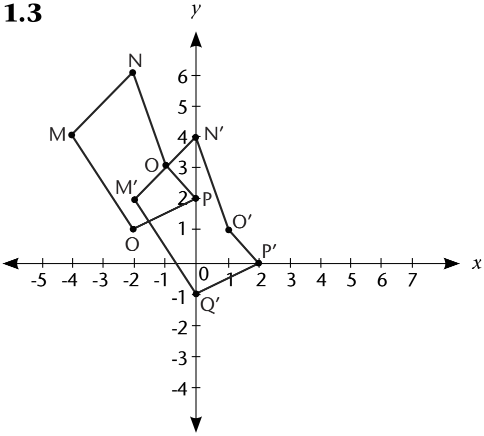
1.1



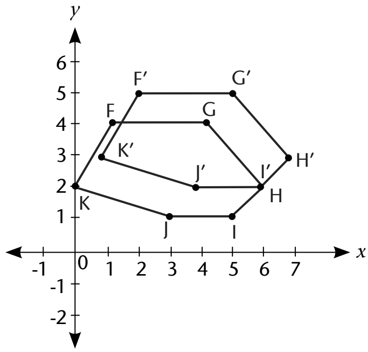
1.2



1.3



1.4



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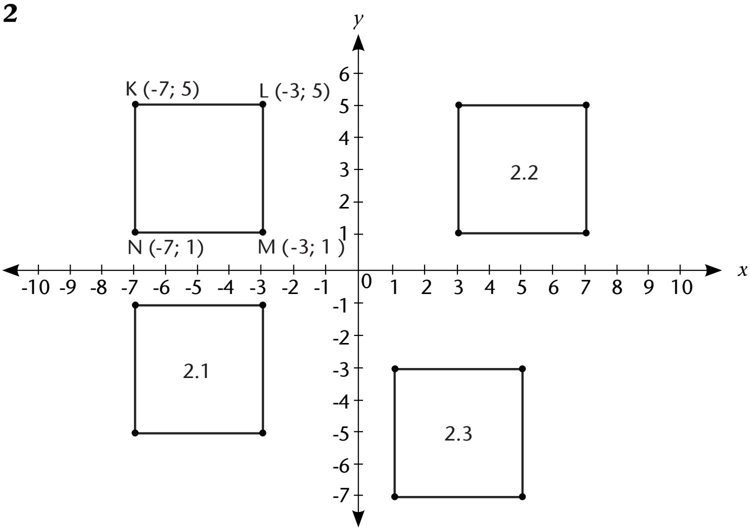
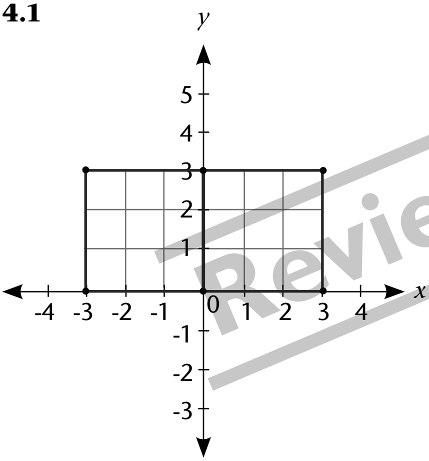
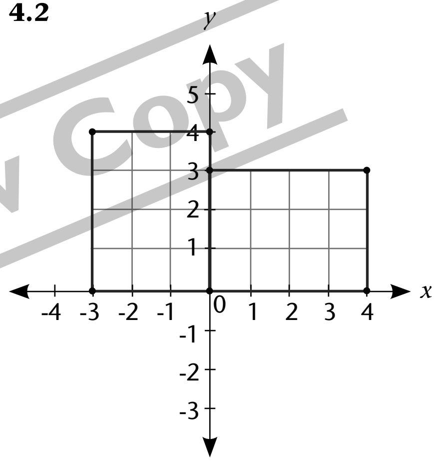


3 T'(−3; 1); V'(−1; 3); W'(−1; 6); X'(−3; 8); Y'(−5; 6); Z'(−5; 3)

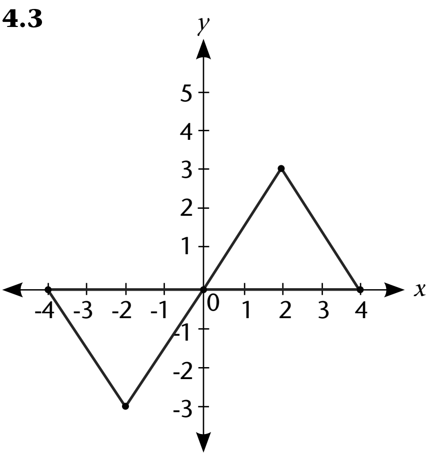
4.1



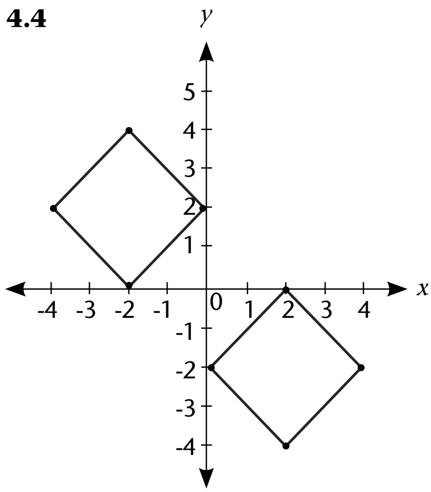
4.2



4.3



4.4



5.1	Reflection across the y-axis	**5.2**	90° anti-clockwise
5.3	180° in either direction	**5.4**	180° in either direction

Remedial

Ensure learners work with grid paper at all times when working with transformations. This helps learners to be more accurate, and helps facilitate the focus on the method rather than the complexities of the construction.

Extension

Encourage stronger learners to draw complex shapes and suggest possible complex transformations. Collect and redistribute the examples around the more confident learners in the class to try.

Unit 3 Enlargements and reductions

Learner's Book page 362

Unit focus

This unit focusses on the following:
- learning about enlarging and reducing by means of a scale factor,
- using proportion to describe the effect of enlargements or reductions on perimeter,
- useing proportion to describe the effect of enlargements or reductions on area, and
- investigateing co-ordinates of vertices of figures that have been enlarged or reduced.

Background information

When figures are enlarged or reduced the shapes are no longer congruent, but similar. Learners need to be familiar with the concept of similar figures and similarity. Enlargements and reductions take place at a set scale factor, from a centre of enlargement or reduction. Learners need to be able to draw similar figures according to the scale factor on the Cartesian plane and determine the scale factor from the drawing. Learners also need to be able to work with area and perimeter in order to investigate the effects of enlargement and reductions by scale factors on the area and perimeter of figures. This unit lays the groundwork for further work on enlargements and reductions in Grade 10 and FET mathematics.

Exercise 1

Learner's Book page 364

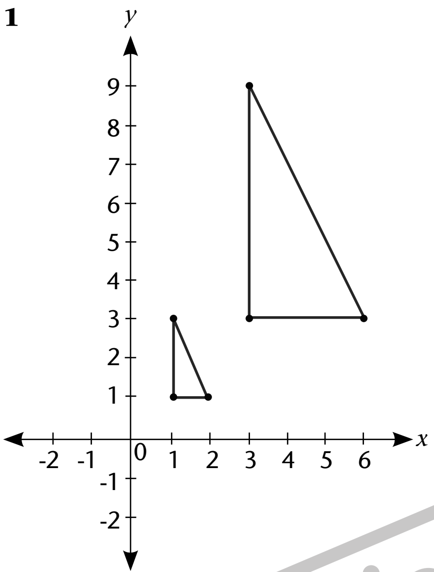
Guidelines on how to implement this activity

Revise the concept of similarity with learners. Ensure learners understand that shapes are similar when their angles are identical and when their corresponding sides are in proportion. Discuss what being in proportion means and ensure learners can accurately calculate this with given examples. Work through the Worked examples on enlarging and reducing figures. Show learners that enlargement occurs when the scale factor is larger than 1, and that reduction happens when the scale factor is less than 1, but greater than 0.

Perform transformations on a few examples together as a class. Investigate what happens to the co-ordinates of the figures when enlarged or reduced. Show learners how to use the co-ordinates to determine the scale factor without necessarily having to perform the transformation.

Suggested answers

1



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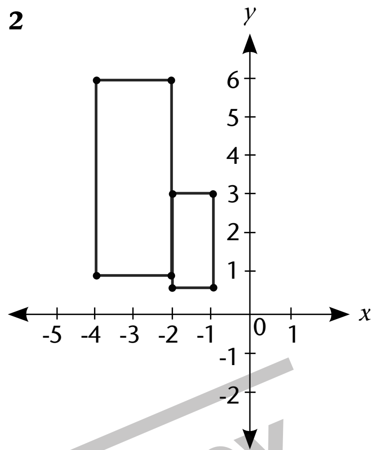

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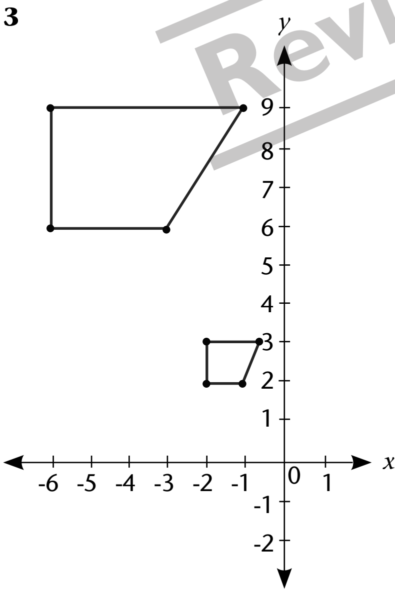

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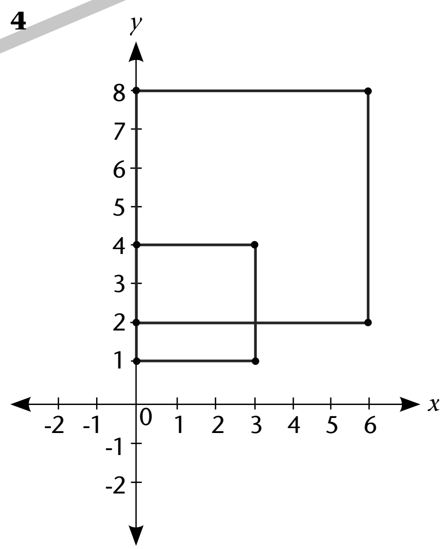


Exercise 2

Learner's Book page 366

Guidelines on how to implement this activity

Revise the concepts of area and perimeter. Discuss formulae for common figures, such as the area of a square, rectangle and triangle. Work through the examples in the Learner's Book to investigate the effect of an enlargement or reduction on the perimeter and area of a figure. Do additional examples where learners hypothesise the new area and perimeter of the enlarged figure, and then perform the transformation to check their hypothesis.

Suggested answers

1 Scale factor is $\frac{1}{3}$

Area of shape B = $21 \times 21 = 441$ cm^2
Area of shape A = = $7 \times 7 = 49$ cm^2

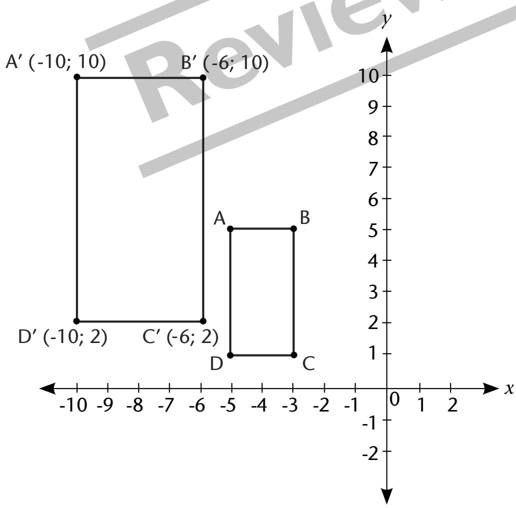
2 Scale factor is 2
Area of shape B = $48 \times 8 = 384$ cm^2
Area of shape A = = $96 \times 16 = 1\ 536$ cm^2

3 Scale factor is 8

4.1 Perimeter = $12 \times 6 = 72$ m
Area = $8 \times 6^2 = 8 \times 36 = 288$ m

4.2 Perimeter = $12 \times \frac{1}{6} = 2$ m
Area = $8 \times \left(\frac{1}{6}\right)^2 = 8 \times \frac{1}{36} = 0{,}22$ m^2

5



6 Area ABCD = $4 \times 2 = 8$ cm^2
Area A'B'C'D' = $8 \times 4 = 32$ cm^2
The area of the image is four times bigger than the area of the original

7 Perimeter ABCD = $2(4 + 2) = 12$ cm
Perimeter A'B'C'D' = $2(8 + 4) = 24$ cm
The perimeter of the image is double the perimeter of the original.

Remedial

Provide additional examples to do together as a class or in small groups before learners tackle the activity on their own. Allow learners to work in groups and encourage learners to explain their thinking and problem solving strategies to one another. This will help learners who may be experiencing problems to better understand the process.

Extension

Use the internet and graphical programs to illustrate transformations on a computer screen or smartboard. It is very useful for learners to see the transformation taking place.

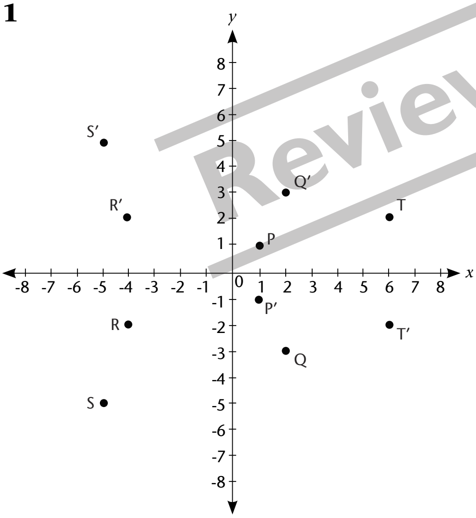
Consolidation

Learner's Book page 369

Before doing this consolidation exercise, encourage learners to review the work covered in this chapter. Advise learners to use the summary and to revise their work. This exercise can be used as an informal assessment task for you to track how learners are coping with the chapter and the concepts covered. The mark allocation provides guidelines on how to assess learners.

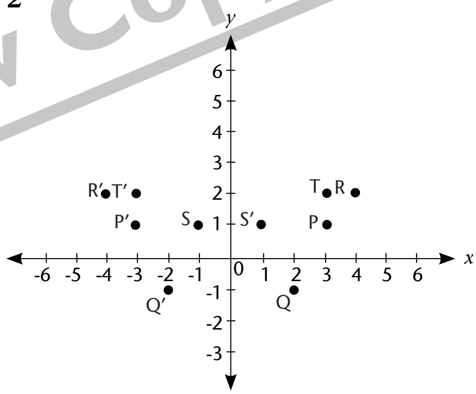
Suggested answers

1



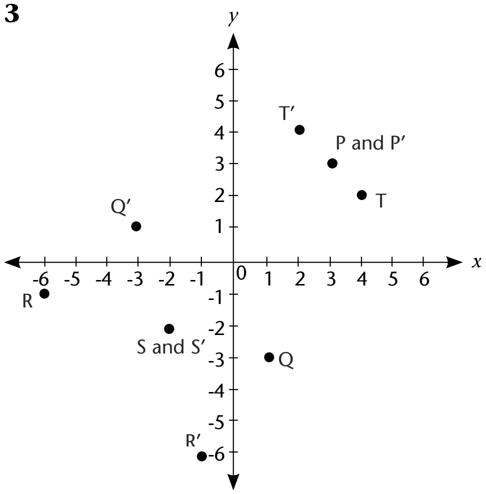
(5)

2



(5)

3

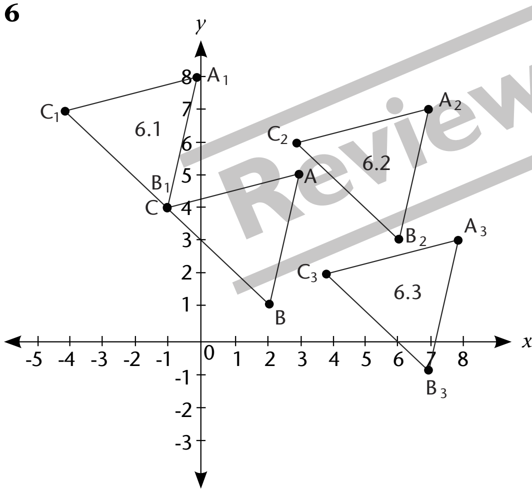


(5)

4.1 (−5; 6) **4.2** (5; −6) **4.3** (6; 5) (3)

5 P to Q: translated up by six units
P to R: translated up by three units and to the right by three units
P to S: translated down by five units and to the right by one unit (4)

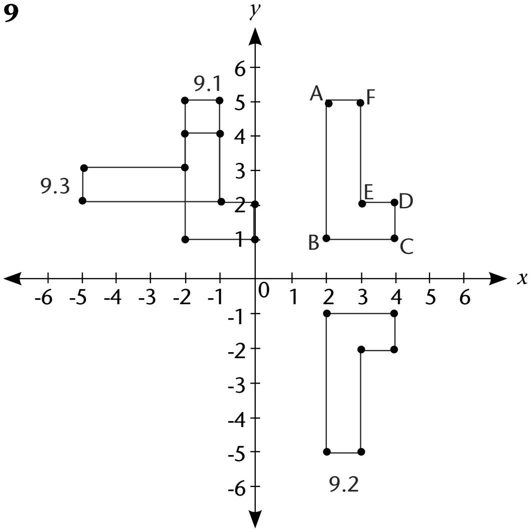
6



(3)(3)(3)

7.1 90° clockwise **7.2** 180°
7.3 90° clockwise **7.4** 90° clockwise (4)
8.1 A'(0; −1) and B'(0; −5) **8.2** A'(−2; −3) and B'(1; 0)
8.3 A'(−2; 3) and B'(−4; 3) **8.4** A'(1; 1) and B'(3; −3) (8)

9



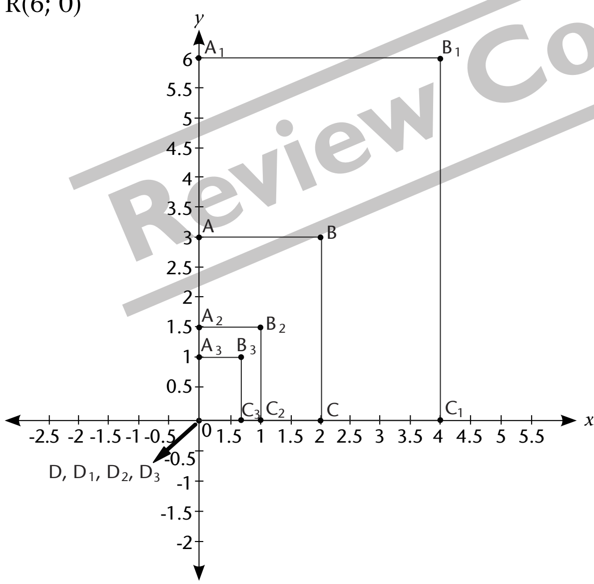
(2 × 3 = 6)

10.1 PQD is a right-angled triangle. (1)

10.2 Translate point D 3 units to the right (point D becomes point R). (1)

10.3 R(6; 0) (1)

11.1



11.2 $A_1B_1 : AB = 4 : 2 = 2 : 1$
$B_1C_1 : BC = 6 : 3 = 2 : 1$ (2)

11.3 Scale factor 2 (2)

11.4 $AB : BC = 2 : 3$
$A_1B_1 : B_1C_1 = 4 : 6 = 2 : 3$ (2)

(2)

11.5 $A_2B_2 : B_2C_2 = 2 : 3$
$A_3B_3 : B_3C_3 = 2 : 3$ (2)

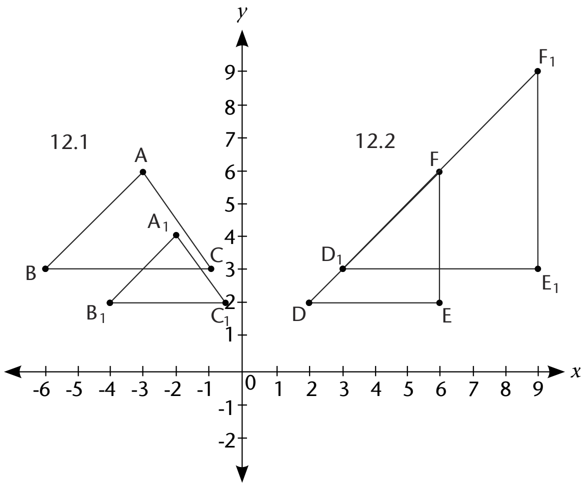
11.6 Yes, you get the same answer as in question 11.4. (2)

11.7 Yes, the new rectangles look similar to the original rectangle. (2)

11.8 All angles are 90 degrees. (3)

11.9 They are not congruent as their sides are different lengths. (2)

12.1 and **12.2**



(3 × 2 = 6)

12.3 DE = 4 units, EF = 4 units, so DE:EF = 1 : 1
D_1E_1 = 6units, E_1F_1 = 6 units, so $D_1E_1 : E_1F_1 = 1 : 1$ (3)

12.4 E = E_1 = 90°
D = D_1 = 45°
F = F_1 = 45° (3)

12.5 The sides of similar triangles are in proportion. (2)

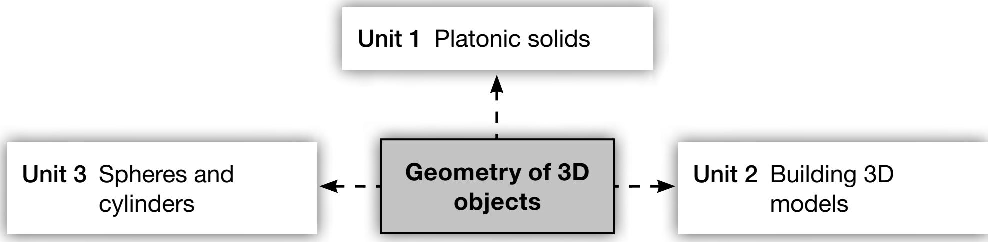
12.6 The angles of similar triangles are equal. (2)

13 AE = 7 units
GH = 8 units
HI = 10 units
IJ = 4 units (3)

[90]

Chapter 14 Geometry of 3D objects

Overview of concepts



Content		Time allocations	LB page
Unit 1	Platonic solids	3 hours	374
Unit 2	Building 3D models	3 hours	377
Unit 3	Spheres and cylinders	3 hours	384

Background information on 3D objects

We live in a 3D world and the study of 3D objects is important in geometry. Learners already have prior knowledge of:

- polygons,
- congruent figures,
- drawing nets of prisms, and
- building models of prisms.

This chapter encourages learners to see the world around them in a mathematical way. The objects and structures in their environment are constructed out of 3D objects. Learners should be encouraged to break down objects they see into definable 3D objects they have learnt about and observe the properties and dimensions of these objects.

Teaching guidelines for teaching 3D objects

Begin by reminding pupils of the following:

- Polygons are named according to the number of their sides (or edges).
- Polygons may be regular or irregular.
- The sum of the interior angles of any polygon with n sides is given by the formula $S = 180°(n - 2)$.
- Congruent figures are identical in all their dimensions.
- Nets are 2D representations of 3D shapes.

Resources

3D objects, especially examples of the platonic solids to work with in class. Card for nets, colour paper and pens, scissors, ruler, glue and each learner should have their own calculator.

Unit 1 Platonic solids

Learner's Book page 374

Unit focus

This unit focusses on the following:
- learning the properties of the five Platonic solids, and
- calculating the Euler characteristic for polyhedrons.

Background information on the Platonic solids and the Euler formula

Platonic solids

3D objects whose faces are polygons are called polyhedrons. There are only five with faces that are all congruent polygons and these are the five Platonic solids: cube, tetrahedron, octahedron, dodecahedron and icosahedron. They are named according to the number of their faces.
For helpful information on the Platonic solids, nets and hints see the website: www.mathsisfun.com.
Despite being named after Plato, the appreciation of solids constructed out of regular congruent figures has been known since 1 500 BC. Such carved figures can be found among the sculptures of the late Neolithic peoples in Scotland and surrounding areas. The Platonic solids are named after Plato because of his extensive dialogue about them in *Timaues*, which he wrote in approximately 360 BC. Plato linked each of the solids to a classical element. The tetrahedron was fire, the cube was earth, the icosahedron was water and the octahedron was air. As these were the only elements thought to exist there was no element assigned to the dodecahedron.
The Platonic solids have inspired mathematicians and scientists through the centuries. In 1596, Johannes Kepler used the Platonic solids in his model of the universe. His theories of the distance between planets being dictated by the solids had to be abandoned, but through his research in this area Kepler developed his three laws of orbital dynamics, which came to determine that planets orbit in ellipses rather than circles. In the twentieth century, the Platonic solids have inspired the work of Robert Moon, who developed the electron shell model and expanded our understanding of particle physics.

Euler's polyhedron formula

Euler's formula states that $V - E + F = 2$. In words, this means that the number of vertices, minus the number of edges, plus the number of faces will equal 2. Euler's formula works for all polyhedron except those that have holes running through them.

The power of Euler's formula is that it shows, without the need for construction, which 3D objects are possible and which are not possible. For example, there is no simple polyhedron with ten faces and seventeen vertices.
Euler's formula was used to determine all the possible Platonic solids and has helped show that the five mentioned above are the only possible Platonic solids that can exist.

Exercise 1

Learner's Book page 376

Guidelines on how to implement this activity

In order to investigate the properties of the Platonic solids, learners need a clear understanding of what is meant by a "face", "vertex" and "edge". This is best demonstrated using a 3D object, such as a rectangular box. Start with a simple 3D object and each aspect using the object as a reference. Ensure each learner understands the differences between an edge and a face.
Have examples of the Platonic solids to show to learners. Ideally these should be 3D objects, but pictures and clear diagrams can also work. Use the objects or pictures to work through the properties of each of the solids. Ensure learners understand what makes the Platonic solids special. The regularity of the shapes must be emphasised. Learners should have as much opportunity to investigate and handle the shapes as possible to help them understand these solids.
Demonstrate Euler's formula using the Platonic solids as examples. Write the names of the Platonic solids on the board and ask learners to come up and fill in the number of faces, vertices and edges for each solid (you may need to show learners diagrams or models to help them with this). Ask learners to calculate the Euler characteristic for each shape on the board.
Learners should memorise this formula and be able to apply it for a variety of 3D objects.

Suggested answers

1.1 $\chi = 4 - 6 + 4 = 2$

1.2 $\chi = 8 - 12 + 6 = 2$

1.3 $\chi = 6 - 12 + 8 = 2$

1.4 $\chi = 20 - 30 + 12 = 2$

1.5 $\chi = 12 - 30 + 20 = 2$

2 All the Platonic solids have a Euler characteristic of 2.

3.1 $\chi = 6 - 10 + 6 = 2$

3.2 $\chi = 16 - 24 + 10 = 2$

3.3 $\chi = 6 - 9 + 5 = 2$

Extension

Set the class a task to see how many different and correct ways they can write Euler's formula. This involves changing the subject of the formula.
Tell learners to test the formula on other solids and find out whether it applies to the Platonic solids only, or also to some other 3D objects.

Unit 2 Building 3D models

Learner's Book page 377

Unit focus

This unit focusses on the following:
- how 3D models can be represented by 2D nets, and
- drawing nets to construct 3D objects.

Background information on building 3D models

Demonstrations of how to fold nets to build models are available on the Internet. One such website is www.learner.org/interactives/geometry. Learners who struggle may benefit by watching the videos available on this website.

Exercise 1

Learner's Book page 383

Guidelines on how to implement this activity

Learners must understand the relationship between a 3D object and a net of the same object. Use a box or rectangular prism and demonstrate how the faces form the net when the object is cut along the edges and flattened. The construction of each of the nets of Platonic solids is covered in the Learner's Book. The class must construct the nets of all five of these solids and build models of the solids.
Learners should work in groups of five so that each group has a complete set of the solids. Ensure learners work neatly and accurately so that they are able to create neat and presentable models.

Suggested answers

Learners construct nets and 3D models. Make sure that learners work neatly and accurately.

4.1

Platonic solid	Faces	Vertices	Edges
Tetrahedron	4	4	6
Cube	6	8	12
Octahedron	8	6	12
Dodecahedron	12	20	30
Icosahedron	20	12	30

4.2 The number of faces, plus the number of vertices, minus the number of edges equals two.
$F + V - E = 2$

Remedial

Learners may need to be reminded of how to calculate the interior angles of a regular pentagon in order to construct the net of the dodecahedron. If the model building activities are done in groups, arrange the groups so that each consists of learners with mixed abilities. In this way they can co-operate and assist each other.

Extension

Ask the class to decorate their models of the solids with colour (colour the nets before building the models). Attach string to the models and hang them up in the classroom, either individually or as a mobile containing many shapes.

Unit 3 Spheres and cylinders

Learner's Book page 384

Unit focus

- learning about spheres,
- learning more about cylinders, and
- constructing cylinders.

Background information on spheres and cylinders

Spheres are 3D objects that are perfectly round. All points on the surface are equidistant from the centre of the sphere. This distance is the radius of the sphere. A sphere has no vertices.
A cylinder has two parallel, circular faces. The curved surface between them is perpendicular to the plane of the circular base and forms a rectangle in the net of the cylinder.
Spheres and cylinders have many practical uses. Balls and marbles are examples of spheres. Tin cans, some drinking glasses, mugs and vases are examples of cylinders.

Exercise 1

Learner's Book page 386

Guidelines on how to implement this activity

Most learners are familiar with objects that are spheres and cylinders, so they should have no trouble understanding their properties. The net of a cylinder is simple to construct and use to build a model. Surface area and volume of a cylinder has been studied in a previous chapter, so simple revision of the formulae should be enough to prepare learners for Exercise 1.

Suggested answers

1.1 It is important that the length of the rectangle is equal to the circumference of the circle so that the net folds exactly to form a cylinder.
1.2 They do not have any properties in common.
1.3 It is made up of only one continuous curved surface.
1.4 Basketball, cricket ball, tennis ball, marble, orange (other reasonable answers should be accepted).
1.5 Drinks cans, tubes inside toilet rolls, poles, canned foods, tennis ball container (other reasonable answers should be accepted).

2.1 $2\pi r = 15$

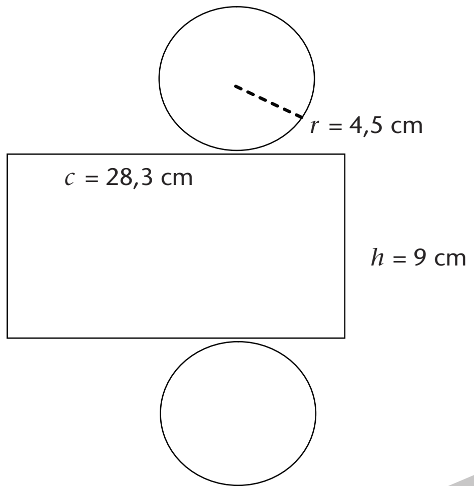
$r = \frac{15}{2\pi} = 2{,}39$ cm

2.2 $2\pi r = 21$

$r = \frac{21}{2\pi} = 3{,}34$ cm

3.1 Radius of cylinder = 4 cm + 5 mm = 4,5 cm
Height of cylinder = 8 cm + 5 mm + 5 mm = 9 cm

3.2



3.3 Vol of cylinder = $\pi(4{,}5)^2(9) = 572{,}56 \text{ cm}^3$

Vol of sphere = $\frac{4}{3}\pi(4)^3 = 268{,}08 \text{ cm}^3$

Volume of air around the sphere in the cylinder $= 572{,}56 \text{ cm}^3 - 268{,}08 \text{ cm}^3$
$= 304{,}48 \text{ cm}^3$

Remedial

Some learners may be confused about why the length of the rectangle in the net of a cylinder has to equal the circumference of the circular face. Point out that the circumference shares an edge with the rectangle and these must therefore be the same length.
If the idea of the radius of a sphere causes confusion, use a polystyrene ball or tennis ball and cut it in half to demonstrate where the radius of a sphere is.

Extension

A ball is a sphere, but what about a soccer ball? It consists of pentagonal and hexagonal faces. Hold a class discussion about whether a soccer ball is a sphere or a polyhedron. If possible, give learners time to research the question.
The more faces a polyhedron has, the closer it becomes to a spherical shape. Challenge the class to a discussion about whether a sphere is a polyhedron with an infinite number of faces or whether it has only one face. Again, some research online may be helpful.
Give the class a project to do some historical research on Plato's studies of the solids that have been named after him.

Consolidation

Learner's Book page 388

Guidelines on how to implement this activity

Before doing this consolidation exercise, encourage learners to review the work covered in this chapter. Advise learners to use the summary and to revise their work. This exercise can be used as an informal assessment task for you to track how learners are coping with the chapter and the concepts covered. The mark allocation provides guidelines on how to assess learners.

Suggested answers

1 C is not the net of a cube. (2)

2 No, because all the faces are not congruent – it has pentagonal and hexagonal faces. (2)

3 No, because all the faces are not congruent – it consists of one square and four triangles, whereas a tetrahedron has four equilateral triangles as its faces. (2)

4 B represents the net of a tetrahedron. (2)

5 The cube is also a right prism. (1)

6 $F + V - 2 = E$
$E = 12 + 20 - 2 = 30$ (2)

7.1 $F + V - E = 2$
$F = 2 - V + 3$
$= 2 - 12 - 30$
$= 20$ (2)

7.2 An icosahedron. (1)

8 8.4 is not a correct statement of Euler's formula ($V + F + 2 = E$) (2)

9 Learners construct a net according to the dimensions in the Learner's Book. Measure their drawing to ensure accurate measurements. (4)

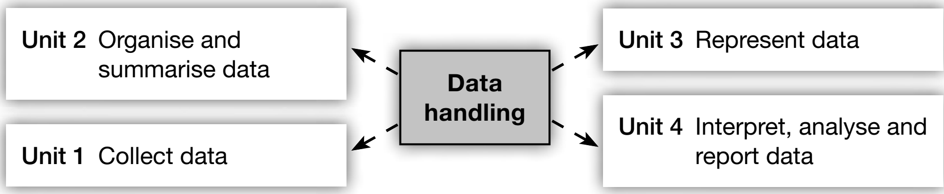
10 Learners construct a net according to the dimensions in the Learner's Book. Measure their drawing to ensure accurate measurements. The length of the rectangle should be 12,57 cm. (6)

[26]

CHAPTER 14

Chapter 15 Data handling

Overview of concepts



Content		Time allocations	LB page
Unit 1	Collect data	2 hours	391
Unit 2	Organise and summarise data	2 hours	394
Unit 3	Represent data	3 hours	398
Unit 4	Interpret, analyse and report data	3,5 hours	403

Background information on data handling

Data handling is a topic that can be enjoyed by learners of all abilities. It is a great leveller as it gives the weaker learners the opportunity to shine alongside their stronger peers. For the weaker learners, data handling provides relief from more difficult and abstract concepts. Much of your learners' enjoyment of this topic will depend on how you present it. If you can keep your treatment of this material fun, practical and accessible to all, you will find that your whole class responds well and with enthusiasm to this work.

Your learners have already worked with data handling in previous grades, so much of the content of this chapter will take the form of revision and consolidation of existing knowledge. Some concepts are new in Grade 9 however, including outliers, scatter plots and organising data according to more than one criterion.

In real life, to be able to speak meaningfully about measures of central tendency and dispersion and to draw conclusions about trends, data sets need to be very large. However, when we discuss data handling at school level, data sets must of necessity be small. This means that a lot of our examples and contexts are contrived and not at all realistic. We do not want to undermine the value to our learners of what they are learning by constantly reminding them that data sets are always much larger in real life. Do not over-think this paradox; you will find that your learners will be quite happy to take at face value the merits of applying concepts to small data sets. What is important is that they get the different concepts clear in their mind and that they are able to apply them correctly.

Generic teaching guidelines for teaching data handling

Your teaching of data handling will be most effective if you keep your treatment of this work practical, relevant and accessible to all your learners.
Visual aids always provide interest. Try to build up a collection of interesting and colourful graphs of all types that you can display on the walls of your classroom. You do not have to limit these to the graphs studied in the mathematics curriculum, although obviously these will be even more relevant to your learners.
The extension opportunities at the end of each unit throughout this chapter are all centred on a single theme, namely a study that your learners will plan, based on a topical issue of their own choosing. How seriously they pursue this study is entirely up to you. It could remain at the planning stage, where ideas are thought up, shared and refined, or it could form the groundwork for a serious practical study in the future. To motivate your learners to take this study seriously, consider offering a small prize or recognition to those learners whose work reflect thought and effort on their part. For example, the winning study could be made into a poster and displayed in the classroom or around the school.

Resources

Examples of data from magazines, newspapers and the internet. Blank graph templates, colour card and pens, rulers, protractors for pie charts, and scissors. Each learner should have their own calculator.

Unit 1 Collect data

Learner's Book page 391

Unit focus

This unit focusses on the following:
- posing questions relating to social, economic and environmental issues,
- selecting and justify appropriate sources for collecting data,
- distinguishing between samples and populations and suggest appropriate samples for investigation, and
- selecting and justifying methods for collecting data.

Background information on collecting data

Your learners have worked with data collection in previous grades, so the ideas in this unit are not new. What is important is that they consolidate their understanding of the terms involved, as well as the idea that it is not always possible to interview a population. A compromise must sometimes be made, and this is done by choosing a sample. A bad choice of sample will lead to meaningless results. It is important that your learners understand the idea of how to select a random sample, as opposed to a biased sample. They should also learn to appreciate the fact that some questions will result in more meaningful data than other questions, so a questionnaire must be planned as carefully as the selection of a random sample.
Make sure that learners understand the meaning of "random sample" and "bias".

Exercise 1

Learner's Book page 391

Guidelines on how to implement this activity

This is a revision exercise that you can use as a baseline to assess how much learners remember about data collection.

Suggested answers

The **population** is the whole group of people that you are studying. When it is not practical to study this whole group of people, you choose a selection of the whole group to study. This is called a **sample**. When you have a list of questions that you want to ask in your survey, this is called a **questionnaire**. A question can have a yes/no response, or it can offer the person a list of options to choose from. The person answering the questions is called a **respondent** and a question that has a list of options to choose from is called a **multiple choice** question.

Exercise 2

Learner's Book page 393

Guidelines on how to implement this activity

Discuss ways in which we can collect data. Refer to different kinds of questions that can appear on a questionnaire. Discuss and compare different kinds of questions. Your learners should appreciate that there are different ways of phrasing questions and that the way in which questions are worded will have a direct effect on the kind of answers they will receive.
Discuss what considerations should be made in order to select a sample. Be sure to address the all-important ideas of random and biased samples. Ask your class to do Exercise 2. Throughout this chapter, consider allowing your learners to work in pairs or in small groups. Your learners will benefit by being allowed to exchange their views with others, especially when questions are open-ended or open to interpretation.
Discuss the answers to Exercise 2 with your class. Do this in the form of a class discussion, where your aim should be to involve as many learners as possible. When answering open-ended questions about data handling, your learners may feel hesitant about offering their own opinions. They may associate the mathematics classroom with a place where answers are either right or wrong. If you can create an atmosphere where all opinions are listened to and considered on their own merits, you should succeed in making learners comfortable in engaging in these class discussions.

Suggested answers

1.1 Suitable.

1.2 Unsuitable as learners may not know whether they are overweight or not, and it may offend learners who are overweight. Rather use check boxes giving ranges of different weights so that learners can select which range they fit into.

1.3 Suitable, but may be more useful to give check boxes with different ages for learners to select.

1.4 Suitable.

1.5 Again, giving check boxes may be more useful (e.g. Have you had: no drinks; 1 – 2 drinks; 3 or more drinks in the past week?).

1.6 Unsuitable as the learners are in high school so none of them would have passed Grade 12.

2.1 All of the learners in your school.

2.2 A biased sample would be a sample consisting of specifically selected learners (i.e. learners who smoke or learners who do not smoke).

2.3 A random sample can be chosen by randomly selecting 10 learners from each class; or by arranging all the learners in alphabetical order and selecting every 4 or 5 learner (depending on the size of the sample required).

3.1 You could count items or you could look up information in newspapers, books, magazines or on the Internet.

3.2 The Internet, a public library, an atlas, a travel bureau, a tourist information centre, a local municipality, a police station, a hospital, newspapers etc.

Remedial

Ask your learners to think about some more questions that would be "safe" to ask their peers. They should make a short list of these questions for homework.
Likewise, ask them to think about some questions that would be controversial to ask their peers. They should make a short list of these questions for homework. Can they think of ways to word these questions that would not cause offence?
Finally, ask them to think of a piece of information that they would like to get from an adult in their lives. Ask them to word this question in at least two different ways to see what kinds of responses they might get. They could try this out at home and make a list of the questions, as well as of the responses.

Extension

Ask your learners to think about how they would conduct a large study on an issue that is topical at the moment. Below are some suggestions that could pertain to their community or their peers:

- recycling awareness (or lack thereof)
- healthcare facilities (or lack thereof)
- career opportunities for school-leavers
- drug/alcohol/nicotine abuse
- domestic violence
- child abuse
- poverty
- crime

These last suggestions may all sound rather serious, which they are. If your learners prefer to study something more light-hearted, let them come up with suggestions of their own, but make sure that their final choices are topical and relevant to their own lives.
As a first step, they should identify the issue that they would wish to study. They should think about this for homework. Ideally they could work with a friend who lives nearby. This study will form the theme for all the extension opportunities throughout this chapter.

Unit 2 Organise and summarise data

Learner's Book page 394

Unit focus

This unit focusses on the following:
- revising and extend your knowledge of organising of data,
- revising and extend your knowledge of summarisation of data, and
- determining measures of central tendency and dispersion.

Background information on organising and summarising data

The focus of this unit is on organising and summarising data by determining measures of central tendency and dispersion. The concept of an outlier is new in Grade 9, but this new concept should be developed alongside a consolidation of their existing knowledge of mean, median, mode and range.

Revision exercise

Learner's Book page 394

Guidelines on how to implement this activity

Use this revision as a diagnostic tool to establish what learners remember from previous grades.

Suggested answers

1.1 The mean, as it includes all five values and gives the average.
1.2 The mode, as it would show the book that is most commonly taken out of the library.
1.3 The median, as it is the middle value of the shoe sizes.
1.4 The median, as it gives the middle value.
1.5 The mean, as it includes all the values and gives the average.
1.6 The mode, as it shows the most common response in a data set.
2.1 Range $= -160 - (-200) = 40$
2.2 No because the range is the difference between the largest and the smallest value, and when you subtract a smaller negative value from a larger negative value you will get a positive answer.

Exercise 1

Learner's Book page 396

Guidelines on how to implement this activity

Start a class discussion about the meaning of an outlier. Work through the Worked example from the Learner's Bookwith your class. Use this Worked example to reinforce your learners' understanding of the mean, median, mode and range of a numerical data set. Point out what effect, if any, the outlier has on the different measures of central tendency and dispersion.

It is important that your learners appreciate the following points about an outlier:
- An outlier affects the range the most, because it is the difference between the highest and lowest values.

- An outlier also affects the mean, because the mean takes every data value into account equally.
- The median and the mode are always the least affected by an outlier.

Revise how to calculate the median of a data set, then ask your class to do Exercise 1. This exercise will give your learners plenty of practice in testing their knowledge of the work that has been done thus far. Walk through your class as they do this exercise and be prepared to remediate any problems as they arise. If your learners do not understand the adjective "modal", simply explain that this refers to the mode.

Suggested answers

1.1 Mean = $\frac{366}{12}$ = 30,5%
Median: 2%; 4%; 5%; 6%; 7%; 30%; 32%; 34%; 46%; 46%; 55%; 99%
The median is halfway between 30% and 32% = 31%
Mode = 46%
Range = 99% − 2% = 97%
Extremes are 2% and 99%
Outlier is 99%
Excluding the outlier:
Mean = $\frac{267}{11}$ = 24,27%
Median is 30%
Mode is 46%
Range = 55% − 2% = 53%

1.2 Mean = $\frac{5,59}{11}$ = 0,5081
Median: −5,32; −4,73;−2,16; 0,11; 0,25; 0,47; 1,52;1,57;2,45; 5,66; 5,77
The median is 0,47
There is no mode
Range = 5,77 – (−5,32) = 11,09
Extremes are −5,32 and 5,77
There are no outliers.

1.3 Mean = $\frac{81\ 424}{8}$ = 10 178
−79 124; 10 760; 12 937; 18 316; 20 376; 22 836; 36 933; 38 390
The median is halfway between 18 316 and 20 376:
Median = $\frac{18\ 316 + 20\ 376}{2}$ = 19 346
There is no mode
Range = 38 390 – (−79 124) = 117 514
Extremes are –79124 and 38 390
Outlier is –79 124
Excluding the outlier:
Mean = $\frac{160\ 548}{7}$ = 22 935,43
Median is 20 376
There is no mode.
Range = 38 390 – 10 760 = 27 630

2

	Mon	Tue	Wed	Thu	Fri	Sat
Customers	36	23	42	23	51	64
Items sold	70	64	103	63	127	185
Income	R560	R534	R810	R509	R1 050	R1 050
Profit	R335	R298	R586	R304	R750	R612

2.1 Modal number of customers = 23

2.2 Mean number of items sold = $\frac{612}{6}$ = 102

2.3 Range of income = R1 050 – R509 = R541

2.4 R298 R304 R335 R586 R612 R750

Median = $\frac{335 + 586}{2}$ = R460,50

2.5 The most successful day was Friday as the profit is highest on Friday.

2.6 You would add the profits from each day and divide the total by six to find the mean profit.

3.1 They have been sorted in chronological order (the order in which they occur in the year).

3.2 There are two modes: $\frac{5}{3}$ and $\frac{8}{5}$

3.3 The modal month is December

3.4

0 | 1 1 2 2 3 3 4 4 4 5 5 7 7 7 7 8 8 8 9
1 | 0 1 1 2 4 4 4 6 6 8 8 9 9
2 | 1 2 3 3 5 6 6 7 8 9
3 | 0 1

3.5.1 Median is halfway between 21 and 22 = 21,5

3.5.2 Mode = 7

3.5.3 Range = 31 – 1 = 30

3.6 There are only one or two possible days in each month starting with a 3 (the 30 and the 31).

4

Team A	12	7	18	9	4	13	21	28	20	0
Team B	4	0	1	125	0	2	6	1	3	8
Team C	30	13	16	9	42	28	13	31	15	13

4.1 Team A
Mean = $\frac{132}{10}$ = 13,2
Median: 0; 4; 7; 9; 12; 13; 18; 20; 21; 28
The median is halfway between 12 and 13 = 13,5
There is no mode.
Range = 28 – 0 = 28
Team B
Mean = $\frac{150}{10}$ = 15
Median: 0; 0; 1; 1; 2; 3; 4; 6; 8; 125
The median is halfway between 2 and 3 = 2,5
The modes are 0 and 1
Range = 125 – 0 = 125
Team C
Mean = $\frac{210}{10}$ = 21
Median: 9; 13; 13; 13; 15; 16; 28; 30; 31; 42
The median is halfway between 15 and 16 = 15,5
The mode is 13
Range = 42 – 9 = 33

4.2 The mean, as it takes the outliers into account, so if a team did not make much effort to sell tickets on some days the mean will take this into account.

4.3 Team C as they have the highest mean and they sold some tickets every day.

4.4 125

Remedial

Some common mistakes learners make in data handling are:

- When calculating the median of a data set, your learners forget to order the data set first. Remind them that the median of a data set is always the middle value of the *ordered* data set.
- When learners order a data set, they write the same number down twice, or leave a number out altogether. One way to safeguard against this is to always count the numbers of values in the ordered data set and then to compare this to the number of values in the original data set. They should always make a habit of doing this.

In Question 4, some of your learners will need to be reminded how a stem-and-leaf diagram works. Make sure that they understand this concept and apply it correctly.
When calculating the mean of a data set, your learners should always check their answer to make sure that it looks reasonable, given the values in the data set. It is an even better idea for them to first estimate the value of the mean before doing the calculation. In this way, they will quickly be alerted to any glaring mistakes.

Extension

At the end of Unit 1, your learners chose a topic that they wish to study. They will now pursue this idea by doing the following:

- They need to decide what group of people (the population) they wish to study. Will they be able to interview every person in the population? Or will they need to select a suitable sample of a size that would be practical to interview?
- As importantly, they need to decide what information they want to find out. It is no good simply compiling a list of questions that all vaguely relate to the topic

under consideration. Without a focal point to their questions, they will end up with a lot of responses that mean very little in themselves.
- For example, if they have chosen to study drug abuse in their community, they may want to find out what drugs are available, how easily they are available, what they cost, what the side-effects of these drugs are, how parents can pick up on the signs that their children are doing drugs, and so on. Obviously the scope of this study is very wide as we have outlined it here, and it may be too dangerous to pursue as a school project, so it may well remain in the realm of theoretical planning at this stage. The important thing is that your learners really engage with the idea of this mini-project. As with much else in life, the more that they put into this work, the more they will enjoy doing it.

Challenge

Learner's Book page 397

Guidelines on how to implement this activity

The Challenge question has been designed with your stronger learners in mind, although it is suitable for all your learners to attempt, should they wish to do so.

Suggested answers

8; 8; 8; 10; 12; 13; 60
8; 8; 9; 10; 11; 13; 60

Unit 3 Represent data

Learner's Book page 398

Unit focus

This unit focusses on the following:
- drawing and interpret bar graphs and double bar graphs, histograms, pie charts, broken-line graphs, and
- learning about scatter plots.

Background information on representing data

The focus of this unit is on drawing a variety of graphs to display and interpret data, including bar graphs and double bar graphs, histograms, pie charts, broken-line graphs and scatter plots. Scatter plots are new in Grade 9, so spend some time on making sure that your learners are familiar with the other kinds of graphs before moving on to scatter plots. This is where the idea of having different graphs on display in your classroom will be helpful to both you and to your class. By labelling each graph clearly with the name of the kind of graph that it represents, your learners will constantly be revising this work.

Exercise 1

Learner's Book page 400

Guidelines on how to implement this activity

Revise the properties of the different kinds of graphs with your learners. Make sure that they understand the differences between these graphs, and refer them back to the useful summary in the Learner's Book if they need to refresh their memory when working with graphs.

Introduce learners to scatter plots. Make sure that your learners understand that a scatter plot is used to represent co-ordinate pairs of $(x; y)$ points. Steer clear of terms like "correlation" and "causation" at this level; the idea is to introduce your learners to the most basic and general concepts that relate to a scatter plot.

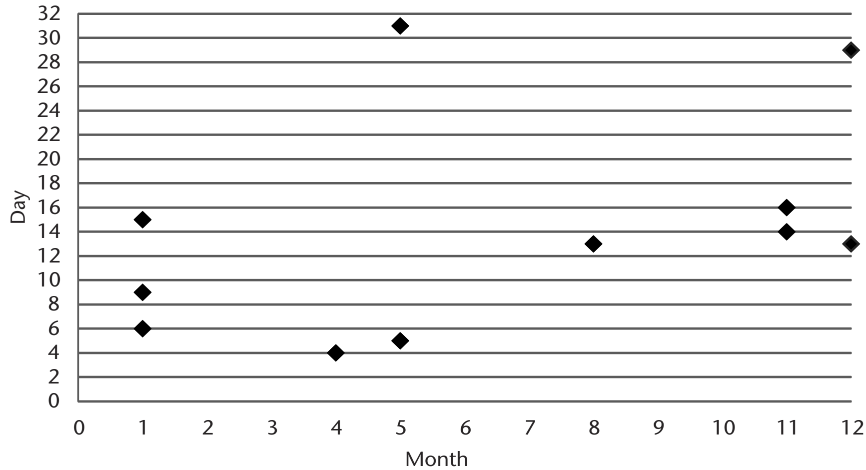
Work through the Worked example with your class. The context is that of a school library teacher who conducted a survey amongst a group of learners to find out how many books they read per month. Her results are summarised as a list of ordered pairs of the form (learner's age; number of books read per month). Spend some time on the resulting scatter plot. Allow your learners to study it carefully and to see how the different points on the scatter plot correspond to the list of ordered pairs. They should be able to read off the co-ordinates of each ordered pair from the scatter plot and should realise that the horizontal value appears before the corresponding vertical value in each ordered pair. Do not expect them to make much of the pattern of dots at this stage; they simply need to be able to relate the points on the plot with the list of ordered pairs.

The second Worked example illustrates the idea of a scatter plot in which a definite trend can be seen. This example is about comparing learners' marks with number of times that they have been in trouble for bad behavior. As one may expect, the learners who score good marks tend to cause less trouble at school than the learners who score poor marks. This can be seen quite clearly in the shape of the dots on the scatter plot. This trend is one of the chief advantages to using a scatter plot, in that it compares up to two variables to see whether or not there might be a relationship between them. This example also illustrates the idea of an outlier which shows up as a dot that does not fit the general trend. In this example, there are in fact two outliers. In terms of this context, the outliers represent learners who are atypical in that they do not "fit the norm".

Once you feel confident that your learners are able to relate sufficiently well to the worked examples, ask them to do Exercise 1. Learners must use rulers to draw the straight lines and should work in pencil so that they can easily erase mistakes – graphs should be neat and accurate. When drawing the pie chart, they should calculate the sizes of the angles accurately. Their calculations should be accurate, even if they do not use actual protractors to measure the sizes of the angles that they draw. If they do not have protractors, they should trace the outline of a suitable round object and then estimate the sizes of the angles that they need to draw.

Suggested answers

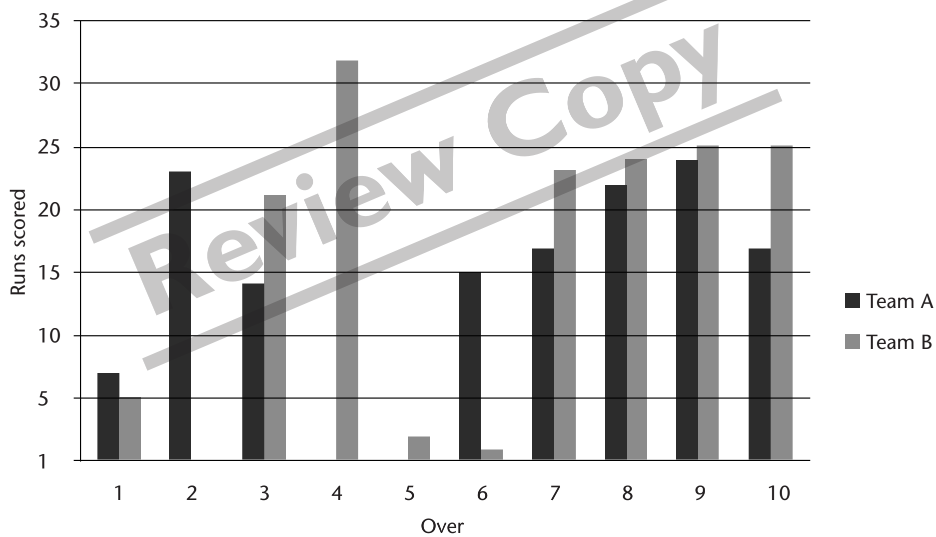
1.1



1.2.1 The most common month is January.

1.2.2 The most common day is the 13 .

2.1



2.2 Mean number of runs per over for team A = $\frac{139}{10}$ = 13,9

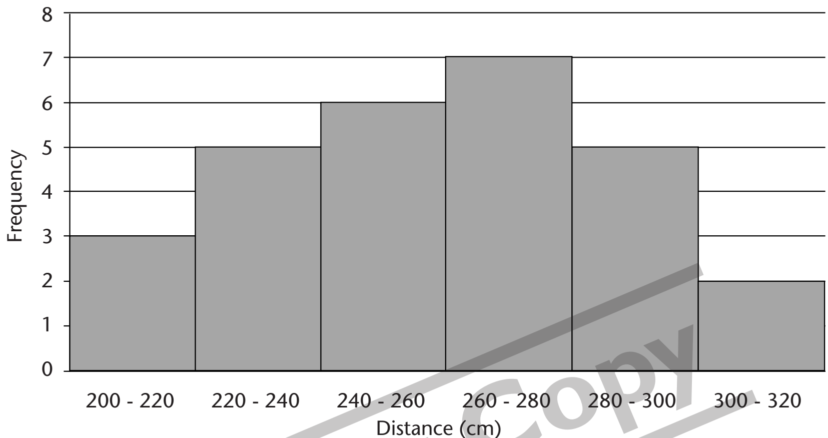
Mean number of runs per over for team B = $\frac{153}{10}$ = 15,3

2.3 Using the mean, the winning team is team B as they have a higher mean. We can also use the total number of runs (153 vs 139), still making team B the winner.

3.1

Distances (cm)	Frequency
[200 – 220)	3
[220 – 240)	5
[240 – 260)	6
[260 – 280)	7
[280 – 300)	5
[300 – 320)	2

3.2



4 Many options possible – learners could choose smaller or larger intervals, but their intervals should all have the same range.

4.1 4; 5; 9; 9; 10; 10; 12; 15; 16; 16; 17; 17; 17; 19; 19; 19; 19; 19; 22; 22; 22; 22; 23; 24; 24; 24; 24; 24; 26; 26; 26; 26; 26; 26; 26; 26; 26; 30; 33; 33

4.2 40 learners

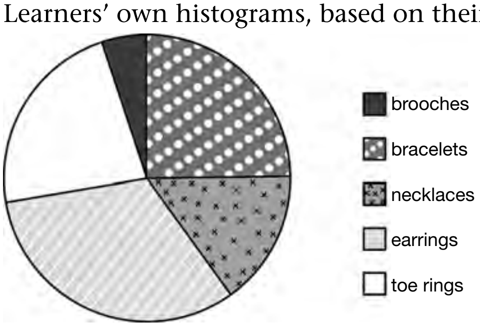
4.3 7,5% of the learners handed their project in late.

4.4 Anna: 4 bars
Jonah: 5 bars
Irshaad: 7 bars
Ludo: 2 bars

4.5 Learners' own frequency tables with intervals of their choice. Bearing the hint in mind, it would be ideal to show this data by week, so the intervals would be 1 – 7; 8 – 14 and so on.

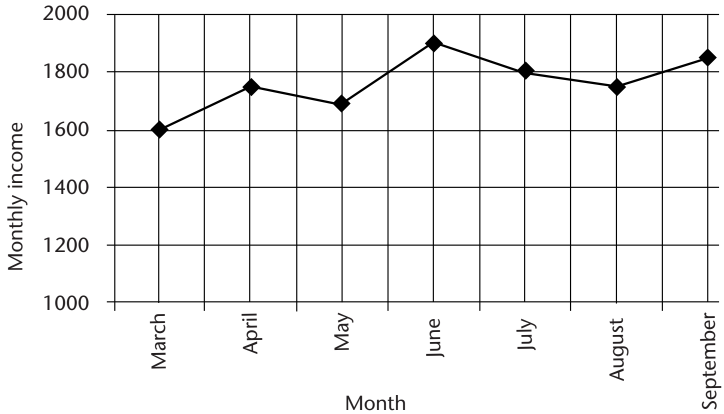
4.6 Learners' own histograms, based on their frequency tables in Question 4.5.

5



CHAPTER 15

6.1



6.2 Her monthly income increased overall from March to September.

7.1 The total number of injuries = 90

Athletics: $\frac{6}{90} \times 360° = 24°$

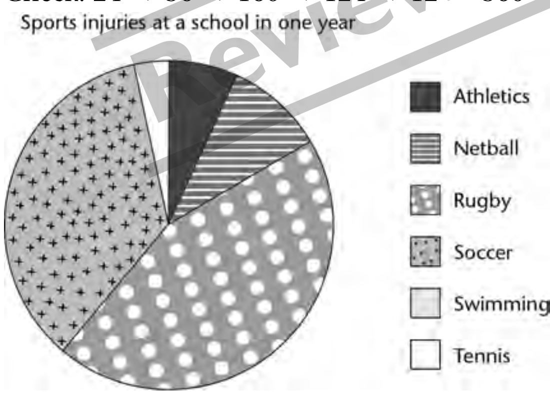
Netball: $\frac{9}{90} \times 360° = 36°$

Rugby: $\frac{40}{90} \times 360° = 160°$

Soccer: $\frac{32}{90} \times 360° = 124°$

Tennis: $\frac{3}{90} \times 360° = 12°$

Check: $24° + 36° + 160° + 124° + 12° = 360°$



7.2 Rugby; Soccer; Netball; Athletics; Tennis; Swimming

7.3 There is no sector for swimming-related injuries.

7.4 Rugby or soccer: $160° + 124° = 284°$; $\frac{284°}{360°} \times \frac{100}{1}\% = 80\%$

7.5 No. The percentages in Question 7.4 were based on sports injuries in one year in one specific school in South Africa and cannot be generalised to apply to the whole of South Africa.

Remedial

- When learners are asked to draw a graph, they should be very clear in their own minds as to what each kind of graph entails. They might, for instance confuse the idea of a bar graph and a histogram. Remind them of the basic differences between the two.
- When learners draw a pie chart, they need to calculate the sizes of the angle of the sectors correctly. They should also check their calculations by making sure that the sum of the angles is 360°. This check should be done before they even start to draw the pie chart.
- If you feel that your class has need of further practice in drawing graphs, look for suitable additional exercises in a Grade 8 textbook. The contexts and the data can be extremely simple; it is the skill in drawing graphs that your learners need to practise at this stage.

Extension

- Learners continue to work on their mini-project. By now they should have a clear idea of the topic that they want to study, as well as of the population that they have in mind, as well as of whether or not they will need to identify a sample.
- If your learners are going to make use of a sample, they need to be clear on how they are going to go about choosing a random sample that fairly represents the population as a whole.
- They should also be refining the questions that they want to ask their population or sample.
- To make this study meaningful, they need to consider the information that they will receive. How best will they collate this information? How best will they group the responses so as to be able to form a coherent overall picture of the responses? Which responses will lend themselves to graphical representations of the data?
- Ask your learners to formulate their thinking clearly, because by the end of the next unit they will be summarising all their thinking on a poster that they will hand in to you. Remind them that you will be offering small prizes to those learners who have managed to put together a clear, coherent and meaningful analysis of the topic they have chosen to study.

Unit 4 Interpret, analyse and report data

Learner's Book page 403

Unit focus

This unit focusses on the following:
- critically reading, interpreting, comparing and analysing data; and
- summarising data in short paragraphs.

Background information on interpreting, analysing and reporting data

In this final unit of this chapter, the focus is on interpreting, analysing and reporting data.

Exercise 1

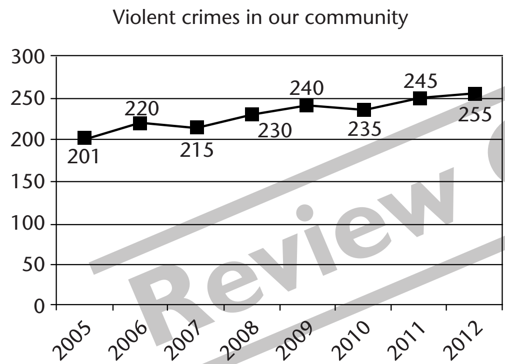
Learner's Book page 404

Guidelines on how to implement this activity

Read through the introductory text with your class then, work through the Worked example with them. Make sure that your learners can all see how Themba manipulated his graph to create a misleading impression. Contrast Themba's graph with the correct version at the end of the example. Do your learners see the differences between them, as well as the similarities? Ask your class to do Exercise 1. Allow them to work in pairs or in small groups at your own discretion. Learners will benefit by discussing these questions with their peers.

Suggested answers

1.1 A broken line graph.
1.2 They started the scale on the vertical axis at 200 instead of at 0.
1.3 They tried to make the impression that violent crime increased dramatically between 2005 and 2011.
1.4



1.5 If the trend continues it would be somewhere between 250 and 265.
2.1 The range was most affected by the percentage of the sixth learner.
2.2 The mode was not affected by the percentage of the sixth learner.
2.3 Mean for five learners $= 63 = \frac{\text{total of percentages}}{5}$
Total $= 63 \times 5 = 315$
Mean for six learners $= 56 = \frac{\text{total of percentages}}{6}$
Total $= 56 \times 6 = 336$
Sixth learner's result $= 336 - 315 = 21\%$
2.4 Yes, because it increases the range significantly.
3.1 Cara's data does not support this as almost 50% of learners dropped litter on the ground.
3.2 No, you cannot be sure.
3.3 If they were the same learners, they may have answered James' questions with what they thought were the 'correct' answers, but their actions show their real attitudes.

3.4 James collected data by asking learners directly about their attitudes towards littering, whereas Cara collected data by observing the actions of learners. Cara's data is more realistic as actions reveal people's attitudes, so she was able to see the behaviour of learners first hand rather than asking them about their attitudes. Learners could have not given James honest answers as they wanted to appear to have the right attitudes. Another possible method of collecting data for this study would be to give learners an anonymous questionnaire, where they would not have to identify themselves.

4.1 Brand X: The graph is based on the first row of the table, the numbers for Brand X.
Brand Y: The graph is based on the third column of the table, the numbers for the children.
Brand Z: The graph was based on the first three columns of the table, the numbers for the three groups of people across all three brands.

4.2 The graph for Brand Z is the truest reflection of the real situation, because it accurately compares all the numbers for the three groups of people across all three brands. It gives the whole picture, instead of only a part of the picture.

4.3 Brand X was the weakest of the three brands in every category, so the only claim that they could make to promote themselves was to compare their own numbers and to emphasise the fact that their brand was more popular amongst women than amongst men and children.
Brand Y was weaker than Brand X, except in the area of the children. Slightly more children preferred Brand Y than the other brands, so Brand Y chose to emphasise this fact.

Remedial

The questions in this exercise are not difficult, but they do require that your learners read the questions carefully and interpret them correctly. For lazy readers, it can be useful to work in pairs or in small groups. They can guide one another to see what is being given and what is being asked.
Reserved learners may feel uncomfortable about voicing their opinions, so you might want to have a general class discussion about the questions in this exercise. Try to keep the atmosphere tolerant and supportive and encourage participation by all.

Extension

Your learners now have the opportunity of impressing you with the planning that they have put into their mini-project. Depending on the amount of available time and also on the format that your learners have prepared, you could ask a few of them to briefly share their thinking with the class as a whole.

Consolidation

Learner's Book page 408

Before doing this Consolidation exercise, encourage learners to review the work covered in this chapter. Advise learners to use the summary and to revise their work. This exercise can be used as an informal assessment task for you to track how learners are coping with the chapter and the concepts covered. The mark allocation provides guidelines on how to assess learners.

Suggested answers

1.1

	1	2	3	4	5	6	Mean	Median	Mode
Laura	32	25	21	25	47	48	33	28,5	25
Bongela	29	48	19	35	30	48	34,8	32,5	No mode
Wesley	25	34	13	32	25	36	27,5	28,5	25

(3)

1.2 The third test as all 3 learners achieved their lowest mark on this test. (2)

1.3 Laura should use the mean, Bongela the mean and Wesley the median. (3)

1.4 Laura should score 47 out of 50. Bongela should score 35 out of 50. Even if Wesley gets 100% for the next test he will still not have an average of 70%. (3)

2.1 The bar graph shows his savings per month – the height of each bar shows his savings each month, while the broken line graph the total monthly savings, rather than the savings each month. (2)

2.2.1 The third month. (1)

2.2.2 There is no bar in the third month. (1)

2.2.3 The point for the third month has not moved vertically from the point for the second month (they are in a horizontal line). (1)

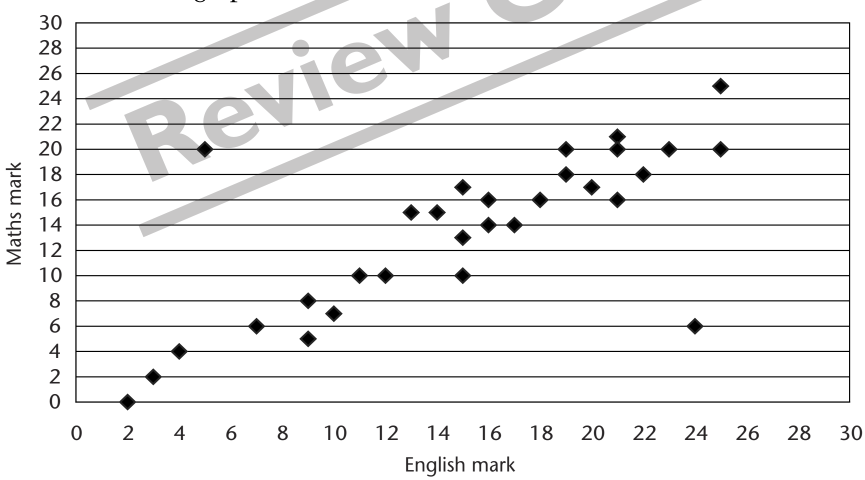
2.3.1 He saved R200 in the fifth month. (1)

2.3.2 The bar graph. (1)

2.4.1 Approximately R550. (1)

2.4.2 The broken-line graph. (1)

3.1



(9)

3.2 The scatter plot shows that there may be a relationship between English and mathematics marks – a learner who does well in English is likely to also do well in mathematics. A learner who does badly in mathematics is likely to also have done badly in English. The scatter plot shows two outliers – the learners with the marks (24; 6) did very well in English and badly in mathematics, while the learner with the marks (5; 20) did badly in English and quite well in mathematics. (8)

4.1 44 learners were interviewed. (1)

4.2

	0 0 0 0 0 0 0 0 0
0	1 1 1 2 3 3 4 4 5 6 6 8 8 8
1	1 4 7 8 9
2	1 3 4 5 6 7 7 9
3	5 7 8 9
4	2 3 6
5	1

Key: 1 4 means 14 (4)

4.3 9 learners (1)

4.4 4 learners (1)

4.5 23 learners (1)

4.6 11 learners (1)

4.7 Stem-and-leaf diagrams show the data in order and they are clear and easy to read. (1)

4.8 More than half of the learners interviewed attend less than 10 religious services a year, while only 4 attend more than 40. The modal attendance is 0 services. It thus seems that not many learners in this sample attend religious services regularly. (3)

[50]

Chapter 16 Probability

Overview of concepts

Unit 1 Two-way tables

Unit 3 Relative frequency ← **Probability** → **Unit 2** Tree diagrams

Content		Time allocations	LB page
Unit 1	Two-way tables	2 hours	411
Unit 2	Tree diagrams	$1\frac{1}{2}$ hours	415
Unit 3	Relative frequency	1 hour	419

Background information on probability

Your learners have already worked with the concept of probability in previous grades. In Grade 8 the focus started moving away from probability experiments, towards the probability of hypothetical events. In Grade 9 the focus remains on the probability of hypothetical events, but now your learners start to consider the probability of compound events, as opposed to single events. In particular, they will learn about two-way tables and tree diagrams.

In this chapter, your learners will:

- represent outcomes of compound events in two-way tables,
- use two-way tables to predict probabilities of compound events,
- represent outcomes of compound events in tree diagrams,
- use tree diagrams to predict probabilities of compound events,
- predict the relative frequency of outcomes of events in simple experiments, and
- compare relative frequency with probability and explain possible differences between them.

Generic teaching guidelines for teaching probability

The topic of probability is always likely to receive a positive reaction from your learners. Your weaker learners will particularly enjoy the change of pace from more challenging mathematical concepts and calculations. The study of probability is a great leveller, allowing all learners an equal opportunity to thrive.

Try always to present the topic of probability as a "fun" topic. Visual aids will also be useful for those learners who struggle to conceptualise things in an abstract way.

Resources

Coins, spinners, dice or dominoes to use for probability experiments. Blank tables and charts for learners to record their experiments. Card and colour pens for vocabulary cards and posters. Each learner should have their own calculator.

Unit 1 Two-way tables

Learner's Book page 411

Unit focus

This unit focusses on the following:
- representing outcomes of compound events in two-way tables
- using two-way tables to predict probabilities of compound events.

Background information on two-way tables

As the name suggests, a two-way table is a table that allows one to represent the outcomes of two events. The only difficulty with this concept is that your learners must be able to read a two-way table and to understand the information that it contains. Some learners will be intimidated by what they perceive as the complex appearance of a two-way table. With a little patient guidance from you, your learners will soon be pleasantly surprised to find that they can easily understand and relate to the information in any given two-way table. The idea behind a two-way table is that all the possible outcomes of two events are listed in the table. The first event is usually listed in the left-most column of the table, while the second event is usually listed in the top row of the table. Each cell within the body of the table represents a joint outcome of the two events.
For a two-way table to be a useful tool for predicting probabilities, it is important that the events represented therein each have ***equally likely*** outcomes. Note that the two events do not both have to have the same number of outcomes.

Revision exercise

Learner's Book page 411

Guidelines on how to implement this activity

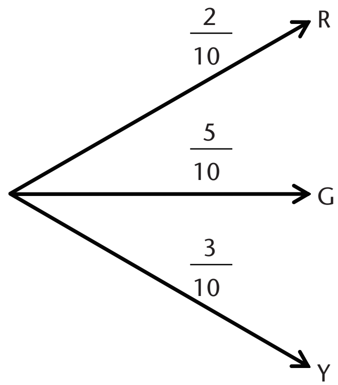
This unit starts off with a revision exercise in which your learners will practise the skills that they have already learnt, as they determine the probabilities of single events with equally likely outcomes. Use this exercise to assess learners' understanding of probability and likelihood.

Suggested answers

1.1 $\frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3} = 33{,}33\%$

1.2 $\frac{1}{2} = 50\%$

1.3



1.3.1 $\frac{3}{10} = 30\%$

1.3.2 $\frac{5}{10} = 50\%$

1.3.3 0%

1.4.1 $\frac{24}{49} = 48{,}98\%$ There are 24 even numbers.
i.e. {2; 4; 6; 8; …; 48}

1.4.2 $\frac{25}{49} = 51{,}02\%$ There are 25 odd numbers.
i.e. {1; 2; 3; 5; 7; 9; …; 49}

1.4.3 $\frac{15}{49} = 30{,}61\%$ There are 15 prime numbers.
i.e. {2; 3; 5; 7; 11; 13; 17; 19; 23; 29; 31; 37; 41; 43; 47}

1.4.4 $\frac{9}{49} = 18{,}37\%$

1.4.5 $\frac{19}{49} = 38{,}78\%$

1.4.6 $\frac{7}{49} = 14{,}29\%$

1.4.7 $\frac{0}{49} = 0\%$

1.4.8 $\frac{1}{49} = 2{,}04\%$

1.4.9 $\frac{9}{49} = 18{,}37\%$

2.1 $\frac{12}{48} = \frac{1}{4}$

2.2 $\frac{6}{48} = \frac{1}{8}$

2.3 $\frac{15}{48} + \frac{9}{48}$
$= \frac{24}{48}$
$= \frac{1}{2}$

2.4 $\frac{0}{48}$

2.5 $\frac{48 - 6}{48}$
$= \frac{42}{48}$
$= \frac{7}{8}$

Remedial

If learners struggled with this exercise, spend some time revising the concepts and allow learners to try the exercise again.

Exercise 1

Learner's Book page 413

Guidelines on how to implement this activity

Once learners have completed the revision exercise, read through the text about two-way tables with your class, then work through the first Worked example with them. This is the first time that they will have seen a two-way table, so take the time to explain very clearly how this table works. Make very sure that all your learners fully understand all the information in the table. Once your learners have mastered how to interpret the information in a two-way table, they will find the questions easy to answer.

Move on to the second worked example. The difference between this example and the previous one is that in the first example, the events had different outcomes. In the second example, both events have the same outcomes (the birth of a girl or a boy). Make sure that your learners understand the difference between Questions 3 and 4. Question 3 is about the probability that the couple will have a boy first, then a girl. Question 4 is about the couple having a boy and a girl, in any order. The fact that the order of the outcomes has a bearing on the resulting probabilities is a new concept to your learners. Encourage learners to do Question 4 on their own.

Suggested answers

1.1

		Second Spinner				
		1	2	3	4	5
First Spinner	A	A1	A2	A3	A4	A5
	B	B1	B2	B3	B4	B5
	C	C1	C2	C3	C4	C5
	D	D1	D2	D3	D4	D5

1.2 $\frac{1}{20} = 5\%$ **1.3** $\frac{3}{20} = 15\%$ **1.4** $\frac{4}{20} = 20\%$ **1.5** $\frac{19}{20} = 95\%$

1.6 The two probabilities add up to 100%. Together the two events represent all the possible outcomes. The events are complementary.
P(getting an A and a 4) = 1 − P(not getting an A and a 4)

2.1

		Second dice					
		1	2	3	4	5	6
First dice	1	1 + 1 = 2	1 + 2 = 3	1 + 3 = 4	1 + 4 = 5	1 + 5 = 6	1 + 6 = 7
	2	2 + 1 = 3	2 + 2 = 4	2 + 3 = 5	2 + 4 = 6	2 + 5 = 7	2 + 6 = 8
	3	3 + 1 = 4	3 + 2 = 5	3 + 3 = 6	3 + 4 = 7	3 + 5 = 8	3 + 6 = 9
	4	4 + 1 = 5	4 + 2 = 6	4 + 3 = 7	4 + 4 = 8	4 + 5 = 9	4 + 6 = 10
	5	5 + 1 = 6	5 + 2 = 7	5 + 3 = 8	5 + 4 = 9	5 + 5 = 10	5 + 6 = 11
	6	6 + 1 = 7	6 + 2 = 8	6 + 3 = 9	6 + 4 = 10	6 + 5 = 11	6 + 6 = 12

2.2 $\frac{6}{36} = \frac{1}{6} = 16{,}6\%$ **2.3** $\frac{15}{36} = \frac{5}{12} = 41{,}6\%$

2.4 $\frac{15}{36} = \frac{5}{12} = 41{,}6\%$

2.5 They all add up to 100%. They represent all possible outcomes.

2.6 $\frac{18}{36} = \frac{1}{2} = 50\%$

3.1

		Second dice					
		1	2	3	4	5	6
First dice	1	$1 \times 1 = 1$	$1 \times 2 = 2$	$1 \times 3 = 3$	$1 \times 4 = 4$	$1 \times 5 = 5$	$1 \times 6 = 6$
	2	$2 \times 1 = 2$	$2 \times 2 = 4$	$2 \times 3 = 6$	$2 \times 4 = 8$	$2 \times 5 = 10$	$2 \times 6 = 12$
	3	$3 \times 1 = 3$	$3 \times 2 = 6$	$3 \times 3 = 9$	$3 \times 4 = 12$	$3 \times 5 = 15$	$3 \times 6 = 18$
	4	$4 \times 1 = 4$	$4 \times 2 = 8$	$4 \times 3 = 12$	$4 \times 4 = 16$	$4 \times 5 = 20$	$4 \times 6 = 24$
	5	$5 \times 4 = 5$	$5 \times 2 = 10$	$5 \times 3 = 15$	$5 \times 4 = 20$	$5 \times 5 = 25$	$5 \times 6 = 30$
	6	$6 \times 1 = 6$	$6 \times 2 = 12$	$6 \times 3 = 18$	$6 \times 4 = 24$	$6 \times 5 = 30$	$6 \times 6 = 36$

3.2 $\frac{8}{36} = \frac{2}{9} = 22{,}2\%$

3.3 $\frac{3}{36} = \frac{1}{12} = 8{,}3\%$

3.4 $\frac{6 + 6 + 8}{36} = \frac{20}{36} = \frac{5}{9} = 55{,}5\%$

3.5 $\frac{27}{36} = 75\%$

No.
It cannot be 50% of the products, because 2,4 and 6 are already even numbers and multiplied by any natural number from 1 to 6 the answer will always be an even number, which is already 50% of the products.
Odd numbers times even numbers will also give further even products.

Remedial

The questions on probability in this chapter are not difficult, but they do require careful reading on correct interpretation. For example, in the case of Question 1.2, your learners are asked to find the probability of getting an A and a 4. Although this is hardly difficult to understand, some of your learners my need it to be pointed out to them that this includes only the outcome A4. Watch out for those learners who count all the outcomes with an A and / or a 4.
Your learners may also need reminding that we always write a probability as a number between 0 and 1. A probability may be written as a common fraction, a decimal fraction or a percentage. They may of course also write a probability of $\frac{1}{4}$ as “1 in 4” if they wish, although try to encourage them to write their answers in one of the forms that we have just mentioned.
If your learners struggle to convert common fractions to percentages, work through some examples on the board. Although the main focus of this chapter is on probability, learners must still get the basic mathematics correct.

Extension

Ask your stronger learners to come up with ideas for a complete multi-part question along the lines of Question 3, but one that is not based on the sum or the product of the numbers on the two dice. Ask them to think outside the box and to try to come up with fresh and new ideas. If your learners come up with something really useful, make a note of their ideas. You could keep these ideas for future questions for tests or exams.

If you like, ask your learners to give mark allocations as well as to provide a full marking memorandum for their question. This will help them to appreciate the effort that actually goes into setting up questions and memoranda for tests and exams. Also, the process of setting up the marking memorandum often causes one to re-think the original question as well as the suitability of the mark allocations. (If this seems like a tall order for Grade 9 learners, remember that this is an *extension opportunity* for the stronger learners!)

Unit 2 Tree diagrams

Learner's Book page 415

Unit focus

This unit focusses on the following:

- representing outcomes of compound events in tree diagrams, and
- using tree diagrams to predict probabilities of compound events.

Background information on tree diagrams

Whereas a two-way table can only represent two events, a tree diagram can represent any number of events. However, in Grade 9, we will use a tree diagram for no more than three compound events.

Exercise 1

Learner's Book page 418

Guidelines on how to implement this activity

Read through the text in the Learner's Book with your class, then work through the first Worked example with them. Take the time to explain very clearly how this example of a tree diagram works. A tree diagram is more visual than a two-way table and is usually more easily understood by the learners. They need to understand, however, that they must always read a tree diagram from left to right. On the left is the "root" of the tree. The first set of branches depicts all the possible outcomes of the first event; one branch for each different outcome. From the end of each branch comes another set of branches that represents the second event. So, each path from the root to the end of the tree represents a unique combined outcome of the compound events. The outcome that is represented by each path is shown to the right of the tree diagram, under the heading "outcome". (A third event could be added to the tree diagram simply by adding another set of branches to each existing outcome.)

Refer your learners back to the first Worked example in the Learner's Book, which depicted the same context as this example. Ask your learners to compare the two-way table in that example with the tree diagram in this example. Have a short class discussion about this. Which representation do your learners prefer?

Move on to the second Worked example. This example involves three events, as opposed to only two. Work through the solutions to each of the questions with your class. Once you have done so, ask your class the following questions:

- Write down all the outcomes for one boy and two girls in any order. (BGG, GBG, GGB)

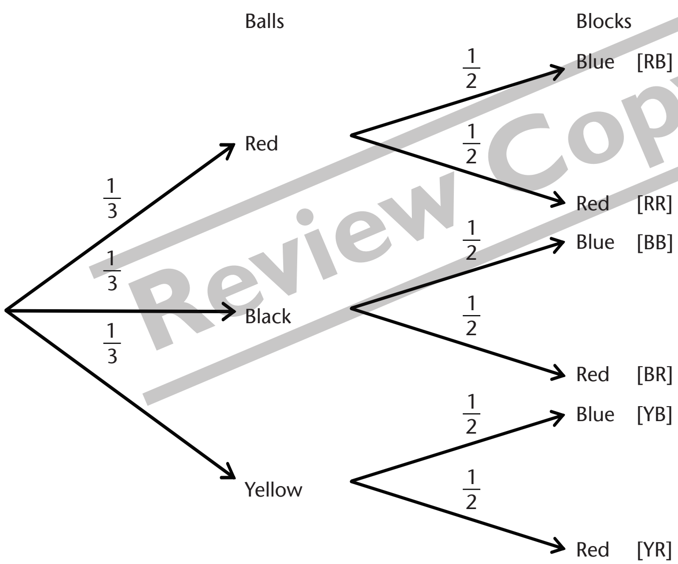
- Do you see the difference between the following outcomes:
  - a boy first, then two girls? (BGG)
  - one boy and two girls in any order? (BGG, GBG, GGB)

Point out to your class that:
- The probability of getting any specific combination of sexes in the desired order is $\frac{1}{8}$.
- The probability of having at least one girl = $\frac{7}{8}$, or 87,5%. This is the same as (1 – the probability of having three boys) = $1 - \frac{1}{8} = \frac{7}{8}$. The events (having three boys) and (having at least one girl) are complementary events.

Refer your learners back to the Worked example on page 413 of the Learner's Book which is also about the sexes of unborn children. Point out to your learners that a two-way table limits us to two compound events and therefore to two children only, whereas a tree diagram is more flexible and allows us to predict the sexes of three or more children. Learners can work in pairs to complete this exercise.

Suggested answers

1.1



1.2.1 The probability of getting a black ball and a blue block

$= \frac{1}{3} \times \frac{1}{2}$

$= \frac{1}{6}$ or 16,6%

1.2.2 The probability of getting a ball and a block of the same colour

$= \frac{1}{3} \times \frac{1}{2}$

$= \frac{1}{6}$ or 16,6%

1.2.3 The probability of getting a ball and a block of different colours

$1 - \frac{1}{6}$ or $5 \times \frac{1}{3} \times \frac{1}{2}$

$= \frac{5}{6}$

$= 83{,}3\%$

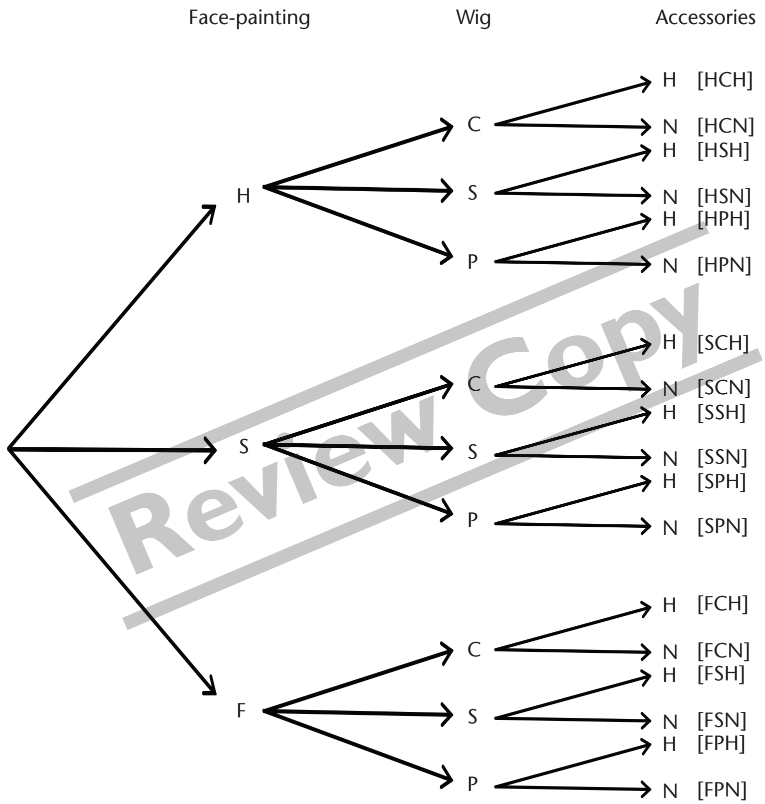
2 Let H = happy face — C = Red curly wig

S = Sad face — S = Lime green straight wig

F = Funny face — P = Neon blue plaits

H = Hat with flower — N = Red plastic nose

2.1 $3 \times 3 \times 2 = 18$



2.2.1 $\frac{6}{18} = \frac{1}{3}$ **2.2.2** $\frac{3}{18} = \frac{1}{6}$ **2.2.3** $\frac{1}{18}$

3.1 $\frac{1}{2} = 50\%$ **3.2** $\frac{1}{4} = 25\%$ **3.3** $\frac{1}{8} = 12{,}5\%$ **3.4** $\frac{1}{2^n}$

Remedial

Learners sometimes struggle with the idea of where to start when drawing a tree diagram. Draw a simple example on the board for them to refer to, or refer them to one of the examples in the Learner's Book.

A tree diagram can easily become clumsy to draw and if learners start running out of space on the page, their efforts can become cramped and untidy. Advise them always to draw a rough sketch of a tree diagram before drawing the final neat version in their workbooks.
Question 4 requires that your learners find specific probabilities and then generalise their results in terms of *n* children. If they struggle with this generalisation, be prepared to remediate this problem by means of a group discussion or even a class discussion, depending on the number of learners who are struggling with this question.

Extension

Give your learners the following problem to solve: A family has *m* children. Each child is allowed to choose a pet out of *n* possible kinds of pet.

- How many different possible outcomes are there?
- What is the probability that all the children will choose the same kind of pet?
- What is the probability that every child will choose a different kind of pet?

Give your learners the following hint: Start with one child who has *n* choices. Then move on to two children with *n* choices each. Generalise your thinking for *m* children with *n* choices each.
Note the following:

- All answers will be in terms of *m* and *n*.
- If there are 3 kinds of pets available (cats, dogs and rabbits), then *n* will be 3. If there are cats, dogs, rabbits and budgies available, then *n* will be 4. So *n* has nothing to do with the number of cats, dogs and so on that are available, but only with the number of ***kinds*** of pets that are available. Your learners may assume that there are enough of each kind of pet to go around, so that each child may choose the same kind of pet (a cat, for example) if they like.

Challenge

Learner's Book page 418

Guidelines on how to implement this activity

Once you feel confident that your learners have understood both examples of tree diagrams, they can do the Challenge. This is intended for those learners who have coped well with this work.

Suggested answers

1 $\frac{1}{2^5}$ = 3,125% **2** $\frac{1}{2^5}$ = 3,125% **3** $\frac{1}{2^5}$ = 3,125%

4 There is only a 1 out of 32 chance (3,125%) to have 5 children in any specific fixed order (one specified order.)

Unit 3 Relative frequency

Learner's Book page 419

Unit focus

This unit focusses on the following:
- predicting the relative frequency of outcomes of events in simple experiments, and
- comparing relative frequency with probability and explain possible differences between them.

Background information on relative frequency

In this unit, your learners will predict the relative frequency of outcomes of events in simple experiments. They will also compare relative frequency with probability and explain possible differences between them. Your learners have already learnt about relative frequency in earlier grades, so this unit is basically a revision of what they already know.

Exercise 1

Learner's Book page 421

Guidelines on how to implement this activity

Read through the text about theoretical and experimental probability in the Learner's Book with your class. It is important that your learners all understand the difference between these two kinds of probability. When we talk about probability generally, we usually mean theoretical probability. We use theoretical probability to *predict* the probability of an outcome.

On the other hand, we use the term experimental probability to describe *what actually happened* in an experiment. Another name for experimental probability is relative frequency. One way to explain this to your class is as follows: A 60% chance of rain means that it may or may not rain, with a slightly better chance of rain than of no rain. This is theoretical probability. The theoretical probability of rain is 60%, or $\frac{6}{10}$, or 0,6. With hindsight, though, we know whether or not it actually rained. The relative frequency or experimental probability of rain would be 1 if it rained, and 0 if it did not.

Work through the Worked example in the Learner's Book with your class. This example is about what actually happened in an experiment that involved tossing a coin 25 times. We know that the theoretical probability of getting heads when tossing a fair coin is 50%. In an actual experiment, however, the number of heads in a given number of trials will vary between 0% and 100%. The important fact for your learners to understand is that as the number of trials in an experiment increases, the closer relative frequency usually becomes to theoretical probability. Learners can complete the exercise on their own.

Suggested answers

1.1

Outcomes of an experiment in which a dice is rolled 20 times					
Number	**Tally**	***f***	**Number**	**Tally**	***f***
1	𝍸 II	7	4	III	3
2		0	5	IIII	4
3	III	3	6	III	3

1.2 2 learners 10 rolls each
4 learners 5 rolls each
5 learners 4 roll each
10 learners 2 rolls each

1.3 4 turns each

1.4.1 $\frac{1}{6}$ = 16,6% **1.4.2** $\frac{3}{6}$ = 50% **1.4.3** $\frac{3}{6}$ = 50%

1.5.1 $\frac{3}{20}$ = 15% **1.5.2** $\frac{6}{20}$ = 30% **1.5.3** $\frac{7}{20}$ = 35%

1.6 Not too many trials were done (only 20). As the number of trials in an experiment increases, the closer the relative frequency will get to the theoretical probability.

2.1

Total obtained when rolling two dice (theoretical probability)											
Total	2	3	4	5	6	7	8	9	10	11	12
f	1	2	3	4	5	6	5	4	3	2	1

2.2

Total obtained when rolling two dice (relative frequency)											
Total	2	3	4	5	6	7	8	9	10	11	12
f	0	2	5	4	5	3	4	5	4	0	4

3

	Theoretical probability	Relative frequency
3.1	$\frac{1}{36}$	$\frac{0}{36}$
3.3	$\frac{2}{36} = \frac{1}{18}$	$\frac{0}{36}$
3.5	$\frac{4}{36} = \frac{1}{9}$	$\frac{4}{36} = \frac{1}{9}$
3.7	$\frac{35}{36}$	$\frac{32}{36} = \frac{8}{9}$
3.9	$\frac{18}{36} = \frac{1}{2}$	$\frac{14}{36} = \frac{7}{18}$
3.11	$\frac{15}{36} = \frac{5}{12}$	$\frac{9}{36} = \frac{1}{4}$

	Theoretical probability	Relative frequency
3.2	$\frac{5}{36}$	$\frac{5}{36}$
3.4	$\frac{0}{36}$	$\frac{0}{36}$
3.6	$\frac{10}{36} = \frac{5}{18}$	$\frac{13}{36}$
3.8	$\frac{18}{36} = \frac{1}{2}$	$\frac{22}{36} = \frac{11}{18}$
3.10	$\frac{12}{36} = \frac{1}{3}$	$\frac{16}{36} = \frac{4}{9}$
3.12	$\frac{11}{36}$	$\frac{8}{36} = \frac{2}{9}$

4 No, they were not always the same.
If the dice had been thrown substantially more than 36 times only, then the relative frequency (experimental probability) would have been closer to the theoretical probability.

Remedial

- Question 1.6 is an interpretative question that requires insight on the part of your learners.
- In Question 2.1, the total number of outcomes is 36. In Question 2.2, the total number of trials is also 36. This means that in Question 3, your learners must calculate all probabilities out of 36. We have kept the total number of outcomes and trials the same in both cases (the theoretical probabilities as well as the relative frequencies) so as not to cloud the issue and to confuse the learners unnecessarily. However, their calculations must be appropriate and correct. In other words, they must calculate all probabilities as fractions or as percentages out of 36.
- Question 4 is another interpretative question along the lines of Question 1.4.

Extension

- Ask your learners to devise their own experiment that involves something other than coins or dice. They should describe their experiment clearly and then write a composite question about it, along the lines of Question 1. They do not need to include sub-questions 1.2 and 1.3.
- If you like, ask your learners to give mark allocations as well as to provide a full marking memorandum for their question.

Consolidation

Learner's Book page 424

Before doing this Consolidation exercise, encourage learners to review the work covered in this chapter. Advise learners to use the summary and to revise their work. This exercise can be used as an informal assessment task for you to track how learners are coping with the chapter and the concepts covered. The mark allocation provides guidelines on how to assess learners.

Suggested answers

1 Let Wafer cone = W Chocolate = C Sugar cone = S Vanilla = V
Banana = B Toffee = T Strawberry = Y

1.1

Ice cream flavours	Cone type	
	W	**S**
C	CW	CS
V	VW	VS
B	BW	BS
T	TW	TS
Y	YW	YS

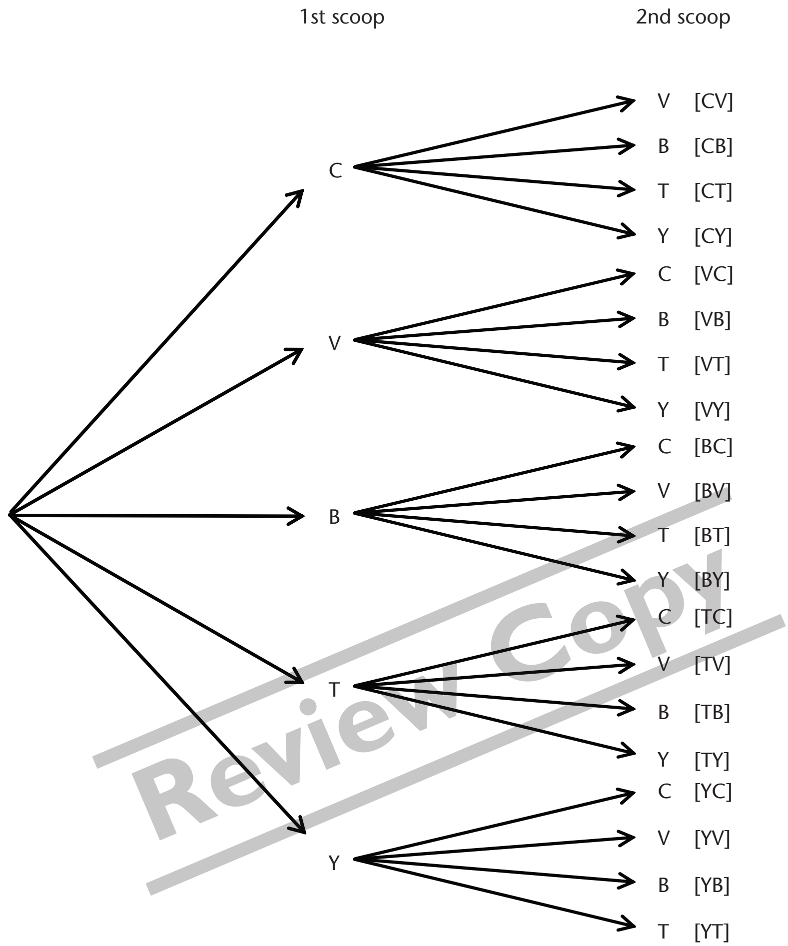
(3)

1.2 $5 \times 2 = 10$ options (1)

1.3.1 $\frac{5}{10} = 50\%$ **1.3.2** $\frac{2}{10} = 20\%$ **1.3.3** $\frac{1}{10} = 10\%$ (3)

2 Use same abbreviations as in Question 1.

2.1 $5 \times 5 - 5 = 20$ options (2)



2.2.1 $\frac{2}{25}$ (2)

2.2.2 $\frac{8}{25}$ (2)

3.1 43 double – scoop ice creams (1)

3.2.1 $\frac{5}{43}$ (2)

3.2.2 $\frac{27}{43}$ (2)

3.3 In Question 2.2.1 and 2.2.2 the probabilities are less for both because flavours are chosen at random and are theoretical probabilities. In Questions 3.2.1 and 3.2.2 real sales are recorded and choice and popularity play a role here. Chocolate, vanilla and strawberry flavours are clearly more popular flavours. (1)

4.1 $\frac{3}{6} = \frac{1}{2} = 50\%$ (1)

4.2 $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4} = 25\%$ (2)

4.3 $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8} = 12{,}5\%$ (2)

4.4 $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16} = 6{,}25\%$ (2)

4.5 $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{32} = 3{,}125\%$ (2)

5.1 The probability halves each time another person is added (50%; 25%; 12,5%; ...). (1)

5.2 Probability of n friends rolling an even number = $\left(\frac{1}{2}\right)^n$ (2)

6.1 $\frac{1}{6} = 16{,}67\%$ (2)

6.2 $\frac{n}{10} = 20\%$

$n = \frac{20}{100} \times 10$

$= 2$ times

6.3 $\frac{n}{20} = 15\%$

$n = \frac{15}{100} \times \frac{20}{1}$

$= 3$ times (4)

6.4 The trend is that the relative frequency gets closer to the theoretical probability of 16,67% as the number of trials increase. We can see how the line flattens out toward the 17% as the number of trials increase. (1)

[38]

CHAPTER 16

Chapter 17 Programme of Assessment

This chapter provides all the resources you need to ensure your learners meet the requirements for promotion.

- Two options for each of the required formal programme of assessment tasks.
- Exemplar exams for learners to use to practise for their exams. These exemplars are in the Learner's book with the memoranda supplied in this section of the Teacher's Guide.
- Control tests for you to use as part of the POA, along with the memoranda.
- A June and a December exam paper, with memoranda.

The following table lays out the programme for you. The shaded cells are only in the Teacher's Guide to ensure the tests and exams are unseen by the learners.

Term	Task		Learner's Book page	Teacher's Guide page
1	Assignment	Option 1 Numbers and fractions	427	313
		Option 2 Algebra	328	314
	Control Test 1			315
2	Investigation	Option 1 Quadrilaterals	429	318
		Option 2 Congruency	431	320
	Control Test 2			321
	Exemplar June exam	Exemplar paper for revision purposes	433	324
	June Exam			327
3	Assignment	Option 1 Algebra	438	336
		Option 2 Graphs	439	336
	Project	Option 1 Project	441	338
		Option 2 Volume and surface area	444	340
	Control Test 3			341
4	Assignment	Option 1 3D objects	446	345
		Option 2 Probability	447	346
	Investigation	Option 1 Transformation	448	346
		Option 2 Data	450	348
	Exemplar December exam	Exemplar paper for revision purposes	452	349
	December exam			355

We suggest that in order to prepare learners adequately for formal assessment you use the allocated Revision time prescribed in the CAPS for revising work.

This Series advises that you use the Consolidation exercises and Summaries at the end of each chapter to revise. The Consolidation exercises have mark allocations to enable informal assessment of how learners are managing the specific content area.

Assignment Option 1: Numbers and fractions

1.1 2; 3; 5; 7; 11; 13; 17; 19; 23; 29; 31; 37; 41; 43; 47; 53 (4)

1.2.1 $78 = 2 \times 3 \times 13$ **1.2.2** $385 = 5 \times 7 \times 11$

1.2.3 $230 = 2 \times 5 \times 23$ **1.2.4** $1\ 131 = 3 \times 13 \times 29$

1.2.5 $2\ 299 = 11 \times 11 \times 19$ **1.2.6** $2\ 261 = 7 \times 17 \times 19$

1.2.7 $10\ 469 = 19 \times 19 \times 29$ **1.2.8** $1\ 147 = 31 \times 37$

1.2.9 $7\ 955 = 5 \times 37 \times 43$ **1.2.10** $2\ 491 = 47 \times 53$ (15)

2.1 1,253; 1,254; 1,255; 1,256; 1,257; 1,258; 1,259; 1,260; 1,261 (2)

2.2 1,2533 (many possible answers) (2)

2.3 1,25333333333… (many possible answers) (2)

3.1 $\frac{3}{4} = \frac{6}{8} = \frac{12}{16}$

$\frac{5}{8} = \frac{10}{16}$

Fraction between $\frac{5}{8}$ and $\frac{3}{4} = \frac{11}{16}$

Other possible answers, such as: $\frac{16}{24}; \frac{17}{24}; \frac{21}{32}; \frac{22}{32}; \frac{23}{32}$ etc. (2)

3.2 $-\frac{7}{5} = -\frac{14}{10}$

$-\frac{6}{5} = -\frac{12}{10}$

Fraction between $-\frac{7}{5}$ and $-\frac{6}{5} = -\frac{13}{10}$ (2)

3.3.1 $1\frac{5}{6} = 1{,}8\dot{3}$ **3.3.2** $\frac{1}{7} = 0{,}\dot{1}4285\dot{7}$ (2)

4.1 $0{,}\dot{6} = \frac{2}{3}$ **4.2** $1{,}\dot{3}\dot{5} = 1\frac{35}{99}$

4.3 $1{,}\dot{1}2\dot{3} = 1\frac{123}{999} = 1\frac{41}{333}$ (3)

5.1 Cost in 4 years $= 12(1 + 0{,}05)^4$

$= \text{R}14{,}59$ (3)

5.2 $\frac{12}{14{,}59} \times 100 = 82{,}25\%$ (3)

TOTAL: 40 Marks

Assignment Option 2 :Algebra

1.1 5 terms (1)

1.2 Yes, because all the variables have exponents that are positive whole numbers and no variables appear in the denominators of terms. (2)

1.3 $-6x^4 + x^3 + 2x^2 + 4x - 3$ (1)

1.4 4^{th} degree (1)

1.5 $-6(-2)^4 + (-2)^3 + 2(-2)^2 + 4(-2) - 3 = -107$ (1)

1.6 $x = 0$ (1)

2.1 $\sqrt{25p^2 - 9p^2} = \sqrt{16p^2}$
$= 4p$ (1)

2.2 $\sqrt{25p^2} - \sqrt{9p^2} = 5p - 3p$
$= 2p$ (1)

3.1 $3x$ (1)

3.2 $9x$ (1)

4.1 $-(3a - 2b)^2 = -9a^2 + 12ab - 4b^2$ (2)

4.2 $\frac{28p^3q - 12p^2r + 4p}{4p} = 7p^2q - 3pr + 1$ (2)

5.1 $x + 5 = 25$
$x = 20$ (1)

5.2 $x - 5 = 25$
$x = 30$ (1)

5.3 $5 - x = 25$
$x = -20$ (1)

5.4 $5x = 25$
$x = 5$ (1)

5.5 $5^x = 25$
$x = 2$ (1)

5.6 $\frac{x}{5} = 25$
$x = 125$ (1)

5.7 $\frac{5}{x} = 25$
$x = \frac{1}{5}$ (1)

5.8 $x^2 = 25$
$x = \pm 5$ (2)

6.1 $x + 6$ (1)

6.2 $x + x + 2 + x + 4 + x + 6 = 3(x + 6) - 1$
$4x + 12 = 3x + 17$
$x = 5$ (3)

6.3 5; 7; 9; 11
$5 + 7 + 9 + 11 = 32$
$3 \times 11 - 1 = 32$ (2)

TOTAL: 30 Marks

Test 1: Numbers and integers; fractions, exponents, patterns and relationships, algebra 1

TOTAL: 50 marks **Time: 1 hour**

Question 1

1.1.1 Determine the HCF of 456 and 1 320 (2)
1.1.2 Determine the LCM of 47 and 61 (1)
1.2 State whether the following are true or false. If false write the correct statement.
1.2.1 Every rational number has a unique successor.
1.2.2 There are no other rational numbers between 1,0 and 1,1.
1.2.3 A terminating decimal fraction is a rational number.
1.2.4 A recurring decimal fraction is not a rational number.
1.2.5 The square root of a perfect square is a rational number. (5)
1.3. Prime factorise 13 500. (2)
1.4 A plane flying at an altitude of 10 000 m above sea level begins its descent at a rate of 150 m/s. How many seconds will it take the plane to reach an altitude of 1 000 m above sea level? (2)
[12]

Question 2

2.1 Draw a number line showing integers from −1 to + 2. Draw one unit for every 5 cm. Show the points representing the following numbers on the number line:
$-0{,}75 \quad -\frac{1}{10} \quad \frac{10}{10} \quad 1{,}5 \quad \frac{2}{3} \quad 1{,}75$ (2)
2.2 Provide the common fraction for each of the following in simplest form
2.2.1 0,875 **2.2.2** 0,005
2.2.3 2,15 **2.2.4** 0,750 (2)
2.3 Simplify:
2.3.1 $\frac{192a^3y^2}{16a^4y}$ (2)
2.3.2 $\frac{4x}{3a} + \frac{2a}{x}$ (2)
2.3.3 $\frac{4ax^2b}{2b^2} \div \frac{b^3c}{c^2}$ (2)
2.3.4 $\sqrt[3]{\frac{27x^3y^9}{64x^9y^{12}}}$ (2)
[12]

Question 3

3. 1 Without using a calculator work out the value of:
3.1.1 $4^{-2} \times (2^{-3}) \times 8^0$ (2)
3.1.2 $3^{10} \div (3^{-4})^{-3}$ (2)
3.2 Write the following in scientific notation:
3.2.1 865 998 000 000 **3.2.2** 0, 000004561 (2)
[6]

CHAPTER 17

Question 4

4.1 The pattern below shows how a prime number plus an even number can produce another prime number:

$41 + 2 = 43$
$43 + 4 = 47$
$47 + 6 = 53$
$53 + 8 = 61$
$61 + 10 = 71$

4.1.1 Write down a definition of a prime number. (1)

4.1.2 Describe the pattern above in words. (2)

4.1.3 Write down the next four equations in the sequence above. Do they also follow this rule? (2)

4.2 Use the following flow diagram to complete the table below:
Input → ÷ (−3) → square the answer → output

Input	Output
30	a
–57	b
c	16
d	400
e	81

(5)

[10]

Question 5

5.1 What is the smallest factor by which you must multiply $5pq^5$ to get a:

5.1.1 perfect square? (1)

5.1.2 perfect cube? (1)

5.2. Calculate the following products:

5.2.1 $(4m + n)(3m - n)$ (1)

5.2.2 $6(x + 7y)(y - 2x)$ (1)

5.3 Determine the value of the following if $x = -2$; $y = 3$ and $z = 0$.

5.3.1 $(3x + 2y)^2$ (2)

5.3.2 $\frac{1}{(x + y + z)}$ (1)

5.4 Solve for x in each of the following:

5.4.1 $\frac{x}{3} = 9$ (1)

5.4.2 $x^2 = 9$ (1)

5.4.3 $3x = 9$ (1)

[10]

TOTAL: 50 Marks

Answers to Term 1 test

Question 1

1.1.1 HCF of 456 and 1 320 is 24. (2)

1.1.2 LCM of 47 and 61 is 2 867. (2)

1.2.1 True. (1)

1.2.2 False – there are many other rational numbers between 1,0 and 1,1, for example 1,01. (1)

1.2.3 True. (1)

1.2.4 False – a recurring decimal fraction is a rational number. (1)

True. (1)

1.3 $13\ 500 = 2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5 = 2^2 \times 3^3 \times 5^3$ (2)

1.4 $S = \frac{D}{T}$

$T = \frac{D}{S}$

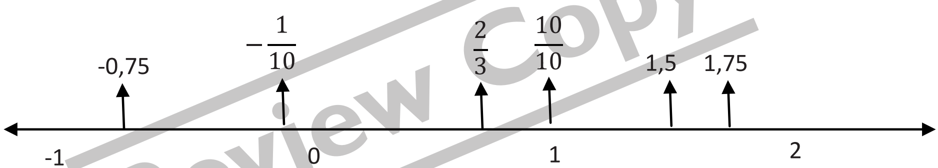
$= \frac{9\ 000}{150}$

$= 60$ s (2)

[12]

Question 2

2.1



(2)

2.2.1 $0{,}875 = \frac{875}{1\ 000} = \frac{7}{8}$ (1)

2.2.2 $0{,}005 = \frac{5}{1\ 000} = \frac{1}{200}$ (1)

2.2.3 $2{,}15 = 2\frac{15}{100} = 2\frac{3}{20}$ (1)

2.2.4 $0{,}750 = \frac{75}{100} = \frac{3}{4}$ (1)

2.3.1 $\frac{192a^3y^2}{16a^4y} = \frac{12y}{a}$ (2)

2.3.2 $\frac{4x}{3a} + \frac{2a}{x} = \frac{4x^2 + 6a^2}{3ax}$ (2)

2.3.3 $\frac{4ax^2b}{2b^2} \div \frac{b^3c}{c^2} = \frac{4ax^2b}{2b^2} \times \frac{c^2}{b^3c} = \frac{2ax^2c}{b^4}$ (2)

2.3.4 $\sqrt[3]{\frac{27x^3y^9}{64x^9y^{12}}} = \frac{3xy^3}{4x^3y^4} = \frac{3}{4x^2y}$ (2)

[14]

Question 3

3.1.1 $4^{-2} \times (2^{-3}) \times 8^0 = \frac{1}{16} \times \frac{1}{8} \times 1$

$= \frac{1}{128}$ (2)

3.1.2 $3^{10} \div (3^{-4})^{-3} = 3^{10} \div 3^{12} = 3^{-2} = \frac{1}{9}$ (2)

3.2.1 $865\ 998\ 000\ 000 = 8{,}65998 \times 10^{11}$ (1)

3.2.2 $0{,}000004561 = 4{,}561 \times 10^{6}$ (1)

[6]

Question 4

4.1.1 A prime number is a whole number with exactly two factors – itself and one. (1)

4.1.2 If you add consecutive even numbers to consecutive prime numbers, you get another prime number each time. (2)

4.1.3 $71 + 12 = 83$
$83 + 14 = 97$
$97 + 16 = 113$
$113 + 18 = 131$
Yes these equations do follow the rule. (2)

4.2

Input	Output
30	$a = 100$
–57	$b = 361$
$c = -12$ or 12	16
$d = -60$ or 60	400
$e = -27$ or 27	81

(5)
[10]

Question 5

5.1.1 $5pq$ (1)

5.1.2 $25p^2q$ (1)

5.2.1 $(4m + n)(3m - n) = 12m^2 - mn - n^2$ (1)

5.2.2 $6(x + 7y)(y - 2x) = 6\left(-13xy + 7y^2 - 2x^2\right)$ (1)
$= -78xy + 42y^2 - 12x^2$

5.3.1 $(3x + 2y)^2 = (3(-2) + 2(3))^2 = 0$ (2)

5.3.2 $\frac{1}{(x + y + z)} = \frac{1}{-2 + 3 + 0} = \frac{1}{1} = 1$ (1)

5.4.1 $\frac{x}{3} = 9$
$x = \pm 3$ (1)

5.4.2 $x^2 = 9$
$x = 27$ (1)

5.4.3 $3^x = 9$
$x = 2$ (1)

[10]
TOTAL: 50 Marks

Investigation Option 1: The properties of quadrilaterals

Quadrilateral	Parallelogram	Rectangle	Rhombus	Square	Kite
Name of Quadrilateral	ABCD	RSTU	NOPQ	EFGH	JKLM
Diagonals equal	No	Yes	No	Yes	No
Both diagonals bisect each other	Yes	Yes	Yes	Yes	No
Only one diagonal bisected	No	No	No	No	Yes
Diagonals perpendicular	No	No	Yes	Yes	Yes
Diagonals bisect all 4 angles	No	No	Yes	Yes	No
Only 2 angles bisected by a diagonal	No	No	No	No	Yes
Each diagonal divides the quadrilateral into two $\equiv\triangle$'s	Yes	Yes	Yes	Yes	No
Only one diagonal divides the quadrilateral into two $\equiv\triangle$'s	No	No	No	No	Yes
Number of lines of symmetry	None	2	2	4	1

Marking guidelines

1 Parallel lines correctly identified and marked, 1 mark per quadrilateral. (4)
Right angles marked at vertices, 1 mark per quadrilateral. (2)
Quadrilateral with no parallel sides correctly identified, 1 mark. (1)

2 Diagonals drawn, correct measurement and indication of which ones are bisected. 1 mark per quadrilateral. (5)

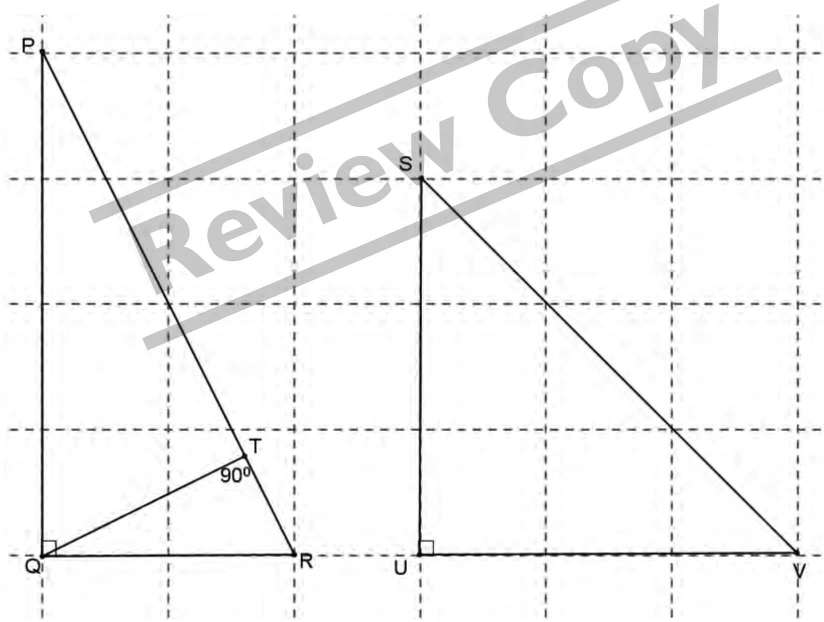
3 Perpendicular diagonals correctly measured and marked on the relevant quadrilaterals. 1 mark per quadrilateral. (3)

4 Table correctly completed, 3 marks per correct column. One or 2 errors, deduct 1 mark,
3 or 4 errors, deduct 2 marks, more than 4 errors in a column, 0 marks for that column. (35)

TOTAL: 50 Marks

Investigation Option 2: Use congruency to investigate properties of triangles

1.1 Construction – check that AB = BC (2)
1.2 Construction – check that $BD \perp AC$ (1)
1.3 Yes, $\triangle ADB \equiv \triangle CDB$ (1)
1.4 $\triangle ADB \equiv \triangle CDB$ because they share a common side, both have a right-angle in the same position and AB = BC (right-angle, hypotenuse, side). (1)
1.5 An altitude to the base of an isosceles triangle divides the triangle into two congruent right-angled triangles. (1)
1.6 Yes, because an equilateral triangle is a special case of an isosceles triangle. (2)
1.7 No, because all three sides are different lengths and all three angles are different sizes. (3)
2.1 Construction. (4)
2.2 Construction. (3)
2.3 Measurements to show that all four triangles are congruent and appropriate case for congruency given (SSS or SAS or SAA) (5)
3.1 Accurate construction: (4)



3.2 Measurements in each triangle to determine whether they are all congruent and appropriate case for congruency used (SSS, SAA, SAS or RHS). (3)
3.3 You can divide $\triangle SUV$ into 9 congruent triangles. (1)
3.4 $\triangle SUV$ can be divided into 9 congruent triangles as the ratio of the perpendicular sides in $\triangle SUV$ is 3 : 3 = 1 : 1 and the perpendicular sides are 3 units long. (1)

TOTAL: 32 Marks

Test 2 Constructions and 2D shapes and lines

TOTAL: 50 Marks **Time: 1 hour**

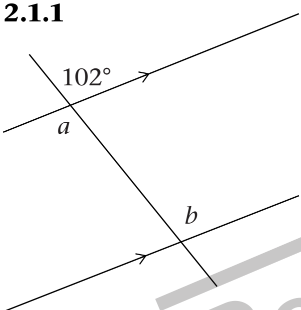
Question 1

1 Construct the following:
1.1 $\triangle PQR$ with QR = 8 cm; PQ = 7 cm and $\hat{Q} = 50°$ (3)
1.2 an angle of 30° without using a protractor (3)
1.3 $\triangle DEF$ with DE = 8,3 cm, DF = 6,4 cm with $\hat{F} = 90°$ (3)
1.4 rhombus ABCD with sides equal to 4 cm and $A\hat{B}C = 125°$ (3)
1.5 a regular pentagon with sides of 3,5 cm (4)
[16]

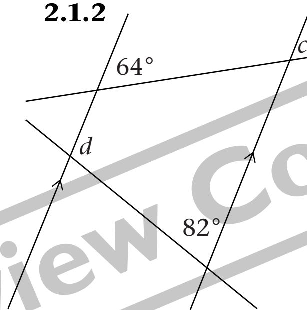
Question 2

2.1 Find the values of the missing variables. Be sure to give reasons for your answer. (6)

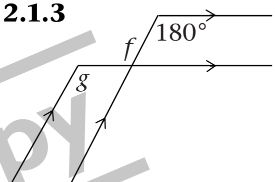
2.1.1



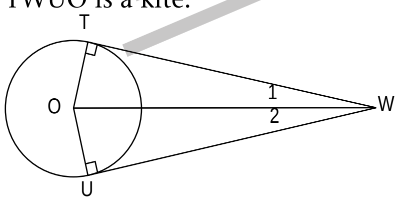
2.1.2



2.1.3

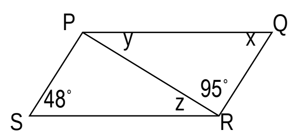


2.2 In the diagram below, O is the centre of the circle. $\hat{T} = \hat{U} = 90°$. Prove that TWUO is a kite.

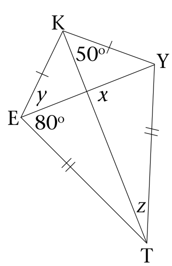


(3)

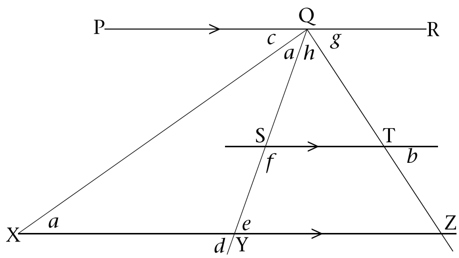
2.3 PQRS is a parallelogram. Calculate the values of x, y and z. (3)



2.4 KYTE is a kite. Calculate the values x, y and z. (4)

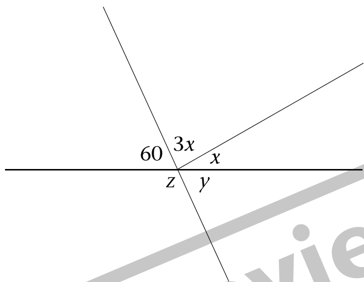


2.5 In the drawing on the left the three lines PQR, ST and XYZ are parallel.

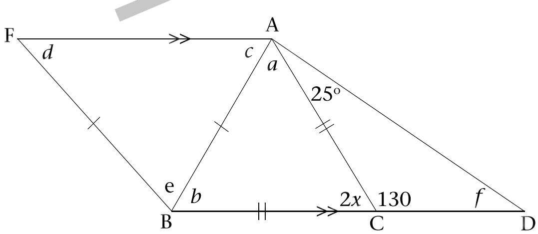


2.5.1 If $a = 32°$ and $b = 58°$, determine the values of c, d, e, f, g and h. Provide reasons for your answers. (6)
2.5.2 Write down a pair of adjacent complementary angles. (1)
2.5.3 What can be deduced from your answer in 2.5.2. (1)

2.6 Determine the values of x, y and z. (3)



2.7 Find, with reasons, the variables in the following diagram. (7)



[34]
TOTAL = 50 Marks

Term 2 test answers

1.1	Accurate construction of $\triangle PQR$ with QR = 8 cm; PQ = 7 cm and $\hat{Q} = 50°$.		(3)
1.2	Accurate construction of an angle of 30° without using a protractor.		(3)
1.3	Accurate construction of $\triangle DEF$ with DE = 8,3 cm, DF = 6,4 cm with $\hat{F} = 90°$.		(3)
1.4	Accurate construction of rhombus ABCD with sides equal to 4 cm and $A\hat{B}C = 125°$.		(3)
1.5	Accurate construction of a regular pentagon with sides of 3,5 cm.		(4)
			[16]
2.1.1	$a = 102°$	(vertical opposite angles)	
	$b = 102°$	(alternate angles, parallel lines)	(2)
2.1.2	$c = 64°$	(corresping angles, parallel lines)	
	$d = 98°$	(co-interior angles, parallel lines)	
	$e = 98°$	(alternate angles, parallel lines)	(2)
2.1.3	$f = 108°$	(alternate angles, parallel lines)	
	$g = 108°$	(alternate angles, parallel lines)	(2)
2.2	TO = TU	(radii of circle)	
	$\triangle TOW \equiv \triangle UOW$	(RHS)	
	Therefore TWUO is a kite as it has two pairs adjacent sides equal (TO = TU, and TW = UW), as well as one pair of opposite angles equal.		(3)
2.3	$x = 48°$	(opposite angles of parallelogram are equal)	
	$y = 180° - 48° - 95° = 37°$	(sum angles triangle)	
	$z = 37°$	(alternate angles, parallel lines)	(3)
2.4	$x = 90°$	(diagonals of a kite intersect at right angles)	
	$y = 40°$	(angle K is bisected; sum angles triangle)	
	$z = 10°$	(angle T is bisected; sum angles triangle)	(4)
2.5.1	$c = 32°$	(alternate angles, PQR ‖ XYZ)	
	$d = 64°$	(corresponding angles, PQR ‖ XYZ)	
	$e = 64°$	(vertical opposite angles)	
	$f = 116°$	(co-interior angles, ST ‖ XYZ)	
	$g = 58°$	(corresponding angles, PQR ‖ ST)	
	$h = 58°$	(corresponding angles, PQR ‖ ST or straight line angles)	(6)
2.5.2	a and h are adjacent, complementary angles.		(1)
2.5.3	Triangle XQZ is a right-angled triangle.		(1)
2.6	$4x + 60° = 180°$	(straight line angles)	
	$x = 30°$		
	$y = 60°$	(vertical opposite angles)	
	$z = 120°$	(vertical opposite angles or straight line angles)	(3)
2.7	$f = 25°$	(sum angles triangle ACD)	
	$x = 25°$	(straight line angles)	
	$a = b = 77,5°$	(sum angles isosceles triangle)	
	$c = 77,5°$	(alternate angles, AF parallel to DB)	
	$d = 77,5°$	(isosceles triangle FB = BA)	
	$e = 25°$	(sum angles triangle FBA)	(7)
			[34]
			TOTAL = 50 Marks

Exemplar June Examinmation Memo

1.1

Number	$\mathbb{N}_0$	$\mathbb{Z}$	$\mathbb{Q}$	$\mathbb{Q}'$	$\mathbb{R}$
–3	No	Yes	Yes	No	yes
$\sqrt{\frac{9}{16}}$	No	No	Yes	No	Yes
3,45	No	No	Yes	No	Yes
$-0,\dot{6}$	No	No	Yes	No	Yes
$\sqrt{6}$	No	No	No	Yes	Yes
2 345	Yes	Yes	Yes	No	Yes

(6)

1.2.1 $\sqrt{53\ 361} = \sqrt{3 \times 3 \times 7 \times 7 \times 11 \times 11} = 3 \times 7 \times 11 = 231$ (3)

1.2.2 $\sqrt[3]{3\ 375} = \sqrt{3 \times 3 \times 3 \times 5 \times 5 \times 5} = 3 \times 5 = 15$ (3)

1.3.1 Directly proportional because the ratio $\frac{x}{y}$ is constant. (2)

1.3.2 Neither (2)

2.1.1 $\frac{88a^2by^5}{4a^5y} = \frac{22by^4}{a^3}$ (1)

2.1.2 $\frac{5xyz^{-2}}{115xy^4z} = \frac{1}{23y^3z^3}$ (2)

[16]

2.2

Common fractions	Decimal fraction	Percentage
$\frac{5}{16}$	0,3125	31,25%
$\frac{1}{8}$	0,125	12,5%
$\frac{95}{100} = \frac{19}{20}$	0,95	95%

(3)

2.3.1 $1\frac{2}{5} - 4\frac{1}{2} = \frac{7}{5} - \frac{9}{2}$

$= \frac{14 - 45}{10} = -\frac{31}{10}$ (2)

2.3.2 $3\frac{2}{3} \times 1\frac{4}{7} = \frac{11}{3} \times \frac{11}{7}$

$= \frac{121}{21}$ (2)

[12]

2.3.3 $4\frac{1}{2} \div 3\frac{3}{4} = \frac{9}{2} \times \frac{4}{15}$

$= \frac{6}{5}$ (2)

3.1.1 $\frac{x^{-2} \times (xy)^4 \times y^{-3}}{x^{-1}y^4} = \frac{x^2y}{x^{-1}y^4}$

$= \frac{x^3}{y^3}$ (2)

3.1.2 $(a^3b^{-2}c)^5 \times (bc^2)^{-1} = a^{15}b^{-11}c^3$

$= \frac{a^{15}c^3}{b^{11}}$ (2)

3.1.3 $\frac{2^{2n-1} \times 2^{n+4}}{2^{3n+3}} = \frac{2^{3n+3}}{2^{3n+3}} = 1$ (2)

3.2.1 0,000006754 = $6{,}754 \times 10^{-6}$ (1)

3.2.2 765 000 000 000 = $7{,}65 \times 10^{11}$ (1)

[8]

4.1.1 $a = 99$ (1)

4.1.2 $b = 16$ (1)

4.1.3 $c = -66$ (1)

4.1.4 $d = 25$ (1)

4.2.1

Number of hexagons (h)	1	2	3	4	5
Number of triangles (t)	6	12	18	24	30
Perimeter of shape (p) (cm)	15	25	35	45	55

(3)

4.2.2 $t = 6h$ (1)

4.2.3 $p = 10h + 5$ (1)

4.2.4 *i)* $p = 10(250) + 5 = 2\ 505$ cm (1)

ii) $10h + 5 = 495$

$h = 49$

$t = 6h$

$t = 6(49)$

$= 294$ triangles (1)

[11]

5.1.1 $16x - 3xy + 5(4x - 2y) = 16x - 3xy + 20x - 10y$

$= 36x - 3xy - 10y$ (2)

5.1.2 $-3k(2k + 2) + k(k - 1) = -6k^2 - 6k + k^2 - k$

$= -5k^2 - 7k$ (2)

5.1.3 $3 + 4a(\mathrm{a} - 2b) - 2 + 5b(\mathrm{a} - 3b) = 3 + 4a^2 - 8ab - 2 + 5ab - 15b^2$

$= 4a^2 - 3ab - 15b^2 + 1$ (2)

5.1.4 $2x - 5[x - (3x + 4) - 3^2] = 2x - 5(-2x - 13)$

$= 2x + 10x + 65$

$= 12x + 65$ (3)

5.1.5 $(6x - 4)(3x + 2) = 18x^2 - 12x + 12x - 8$

$= 18x^2 - 8$ (2)

5.1.6 $(\mathrm{a} - \mathrm{b})^2 = a^2 - 2ab + b^2$ (1)

5.2.1 $x + 3(x + 1) = 2x + 8$

$x + 3x + 3 = 2x + 8$

$2x = 5$

$x = 2{,}5$ (2)

5.2.3 $\frac{x}{2} - \frac{1}{4} = \frac{7}{12}$

$6x - 3 = 7$

$6x = 10$

$x = \frac{5}{3}$ (2)

[16]

6.1 Accurate construction. (3)

6.2 Accurate construction. (3) $K\hat{L}J = 90°$ (1) (4)

6.3 Interior angle = 120° . (1) Accurate construction. (3) (4)

[11]

7.1.1 $\triangle ABC \equiv \triangle EFD$ (SAS) (2)

7.1.2 Not congruent. The equal angle and side are not in the corresponding positions. (2)

7.1.3 $\triangle ABD \equiv \triangle ACD$ (SSS) (2)

7.1.4 $\triangle ABC \equiv \triangle FED$ (SAA) (2)

7.1.5 $\triangle ABC \equiv \triangle EFD$ (RHS) (2)

7.1.6 Not congruent. The equal sides are not in the corresponding positions. (2)

7.2.1 $a = 146°$ (opposite angles of a kite) (1)

$b = 360° - 2(146°) - 47°$ (sum angles of a quadrilateral)

$b = 21°$ (1)

7.2.2 c = 180° − 116° (co-interior angles, parallel lines)

$c = 64°$ (1)

$d = 180° - 49°$ (co-interior angles, parallel lines)

$d = 131°$ (1)

7.2.3 $e = 180° - 90° - 46°$ (diagonals of kite intersect at right angles; one pair of opposite angles of kite equal; sum angles triangle)

$e = 44°$ (1)

$f = 180° - 90° - 14°$ (diagonals of kite intersect at right angles; one pair of opposite angles of kite equal; sum angles triangle)

$f = 76°$ (1)

[18]

8.1.1 $QR^2 = (3{,}4)^2 - 3^2$

$= 2{,}56$

$QR = 1{,}6$ m (1)

Perimeter $\triangle PQR = 3 + 1{,}6 + 3{,}4 = 8$ m (1)

8.1.2 Area $= \frac{1}{2}bh$

$= \frac{1}{2} \times 2{,}5 \times 3$

$= 3{,}75\ m^2$ (3)

8.2 Area of parallelogram $= 6{,}5 \times 2{,}5 = 16{,}25\ m^2$

Area of circle $= \pi \times 1{,}25^2 = 4{,}9\ m^2$

Shaded area $= 16{,}25m^2 - 4{,}9\ m^2 = 11{,}35\ m^2$ (3)

[8]

TOTAL: 100 Marks

June Examination

Total: 100 marks **Time: 2 hours**

Question 1

1.1 Use prime factors to determine the HCF of 105 and 168. (2)

1.2 Swazi, Thato and Ursula are partners in a business, STU trading company. Swazi invested R20 000, Thato invested R28 000 and Ursula invested R24 000 as capital when they started the business.

1.2.1 Express the capital investment of the three partners as a ratio, in simplest form.

1.2.2 If the three partners share the profits in the same ratio as that of their capital investment, what share will each get of a profit of R12 600? (4)

1.3 Vuyo deposits R10 000 into a bank savings account at 9% p.a. compounded annually for four years.

1.3.1 Calculate how much money is in the account at the end of the four years.

1.3.2 Vuyo invests another R10 000 in a different savings account for three years. This account pays interest at a rate of 5% compounded every *half* year. Just write the formula to calculate how much money is in this account after the three years. (3)

1.4 For each table below, determine whether it displays a *direct proportion*, an *indirect proportion* or none of the two. Give a reason for each answer:

1.4.1

x	0,5	4	7,5	10	12,5
y	1,5	12	22,5	30	37,5

1.4.2

x	–12	–8	0	4	12
y	–9	–6	0	3	9

1.4.3

x	–5	–3	2,5	4	10
y	2	$3\frac{1}{3}$	–4	–2,5	–1

(3)

[12]

Question 2

2.1 Complete the following table of equivalent forms:

Common fraction	Decimal	Percentage
$\frac{4}{5}$		
	$0,\dot{6}$	
		88%
$\frac{1}{8}$		
	0,165	

(5)

2.2 Calculate:

2.2.1 $3\frac{1}{3} \div 2\frac{1}{7}$ (2)

2.2.2 $4\frac{1}{6} - 2\frac{4}{5}$ (2)

2.2.3 $\sqrt[3]{\frac{1}{27}} + \sqrt{\frac{25}{81}}$ (2)

[11]

Question 3

3.1 Simplify so that the answers only contain positive exponents:

3.1.1 $3x^2(xy^2)(4x^{-3})$ **3.1.2** $-2(xy)(-4x^4y^{-3})(2y^9)$

3.1.3 $x^{-1}y^3z^2 \times x^3yz^{-2} \times xy^{-0}z^{-1}$ **3.1.4** $12a^7b^{-5}c \div 9a^5b^{-5}c^3$

3.1.5 $[2(a^{-2}\,b^3)]^3$ (5)

3.2 Solve the equation $2^{1-x} = 8$ (2)

3.3 Write 0,00090125 in scientific notation. (1)

[8]

Question 4

4.1 Write down the next three terms in each of the following patterns:

4.1.1 –4,5; –3; –1,5; 0; … (1)

4.1.2 $a + 8b$; $2a + 7b$; $3a + 6b$; … (1)

4.2 In each of the patterns below, you can work out any term by applying a rule to the previous term. Describe the rule for each pattern.

4.2.1 256; 64; 16; 4; ... (1)

4.2.2 2; 5; 11; 23; 47; ... (1)

4.3 The general term of a pattern is given by the following rule: Multiply the term number by 2, then subtract 5.

4.3.1 Write down this rule algebraically, in the form $T_n = \ldots$ (1)

4.3.2 Use your rule to calculate T_{20}. (1)

4.3.3 If $T_n = 199$, what is the value of n? (1)

4.4 Sam uses matches to build the outlines of squares. He starts out with a square of which each side contains one match. For the next square, he doubles the number of matches in each side of the square, and so on. The number of matches that he uses to build these outlines are shown in the table below:

Square	1	2	3	4	5
Number of matches in one side of the square	1	2	4	8	16
Number of matches in the outline of the square	4	8	16	32	64

4.4.1 Describe the number pattern in the second row of the table. (1)

4.4.2 Describe the number pattern in the third row of the table. (1)

4.4.3 If n is the square number and T_n is the number of matches in one side of the square, which formula below describes T_n in terms of n?

A $T_n = n$ B $T_n = n^2$ C $T_n = 2^n$

D $T_n = 2^{n-1}$ E $T_n = 2^{n+1}$ (1)

4.4.4 If n is the square number and Tn is the number of matches in the outline of the square, which formula below describes Tn in terms of n?

A $T_n = 4n$ B $T_n = 4n^2$ C $T_n = 4^n$

D $T_n = 2^{n-1}$ E $T_n = 2^{n+1}$ (1)

[11]

Question 5

5.1 Simplify the following expressions by first removing the brackets and then collecting the like terms:

5.1.1 $4(2m - 3) + 10$ (1)

5.1.2 $5x - 2(x - 1)$ (1)

5.1.3 $3(4a - 3) + 7a$ (1)

5.1.4 $-3a^2b^6(-2a^{-1}b^{-2})^3$ (3)

5.1.5 $\left(-\frac{x}{2}\right)^2 - 3(x^2 - 4)$ (3)

5.2 Change each of the following word problems into a mathematical equation. Use x for the unknown value. Do not solve the equation, only write down the equation.

5.2.1 A number is increased by 9. The answer is 24 (1)

5.2.2 Twice a number plus 30 gives 50. (1)

5.2.3 Half a number is subtracted from 10. The answer is 4. (1)

5.3 Solve each of the following equations:

5.3.1 $3x + 25 = x + 49$ (1)

5.3.2 $\frac{y}{2} - \frac{1}{4} = \frac{9}{4}$ (2)

5.3.3 $\frac{p^3}{2} = 32$ (2)

5.3.4 $-4x + 3 = x - 9$ (2)

5.3.5 $4(x + 3) = 2(x - 1)$ (2)

[21]

Question 6

6.1 Construct each of the following steps accurately, using a ruler and compasses. Each step of your construction must be visible, so do not erase any arcs drawn as part of the construction. A rough diagram of ABCD is given alongside to guide you.

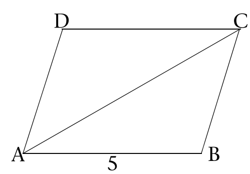
6.1.1 Construct a line AB equal to 5 cm in length. (1)

6.1.2 At A construct an angle of 60°. (2)

6.1.3 Draw the line AC 8 cm in length, where $C\hat{A}B = 60°$. (1)

6.1.4 At C construct another angle $A\hat{C}D$ equal to 60°. (1)

6.1.5 Draw the line CD equal to 5 cm in length. (1)



6.1.6 Join AD and BC. Measure AD and BC. (2)

6.1.7 What type of quadrilateral is ABCD? Why? (2)

[10]

Question 7

7.1 Complete each of the following statements:

7.1.1 A triangle with all angles less than 90° and two sides equal in length is called an ……………., …………– angled triangle. (1)

7.1.2 A triangle with one angle equal to 55° and one angle equal to 35° is a ……..–………… triangle. (1)

7.1.3 A parallelogram with adjacent sides equal is a ……………… (1)

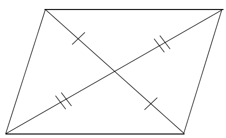
7.1.4 An isosceles trapezium has one pair of opposite sides …………… and one pair ……….. (1)

7.1.5 The diagonals of a kite intersect at ………. (1)

[5]

Question 8

8.1 Identify each of the following quadrilaterals, based on the information marked on the diagram. In each case, give full reasons for your answer.



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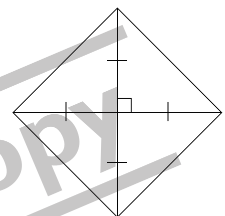


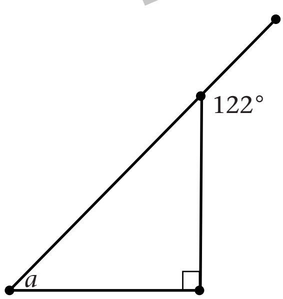
(2)

[4]

Question 9

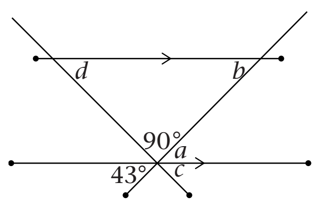
9.1 In each of the following, find the size of the unknown angles marked with small letters.
Give a reason for each answer.

9.1.1



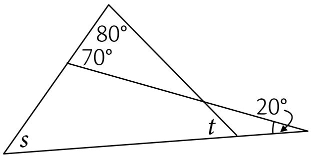
(1)

9.1.2



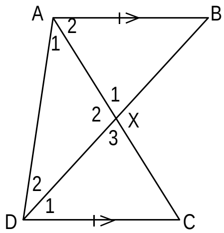
(4)

9.1.3



(2)

9.2 If AB = DC and AB || DC, prove that BX = DX



(4)
[11]

Question 10

10.1 Calculate, correct to two decimal places:

10.1.1 the area of parallelogram ABCD with sides AB = CD = 125 mm and the distance between AB and CD, h = 48 mm. Give the answer in cm^2. (2)

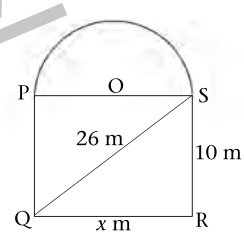
10.1.2 the area of a circle with diameter 38 m. (2)

10.2 PQRS is a rectangle with a semi-circle segment on top. Its diagonal QS = 26 m and its side RS = 10 m. Calculate, correct to one decimal place:

10.2.1 the length of QR (1)

10.2.2 the perimeter of the shape (exclude diameter PS). (2)

[7]



TOTAL: 100 Marks

June Examination Memo

Question 1

1.1 $105 = 3 \times 5 \times 7$
$168 = 2 \times 2 \times 2 \times 3 \times 7$
HCF = 21 (2)

1.2.1 S : T : U = 20 000 : 28 000 : 24 000 = 5 : 7 : 6 (2)

1.2.2 Total shares = 5 + 7 + 6 = 18

Swazi's share = $\frac{5}{18} \times 12\ 600$ = R3 500

Thato's share = $\frac{7}{18} \times 12\ 600$ = R4 900

Ursula's share = $\frac{6}{18} \times 12\ 600$ = R4 200 (2)

1.3.1 Amount = $10\ 000(1 + 0{,}09)^4$ = R14 115,82 (2)

1.3.2 Amount = $P\left(\frac{1 + i\%}{2}\right)^{2n} = 10\ 000\left(1 + \frac{0{,}05}{2}\right)^{2\times3}$ (1)

1.4.1 Direct proportion because the ratio $\frac{x}{y}$ is constant. (1)

1.4.2 Direct proportion because the ratio $\frac{x}{y}$ is constant. (1)

1.4.3 Indirect proportion because the product xy is constant. (1)

[12]

Question 2

2.1

Common fraction	Decimal	Percentage
$\frac{4}{5}$	0,8	80%
$\frac{2}{3}$	$0,\dot{6}$	66,67%
$\frac{88}{100} = \frac{22}{25}$	0,88	88%
$\frac{1}{8}$	0,125	12,5%
$\frac{165}{1\ 000} = \frac{33}{200}$	0,165	16,5%

(5)

2.2.1 $3\frac{1}{3} \div 2\frac{1}{7} = \frac{10}{3} \times \frac{7}{15}$
$= \frac{14}{9} = 1\frac{5}{9}$ (2)

2.2.2 $4\frac{1}{6} - 2\frac{4}{5} = \frac{25}{6} - \frac{14}{5}$
$= \frac{125}{30} - \frac{84}{30}$
$= \frac{41}{30} = 1\frac{11}{30}$ (2)

2.2.3 $\sqrt[3]{\frac{1}{27}} + \sqrt{\frac{25}{81}} = \frac{1}{3} + \frac{5}{9}$
$= \frac{3}{9} + \frac{5}{9} = \frac{8}{9}$ (2)

[11]

Question 3

3.1.1 $3x^2(xy^2)(4x^{-3}) = 12x^0y^2 = 12y^2$ (1)

3.1.2 $-2(xy)(-4x^4y^{-3})(2y^9) = 16x^5y^7$ (1)

3.1.3 $x^{-1}y^3\,z^2 \times x^3yz^{-2} \times xy^{-0}z^{-1} = x^3y^4z^{-1} = \frac{x^3y^4}{z}$ (1)

3.1.4 $12a^7\,b^{-5}\,c \div 9a^5b^{-5}c^3 = \frac{4a^2}{3c^2}$ (1)

3.1.5 $[2(a^{-2}\,b^3)]^3 = 8a^{-6}b^9 = \frac{8b^9}{a^6}$ (3 or 5??) (1)

3.2 $2^{1-x} = 2^3$
$1 - x = 3$
$x = -2$ (2)

3.3 $0{,}00090125 = 9{,}0125 \times 10^{-4}$ (1)

[8]

Question 4

4.1.1 1,5; 3; 4,5 (1)
4.1.2 $4a + 5b$; $5a + 4b$; $6a + 3b$ (1)
4.2.1 Divide the previous term by 4. (1)
4.2.2 Multiply the previous term by 2 and then add 1. (1)
4.3.1 $T_n = 2n - 5$ (1)
4.3.2 $T_{20} = 2(20) - 5 = 35$ (1)
4.3.3 $2n - 5 = 199$
$2n = 204$
$n = 102$ (1)
4.4.1 Starting from 1, each number is double the previous number. (1)
4.4.2 Starting from 4, each number is double the previous number. (1)
4.4.3 D $T_n = 2^{n-1}$ (1)
4.4.4 E $T_n = 2^{n+1}$ (1)

[11]

Question 5

5.1.1 $4(2m - 3) + 10 = 8m - 2$ (1)

5.1.2 $5x - 2(x - 1) = 3x + 2$ (1)

5.1.3 $3(4a - 3) + 7a = 19a - 9$ (1)

5.1.4 $-3a^2b^6(-2a^{-1}b^{-2})^3 = 24a^{-1}b^0 = \frac{24}{a}$ (1)

5.1.5 $\left(-\frac{x}{2}\right)^2 - 3(x^2 - 4) = \frac{x^2}{4} - 3x^2 + 12 = -2\frac{3}{4}x^2 + 12$ (1)

5.2.1 $x + 9 = 24$ (1)

5.2.2 $2x + 30 = 50$ (1)

5.2.3 $10 - \frac{1}{2}x = 4$ (1)

5.3.1 $3x + 25 = x + 49$
$2x = 24$
$x = 12$ (1)

5.3.2 $\frac{y}{2} - \frac{1}{4} = \frac{9}{4}$
$2y - 1 = 9$
$2y = 10$
$y = 5$ (2)

5.3.3 $\frac{p^3}{2} = 32$
$p^3 = 64$
$p = 4$ (2)

5.3.4 $-4x + 3 = x - 9$
$-5x = -12$
$x = \frac{12}{5}$ (2)

5.3.5 $4(x + 3) = 2(x - 1)$
$4x + 12 = 2x - 2$
$2x = -14$
$x = -7$ (2)

[21]

Question 6

6.1.1 Accurate construction of AB = 5 cm. (1)
6.1.2 Accurate angle of 60°. (2)
6.1.3 Accurate construction of AC = 8 cm. (1)
6.1.4 Accurate angle of 60°. (1)
6.1.5 Accurate construction of CD = 5 cm. (1)
6.1.6 AD and BC drawn. (1)
Measurements given and are equal. (1)
6.1.7 ABCD is a parallelogram as its opposite sides are equal and its diagonals are equal in length. (2)

[10]

Question 7

7.1.1 A triangle with all angles less than 90° and two sides equal in length is called an **isosceles acute**-angled triangle. (1)
7.1.2 A triangle with one angle equal to 55° and one angle equal to 35° is a **right-angled** triangle. (1)
7.1.3 A parallelogram with adjacent sides equal is a **rhombus**. (1)
7.1.4 An isosceles trapezium has one pair of opposite sides **equal** and one pair **of equal angles**. (1)
7.1.5 The diagonals of a kite intersect at **right angles**/90°. (1)

[5]

Question 8

8.1 A is a parallelogram as its diagonals bisect each other. (2)
B is a square as its diagonals are equal in length and bisect at right angles. (2)
[4]

Question 9

9.1.1 $a = 122° - 90° = 32°$ (exterior angles triangle) (1)
9.1.2 $a = 43°$ (vertical opposite angles)
$b = 43°$ (alternate angles, parallel lines)
$c = 180° - 43° - 90° = 47°$ (straight line angles)
$d = 47°$ (corresponding angles, parallel lines) (4)

9.1.3 $s = 70° - 20° = 50°$ (exterior angles triangle)
$t = 180° - 80° - 50° = 50°$ (sum angles triangle) (2)

9.2 In triangles ABX and CDX:
$\hat{A}_2 = \hat{C}$ (alternate angles, AB || CD)
$\hat{B} = \hat{D}_1$ (alternate angles, AB || CD)
AB = CD (given)
$\therefore \triangle ABX \equiv \triangle CDX$ (AAS)
$\therefore$ BX = DX (4)
[11]

Question 10

10.1.1 area $= 125 \times 48 = 6\ 000\ mm^2$
$= 60\ cm^2$ (2)
10.1.2 area $= \pi \times 19^2 = 1\ 134{,}11\ m^2$ (1)
10.2.1 $QR^2 = 26^2 - 10^2$
$= 576$
QR = 24 m (1)
10.2.2 Circumference of semi-circle $= \frac{\pi \times 6^2}{2} = 56{,}55$ m.

Perimeter = 56,55 + 10 + 24 + 10 = 100,55 m. (2)
[6]
TOTAL: 100 Marks

Assignment Option 1: Algebra

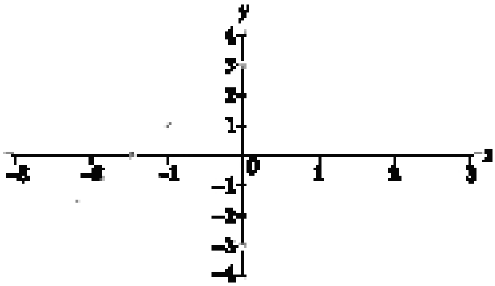
1.1 LHS = $2^3 - 1^3 = 7$
RHS = $(2 - 1)\{2^2 + (2)(1) + 1^2\} = (1)(7) = 7$
$\therefore$ LHS = RHS (2)
Similarly, learners show that the equation is true for:
1.2 $x = -4$ and $y = 3$ (2)
1.3 $x = -6$ and $y = -5$ (2)
1.4 $x = 6{,}1$ and $y = -3{,}9$ (2)
2 No. We have only proved the equation true for certain values of x and y. (1)
3 RHS $= (x - y)(x^2 + xy + y^2)$
$= x^3 + x^2y + xy^2 - x^2y - xy^2 - y^3$
$= x^3 - y^3$
= LHS (4)
4.1 LHS = $2^3 + 1^3 = 9$
RHS = $(2 + 1)\{2^2 - (2)(1) + 1^2\} = (3)(3) = 9$
$\therefore$ LHS = RHS (2)
Similarly, learners show that the equation is true for:
4.2 $x = -4$ and $y = 3$ (2)
4.3 $x = -6$ and $y = -5$ (2)
4.4 $x = 6{,}1$ and $y = -3{,}9$ (2)
5 RHS $= (x + y)(x^2 - xy + y^2)$
$= x^3 - x^2y + xy^2 + x^2y - xy^2 + y^3$
$= x^3 + y^3$
= LHS (4)

TOTAL: 25 Marks

Assignment Option 2: Graphs

1.1 $2y - 6x - 8 = 0$
$y = 3x + 4$ (2)
1.2 3 (1)
1.3 4 (1)
1.4 For the x-intercept, let $y = 0$:
$0 = 3x + 4$
$3x = -4$
$x = -\frac{4}{3}$
$= -1{,}\dot{3}$ (2)
1.5 $3(2) + 4 = 10$
Therefore the point (2; 2) does not lie on the line. (3)

2.1 and **2.2**



(6)

2.3 Both lines have the same *x*-intercept. Both graphs have *y*-intercepts that are the same value, but different signs. Both graphs have gradients that are the same value, but different signs. (1)

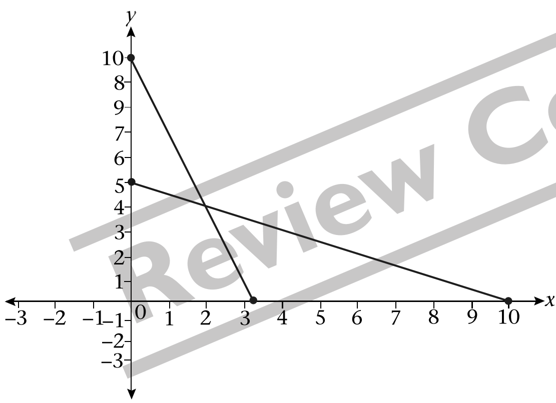
3.1 $y = x + 5$

3.2 $y = -2x + 2$ (4)

3.3 $y = \frac{1}{3}x + 1$

3.4 $y = -3x - 2$ (6)

4



The point of intersection is (2; 4) (4)

5.1 $x + y = 12 \ldots (1)$

$x - y = 2 \ldots (2)$

Rearrange (1): $y = 12 - x \ldots (3)$

Substitute (3) into (2): $x - (12 - x) = 2$

$2x = 14$

$x = 7$

Substitute $x = 7$ into (3): $y = 12 - 7 = 5$ (3)

5.2 $2x + y = 16 \ldots (1)$

$x + 2y = 11 \ldots (2)$

Rearrange (2): $x = 11 - 2y \ldots (3)$

Substitute (3) into (1): $2(11 - 2y) + y = 16$

$-3y = -6$

$y = 2$

Substitute $y = 2$ into (3): $x = 11 - 2(2) = 7$ (3)

5.3 $2x + 3y = 7 \ldots (1)$

$4x - 3y = 5 \ldots (2)$

Add (1) and (2): $6x = 12$

$x = 2$

Substitute $x = 2$ into (1): $2(2) + 3y = 7$

$3y = 3$

$y = 1$ (3)

5.4 $5x + y = 19 \ldots$ (1)

$2x + 4y = 22 \ldots$ (2)

Rearrange (1): $y = 19 - 5x \ldots$ (3)

Substitute (3) into (2): $2x + 4(19 - 5x) = 22$

$-18x = -54$

$x = 3$

Substitute $x = 3$ into (3): $y = 19 - 5(3) = 4$ (3)

6.1 E (1)

6.2 F is parallel to C. (1)

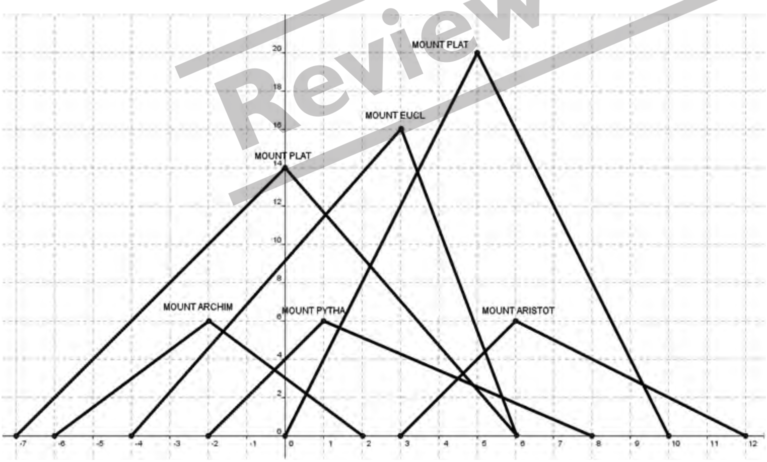
6.3 No, no lines have the same gradient as B. (1)

6.4 F (2)

6.5 A and G are perpendicular because $5 \times -\frac{1}{5} = -1$ (3)

TOTAL: 50 Marks

Project Option 1 Graphs



1.1 Mount Sol: 2 and $-\frac{7}{3}$ Mount Archim: $\frac{3}{2}$ and $-\frac{3}{2}$

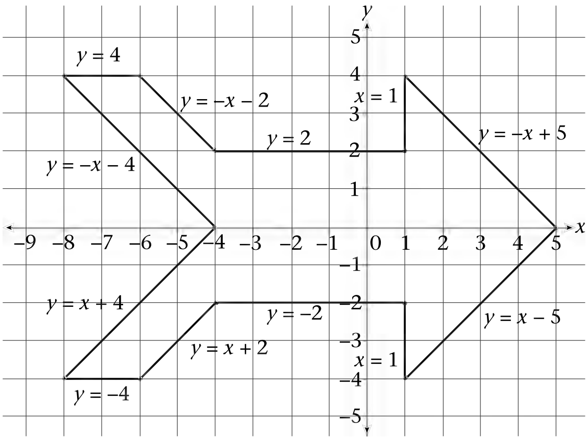
Mount Pytha: 2 and $-\frac{6}{7}$ Mount Eucl: $\frac{16}{7}$ and $-\frac{16}{3}$

Mount Plat: 4 and -4 Mount Aristot: 2 and -1 (6)

1.2 Mount Eucl west-facing slope would be the most difficult to climb. (1)

1.3 Mount Plat is the highest mountain and Mounts Archim, Pytha and Aristot are the lowest. (2)

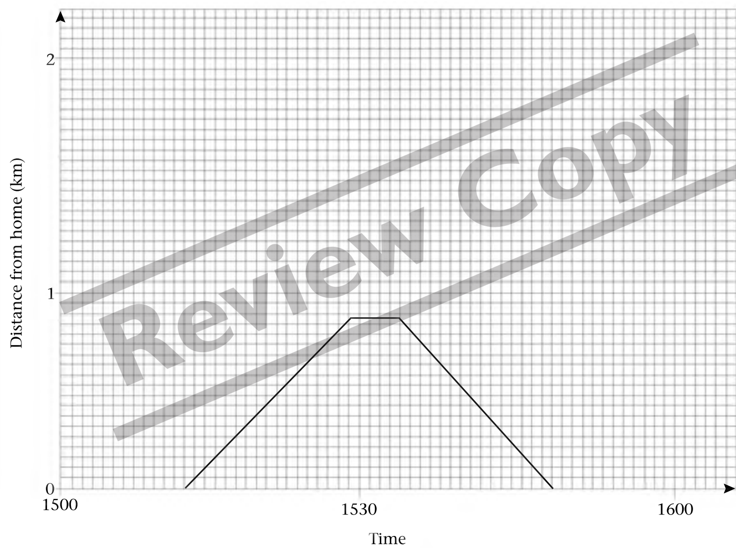
2.1



(12)

2.2 Learners' own design (8)

3



(8)

4.1 The car spent 24 seconds at the first traffic light and 72 seconds at the second traffic light. (4)

4.2 The car travelled fastest between E and F. (1)

4.3 1 300 m. (3)

4.4 480 seconds. (2)

4.5 Speed $= \frac{\text{distance}}{\text{time}} = \frac{1\ 300}{480} = 2{,}7$ m/s (8)

TOTAL: 50 Marks

Project Option 2: 3D objects

1 $280^2 + 220^2 = 126\ 800$
$356^2 = 126\ 736$ (7)

2.1 Surface area $= 440 \times 440 + 4\left(\frac{1}{2} \times 440 \times 356\right) = 506\ 880$ cubits^2 (7)

2.2 cubit = 50 cm, thus 1 square cubit = 2 500 square cm
506 880 cubits^2 $= 506\ 880 \times 2\ 500 = 1\ 267\ 200\ 000$ cm^2 (7)

3 Volume $= \frac{1}{3} \times 440 \times 440 \times 280 = 18\ 069\ 333{,}33$ cubits^3
1 cubit = 50 cm = 0,5 m
1 cubit cubed = 0,125 metre cubed
18 069 333,33 cubits^3 $= 18\ 069\ 333{,}33 \times 0{,}125 = 2\ 258\ 666{,}67$ m^3 (7)

4 Mark using rubric. (14)

	7	6	5	4	3	2	1
Correct application of Pythagoras							
Calculation of total surface area							
Calculation of volume							
Conversion to metric units							
Written report for accuracy							
Written report for interest							

TOTAL: 42 Marks

Test 3

Total: 50 marks **Time: 1 hour**

Question 1

1.1 Sibongile goes on a business trip and leaves home at 6 a.m. She returns home at 9 p.m. During her trip, Sibongile spends 40% of her time travelling. Calculate the number of hours Sibongile spent travelling. (2)

1.2 Sibongile's company uses the following formulae to calculate allowances for travel and meals: Allowance (R) = $15d + \frac{k - 5d}{3}$ where d represents the number of days away from home and k represents the distance (in km) travelled.

1.2.1 What would Sibongile's allowance be if she spent four days on a business trip and travelled 740 km? (2)

1.2.2 Determine how many kilometres Sibongile would have travelled if she received an allowance of R195 for three days travel. (2)

[6]

Question 2

2.1 Factorise the following:

2.1.1 $x^2 - 6x + 5$ (2)

2.1.2 $a^2 + 2a - 8$ (2)

2.1.3 $y^4 - 16$ (2)

2.1.4 $6x^2 + 72x + 120$ (3)

2.2 Simplify using factorisation:

2.2.1 $\frac{3x^2 - 9x - 12}{3x - 12}$ (3)

2.2.2 $\frac{5x^2 - 125}{x + 5}$ (2)

2.3 Solve the following equations:

2.3.1 $3(x - 2) = 2x + 1$ (1)

2.3.2 $x^2 - 8x = 9$ (2)

2.4 The base of a triangle is x cm and its perpendicular height is 3 cm less than the base. The area of the triangle is 20 cm^2.

2.4.1 Express the perpendicular height of the triangle in terms of x. (1)

2.4.2 Write down a formula to calculate the area of a triangle in terms of its base (b) and its perpendicular height (h). (1)

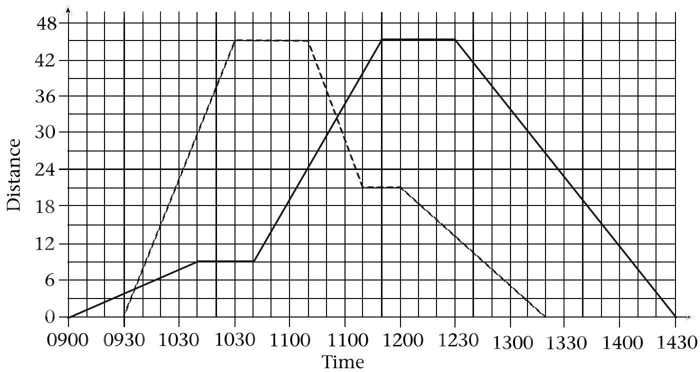
2.4.3 Set up an equation (in terms of x) that expresses this situation. (1)

2.4.4 Solve your equation and find the base and the perpendicular height of the triangle. Show all your calculations and test your answers. Make sure that your answers make practical sense. (3)

[23]

Question 3

3.1. The following distance-time graph shows the journeys of a van and a car travelling from Cape Town to Worcester and back to Cape Town.



3.1.1 How long after the departure of the van did the car start out on its journey? (1)

3.1.2 Which of the two vehicles had the greater speed, the car or van? Justify your answer by referring to the gradients of the graphs. (2)

3.1.3 How far had the car travelled when it overtook the van for the second time? (1)

3.1.4 Calculate, in kilometres per hour, the average speed of the car between 9:50 and 10:00. (2)

3.1.5 During which time period was the van travelling at its greatest speed? (1)

3.1.6 Compare the total time the trip took for the car and the van. (2)

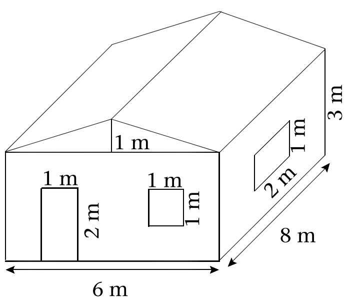
3.1.7 Determine the average speed of the van and the car for the entire trip. (2)

3.2 Plot the following graphs onto the Cartesian plane, using any method:
$y = \frac{3}{4}x - 3$ and $3y - 4x = 12$ (4)

[15]

Question 4

4 Given the following diagram of a house. Each opposite side is the same (this means that there is a door and window at the back and there is a window on the left side that we can't see).



4.1 Determine the total area of the outside of the house. (3)

4.2 Determine the volume inside the house. (3)

[6]

TOTAL: 50 Marks

Answers to Term 3 test

1 $40\% \times 15 = \frac{40}{100} \times 15$

$= 6$ hours (2)

1.2.1 Allowance (R) $= 15d + \frac{k - 5d}{3}$

$= 15(4) + \frac{740 - 5(4)}{3}$

$= \text{R}300$ (2)

1.2.2 Allowance (R) $= 15d + \frac{k - 5d}{3}$

$195 = 15(3) + \frac{k - 5(3)}{3}$

$150 = \frac{k - 15}{3}$

$k - 15 = 450$

$k = 465$ km (2)

[6]

2.1.1 $x^2 - 6x + 5 = (x - 5)(x - 1)$ (2)

2.1.2 $a^2 + 2a - 8 = (a + 4)(a - 2)$ (2)

2.1.3 $y^4 - 16 = (y^2 - 4)(y^2 + 4)$

$= (y - 2)(y + 2)(y^2 + 4)$ (2)

2.1.4 $6x^2 + 72x + 120 = 6(x^2 + 12x + 20)$

$= 6(x + 10)(x + 2)$ (3)

2.2.1 $\frac{3x^2 - 9x - 12}{3x - 12} = \frac{3(x^2 - 3x - 4)}{3(x - 4)}$

$= \frac{3(x - 4)(x + 1)}{3(x - 4)}$

$= x + 1$ (3)

2.2.2 $\frac{5x^2 - 125}{x + 5} = \frac{5(x^2 - 25)}{x + 5}$

$= \frac{5(x - 5)(x + 5)}{x + 5}$

$= 5(x - 5)$ (2)

2.3.1 $3(x - 2) = 2x + 1$

$3x - 6 = 2x + 1$

$x = 7$ (1)

2.3.2 $x^2 - 8x = 9$

$x^2 - 8x - 9 = 0$

$(x - 9)(x + 1) = 0$

$x = 9$ or $x = -1$ (2)

2.4.1 $x - 3$ (1)

2.4.2 area $= \frac{1}{2}bh$ (1)

2.4.3 $20 = \frac{1}{2}(x)(x - 3)$ (1)

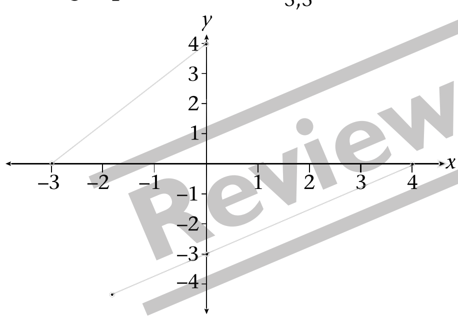
CHAPTER 17

2.4.4 $20 = \frac{1}{2}(x)(x - 3)$
$40 = x^2 - 3x$
$x^2 - 3x - 40 = 0$
$(x - 8)(x + 5) = 0$
$x = 8$ or $x = -5$
But x must be a positive number, so $x = 8$ only.
Therefore base = 8 cm
Height = 5 cm (3)

[23]

3.1.1 30 minutes/half an hour. (1)
3.1.2 The car had the greater speed as the gradient of its graph is steeper. (2)
3.1.3 57 km (45 km + 12 km) (1)
3.1.4 Speed = $\frac{\text{D}}{\text{T}} = \frac{6\text{ km}}{\frac{1}{6}\text{h}}$ = 36 km/h (2)
3.1.5 Between 10:40 and 11:50 (1)
3.1.6 The car took 3 hours and 50 minutes and the van took 5 hours and 30 minutes. (2)
3.1.7 Average speed of car = $\frac{90}{3{,}83}$ = 23,5 km/h
Average speed of van = $\frac{90}{5{,}5}$ = 16,4 km/h (2)

3.2 (4)



[15]

4.1 Area of front face = $6 \times 3 - (1 \times 1 + 2 \times 1) + \left(\frac{1}{2} \times 6 \times 1\right)$
$= 18\text{ m}^2$
Area of side face = $8 \times 3 - 2 \times 1$
$= 22\text{ m}^2$
Diagonal of roof = $\sqrt{1^2 + 3^2} = \sqrt{10}$ m
Area of roof panel = $8 \times \sqrt{10} = 25{,}3\text{ m}^2$
Total area = 2 × front face + 2 × side face + 2 × roof panel
$= 2(18) + 2(22) + 2(25{,}3) = 130{,}6\text{ m}^2$ (3)
4.2 Volume of rectangular prism = $8 \times 6 \times 3 = 144\text{ m}^3$
Volume of triangular prism = $\frac{1}{2} \times 6 \times 1 \times 8 = 24\text{ m}^3$
Total volume = $144\text{ m}^3 + 24\text{ m}^3 = 168\text{ m}^3$ (3)

TOTAL: 50 Marks

Assignment Option 1: 3D objects

1.1	A – Tetrahedron B – Octahedron C – Dodecahedron D – Icosahedron	(4)
1.2	Cube	(1)
1.3	Accurate construction of a cube:	
	All vertices right angles	(2)
	All sides (edges) equal	(2)
	Correct number of faces, namely 6	(1)
	Faces in correct positions to be folded into a cube	(1)
1.4	4 faces, 4 vertices, 6 edges	(3)
	Euler's formula	(1)
	$F + V - E = 2$ or $F + V - 2 = E$	(2)
	$F + V - E = 2$ $V = E - F + 2$ $V = 42 - 30 + 2 = 14$ vertices	(3)
2.1	In the net of a cylinder the length of the rectangular face must equal the **circumference** of the circular face.	(1)
2.2	Accurate construction of the net of a cylinder:	
	Calculation of circumference of circular face	(2)
	Rectangle has length equal to circumference and breadth 6 cm	(1)
	Vertices of rectangle are right angles	(1)
	Circular faces have correct radius	(1)
	Two circles and one rectangle in suitable positions to form cylinder	(1)
2.3	A sphere is a 3-dimensional object with every point on its surface the same distance from its centre.	(2)
2.4	3 cm	(1)

TOTAL: 30 Marks

Assignment Option 2: Probability

1 $0; \frac{1}{5}; 0{,}3; \frac{1}{3}; \frac{9}{20}; 50\%; \frac{3}{4}; 0{,}81; \frac{23}{25}; 100\%$ (4)

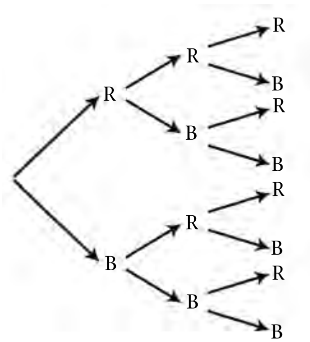
2.1 $360° \div 8 = 45°$ (1)

2.2.1 $\frac{1}{8}$ **2.2.2** $\frac{3}{8}$ **2.2.3** $\frac{1}{4}$ (3)

2.3.1 $\frac{1}{4}$ **2.3.2** $\frac{1}{8}$ **2.3.3** $\frac{1}{4}$ (3)

3.1 A two-way table is only suitable for displaying the outcomes of two events. Choosing three playing cards involves three events, so I would choose a tree diagram. (2)

3.2



P (choosing three cards of the same colour) = $2\left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right) = 2(\frac{1}{8}) = \frac{1}{4}$ (6)

4.1.1 $\frac{6}{11}$ **4.1.2** $\frac{5}{11}$ (2)

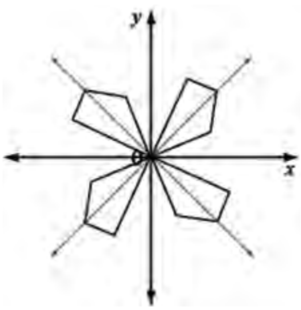
4.2.1 $\frac{1}{2}$ **4.2.2** $\frac{1}{2}$ (2)

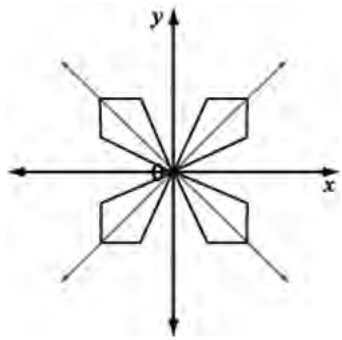
4.3 The answers to Questions 4.1 and 4.2 are different, because the youngest girl is not part of the second draw. There are only 10 children left in the second draw. (2)

TOTAL: 25 Marks

Investigation Option 1: Using transformations in design

1.1 – **1.3** Learners should end up with a drawing like one of these.





1.4 The learners will have different co-ordinates.

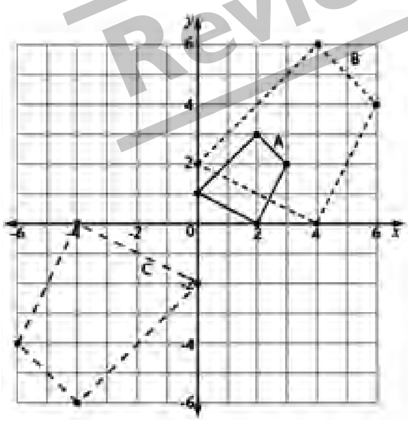
1.5 – **1.8** On the drawing.

1.9 Learners should see that the co-ordinates $(x; y)$ of a point in the first quadrant becomes $(y; -x)$ under a clockwise rotation, and $(-y; x)$ under an anticlockwise rotation through 90^o.

2.1 – **2.2** Learners should have a quadrilateral in the first quadrant as in the drawing.

2.3 – **2.4** Their sets of co-ordinates will differ according to their drawings.

2.5 Learners should have a drawing like the one shown below.



2.6 – **2.8** Learners should notice that the enlargement lies in the third quadrant and is a rotated version through 180^o of the positive enlargement.

Marking guidelines for transformation investigation

Question 1 (20 marks)

- Lines $y = x$ and $y = -x$ drawn correctly (1)
- Kite or quadrilateral drawn so that the vertex of the smallest angle is on the origin and the longest diagonal lies on the line $y = x$. (1)
- Co-ordinates of the vertices of the shape correctly written down (2)
- Shape is rotated clockwise through 90° (1)
- The co-ordinates of the vertices of the new position of the shape are written down correctly. (2)
- Clockwise rotation is repeated two more times and co-ordinates of the vertices of each new position of the shape are written down correctly. (4)
- Shape is rotated anti-clockwise through 90° (1)
- The co-ordinates of the vertices of the new position of the shape are written down correctly. (2)
- Anti-clockwise rotation is repeated two more times and co-ordinates of the vertices of each new
- position of the shape are written down correctly. (4)
- Valid conclusion made about effect of rotation through 90° on the co-ordinates of a shape. (2)

[20]

Question 2 (10 marks)

- Any quadrilateral, A, drawn in the first quadrant (1)
- Co-ordinates of the vertices of quadrilateral written down correctly (2)
- Correct co-ordinates of the image B given if the quadrilateral A is enlarged by a factor 2. (2)
- Enlargement B of quadrilateral A drawn correctly (2)
- Evidence shown of investigation of enlargement of quadrilateral A by scale factor –2 (1)
- Valid statement written down about scale factors –1; –3; –4 etc (1)
- Valid conclusion made about an enlargement with a negative scale factor (1)

[10]

TOTAL: 30 Marks

Investigation Option 2: Data rubric

Assessment criterion: Did the learner	**Total possible marks**	**Learner's mark**
identify the population that s/he chose?	1	
identify the sample that s/he chose?	2	
explain how s/he chose that particular sample?	2	
make a credible attempt to choose a sample that fairly represented the whole population?	2	
plan a coherent, well-thought-out questionnaire?	4	
provide a neat list of the raw data and/or provide data that was correctly and sensibly summarised?	10	
draw an accurate pie chart of data that was appropriate for a pie chart?	5	
draw a scatter plot of data that was appropriate for a scatter plot?	5	
draw a third kind of graph of data appropriate for that kind of graph?	5	
draw meaningful conclusions from the results of their survey, and make sensible predictions, where appropriate?	4	
TOTAL:	40	

December Exemplar Examination Memo

Question 1

1.1.1 $\sqrt{2\ 401} = \sqrt{7 \times 7 \times 7 \times 7} = 49$ (3)

1.1.2 $729 = 3 \times 3 \times 3 \times 3 \times 3 \times 3$

$405 = 5 \times 3 \times 3 \times 3 \times 3$

The HCF of 729 and 405 is 81 (2)

1.2 $-2\frac{1}{2}\left(3\frac{2}{3} - 2\frac{5}{6}\right) = -\frac{5}{2}\left(\frac{11}{3} - \frac{17}{6}\right)$

$= -\frac{5}{2}\left(\frac{22 - 17}{6}\right)$

$= -\frac{5}{2} \times \frac{5}{6}$

$= -\frac{25}{12}$

$= -2\frac{1}{12}$ (2)

1.3.1 $\frac{(2^3ab^2c)^2}{(4ab)^3} = \frac{bc^2}{a}$ (2)

1.3.2 $\frac{(-9x^2y^4) \times 6xy^{-1}}{3xy \times (-3x^2)} = \frac{-54x^3y^3}{-9x^3y} = 6y^2$ (3)

[12]

Question 2

2.1.1

Shape number (g)	1	2	3	4	5	6	7	8
Number of dominoes (d)	1	3	5	7	9	11	13	15
Perimeter of the shape (p) (cm)	18	42	66	90	114	138	162	186

(3)

2.1.2 (1) $24n - 6$ (1)

2.1.3 $24n - 6 = 24(150) - 6 = 3\ 594$ cm (1)

2.2.1 $P = 40t + 5t^2 = 40(5) + 5(25) = R325$ (1)

2.2.2 $900 = 40t + 5t^2$

$5(t^2 + 8t - 180) = 0$

$5(t + 18)(t - 10) = 0$

$t = -18$ or $t = 10$ (3)

The trip was 10 days long. **[9]**

Question 3

3.1.1 $\frac{36x^4 - 1}{6x^2 + 1} = \frac{(6x^2 - 1)(6x^2 + 1)}{6x^2 + 1}$

$= 6x^2 - 1$ (2)

3.1.2 $\frac{a^2 - ab}{a} + \frac{b^2 - ab}{b} = \frac{a(a - b)}{a} + \frac{b(b - a)}{b}$

$= (a - b) + (b - a) = 0$ (2)

3.1.3 $\frac{2x}{x+5} \times \frac{x^2-25}{4x^2-6x} = \frac{2x}{x+5} \times \frac{(x-5)(x+5)}{2x(2x-3)}$

$= \frac{x-5}{2x-3}$ (3)

3.1.4 $\frac{2a+3c}{2a+10b} \div \frac{4a^2-9c^2}{a^2-25b^2} = \frac{2a+3c}{2(a+5b)} \times \frac{(a-5b)(a+5b)}{(2a-3c)(2a+3c)}$

$= \frac{a-5b}{2(2a-3c)}$ (3)

3.2.1 $6(13-x)+2x(x-13)=0$

$78-6x+2x^2-26x=0$

$2x^2-32x+78=0$

$2(x-13)(x-3)=0$

$x=13$ or $x=3$ (2)

3.2.2 $8x^2-1=24-x^2$

$9x^2-25=0$

$(3x-5)(3x+5)=0$

$x=\frac{5}{3}$ or $x=-\frac{5}{3}$ (2)

3.2.3 $12(2-x)=2x-x^2$

$24-12x=2x-x^2$

$x^2-14x+24=0$

$(x-12)(x-2)=0$

$x=12$ or $x=2$ (3)

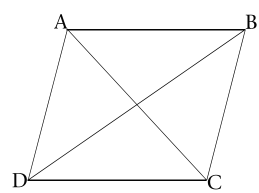
[17]

Question 4

4.1 Accurate construction of an angle of 30°. (2)

4.2 Accurate construction of triangle KLM with KL = 6,4 cm, $\hat{L} = 90°$, and LM = 5,29 cm. (3)

4.3 Given rhombus ABCD with diagonals AC and BD:



In $\triangle$ABC and $\triangle$ADC:

AB = DC (sides of a rhombus)

BC = AD (sides of a rhombus)

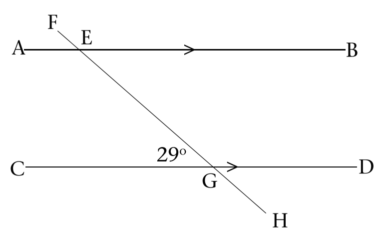
AC is common

$\therefore \triangle ABC \equiv \triangle ADC$ (SSS)

$\therefore B\hat{A}C = D\hat{A}C$ and $A\hat{C}D = A\hat{D}B$

Similarly, $\therefore \triangle ADB \equiv \triangle DDB$ (SSS) and $\therefore A\hat{D}B = C\hat{D}B$ and $A\hat{B}D = C\hat{B}D$ (3)

4.4



$B\hat{E}G = 29°$ (alternate angles, AB $\parallel$ CD)

$A\hat{E}F = 29°$ (vertical opp angles)

$A\hat{E}G = 151°$ (straight line angles)

$F\hat{E}B = 151°$ (vertical opposite angles OR straight line angles)

$D\hat{G}H = 29°$ (vertical opposite angles)

$C\hat{G}H = 151°$ (straight line angles)

$E\hat{G}D = 151°$ (vertical opposite angles OR straight line angles) (4)

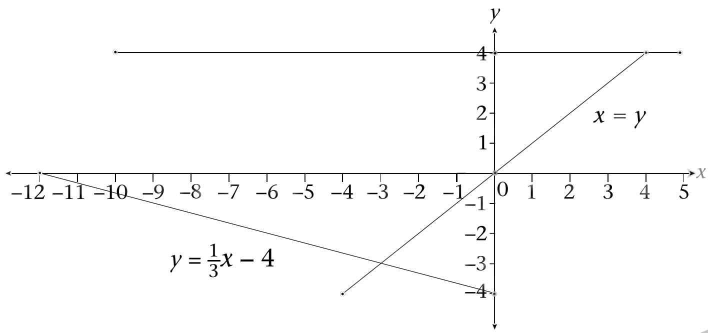
CHAPTER 17

4.5 $a = 73°$ (sum angles triangle ABC)
$b = 17°$ (alternate angles, AB || CD)
$c = 36°$ (straight line angles)
$d = 66°$ (sum angles triangle CEF) (5)

[17]

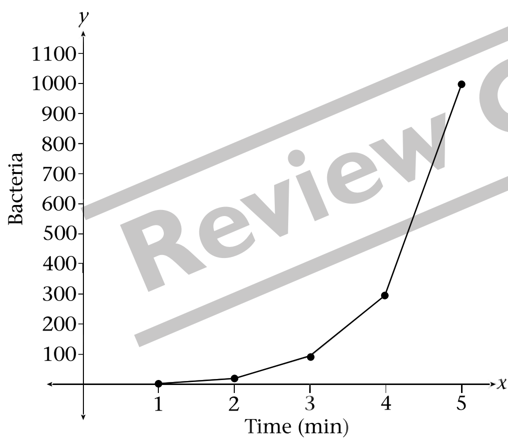
Question 5

5.1



(5)

5.2.1



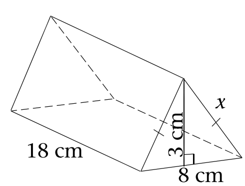
(3)

5.2.2 An exponential graph. (1)

5.2.3 After 10 minutes there will be 4^{10} bacteria = 1 048 576 bacteria. (2)

[11]

Question 6



6.1.1 $x = \sqrt{3^2 + 4^2}$ (Pythag)

$x = 5$ cm

Surface area $= 2\left(\frac{1}{2} \times 8 \times 3\right) + 2(18 \times 5) + 18 \times 8$

$= 24 + 180 + 144$

$= 348 \text{ cm}^2$

Volume $= \frac{1}{2}(8)(3) \times 18$

$= 216 \text{ cm}^3$ (2)

6.1.2 Surface area $= 2(12 \times 6) + 2(15 \times 6) + 2(12 \times 15)$
$= 684 \text{ cm}^2$
Volume $= 12 \times 6 \times 15$
$= 1\,080 \text{ cm}^3$ (2)

6.2.1 Volume of metal removed by the drill $= \pi \times 2^2 \times 11$
$= 138{,}23 \text{ mm}^3$ (3)

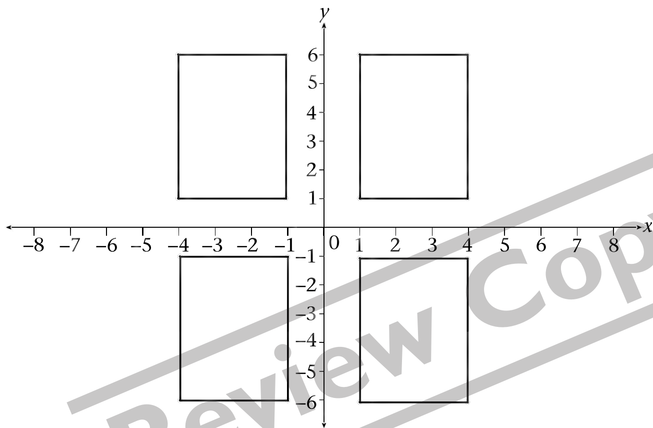
6.2.2 Volume of metal in the collar $= \pi \times 4^2 \times 11 - 138{,}23$
$= 414{,}69 \text{ mm}^3$ (2)

[9]

Question 7

7.1 (1;1) (1;6) (4;6) (4;1) (1)

7.2 – 7.4



(3)

7.5 A rotation through 180° around the origin. (1)

Question 8

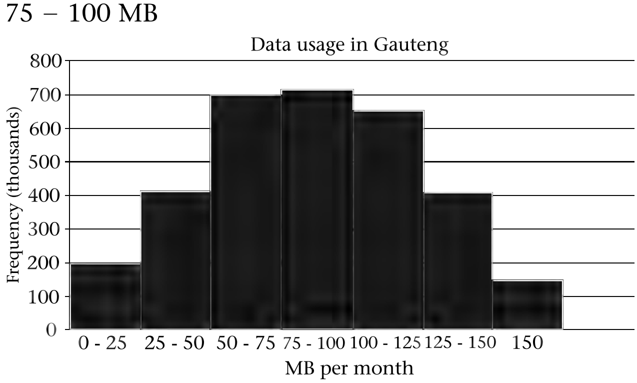
8.1 Continuous data (1)

8.2 3 237 000 people (1)

8.3 The population of Gauteng (1)

8.4 75 – 100 MB (1)

8.5



(3)

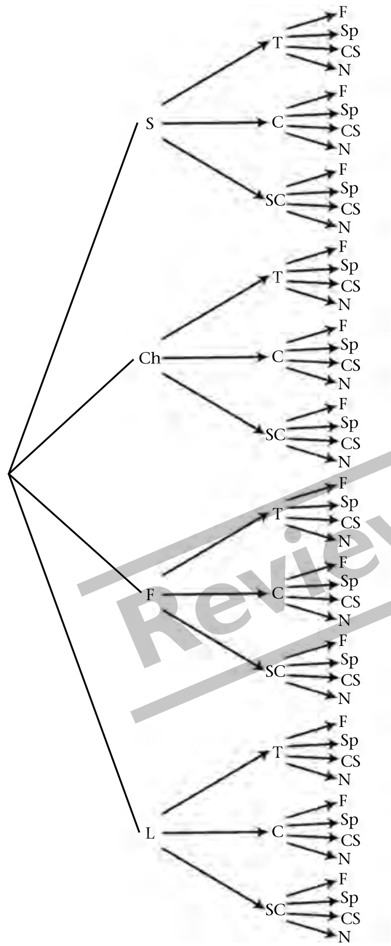
8.6 Few people use below 50 MB or above 125 MB per month and most people use between 50 and 125 MB per month. The most common (modal) category is 75 – 100 MB per month and the least common category is 150 MB per month. (2)

8.7 They can sell packages of data of around 75 – 100 MB per month. (1)

[10]

Question 9

9.1.1



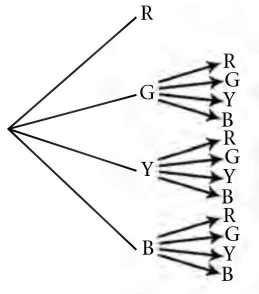
There are 48 possible combinations. (2)

9.1.2 i) P(chocolate) $= \frac{12}{48} = \frac{1}{4}$ (1)

9.1.2 ii) P(strawberry and sugar cone) $= \frac{4}{48} = \frac{1}{12}$ (1)

9.1.2 iii) P(fudge, tub, caramel sauce) $= \frac{1}{48}$ (1)

9.2.1 (2)



9.2.2 $4 \times \frac{1}{4} \times \frac{1}{7} = \frac{1}{4}$ (2)

9.2.3 $\frac{1}{4} \times \frac{2}{7} \times \frac{1}{6} = \frac{2}{168} = \frac{1}{84}$ (1)

[10]

TOTAL: 100 Marks

December Examination

TOTAL: 100 Marks **Time: 2 hours**

Question 1

1.1 Calculate:

1.1.1 $12{,}5 - (-2{,}5)$ **1.1.2** $\left(-1\frac{1}{3}\right) \div \left(-\frac{1}{3}\right)$

1.1.3 $(-5) \times (-8) \div (-\frac{1}{2})$ (3)

1.2 Write 1 296 in terms of its prime factors. (3)

[6]

Question 2

2 Look at the table below:

x	1	2	3	4	5
y	3	6	11	18	27

2.1 Describe in your own words the number pattern that you see in the second row of the table. (2)

2.2 Which of the following equations describes how to calculate y in terms of x? (1)

A $y = 3x$ **B** $y = x + 2$

C $y = x^2 + 2$ **D** $y = 2x^2 + 1$

2.3 If $x = 7$, what will be the value of y? (1)

2.4 If $y = 102$, what will be the value of x? (2)

[6]

Question 3

3.1 Find each of the following products:

3.1.1 $(2x + 7)(3x - 1)$ (1)

3.1.2 $2(p + 4)^2$ (1)

3.2 The length of a rectangle is three times its breadth. Let the breadth be x cm.

3.2.1 Write down an expression to describe the perimeter of the rectangle. (1)

3.2.2 If the perimeter of the rectangle is 48 cm, write down an equation to describe this situation. (1)

3.3 Solve your equation for x. (1)

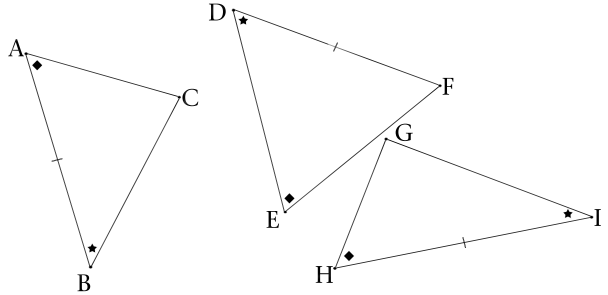
3.4 Use your result to find the area of the rectangle in cm^2. (1)

[6]

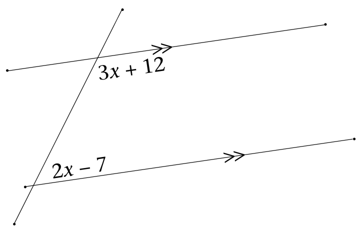
Question 4

4.1 Use only your ruler and compass to construct a triangle ABC with angle $\hat{B} = 30°$, $\hat{C} = 60°$ and BC = 4 cm. Show all steps of the construction. (3)

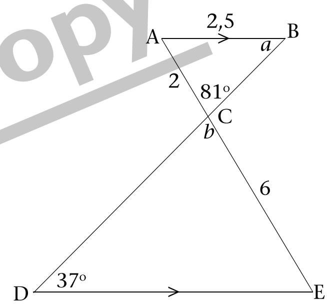
4.2 Identify which of the following triangles are congruent. Give a reason for your answer. (2)



4.3 Use an equation to find the value of x in this diagram. Give a reason. (3)



4.4 Refer to the diagram alongside:



4.4.1 write down the size of the angles marked a and b, with reasons (2)

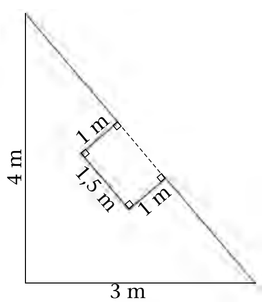
4.4.2 complete the following statement:
$\triangle$........|||$\triangle$........ (A;A;.......) (1)

4.4.3 find the length of DE. (2)

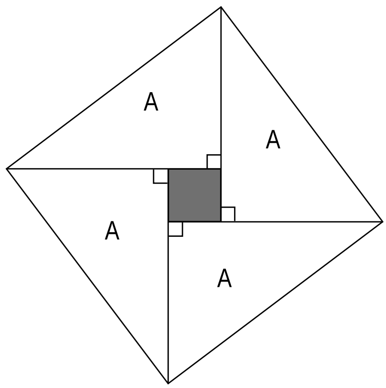
[13]

Question 5

5.1 Determine the area of the plot of land marked with solid lines in the drawing below. (3)



5.2 A plot is made up of land and water. The shaded square in the diagram below shows the water. Four congruent right-angled triangles (each labelled A) represent the land. On the triangles, the sides that meet at right angles have lengths 3 m and 4 m. Calculate the area of the land. (3)



[6]

Question 6

6 The cost of hiring a van is R700 per day and R20 for every kilometre travelled.

6.1 Write an equation of the cost, C, in Rands, if the van is hired for d days and used for k kilometres. (2)

6.2 Write an equation of the cost, C, in Rands if the van is hired for two days and used for k kilometres. (2)

6.3 Umar hired the van for three days and he had to pay a bill of R3 000. How many kilometres did he travel? (2)

[6]

Question 7

7.1 Some Grade 9 learners did the following factorisations. For each one, say whether or not it is correct. For each incorrect factorisation, do the factorisation correctly.

7.1.1 $4x^2y + 8xy - 12y^2 = 4xy(x + 2 - 3y)$ (2)

7.1.2 $x^2 - 16 = (x + 8)(x - 8)$ (2)

7.1.3 $x^2 - 81 = (x + 3)^2(x - 3)^2$ (2)

7.1.4 $2x^2 + 18x + 16 = 2(x + 8)(x + 1)$ (2)

7.1.5 $x^2 - 5x + 6 = (x - 2)(x - 3)$ (2)

7.2 Solve for x in each of the following:

7.2.1 $242 - 2x^2 = 0$ (2)

7.2.2 $x(x - 5) + 9(5 - x) = 0$ (2)

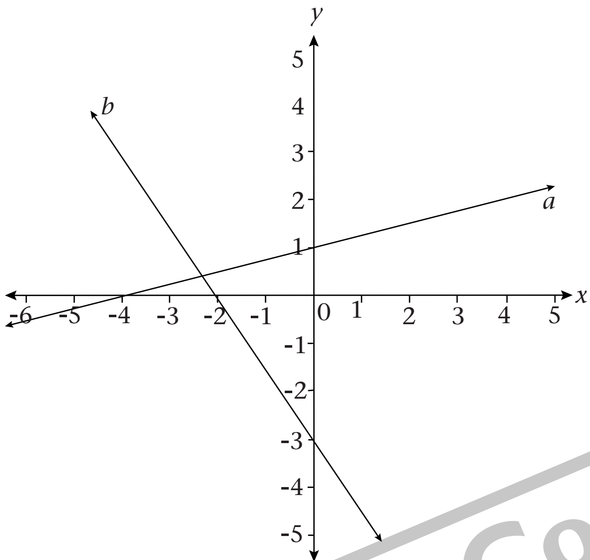
7.2.3 $x^2 - x - 49 = -7$ (2)

[16]

Question 8

8.1 Use the intercept-intercept method to plot the following graphs on a set of axes: $y = 3x + 6$ and $2x + 3y = 12$ (5)

8.2 Determine the equations of lines a and b in the diagram below. (3)



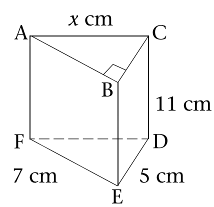
[8]

Question 9

9.1 The triangular prism ABCDEF has dimensions as shown. Calculate, correct to two decimal places:

9.1.1 the length of AC (1)

9.1.2 the total surface area of the prism. (2)

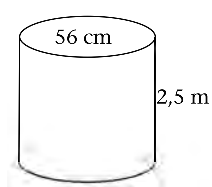


9.2 The inside diameter of a cylindrical water tank is 56 cm and its height inside is 2,5 m. Calculate, correct to two decimal places:

9.2.1 the capacity of the tank in kilolitres (2)

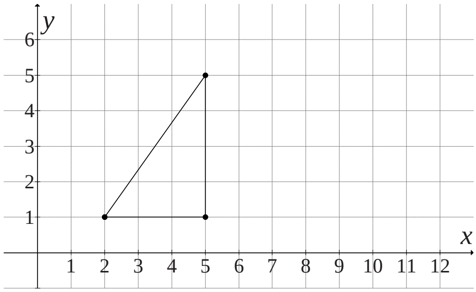
9.2.2 the diameter of a tank with the same height but with double the capacity as the first one. (2)

[7]



CHAPTER 17

Question 10



10.1 Enlarge △DEF by a scale factor $\frac{5}{2}$ by drawing △D′E′F′ next to it. The centre of enlargement is the origin. (2)

10.2 Measure the sides of △DEF and D′E′F′ and then calculate the ratios DE : EF : DF and D′E′ : E′F′ : D′F′. (2)

10.3 Measure the angles of △DEF and △D′E′F′ and write down the measurements. (2)

[6]

Question 11

11.1 In the following table, match the solid in Column A with the description in Column B. You only need to write down the number from A with the matching letter from B.

COLUMN A		COLUMN B	
1	TETRAHEDRON	A	6 SQUARE FACES
2	HEXAHEDRON	B	20 EQUILATERAL TRIANGULAR FACES
3	OCTAHEDRON	C	10 REGULAR PENTAGONAL FACES
4	DODECAHEDRON	D	8 EQUILATERAL TRIANGULAR FACES
5	ICOSAHEDRON	E	4 EQUILATERAL TRIANGULAR FACES

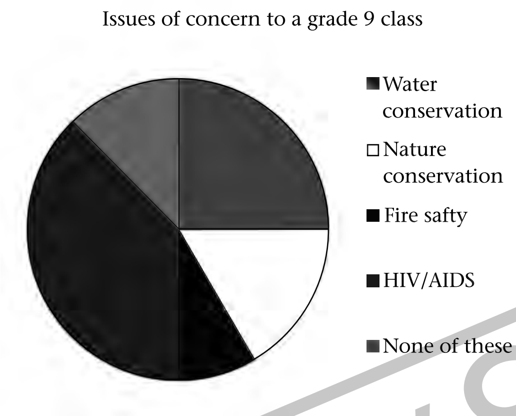
(5)

11.2 Describe the identifying characteristics of a sphere. (1)

[6]

Question 12

12.1 Kagiso interviewed all the learners in his class to find out whether or not they were concerned about any of the following issues:
Water conservation
Nature conservation
Fire safety
HIV/AIDS
He asked every learner that he interviewed to vote for one issue only. He summarised his findings in the pie chart below:



12.1.1 From the pie chart, can you tell how many learners Kagiso interviewed? Give a reason for your answer. (1)

12.1.2 If four learners were concerned about fire safety, estimate how many learners were concerned about each of the other issues. Explain all your reasoning and show any calculations that you make. (2)

12.1.3 Which issue caused the most concern in the class? (1)

12.2 The data below shows the weekly rainfall in millimetres in two towns for one year:

Calvinia: 0; 0; 0; 8; 13; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 10; 0; 0; 0; 0; 3; 0; 0; 0; 0; 0; 0; 20; 22; 0; 0; 0; 0; 49; 52; 0; 0; 0; 0; 0; 0

George: 10; 0; 4; 6; 8; 0; 0; 0; 22; 36; 12; 12; 5; 5; 3; 8; 17; 0; 0; 4; 9; 0; 10; 4; 0; 22; 5; 0; 0; 5; 15; 0; 0; 2; 8; 10; 56; 82; 10; 20; 0; 4; 4; 20; 10; 108; 96; 3; 7; 8; 9; 26

12.2.1 Calculate the mean, median and mode for each town. (3)

12.2.2 Write a short report about the rainfall patterns in each town. Compare the means of the towns, the modes of the towns and the medians of the towns. For each town, which measure of central tendency best describes the rainfall pattern in that town? (2)

[9]

Question 13

Mr and Mrs Molefe are expecting triplets. You are going to help them to determine the probability of the sexes of their unborn babies.

13.1 Draw up a tree diagram that shows all the possibilities of the sexes of their triplets. (2)

13.2 Use your tree diagram to predict the probability that they will have:

13.2.1 all three babies of the same sex (1)

13.2.2 at least one of each (a girl and a boy) (1)

13.2.3 two boys and one girl. (1)

[5]

TOTAL: 100 Marks

December Examination Memo

Question 1

1.1.1 $12{,}5 - (-2{,}5) = 15$

1.1.2 $\left(-1\frac{1}{3}\right) \div \left(-\frac{1}{3}\right) = -\frac{4}{3} \times -\frac{3}{1} = 4$

1.1.3 $(-5) \times (-8) \div \left(-\frac{1}{2}\right) = -80$ (3)

1.2 $1\,296 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3$
$= 2^4 \times 3^4$ (3)

[6]

Question 2

x	1	2	3	4	5
y	3	6	11	18	27

2.1 The pattern starts with 3 and increases by 3, after which it increases by successive odd numbers (or by two more each time). (2)

2.2 C $y = x^2 + 2$ (1)

2.3 $y = x^2 + 2 = 7^2 + 2 = 51$ (1)

2.4 $x^2 + 2 = 102$
$x^2 - 100 = 0$
$(x - 10)(x + 10) = 0$
$x = 10$ or $x = -10$
As x can only take on positive values, $x = 10$ (2)

[6]

Question 3

3.1.1 $(2x + 7)(3x - 1) = 6x^2 + 19x - 7$ (1)

3.1.2 $2(p + 4)^2 = 2(p^2 + 8p + 16)$
$= 2p^2 + 16p + 32$ (1)

3.2.1 Perimeter of the rectangle $= 2(x + 3x) = 8x$ cm (1)

3.2.2 $8x = 48$ (1)

3.3 $x = 6$ cm (1)

3.4 Area $= 6 \times 18 = 108$ cm^2 (1)

[6]

Question 4

4.1 Accurate construction of triangle ABC with angle B = 30°, angle C = 60° and BC = 4 cm. (3)

4.2 $\triangle ACB \equiv \triangle HGI$ (SAA) (2)

4.3 $3x + 12° + (2x - 7°) = 180°$ (co-interior angles, parallel lines)
$5x - 5° = 180°$
$x = 37°$ (3)

4.4.1 $a = 37°$ (alternate angles, $AB \parallel DE$)
$b = 81°$ (vertically opposite angles) (2)

4.4.2 $\triangle ABC ||| \triangle EDC$ (AAA) (1)
4.4.3 DE $= 2{,}5 \times 3 = 7{,}5$ units (2)
[13]

Question 5

5.1 Area $= \left(\frac{1}{2} \times 4 \times 3\right) - (1{,}5 \times 1)$
$= 4{,}5 \text{ m}^2$ (3)

5.2 Hypotenuse of triangles = 5 cm (Pythagoras' theorem).
Area of big square $= 5 \times 5 = 25 \text{ m}^2$
Area of congruent triangles $= 4 \times \left(\frac{1}{2} \times 4 \times 3\right) = 24 \text{ m}^2$
Area of the land $= 25 \text{ m}^2 - 24 \text{ m}^2 = 1 \text{ m}^2$ (3)
[6]

Question 6

The cost of hiring a van is R700 per day and R20 for every kilometre travelled.

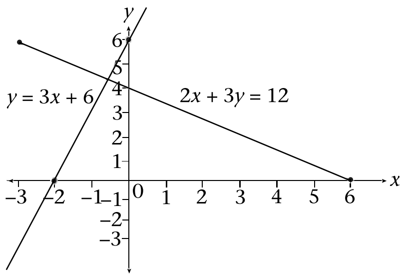
6.1 $C = 20k + 700d$ (2)
6.2 $C = 20k + 1\,400$ (2)
6.3 $20k + 700(3) = 3\,000$
$20k = 900$
$k = 45$ km (2)
[6]

Question 7

7.1.1 Incorrect: $4x^2y + 8xy - 12y^2 = 4y(x^2 + 2x - 3y)$ (2)
7.1.2 Incorrect: $x^2 - 16 = (x + 4)(x - 4)$ (2)
7.1.3 Incorrect: $x^2 - 81 = (x + 9)(x - 9)$ (2)
7.1.4 Correct (2)
7.1.5 Correct (2)
7.2.1 $242 - 2x^2 = 0$
$2(121 - x^2) = 0$
$(11 - x)(11 + x) = 0$
$x = 11$ or $x = -11$ (2)
7.2.2 $x(x - 5) + 9(5 - x) = 0$
$x(x - 5) - 9(x - 5) = 0$
$(x - 5)(x - 9) = 0$
$x = 5$ or $x = 9$ (2)
7.2.3 $x^2 - x - 49 = -7$
$x^2 - x - 42 = 0$
$(x - 7)(x + 6) = 0$
$x = 7$ or $x = -6$ (2)
[16]

Question 8

8.1 (5)



8.2 *a* $y = \frac{1}{4}x + 1$

b $y = -\frac{3}{2}x - 3$ (3)

[8]

Question 9

9.1.1 $AC = \sqrt{7^2 + 5^2} = 8{,}60$ cm (1)

9.1.2 Total surface area $= 2\left(\frac{1}{2} \times 5 \times 7\right) + 11 \times 5 + 11 \times 7 + 11 \times 8{,}6$

$= 261{,}60 \text{ cm}^2$ (2)

9.2.1 Radius $= 56 \div 2 \div 100 = 0{,}28$ m

Capacity of tank $= \pi \times 0{,}28^2 \times 2{,}5$

$= 0{,}61575\ldots \text{m}^3$

$= 0{,}62$ kilolitres (2)

9.2.2 $1{,}24 = \pi \times r^2 \times 2{,}5$

$r^2 = 0{,}15788$

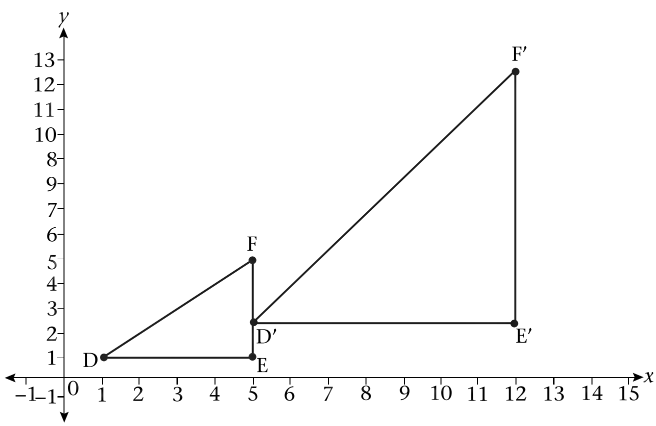
$r = 0{,}397$

$d = 0{,}79 \text{ m} = 79 \text{ cm}$ (2)

[7]

Question 10

10.1 (2)



CHAPTER 17

10.2 DE = 3 cm; EF = 4 cm and DF = 5 cm
D'E' = 7,5 cm; E'F' = 10 cm and D'F' = 12, 5 cm
DE : EF : DF = 3 : 4 : 5
D'E' : E'F' : D'F' = 3 : 4: 5 (2)

10.3 $\hat{D} = \hat{D}' = 53°$
$\hat{E} = \hat{E}' = 90°$
$\hat{F} = \hat{F}' = 37°$ (2)

[6]

Question 11

11.1 1E 2A 3D 4C 5B (5)

11.2 A sphere is a 3 dimensional object in which every point on the surface is equidistant from the centre of the object. (1)

[6]

Question 12

12.1.1 No, because the pie chart only shows the proportion between the categories, not the exact values. (1)

12.1.2 Fire safety makes up less than an eighth and approximately a twelfth of the total (a third of one quarter). Four learners thus make up a twelfth of the total.
Water conservation – a quarter of the total would be 12 learners
Nature conservation – 8 learners
None of these – an eighth of the total would be 6 learners
HIV/AIDS – three eighths of the total would be 18 learners (2)

12.1.3 HIV/AIDS (1)

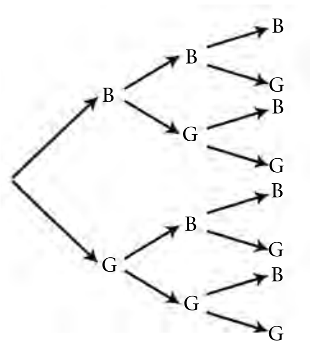
12.2.1 Calvinia:
Mean = $\frac{177}{52}$ = 3,4 mm
Median = 0 mm
Mode = 0 mm
George:
Mean = $\frac{705}{52}$ = 13,6 mm
Median = 6,5 mm
Mode = 0 mm (3)

12.2.2 The two towns have the same modes (0 mm). The median of Calvinia is 0 mm while the median of George is 6,5 mm. There is a large difference in the means – Calvinia has a mean of 3,4 mm and George has a mean of 13,6 mm. Generally, George has more rainfall than Calvinia. The best measure of central tendency for Calvinia is the median and the best measure for George is the mean. (2)

[9]

Question 13

13.1



(2)

13.2.1 P(all three babies of the same sex) = $\frac{1}{8} + \frac{1}{8} = \frac{1}{4}$ (1)

13.2.2 P(at least one of each) = $\frac{6}{8} = \frac{3}{4}$ (1)

13.2.3 P(two boys and one girl) = $\frac{3}{8}$ (1)

[5]

TOTAL: 100 Marks

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